Allotment and Subcontracting in Procurement Bidding

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Abstract

Allotment and subcontracting are the two alternative mechanisms enabling the participation of SMEs in procurement. We compare these two alternatives in the context of a procurement contract awarded by a first-price sealed-bid auction. When the winning large firm is constrained with respect to the degree of subcontracting, we show that only a reduction of the chosen SME's profit can reduce the expected cost of the contract. However, when the large firm is allowed to choose the subcontracting level, subcontracting can be a Pareto dominating mechanism, i.e. simultaneously increasing both firms' profits and reducing the expected total cost of the contract.

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1 Introduction

The World Bank estimates that current spending and growth investment activity by governments account for 18% of GDP in developed countries, 13% of GDP in developing nations and 19% of GDP in transitional economies.[1] In most countries, microenterprises and small-scale enterprises account for the majority of firms and a large share of employment and it may be the policy of governments to provide maximum practicable opportunities in their acquisitions from small business. For example, SMEs account for over 65 % of turnover generated by the private sector in the European Union, but the share of public contracts won directly (not taking subcontracting into account) by SMEs remains low (less than 25 %). For some years already the European Commission has paid special attention to the access and participation of SMEs in the public procurement market. The Commission has called for greater participation by SMEs with a view to strengthening their competitiveness and enabling them to contribute more towards growth, employment and competitiveness in the European economy. Similar concerns can be found in the US Federal Acquisition Regulation (FAR). Indeed, section 19.202-1 of the FAR is devoted to "encouraging small business participation in acquisitions"

> "Small business concerns shall be afforded an equitable opportunity to compete for all contracts that they can perform to the extent consistent with the Government's interest. When applicable, the contracting officer shall take the following actions:
>
> (a) Divide proposed acquisitions of supplies and services (except construction) into reasonably small lots (not less than economic production runs) to permit offers on quantities less than the total requirement. [...]
>
> (d) Encourage prime contractors to subcontract with small business concerns".

[1] Source: Oxford Analytica Academic Database.

As noted in a report to the US Small Business Administration,[2] despite clauses in the FAR calling on contract officers to make special efforts to sustain small business participation in procurement, budget cuts and directives to streamline the procurement process may be leading contracting officers to consolidate small purchases into larger contracts in the name of a limited efficiency. These kinds of procurement "efficiencies" impact small businesses negatively because the requirements of larger, multi-faceted contracts can easily outstrip the financial or administrative capabilities of a small business, precluding them from competing. Furthermore, the opportunity for small businesses to subcontract from the larger companies winning the bundled contracts may also diminish because of a tendency for larger firms to use their own resources on the contracts they win. Evidence of the negative impact of contract bundling on small business was first presented in the U.S. Small Business Administration's 1993 report.[3]

Similarly, the sixth European Observatory for SMEs shows that the most important reason why SMEs do not try to participate in European tenders is that the projects are too large for their enterprises. It also reveals that there are considerable country differences with regard to the participation of SMEs in the public procurement market. In Sweden, Italy and Portugal the percentage of SMEs trying to participate in European tenders is lower than 10%, whereas in France it is 45%. In Belgium and Luxembourg about one third of the SMEs attempt to participate.

The primary way of enabling SMEs' participation in public procurement is to divide proposed acquisitions of supplies and services into reasonably small lots to permit offers on quantities less than the total requirement. This allotment enables wide small business participation. Further, some countries have established programmes to encourage the use of SMEs in subcontracting with large businesses. In such programmes, the government awards

[2] "The Impact of Contract Bundling on Small Business", report by Eagle Eye Publishers, Inc. to the US Small Business Administration's Office of Advocacy, (2000).

[3] U.S. Small Business Administration, Study of the Impact of Contract Bundling on Small Business Concerns and Practical Recommendations (Report to the Committee on Small Business of the United States Senate and the Committee on Small Business of the United States House of Representatives, 14 May 1993), 77 pages.

a contract to a large firm with the requirement or goal that the large firm purchase x% of the value of its intermediate inputs from SMEs. Subcontracting programmes can thus be viewed as an alternative means of involving SMEs in public procurement activities. Hence, subcontracting and allotment of a procurement contract are the two alternative ways for SMEs to access public procurement.

Should the public buyer promote the participation of SMEs by an allotment of procurement contracts or encourage subcontracting practices when contracts are awarded by means of a first-price sealed-bid auction ? This is the main concern of this paper.

To the best of our knowledge, this question has never been addressed in the literature though subcontracting and allotment have already been considered separately.

The literature on subcontracting essentially focuses on the agency relation between a firm and its subcontractors (e.g. Yun (1999) or Kawasaki and McMillan (1987)) and on the impact of the possibility of subsequent subcontracting among rivals on the competition between two firms (e.g. Kamien *et al.* (1989) or Gale *et al.* (2000)). We depart from these analyses, assuming that large firms do not subcontract among rivals but more realistically with SMEs.[4] Besides, no positive studies of the allotment procedure have been done[5].

In this article, we consider the procurement of a fixed-price contract awarded by means of a first-price sealed-bid auction. The aim of this paper is to compare the allotment and subcontracting procedures in order to exhibit some implications of these procedures for the minimization of the total cost of the contract and also for small business and large firms' profits. More precisely, we derive conditions under which both the winning large firm and the winning or chosen SME can be better off with the allotment procedure or with the subcontracting procedure. We also show, counter-intuitively, that the public buyer and the large firms can benefit from the asymmetric information between SMEs and large firms. Furthermore, when the winning large firm is constrained on the subcontracting level, we show that only a reduction of the chosen SME's profit can reduce the expected cost of the contract.

[4] Laffont and Tirole (1993) have addressed the question of the value of delegating subcontracting to a regulated firm in a normative approach.

[5] For a normative approach of optimal allotment rules, see Morand (2003).

However, when the large firm is allowed to choose the subcontracting level, subcontracting can be a Pareto dominating mechanism, i.e. simultaneously increasing both firms' profit and reducing the expected total cost of the contract. It gives strong support that the public buyer should not constrain the level of subcontracting and that contrarily to common view SMEs are not necessarily better off when a part of a contract has been specifically allotted to them.

The next section presents the model and its assumptions. Section 3 characterizes the allotment procedure whereas the subcontracting procedure is discussed in section 4. Section 5 offers a first comparison of both procedures when the subcontracting level is imposed. Section 6 deals with the same comparison in a more general context, i.e. when large firms are allowed to choose the subcontracting level. A final section contains concluding remarks. Most of the proofs are relegated to the Appendix.

2 Outline of the model

We consider a procurement contract awarded by a first-price sealed-bid auction. We assume that the supply-side of the market is composed of n large firms (hereafter LFs) and m small business firms (hereafter SBFs). All firms are assumed to be risk-neutral.

Let $p \in [0,1]$ be the proportion of the contract undertaken by a small business firm (hereafter SBF), either by a subcontracting or an allotment procedure.

Each large firm (hereafter LF) $i = 1, ..., n$ and each SBF $j = 1, ..., m$ has private information about its own efficiency parameter θ_i and θ_j entering into its respective cost function $c_i(\theta_i, p)$ $\forall i = 1, ..., n$ and $c_j(\theta_j, p)$ $\forall j = 1, ..., m$. However, it is common knowledge that θ_i and θ_j are i.i.d. on the interval $[\theta^-, \theta^+]$, according to probability density f, with cumulative F.[6]

Concerning cost functions, we naturally have $\frac{\partial c_i(\theta_i,p)}{\partial p} < 0$ and $\frac{\partial c_j(\theta_j,p)}{\partial p} > 0$. We further assume that $\frac{\partial^2 c_i(\theta_i,p)}{\partial p^2} > 0$, $\frac{\partial c_i(\theta_i,p)}{\partial \theta_i} > 0$, $\frac{\partial^2 c_j(\theta_j,p)}{\partial p^2} > 0$ and $\frac{\partial c_j(\theta_j,p)}{\partial \theta j} > 0$. We also assume that $\frac{\partial^2 c_i(\theta_i,p)}{\partial p \partial \theta_i} < 0$, which means that the larger the part of the contract undertaken by the LF, the

[6] θ^- is assumed to reflect the parameter of the most efficient firm.

greater is the impact of an increase in efficiency. Similarly, concerning SBFs' cost function, we assume that $\frac{\partial^2 c_j(\theta_j,p)}{\partial p \partial \theta j} > 0$.

3 The allotment procedure

In this section, we consider that by law, the contract has to be sub-divided into two lots, so that SBFs can have direct access to one of the lots. Then, n LFs compete for the award of a part $(1 - p_a)$ of the contract and m SBFs compete for the award of the reminder p_a. With this allotment (a) procedure, we assume that subcontracting is not allowed.

The expected profit of LF i in a first-price sealed-bid auction is

$$E\pi_i^a = (b_i - c_i(\theta_i, p_a))\,(1 - F(b^{-1}(b_i)))^{n-1}. \tag{1}$$

Then, each LF i chooses a bid b_i to maximize $E\pi_i^a$. The derivative of the expected profit with respect to θ_i is

$$\frac{dE\pi_i^a}{d\theta_i} = -\frac{\partial\,(c_i(\theta_i, p_a))}{\partial \theta_i}(1 - F(b^{-1}(b_i)))^{n-1}. \tag{2}$$

Following standard derivations,[7] we obtain the optimal bidding strategy of LF i

$$b_i^a(\theta_i) = c_i(\theta_i, p_a) + \frac{\int_{\theta_i}^{\theta^+} \frac{\partial c_i(\theta,p_a)}{\partial \theta}(1 - F(\theta))^{n-1} d\theta}{(1 - F(\theta_i))^{n-1}} \quad \forall i = 1, ..., n.$$

The optimal bidding strategy is equal to the true cost plus a strategic mark-up. Contrary to conventional developments in auction theory, the strategic mark-up is not directly related to the efficiency parameter but to the cost function and so is affected by the marginal variation of this cost induced by the variation of the efficiency parameter. This is the *marginal competition effect*, which turns out to be crucial in the following.[8] Note that by assumption, $\frac{\partial c_i(\theta,p_a)}{\partial \theta_i} > 0$ $\forall_i$. Actually, the more the cost is sensitive to the efficiency parameter, the greater is the difference between the winner and the second bidder and so the more the winner can increase its strategic mark-up.

[7] See the appendix.

[8] This effect was first depicted by Marechal and Morand (2003), but in a less general framework.

Similarly, the optimal bidding strategy of SBF j is

$$b_j^a(\theta_j) = c_j(\theta_j, p_a) + \frac{\int_{\theta_j}^{\theta^+} \frac{\partial c_j(\theta, p_a)}{\partial \theta}(1-F(\theta))^{m-1} d\theta}{(1-F(\theta_j))^{m-1}} \; \forall j = 1, ..., m.$$

Let us denote $g(\theta_i) = n(1-F(\theta_i))^{n-1} f(\theta_i)\, d\theta_i$ as the density function of the lowest cost of the n LFs, and $h(\theta_j) = m(1-F(\theta_j))^{m-1} f(\theta_j)\, d\theta_j$ as the density function of the lowest cost of the m SBFs.

Given $b_i^a(\theta_i)$ and $b_j^a(\theta_j)$, the expected total cost of the project is

$$ETC^a = \int_{\theta^-}^{\theta^+} \left(b_i^a(\theta_i)\right) g(\theta_i) d\theta_i + \int_{\theta^-}^{\theta^+} \left(b_j^a(\theta_j)\right) h(\theta_j) d\theta_j. \tag{3}$$

From the point of view of the public buyer, the expected profit of the winning LF is

$$E\pi_i^{a(1)} = \int_{\theta^-}^{\theta^+} \left(\int_{\theta_i}^{\theta^+} \frac{\partial c_i(\theta, p_a)}{\partial \theta}(1-F(\theta))^{n-1} d\theta \right) g(\theta_i) d\theta_i. \tag{4}$$

The expected profit of the winning SBF is

$$E\pi_j^{a(1)} = \int_{\theta^-}^{\theta^+} \left(\int_{\theta_j}^{\theta^+} \frac{\partial c_j(\theta, p_a)}{\partial \theta}(1-F(\theta))^{m-1} d\theta \right) h(\theta_j) d\theta_j. \tag{5}$$

Remark 1 *For both the winning LF and the winning SBF, the expected profit only depends on the marginal competition effect. Indeed, in an auction, the profit derived by the winner is equal to the expected strategic mark-up.*

4 The subcontracting procedure

In this section, we consider that the contract is not sub-divided so that SBFs only have access to public procurement by means of subcontracting.

We consider that the proportion of the contract to be subcontracted is not constrained by regulation.[9] So, only LFs compete for the award of the whole contract but may subcontract a part of this contract. According to *e.g.* the US subcontracting regulation for contracts that is expected to exceed \$500,000 (1,000,000 for construction),[10] we assume that each LF is

[9] We further relax this assumption, considering that the subcontracting level is imposed by procurement rules. This will enable us to highlight some specific results.

[10] Exceptions to this rule can be found in Section 19.702 of the US Federal Acquisition Regulation.

required to submit the potential subcontracting plan before the award of the contract. Then, each LF i has to choose both the subcontracting level $p(\theta_i)$ and a bid b_i.

The bargaining process between the winning LF and the chosen SBF is modelled in a very simple way. The payment received by the SBF depends on the bargaining power of both firms. Furthermore, although the information is asymmetric, we assume that the bargaining process results in the most efficient SBF being chosen.

Therefore, we can consider that the LF pays the SBF at a cost $c_j(\widehat{\theta}, p(\theta_i))$, where $\widehat{\theta} > \theta_j^{(1)}$ the expected true efficiency parameter of the chosen (and most efficient) SBF. Roughly speaking, the closer $\widehat{\theta}$ is to $\theta_j^{(1)}$, the greater is the bargaining power of the LF. However, under asymmetric information, no bargaining process can perform better than an optimal auction. Hence, $c_j(\widehat{\theta}, p(\theta_i))$ is bounded below by $c_j(\theta_j^{(2)}, p(\theta_i))$, the second lowest cost of the m SBFs, which would correspond to the result of an auction mechanism.

Note that we will also consider a context of complete information between LFs and SBFs as a benchmark situation. This case corresponds to a context in which LFs know, contrary to the public buyer, the true costs of the SBFs. The winning LF can then choose the most efficient SBF, characterized by $\theta_j^{(1)}$, and this latter receives no rents.

When a LF is selected, it has to support a total cost which is equal to the sum of its own cost (the non-subcontracted part of the contract) plus the negotiated cost of the chosen subcontractor (as long as a part of the contract is subcontracted).

The expected profit of LF i when it chooses to subcontract (s) a part $p(\theta_i)$ is

$$E\pi_i^s(\theta_i) = \left(b_i - c_i(\theta_i, p(\theta_i)) - c_j(\widehat{\theta}, p(\theta_i))\right)(1 - F(b^{-1}(b_i)))^{n-1}. \tag{6}$$

The optimal subcontracting level $p(\theta_i)$ maximizes (6). The derivative of $E\pi_i^s(\theta_i)$ with respect to θ_i is

$$\frac{dE\pi_i^s(\theta_i)}{d\theta_i} = \left[-\frac{\partial p(\theta_i)}{\partial \theta_i} \underbrace{\left(\frac{\partial c_i(\theta_i, p(\theta_i)}{\partial p(\theta_i)} + \frac{\partial c_j(\widehat{\theta}, p(\theta_i))}{\partial p(\theta_i)} \right)}_{=0} - \underbrace{\frac{\partial c_i(\theta_i, p(\theta_i)}{\partial \theta_i}}_{>0} - \underbrace{\frac{\partial c_j(\widehat{\theta}, p(\theta_i))}{\partial \theta_i}}_{\lesseqgtr 0} \right] (1 - F(b^{-1}(b_i)))^{n-1}.$$

The term $\left(\frac{\partial c_j(\widehat{\theta},p(\theta_i))}{\partial \theta_i}\right)$ can either be positive, negative or equal to zero, since a more efficient LF may be a better or a poorer bargainer. Obviously, the term $\left(\frac{\partial (c_i(\theta_i,p(\theta_i))}{\partial p(\theta_i)} + \frac{\partial c_j(\widehat{\theta},p(\theta_i))}{\partial p(\theta_i)}\right)$ vanishes since $p(\theta_i)$ maximizes $E\pi_i^s(\theta_i)$.

Following the derivations of the previous section, we obtain the optimal bidding strategy[11] of LF i

$$b_i^s(\theta_i) = c_i(\theta_i, p(\theta_i)) + c_j(\widehat{\theta}, p(\theta_i)) + \frac{\int_{\theta_i}^{\theta^+} \left(\frac{\partial (c_i(\theta,p(\theta))}{\partial \theta} + \frac{\partial c_j(\widehat{\theta},p(\theta))}{\partial \theta}\right)(1 - F(\theta))^{n-1} d\theta}{(1 - F(\theta_i))^{n-1}} \quad \forall i = 1, ..., n.$$

Note that, compared to the previous bidding strategy $b_i^a(\theta_i)$, the marginal competition effect still appears but in the context of a new specific effect. Indeed, $\frac{\partial c_j(\widehat{\theta},p(\theta_i))}{\partial \theta_i}$ reflects the impact of the LF's efficiency on the bargaining result.

- If $\frac{\partial c_j(\widehat{\theta},p(\theta_i))}{\partial \theta_i} < 0$, a more inefficient LF is supposed to be a better bargainer (bargaining specialized).
- If $\frac{\partial c_j(\widehat{\theta},p(\theta_i))}{\partial \theta_i} > 0$, a more inefficient LF is then a poorer bargainer (production specialized).
- Finally, the case where $\frac{\partial c_j(\widehat{\theta},p(\theta_i))}{\partial \theta_i} = 0$ reflects the assumption that the LF's efficiency does not affect the bargaining result.

[11] In order to satisfy $\frac{db_i^s(\theta_i)}{d\theta_i} > 0 \; \forall \theta_i$, we assume in the following that $\frac{\partial (c_i(\theta_i,p(\theta_i))}{\partial \theta_i} + \frac{\partial c_j(\widehat{\theta},p(\theta_i))}{\partial \theta_i} > 0 \; \forall \theta_i$.

The expected total cost of the contract is

$$ETC^s = \int_{\theta^-}^{\theta^+} \left(b_i^s(\theta_i)\right) g(\theta_i) d\theta_i. \tag{7}$$

The expected profit of the winning LF is

$$E\pi_i^{s(1)} = \int_{\theta^-}^{\theta^+} \left[\int_{\theta_i}^{\theta^+} \left(\frac{\partial c_i(\theta, p(\theta)}{\partial \theta} + \frac{\partial c_j(\widehat{\theta}, p(\theta))}{\partial \theta} \right) (1 - F(\theta))^{n-1} d\theta \right] g(\theta_i) d\theta_i. \tag{8}$$

The expected profit of the chosen SBF is

$$\begin{aligned} E\pi_j^{s(1)} &= \int_{\theta^-}^{\theta^+} \left(\int_{\theta^-}^{\theta^+} \left(c_j(\widehat{\theta}, p(\theta_i)) - c_j(\theta_j, p(\theta_i)) \right) g(\theta_i) d\theta_i \right) h(\theta_j) d\theta_j \\ &= \int_{\theta^-}^{\theta^+} \left[c_j(\widehat{\theta}, p(\theta_i)) - c_j(\theta_j^{(1)}, p(\theta_i)) \right] g(\theta_i) d\theta_i. \end{aligned} \tag{9}$$

Before turning to consider the main concern of our paper, an interesting question to investigate is whether the public buyer and/or the LFs and the SBFs are better off under asymmetric information. Note that the chosen SBF gets no rents when, contrary to the public buyer, the LF knows the SBFs' costs. Then obviously, from (9), SBFs are better off under asymmetric information, since $E\pi_j^{s(1)} > 0$ when $\widehat{\theta} > \theta_j^{(1)}$. Somewhat counter-intuitively we have the following lemma

Lemma 1 *The winning LF can be better off when information is asymmetric (i.e. when SBFs have private information about their cost) if it is production specialized. In this case, both the chosen SBF and the winning LF can be better off when information is asymmetric between them.*

Proof. See the Appendix.

This lemma can be easily interpreted. Since each LF incorporates the negotiated cost of the subcontractor into its bidding strategy, the global negotiated cost is neutral for the LF; only the marginal competition effect of efficiency on negotiated cost matters. Under complete information between LFs and SBFs, this latter vanishes. So the winning LF can be better off if asymmetric information unables it to increase its mark-up.

Furthermore, the goal of minimizing total costs can be better achieved under incomplete information, according to the following

Lemma 2 *The expected cost of the contract may be lower under incomplete information than under complete information if the winning LF is highly bargaining specialized.*

Proof. See the Appendix.

Indeed, when the LF is bargaining specialized, it gets more rents under incomplete information than under complete information, which increases expected total costs.

Let us now compare in the following the two procurement procedures.

5 A first comparison

In order to generate some interesting first results, we now consider that the subcontracting level is imposed by the public buyer. Roughly speaking, this first comparison will highlight the problem of delegation. Indeed, the public buyer has to choose between two procedures: either the winning LF is delegated to subcontract a given part of the project or the contract is divided so that SBFs can directly compete for the award of *the same given part* of the contract. Note that this first comparison does not incorporate the indirect impact of the modification of the subcontracting level which is analyzed in the next section.

When subcontracting is imposed, the analysis can be derived in a straightforward fashion from the previous section, substituting $p(\theta_i) = p_a$.

The optimal bidding strategy of LF i becomes

$$\begin{aligned} b_i^s(\theta_i) &= c_i(\theta_i, p_a) + c_j(\widehat{\theta}, p_a) \\ &\quad + \frac{\int_{\theta_i}^{\theta^+} \left(\frac{\partial c_i(\theta, p_a)}{\partial \theta} + \frac{\partial c_j(\widehat{\theta}, p_a)}{\partial \theta} \right) (1 - F(\theta))^{n-1} d\theta}{(1 - F(\theta_i))^{n-1}}, \ \forall i = 1, ..., n \end{aligned}$$

with the assumption that $\frac{\partial c_i(\theta_i, p_a)}{\partial \theta_i} + \frac{\partial c_j(\widehat{\theta}, p_a)}{\partial \theta_i} > 0$ to ensure that $b_i^s(\theta_i)$ is increasing.

5.1 Comparison of profits with imposed subcontracting

From (8) and (4), when $p(\theta_i) = p_a$, the difference in the winning LF's expected profits derived under the two procedures is

$$E\pi_i^{s(1)} - E\pi_i^{a(1)} = \int_{\theta^-}^{\theta^+} \left(\int_{\theta_i}^{\theta^+} \frac{\partial c_j(\widehat{\theta}, p_a)}{\partial \theta} (1 - F(\theta))^{n-1} d\theta \right) g(\theta_i) d\theta_i. \qquad (10)$$

Lemma 3 *If the bargaining power of the LF does not depend on its own efficiency, then allotting a part of the contract or imposing upon the LF to subcontract the same part yields the same profit for the winning LF.*

Proof. If $\frac{\partial c_j(\widehat{\theta},p_a)}{\partial \theta_i} = 0$, then $E\pi_i^{s(1)} = E\pi_i^{a(1)}$. *Q.E.D.*

Corollary 1 *If* $\frac{\partial c_j(\widehat{\theta},p_a)}{\partial \theta_i} < 0$, *i.e. a more inefficient LF is a better bargainer (bargaining specialized), then the winning LF gets a higher profit with allotment. If* $\frac{\partial c_j(\widehat{\theta},p_a)}{\partial \theta_i} > 0$, *i.e. a more inefficient LF is a poorer bargainer (production specialized), then the winning LF gets a higher profit with subcontracting.*

From the point of view of the winning LF, only the marginal competition effect matters. Even if a LF is a poorer bargainer, the global competition effect does not matter since its bid covers the cost of the SBF. The key element is the gap between the LF's cost and the second expected net cost, i.e. its cost plus the cost paid to the subcontractor. In a nutshell the second bidder is less efficient than the first one and therefore if a less efficient firm is a better bargainer the gap between both net costs is reduced. This leads to a reduction in markup for the winner. Conversely, the same argument applies when a less efficient firm is a poorer bargainer.

We now turn to the comparison of expected profits of the winning SBF. We have

$$\begin{aligned} & E\pi_j^{s(1)} - E\pi_j^{a(1)} \qquad (11) \\ = & \int_{\theta^-}^{\theta^+} \left(c_j(\widehat{\theta}, p_a) - c_j(\theta_j, p_a) - \int_{\theta_j}^{\theta^+} \frac{\partial c_j(\theta, p_a)}{\partial \theta} (1 - F(\theta))^{m-1} d\theta \right) h(\theta_j) d\theta_j. \end{aligned}$$

Equation (11) shows that a SBF prefers to be a subcontractor than to participate in the auction if the gain from the bargaining process is higher than the expected strategic mark-up obtained in the auction.

Recall that standard developments in auction theory show that the winner's expected profit, in a first-price sealed-bid auction, corresponds to the difference between its own cost and the (conditional) expected second cost.[12] Equation (11) can, therefore, be rewritten as

$$\begin{aligned} E\pi_j^{s(1)} - E\pi_j^{a(1)} &= c_j(\widehat{\theta}, p_a) - c_j(\theta_j^{(1)}, p_a) - \left(c_j(\theta_j^{(2)}, p_a) - c_j(\theta_j^{(1)}, p_a) \right) \\ &= c_j(\widehat{\theta}, p_a) - c_j(\theta_j^{(2)}, p_a), \end{aligned}$$

where $c_j(\theta_j^{(2)}, p_a)$ reflects the expected second lowest cost among m SBFs.

Under our framework (with symmetric, risk-neutral bidders and independent private values), it is well known that the optimal bargaining institution is an auction, and that a first-price auction is revenue-equivalent to other auction mechanisms. Hence, in a context of asymmetric information, we have $c_j(\widehat{\theta}, p_a) \geq c_j(\theta_j^{(2)}, p_a)$, since an auction is the most powerful tool to extract rents. Consequently, a SBF always prefers to be a subcontractor than to participate in an auction (obviously, SBFs are better off with the allotment procedure if LFs know the SBFs' costs).

5.2 Comparison of total costs with imposed subcontracting

From (3) and (7), the comparison between expected total costs, when the public buyer uses either the subcontracting or the allotment rule, yields

$$\begin{aligned} & ETC^s - ETC^a \\ = & \int_{\theta^-}^{\theta^+} \left(c_j(\widehat{\theta}, p_a) + \frac{\int_{\theta_i}^{\theta^+} \frac{\partial c_j(\widehat{\theta}, p_a)}{\partial \theta} (1 - F(\theta))^{n-1} d\theta}{(1 - F(\theta_i))^{n-1}} \right) g(\theta_i) d\theta_i \\ & - \int_{\theta^-}^{\theta^+} \left(c_j(\theta_j, p_a) + \frac{\int_{\theta_j}^{\theta^+} \frac{\partial c_j(\theta, p_a)}{\partial \theta} (1 - F(\theta))^{m-1} d\theta}{(1 - F(\theta_j))^{m-1}} \right) h(\theta_j) d\theta_j. \end{aligned}$$

Since the subcontracted part of the contract corresponds to the allotted part, the cost of the winning LF is the same under both procedures. Nevertheless, the cost of the winning SBF differs, because it either represents the cost the LF negotiates and incorporates into its own bid, or the expected second cost among the m SBFs when the contract is allotted. Therefore, only rents conceded to the winning LF and the chosen SBF modify the expected total cost of the contract. We then have the following lemma

[12] See *e.g.* Krishna (2002) and Klemperer (1999) for surveys of auction theory.

Lemma 4 *If there is asymmetric information between LFs and SBFs (about SBFs' costs) and if LFs' efficiency does not modify their bargaining power, then allotting a part of the contract reduces the total expected cost compared to imposing upon the LF the requirement to subcontract the same part.*[13]

Proof. See the Appendix.

Since $p(\theta_i) = p_a$, the expected total cost is only affected by rents conceded to both LFs and SBFs. When the bargaining power of the LF does not depend on its own efficiency, from lemma 3, the profit of the winning LF is the same. So the expected total cost differs only by the SBF's rents and under asymmetric information, the allotment procedure performs better in reducing expected total costs. Obvioulsy, under complete information between LFs and SBFs, the SBF's rents vanish. So, the cost minimizing goal is better achieved when the public buyer delegates the selection of the SBF.

The comparison between expected profits of the chosen SBF and expected costs of the contract highlights the trade-off between cost efficiency concerns and a public policy which favors SBFs. Indeed, we have the following proposition

Proposition 1 *When the LF is constrained on the subcontracting level, only a reduction of the chosen SBF's profit can reduce the expected cost of the contract. Hence, there does not exist any Pareto dominating mechanism.*

Proof. See the Appendix.

6 Subcontracting vs allotment

The previous results clearly rely on the assumption that the subcontracting level is imposed by law, and corresponds to the allotted part. In order to highlight the impact of the choice of

[13] Assuming complete information between LFs and SBFs on SBFs' costs may also require the assumption of complete information among SBFs. In this case, the result of the bidding process among the SBFs would actually remain unchanged in terms of expected cost which will in each case correspond to the second lowest cost.

the subcontracting level by the LF, we now return to the general case where the law enables LFs to choose the subcontracting level $p(\theta)$.

6.1 Comparison of profits

Let us first analyze the expected profits of the winning LF. Comparing expected profits in each situation, we get

$$\begin{aligned} & E\pi_i^{s(1)} - E\pi_i^{a(1)} \\ = \; & \int_{\theta^-}^{\theta^+} \left[\int_{\theta_i}^{\theta^+} \left(\frac{\partial c_j(\widehat{\theta}, p(\theta))}{\partial \theta} \right) (1 - F(\theta))^{n-1} d\theta \right] g(\theta_i) d\theta_i \\ & + \int_{\theta^-}^{\theta^+} [\int_{\theta_i}^{\theta^+} \left(\frac{\partial \left[c_i(\theta, p(\theta)) - c_i(\theta, p_a) \right]}{\partial \theta} \right) (1 - F(\theta))^{n-1} d\theta] \; g(\theta_i) d\theta_i . \end{aligned} \tag{12}$$

The second line of (12) refers to the impact of the marginal competition effect of the bargaining power. This effect already existed in the case where subcontracting was imposed, and may either be positive or negative depending on specific assumptions about the bargaining technology. The third line of (12) reflects the impact of the choice of the subcontracting level. This impact may reinforce or reverse the previous effect depending on the condition $p(\theta_i) \lesseqgtr p_a$.

We then have the following lemma

Lemma 5 *If the bargaining power of the LF does not depend on its own efficiency or if a more inefficient LF is a better bargainer (bargaining specialized), then the winning LF is better off with the allotment procedure than with the subcontracting procedure if the LF is induced to subcontract a larger part of the contract relative to the allotted part.*

Proof. See the Appendix.

If these conditions are not all satisfied, the results are less clear-cut: the winning LF may be better off with subcontracting or with allotment depending on both the sensibility of the optimal choice of p and the marginal competition effect of the bargaining power. Indeed, from (12), we have $E\pi_i^{s(1)} > (<) \, E\pi_i^{a(1)}$ if

$$\underbrace{\frac{\partial c_j(\widehat{\theta}, p(\theta_i))}{\partial \theta_i} + \frac{\partial \left[c_i(\theta_i, p(\theta_i))\right]}{\partial \theta_i}}_{>0 \text{ by assumption}} > (<) \, \frac{\partial \left[c_i(\theta_i, p_a)\right]}{\partial \theta_i}. \tag{13}$$

Consider now the case of the winning (or chosen in the case of subcontracting) SBF. The difference in the expected profits derived from each procedure is

$$\begin{aligned} & E\pi_j^{s(1)} - E\pi_j^{a(1)} \qquad (14) \\ = & \int_{\theta^-}^{\theta^+} \left[\int_{\theta^-}^{\theta^+} \left(c_j(\widehat{\theta}, p(\theta_i)) - c_j(\theta_j, p(\theta_i)) \right) g(\theta_i) d\theta_i - \int_{\theta_j}^{\theta^+} \frac{\partial c_j(\theta, p_a)}{\partial \theta} (1 - F(\theta))^{m-1} d\theta \right] \\ & h(\theta_j) d\theta_j. \end{aligned}$$

As depicted by the second line of (14), the difference between expected profits depends on both the global comparison of costs and the marginal competition effect. Intuitively, the winning SBF is better off with subcontracting if the gain from the bargaining on costs is higher than the expected strategic mark-up in the auctioning of the allotted part.

6.2 Pareto dominating mechanism

The last section enables us to show that both the winning LF and the chosen SBF can simultaneously prefer either the subcontracting procedure or the allotment procedure. In contrast to proposition 1, since the LF can choose the subcontracting level, this result is not necessarily at odds with an increase in expected total costs. Thus, we have the following proposition

Proposition 2 *When the LF is allowed to choose the subcontracting level, subcontracting can be a Pareto dominating mechanism, i.e. simultaneously increasing both firms' profit and reducing the expected total cost.*

We now give an intuitive explanation of this main result (a formal proof is given in the appendix). The expected total cost for the public buyer is equal to the expected true costs ($ETrC$) plus the profits conceded to the firms. Let us write the expected total costs respectively for the subcontracting procedure

$$ETC^s = ETrC^s + E\pi_i^{s(1)} + E\pi_j^{s(1)},$$

and the allotment procedure

$$ETC^a = ETrC^a + E\pi_i^{a(1)} + E\pi_j^{a(1)}.$$

Since the LF chooses the subcontracting level which minimizes $ETrC$, we necessarily have

$$ETrC^s < ETrC^a.$$

Then, subcontracting can be Pareto improving if

$$E\pi_i^{s(1)} > E\pi_i^{a(1)} \text{ and } E\pi_j^{s(1)} > E\pi_j^{a(1)},$$

and simultaneously

$$\begin{aligned} ETrC^s + E\pi_i^{s(1)} + E\pi_j^{s(1)} &< ETrC^a + E\pi_i^{a(1)} + E\pi_j^{a(1)} \\ \text{or } \underbrace{ETrC^a - ETrC^s}_{>0} &> \underbrace{E\pi_i^{s(1)} - E\pi_i^{a(1)}}_{>0} + \underbrace{E\pi_j^{s(1)} - E\pi_j^{a(1)}}_{>0}, \end{aligned}$$

that is if the increase in both firms' profits does not offset the difference between expected true costs.

Obviously, this result could not be obtained with imposed subcontracting since we would always have $ETrC^a = ETrC^s$.

Further, the allotment procedure can never be a Pareto improving mechanism since it would require the following inequality to be satisfied

$$\underbrace{ETrC^a - ETrC^s}_{>0} < \underbrace{E\pi_i^{s(1)} - E\pi_i^{a(1)}}_{<0} + \underbrace{E\pi_j^{s(1)} - E\pi_j^{a(1)}}_{<0},$$

which clearly cannot be true.

7 Conclusion

This paper has focused on the comparison of two aternative mechanisms enabling the participation of SMEs in procurement. We highlight the impact of the choice of the subcontracting level and of the bargaining process on the bidding strategy of LFs. Compared to traditional developments in auction theory, these marginal competition effects either increase or decrease the familiar strategic mark-up. It enables us to derive some new insights into the design of procurement rules.

We have derived conditions under which both the winning LF and the winning or chosen SME can be better off with the allotment procedure or with the subcontracting procedure. Specificaly, if the bargaining power of the LF does not depend on its own efficiency or if a more inefficient LF is a better bargainer, then the winning LF is better off with the allotment procedure than with the subcontracting procedure if, compare to the allotted part, the LF is induced to subcontract a larger part.

We have also explained why the public buyer and LFs can benefit from the asymmetric information between SMEs and LFs. Furthermore, when the winning LF is constrained on the subcontracting level, we have shown that only a reduction of the chosen SME's profit can reduce the expected cost of the contract. Nevertheless, when the LF is allowed to choose the subcontracting level, subcontracting can be a Pareto dominating mechanism, i.e. simultaneously increasing both firms' profit and reducing the expected total cost of the contract.

An extension of this model could be to consider an allotted contract but to allow the winning LF to subcontract a part of the residual project. In this context, SMEs would have two ways to access public procurement.

8 Appendix

Derivation of the optimal bidding strategy of LF i

Since we search for a symmetric Nash equilibrium of the bidding function, we have $b_i = b(\theta_i)\ \forall \theta_i$. Then, from (2), we have

$$\frac{dE\pi_i^a}{d\theta_i} = -\frac{\partial\left(c_i(\theta_i, p_a)\right)}{\partial\theta_i}(1-F(\theta_i))^{n-1}.$$

By integration, and with $E\pi_i^a\left(\theta^+\right) = 0$, we obtain

$$E\pi_i^a\left(\theta_i\right) = \int_{\theta_i}^{\theta^+}\left(\frac{\partial\left(c_i(\theta, p_a)\right)}{\partial\theta}(1-F(\theta))^{n-1}\right)d\theta,$$

From this equation and (1), we obtain the optimal bidding strategy of LF i. *Q.E.D.*

Proof of lemma 1. From (8), we can consider the difference between the expected profits of the winning LF under asymmetric information (ai) and under complete information (ci). Indeed we have

$$\begin{aligned} E\pi_i^{s(ai)(1)} &> E\pi_i^{s(ci)(1)} \\ \Leftrightarrow &\int_{\theta^-}^{\theta^+}\left[\int_{\theta_i}^{\theta^+}\left(\frac{\partial c_j(\widehat{\theta}, p(\theta))}{\partial\theta} - \frac{\partial c_j(\theta_j^{(1)}, p(\theta))}{\partial\theta}\right)(1-F(\theta))^{n-1}\,d\theta\right]g(\theta_i)d\theta_i > 0. \end{aligned} \tag{A1}$$

Obviously, under complete information, $\frac{\partial c_j(\theta_j^{(1)}, p(\theta_i))}{\partial\theta_i} = 0$ since whatever the efficiency of the winning LF is. So, this latter can always pay the subcontractor at a cost $c_j(\widehat{\theta}, p(\theta)) = c_j(\theta_j^{(1)}, p(\theta))$. Then, a condition for (A1) to be satisfied is

$$\frac{\partial c_j(\widehat{\theta}, p(\theta_i))}{\partial\theta_i} > 0, \tag{A2}$$

i.e. if the LF is production specialized. *Q.E.D.*

Proof of lemma 2. From (7), the difference between the expected total costs under incomplete and under complete information is

$$\begin{aligned} &ETC^{s(ai)} - ETC^{s(ci)} \\ = &\int_{\theta^-}^{\theta^+}\left[c_j(\widehat{\theta}, p(\theta_i)) - c_j(\theta_j^{(1)}, p(\theta_i)) + \frac{\int_{\theta_i}^{\theta^+}\left(\frac{\partial c_j(\widehat{\theta}, p(\theta_i))}{\partial\theta_i}\right)(1-F(\theta))^{n-1}d\theta}{(1-F(\theta_i))^{n-1}}\right] \\ &g(\theta_i)d\theta_i. \end{aligned}$$

Since

$$c_j(\widehat{\theta}, p(\theta_i)) - c_j(\theta_j^{(1)}, p(\theta_i)) > 0,$$

a condition for $ETC^{s(ai)} < ETC^{s(ci)}$ to hold is $\frac{\partial c_j(\widehat{\theta}, p(\theta_i))}{\partial \theta_i} < 0$, which is in contradiction with (A2). So, we cannot have both $ETC^{s(ai)} < ETC^{s(ci)}$ and $E\pi_i^{s(ai)} > E\pi_i^{s(ci)}$.

Further, we can have $ETC^{s(ai)} < ETC^{s(ci)}$ if

$$\begin{aligned}
&\int_{\theta^-}^{\theta^+} \left[\frac{\int_{\theta_i}^{\theta^+} \left(-\frac{\partial c_j(\widehat{\theta}, p(\theta_i))}{\partial \theta_i} \right) (1 - F(\theta))^{n-1} d\theta}{(1 - F(\theta_i))^{n-1}} \right] g(\theta_i) d\theta_i \\
&> \int_{\theta^-}^{\theta^+} \left[c_j(\widehat{\theta}, p(\theta_i)) - c_j(\theta_j^{(1)}, p(\theta_i)) \right] g(\theta_i) d\theta_i
\end{aligned}$$

i.e. if $\frac{\partial c_j(\widehat{\theta}, p(\theta_i))}{\partial \theta_i}$ is not too high. *Q.E.D.*

Proof of Lemma 4. Assuming $\frac{\partial \left(c_j(\widehat{\theta}, p_a) \right)}{\partial \theta} = 0$, we get

$$\begin{aligned}
& ETC^s - ETC^a \\
= \;& c_j(\widehat{\theta}, p_a) - \int_{\theta^-}^{\theta^+} \left(c_j(\theta, p_a) + \frac{\int_{\theta}^{\theta^+} \frac{\partial c_j(\theta, p_a)}{\partial \theta} (1 - F(\theta))^{m-1} d\theta}{(1 - F(\theta)^{m-1}} \right) g(\theta) d\theta \\
= \;& c_j(\widehat{\theta}, p_a) - c_j(\theta_j^{(2)}, p_a).
\end{aligned}$$

As with the comparison of the winning SBF's profit, the comparison of expected total costs is based on the difference between negotiated and auctioned lots. Clearly, we obtain $ETC^s < ETC^a$ if the winning LF knows the SBFs' costs. If these costs are private information, then obviously $ETC^s > ETC^a$. *Q.E.D.*

Proof of proposition 1. From (3) and (7), we have

$$\begin{aligned}
ETC^s - ETC^a \;=\;& c_j(\widehat{\theta}, p_a) - c_j(\theta_j^{(2)}, p_a) \\
&+ \int_{\theta^-}^{\theta^+} \left(\frac{\int_{\theta_i}^{\theta^+} \frac{\partial c_j(\widehat{\theta}, p_a)}{\partial \theta} (1 - F(\theta))^{n-1} d\theta}{(1 - F(\theta_i))^{n-1}} \right) g(\theta_i) d\theta_i.
\end{aligned}$$

Recall that

$$E\pi_j^{s(1)} - E\pi_j^{a(1)} = c_j(\widehat{\theta}, p_a) - c_j(\theta_j^{(2)}, p_a),$$

and that

$$E\pi_i^{s(1)} - E\pi_i^{a(1)} = \int_{\theta^-}^{\theta^+} \left(\int_{\theta_i}^{\theta^+} \frac{\partial c_j(\widehat{\theta}, p_a)}{\partial \theta} (1 - F(\theta))^{n-1} d\theta \right) g(\theta_i) d\theta_i.$$

Thus, subcontracting can be a Pareto dominating mechanism if the following conditions are simultaneously satisfied

$$\begin{cases} ETC^s < ETC^a \\ E\pi_j^{s(1)} > E\pi_j^{a(1)} \\ E\pi_i^{s(1)} > E\pi_i^{a(1)} \end{cases}. \tag{A3}$$

Obviously,

$$\begin{aligned} ETC^s &< ETC^a \Leftrightarrow \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (A4) \\ c_j(\widehat{\theta}, p_a) - c_j(\theta_j^{(2)}, p_a) &< -\int_{\theta^-}^{\theta^+} \left(\frac{\int_{\theta_i}^{\theta^+} \frac{\partial c_j(\widehat{\theta}, p_a)}{\partial \theta}(1-F(\theta))^{n-1} d\theta}{(1-F(\theta_i))^{n-1}} \right) g(\theta_i) d\theta_i, \end{aligned}$$

and

$$E\pi_j^{s(1)} > E\pi_j^{a(1)} \Leftrightarrow c_j(\widehat{\theta}, p_a) - c_j(\theta_j^{(2)}, p_a) > 0. \tag{A5}$$

(15) and (A5) imply

$$\int_{\theta^-}^{\theta^+} \left(\frac{\int_{\theta_i}^{\theta^+} \frac{\partial c_j(\widehat{\theta}, p_a)}{\partial \theta}(1-F(\theta))^{n-1} d\theta}{(1-F(\theta_i))^{n-1}} \right) g(\theta_i) d\theta_i < 0 \Leftrightarrow \frac{\partial c_j(\widehat{\theta}, p_a)}{\partial \theta} < 0. \tag{A6}$$

However,

$$\begin{aligned} E\pi_i^{s(1)} &> E\pi_i^{a(1)} \Leftrightarrow \int_{\theta^-}^{\theta^+} \left(\int_{\theta_i}^{\theta^+} \frac{\partial c_j(\widehat{\theta}, p_a)}{\partial \theta}(1-F(\theta))^{n-1} d\theta \right) g(\theta_i) d\theta_i > 0 \\ &\Leftrightarrow \frac{\partial c_j(\widehat{\theta}, p_a)}{\partial \theta} > 0. \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad (A7) \end{aligned}$$

Since (A6) and (15) cannot both be satisfied, subcontracting cannot be a Pareto improving mechanism. The same reasoning applies to the allotment procedure. *Q.E.D.*

Proof of lemma 5

Recall that comparing expected profits in each situation, we get

$$\begin{aligned} & E\pi_i^{s(1)} - E\pi_i^{a(1)} \\ = & \int_{\theta^-}^{\theta^+} \left[\int_{\theta_i}^{\theta^+} \left(\frac{\partial c_j(\widehat{\theta}, p(\theta))}{\partial \theta} \right) (1 - F(\theta)^{n-1} d\theta \right] g(\theta_i) d\theta_i \\ & + \int_{\theta^-}^{\theta^+} [\int_{\theta_i}^{\theta^+} \left(\frac{\partial \left[c_i(\theta, p(\theta)) - c_i(\theta, p_a) \right]}{\partial \theta} \right) (1 - F(\theta)^{n-1} d\theta] \; g(\theta_i) d\theta_i. \end{aligned}$$

The sign of $\left(\frac{\partial[c_i(\theta_i,p(\theta_i))-c_i(\theta_i,p_a)]}{\partial\theta_i}\right)$ is related to the sign of the cross derivative $\frac{\partial^2 c_i(\theta_i,p)}{\partial p\partial\theta_i}$. More precisely, since we assume that $\frac{\partial^2 c_i(\theta_i,p)}{\partial p\partial\theta_i}<0$, we have

$$\frac{\partial\left[c_i(\theta_i,p(\theta_i))-c_i(\theta_i,p_a)\right]}{\partial\theta_i}<(>)\,0 \text{ if } p(\theta_i)>(<)\,p_a.$$

If $\frac{\partial c_j(\widehat{\theta},p(\theta))}{\partial\theta}\leq 0$ and $p(\theta_i)>p_a$, then $E\pi_i^{a(1)}>E\pi_i^{s(1)}$. *Q.E.D.*

Proof of proposition 2. The difference in expected total costs of the contract, when the LF is allowed to choose the subcontracting level, is

$$\begin{aligned}&ETC^s-ETC^a \qquad\qquad (A8)\\ =&\int_{\theta^-}^{\theta^+}\left(\begin{array}{c}c_i(\theta_i,p(\theta_i))+c_j(\widehat{\theta},p(\theta_i))-c_i(\theta_i,p_a)\\ +\frac{\int_{\theta_i}^{\theta^+}\left(\frac{\partial[c_i(\theta,p(\theta))-c_i(\theta,p_a)]}{\partial\theta}+\frac{\partial c_j(\widehat{\theta},p(\theta))}{\partial\theta}\right)(1-F(\theta))^{n-1}d\theta}{(1-F(\theta_i))^{n-1}}\end{array}\right)g(\theta_i)d\theta_i\\ &-\int_{\theta^-}^{\theta^+}\left(c_j(\theta_j,p_a)+\frac{\int_{\theta_j}^{\theta^+}\frac{\partial c_j(\theta,p_a)}{\partial\theta}(1-F(\theta))^{m-1}d\theta}{(1-F(\theta_j))^{m-1}}\right)h(\theta_j)d\theta_j.\end{aligned}$$

Consider the special case of $n=m$ and $\theta_i=\theta_j$,[14] then $h(\theta_j)=g(\theta_i)$ and (15) becomes

$$\begin{aligned}&ETC^s-ETC^a \qquad\qquad (A9)\\ =&\int_{\theta^-}^{\theta^+}\left[\begin{array}{c}c_i(\theta_i,p(\theta_i))+c_j(\widehat{\theta},p(\theta_i))-c_i(\theta_i,p_a)-c_j(\theta_j,p_a)\\ +\frac{\int_{\theta_i}^{\theta^+}\left(\underbrace{\frac{\partial c_j(\widehat{\theta},p(\theta))}{\partial\theta}+\frac{\partial c_i(\theta,p(\theta))}{\partial\theta}}_{>0\text{ by assumption}}-\frac{\partial c_i(\theta,p_a)}{\partial\theta}-\frac{\partial c_j(\theta,p_a)}{\partial\theta}\right)(1-F(\theta))^{n-1}d\theta}{(1-F(\theta_i))^{n-1}}\end{array}\right]g(\theta_i)d\theta_i.\end{aligned}$$

From (15), sufficient conditions for $ETC^s<ETC^a$ to be satisfied are

$$\left\{\begin{array}{c}\frac{\partial c_j(\widehat{\theta},p(\theta_i))}{\partial\theta_i}+\frac{\partial c_i(\theta_i,p(\theta_i))}{\partial\theta_i}<\frac{\partial c_i(\theta_i,p_a)}{\partial\theta_i}+\frac{\partial c_j(\theta_j,p_a)}{\partial\theta_i}\\ c_i(\theta,p(\theta))+c_j(\widehat{\theta},p(\theta))-c_i(\theta,p_a)-c_j(\theta,p_a)<0\end{array}\right.. \qquad (A10)$$

Given (13) and (A10), we have both $ETC^s<ETC^a$ and $E\pi_i^{s(1)}>E\pi_i^{a(1)}$ if

$$\left\{\begin{array}{c}\frac{\partial c_i(\theta_i,p_a)}{\partial\theta_i}<\frac{\partial c_j(\widehat{\theta},p(\theta_i))}{\partial\theta_i}+\frac{\partial c_i(\theta_i,p(\theta_i))}{\partial\theta_i}<\frac{\partial c_i(\theta_i,p_a)}{\partial\theta_i}+\frac{\partial c_j(\theta_j,p_a)}{\partial\theta_i}\\ c_i(\theta,p(\theta))+c_j(\widehat{\theta},p(\theta))-c_i(\theta,p_a)-c_j(\theta,p_a)<0\end{array}\right.. \qquad (A11)$$

[14] Similar (but tedious) derivations can be made for more general situations. Our aim here is to provide some sufficient conditions.

Then, subcontracting can be a Pareto improving procedure if conditions (A11) and $E\pi_j^{s(1)} > E\pi_j^{a(1)}$ are simultaneously satisfied. Given (14), $E\pi_j^{s(1)} > E\pi_j^{a(1)}$ if

$$\int_{\theta^-}^{\theta^+} \left[c_j(\widehat{\theta}, p(\theta)) - c_j(\theta, p(\theta))\right] g(\theta)d\theta > \int_{\theta_j}^{\theta^+} \left(\frac{\partial c_j(\theta, p_a)}{\partial \theta}(1 - F(\theta))^{m-1}d\theta\right) h(\theta)d\theta,$$

i.e. if the gain from the bargaining on costs is higher than the expected strategic mark-up in the auctioning of the allotted part. Then, sufficient conditions for a Pareto improvement are

$$\begin{cases} \frac{\partial c_i(\theta_i,p_a)}{\partial \theta_i} < \frac{\partial c_j(\widehat{\theta},p(\theta_i))}{\partial \theta_i} + \frac{\partial c_i(\theta_i,p(\theta_i))}{\partial \theta_i} < \frac{\partial c_i(\theta_i,p_a)}{\partial \theta_i} + \frac{\partial c_j(\theta_i,p_a)}{\partial \theta_i} \\ c_i(\theta_i, p(\theta_i)) + c_j(\widehat{\theta}, p(\theta_i)) - c_i(\theta_i, p_a) - c_j(\theta_j, p_a) < 0 \\ \int_{\theta^-}^{\theta^+} \left[c_j(\widehat{\theta}, p(\theta)) - c_j(\theta, p(\theta))\right] g(\theta)d\theta > \int_{\theta_j}^{\theta^+} \left(\frac{\partial c_j(\theta,p_a)}{\partial \theta}(1 - F(\theta))^{m-1}d\theta\right) h(\theta)d\theta \end{cases}.$$

Q.E.D.

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