ON MULTIDIMENSIONAL INEQUALITY WITH VARIABLE DISTRIBUTION MEAN

ERNESTO SAVAGLIO

1. EXTENDED ABSTRACT

Economic literature on inequality measurement is mainly concerned with the comparison of *univariate* indices of well-being, which record the differences in distribution of income (and/or wealth) within and between populations. However, such an approach is considered an inadequate basis for comparing individual disparities because people differ in many aspects besides income. The analysis of different individual attributes is indeed crucial to understand and evaluate inequality among persons. Therefore, a very recent research trend is focused on the development of criteria for ranking *multivariate* distributions of individual attributes.¹ Unfortunately, few progress has been made on extending the theory of inequality measurement from univariate to the multivariate case (see e.g. [1], [2], [3], [5], [6] [8]), the works on multidimensional disparity comparisons are rather sparse and the problem is really complex. In the present work, we follow the literature trend analyzing inequality in a context of more than one (income) variable and addressing the problem to compare multidimensional alternative distributions. In order to achieve our aim, we extend the notion of generalized Lorenz (or dually weak majorization) preorder (see [4], [7]) to a context of multiple individual attributes. In particular, we compare multivariate distributions in terms of inequality when the means of their marginals differ. We represent a multidimensional distribution as a matrix, whose generic entry consists in the quantity of the k th good, $k = 1, \dots, m$, allocated to the i th group of individuals, $i = 1, \dots, n$. A preorder of different distribution matrices is defined according to their level of inequality and the properties of the preorder are provided. Using certain tools of convex analysis, we show that such a preorder can be replaced by the order defined as the inclusion of the columns (and of course of the rows) of a distribution matrix in the convex hull defined by the set of all convex combinations of the rows (and of course of the rows) of another distribution matrix and analogously by a social evaluation function that records the level of inequality of alternative individual distributions of goods.

Finally, we compare our preorder of matrix distributions with the main inequality criteria for ranking multidimensional distributions discussed in economic literature (see e.g. [1], [2], [3], [4], [5]).

Date: March 10, 2007.

Key words and phrases. Multidimensional inequality, price majorization, generalized Lorenz preorder

JEL classification: D31; D63; I31.

¹See Savaglio [6] for a survey.

REFERENCES

- [1] Kolm, S.C. (1977) - Multidimensional Egalitarianisms - *Quarterly Journal of Economics* 91, 1-13.
- [2] Koshevoy, G. (1995) - Multivariate Lorenz majorization - *Social Choice and Welfare* 12, 93-102.
- [3] Koshevoy, G. (1998) - The Lorenz zonotope and multivariate majorizations - *Social Choice and Welfare* 15, 1-14.
- [4] Marshall A. W. and I. Olkin (1979) - *Inequalities: Theory of Majorization and its Applications* - Academic Press.
- [5] Savaglio, E. (2006) - On multidimensional inequality with variable population size - *Economic Theory* 28, 85-94.
- [6] Savaglio, E. (2006) - Three approaches to the analysis of multidimensional inequality - in Farina, F. and E. Savaglio (eds), *Inequality and Economic Integration*, Routledge, London.
- [7] Shorrocks, A. F. (1983) - Ranking income distributions - *Economica*, 50, 3-17.
- [8] Trannoy, A. (2006) - Multidimensional egalitarianism and the dominance approach: a lost paradise? - in Farina, F. and E. Savaglio (eds), *Inequality and Economic Integration*, Routledge, London.

DMQTE, UNIVERSITY "G. D'ANNUNZIO" OF PESCARA: VIALE PINDARO, 42 - 65100 PESCARA (ITALY), AND DEPARTMENT OF ECONOMICS, UNIVERSITY OF SIENA: PIAZZA SAN FRANCESCO, 7 - 53100 SIENA (ITALY).

E-mail address: `ernesto@unich.it`