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ON REAL ECONOMIC FREEDOM

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Abstract

A priori, real economic freedom is purchasing (and selling) power. Yet, Xu's theorem comforts ranking the freedom of choice provided by budget sets as their volume in deriving it from three axioms. However, one and a half of these axioms can be discussed. In contrast, the standard measure of purchasing power leads one to order the freedom provided by budget sets as the distance to the origin of the intersection of the budget hyperplanes with a given ray from the origin. Hence, equal budget freedoms correspond to pencils of budget hyperplanes. Applied to labour and earnings of individuals with different wage rates, this equal freedom yields the distributive principle of equal labour income equalization.

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1. Economic freedom

Economic freedom often means freedom of exchange. Freedom of exchange is a part of basic freedoms or basic rights, which constitute the legal and moral basis of our societies. It has often been reproached to these rights that they may leave you with little actual freedom if you do not have the means to make use of them. In particular, even if all "men are free and equal in rights" (the 1789 Declaration), this allows for very unequal actual freedom. "Rich and poor are equally free to sleep under bridges" (Anatole France). Marx denounces these basic rights

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as being only "formal freedom" and not providing "real freedom".[1] Yet, he thus inspired suppressions of "formal freedoms" with the historical dramatic consequences we know. Real freedom requires both formal freedom and other means, that is, with free exchange, income or means to acquire it such as a sufficient wage rate. The budget sets are the domains of possible choice which provide the corresponding "freedom of choice."

One can certainly admit that, with given prices, a higher income provides a higher such freedom. At least, it provides a possibility set that includes the other in adding possibilities to have more of all goods. As common language puts it: "purchasing power" is higher. Yet, what can be said when prices are not the same and the budget sets are not related by inclusion? This situation is a very common problem for economists and economic statisticians. They face it in choosing a price index for computing a "purchasing power" in dividing income by this index. The result is also "real" in another sense of the term, now opposed to "nominal", i.e., it does not change if incomes and prices are multiplied by the same number (for whatever reason). More precisely, the price index is a linear function of prices, with coefficients that represent quantities of goods in a "basket" the choice of which is the subject of the theory of price indexes. The result thus measures the purchasing power of an income facing a set of prices, hence of the corresponding budget set, as the number of such baskets that this income can buy when facing these prices. That is to say, this classical real freedom is the distance to the origin of the intersect of the budget hyperplane with the ray from the origin bearing the vector of the coefficients of the price index. One consequence is that the equality of such freedoms means that the budget hyperplanes pass through the same point which represents the vector of the price-index basket multiplied by the same number (they constitute a "pencil" of hyperplanes).

This comparison of purchasing powers is often done for the different prices at different dates or in different places. Yet, there is a still more important application, since freedom is a priori that of individuals, and different individuals generally face different prices in free exchange for the good which can be said to be the most important one, labour, whose price, the wage rate, depends on the different productive capacities and on the demands for them, and is also the market price of the good leisure. This leads to the determination of given incomes, or income transfers, that maintain an equal purchasing power, or real economic

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[1] Marx also objected to the individualism of basic rights, thus echoing a reproach made by Robespierre as early as 1789.

freedom, for individuals endowed with different given productive capacities, hence facing different wage rates. The result in the simplest case turns out to be that each individual i receives the net transfer $t_i=k\cdot(\overline{w}-w_i)$, where w_i is individual i's wage rate, $\overline{w}=(1/m)\Sigma w_i$ is the average wage rate for m individuals, k is a number defining the price index, and t_i is a subsidy if $t_i>0$ and a tax of $-t_i$ if $t_i<0$ (see Section 4 below).[2]

However, a remarkable recent article of Yongsheng Xu (2004) proposes a strong axiomatic basis for the application to competitive budget sets of an old alternative proposal, which consists of ranking the freedom offered by domains of choice of quantities of goods by their volume. This is at odds with the foregoing result based on standard practice. For instance, in the case of two goods, the budget lines corresponding to equal real freedom pass through the same point with the standard comparison and are tangent to the same rectangular hyperbola with the volume ranking.

In 1972, Jean-Marc Oury, then a student, proposed to me, as topic of dissertation, the measure of the freedom of choice of a bundle of commodities by the volume of the possibility set. I rather discouraged him. One of my reasons was that this amounts to measuring real income with a price index which is the geometric mean of prices, and this neat property has strange consequences shortly recalled – and needs to be justified.[3] However, Xu derives his conclusion from axioms, not from consequences.[4] The volume ranking was nevertheless among the rankings or measures of freedom of choice whose properties I analyzed.[5] Yet, the contradiction has to be elucidated at the level of the axioms and of their meaning.[6]

Most generally, there are three kinds of freedoms, which can be labelled negative freedom, positive freedom, and mental freedom. They are often closely interrelated. Negative

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[2] And Kolm 2004a.

[3] Moreover, forbidding access to any one good makes you less free than any other kind of constraint. Yet, this is not a budget constraint, and hence this could be objected to Oury but cannot to Xu.

[4] Classical epistemology states that rules should be judged jointly according to their statements, their conditions (including axioms), and their consequences (see, e.g., Plato's "dialectic" in *Republic* or Rawls's "reflective equilibrium").

[5] See Kolm 2004a, Chap. 24 (also 1993).

[6] Discouraged, Jean-Marc Oury abandoned his Ph.D. Hired by the French industry mogul Guy Dejouany at the Compagnie Générale des Eaux, he became in charge of the real-estate branch where he drove the Maisons Phenix to buoyancy and then to failure. This did not discourage Dejouany from hiring other bright young people from the best school. The next one was Jean-Marie Messier, who had Oury fired, took over the firm, and did exactly the same thing in the media business while renaming the firm Vivendi Universal.

freedom or social freedom is freedom from the forceful interference of other humans. It is defined by the nature of the constraints and it consists of the classical basic rights or liberties. In economics, it consists of free enterprise and free exchange and market. People are of course constrained not to forcefully interfere with others, in particular to respect the consequences of free actions or agreements (such as rights so created). Social freedom raises no issue of rivalry and can be at satiety (remaining rivalry concerns the allocation of rights concerning given resources). Positive freedom is defined by the domains of free choice, and it also is the real freedom of the discussions referred to above. In a market system, it is aptly called purchasing power (and selling power). Mental freedom is the freedom to determine one's emotions and preferences. The "autonomy" of Rousseau and Kant is a type of it. In economics it would refer to issues such as an absence of manipulation by advertisement, deliberative choices, or choice of criteria of fairness.[7] The present topic fully discards preferences and hence mental freedom. It is concerned with the second kind of liberty, positive freedom, in the context of the economic manifestation of the first, free markets. That is, it concerns the purchasing power of budget sets, or *budget freedom*.[8]

Section 2 considers Xu's theorem. The consequence of the alternative of classical price indexes is shown in Section 3. Section 4 applies this result to the question of macrojustice.[9]

2. Freedom as volume

The freedom of choice offered by domains of choice (possibility or opportunity sets) D will be ordered by an ordering assumed here to be representable by an ordinal freedom function

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[7] One can also build an economic model of consciously influencing one's preferences (see Kolm 1982, chap.23, 1985).

[8] The vocabulary of freedom has both this established base and fluctuations. For instance, what Isaiah Berlin (1958) calls positive freedom is largely mental freedom violated by ideologies. General mental freedom, including the basic issue of mastering one's desires which in principle could have drastic economic consequences, is the topic of my book *Happiness-Freedom* (*Deep Buddhism and Modernity*) (1982, in French).

[9] I wish to thank Nicolas Gravel for very usefull corrections and suggestions.

$F(D)$ (this representability is no restriction for the present problem).[10] One property is $D' \subseteq D \Rightarrow F(D') \leq F(D)$.[11]

We consider particular D's which are budget sets. Denote as i one of n goods, $x_i \geq 0$ its quantity, $x=\{x_i\} \in \Re^n_+$ a bundle of these goods, $p_i>0$ a constant price of good i, $p=\{p_i\}$ the price vector, and y an income. In the space of x, $\Sigma p_i x_i = y$ is the equation of the budget hyperplane, and $\Sigma p_i x_i \leq y$ with $x_i \geq 0$ for all i defines the budget set. Then, $a_i = y/p_i$ is the x_i of the intersect of the budget hyperplane with the axis of the x_i, and $a=\{a_i\}$ denotes the set or vector of the a_i's.

The budget set is defined by the pair (y, p), and the freedom function can be written as $F(y, p)$. A priori, this function represents a real issue (not a nominal one), and hence it is homogeneous of degree zero in y and p. Hence,

$$F(y, p)=F(1, 1/a_1, ..., 1/a_n)=\phi(a).$$

Function F will be taken as increasing in y (and decreasing in the p_i) since an increase in y (and a decrease in a p_i) add bundles containing larger quantities of all goods than previous possible ones. Hence function ϕ is increasing in the a_i.

The following remarks in this section are standard or obvious.

If functions F and ϕ are such that their ordering of the a does not change if the same coordinate of all a is multiplied by the same positive number, for all coordinates and numbers, that is,

$$\phi(a) \geq \phi(a') \Rightarrow \phi(a_1,...,\lambda a_i,..., a_n) \geq \phi(a'_1,...,\lambda a'_1,..., a'_n) \quad (1)$$

for all a, i, and $\lambda>0$, then ϕ is of the form

$$\phi(a)=\phi_1(\Pi a_i^{\alpha_i}) \quad (2)$$

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[10] Xu's ordering is so representable. He has studied various possible general properties of freedom orderings and functions in Pattanaik and Xu (2000).

[11] This property seems unavoidable if the various costs of choice are considered a different issue from freedom (including material costs, mental costs, disliking responsibility, and anguish of choice – emphasized by Kierkegaard and Sartre).

where Π denotes the product, $\alpha_i>0$ are numbers, and ϕ_1 is an increasing function; and conversely.[12]

Then, function $\phi(a)$ with this form is symmetrical if and only if all the α_i are equal: $\alpha_i=\alpha>0$ for all i. Then,

$$\phi(a)=\phi_2(\Pi a_i)$$

where ϕ_2 is an increasing function.

Yet, the volume of the budget set is $V=(1/n!)\,\Pi a_i$. Hence,

$$\phi(a)=\phi_3(V)$$

where ϕ_3 is an increasing function.

Xu postulates the symmetry of the function $\phi(a)$. This amounts to the symmetry of the freedom function $F(y, p)$ in the prices. Then, your freedom of choice is not changed by a permutation of the prices of the goods. It is not changed if you pay a sandwich for the price of a car and conversely. The mad store manager who assigns randomly his price tags to the goods leaves his customers as free. Moreover, if the price of one good doubles, your freedom is the same whatever the good, for instance a good of which you consume much or one which has no interest for you. Xu would not accept the argument that you could become less free because your favourite goods become more expensive, since this would depend on your preferences (this is assumed to apply also to the case where the good you *need* most becomes more expensive). However, this conception does not belong to all the actual uses of the terms "free" and "freedom," as we will see in conclusion.

Yet, the main problem may be with condition (1), Xu's "invariance of scaling effects." This problem is that functions F and ϕ are not a priori unit-invariant. Indeed, the proposed justification of condition (1) is that the ranking of freedom should not depend on the units in which the quantities of goods are measured. If the unit of good i becomes λ times smaller, then number a_i should become λ times larger, and nothing real is changed. However, this is

[12] More explicitly, condition (1) is the following. Let vector a take four values a^1, a^2, a^3, a^4, such that $a_i^3=\lambda a_i^1$, $a_i^4=\lambda a_i^2$, and, for all $j\neq i$, $a_j^3=a_j^1$ and $a_j^4=a_j^2$. Then $\phi(a^1)\geq\phi(a^2)\Rightarrow\phi(a^3)\geq\phi(a^4)$ for any a^1, a^2, i, and $\lambda>0$ if and only if (2) holds.

not described by condition (1) because, in fact, functions F and ϕ should change for taking account of this change in unit. In technical terms, they incur the corresponding contravariant transformation. Precisely because they represent real issues and not nominal ones. If one wants to make explicit this issue of neutrality with respect to the choice of units of measurement, one has to order not sets $a=\{a_i\}$ of the numbers a_i, but sets of numbers $\{a_i/b_i\}$ where b_i is a given (arbitrary) quantity of the good i: when units change, both a_i and b_i are multiplied by the same number, and a_i/b_i does not change (b_i is a real – not nominal – unit of measure of good i). An example will be met shortly.[13]

This can also be seen in another strong consequence of the proof under discussion, namely, all utility functions are Cobb-Douglas provided it does not matter whether bread is measured in kilos or in grams (and the like). Indeed, assume the a_i denote now the quantities of goods i consumed by an individual, and assume that the ordering under consideration is the preference ordering of this individual. Consider Xu's axioms except "symmetry." The individual prefers higher a_i. Assume she is indifferent to the units of measure of the quantities of all goods and similarly translate this as the "scaling effect." Then, the individual's ordinal utility function has form (2), a specification of which is $\Pi\, a_i^{\alpha_i}$, a Cobb-Douglas utility function.

3. Freedom as pointed distance

[13] Xu works with orderings, and hence his orderings can incur the contravariant transformation. His ordering are representable by ordinal freedom functions. The kind of issue met is not unfrequent in normative economics. Another case has exactly the same structure. Suppose you want to justify Nash bargaining solution. Then, take ϕ to be a social welfare function, and $a_i=u_i(x)–u_i(x_0)$ where u_i is a cardinal utility function, x the social state, and x_0 a particular reference state. Then a_i is defined up to an arbitrary multiplicative factor independently for each i. If this is interpreted as implying condition (1), this leads to the form $\phi=\phi_1$. A appeal to symmetry may then produce $\phi=\phi_2$, and hence Πa_i as a specification of the social welfare function (Nash's solution for n=2). Yet, this appeal to symmetry has no justification, and the appeal to condition (1) bypasses the contravariant transformation. In another story, ϕ is again a social welfare function, a_i is a cardinal utility of individual i, and ϕ is also required to be cardinal (for instance, they are the corresponding von Neumann-Morgenstern specifications). Then, each of these functions being defined up to an increasing affine function, plus an appeal to symmetry, would require that ϕ is a utilitarian sum. Yet, this omits the contravariant transformation, and the symmetry has no justification (see Kolm 1996a, Chap. 14, and Maskin's derivation of utilitarianism).A similar fallacy underlies the argument, which has been proposed, that an index of income inequality should be "scale-invariant" or homogeneous of degree zero because it should not change if the unit of measure of incomes changes (measure in dollars or in cents). Yet, an other index need not be unit-neutral and can incur the contravariant transformation when the unit for measuring incomes change. Note that the very term "scale-invariant" may suggest the mistake (homogeneity of degree zero as a real property, that is dependence on ratios – here of incomes – only, characterizes the measures that engineers and physicists call "intensive").

With this volume ranking of budget freedom, this freedom is also ranked as Πa_i and hence as $(\Pi a_i)^{1/n}=y/(\Pi p_i)^{1/n}$. This expression is homogeneous of degree zero in y and the p_i. The denominator – the geometric mean of the prices – is homogeneous of degree one in the p_i. It is a kind of price index and, then, the expression is a measure of real income.

More generally, a price index is a linearly homogeneous function of prices $\pi(p)$, and $y/\pi(p)$ is the corresponding real income. Budget freedom is ranked by real income when the freedom function F can be written as

$$F(y,p)=\varphi[y, \pi(p)],$$

since the homogeneity of degree zero of F in y and the p_i implies

$$\varphi[y, \pi(p)]=\varphi[y/\pi(p),1]=f[y/\pi(p)]$$

where f is an increasing function. Since real income is also called purchasing power, budget freedom is ranked as the corresponding purchasing power.

The volume ranking of budget freedoms takes the geometric mean of the prices as price index (Kolm 2004a).

In contrast, the standard price index is a linear form $\pi(p)=\Sigma b_i p_i$ with $b_i \geq 0$ for all i and $b_i>0$ for at least some i. It seems that this is the only meaningful form of a price index, because this is the cost of buying the set of quantities b_i of the goods, the income necessary for this purchase at these prices. It also seems that, for this reason, all the price indexes actually used are applications of this form.

With this index, budget freedom is ranked as

$$\beta=y/\Sigma b_i p_i.$$

Hence,

$$\Sigma \beta b_i p_i=y.$$

This shows that the point of coordinates βb_i is on the budget hyperplane of equation $\Sigma p_i x_i=y$. If $b=\{b_i\}$ denotes the vector of the coefficients of the price index b_i, this point is βb.

Hence, *budget freedom is ranked as the distance to zero of the intersect of the budget hyperplane with the ray from the origin bearing the vector b of the coefficients of the price index* (figure 1).

FIGURE 1

In particular, budget sets with equal budget freedom are budget sets whose budget hyperplanes pass through the same point βb (figure 2).

FIGURE 2

Conversely, if a number of budget hyperplanes pass through the same point of the non-negative orthant, they can be said to determine equal budget freedoms, relatively to a vector of coefficients of the price index in the direction of this point.

That is, equal budget freedom corresponds to the pencils of budget hyperplanes.[14]

This result also amounts to the classical principle of free choice from an equal (identical) allocation – which is represented by the common point of the budget hyperplanes.[15]

However, the issue and result are important when situations with different prices are compared. This can be different times, or places, or individuals when some prices are specific to them (such as their wage rate which is the market price of their leisure or labour). The choice of price indexes for comparing real income across times or places is the classical topic of the theory of price indexes and of a vast literature. The corresponding question for individuals and wage rates will shortly be noted.

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[14] With the general freedom function $F(y, p)$, equal-freedom hyperplanes (the hyperplanes limiting equal-freedom budget sets) have an envelop, and these envelops for various freedoms do not intersect and are the graphs of quasi-concave functions which can be written as $E(x)=F$ (see Appendix B). These envelops degenerate into points βb for a linear price index. They are rectangular hyperbolas for volume measures and n=2.

[15] See Kolm 1971.

The obtained form can illustrate the preceding remark about unit neutrality and contravariant transformations. Indeed,

$$y/\Sigma b_i p_i = (\Sigma b_i/a_i)^{-1}. \quad (3)$$

Each coefficient b_i has the dimension of a quantity of good i. When the units of good i are divided by λ, both a_i and b_i are multiplied by λ, and expression (3) does not change.

The ranking of budget sets by their coefficients β permits the comparison of economic situations with one budget set for each individual, say β^j for individual j. The logic is that of the comparison of profiles of co-ordinal indices (ordinal and interpersonnaly comparable). The following properties are meaningful.[16] Various counting of β^j that are larger or smaller than, or equal to, others. Equality of the β^j (hence of the freedom of budget sets) in a situation, and across situations for the same or different individuals. Pareto-like dominance.[17] Permutation of the β^j among individuals (symmetry of the evaluation if the permutation does not affect it because the β^j are the only relevant characteristics of the agents). "Fundamental" Pareto-like dominance (Pareto-like dominance after some permutation of the β^j in any state). Truncations (increase in the lowests or decrease in the highests). Leximin.

It may also be interesting to be more specific than ordinal, and to consider a conception of freedom which is not only ranked by real income but also measured by it. Then, you would be twice freer if you are twice richer and hence if, with the same prices, you can buy twice more of each good. These freedoms can be added and national or global freedom would be national or global income. Inequality in freedom then is measured as income inequality, and unfreedom as poverty.

4. Application: macrojustice as equal economic freedom

On ethical grounds, equal freedom of choice should be an aim when freedom of choice is the only direct value of the relevant conception of justice. This value is freedom of choice rather the outcome of the choice when the choosing individuals are deemed accountable for their choice given their freedom of choice provided by their domains of possibilities. This accountability is usually justified by their responsibility. Moreover, the direct value of justice takes no account of any aspect of intensity of preferences in a number of cases. One of these

[16] Kolm 1971, Part 3, and 2004a, Chapter 23.

[17] I.e., the same comparison as Pareto's for ordinal utilities.

cases is very important: the overall distributive justice (in "macrojustice") implemented by the main fiscal tools such as the income tax. Indeed, nobody thinks that someone should pay a higher income tax than someone else because she is less able to enjoy the euros taken away or more able to enjoy the euros left (this discards both utilitarian and egalitarian welfarist conceptions, and all intermediate ones). In these conditions, freedom of choice is the direct value of justice. This implies that it should ideally be equal among the individuals, from a reason of rationality in the basic sense of providing a reason, since there is no relevant item which could justify non-equal solutions.[18]

However, the result obtained above is non-trivial only when prices differ among the compared cases. For individuals and perfect competition, this happens when the wage rates are among the prices, and hence leisure and labour are among the goods. This is precisely the case for the determination of the just disposable income, just noted. The issue then is equal freedom of choice – notably of work and earnings –, and the result consists of the corresponding optimum transfers.

Consider m individuals indexed by i. There are two goods, income which enables one to buy consumption goods, and leisure or labour. Individual i has income c_i, leisure λ_i, labour ℓ_i, with $\lambda_i+\ell_i=1$ by choice of units, and given productivity and wage rate w_i. Her earned income is $w_i\ell_i$. She can receive transfer t_i, which is a tax of $-t_i$ if $t_i<0$.

Individual i's income is

$$c_i=w_i\ell_i+t_i.$$

Her *total income*, including the value of leisure λ_i at its market price w_i, is

$$y_i=c_i+w_i\lambda_i=w_i+t_i$$

and corresponds to the income of the previous sections. This is the equation of individual i's budget line in the space of leisure λ_i and income c_i (figure 3).

FIGURE 3

[18] This logical necessity of *prima facie* or ideal equality in the relevant items is fully explained notably in Kolm 1998, Foreword, Section 5 (see also Kolm 1996a, Chapter 2).

From the above, equal budget freedom implies that all budget lines pass through the same point. Denote as k and η the labour and the income corresponding to this point, respectively (the leisure is $1–k$). We have, for all i,

$$\eta=w_i k+t_i.$$

Hence,

$$m\eta = k\Sigma w_i+\Sigma t_i.$$

If the system is financially closed, $\Sigma t_i=0$. Then, $\eta=k\overline{w}$ where $\overline{w}=(1/m)\,\Sigma w_i$ is average wage rate, and $t_i=k\cdot(\overline{w}-w_i)$.

The t_i constitute a redistribution which amounts to redistributing equally the product of the same labour k of all individuals (with their different productivities). This is *equal labour income equalization* (ELIE). It produces *equal pay for equal work* for the equalization labour k. It also amounts to the grant of an *equal universal basic income* (of $k\overline{w}$) to everyone, *financed by an equal sacrifice* of each in terms of labour (k) – since each individual i then pays $w_i k$. Equivalently, this distribution amounts to each individual transferring to each other the proceeds of the same labour of hers (k/m) – this is *general balanced labour reciprocity*. The final outcome also amounts to all individuals *freely choosing* their labour and earnings *from an equal allocation* of both leisure $1–k$ and income $k\overline{w}$. Moreover, $y_i=w_i+t_i=k\overline{w}+(1-k)w_i$ shows that the operation is a *concentration* of the total incomes (a linear uniform concentration towards the mean) from their values $y_i=w_i$ for $k=0$; this is the structure that most uncontroversially reduces inequality.[19]

Since human capacities provide an extremely large part of the economic value of the natural resources (labour provides the largest part of national income and it also largely created capital), this ELIE solution allocates the bulk of resources. It constitutes overall distributive justice, a part of macrojustice, along with social freedom. The case $k=0$ is classical full self-ownership. The equalization labour k is a coefficient of equalization, redistribution, community of human resources, reciprocity, and a minimum income in so far as individuals are not responsible for their low income ($c_i > k\overline{w}$ if $\ell_i > k$, and $c_i = k\overline{w}$ if $w_i=0$).[20] The amounts presently redistributed are those that would correspond to an equalization labour of one to two days per week. One has $c_i=\overline{w}\,k+w_i\cdot(\ell_i-k)$ and hence, with

[19] Cf. Kolm 1999b.
[20] Approximately, $k\overline{w}$ is to average earnings as k is to average labour duration.

ℓ_i>*k* for a normal full labour, the result amounts to individuals receiving according to their *work* (*desert*) for the equalization labour *k*, and according to their *work and capacities* (*merit*) for the rest of their labour.[21]

The prices are 1 for income and *w* for leisure-labour (i.e., w_i for individual *i*). Since the common possible allocation is $k\overline{w}$ for income with *k* for labour (1!*k* for leisure), the price index can be taken as $k\overline{w} + (1-k)w = y$ (y_i for individual *i*, her total income). Its choice amounts to the choice of the equalization labour or coefficient *k*, an issue which has been analyzed in depth.[22]

The particular case $k=t_i=0$ and an absence of transfers is very important because it represents an essential historical social ethics, or ideology, and it is particularly important here because its most common justification rests on a particular assumption about the borderline between the various concepts of economic freedom. This is *classical* economic liberalism promoting free market and free enterprise and rejecting distributive public transfers. A higher wage rate w_i then permits the individual to have a higher income (consumption goods) for any given labour, or higher leisure for a given income, and hence freedoms of choice for different productivities are unequal from the inclusion of domains. Yet, this ethic classically justifies itself by freedom of action and exchange which apply basic rights and negative or social freedom, and would be equally full for all. However, there can be such freedoms with $k\neq0$ and lump-sum transfers $t_i\neq0$. A classical liberal line of defense would then be that if someone has to pay a net tax (such as a $t_i<0$) out of earned income, she is in fact forced to work to pay this liability, and forced labour is an infringement of social (negative, protective) freedom. Hence, this latter freedom would finally preclude all distributive transfers. However, the direct effect of this tax is that it reduces the freedom of choice of bundles of income (goods) and leisure (in particular in precluding $\ell_i=0$ for a lump-sum tax). Yet, a higher productivity (w_i) by itself raises this freedom of choice since it enables

[21] Kolm (2004a) provides the answers to the questions concerning : intensity of labour and effort; education and training; multidimensional labour; non-linear production functions of labour; involuntary unemployment; information about wage rates and given productivities; determination of the equalization labour and coefficient *k*; comparison with the relevant policies, proposals, and philosophies; method of practical implementation by reform of the present fiscal distributive tools; etc. See also Kolm (1993, 1996a, 1996b, and 2004b), and Maniquet (1998) whose question is a priori not freedom.

[22] Part IV of Kolm 2004a.

one to have more income with the same labour (or more leisure for the same income). Then, the tax may only provide some compensation of this inequality if it is higher for more productive individuals and subsidizes less productive ones. This means that the advantage or handicap of having a higher or lower productivity is more or less “socialized” and shared. This can only redistribute the value of the right to use one’s capacities, since this right itself has to remain to the holder of the capacity if social freedom is respected. Classical economic liberalism amounts to attributing also all this value to the holder of the capacity. That is, what it really is is full self-ownership. This entails social freedom, but the converse does not hold with the most cogent conception. These remarks constitute an essential part of economic and social ethics.

5. Non-linear frontier

There is no reason to limit oneself to cases of agents facing given unit prices. The budget sets often have other forms describing non-constant returns to scale, effects of scarcity, price rebates, and so on. We continue to consider the standard economic case of divisible quantities and free disposal. Hence, if a domain of possible choice includes $x = \{x_i\} \in \Re_+^n$ where x_i is a quantity of good *i*, it also includes all $x' = \{x'_i\} \in \Re_+^n$ where $0 \leq x'_i \leq x_i$ for all *i*. A possibility set is the closed subset $B \subset \Re_+^n$ delimited by a *n*-dimensional hypersurface $P \subset \Re_+^n$ of equation $P(x)=0$ with $\partial P/\partial x_i \geq 0$ for all *i*. That is, $B=\{x\colon\ x \in \Re_+^n$ and $[\exists x' \in P : x \square x']\}$.

The foregoing result says that, when *P* is a hyperplane, the freedom of choice provided by the possibility set *B* is ranked (and, possibly, measured) by the largest multiple of a given bundle *b* you can choose. That is, by the number β such that $\beta b \in P$. Since the exclusive focus is on the points of the ray from the origin αb for all $\alpha > 0$, a priori the same ranking (or measure) can be adopted in this case when *P* is not a hyperplane. One then has $B' \subseteq B \Rightarrow \beta' \leq \beta$, and hence $F(B') \leq F(B)$ (yet, it is possible that both $B' \subset B$ and $\beta' = \beta$, since points αb only are relevant for F).

Equal freedom of several possibility sets again mean that the frontiers *P* have a common point on the ray αb.

In the case of overall distribution in macrojustice, where equal freedom of choice is the relevant principle as noted above, the production function of individual i is the income $f_i(P_i)$ she can have with labour P_I, which may not be of the form w_iP_i. The possibility frontier is $y_i = f_i(\ell_i) + t_i$ where t_i □0 is a distributive transfer, with Σt_i=0. Individual i freely chooses her labour P_i and income y_i in this domain. Equal freedom means that the frontiers have a common point with P_i=k for all i, and therefore $y_i = \eta = f_i(k) + t_i$ for all i, whence $m\eta = \Sigma f_i(k)$ and $\eta = (1/m)\Sigma f_i(k) = \bar{f}(k)$, where m is the number of individuals. Then, the distributive transfers are $t_i = \bar{f}(k) - f_i(k)$. This is again an equal labour income equalization with the equalization labour k. This also is, again, the universal allocation of the same basic income $\bar{f}(k)$ financed by an equal sacrifice of labour k by each (i.e., a production $f_i(k)$ for individual i). The determination of k (that is, of vector b of previous paragraphs) has been analyzed in depth.[23]

This possible non-linearity of the constraint permits the consideration of constraints bearing on quantitites. This is very important for the present application since this is the case of involuntary unemployment where a constraint of the form $\ell_i \leq \ell_i^0$, where $\ell_i^0 \geq 0$ is a given constant, is the available emplyment for individual i. Involuntary unemployment for individual i is in general full if ℓ_i^0=0 and only partial if $\ell_i = \ell_i^0 > 0$. Full involuntary unemployment cannot be taken into account by the volume ranking of freedom, because it gives a zero volume and hence a freedom inferior to all other cases, even if disposable income y_i is high because of large transfers. Involuntary unemployment can be taken into account in replacing an impossoibility to work more (or at all) by an impossibility to earn more in working more (or to earn in working). That is, function $f_i(\ell_i)$ is "truncated" in being replaced by $f_i(\ell_i^0)$ for all $\ell_i > \ell_i^0$. The result is that $y_i = \bar{f}(k)$ if individual i is involuntarily unemployed and $\ell_i^0 \leq k$ (hence $\ell_i = \ell_i^0 \leq k$), and in particular if she is fully involuntarily unemployed ($\ell_i = \ell_i^0 = 0$).

Finally,all the foregoing extends to the case where labour P_i, and hence the equalization labour k, are multidimensional (duration, education and training, intensity, etc.), with the same formulation.

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[23] See Kolm 2004a, Part IV.

6. Conclusion

The considerations of this note belong to the issue of characterizing freedom independently of the preferences of the actor, as Xu emphasizes.[24] This is one topic in freedom studies, but not the only one. In other cases, considering preferences is essential. This is for instance the case for potential freedom where the allocations are compared by the inclusion comparison of possible domains of choice that can lead to choosing them.[25] More generally, freedom is relative to a set of constraints and possibilities, and it may be relevant, or not, to include personal characteristics in them, notably properties of capacities to enjoy or be satisfied (eudemonistic capacities) determined by tastes and influencing preferences or represented by them. This depends on what one wants to do with the concept – note that Sections 4 and 5 do not consider such capacities but consider productive personal capacities (the wage rate). One can even consider the "freedom to obtain a given utility level" where the freedom functions (specific to each individual) are the indirect (Roy) utility functions for linear (constant-price) budget constraints. A similar representation holds for a production function in the space of obtainable inputs for the "freedom to obtain a given output." More generally, there even is a case for identifying freedom with happiness since increased freedom that enables you to choose alternatives that you enjoy more can make you happier, whereas unhappiness is undesired and hence is a constraint. This equivalence (in the form of an equivalence between liberation and the diminishing of insatisfaction or suffering) is a basis of the philosophical psychology of advanced buddhism.[26] Relatedly, mental freedom implies more or less choosing one's principles of action, desires – and hence preferences –, emotions, etc. As a general rule, the relevant specification of the concept of freedom depends on what one wants to do with it. In particular, discarding preferences has to be justified for each problem under consideration. We have noted that this is the case for an issue of major importance, since this discarding is a unanimous opinion for overall distributive justice in macrojustice (for instance

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[24] The next general step consists of characterizing fredom of choice when the actions chosen by various individuals can interfere, that is, the domain of choice of one agent can depend on the actions chosen by others – a "game-theoretic" situation. Normative considerations notably demand to define equal freedom in this case. The basic concept turns out to be symmetrical reciprocity, that is: "if I can choose *a* when you choose *b*, then you can choose *a* when I choose *b*", equivalent to the symmetry of the social possibility set in the space of the actions of all agents (and symmetrical game forms). (See Kolm 1993).

[25] Kolm 1999a.

[26] See Kolm 1982.

for the income tax): *de facto*, everybody thinks that direct reference to a kind of "utility" is irrelevant for this topic. This unanimous view should probably be endorsed, and at any rate it will be implemented since everybody shares this opinion. Yet, various aspects of preferences are relevant for more specific issues, and this can be associated with aspects of freedom.

7. Appendix. Extension to related concepts of budget freedom

The foregoing concepts extend in various ways.

1) Potential budget freedom

Let x_j^i the quantity of good *j* held by individual *i*, $x^i = \{x_j^i\}_j \in \Re_+^n$ individual *i*'s allocation, and $X = \{x^i\}$ the total allocation. We want to judge *X*, and the following ethics is a possible one which is sometimes relevant. The direct value of justice is budget freedom, yet the individuals' choices can be potential only. Let $u^i(x^i)$ be an ordinal quasi-concave utility function of individual *i*, non-decreasing in x_j^i for all *j*. For a given x^i, an implicit budget hyperplane $H^i(x^i)$ is an hyperplane tangent to the hypersurface $u^i(\xi^i)= u^i(x^i)$ where the $\xi^i = \{\xi_j^i\} \in \Re_+^n$ are individual *i*'s allocations, and an implicit budget set $B^i(x^i)$ is the set of ξ^i such that there exists a $\xi^i{}' \in H(x^i)$ such that $\xi_j^i{}' \geq \xi_j^i$ for all *j*, and $\xi_j^i \geq 0$ for all *j*. Hence, x^i is one of the possible choices of individual *i* if she were choosing in the budget set $B^i(x^i)$.

The freedom provided by the domain of choice $B^i(x^i)$ is the *implicit or potential budget freedom of individual i endowed with allocation* x^i. It is sometimes relevant to judge the total allocation *X* in taking potential freedom as the direct value of justice.

Then, with the foregoing concepts and a linear price index of coefficients b_j whose vector is $b = \{b_j\}$, the intersection of hyperplane $H^i(x^i)$ with the ray from the origin in the direction of *b* is $\beta^i(x^i) \cdot b$. The numbers $\beta^i(x^i)$ provide an ordinal ranking of potential budget freedoms, both across individuals *i* and across states *x*. This defines potential budget freedoms that are equal or larger, and permits comparing total allocations by relations of dominance, fundamental dominance, truncations, leximin, etc. (see Kolm 1971, Part 3, and 2004a, Chapter 23). If one accepts to measure freedom by real income, the β^i are these measures, and all the comparisons of income distributions or inequalities can be used. However, these comparisons or measures of freedom depend on the chosen price index coefficients b, and one can have $u^i(x^{i\prime}) > u^i(x^i)$ with $\beta^i(x^{i\prime}) < \beta^i(x^i)$ (yet, individual i is then accountable for her preferences and hence for her utility function).

2) More general freedom functions and indexes, characteristic envelops

With an ordinal freedom function $F(D)$, the domains D that provide equal freedom constitute a family. With a D defined in the space of bundles of quantities of goods j, $x = \{x_j\}$, they have an envelop. With linear budget freedom (facing constant prices p) with freedom function $F(y,p)$, the points of this envelop turn out to be with $x_j = !F_j/F_y$ if F is differentiable with $F_y = \mathrm{M}F/\mathrm{M}y$ and $F_j = \mathrm{M}F/\mathrm{M}p_j$. The envelops corresponding each to a level of freedom have no common point, with more freedom towards higher quantities, and form the lower boundary of convex sets (they are representable by a quasi-concave function $J(x;F)=0$ for each level of F, which can be written as a non-decreasing quasi-concave function $E(x)=F(y,p)$, with grad$E=p/y$).

With a general price index $\pi(p)$, and hence budget freedoms ordered as the real income $\eta = y/\pi(p)$, the point on the envelop for prices p has coordinates $x_j = \eta\pi_j$ where $\pi_j = \mathrm{M}\pi/\mathrm{M}p_j$, assumed to exist, and hence is $x = \eta$ grad π. A resulting property is that, at this point, the share of the value of the good j in income is $p_j x_j / y = p_j \pi_j / \pi$, that is, the elasticity of the price index for price p_j.

For a linear price index $\pi = \Sigma b_i p_i$, $\pi_j = b_j$ and the envelop reduces to the common point ηb of the budget hyperplanes of equal-freedom budget sets. The share of good j in income at this allocation then is the share of its price in the price index, $p_j x_j / y = b_j p_j / \Sigma b_i p_i$.

With a geometric mean as price index, that is, a volume ranking of budget freedom, there comes $p_j x_j = y/n$ where n is the number of goods: at each point of the envelop, the value of all goods have an equal share of the budget (for $n=2$, the equal-freedom envelops are rectangular hyperbolas).

Both the linear index and the volume ranking are particular cases of a price index of the form $\pi = \left[\Sigma(b_i p_i)^\alpha\right]^{1/\alpha}$ when, respectively, α=1 and α 6 4. Then,

$$\pi_j = b_j^\alpha p_j^{\alpha-1}\pi^{1-\alpha} = b_j^\alpha(\pi / p_j)^{1-\alpha} = (p_j b_j / \pi)^\alpha (\pi / p_j),$$

and the share of the value of the good *j* in income on the envelop is

$$p_j x_j / y = (p_j b_j / \pi)^\alpha = (p_j b_j)^\alpha / \Sigma(p_i b_i)^\alpha .$$

Another extension of the volume takes $\pi(p) = \Pi p_i^{\gamma_i}$ with $\gamma_i > 0$ for all *i* and $\Sigma\gamma_i = 1$, and budget freedoms are ranked as $\Pi a_i^{\gamma_i}$. The share of good *j* in income on the envelop is $p_j x_j / y = \gamma_j$. The volume case takes $\gamma_j = 1/n$.

However, as far as tangible (economic) meaning is concerned – in contrast with mathematical meaning –, it seems that linear indices only can really make sense.

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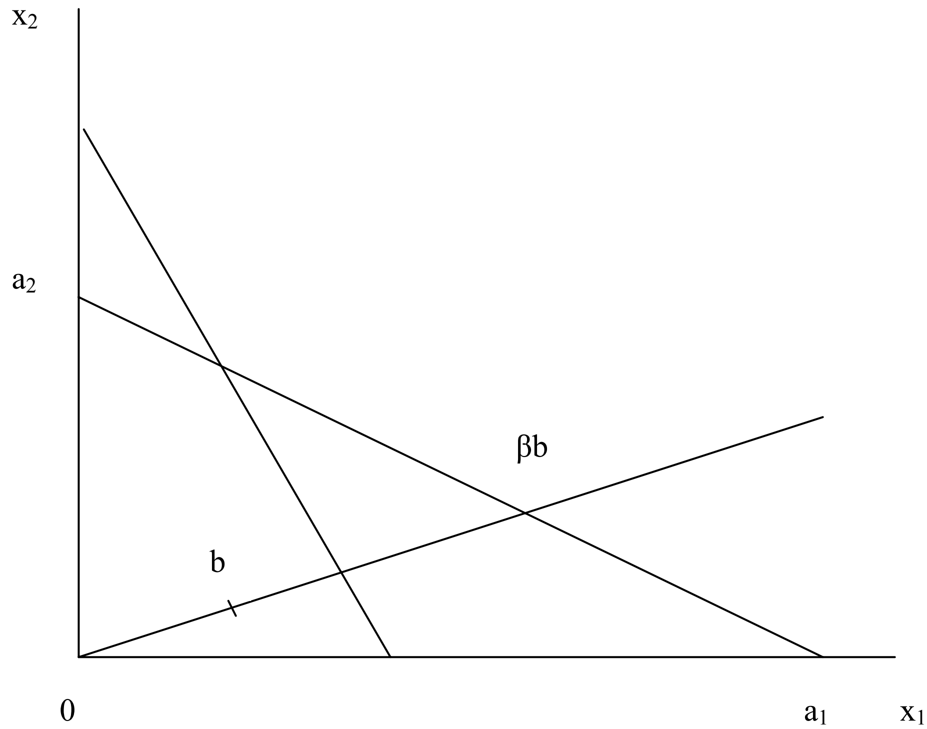

Figure 1

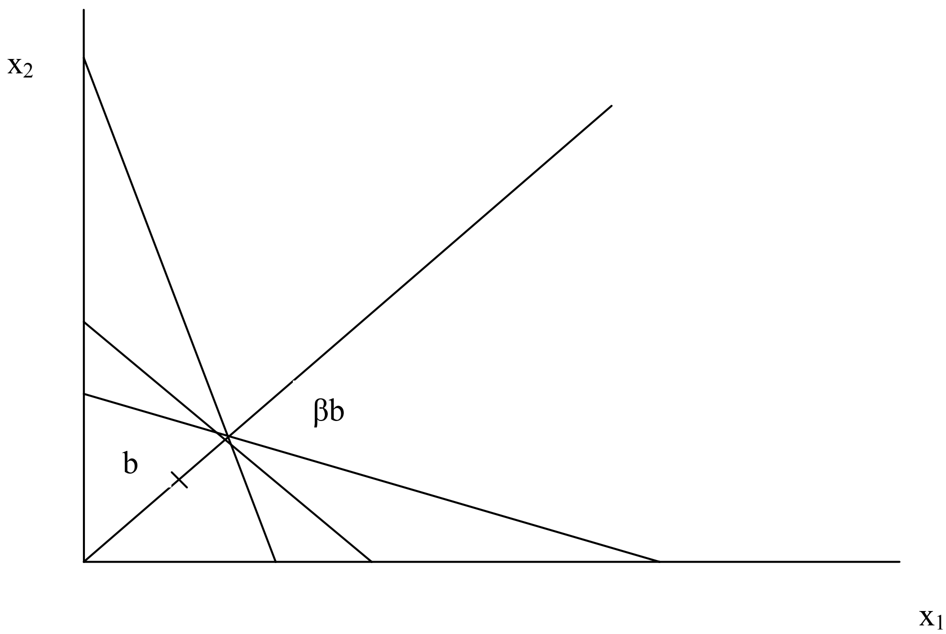

Figure 2

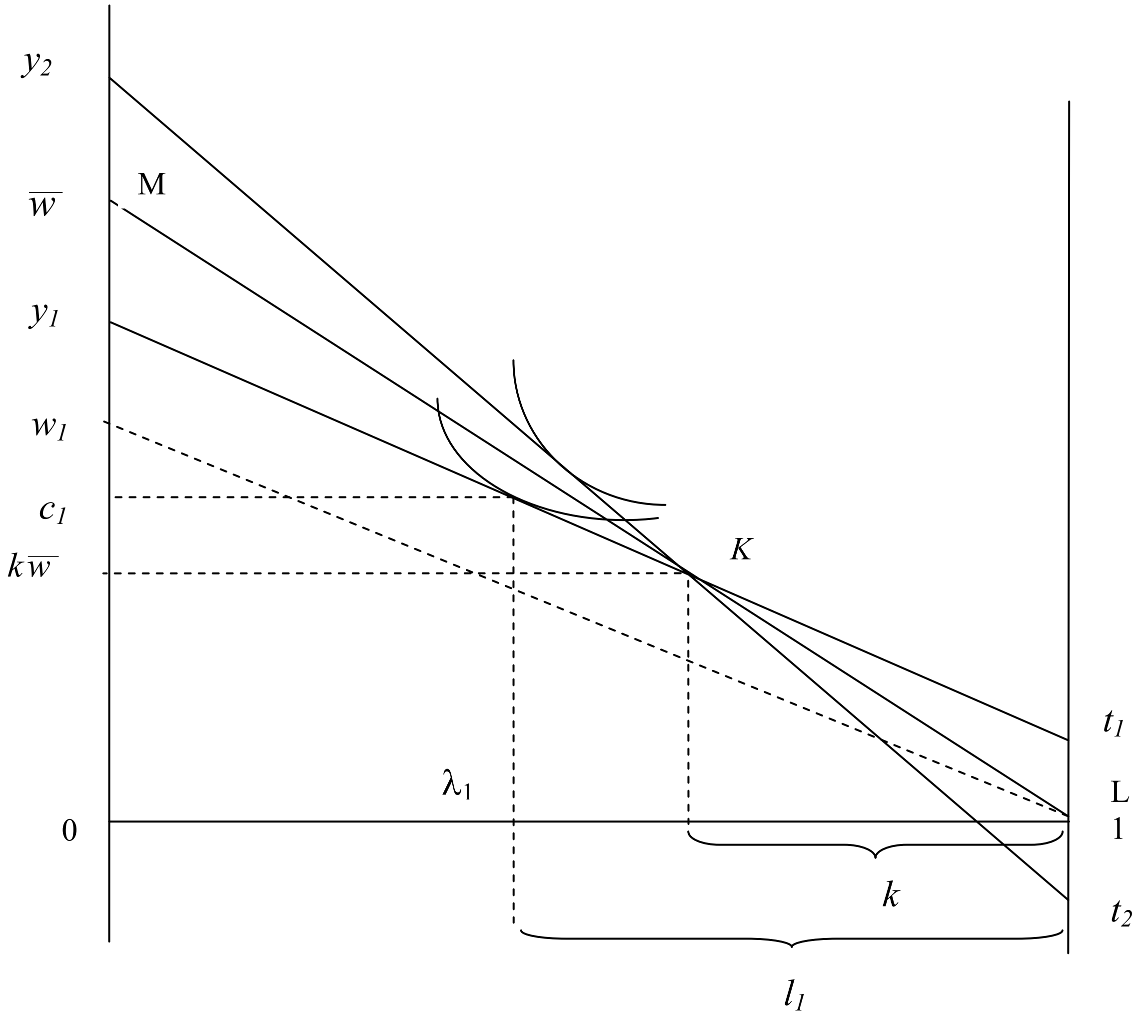

Figure 3