# Spatial Price Discrimination, Buyer Mobility and Mergers

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**Abstract**

We present a Cournot model with symmetric firms selling (homogeneous) products in a high demand and a low demand market region. Buyers are mobile but restricted by travel costs, so that imperfect arbitrage occurs when prices differ in both market regions. Market equilibria are distorted away from Cournot outcomes in order to prevent high demand consumers to buy on the lower demand market. Moreover, a merger can lead to a high-price equilibrium outcome in which only the high-demand market region is served. This "market withdrawal effect" of a merger becomes more likely (i) the lower the firms' ability to segment consumers (i.e., the more mobile buyers are) and (ii) the higher the concentration of the industry. We show that the possibility that a mergers trigger a complete withdrawal from the low demand country is important for assessing the competitive effects of mergers and their impact on overall welfare.

*JEL-Classification: D43, L13, L41*

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# 1 Introduction

In this paper we analyze merger incentives when firms sell their (homogenous) products in different regions or countries. We assume that the supply side is perfectly integrated, so that firms can serve different markets without incurring any additional costs. In contrast to standard approaches of international oligopoly theory (see, e.g., Brander 1995) and international merger control (e.g. Barros and Cabral 1994), we make the possibility of demand substitution between markets explicit.[1] This allows us to analyze how buyer mobility, or equivalently, the extent of arbitrage between different market regions may impact on firms' merger incentives.

Our main point is that demand-side market mobility may give rise to hitherto unexplored incentives to merge which result from firms' incentives to suppress sales in the low demand country. While there may exist an equilibrium in which both markets are served, firms can be better off by serving only the high demand country. We show that serving only the high demand country is more likely to be an equilibrium the more concentrated the supply side becomes. We therefore argue that a merger can lead to a "coordinated effect" such that all firms in the industry coordinate on the pay-off dominant equilibrium in which only the high demand market is served. This type of "coordination" is very different from standard reasoning about coordinated effects of mergers, which has so far relied on collusive conduct supported by repeated interaction (see, e.g., Ivaldi et al. 2003 for an overview). While the standard coordinated effects doctrine has been criticized on various grounds an alternative model of how a merger may coordinate firms on a less competitive equilibrium has not been developed so far.

Our paper contributes to two strands of literature, which are fairly unconnected so far: first the merger literature and second the literature on third-degree price discrimination.

With regards to merger incentives, our paper contributes to the large literature on mergers in Cournot markets. In their seminal work, Salant, Switzer and Reynolds (1983) proved that a bilateral merger is typically not profitable when firms compete in a homogeneous product market. They assumed symmetric firms and constant marginal costs. Both assumptions, and with this the controversial "merger paradox", have been criticized in the subsequent literature.

[1] See Neven and Röller (2003) for an exception.

Salant et al.'s rudimentary merger model has been enriched by many supply-side features, as synergies (e.g. Farrell and Shapiro 1990 and for a review see Röller, Stennek and Verboven 2003), firm specific marginal costs (Levin 1990 and Cheung 1992), quadratic costs (Perry and Porter 1985), Stackelberg-leadership (Daughety 1990),[2] multi-product rivalry (Lommerud and Sörgard 1997), purchasing power (von Ungern-Sternberg 1996), product differentiation (Inderst and Wey 2004), strategic divisionalization (Baye, Crocker, and Ju 1996), and spatially differentiated suppliers (McAfee, Simons, and Williams 1992). Again, relating to the supply side, Davidson and Deneckere (1985) studied Bertrand behavior and Kwoka (1989) examined the effects of Maverick behavior.

Quite interestingly, this entire literature has exclusively focused on supply-side aspects, while demand-side sources of adverse competitive effects of mergers have been neglected. International aspects of mergers have been addressed, e.g., by Barros and Cabral (1994), but this literature, again, has focused on the effects of supply-side market integration (through imports and exports), while disregarding the possibility of demand mobility.

Our analysis of price discrimination across regions is also related to the literature of third degree price discrimination in oligopoly (see, e.g., Neven and Phlips 1985, Holmes 1989, and Varian 1989 and Stole 2003 for surveys). This literature has focused on the welfare effects of price discrimination when compared with a regime which bans price discrimination.

In the following we first set out the general model in section 2. In section 3 we characterize the properties of the possible Cournot equilibria. Section 4 considers a numerical example which indicates that the firms' incentives to merge can in fact be higher then in the standard Cournot models.

## 2 The Model

We consider a Cournot oligopoly model with $n$ firms offering a homogeneous product in two countries $j = h, l$. All firms have no fixed costs and constant marginal costs which we normalize to zero.

Additionally, firms have no transportation costs, i.e., the firms' costs for supplying their products in each country are the same.

[2] See also Selten 1973 for a similar reasoning in the context of cartel formation.

In each country $j$ there is a unit mass of $j-$consumers. Consumers can decide in which country they buy and what quantity they demand. Considering first the demand decisions we assume that all $j-$consumers have the same quasi-linear utility function

$$U_j(x,p) = u_j(x) - px \tag{1}$$

where $x$ denotes the quantity bought and $p$ the price. We assume $u_j'(x) > 0 = u_j(0) > u_j''(x)$ and $u_j'''(x) \leq 0$.[3] Furthermore, $h-$consumers have a higher marginal utility for all positive quantities, i.e.,

$$u_h'(x) > u_l'(x) \text{ and } u_h''(x) \leq u_l''(x),\ u_h'''(x) \geq u_l'''(x) \text{ for all } x > 0. \tag{2}$$

Note that (2) leads to $d(pX_h(p))/dp > d(pX_l(p))/dp$ for all $p$ with $X_l(p) > 0$. Using (1) and defining $X_j(p) := \arg\max_x [u_j(x) - px]$, consumers' indirect utility functions are given by $V_j(p) := u_j(X_j(p)) - pX_j(p)$.

Turning to the decision in which country to buy we assume that consumers have different travelling or switching costs $t$. These costs depend on a consumer specific parameter $\theta_j$ and on a shift parameter $\alpha$ :

$$\begin{aligned} t(\alpha,\theta) \text{ with } t(\alpha,0) &= 0 < t_\theta(\theta_j,\alpha),\ t_{\theta\theta}(\theta_j,\alpha) = 0 \\ \text{and } t_\alpha(\theta_j,\alpha) &< 0 \text{ for } \theta_j > 0. \end{aligned} \tag{3}$$

While $\theta_j$ is distributed in the interval $[0,1]$ according to the distribution function $F(\theta_j)$ with $f(\theta_j) := F'(\theta_j) = 1$, we can interpret $\alpha$ as a measure of mobility. The higher $\alpha$ the lower the consumers' costs to buy in the other country. Letting $p_j$ denote the price in country $j$, the net utility of a $j-$consumer with $\theta_j$ is given by

$$\begin{array}{ll} V_j(p_j) & \text{if he buys in country } j \\ V_j(p_k) - t(\alpha,\theta_j) & \text{if he buys in country } k \neq j,\ k = l,h. \end{array} \tag{4}$$

Using (4) and defining

$$\lambda_j(p_j,p_k,\alpha) := \min\left\{\max\left\{0, \theta |\, V_j(p_j) = V_j(p_k) - t(\theta,\alpha)\right\}, 1\right\}. \tag{5}$$

we obtain the following aggregate demand functions $X_h^D(p_h,p_k,\alpha)$ and $X_l^D(p_h,p_l,\alpha)$ in country $h$ and $l$, respectively:

$$X_h^D(p_h,p_l,\alpha) \ := \begin{cases} (1-\lambda_h(p_h,p_l,\alpha))X_h(p_h) & \text{for} \quad p_h \geq p_l \\ \lambda_l(p_h,p_l,\alpha)X_l(p_h) + X_h(p_h) & \text{for} \quad p_h \leq p_l \end{cases} \tag{6}$$

$$X_l^D(p_l,p_h,\alpha) \ := \begin{cases} \lambda_h(p_h,p_l,\alpha)X_h(p_l) + X_l(p_l) & \text{for} \quad p_h \geq p_l \\ (1-\lambda_l(p_h,p_l,\alpha))X_l(p_l) & \text{for} \quad p_h \leq p_l \end{cases} \tag{7}$$

[3] Assuming $u_j'''(x) \leq 0$ ensures that the consumers' demand function are concave.

With $x_j$ as the total quantity supplied in country $j$ the inverse demand functions $P_h(x_h, x_l, \alpha)$ and $P_l(x_h, x_l, \alpha)$ implied by (6) and (7) satisfy[4]

$$x_h = X_h^D(P_h, P_l, \alpha) \text{ and } x_l = X_l^D(P_h, P_l, \alpha). \tag{8}$$

# 3 Analysis of the Model

We assume that firms simultaneously choose the quantities they supply in both countries. Letting $x_j^i$ denote the quantity which firm $i$ supplies on market $j$ and defining

$$x_j^{-i} := \sum_{m=1, m\neq i}^{n} x_j^m > 0. \tag{9}$$

the firms' profit functions $\Pi_i(x_h^i, x_l^i, x_h^{-i}, x_l^{-i})$ are given by

$$\Pi_i(x_h^i, x_l^i, x_h^{-i}, x_l^{-i}, \alpha) = P_h(x_h, x_l, \alpha)x_h^i + P_l(x_l, x_h, \alpha)x_l^i \tag{10}$$

Differentiating (10) with respect to $x_j^i$ leads to the following first order conditions

$$\frac{\partial \Pi_i}{\partial x_j^i} = \frac{\partial P_j}{\partial x_j} x_j^i + P_j + \frac{\partial P_k}{\partial x_j} x_k^i \leq 0; \; \frac{\partial \Pi_i}{\partial x_j^i} x_j^i = 0 \tag{11}$$

where $\partial P_j / \partial x_j$ and $\partial P_k / \partial x_j$ follow from differentiating (8) and applying the implicit function theorem.

Inspection of (11) shows that there may exist two types of Cournot-Nash equilibria: *$HL$-equilibria* in which strictly positive quantities are offered in both countries and *$H$-equilibria* in which the firms supply strictly positive quantities only in the high demand country $h$.[5] In the following we characterize the main properties of these two types of equilibria.

## 3.1 $H$-equilibria: Only $h$ country is served

Consider first the equilibrium in which firms do not supply any quantities in the low demand country $l$. Then, the first order conditions for the optimal quantities $x_h^{i*}$ can be written as

$$\left.\frac{\partial \Pi_i}{\partial x_h^i}\right|_{x_l=0} = \frac{\partial P_h}{\partial x_h} x_h^i + P_h = 0 \tag{12}$$

[4] In the following we will omit the arguments of the functions where this does not lead to any confusion

[5] The existence of an equilibrium with $x_h^* = 0 < x_l^*$ is ruled out by the assumption $u_h'(x) > u_l'(x)$ for all $x$.

and we get the following properties[6]

**Lemma 1** *If a H-equilibrium exists, it is unique and symmetric, i.e., the equilibrium quantities can be characterized by*

$$x_h^*(n) = nx_h^{i*}(n) \text{ and } x_l^{i*} = 0 \text{ for all } i = 1, ..n.$$

*Furthermore,* $P_h(x_h^*, 0, \alpha) > \bar{p}_l := \sup\{p | X_l(p) > 0\}$*. and the equilibrium profits* $\Pi_i^*(n) := \Pi_i(x_h^{i*}, 0, x_h^{-i*}, 0, \alpha)$ *are decreasing in n, while*

$$\frac{d}{dn}[nx_h^*(n)] > 0 > \frac{d}{dn}x_h^{i*}(n).$$

Lemma 1 establishes the standard properties of Cournot equilibria. It also reveals that an equilibrium with $x_h^* > 0 = x_l^*$ implies $P_h(x_h^*, 0, \alpha) > \bar{p}_l$. Intuitively, with $P_h(x_h^*, 0, \alpha) \leq \bar{p}_l$ the price in country $h$ is so low that each firm has an incentive to increase its revenues by serving the $l$-consumers who initially do not buy in country $h$.

Turning to existence, a $H$-equilibrium exists if and only if deviating by supplying strictly positive quantities in country $l$ is not worthwhile, i.e., if and only if

$$\Pi_i(x_h^i, x_l^i, x_h^{-i*}, 0, \alpha) \leq \Pi_i^*(n) \text{ for all } x_h^i, x_l^i > 0. \tag{13}$$

Analyzing the first order conditions for $\max_{x_h^i, x_l^i} \Pi_i(x_h^i, x_l^i, x_h^{-i*}, 0, \alpha)$, i.e.,

$$\frac{\partial \Pi_i}{\partial x_h^i} = \frac{\partial P_h}{\partial x_h}x_h^i + P_h + \frac{\partial P_l}{\partial x_l^i}x_l^i = 0 \text{ and } \frac{\partial \Pi_i}{\partial x_l^i} = \frac{\partial P_h}{\partial x_l}x_h^i + P_l + \frac{\partial P_l}{\partial x_l^i}x_l^i = 0 \tag{14}$$

the following lemma characterizes the optimal deviation behavior:

**Lemma 2** *Given* $x_h^{-i*}$ *and* $x_l^{-i*} = 0$, $\Pi_i(x_h^i, x_l^i, x_h^{-i*}, 0, \alpha)$ *attains a (local) maximum with* $x_h^{id} > 0$ *and* $x_l^{id} > 0$ *only if*

$$\begin{aligned} i) &: \quad \bar{p}_l > P_l(x_l^{id}, x_h^{id} + x_h^{-i*}, \alpha) \text{ and} \\ ii) &: \quad P_h(x_h^*, 0, \alpha) > P_h(x_h^{id} + x_h^{-i*}, x_l^{id}, \alpha) > P_l(x_l^{id}, x_h^{id} + x_h^{-i*}, \alpha) \end{aligned}$$

*Additionally,* $P_h X_h(P_h) > P_l X_h(P_l)$ *holds.*

Obviously, $P_l(x_l^{id}, x_h^{id} + x_h^{-i*}, \alpha) < \bar{p}_l$ reflects the fact that deviation can be worthwhile only if more consumers are served. Part $ii$) shows the basic trade-off

[6] The proofs of all lemmata are relegated to the Appendix.

implied by offering positive quantities in the low demand country $l$. With $P_h > P_l$ some $h-$consumers buy in country $l$ which lowers the price in the high demand country $h$. The lower the travel costs, i.e., the higher $\alpha$, the less attractive deviation should be. On the other hand, considering the price level in country $h$ we should expect that the lower the price $P_h(x_h^*(n), 0, \alpha)$ the higher the incentive to deviate. Taking into account $dx_h^*(n)/dn > 0$ and thus $dP_h/dn < 0$ the incentives to deviate should therefore be positively correlated with the number of firms. Analyzing the impact of $\alpha$ and $n$ more carefully and defining the maximal attainable profits from deviation by $\Pi_i^d(n, \alpha)$ we obtain

**Lemma 3**

$$\begin{aligned} i) &: \quad \frac{\partial \Pi_i^d(n,\alpha)}{\partial \alpha} < 0 \; and \; \frac{\partial \Pi_i^d(n,\alpha)}{\partial n} < 0 \\ ii) &: \quad sign\left[\frac{\partial \Pi_i^d(n,\alpha)}{\partial n} - \frac{d\Pi_i^*(n)}{dn}\right] > 0 \end{aligned}$$

Combining parts $i)$ and $ii)$ of lemma 3 allows us to characterize the necessary and sufficient conditions for the existence of a $H$-equilibrium.

**Proposition 1** *With $\Pi_i^d(1, \alpha) < \Pi_i^*(1)$ a $H$-equilibrium exists if and only if*

$$n \leq n^k(\alpha) := \left\{ n | \, \Pi_i^d(n, \alpha) = \Pi_i^*(n) \right\}.$$

*Furthermore, $n^{k\prime}(\alpha) > 0$.*

**Proof.** Parts $i)$ and $ii)$ of lemma 3 imply that with $\Pi_i^d(1, \alpha) < \Pi_i^*(1)$ there exists a unique $n > 1$ such that $\Pi_i^d(n, \alpha) = \Pi_i^*(n)$. $n^{k\prime}(a) > 0$ follows from the implicit function theorem. ■

Employing lemma 1 we additionally get that $P_h(x_h^*(n^k(\alpha)), 0, \alpha) > \bar{p}_l$ holds. The positive sign of $n^{k\prime}(\alpha)$ is due to the fact that the higher $\alpha$ the lower the switching costs and the more a deviating firm would loose from the consumers' switching behavior.

### 3.2 $HL$-equilibria: Both countries served

Turning to the case in which firms supply positive quantities in both countries, the relevant first order conditions are

$$\frac{\partial \Pi_i}{\partial x_h^i} = \frac{\partial P_h}{\partial x_h} x_h^i + P_h + \frac{\partial P_l}{\partial x_l^i} x_l^i = 0 \text{ and } \frac{\partial \Pi_i}{\partial x_l^i} = \frac{\partial P_h}{\partial x_l} x_h^i + P_l + \frac{\partial P_l}{\partial x_l^i} x_l^i = 0. \quad (15)$$

Analyzing (15) we obtain the followinbg two lemmata:

**Lemma 4** *Any* $HL$*-equilibrium with* $x_h^c,\ x_l^c > 0$ *is symmetric, i.e.,* $x_j^{ic} = 1/n\ x_j^c$ *for* $j = l, h$*. The quantities* $x_h^c$ *and* $x_l^c$ *are such that* $P_h(x_h^*, 0, \alpha) > P_h(x_h^c, x_l^c, \alpha) > P_l(x_l^c, x_h^c, \alpha)$ *and* $\bar{p}_l > P_l(x_l^c, x_h^c, \alpha)$ *as well as*

$$\begin{aligned} P_h X_h(P_h) - P_l X_h(P_l) &> 0 \text{ and} \\ (1-\lambda_h) P_h X_h'(P_h) + x_h^{ic} &> 0 > P_l \left( X_l'(P_l) + \lambda_l X_h'(P_l) \right) + x_l^{ic}. \end{aligned}$$

*holds.*

**Lemma 5** *If a* $HL$*-equilibrium exists, it is unique.*

While $P_h(x_h^*, 0, \alpha) > P_h(x_h^c, x_l^c, \alpha)$ is intuitive since $P_h(x_h^c, x_l^c, \alpha) > P_l(x_l^c, x_h^c, \alpha)$ induces some $h-$consumers to buy in country $l$, $P_h X_h(P_h) - P_l X_h(p_l) > 0$ together with $(1-\lambda_h) P_h X_h'(P_h) + x_h^{ic} > 0 > p_l \left( X_l'(P_l) + \lambda_l X_h'(P_l) \right) + x_l^i$ indicates that the firms try to avoid switching by offering relatively high quantities in country $h$. More precisely, given the number of $h-$consumers who switch to country $l$, the equilibrium quantities offered in country $h$ are higher than the corresponding Cournot quantities while the opposite is true for the quantities offered in country $l$. While this reduces the firms' losses from the induced switching of $h$-consumers, $P_h(x_h^*, 0, \alpha) > P_h(x_h^c, x_l^c, \alpha)$ nevertheless implies that the firms' profits $\Pi_i(x_h^{ic}, x_l^{ic}, x_h^{-ic}, x_l^{-ic}, n, \alpha)$ are lower than in the standard model in which buyers are perfectly immobile.

# 4 Coordinated Effects

Having characterized the possible equilibria we can now turn to the analysis of the firms' incentives to merge. Employing the fact that buyers mobility reduces the firms' profits when both markets are served, the firms' profits may be higher when only country $h$ is served. This observation together with the result that a $H$-equilibrium is more likely to exists the lower the number of firms (see proposition 1) establishes the basic merger incentive mentioned in the introduction.

Assuming $\Pi_i(x_h^{ic}, x_l^{ic}, x_h^{-ic}, x_l^{-ic}, n, \alpha) < \Pi_i^*(n)$ and $n-1 < n^k(\alpha)$ we may in fact have

$$\Pi_i^*(n-1) > 2\Pi_i(x_h^{ic}, x_l^{ic}, x_h^{-ic}, x_l^{-ic}, n, \alpha). \qquad (16)$$

As long as (16) holds, a merger of two firms is profitable if firms can also coordinate on the $H$-equilibrium. Obviously, coordination is only feasible if the $H$-equilibrium exists. Coordination is profitable if the firms are better off under the $H$-equilibrium, i.e., if

$$\Pi_i^*(n-1) > \Pi_i(x_h^{ic}, x_l^{ic}, x_h^{-ic}, x_l^{-ic}, n-1, \alpha) \tag{17}$$

holds. Taking these two aspects—feasibility and profitability—into account, a merger can be uniquely traced back to coordination effects as long as (16), (17) and

$$\Pi_i(x_h^{ic}, x_l^{ic}, x_h^{-ic}, x_l^{-ic}, n-1, \alpha) < 2\Pi_i(x_h^{ic}, x_l^{ic}, x_h^{-ic}, x_l^{-ic}, n, \alpha) \tag{18}$$

hold. Note that (18) and (16) imply (17).

Since the analysis of (16) and (18) involves a rather complicated comparison of the firms' profits under different equilibrium regimes, the following analysis is based on a simple numerical example. While this restricts the generality of the results it allows us to derive some explicit results with respect to the relations which $\alpha$ and $n$ have to obey such that (16) and (18) are satisfied.

### 4.1 Functional Forms

We assume the following specification of consumers' utility and travel costs:

$$u_h(x) = x - \frac{1}{2}x^2;\; u_l(x) = ax - \frac{b}{2}x^2;\; t(\theta, \alpha) = \frac{1}{\alpha}\theta;\; f(\theta) = 1. \tag{19}$$

Using (19) and considering the equilibrium in which only country $h$ is served, we get

$$x_h^* = \frac{n}{1+n};\; P_h(x_h^*, 0, \alpha) = \frac{1}{1+n} \text{ and } \Pi_i^*(n) = \frac{1}{(1+n)^2}. \tag{20}$$

Note that (20) and lemma 1 immediately imply that an equilibrium with $(x_h^*, 0)$ does not exist for $n \geq 1/a - 1$.

Turning to the deviation profit $\Pi_i^d(n, \alpha)$ and using (20) and $p_h = P_h(x_h^{id} + x_h^{-i*}, x_l^{id}, \alpha) > p_l = P_l(x_l^{id}, x_h^{id} + x_h^{-i*}, \alpha)$ we get the following expressions for

$\Pi_i^d(n,\alpha)$ and the respective first order conditions

$$\begin{aligned}
\Pi_i^d(n,\alpha) &= \frac{(1+n)\ (a-p_l)\,p_l + bp_h\,(2-(n+1)p_h)}{b\,(1+n)} \\
&\quad -\frac{\alpha}{2}(p_h-p_l)^2\,(2-p_h-p_l)\,(1-p_h-p_l) && (21)\\
\frac{\partial \Pi_i^d}{\partial x_h} &= 0 \Leftrightarrow && (22)\\
0 &= 4+(1+n)\,(-4\,p_h-\alpha\,(p_h-p_l)\,(4-3p_l+p_h\,(-9+4p_h+4p_l))) \\
\frac{\partial \Pi_i^d}{\partial x_l} &= 0 \Leftrightarrow && (23)\\
0 &= 2a-4p_l+\alpha b\,(p_h-p_l)\,(4+p_l\,(-9+4p_l)+p_h\,(-3+4p_h))
\end{aligned}$$

If both countries are served, we can use proposition **??** to calculate the relevant profit functions and first order conditions. Employing symmetry and $p_h = P_h(x_h^c, x_l^c, \alpha) > p_l = P_l(x_l^c, x_h^c, \alpha)$ and $p_l < a$ equilibrium profits $\Pi_i(x_h^{ic}, x_l^{ic}, x_h^{-ic}, x_l^{-ic}, n, \alpha)$ are given by

$$\begin{aligned}
\Pi_i(\cdot) &= \frac{1}{2nb}\left[2b\,(1-p_h)\,p_h + 2p_l\,(a-p_l)\right. && (24)\\
&\quad \left. -b(p_h-p_l)^2\,(2-p_h+p_l)\,(1-p_h-p_l)\,\alpha\right]
\end{aligned}$$

where $p_h = P_h(x_h^c, x_l^c, \alpha) > p_l = P_l(x_l^c, x_h^c, \alpha)$ are determined by the following two equations

$$\begin{aligned}
0 &= -2+2\,(1+n)\,p_h+\alpha\,(p_h-p_l)\,[(-1+p_h)\,(-2+p_h+p_l) && (25)\\
&\quad +n\ (2-2p_l+3p_h\ (-2+p_h+p_l))] \\
0 &= 2a-2\,(1+n)\,p_l+\alpha b\,(p_h-p_l)\,[(-1+p_l)\,(-2+p_h+p_l) && (26)\\
&\quad +n\,(2+3\,(-2+p_l)\,p_l+p_h\,(-2+3p_l))]
\end{aligned}$$

## 4.2 Results

Using $a = 0.1$ and considering $b = 0.2$ as well as $b = 0.5$, (20)—(26) lead to the following results: $x_h^c$ and $x_l^c$ are uniquely determined for all $n \geq 1$ and $\alpha > 0$. Hence we have $x_h^{ic}(n,\alpha)$, $x_l^{ic}(n,\alpha)$ and we can define $\Pi_i^c(n,\alpha) := \Pi_i(x_h^{ic}, x_l^{ic}, x_h^{-ic}, x_l^{-ic}, n, \alpha)$.

Concerning our main point, i.e., the firms' incentives to merge, analyzing (24)—(26) reveals

$$2\Pi_i^c(n,\alpha) > \Pi_i^c(n-1,\alpha) \text{ for all } n \geq 3, \tag{27}$$

which implies that—without coordination effects—a merger is not profitable if $n \geq 3$.

However, comparing the firms' profits in the $HL$-equilibrium and the $H$-equilibrium—see (16)—shows that there exist a unique $n^f(\alpha)$ such that for all $n < n^f(\alpha)$ a merger is profitable as long as it also implies that the firms switch from the $HL$-equilibrium to the $H$-equilibrium, i.e.,

$$n < n^f(\alpha) \Leftrightarrow \Pi_i^*(n-1) > 2\Pi_i(x_h^c, x_l^c, n, \alpha). \tag{28}$$

Starting with $b = 0.2$ and neglecting integer constraints the graphs for $n^k(\alpha)$ and $n^f(\alpha)$ are shown in Figure 1.

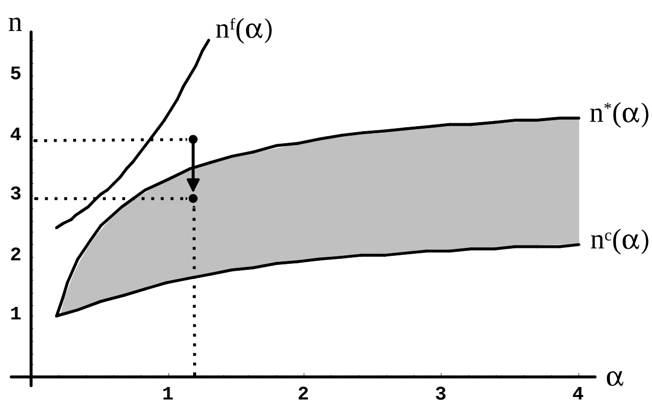


Figure 1: $n^f(\alpha)$, $n^k(\alpha)$ and $n^c(\alpha)$ for $b = 0.2$

Since $n \leq n^k(\alpha)$ implies that a $HL$-equilibrium exists, figure 1 shows that coordinated effects increase the firms' incentives to merge. Considering $\alpha \approx 1.2$ and $n = 4$ a merger between two firms allows to coordinate on the $H$-equilibrium and is thus profitable.

With $b = 0.5$ we obtain the graphs depicted in Figure 2.

Comparing figures 1 and 2 shows that an increase in $b$ leads to an upward shift of $n^k(\alpha)$. The higher $b$ the steeper the demand curve of $l$-consumers and thus the lower the profits from deviating. With $\alpha \approx 2.8$ and $n = 6$ a merger of two firms is profitable.

However, considering relative low values of $\alpha$ reveals that although a merger may allow coordination on a $H$-equilibrium, such a merger may not be profitable due

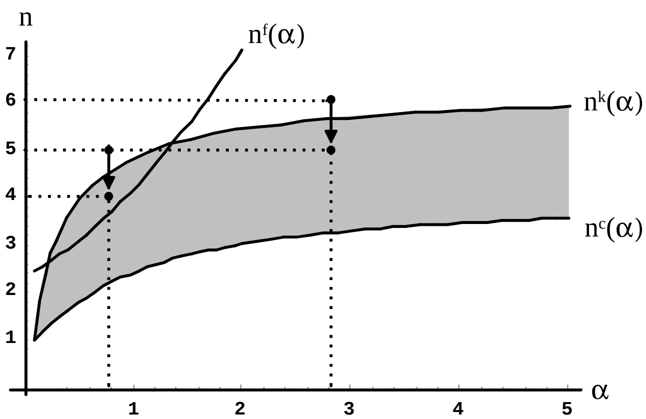


Figure 2: $n^f(\alpha)$, $n^k(\alpha)$ and $n^c(\alpha)$ for $b = 0.5$

to the losses the firms would have to bear by not serving the $l$-consumers. This situation occurs for all $\alpha$ and $n$ such that $n > n^k(\alpha) > n - 1$ and $n > n^f(\alpha)$. For example, with $\alpha \approx 0.8$ and $n = 5$ a merger is not profitable.

# 5 Conclusion

We have presented a standard Cournot model with symmetric firms which sell their (homogeneous) products in a high demand and a low demand market region. Buyers are mobile but restricted by travel costs, so that imperfect arbitrage occurs when prices differ in both market regions. We showed that spatial price discrimination together with buyer mobility give rise to new strategic and competitive effects of mergers. With mobile buyers market equilibria are distorted away from Cournot outcomes, and a merger reduces this distortion by widening the price differential between both market regions. Most importantly, a merger can lead to an equilibrium outcome in which only the high-demand market region is served. As this equilibrium can be pay-off dominant, we interpret the realization of this equilibrium as a "coordinated effect" of a merger. This kind of coordinated effect becomes more likely (i) the lower the firms' ability to segment consumers (i.e., the more mobile buyers become) and (ii) the higher the concentration of the industry.

Our model points at so far neglected dangers of mergers in industries such as the pharmaceutical industry, where coordinated effects of mergers can imply disastrous effects on developing countries' consumer welfare.

# 6 Appendix

**Proof of Lemma 1** Assume $P_h(x_h^*, 0, \alpha) > \bar{p}_l$ and note that this also implies that only $h-$consumers buy and that $\Pi_i(x_h^{i*}, 0, x_h^{-i*}, 0, \alpha)$ does not depend on $\alpha$. Inspection of (12) leads to symmetry. Employing standard arguments with respect to the slope of the firms' reaction functions establishes uniqueness. Hence, we have $x_h^{i*}(n)$ and $x_h^* = nx_h^{i*}(n)$. Furthermore, it is easy to show that $x_h^{i*}(n)$ and $\Pi_i^*(n)$ have the following (standard) properties

$$x_h^{i*\prime}(n) < 0 < x_h^{*\prime}(n) \text{ and} \tag{29}$$

$$\frac{d\Pi_i^*(n)}{dn} = -P_h(nx_h^*(n), 0, \alpha)\frac{d}{dn}\left[(n-1)x_h^*(n)\right] < 0 \tag{30}$$

where (30) follows from (29) and the envelope theorem. For later reference note also that we have

$$\left.\frac{\partial \Pi_i}{\partial x_h^i}\right|_{x_l=0} \gtreqless 0 \Leftrightarrow P_h(x_h^i + x_h^{-i}, 0, \alpha)X_h'(P_h(x_h^i + x_h^{-i}, 0, \alpha)) + x_h^i \lesseqgtr 0. \tag{31}$$

To show that $P_h(x_h^*, 0, \alpha) > \bar{p}_l$ must hold assume first that $P_h(x_h^*, 0, \alpha) = \bar{p}_l$ holds. $P_h(x_h^*, 0, \alpha) = \bar{p}_l$ implies that the marginal revenue of increasing $x_l^i$ is strictly positive while the loss from $h-$consumers switching to country $l$ is only of second order. Hence, firms would have an incentive to deviate by choosing $x_l^i > 0$ and $P_h(x_h^*, 0, \alpha) = \bar{p}_l$ can not hold in equilibrium. Assuming $P_h(x_h^*, 0, \alpha) < \bar{p}_l$ and using w.l.o.g. $P_l(x_h^*, 0, \alpha) = \bar{p}_l$, a fraction $\lambda_l(P_h(x_h^*, 0, \alpha), \bar{p}_l, \alpha)$ of $l-$consumers would buy on market $h$. Let $\tilde{X}_l := \lambda_l(P_h(x_h^*, 0, \alpha), \bar{p}_l, \alpha)X_l(P_h(x_h^*, 0, \alpha))$ and consider the following change in firm $i's$ supply: Instead of supplying $x_h^{i*} > 0$ and $x_l^{i*} = 0$ firm $i$ chooses $\tilde{x}_h^i = \max\{0, x_h^{i*} - \tilde{X}_l\}$ and $\tilde{x}_l^i = \tilde{X}_l$. Since this implies $\tilde{x}_h^i + \tilde{x}_l^i \geq x_h^{i*}$ and

$$P_h(\tilde{x}_h^i + x_h^{-i}, \tilde{x}_l^i, \alpha) \geq P_h(x_h^*, 0, \alpha) \text{ and } P_l(\tilde{x}_l^i, \tilde{x}_h^i + x_h^{-i}, \alpha) > P_h(x_h^*, 0, \alpha) \tag{32}$$

firm $i$'s profit is higher when it chooses $(\tilde{x}_h^i, \tilde{x}_l^i)$ instead of $(x_h^{i*}, 0)$. Therefore, in any equilibrium with $x_h^* > 0 = x_l^*$ we must have $P_h(x_h^*, 0, \alpha) > \bar{p}_l$.

**Proof of Lemma 2** $P_l(x_l^{id}, x_h^{id} + x_h^{-i*}, \alpha) < \bar{p}_l$ is implied by lemma 1. The proof of part $ii)$ proceeds in several steps: We first show $P_h(x_h^{id} + x_h^{-i*}, x_l^{id}, \alpha) >$

$P_l(x_l^{id}, x_h^{id} + x_h^{-i*}, \alpha)$ and then turn to $P_h(x_h^*, 0, \alpha) > P_h(x_h^{id} + x_h^{-i*}, x_l^i, \alpha)$ and $P_h X_h(P_h) > P_l X_h(P_l)$.

Assume to the contrary that $P_h(x_h^{id} + x_h^{-i*}, x_l^{id}, \alpha) < P_l(x_l^{id}, x_h^{id} + x_h^{-i*}, \alpha)$. Solving (14) we get that the optimal quantities $x_h^{id}$ and $x_l^{id}$ are implicitly defined by

$$\begin{aligned} t_\theta(\theta_l, \alpha) &= \frac{X_l(P_h)\,(P_h X_l(P_h) - P_l X_l(P_l))}{P_h\,(X_h'(P_h) + \lambda_l X_l'(P_h)) + x_h^i} \\ &= -\frac{X_l(P_l)\,(P_h X_l(P_h) - P_l X_l(P_l))}{(1-\lambda_l)(P_l X_l'(P_l) + x_l^i)} > 0 \end{aligned} \tag{33}$$

where we used $x_h^i + x_h^{-i*} \equiv X_h(P_h) + \lambda_l(P_h, P_l, \alpha) X_l(P_h)$ and $x_l^i \equiv (1 - \lambda_l(P_h, P_l, \alpha))X_l(P_l)$ as well as $\partial\lambda_l / \partial P_h = -\, X_l(P_h) / \, t_\theta(\theta_l, \alpha)$ and $\partial\lambda_l / \partial P_l = X_l(P_l) / \, t_\theta(\theta_l, \alpha)$.

Analyzing (33) and taking into account (2) and $t_\theta(\cdot) > 0$ shows that $P_h X_l(P_h) - P_l X_l(P_l) < 0$ implies $P_h\,(X_h'(P_h) + \lambda_l X_l'(P_h)) + x_h^{id} < 0$ and $P_l X_l'(P_l) + x_l^{id} > 0$ which contradicts $P_h < P_l$. With $P_h X_l(P_h) - P_l X_l(P_l) > 0$ we arrive at $P_h\,(X_h'(P_h) + \lambda_l) X_l'(P_h)) + x_h^{id} > 0$ and $P_l X_l'(P_l) + x_l^{id} < 0$. However, considering quantities $\hat{x}_h^i$ and $\hat{x}_l^i$ such that

$$\begin{aligned} P_h(\hat{x}_h^i + x_h^{-i*}, \hat{x}_l^i, \alpha) &= P_l(\hat{x}_l^i, \hat{x}_h^i + x_h^{-i*}, \alpha) = \max[P_h(x_h^{id} + x_h^{-i*}, x_l^{id}, \alpha), p_l^m] \\ \text{with} &: \; p_l^m := \arg\max p_l X_l(p_l) \end{aligned}$$

reveals that a deviation which leads to $P_h\,(X_h'(P_h) + \lambda_l) X_l'(P_h)) + x_h^{id} > 0$ and $P_l X_l'(P_l) + x_l^{id} < 0$ can not be optimal.

Assuming, $P_h = P_l$ leads to $\lambda_l = \lambda_h = 0$ which also implies that the first conditions (14) can be written as

$$x_h^i + P_h X_h'(P_h) = 0 \text{ and } P_l X_l'(P_l) + x_l^i = 0. \tag{34}$$

which—by using (2) and $P_h = P_l$—leads again to a contradiction. Thus, we must have $P_h(x_h^{id} + x_h^{-i*}, x_l^{id}, \alpha) > P_l(x_l^{id}, x_h^{id} + x_h^{-i*}, \alpha)$.

Turning to $P_h(x_h^*, 0, \alpha) > P_h(x_h^{id} + x_h^{-i*}, x_l^i, \alpha)$ and $P_h X_h(P_h) > P_l X_h(P_l)$ and using $P_h(x_h^{id} + x_h^{-i*}, x_l^{id}, \alpha) > P_l(x_l^{id}, x_h^{id} + x_h^{-i*}, \alpha)$ and $\bar{p}_l > P_l(x_l^{id}, x_h^{id} + x_h^{-i*}, \alpha)$, the first order conditions (14) can be written as

$$\begin{aligned} t_\theta(\theta_h, \alpha) &= -\frac{X_h(P_l)\,(P_l X_h(P_l) - P_h X_h(P_h))}{(1-\lambda_h) P_h X_h'(P_h) + x_h^i} \\ &= \frac{X_h(P_l)\,(P_l X_h(P_l) - P_h X_h(P_h))}{P_l\,(X_l'(P_l) + \lambda_l X_h'(P_l)) + x_l^i} > 0. \end{aligned} \tag{35}$$

where we used $x_h^i + x_h^{-i} \equiv (1-\lambda_h(P_h,P_l,\alpha))X_h(P_h)$ and $x_l^i \equiv \lambda_h(P_h,P_l,\alpha)X_h(P_l) + X_l(P_l)$ as well $\partial\lambda_h/\partial P_h = X_h(P_h)/t_\theta(\theta_h,\alpha)$ and $\partial\lambda_h/\partial P_l = -X_l(P_h)/t_\theta(\theta_h,\alpha)$.

To prove $P_hX_h(P_h) > P_lX_h(P_l)$, assume to the contrary that $P_lX_h(P_l) - P_hX_h(P_h) > 0$. Then, (35) implies $(1-\lambda_l)P_lX_l'(P_l) + x_h^{id} < 0$ and $P_l\left(X_l'(P_l) + \lambda_l)X_h'(P_l)\right) + x_l^{id} > 0$ which can not be optimal since firm $i$ can increase its profit by simply increasing $x_h^i$ and decreasing $x_l^i$. Hence we must have $P_lX_h(P_l) - P_hX_h(P_h) < 0$ which also implies $(1-\lambda_h)P_hX_h'(P_h) + x_h^{id} > 0$ and therefore $P_h(x_h^*, 0, \alpha) > P_h(x_h^{id} + x_h^{-i*}, x_l^{id}, \alpha)$ (see (31)).

**Proof of Lemma 3** Part *i*) is based on applying the envelope theorem which yields

$$\frac{\partial\Pi_i^d(n,\alpha)}{\partial\alpha} = \frac{\partial\lambda_h}{\partial\alpha}\left[P_lX_h(P_l) - P_hX_h(P_h)\right] < 0 \text{ by lemma 2} \tag{36}$$

$$\frac{\partial\Pi_i^d(n,\alpha)}{\partial n} = -P_h(x_h^{id} + (n-1)x_h^*, x_l^{id}, \alpha)\frac{d}{dn}\left[(n-1)x_h^*(n)\right] < 0. \tag{37}$$

Employing (30) and (37) leads to

$$\begin{aligned} &\frac{\partial\Pi_i^d(n,\alpha)}{\partial n} - \frac{d\Pi_i^*(n)}{dn} \\ = \quad &\left[-P_h(x_h^{id} + (n-1)x_h^{i*}, x_l^{id}, \alpha) + P_h(x_h^*, 0, \alpha)\right]\frac{d}{dn}\left[(n-1)x_h^*(n)\right] > 0 \end{aligned} \tag{38}$$

where the sign follows from lemma 2.

**Proof of Lemma 4** Symmetry is implied by (15). The rest of the proof is similar to the proof of lemma 2 and proceeds in several steps: We first show that an equilibrium with $x_h^c, x_l^c > 0$ must lead to $P_h(x_h^c, x_l^c, \alpha) > P_l(x_l^c, x_h^c, \alpha)$ and $\bar{p}_l > P_l(x_l^c, x_h^c, \alpha)$. We then turn to $P_h(x_h^*, 0, \alpha) > P_h(x_h^c, x_l^c, \alpha)$ and $P_hX_h(P_h) - P_lX_h(p_l) > 0$.

To start with, assuming $P_h(x_h^c, x_l^c, \alpha) > P_l(x_l^c, x_h^c, \alpha) > \bar{p}_l$ we can rewrite the first order conditions as

$$\begin{aligned} t_\theta(\theta_h,\alpha) &= -\frac{X_h(P_h)\left(P_lX_h(P_l) - P_hX_h(P_h)\right)}{(1-\lambda_h)(P_hX_h'(P_h) + x_h^i)} \\ &= \frac{X_l(P_h)\left(P_lX_h(P_l) - P_hX_h(P_h)\right)}{\lambda_h(P_lX_l'(P_l) + x_l^i)} > 0. \end{aligned} \tag{39}$$

With $P_lX_h(P_l) - P_hX_h(P_h) < 0$ (39) leads to $P_hX_h'(P_h) + x_h^{ic} > 0 > P_lX_h'(P_l) + x_l^{ic}$ which contradicts $P_h(x_h^c, x_l^c, \alpha) > P_l(x_l^c, x_h^c, \alpha)$. Thus, we must also have

$P_l X_h(P_l) - P_h X_h(P_h) > 0$ and $P_h X_h'(P_h) + x_h^{ic} < 0 < P_l X_l'(P_l) + x_l^{ic}$ which implies that $P_h(x_h^c, x_l^c, \alpha)$ is higher and $P_l(x_l^c, x_h^c, \alpha)$ is lower than the Cournot price $P_h(x_h^*, 0, \alpha)$. Note further that every firm $i$ serves a fraction $1/n\lambda_h$ of $h-$consumers in country $l$. Using this observation consider a deviation of firm $i$ such that it offers quantities $\tilde{x}_h^i > x_h^{ic}$ and $x_l^{ic} = 0$ such that $1/n\lambda_h$ $h$-consumers would switch back and buy in country $h$. Taking into account that switching implies costs we get $P_h(x_h^c, x_l^c, \alpha) > P_h(\tilde{x}_h^i + x_h^{-ic}, x_l^{-ic}, \alpha)$ and $P_h(\tilde{x}_h^i + x_h^{-ic}, x_l^{-ic}, \alpha)\tilde{x}_h^i > P_h(x_h^c, x_l^c, \alpha)x_h^{ic} + P_l(x_l^c, x_h^c, \alpha)x_l^{ic}$. Therefore, $P_h(x_h^c, x_l^c, \alpha) > P_l(x_l^c, x_h^c, \alpha) > \bar{p}_l$ implies that firm $i$ can profitably deviate by economizing on the consumers' switching costs.

Considering the case with $\bar{p}_l > P_l(x_l^c, x_h^c, \alpha) > P_h(x_h^c, x_l^c, \alpha)$ and analyzing the respective first order conditions shows that $x_h^c$ and $x_l^c$ must satisfy

$$t_\theta(\theta_l, \alpha) = \frac{X_l(P_h)\left(P_h X_l(P_h) - P_l X_l(P_l)\right)}{P_h(X_h'(P_h) + \lambda_l X_l'(P_h)) + x_h^i} = -\frac{X_l(P_l)\left(P_h X_l(P_h) - P_l X_l(P_l)\right)}{(1-\lambda_l)P_l X_l'(P_l) + x_l^i} > 0. \tag{40}$$

Assuming $P_h X_l(P_h) - P_l X_l(P_l) < 0$ leads to $P_h(X_h'(p_h) + \lambda_l X_l'(P_h)) + x_h^{ic} < 0 < (1-\lambda_l)P_l X_l'(P_l) + x_l^{ic}$ and thus to a contradiction since firm $i$ can increase its profit by increasing $x_h^i$ and decreasing $x_l^i$. Hence, we must have $P_h X_l(P_h) > P_l X_l(P_l)$ and thus $P_h(X_h'(P_h) + \lambda_l X_l'(P_h)) + x_h^{ic} > 0 > (1-\lambda_l)P_l X_l'(P_l) + x_l^{ic}$. Again, considering the fact that firm $i$ serves a fraction $1/n\lambda_l$ of $l$-consumers in country $h$ and that $0 > (1-\lambda_l)P_l X_l'(P_l) + x_l^{ic}$ implies that the price in country $l$ exceeds the respective Cournot price we can apply the same argument as above. That is, firm $i$ can decrease the quantity it supplies in country $h$ and increase the quantity offered in country $l$ such that a fraction $1/n\lambda_l$ of $l$-consumers would switch back and buy in country $l$. This would lead to $P_l(x_l^c, x_h^c, \alpha) > P_l(\tilde{x}_h^i + x_h^{-ic}, x_l^{-ic} + \tilde{x}_l^i, \alpha)$ and to $P_l(\tilde{x}_h^i + x_h^{-ic}, x_l^{-ic} + \tilde{x}_l^i, \alpha)\tilde{x}_l^i > P_h(x_h^c, x_l^c, \alpha)x_h^{ic} + P_l(x_l^c, x_h^c, \alpha)x_l^{ic}$ and thus to higher profits.

Since $P_l(x_l^c, x_h^c, \alpha) \geq \bar{p}_l > P_h(x_h^c, x_l^c, \alpha)$ and $P_l(x_l^c, x_h^c, \alpha) = P_h(x_h^c, x_l^c, \alpha)$ can be excluded by the same reasoning and inspection of the first order conditions, respectively, an equilibrium with $x_h^c,\ x_l^c > 0$ must imply $P_h(x_h^c, x_l^c, \alpha) > P_l(x_l^c, x_h^c, \alpha)$ and $\bar{p}_l > P_l(x_l^c, x_h^c, \alpha)$.

The proof of $P_h(x_h^c, x_l^c, \alpha) < P_h(x_h^*, 0, \alpha)$ and $P_h X_h(P_h) - P_l X_h(P_l) > 0$ again follows the same reasoning as the proof of lemma 2. That is, with $P_h > P_l$,

$\bar{p}_l > P_l$ and (15) we have

$$\begin{aligned} t_\theta(\theta_h, \alpha) &= -\frac{X_h(P_h)\left(P_l X_h(P_l) - P_h X_h(P_h)\right)}{(1-\lambda_h) P_h X_h'(P_h) + x_h^i} \qquad (41) \\ &= \frac{X_h(P_l)\left(P_l X_h(P_l) - P_h X_h(P_h)\right)}{P_l\left(X_l'(P_l) + \lambda_l X_h'(P_l)\right) + x_l^i} > 0. \end{aligned}$$

Assuming $P_l X_h(P_l) - P_h X_h(P_h) > 0$ leads to a contradiction since firm $i$ can increase its profit by increasing $x_h^i$ and decreasing $x_l^i$. Hence, we must have $P_h X_h(P_h) > P_l X_h(P_l)$ and thus $(1-\lambda_h) P_h X_h'(P_h) + x_h^{ic} > 0$ which also leads to $P_h(x_h^*, 0, \alpha) > P_h(x_h^c, x_l^c, \alpha)$ (see (31)). Finally, $P_h X_h(P_h) > P_l X_h(P_l)$ and (41) imply $P_l(X_l'(P_l) + \lambda_l X_h'(P_l)) + x_l^{ic} < 0$.

**Proof of Lemma 5** Using lemma 4, and defining

$$V_h(P_h, n) \;:=\; \frac{P_h X_h'(P_h) + \frac{X_h(P_h)}{n}}{X_h(P_h)}, \; V_l(P_l, n) := \frac{P_l X_h'(P_l) + \frac{X_h(P_l)}{n}}{X_h(P_l)}, \qquad (42)$$

$$\tilde{V}_l(P_l, n) \;:=\; \frac{P_l X_l'(P_l) + \frac{X_l(P_l)}{n}}{X_l(P_l)} \qquad (43)$$

and $U_h(P_h) := P_h X_h(P_h)$ and $U_l(P_l) := P_l X_h(P_l)$, equations (41) can be transformed to (ommiting arguments)

$$(1-\lambda_h) V_h - \frac{X_h(P_h)}{t_\theta}(U_h - U_l) = 0, \qquad (44)$$

$$(1-\lambda_h) V_h + \lambda_h V_l + \tilde{V}_l = 0. \qquad (45)$$

Using (44), holding $n$ and $\alpha$ constant and interpretating $P_h$ as a function $\Phi$ of $P_l$ the implicit function theorem leads to

$$\Phi'(P_l) = \frac{U_l' + V_h X_l}{U_h' + V_h X_h + \frac{1}{t_\theta} V_{hP_h}(\lambda_h - 1)}. \qquad (46)$$

Employing lemma 4 we get that $\Phi'(P_l) > 0$ must hold as long as (44) and (45) are satisfied (this is due to $U_h', U_l', V_h > 0$ and $X_h''(p) \le 0 \Rightarrow V_{hP_h} < 0$).

Using (45) and interpretating $P_h$ as a function $\Psi$ of $P_l$ we get

$$\Psi'(P_l) = \frac{X_l(V_h - V_l) + \frac{1}{t_\theta}(\lambda_h V_{lP_l} + \tilde{V}_{lP_l})}{X_h(V_h - V_l) + \frac{1}{t_\theta} V_{hP_h}(\lambda_h - 1)}. \qquad (47)$$

Since $P_h > P_l$ implies $V_h - V_l < 0$ and since $V_{lP_l}, \tilde{V}_{lP_l} < 0$ (because of $X_l''(p) < 0$) we obtain

$$\begin{aligned} \Psi'(P_l) &< 0 \Leftrightarrow X_h(V_h - V_l) + \frac{1}{t_\theta} V_{hP_h}(\lambda_h - 1) > 0 \qquad (48) \\ &\Leftrightarrow V_{hP_h}(U_l - U_h) + X_h V_h (V_h - V_l) > 0 \end{aligned}$$

where the second line of (48) follows from (44) and (45). Since $U_l - U_h = V_h - V_l = 0$ for $P_h = P_l$, (48) implies $\Psi'(P_l) < 0$ if

$$\frac{V_{hP_h}}{X_h V_h} - \frac{V_{lP_l}}{U_l'} < 0. \tag{49}$$

Evaluating (49) and restricting the analysis to prices $P_h$ such that $V_h > 0$ reveals

$$\left.\frac{V_{hP_h}}{X_h V_h} - \frac{V_{lP_l}}{U_l'}\right|_{P_h = P_l} < 0. \tag{50}$$

Furthermore, differentiating $V_{lP_l}/U_l'$ with respect to $P_l$ shows that $X_h'''(p) \leq 0 \Rightarrow \frac{d}{dP_l}\left[V_{lP_l}/U_l'\right] < 0$ as long as $V_l > 0$. Thus, if (44) and (45) hold we must also have $\Psi'(P_l) < 0$. Combining this finding with $\Phi'(P_l) > 0$ (as long as (44) and (45) are satisfied) implies that if an *HL* equilibrium exists it is unique.

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