Financing Experimentation*

Mikhail Drugov[†] and Rocco Macchiavello[‡]

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Abstract

When people start a new activity, they may not know how good they will be at it and, therefore, must experiment in order to learn. What happens when the experimentation is financed by a lender and there is an agency problem between the lender and the borrower? In a standard setting, experimentation is more profitable for a longer time horizon or a lower payoff of the known activity. In contrast, financing experimentation might be harder in these cases, that is, precisely when it is more valuable. The optimal contract resembles typical microfinance schemes observed in practice.

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[†] Oxford University. E-mail: mikhail.drugov@nuffield.ox.ac.uk.

[‡] Oxford University and CEPR. E-mail: rocco.macchiavello@nuffield.ox.ac.uk.

"Each of us has much more hidden inside us than we have had a chance to explore. Unless we create an environment that enables us to discover the limits of our potential, we will never know what we have inside of us."

Muhammad Yunus, Founder of Grameen Bank

1 Introduction

When people start a new activity, they might not know how profitable it is, or how good they will be doing it. They can only learn by trying it out. In other words, people must experiment to learn about the activity or about themselves. An important case of such a situation is a person starting a business. This may be a poor woman in a slum in India trying to open a small shop, or an IT-entrepreneur in Silicon Valley hoping to found the next Google. In any case, if initial capital has to be borrowed, the lender – be it a microfinance institution in India or a venture capitalist in the US – finances the experimentation.

What happens when the experimentation is financed by a lender? The lender should take into the account that the borrower might misbehave, for example, by shirking or by diverting the loan; also, the borrower might (privately) acquire some information relevant to the continuation of the project. In order to study such a setting, this paper builds a simple model that embeds a two-period experimentation problem into a lending relationship.

A standard experimentation setting arises, broadly speaking, when certain activities undertaken today generate valuable information that can be used in future decision making.[1] In its simplest form, standard experimentation involves, in at least two periods, a choice between one activity with known returns (the so-called known arm), and another activity with initially unknown returns (the so-called unknown arm). Experimentation is then a particular form of investment: it involves a trade-off between short term costs of generating information and long term benefits of using it. Therefore, the longer the period during which the information to be learned can be used for the decision making, the more the decision maker finds it attractive to experiment.

The paper studies a two-period model in which in each period an agent can start a

[1] See Dirk Bergemann & Juuso Välimäki (2008) for a survey.

project. Initially, both the agent and the lender are uninformed about the effort costs needed to complete the project. Upon starting the project, the agent learns her effort costs. While it is optimal to complete the project regardless of the agent's effort costs since the investment is already sunk, the agent might decide not to exert the effort and to divert the capital for private benefit. In the second period the agent can obtain another loan, depending on the first-period outcome and her communication with the lender.

In contrast to the standard experimentation setting, obtaining credit to finance the experimentation might become harder as the horizon becomes longer, i.e., when the experimentation is more valuable. By experimenting, the borrower privately learns about herself and, therefore, in addition to the standard moral hazard problem associated with borrowing, there is an adverse selection dimension which emerges after the loan has been disbursed. If the horizon is long, the selection of the right type of borrower becomes difficult since entrepreneurs have high incentives to fake short-term performance in order to enjoy moral hazard rents in the future. For the same reason, a lower payoff of the known arm, i.e., the outside option, makes financing experimentation more difficult. In other words, the future rents which are helpful in solving the moral hazard problem (see, e.g., William P. Rogerson (1985) and Patrick Bolton & David S. Scharfstein (1990)) come at the cost of rendering the adverse selection problem more severe.

An important application of our setting is microfinance, that is, the provision of small uncollateralized loans to poor borrowers in developing countries. We show that the optimal contract in our model is similar to the loan contracts typically offered. In particular, the model predicts that retained earnings from the beginning of the relationship can be used to select the right entrepreneur at a later stage. Retained earnings are used to endogenously build up collateral through compulsory saving requirements (CSRs), a pervasive but understudied feature of microfinance schemes (see, e.g., Jonathan Morduch (1999)).[2]

Finally, the paper presents an extension of the model in which the size of the projects in the two periods is endogenous. The growth path can be either steeper or flatter than the first best growth path. In the former case, the initial project is smaller than optimal, but later the project size catches up with the optimal one. In the latter case, which

[2] Most of the theoretical work on microfinance has focused on joint liability, a far less common contractual element of those schemes (see surveys in Maitreesh Ghatak & Timothy Guinnane (1999) and Dean Karlan & Jonathan Morduch (forthcoming)).

happens when the horizon is long, the size of the initial project is distorted upward in order to increase retained earnings needed to select the entrepreneur in the second period, when the project size is distorted downwards.

This paper belongs to a growing literature that combines experimentation and agency problems. Early examples are Dirk Bergemann & Ulrich Hege (1998) and Dirk Bergemann & Ulrich Hege (2005), where the agent can either explore an innovative project or shirk, in which case the project outcome (failure) is not informative. Gustavo Manso (2009) studies a richer framework in which the agent can also exploit a known activity and shows that motivating experimentation requires dramatically different incentives from standard pay-for-performance schemes. These papers focus on how experimentation affects agency problems, while our paper highlights how agency problems change the nature of experimentation. The two approaches are thus complementary.

In Jacques Crémer & Fahad Khalil (1992) and Jacques Crémer, Fahad Khalil & Jean-Charles Rochet (1998), the agent may become informed at a cost, and the principal adjusts the contract to provide the agent with optimal incentives for information acquisition. In Steven D. Levitt & Christopher M. Snyder (1997) and Roman Inderst & Holger Mueller (2009) the agent's effort affects not only the success probability given the state of the world but also the state of the world and this interacts with the eliciting of the signal from the agent.[3] These papers, however, are essentially static and do not consider the intertemporal trade-offs involved.

The rest of the paper is organized as follows. Section 2 presents the model and derives the main results of the paper. Section 3 studies the optimal contract and interprets microfinance contracts in the light of the model. Section 4 analyzes the extension with endogenous project size and discusses some assumptions of the model. Section 5 concludes. All the proofs are in the Appendix.

[3] Other models mixing moral hazard and adverse selection are discussed in Jean-Jacques Laffont & David Martimort (2002) (ch. 7) and Patrick Bolton & Mathias Dewatripont (2005) (ch. 6).

2 The Model

2.1 Setup

There is an agent that lives for two periods, $\tau = 1, 2$. In each period the agent has the opportunity to undertake a project that needs an initial capital investment of 1 and yields return r when completed. A project that is not completed fails and yields 0.

The agent has no assets and no access to a saving technology, so she needs to borrow 1 unit of capital in order to start the project. She is protected by limited liability. The agent and lenders have a common discount factor $\delta \in [0, \infty)$ across the two periods. The contracts are described in Section 2.3.

To complete the project the agent needs to appropriately invest the unit of capital and to exert effort. The agent can divert a share $\psi \leq 1$ of the initial investment for private consumption. If she does so, the project fails. The parameter ψ reflects the difficulty for the lender of monitoring the investment.

There are two types of agent, good G and bad B, which remain constant over the two periods. The cost of effort for the good agent is $e_G = 0$, and $e_B = e > 0$ for the bad agent.[4] Initially, both the agent and the lenders are uninformed about the type of agent and have a common prior ρ about the probability of the agent being the good type. The agent privately learns her type upon starting the project in period 1 but does not if she doesn't start the project.[5] After having learned her type, she decides whether to exert effort and whether to divert the capital. The complete description of the timing of events is postponed until Section 2.3.

Whenever effort is exerted and investment is not diverted, the project succeeds and yields r, which is observable and verifiable. In any period in which the agent does not start the project, she takes an outside option $u > 0$.

We make the following parametric assumptions:

Assumption 1 $r - 1 < u + e$.

Assumption 2 $u < \psi$.

Assumption 3 $\max\{1, e\} < r - \psi$.

[4] The model can be also interpreted with the effort cost being a characteristic of the project, rather than of the agent.

[5] In Section 4.3 we discuss the case where the agent obtains a signal about her type.

The first assumption implies that it is not optimal to invest if the agent is (known to be) bad: the opportunity costs of investment $1 + u$ are higher than revenues r net of effort costs e.

The second assumption implies that the agent always prefers to start the project with borrowed money rather than take her outside option u.

Finally, the third assumption has two implications. First, $r - 1 > \psi$ implies that the project generates enough revenues to solve the moral hazard problem of the good type. Second, $r > \psi + e$ implies that, once the project is started and the initial outlay of 1 unit of capital is sunk, it is optimal to complete the project regardless of the agent's type.[6]

2.2 Optimal Experimentation by a Self-Financed Agent

Let us first consider the benchmark case in which the agent has enough wealth so that she does not need to borrow. In this case the agent is the residual claimant of the project: there are no incentive problems and, therefore, the first-best allocation is chosen.

Once she has started the project in period 1, the agent exerts effort and completes the project regardless of her type (Assumption 3). In period 2, she invests and completes the project again if she has learned that she is of the good type, since $r - 1 > u$. If she has learned that she is of the bad type she prefers to take her outside option (Assumption 1). This is the first-best allocation.

Investment in period 1 can be thought of as experimentation: its costs are borne in period 1 while the benefits are realized in period 2. After the agent has learned her type, she will be able to make an informed decision. The costs of experimentation are given by the difference between the opportunity cost u and the expected surplus created by the project in period 1, i.e., $r - 1 - (1 - \rho)e$. The benefits of experimentation are due to better decision-making in period 2. With probability ρ, the information gathered through experimentation leads the agent to start a project, instead of taking the outside option. With probability $1 - \rho$, instead, the agent learns she is of the bad type and takes her outside option. In this case, the information gathered through experimentation does not change her decision. The value of information therefore equals $\delta\rho(r - 1 - u)$. Experimentation is optimal if its costs are lower than its benefits.

[6] Note that if both types complete the project in period 1 the project's outcome is uninformative about the type of the agent. The alternative scenario in which it is optimal to let the bad type fail in period 1 is discussed in Section 4.3.

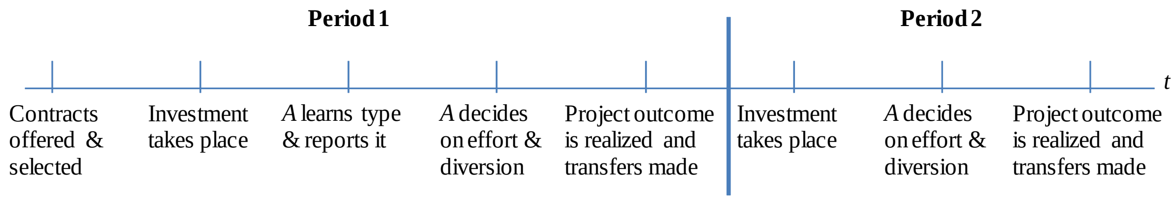

Figure 1: Timing of events

Lemma 1 *If the agent does not need to borrow, experimentation (investment in period* 1*) is optimal if and only if* $\delta \geq \delta_E$*, where*

$$\delta_E \equiv \frac{u + e(1-\rho) - (r-1)}{\rho(r-1-u)}. \tag{1}$$

As in standard experimentation models, starting the project in period 1 becomes profitable if the future is sufficiently important, i.e., if the discount factor is high enough. Lemma 1 also shows that the agent starts the project in period 1 if she is sufficiently confident about being of the good type (high ρ), if the opportunity costs are not too high (low u) and if the project yields high returns (high $r-1$).

2.3 Contracts and Timing of Events

We now describe contracts and the structure of the credit market. Lenders compete in the market and make zero profits in expectation. They have full commitment power and offer two-period contracts. The project is financed in period 1. We assume that the agent cannot change her lender in period 2 but she can take her outside option u.[7]

The timing of events is the following. Immediately after the agent learns her type, she sends a message $m \in \{G, B\}$ to the lender.[8] According to the message, the contract specifies the agent's actions in period 1, a transfer conditional on the project outcome in period 1 and a re-financing policy in period 2. The contract also specifies a transfer in period 2 conditional on project outcomes in periods 1 and 2. The timing of events is summarized in Figure 1.

[7] Other contractual assumptions are discussed in Section 4.2.

[8] Since lenders have commitment power and contracts are exclusive, the Revelation Principle applies and we can focus on direct revelation mechanisms.

2.4 Implementing the First Best

This section studies *when* the first-best allocation can be financed, while the next section shows *how* a particular simple contract implements it. An allocation can be financed if there exists a contract that gives appropriate incentives to the agent and that satisfies the lender's zero-profit constraint.

Conditional on starting the project in period 1, the first-best allocation generates total revenues generated by the projects and plus the outside option taken by the bad type in period 2, i.e.,

$$R = r - 1 + \delta\rho(r - 1) + \delta(1 - \rho)u. \tag{2}$$

Competition among lenders ensures that the agent receives R in expectation over the two periods.

The main constraints of the model are the two truth-telling constraints, that is, the agent's incentives to reveal her type truthfully. In contrast to most dynamic agency models with commitment, both the bad and the good type might have incentives to misreport their type.

Consider first the truth-telling constraint of the good type. In period 1, the project should be completed independently of the type of agent since, at that stage, the initial outlay of 1 unit of capital is sunk (Assumption 3). It is necessary to prevent both types from stealing ψ and to compensate the bad agent for her effort costs e. This, however, gives an incentive to the good type to pretend to be the bad type. In period 2, the good type must also be compensated for not taking the outside option. Therefore, the first-best allocation can be financed only if

$$R \geq \psi + e + \delta u. \tag{3}$$

This expression can be rewritten as

$$\delta \geq \underline{\delta}^{FB} \equiv \frac{\psi + e - (r - 1)}{\rho\,(r - 1 - u)}. \tag{4}$$

Threshold $\underline{\delta}^{FB}$ is higher than the one of the self-financing agent, δ_E. The incentive problems create rents that render the experimentation more expensive. The logic for when experimentation is optimal is the same as in (1), the difference being that the cost of experimentation is higher by an amount equal to $\psi - u + \rho e$. This is because the contract needs to pay rent $\psi + e$ to both types, which is in excess of the expected opportunity costs of a self-financed agent, $u + (1-\rho)e$. While this rent increases the costs of experimentation, it does not change the nature of the problem.

Turn now to the truth-telling constraint of the bad type. By Assumption 1, it is not optimal to lend to the bad type in period 2. The contract then has to induce the bad type to take the outside option in period 2. The good type, however, should complete the project in period 2 for which rent ψ is necessary. The bad type has incentives to seek financing in period 2 in order to obtain this rent since it is higher than the outside option u. The first best can be financed only if

$$R \geq \delta\psi. \tag{5}$$

This expression can be rewritten as

$$\delta \leq \overline{\delta}^{FB} \equiv \frac{r-1}{(1-\rho)(\psi-u) - \rho(r-1-\psi)}. \tag{6}$$

The logic of (6) is the following. In period 2, the lender needs to pay $\psi - u$ to prevent the bad type from obtaining a project. The lender faces a deficit of $(1-\rho)(\psi-u) - \rho(r-1-\psi)$ which has to be financed by the first-period profits $\frac{r-1}{\delta}$ (in the second-period terms).[9] A higher δ, therefore, reduces the profits available to pay the second period rent and, therefore, makes it harder to finance experimentation. This is in stark contrast both to the implication of the truth-telling constraint of the good type (4) and to the case of a self-financing agent (1).

Combining (3) and (5), it follows that a necessary condition for the first best allocation to be financed is

$$R \geq \max\{\psi + e + \delta u, \delta\psi\}. \tag{7}$$

This implies that (4) is the relevant constraint if $\delta \leq \delta_\psi = \frac{\psi+e}{\psi-u}$ while (6) is the relevant constraint if $\delta > \delta_\psi$. The threshold δ_ψ has an intuitive explanation: it is equal to the ratio of the first-period rent of the good type $\psi + e$ to the second-period rent of the bad type $\psi - u$.

[9] If, however, $\rho(r-1-\psi) > (1-\rho)(\psi-u)$, the project in period 2 generates enough surplus to separate the two types and (6) is irrelevant.

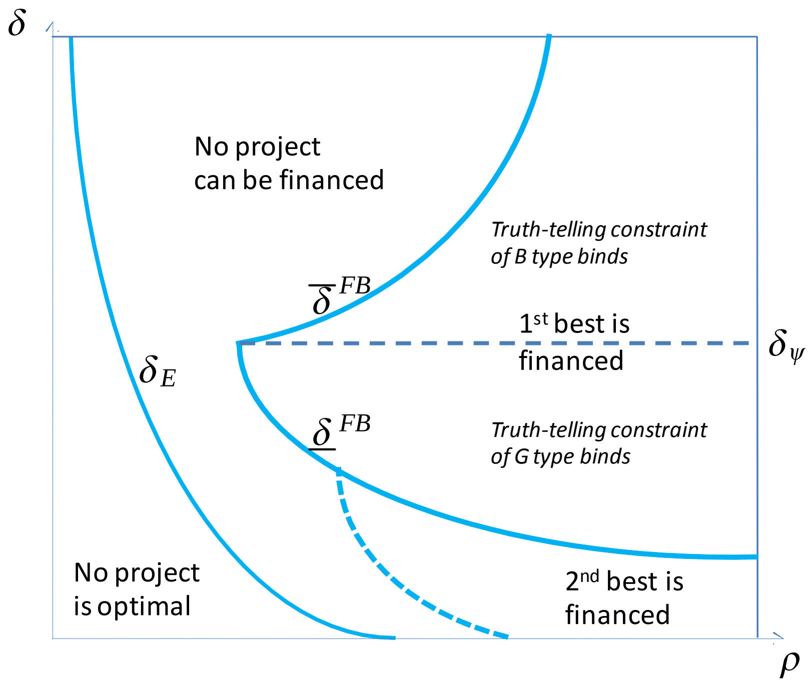

Figure 2: Characterization of implementable allocations

It turns out that the condition (7) is not only necessary but also sufficient for the first best to be financed. This leads to the following proposition.

Proposition 1 *The first-best allocation can be financed if and only if* $\underline{\delta}^{FB} \leq \delta \leq \overline{\delta}^{FB}$.

Proof. See the Appendix. ■

Figure 2 illustrates Proposition 1. In a standard experimentation model, the profits earned in period 2 are used to finance the experimentation costs of period 1. When the agent has to borrow, a dynamic moral hazard problem arises. When δ is low, the main source of the agent's rent comes from period 1 and, therefore, the lender uses second-period profits to pay this rent. A higher δ increases the value of period 2 profits and makes contracting easier, as in standard dynamic moral hazard models.

These future rents, however, come at a cost. When δ is sufficiently high the truth-telling constraint of the bad type is binding since the second-period rents are very attractive. The bad agent is then tempted to "take the money and run". Then, a high δ makes contracting harder. A useful analogy is dynamic adverse selection models (see, e.g., Jean-Jacques Laffont & Jean Tirole (1987) and Jean-Jacques Laffont & Jean Tirole (1988)) where the ratchet effect, brought about by the absence of commitment, makes the principal pay a high rent to the good type in period 1 which then attracts the bad type. In our case, the source of the rent is the possibility of diverting the investment in period 2.

In other words, while it is generally perceived that future rents associated with a project are helpful to solve moral hazard, this paper shows that under initial uncertainty about these rents, they might attract undesirable borrowers and, therefore, lower the ex ante borrowing capacity. Interestingly, these rents are increasing in the net present value of the project, implying that more profitable projects might be harder to finance.

This logic is well illustrated by the comparative statics with respect to the discount factor δ and the outside option u. If $\delta < \delta_\psi$, a higher δ expands the interval of ρ for which the project can be financed. If $\delta > \delta_\psi$, a higher δ shrinks this interval. Similarly, the comparative statics with respect to the outside option u is non-monotonic. When $\delta < \delta_\psi$, a higher u reduces the costs of being denied access to credit in period 2 which makes lending more difficult, see (4). This shrinks the region in Figure 2 where financing the project is possible. When $\delta > \delta_\psi$, a higher u reduces the rent needed to keep the bad type out in period 2. This expands the region where financing the project is possible. In contrast to the case of a self-financed agent (1), a higher outside option makes lending easier, see (6).[10]

Finally, when e is too high or δ is too low, it may not be possible to finance the first-best allocation. Then, it might be optimal to let the bad type fail in period 1 and finance the project in period 2, conditional on the successful completion of the project in period 1. In other words, the lender shuts down the bad type in period 1. This is the second-best allocation.

The contract then only needs to solve the moral hazard problem of the good type. It can be shown that, for δ sufficiently low, the second-best allocation can be financed if

$$\delta \geq \underline{\delta}^{SB} \equiv \frac{1 - \rho(r - \psi)}{\rho\,(r - 1 - u)}. \tag{8}$$

The similarity between (3) and (8) illustrates the choice of the financed allocation. In both allocations, the lender uses second-period profits earned on the good type to cover the first-period loss. He then chooses the allocation that minimizes this loss. In the

[10] The remaining comparative statics has the expected sign. It is easier to finance the project in period 1 if the project is more profitable (higher $r - 1$) and monitoring is better (lower ψ). The cost of effort e affects only the rent of the good type in period 1 and, therefore, it matters only if the truth-telling constraint of the good type is binding, that is, when δ is low (it also makes this case more likely).

second-best allocation the lender economizes on rent e paid to the two types but looses the revenues $r-\psi$ from the bad type. The resulting comparative statics follows the standard logic: a higher δ and lower u expand the region in which the project can be financed.

3 Indirect Mechanism

The previous section showed when it is possible to finance the first-best allocation. Because of the linear structure of the model, however, the structure of payments in the optimal contract is not uniquely determined. Also, it is important to know whether there are realistic contracts that replicate the direct mechanism and implement the first best. This section answers both questions.

3.1 Structure of the Optimal Contract

Consider the case in which the agent has a utility function which is concave in consumption, and separable in effort and consumption, i.e., her utility can be written as $U(c)-e$, with $U'(\cdot)>0$ and $U''(\cdot)<0$. To pin down the structure of payments induced by the contract, we examine the limit when $U(\cdot)$ converges pointwise to a linear function.[11]

The optimal contract then minimizes the spread of consumption across the two types and across the two periods for each type, subject to the incentive compatibility constraints needed for the implementation of the first best. Denote by c_τ^i, $i=G,B$, $\tau=1,2$, the consumption of type i in period τ. The next Proposition describes the consumption profile induced by the optimal contract.

Proposition 2 *The optimal contract achieves*

i) perfect consumption smoothing across types, $c_1^B+\delta c_2^B=c_1^G+\delta c_2^G=R$*;*

ii) perfect consumption smoothing across periods for the bad type, $c_1^B=c_2^B=\frac{R}{1+\delta}>u$*;*

iii) perfect consumption smoothing for the good type $c_1^G=c_2^G=\frac{R}{1+\delta}$ *if and only if* $R\geq(1+\delta)\psi$*. Otherwise, the optimal contract involves* $c_1^G<c_2^G=\psi$*.*

Proof. See Appendix. ■

[11] The limit is taken for expositional purposes only; all the insights are robust to larger degrees of risk aversion.

Several aspects of the optimal contract are noteworthy. First, the optimal contract provides full consumption insurance to the borrower against "bad" realizations of her entrepreneurial talent.

Second, the contract provides perfect consumption smoothing across the two periods for the bad type. Incentives to the bad type have to be given only in period 1, and this can be achieved by promising enough rent over the whole relationship. The timing of the payments does not matter for incentive reasons, and consumption smoothing can always be achieved at no cost. Furthermore, the fact that the project can be financed implies that $R > (1 + \delta)u$, i.e., in each period the bad type consumes more than her outside option u.

Third, the contract might fail to achieve perfect consumption smoothing for the good type. Indeed, since the good type has to obtain at least ψ in period 2 to complete the project, perfect consumption smoothing is possible only if ψ can also be paid in period 1, i.e., if $R \geq (1+\delta)\psi$. This inequality is harder to satisfy for higher δ. This implies that the good type achieves perfect consumption smoothing if δ is low enough. If δ is high enough, the good type never achieves perfect consumption smoothing unless ρ is sufficiently high. A higher ρ increases R and, therefore, always leads to a higher consumption for the good type in period 1. A higher δ, while also increasing R, makes it more costly to satisfy $c_2^G \geq \psi$ and, thus, lowers c_1^G if perfect consumption smoothing cannot be achieved.

3.2 An Optimal Contract: Application to Microfinance

Is there an indirect mechanism that implements the optimal allocation and consumption levels and resembles a real world contract? This section describes a such a contract and relates it to observed contractual practices in microfinance.

The contract considered is a quadruple $\mathcal{C} \equiv \{d_1, d_2, s^G, i\}$ defined as follows. The agent borrows one unit of capital at the beginning of period 1. The loan is repaid in the amount d_1 at the end of period 1. In period 2, the borrower can apply for further funding. The contract specifies that she obtains further funding if she has a saving balance equal to at least s^G, to be pledged as collateral. If the borrower seeks and obtains funding in period 2, she borrows one unit of capital and is expected to repay d_2 on that loan. If she defaults, she loses her savings. Finally, i is the interest rate paid by the lender on the savings account.

As noted above, in period 2 the bad type consumes more than the income she derives from taking the outside option, $c_2^B = \frac{R}{1+\delta} > u$. It is therefore natural to interpret the consumption in excess of income in period 2 as a positive saving balance, s^B. Note that, in contrast to s^G, s^B is not part of the contract with the lender.

In sum, the agent borrows one unit of capital at the beginning of period 1 and learns her type. The optimal contract always induces investment (and no default). If the agent is of the good type, she saves s^G and obtains further funding in period 2. If, instead, she learns she is of the bad type, she saves s^B and takes her outside option in period 2. First-period consumptions are then defined by

$$c_1^G = r - d_1 - s^G \text{ and } c_1^B = r - d_1 - s^B$$

for the good and bad type, respectively. Similarly, second-period consumptions are defined by

$$c_2^G = r - d_2 + (1+i)s^G \text{ and } c_2^B = (1+i)s^B + u.$$

There is no loss of generality in setting the interest rate on savings $(1+i)$ equal to $\frac{1}{\delta}$. The remaining terms of the contract can be computed substituting for the consumption values derived in Proposition 2.

This contract displays several interesting features. First, since c_1^B and c_2^B are both increasing in ρ, the savings of the bad type s^B increase with ρ and, therefore, the repayment d_1 must decrease with ρ. Better clients consume (and save) more and repay less for the first loan, since there is a higher surplus produced in period 2.

Second, when it is possible to achieve perfect consumption smoothing for the good type, $c_1^G = c_1^B$ implies $s^G = s^B$. In other words, the requirement to have a minimum amount of savings in order to obtain the loan in period 2 does not bite. Both types save at the qualifying level s^G, and the bad type could obtain a loan in period 2 but prefers to take the outside option.

When perfect consumption smoothing is not possible for the good type, $c_1^G < c_1^B$ implies $s^G > s^B$. The optimal lending contract, therefore, requires the borrower to save a larger amount in order to continue borrowing in period 2. The contract uses retained earnings to screen out the bad type by endogenously building up collateral through compulsory saving requirements (CSRs). Saving requirements are compulsory in the sense that the continuing borrower, if given an income of equivalent net present value, would prefer to save a smaller amount and smooth consumption.

An important example of a loan contract with compulsory saving requirements is found in microfinance, broadly defined as the provision of small uncollateralized loans to poor borrowers in developing countries.

CSRs are an extremely common feature of microcredit schemes. For instance, the three largest microfinance institutions in Bangladesh (Grameen Bank, BRAC and ASA) have been collecting compulsory regular savings from their clients from the very start of their programs (see, e.g., Asif Dowla & Dewan Alamgir (2003)). All of the five major microfinance institutions described by Morduch (1999) use combinations of borrowing and saving. CSRs are the payments that are required for participation in the scheme, are part of loan terms, and are required in place of collateral. The amount, timing, and access to these deposits are determined by the policies of the institution rather than by the clients who are typically allowed to withdraw at the end of the loan term, after a predetermined amount of time, or when they terminate their membership.[12]

When the second-best allocation is financed, CSRs are never needed. Indeed, the bad type reveals herself by defaulting in period 1. Therefore, the model implies that CSRs are only observed when the contract induces all borrowers to repay their loans. This suggests a connection between extremely high repayment rates and the prevalent use of CSRs observed in microfinance.

In sum, the model highlights two conceptually distinct roles for savings offered by microfinance institutions: *i*) consumption smoothing, and *ii*) creation of collateral to facilitate identification of trustworthy borrowers in period 2, maintaining portfolio quality.[13]

[12] The model assumes verifiable output and therefore no constraint is associated with repayment of d_1. In practice, microlending institutions collect deposits through small instalments at frequent meetings. This practice makes it easier to enforce CSRs. Under non-verifiable output, repayment is guaranteed by setting $d_1 \leq s^B$ and, therefore, s^B will also be a part of the contract.

[13] The logic of the model applies in other contexts as well. In lending to small and medium-sized firms, for instance, it is common practice to require a positive balance on a deposit account as a condition for maintaining a credit line. Also, the refinancing policy in period 2, which pays a transfer to a borrower that abandons the project, is reminiscent of "golden parachutes", through which executives receive significant benefits if employment is terminated.

4 Extensions

This Section discusses projects of variable scale as well as alternative market structures and learning processes.

4.1 Variable Scale

This Section endogenizes the project size and shows the project growth path to be either steeper or flatter than the one chosen by a self-financed agent. Moreover, a flatter growth path is associated with underinvestment in period 2 and overinvestment in period 1.

Suppose that a project with an initial investment k has an output of

$$r(k) = \begin{cases} f(k) \text{ if } k \geq \underline{k} \\ 0 \text{ otherwise} \end{cases}$$

where $f(k)$ is increasing and concave. The size of the project has to be at least $\underline{k}$, so that the agent cannot learn her type by starting an arbitrarily small project. Both ψ and e are now in per unit terms: the agent can divert ψk and needs to exert effort ek for the project to succeed when she is of the bad type. Assumptions 1-3, modified accordingly, hold for any $k \geq \underline{k}$. Denote $k^* = \arg\max f(k) - k$, the efficient project size if the agent is good.

Conditional on starting a project, the first-best investment levels k_τ^* chosen by a self-financed agent in period $\tau = \{1, 2\}$ are given by

$$f'(k_1^*) = 1 + (1 - \rho)e \text{ and } k_2^* = k^*.$$

It immediately follows that $k_2^* > k_1^*$: the model captures a natural tendency for the project to grow. Because of effort costs that are increasing in the project size, the optimal investment path requires starting small and later, when the agent is confident that she is the good type, increasing the project size to its efficient level.

Turn now to the agent that has to borrow. Competition among lenders ensures that equilibrium contracts maximize the borrower's utility subject to the zero profit constraint for the lender and borrower's incentive compatibility constraints. The region where the first-best investment path k_1^* and k_2^* can be financed is similar to the one described in Proposition 1, when the size of projects is fixed. There are two regions separated by

$\delta_\psi^{k^*} = \frac{(\psi+e)k_1^*}{\psi k_2^* - u}$. In the upper region, the bad type's truth-telling constraint binds and a higher δ makes this constraint harder to satisfy and, therefore, may prevent financing. In the lower region, the good type's truth-telling constraint is binding and a higher δ relaxes it, making financing easier.

The next Proposition characterizes the distortions in the size of the projects when the first-best investment path k_1^* and k_2^* cannot be financed but both types still complete the project in period 1.[14]

Proposition 3 *When the first-best investment levels k_1^* and k_2^* cannot be financed and the contract induces both types to complete the project in period* 1*, there exists threshold δ_ψ^k such that the optimal sizes of projects in periods* 1 *and* 2*, $k_1$and k_2, are given by:*

- $k_1 < k_1^*$ *and* $k_2 = k_2^*$ *if* $\delta < \delta_\psi^k$*;*
- $k_1 > k_1^*$ *and* $k_2 < k_2^*$ *if* $\delta > \delta_\psi^k$*.*

Proof. See Appendix. ■

When $\delta < \delta_\psi^k$, the binding constraint is the truth-telling constraint of the good type. The size of the first-period project is then distorted downwards, $k_1 < k_1^*$, since this decreases the bad type's effort costs and, therefore, relaxes the constraint. There is no need to distort the size of the second-period project, $k_2 = k_2^*$. The growth profile of the projects is steeper than the optimal one.

When $\delta > \delta_\psi^k$, the truth-telling constraint of the bad type is binding. The size of the second-period project is then distorted downwards, $k_2 < k_2^*$, in order to decrease the good type's rent and to relax the constraint. Interestingly, the size of the first-period project is distorted upwards, $k_1 > k_1^*$. The high rent in period 2 makes the bad type willing to exert high effort in period 1. It is then optimal to expand the size of the project k_1 to increase the monetary revenues available to pay the necessary rents in period 2. The growth profile of the projects is then flatter than the optimal one.[15]

[14] As discussed at the end of Section 2.4, it might be optimal to let the bad type fail in period 1, that is, to implement the second-best allocation.

[15] A further interesting feature of the model with variable scale is that the threshold δ_ψ^k separating the two regions depends on ρ. In a richer model with a longer horizon and imperfect learning, the relationship between the borrower and the lender may, overtime, shift across regions characterized by different agency problems.

4.2 Market Structure

The main mechanism of the model survives if assumptions on market structure and available contracts are relaxed.

A monopolistic lender maximizes his profits subject to agent participation and incentive compatibility constraints, trading-off efficiency and rents. This makes a difference when the lender's profits from implementing the first best are close to zero. In particular, close to the lower frontier of the region where the first best is implementable, a monopolistic lender would finance the second-best allocation in which the bad type fails in period 1 saving on rent e obtained by the good type (as described at the end of Section 2.4).

If the agent is allowed to change lenders in period 2, the analysis becomes more involved since the original lender has to counteract the offers of other lenders in period 2. In particular, the lender might allow for inefficient continuation, that is, refinance the bad type in period 2 to make it harder for alternative lenders to poach the good type. The access to alternative lenders in period 2 makes financing more difficult ex ante.

When the agent has access to a saving technology, she might learn her type, divert ψ and apply for a loan to another lender in period 2. The original lender will have then to counteract this possibility. Note that this deviation is more profitable when ρ is higher, since alternative lenders offer better terms for a loan in period 2. Potentially, a higher ρ then makes financing experimentation harder.

4.3 Learning Process

The learning process in the model is very stark: the agent perfectly learns her type upon starting the project, while the lender learns nothing. The crucial underlying assumption, however, is only that the agent acquires some private information that the lender wants her to reveal. Making the learning process more realistic preserves the basic logic of the model and adds new insights.

Suppose that the agent acquires an imperfect signal about her type.[16] In contrast to a standard model of experimentation, in which better learning, in the form of a more precise signal, always makes experimentation more profitable, it can be shown that better information might increase the agent's rent and make financing experimentation harder.

[16] Such a formulation arises more naturally in a model in which the borrower learns about the probability of success.

On the one hand, more information improves the allocation of projects in period 2 as fewer errors are committed. On the other hand, better information makes it harder to satisfy the truth-telling constraint of the good type: upon receiving a negative signal, the bad type becomes more pessimistic about her costs and this increases the rent that has to be paid to the good type.

The lender might also receive a signal about the agent's type. For example, if the two types have different probabilities of success, then the project outcome is informative. However, given the agent's limited liability, the agent still earns some rent and this rent can be decreased by eliciting the agent's private information.

5 Conclusion

Exploration of unknown activities lies at the heart of this model. What happens when such activities are financed by a lender? The paper has shown that introducing agency problems changes the nature of experimentation. In particular, in contrast to a standard model of experimentation, financing experimentation might be harder the longer the time horizon and the lower the payoff of the known activity, i.e., when the net present value of the project is higher. The optimal contract resembles typical microfinance schemes observed in practice.

A promising direction for future work is to study venture capital financing where experimentation is also crucial. However, there seems to be an important conceptual difference between VC financing and microfinance. In our model, upon starting a project, it is efficient for both types to exert effort and complete the project. This might be a good description of typical microfinance loans, which are disbursed to start small projects in small scale farming, tailoring, shop keeping, etc. The fact that most of these activities are very common suggests that at some reasonable effort cost any borrower can successfully complete the project. However, this assumption might not be appropriate in VC where a bad idea will never achieve success, regardless of the effort put into the project. If bad ideas always fail, in this model the only concern is to provide enough incentives for entrepreneurs with good ideas, which, as shown above, does not fundamentally change the nature of the experimentation. The model should then be adjusted for a more realistic learning process as well as typical VC issues such as the reallocation of control rights and the active monitoring by the lender at the various stages of the project.

6 Appendix: Proofs

Proof of Proposition 1. The proof is organized in two steps. First, we find the cost-minimizing contract, that is, the contract that finances the first best with the least possible transfers. Second, we find for which parameter values this contract allows the lender to earn non-negative profits.

First step. Denote $t_{i,\tau}$, $i = G, B$, $\tau = 1, 2$, the transfer that the type i receives in period τ, and $T_i = t_{i,1} + \delta t_{i,2}$ the total transfer to the type i. Let us show that the cost-minimizing contract takes the following form:

$$\begin{cases} T_G^* = \psi + e + \delta u \\ T_B^* = \psi + e \end{cases}, \text{ if } \delta \leq \delta_\psi \text{ and } \begin{cases} T_G^* = \delta\psi \\ T_B^* = \delta(\psi - u) \end{cases}, \text{if } \delta \geq \delta_\psi, \tag{9}$$

where $\delta_\psi = \frac{\psi+e}{\psi-u}$.

The agent can deviate by i) misreporting her type, ii) diverting the investment (and not exerting the effort) in period 1, iii) diverting the investment in period 2 (when the agent is of the good type). The contract has therefore to satisfy the following constraints:

for the good type :

$$\begin{aligned} &T_G \geq \psi + \delta u && IC_{G,1} \\ &T_G \geq T_B + \delta u && TT_G \\ &t_{G,2} \geq \psi && IC_{G,2} \\ &t_{G,\tau} \geq 0 && LL_{G,\tau} \end{aligned}$$

for the bad type :

$$\begin{aligned} &T_B + \delta u \geq \psi + e + \delta u && IC_{B,1} \\ &T_B + \delta u \geq t_{G,1} + \delta \max\{t_{G,2} - e, \psi\} && TT_B \\ &t_{B,\tau} \geq 0 && LL_{B,\tau} \end{aligned}$$

Rewrite TT_B as

$$\left.\begin{aligned} T_B + \delta u &\geq T_G + \delta \max\{-e, \psi - t_{G,2}\} \\ &= T_G - \delta \min\{e, t_{G,2} - \psi\} \end{aligned}\right. \quad TT_B$$

Combining TT_G and $IC_{B,1}$ gives $T_G \geq T_B + \delta u \geq \psi + e + \delta u > \psi + \delta u$ and, therefore, $IC_{G,1}$ is implied.

Also note that cash constraints for the bad type, $LL_{B,\tau}$, never bind. Since this type does not undertake the project in period 2, T_B can be split into $t_{B,1}$ and $t_{B,2}$ in any way.

There are two cases depending on the relative values of e and $t_{G,2} - \psi$.

- Case 1: $t_{G,2} \leq e + \psi \Rightarrow \min\{e, t_{G,2} - \psi\} = t_{G,2} - \psi$. Setting $t_{G,2} = \psi$ does not hurt and it can be useful to relax $LL_{G,1}$. Then $\min\{e, t_{G,2} - \psi\} = t_{G,2} - \psi = 0$ and we have the following system of constraints:

$$\begin{aligned} T_G &\geq \delta\psi & \quad LL_{G,1} \\ T_G &\geq T_B + \delta u & \quad TT_G \\ T_B &\geq \psi + e & \quad IC_{B,1} \\ T_B &\geq T_G - \delta u & \quad TT_B \end{aligned}$$

  From TT_G and TT_B, $T_G = T_B + \delta u$ and there are two possible cases depending on which of the other two constraints, $LL_{G,1}$ or $IC_{B,1}$, binds:

  - Case 1a: $IC_{B,1}$ binds and thus $T_B = \psi + e$ and $T_G = \psi + e + \delta u$. We need only to check that $T_G \geq \delta\psi$, that is, $\delta \leq \delta_\psi$.
  - Case 1b: $LL_{G,1}$ binds and thus $T_G = \delta\psi$ and $T_B = \delta(\psi - u)$. $IC_{1,B}$ is satisfied if and only if $\delta \geq \delta_\psi$.

- Case 2: $t_{G,2} \geq \psi + e \Rightarrow \min\{e, t_{G,2} - \psi\} = e$. This is never optimal, that is, we cannot do better than in case 1. Indeed, increasing $t_{G,2}$ (and, therefore, T_G) is only useful in order to relax TT_B. However, exactly the same effect can be achieved by increasing $t_{G,1}$. Moreover, increasing $t_{G,2}$ is strictly worse when the constraint $T_G \geq \delta\psi$ binds (case 1a).

Second step. The project revenues have to cover the costs of the cost-minimizing contract (9),

$$(r-1)(1+\delta\rho) \geq \rho T_G^* + (1-\rho) T_B^*.$$

Region 1. When $\delta \leq \delta_\psi$, $T_B^* = \psi + e$ and $T_G^* = \psi + e + \delta u$

$$(r-1)(1+\delta\rho) \geq \psi + e + \rho\delta u$$

which is satisfied if and only if $\delta \geq \underline{\delta}^{FB} \equiv \frac{\psi+e-(r-1)}{\rho(r-1-u)}$.

Region 2. When $\delta \geq \delta_\psi$, $T_B^* = \delta(\psi - u)$ and $T_G^* = \delta\psi$

$$(r-1)(1+\delta\rho) \geq \delta\psi - \delta(1-\rho)u$$

which is satisfied if and only if $\delta \leq \overline{\delta}^{FB} \equiv \frac{r-1}{\psi-\rho(r-1)-(1-\rho)u}$ when $\rho < \frac{\psi-u}{r-1-u}$ and always when otherwise. In the latter case, define $\overline{\delta}^{FB} = \infty$.

Combining the two regions gives the result. ■

Proof of Proposition 2. The proof is organized in two steps. First, we find the cost-minimizing contract when the agent has a concave utility function. Second, we derive the consumption profile by distributing the project revenues in excess of the cost-minimizing contract and taking the limit where the utility function becomes linear.

Denote by $U(\cdot)$ the utility function of the agent. At the second step, we will be interested in the limit where it converges pointwise to the linear function $U(x) = x$.

First step. Denote $\underline{x} = U(x)$. Let us show that the cost-minimizing contract takes the following form:

$$\begin{array}{ccc} \text{if } \delta \leq \delta_s^U & \text{if } \delta \in \left[\delta_s^U, \delta_\psi^U\right] & \text{if } \delta \geq \delta_\psi^U \\ \left\{\begin{array}{l} \underline{t_{G,1}^*} = \frac{\underline{\psi}+e+\delta\underline{u}}{1+\delta} \\ \underline{t_{G,2}^*} = \frac{\underline{\psi}+e+\delta\underline{u}}{1+\delta} \\ \underline{t_{B,1}^*} = \frac{\underline{\psi}+e+\delta\underline{u}}{1+\delta} \\ \underline{t_{B,2}^* + u} = \frac{\underline{\psi}+e+\delta\underline{u}}{1+\delta} \end{array}\right. & \left\{\begin{array}{l} \underline{t_{G,1}^*} = \underline{\psi} + e - \delta(\underline{\psi} - \underline{u}) \\ \underline{t_{G,2}^*} = \underline{\psi} \\ \underline{t_{B,1}^*} = \frac{\underline{\psi}+e+\delta\underline{u}}{1+\delta} \\ \underline{t_{B,2}^* + u} = \frac{\underline{\psi}+e+\delta\underline{u}}{1+\delta} \end{array}\right. & \left\{\begin{array}{l} \underline{t_{G,1}^*} = 0 \\ \underline{t_{G,2}^*} = \underline{\psi} \\ \underline{t_{B,1}^*} = \frac{\delta\underline{\psi}}{1+\delta} \\ \underline{t_{B,2}^* + u} = \frac{\delta\underline{\psi}}{1+\delta} \end{array}\right. \end{array} \quad (10)$$

where $\delta_\psi^U = \frac{\underline{\psi}+e}{\underline{\psi}-\underline{u}}$ and $\delta_s^U = \frac{e}{\underline{\psi}-\underline{u}}$.

In order to derive the cost-minimizing contract, we proceed in a way similar to the one in the proof of Proposition 1. The constraints are now

$$
\begin{array}{ll}
\underline{t_{G,1}} + \delta \underline{t_{G,2}} \geq \underline{\psi} + \delta \underline{u} & IC_{G,1} \\
\underline{t_{G,1}} + \delta \underline{t_{G,2}} \geq \underline{t_{B,1}} + \delta \underline{t_{B,2} + u} & TT_G \\
t_{G,2} \geq \psi & IC_{G,2} \\
\underline{t_{B,1}} + \delta \underline{t_{B,2} + u} \geq \underline{\psi} + e + \delta \underline{u} & IC_{B,1} \\
\underline{t_{B,1}} + \delta \underline{t_{B,2} + u} \geq \underline{t_{G,1}} + \delta \max\{\underline{t_{G,2}} - e, \underline{\psi}\} & TT_B \\
t_{i,\tau} \geq 0 & LL_{i,\tau}
\end{array}
$$

Combining TT_G and $IC_{B,1}$ gives $\underline{t_{G,1}} + \delta \underline{t_{G,2}} \geq \underline{t_{B,1}} + \delta \underline{t_{B,2} + u} \geq \underline{\psi} + e + \delta \underline{u} > \underline{\psi} + \delta \underline{u}$ and, therefore, $IC_{G,1}$ is implied.

Also note that the limited liability constraints for the bad type, $LL_{B,\tau}$, never bind since this type does not undertake the project in period 2. In order to minimize the cost, it is optimal to set $t_{B,1} = t_{B,2} + u$.

There are two cases depending on the relative values of $\underline{t_{G,2}} - e$ and $\underline{\psi}$.

- Case 1: $\underline{t_{G,2}} - e \leq \underline{\psi} \Rightarrow \max\{\underline{t_{G,2}} - e, \underline{\psi}\} = \underline{\psi}$. We have the following system of constraints:

$$
\begin{array}{ll}
t_{G,1} \geq 0 & LL_{G,1} \\
t_{G,2} \geq \psi & IC_{G,2} \\
\underline{t_{G,1}} + \delta \underline{t_{G,2}} \geq \underline{t_{B,1}} + \delta \underline{t_{B,2} + u} & TT_G \\
\underline{t_{B,1}} + \delta \underline{t_{B,2} + u} \geq \underline{\psi} + e + \delta \underline{u} & IC_{B,1} \\
\underline{t_{B,1}} + \delta \underline{t_{B,2} + u} \geq \underline{t_{G,1}} + \delta \underline{\psi} & TT_B
\end{array}
$$

  Suppose first that $IC_{G,2}$ binds. We will later check that the implied $t_{G,1} \leq \psi$ since otherwise it would be optimal to smooth the consumption of the good type.

  From TT_G and TT_B, $\underline{t_{G,1}} + \delta \underline{\psi} = \underline{t_{B,1}} + \delta \underline{t_{B,2} + u}$ and there are two possible cases depending on which of the constraints, $LL_{G,1}$ or $IC_{B,1}$, binds:

  - Case 1a: $IC_{B,1}$ binds and thus $\underline{t_{B,1}} + \delta \underline{t_{B,2} + u} = \underline{\psi} + e + \delta \underline{u}$ and $\underline{t_{G,1}} = \underline{\psi} + e + \delta \underline{u} - \delta \underline{\psi}$. We need only to check that $\underline{t_{G,1}} \geq 0$, that is, $\delta \leq \delta^U_\psi$.

    Check that $t_{G,1} \leq \psi$ and obtain that it is true if and only if $\delta \geq \delta^U_s$.
  - Case 1b: $LL_{G,1}$ binds and thus $\underline{t_{G,1}} = 0$ and $\underline{t_{B,1}} + \delta \underline{t_{B,2} + u} = \delta \underline{\psi}$. $IC_{1,B}$ is satisfied if and only if $\delta \geq \delta^U_\psi$.

When $\delta < \delta_s$, $IC_{G,2}$ does not bind and $t_{G,1} = t_{G,2}$. TT_G will bind since there is no reason to give more rent to the good type than necessary. Then, from binding $IC_{B,1}$ and TT_G, $\underline{t_{G,1}} = \underline{t_{G,2}} = \frac{\underline{\psi}+e+\delta u}{1+\delta} < \underline{\psi} + e$. (Note that TT_B does not bind.)

- Case 2: $\underline{t_{G,2}} - e > \underline{\psi} \Rightarrow \max\{\underline{t_{G,2}} - e, \underline{\psi}\} = \underline{t_{G,2}} - e$. This case is always worse than case 1. Indeed, increasing $t_{G,2}$ is only useful in order to relax TT_B. However, the same effect can be achieved by increasing $t_{G,1}$. Moreover, since $\underline{t_{G,1}}$ is at most $\frac{\underline{\psi}+e+\delta u}{1+\delta} < \underline{\psi} + e < \underline{t_{G,2}}$, it is better to increase $t_{G,1}$ than $t_{G,2}$ in order to smooth the consumption of the good type as much as possible.

Second step. Suppose now that

$$(r-1)(1+\delta\rho) > \rho\left(t^*_{G,1} + \delta t^*_{G,2}\right) + (1-\rho)\left(t^*_{B,1} + \delta t^*_{B,2}\right).$$

What is the optimal way to distribute this surplus across types and periods? Competition among lenders makes them maximize the expected agent's utility $\rho\left[U(c^G_1) + \delta U(c^G_2)\right] + (1-\rho)\left[U(c^B_1) - e + \delta U(c^B_2)\right]$ subject to the incentive constraints. Denote by $\Delta_{i,\tau}$ the increase in the transfer (and, therefore, consumption) of type i in period τ; the consumption is then $c_{i,\tau} = t^*_{i,\tau} + \Delta_{i,\tau}$.

The cost-minimizing contract (10) always gives the same total (consumption) utility to the two types, $U\left(t^*_{G,1}\right) + \delta U\left(t^*_{G,2}\right) = U\left(t^*_{B,1}\right) + \delta U\left(t^*_{B,2}\right)$. Keeping this equality is optimal and does not violate any incentive constraints,

$$U\left(t^*_{G,1} + \Delta_{G,1}\right) + \delta U\left(t^*_{G,2} + \Delta_{G,2}\right) = U\left(t^*_{B,1} + \Delta_{B,1}\right) + \delta U\left(t^*_{B,2} + u + \Delta_{B,2}\right).$$

In the limit when the utility function becomes linear, the total consumption of the two types is the same and equals the total expected revenue of the project, R.

The cost-minimizing contract (10) always achieves perfect consumption smoothing for the bad type; and it is optimal to sustain it, $\Delta_{B,1} = \Delta_{B,2}$. In the limit, the consumption of the bad type is $c_{B,1} = c_{B,2} = \frac{R}{1+\delta}$.

For the good type, there are three cases:

- $\delta \leq \delta^U_s$: perfect consumption smoothing is already achieved with the cost-minimizing contract. Any surplus should be distributed equally across periods, and in the limit $c_{G,1} = c_{G,2} = \frac{R}{1+\delta}$. Note that $\delta^U_s \to \delta_s$ as U becomes linear.

- $\delta \in \left[\delta_s^U, \delta_\psi^U\right]$: $t^*_{G,1} < t^*_{G,2} = \psi$ and the surplus should be used first to increase $c_{G,1}$ up to $c_{G,2} = \psi$. In the limit, $S = \delta(\psi - u) - e$ is needed to achieve perfect consumption smoothing. If the surplus is higher, it is distributed equally across the two periods. Note that $\delta_\psi^U \to \delta_\psi$ as U becomes linear.

- $\delta \geq \delta_\psi^U$: this case is similar to the previous one, $t^*_{G,1} = 0 < t^*_{G,2} = \psi$. In the limit, $S = \psi$ is needed to achieve perfect consumption smoothing. If the surplus is higher, it is distributed equally across the two periods.

■

Proof of Proposition 3. Following a logic similar to the proof of Proposition 1, there are two regimes, depending on which truth-telling constraint is binding. Given project sizes k_1 and k_2, the truth-telling constraints yield minimum transfers with the same form as in (9). Substituting the corresponding expressions into the objective function, the problem can be written as

$$\begin{aligned}
&\max_{k_1,k_2} W = f(k_1) - k_1 + \rho\delta\left(f(k_2) - k_2\right) + (1-\rho)(-ek_1 + \delta u)\\
&\text{s.t.}\\
&ZP_1 : (\psi + e)k_1 + \rho\delta u \leq f(k_1) - k_1 + \rho\delta\left(f(k_2) - k_2\right), \text{ if } \delta \leq \widetilde{\delta}_\psi,\\
&ZP_2 : \delta\psi k_2 - (1-\rho)\delta u \leq f(k_1) - k_1 + \rho\delta\left(f(k_2) - k_2\right), \text{ if } \delta \geq \widetilde{\delta}_\psi.
\end{aligned} \tag{11}$$

Suppose we are in the first case. Write the Lagrangian

$$\begin{aligned}
\max_{k_1,k_2,\lambda_1} \mathcal{L}_1 \;=\;& f(k_1) - k_1 + \rho\delta\left(f(k_2) - k_2\right) + (1-\rho)(-ek_1 + \delta u)\\
&-\lambda_1\left[(\psi + e)k_1 + \rho\delta u - \left(f(k_1) - k_1 + \rho\delta\left(f(k_2) - k_2\right)\right)\right].
\end{aligned}$$

As ZP_1 is binding (and first best cannot be financed), $\lambda_1 > 0$. The first-order conditions are

$$\begin{aligned}
\tfrac{\partial\mathcal{L}_1}{\partial k_1} &= \left(f'\left(k_1\right) - 1\right)\left(1 + \lambda_1\right) - (1-\rho)\,e - \lambda_1\left(\psi + e\right) = 0,\\
\tfrac{\partial\mathcal{L}_1}{\partial k_2} &= \rho\delta\left(f'\left(k_2\right) - 1\right)\left(1 + \lambda_1\right) = 0.
\end{aligned}$$

Then, $k_2 = k_2^*$ and since $\frac{(1-\rho)e + \lambda_1(\psi+e)}{(1+\lambda_1)} > (1-\rho)\,e$, $k_1 < k_1^*$.

If we are in the second case the Lagrangian is

$$\begin{aligned}\max_{k_1,k_2,\lambda_2} \mathcal{L}_2 &= f(k_1) - k_1 + \rho\delta\left(f(k_2) - k_2\right) + (1-\rho)(-ek_1 + \delta u) \\ \tau \quad & \quad -\lambda_2\left[\delta\psi k_2 - (1-\rho)\delta u - \left(f(k_1) - k_1 + \rho\delta\left(f(k_2) - k_2\right)\right)\right].\end{aligned}$$

As ZP_2 is binding (and first best cannot be financed), $\lambda_2 > 0$. The first-order conditions are

$$\frac{\partial\mathcal{L}_2}{\partial k_1} = \left(f'(k_1) - 1\right)(1+\lambda_2) - (1-\rho)\,e = 0,$$
$$\frac{\partial\mathcal{L}_2}{\partial k_2} = \rho\delta\left(f'(k_2) - 1\right)(1+\lambda_2) - \lambda_2\delta\psi = 0.$$

Then, since $0 < \frac{(1-\rho)e}{1+\lambda_2} < (1-\rho)\,e$, $k_2^* > k_1 > k_1^*$ and since $\frac{\lambda_2\psi}{\rho(1+\lambda_2)} > 0$, $k_2 < k_2^*$.

Finally, the threshold $\widetilde{\delta}_\psi$ is implicitly defined by the equation $\frac{k_1^{reg2}(\rho,\delta)}{\psi k_2^{reg2}(\rho,\delta)-u} = \frac{k_1^{reg1}(\rho,\delta)}{\psi k_2^{reg1}(\rho,\delta)-u}$ where k_τ^{reg1} and k_τ^{reg2} are the (distorted) project sizes in the two regimes studied above, for $\tau = \{1,2\}$. This concludes the proof. ■

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