Minsky's Financial Instability Hypothesis and the Leverage Cycle*

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Abstract

Busts after periods of prolonged prosperity have been found to be catastrophic. Financial institutions increase their leverage and shift their portfolios towards projects that were previously considered risky. This results from institutions rationally updating their expectations and becoming more optimistic about the future prospects of the economy. Default is inevitably harsher when a bad shock occurs after periods of good news. Commonly used measures to forecast risk in the system, such as VIX, fail to do so, as they are biased by optimistic expectations. Competition among financial institutions for better relative performance exacerbates the boom-bust cycle. We explore the relative advantages of alternative regulations in reducing financial fragility, and suggest a novel criterion.

Keywords: Financial Instability, Minsky, Leverage, Optimism, Relative Performance

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1 Introduction

Business cycles have received much attention in the economics literature. Stabilizing productivity and demand shocks has been at the center of macroeconomic policy. The financial crisis of 2007-2008 has shed light on a another theory focusing on credit cycles, which argues that net expansion in credit leads to economic expansions, while net contraction causes recessions, and if it persists, depressions. The pioneer of this view was Irving Fisher, who suggested a Debt-Deflation theory of Great Depressions in his 1933 paper. His analysis is based on two fundamental principles, over-indebtedness and deflation. He argued that over-indebtedness can result in deflation in future periods and that can cause liquidation of collateralised debt. Debt is denominated in nominal terms and thus is constant, whereas the value of the collateral that secures this debt depends on demand and supply in the relevant markets. This theory brings financial intermediation to the center of attention. The purpose of this paper is to examine the interaction between the leverage cycle and financial stability.

A number of papers have followed the debt-deflation theory of financial amplification to analyse the effect of collateral constraints on borrowing, production and eventually financial stability. In a seminal paper, Bernanke and Gertler (1989) modeled a collateral-driven credit constraint, arising from strong informational asymmetries, whereby the firm is only able to obtain fully collateralised loans. Hence, the value of the firm's assets has to be greater than the value of the loan or at the limiting case equal to it. Due to the important assumption of scarcity of assets and capital, the amount of credit to the firm shrinks in the presence of deflationary pressures on the prices of its assets. This introduces an external finance premium, which increases with a decrease in the relative price of capital. In turn, an increase in the cost of capital will result in a decrease in investment and capital usage and a reduction in GDP. Mendoza (2006, 2010) and Mendoza and Smith (2006) develop a debt-deflation theory of Sudden Drops. Geanakoplos (2003) and Fostel and Geanakoplos (2008) show how the arrival of bads news about the future economic prospects results in a reduction in the price of assets used as collateral and leads to a drying up of liquidity and fire-sales externalities. Gromb and Vayanos (2002) and Brunnermeier and Pedersen (2009) show how the borrowing capacity of agents, i.e. funding liquidity, and the pricing of assets, i.e. market liquidity, interact and how an idiosyncratic liquidity shock can lead to fire-sales and the unravelling of the whole market.

Other papers, which model fire-sales due to adverse productivity or funding shocks to capture debt-deflationary effects on asset prices leading to loss spirals and financial instability, include Shleifer and Vishny (1992), Kiyotaki and Moore (1997), Kyle and Xiong (2001), Morris and Shin (2004). Adrian and Shin (2009) examine the importance of this channel empirically for financial institutions.

The purpose of this paper is to analyse the risk-taking behaviour of financial institutions over the leverage cycle. The main question we tackle is whether high leverage is accompanied by more risk-taking. Our thesis is akin to Minsky's Financial Instability Hypothesis (1992). The second theorem[1] of the hypothesis states that over periods of prolonged prosperity and optimism about future prospects, financial institutions invest more in riskier assets, which can make the economic system more vulnerable in the case that default materializes. Our findings support Minsky's view. Expectations over the cycle are the main driving force behind over-leveraging and investing in riskier projects. When agents observe a good realization they update their expectations upwards. Not only financial institutions, but also their creditors are Bayesian learners and learn the true riskiness of portfolios over time. They are not asymmetrically informed, but both have incomplete information about the true probability of good and bad outcomes. Thus, after a period of good realization of returns, riskier projects become more appealing to financial institutions, which increase their borrowing to expand their balance sheet. Creditors are willing to provide credit at lower interest rates, since the prospects of the economy have improved from their perspective as well. This results in much higher defaults and financial instability once a bad state realizes. Overall, we examine the effect of leverage, as a path-dependent process, on financial stability by linking learning to risk-taking behaviour.

We also analyse the effect of banking competition on risk-taking behaviour. Following Bhattacharya *et al.* (2007), we assume that financial institutions compete for funding from investors and thus care about their relative performance. We show that they will rebalance their portfolios towards riskier

[1] We do not address his first theorem that "the economy has financing regimes under which it is stable, and financing regimes in which it is unstable[...] furthermore, if an economy with a sizeable body of speculative financial units is in an inflationary state, and the authorities attempt to exorcise inflation by monetary constraint[...]units with cash flow shortfalls will be forced to try to make position by selling out position. This is likely to lead to a collapse of asset values". The theorem is closely related to Fisher Debt-Deflation theory. Lin *et al.* (2010) examine the debt deflation effects of monetary policy on collateral values, default and aggregate output.

assets, but with higher expected profitability, after a shorter period of good news when competition for expected returns is higher. The expectations channel is more pronounced in our analysis than the relative performance one. However, our results on the latter sheds some light on why financial institutions expanded their balance sheets so rapidly before the financial crisis of 2007. The rationale behind the relative performance effect can come from the fact that managerial incentives are tied to higher performance or from reputational consideration and access to the capital markets. Empirical evidence in support of this hypothesis can be found in the mutual funds literature, for example in Chevalier and Elison (1997) and Dasgupta and Prat (2006).

Cogley and Sargent (2008) use a similar learning model to provide an explanation for the equity premium puzzle by modelling a period of persistent pessimism caused by the Great Depression. They also consider agents that have incomplete information about the true transition probabilities across good and bad states of consumption growth. Boz and Mendoza (2010) also consider a learning model in which agents update their expectations about the leverage constraint that will prevail in the future. They show the interaction between borrowing constraints and the mispricing of risk. A sequence of periods characterised by low borrowing constraints induces optimistic expectations about the continuation of such regimes and leads to the underpricing of risk, high leverage, over inflated collateral values and a sharp collapse after a realization of a tighter constraint.

In our model, borrowing regimes are endogenous and depend on the willingness of financial institutions to invest in projects and on the interest rates creditors charge for extending credit. The interest rates are endogenously set to capture beliefs about future credit risk. Projects' payoffs do not change over time. What changes is the perceived belief about the likelihood of good realizations. This allows us to examine shifts in banking portfolios from safer to riskier assets and evaluate the measures used to identify the leverage cycle. The implications of our approach are the same as those in the literature on fire-sales once a bad shock realizes. Default materializes and the financial system becomes fragile. In this sense, we extend the existing theories by arguing that it is optimistic expectations that result in over-leveraging and exacerbate the negative effect of bad realizations on financial stability. It is not bad news that threaten the resilience of the system. It is prolonged prosperity that makes bad news have such a negative effect.[2] This has important implications for the appropriate policy responses. As we show, restricting leverage or regulating margins (see Geanakoplos (2010)) are not enough to stabilize the system. Financial institutions will divert funds from safer projects to riskier ones to meet the increased haircuts. Creditors will not penalize them to the desired extent, since their expectations have been boosted as well. This is the underlying reason why commonly used measures, such as VIX or the TED spread, failed to capture the building up of risk before the crisis. They are very sensitive to prevailing expectations. A measure capturing the shift in portfolios holdings towards riskier projects would be more effective in capturing the credit cycle, given that projects' relative riskiness is preserved when expectations are boosted, although all of them may look safer.

The rest of the paper proceeds as follows. Section 2 presents the baseline model, while section 3 extends it to incorporate a credit market and default, and presents the implications for the leverage cycle. Sections 4 and 5 discuss possible policy responses and empirical implications of our approach. Section 6 concludes. Some figures and tables are relegated to the Appendix.

2 Baseline Model

Consider a multi-period economy with two financial institutions[3], $i \in I = \{\Gamma, \Delta\}$. At any date t, the economy can be in one of two states, denoted by u ("up"/good state) and d ("down"/bad state) respectively. For example, the "up" state at time t is denoted by $s_t = s_{t-1}u$. The set of all states is $s_t \in S = \{0, u, d, \ldots, uu, ud, du, dd, \ldots, s_t u, s_t d, \ldots\}$. The probability that a good state occurs is constant at any point in time and denoted by θ. For simplicity we assume that $\theta \in \{\theta_1, \theta_2\}$ with $1 > \theta_1 > \theta_2 > 0$. However, agents do not know this probability and try to infer to infer it by observing past realizations of good and bad states. Agents have priors $\Pr(\theta = \theta_1)$ and $Pr(\theta = \theta_2)$ that the true probability is θ_1 or θ_2 respectively. Their subjective belief in state s_t of a good state occurring at t+1 is denoted by π_{s_t} and that of the bad $1 - \pi_{s_t}$. These probabilities de-

[2] Acharya and Viswanathan (2010) argue as well that it is the build-up of leverage in good times that makes adverse shock have intense effects due to fire-sales. In this paper, we argue that leveraging up is only one side of the problem. Increased holdings of riskier assets due to prevailing optimism accompanied by higher leverage is the main reason behind catastrophic events.

[3] From now on we will refer to them as banks in the broad sense, since we do not model in any way the capital requirements that they face.

pend on the whole history of realizations up to t. In other words, $\pi_{s_t} = Pr_{s_t}(s_{t+1} = s_t u | s_0, \ldots, s_t)$. Given our notation, state s_t completely summarizes the history of realizations up to t. Thus, $\pi_{s_t} = Pr_t(s_{t+1} = s_t u | s_0, \ldots, s_t) = Pr_{s_t}(s_{t+1} = s_t u | s_t)$. We assume that past realizations of the states of the world are observable by all agents, thus there is no information asymmetry on top of the incomplete information structure.

Consequently, agents' subjective belief is given by:

$$\pi_{s_t} = Pr_{s_t}(\theta = \theta_1 | s_t) \cdot \theta_1 + Pr_{s_t}(\theta = \theta_2 | s_t) \cdot \theta_2$$

Agents are Bayesian updaters and try to learn from past realizations the true probability θ. Their conditional probability given past realizations is:

$$\begin{aligned} Pr_t(\theta = \theta_1 | s_t) &= \frac{Pr_t(s_t | \theta = \theta_1) \cdot Pr(\theta = \theta_1)}{Pr(s_t)} \\ &= \frac{Pr_t(s_t | \theta = \theta_1) \cdot Pr(\theta = \theta_1)}{Pr_t(s_t | \theta = \theta_1) \cdot Pr_t(\theta = \theta_1) + Pr_t(s_t | \theta = \theta_2) \cdot Pr(\theta = \theta_2)} \\ &= \frac{\theta_1^n (1-\theta_1)^{t-n} \cdot Pr(\theta = \theta_1)}{\theta_1^n (1-\theta_1)^{t-n} \cdot Pr(\theta = \theta_1) + \theta_2^n (1-\theta_2)^{t-n} \cdot Pr(\theta = \theta_2)} \end{aligned}$$

where n is the number of good realization up to time t. Then,

$$\begin{aligned} \pi_{s_t} = & \frac{\theta_1^n (1-\theta_1)^{t-n} \cdot Pr(\theta = \theta_1)}{\theta_1^n (1-\theta_1)^{t-n} \cdot Pr(\theta = \theta_1) + \theta_2^n (1-\theta_2)^{t-n} \cdot Pr(\theta = \theta_2)} \theta_1 \\ & + \frac{\theta_2^n (1-\theta_2)^{t-n} \cdot Pr(\theta = \theta_2)}{\theta_1^n (1-\theta_1)^{t-n} \cdot Pr(\theta = \theta_1) + \theta_2^n (1-\theta_2)^{t-n} \cdot Pr(\theta = \theta_2)} \theta_2 \end{aligned} \tag{1}$$

As the number of good realizations increases the subjective probability of the good state realizing in the following period increases as well, i.e. given that $s_t = s_{t-1} u$, then $\pi_{s_t} > \pi_{s_{t-1}}$. Assume that the priors are the same, that is $Pr(\theta = \theta_1) = Pr(\theta = \theta_2)$.

To prove our claim that agents become more optimistic after they observe good outcomes in the past, we just need to show that $Pr_{s_t}(\theta = \theta_1 | s_t) > Pr_{s_{t-1}}(\theta = \theta_1 | s_{t-1})$ and $Pr_{s_t}(\theta = \theta_2 | s_t) < Pr_{s_{t-1}}(\theta =$

$\theta_2 \,|s_{t-1})$ given that $s_t = s_{t-1}u$.

Proof.

$$
\begin{aligned}
&Pr_{s_t}(\theta = \theta_1 \,|s_t) > Pr_{s_{t-1}}(\theta = \theta_1 \,|s_{t-1}) \\
&\frac{\theta_1^{n+1}(1-\theta_1)^{t+1-(n+1)}}{\theta_1^{n+1}(1-\theta_1)^{t+1-(n+1)} + \theta_2^{n+1}(1-\theta_2)^{t+1-(n+1)}} > \frac{\theta_1^{n}(1-\theta_1)^{t-n}}{\theta_1^{n}(1-\theta_1)^{t-n} + \theta_2^{n}(1-\theta_2)^{t-n}} \\
&\left(\frac{\theta_2}{1-\theta_2}\frac{1-\theta_1}{\theta_1}\right)^n \left(\frac{1-\theta_2}{1-\theta_1}\right)^t > \left(\frac{\theta_2}{1-\theta_2}\frac{1-\theta_1}{\theta_1}\right)^{n+1} \left(\frac{1-\theta_2}{1-\theta_1}\right)^{t+1} \\
&1 > \frac{\theta_2}{1-\theta_2}\frac{1-\theta_1}{\theta_1}\frac{1-\theta_2}{1-\theta_1} \\
&1 > \frac{\theta_2}{\theta_1}
\end{aligned}
$$

□

At each date $t \in \{0, 1, \dots\}$ each bank receives equity capital of 1 unit and it faces two investment opportunities. It can invest either in a safer project, denoted by L (standing for "*low*" risk), or in riskier one, denoted by H (standing for "*high*" risk). Both projects require one unit of investment and expire in one period. The safe project has an expected return of $\mu_{s_t}^L$ and standard deviation $\sigma_{s_t}^L$ when evaluated at s_t, while the riskier one an expected return of $\mu_{s_t}^H$ and standard deviation $\sigma_{s_t}^H$. The safer project yields a payoff X_u^L in the good state and X_d^L in the bad. Equivalently, the payoffs for the riskier project are X_u^H and X_d^H. We assume that $X_u^H > X_u^L > 1 > X_d^L > X_d^H > 0$, such that the riskier project is more profitable if the good state realizes. Note that the expected returns and standard deviations for the two projects change over time due to the fact that expectations about the good state being realised change, while the ex-post payoffs are the same.

Given its capital, the bank can invest in only one type of project. In the baseline model we make the assumption that the two projects are mutually exclusive, indivisible and that every bank can only invest in one unit of a project in each period. We assume that the capital not used to fund new investments is returned to shareholders, and we do not include it in our baseline analysis. The reason that we are making these assumptions is that our focus lies in the conditions that have to hold for banks to switch from a safe investment to a riskier one, once some of the uncertainty is resolved and expectations about future returns are updated. We relax these assumptions in section 3, where we allow banks to form a portfolio of the two projects, use the capital they have accumulated from investment proceeds in previous periods and also leverage up through the credit market.

Each bank $i \in I = \{\Gamma, \Delta\}$ has the following payoff/utility function in state s_t:

$$\tilde{U}^i_{s_t} = \tilde{\Pi}^i_{s_t} - \gamma^i \cdot \left(\tilde{\Pi}^i_{s_t}\right)^2 + \omega^i \left(\tilde{R}^i_{s_t} - \tilde{R}^{i'}_{s_t}\right)$$

where $i' = \Delta$ if $i = \Gamma$ and $i' = \Gamma$ if $i = \Delta$, γ^i is the risk aversion coefficient of bank i, $\tilde{\Pi}^i_{s_t}$ are the realised profits of the bank in s_t, $\tilde{R}^i_{s_t}$ and $\tilde{R}^{i'}_{s_t}$ are the realised returns on banks' i and i' investments at t respectively, and ω^i the coefficient of relative performance of bank i. We introduce such a strategic consideration by including a linear term in each financial institution's objective function, which is proportional to the difference between the institution's own return on its investments and the return on the investments of its competitors. The severity of competition is captured by a scalar, which we call relative performance coefficient following Bhattacharya *et al.* (2007). This coefficient can be institution specific and is greater than zero. The higher it is, the more institutions try to outperform their competitors.

The expected payoff at date t and state s_t is:

$$\begin{aligned}\sum_{k=t}^{\infty} \mathbb{E}_{s_t} \tilde{U}^i_{s_{k+1}} &= \sum_{k=t}^{\infty} \mathbb{E}_{s_t} \tilde{\Pi}^i_{s_{k+1}} - \gamma^i \mathbb{E}_{s_t} (\tilde{\Pi}^i_{s_{k+1}})^2 + \omega^i \left(\mathbb{E}_{s_t} \tilde{R}^i_{s_{k+1}} - \mathbb{E}_{s_{k+1}} \tilde{R}^{i'}_{s_{k+1}}\right) = \\ &= \sum_{k=t}^{\infty} \mathbb{E}_{s_t} \tilde{\Pi}^i_{s_{k+1}} - \gamma^i \left(\mathbb{E}_{s_t} \tilde{\Pi}^i_{s_{k+1}}\right)^2 - \gamma^i Var\left(\tilde{\Pi}^i_{s_{k+1}}\right) + \omega^i \left(\mathbb{E}_{s_t} \tilde{R}^i_{s_{k+1}} - \mathbb{E}_{s_t} \tilde{R}^{i'}_{s_{k+1}}\right)\end{aligned}$$

where $\mathbb{E}_{s_t}$ is the expectation in state s_t, under the probability measure π_{s_t}, when the investment decision is made, $\mathbb{E}_{s_t} \tilde{\Pi}^i_{s_{k+1}} = 1 + \mu^j_{s_k}$, $j = L, H$ and $\mathbb{E}_{s_t} \tilde{R}^i_{s_{k+1}} = \mu^j_{s_k}$, since we have normalized the investment to 1. ω^Γ can be different from ω^Δ. It is clear that there are many subgames that correspond to the investment decision at different states s_t. Due to the assumptions made above, investment decisions at each point in time can be considered separately from decisions made at different points in time, since the budget constraints do not overlap. In other words, banks look only one period ahead. The resulting equilibria will thus be subgame perfect. The analysis is more complicated when we consider credit and portfolio holdings in section 3, and there may exist equilibria that are not subgame perfect and should be eliminated.

We first find the upper limit for a bank's risk-aversion, such that it chooses to invest in one of the two projects and not hold onto its capital. The individual rationality constraint for bank b is:

$$1+\mu_{s_t}^{j}-\gamma^{i}\cdot\left(1+\mu_{s_t}^{j}\right)^{2}-\gamma^{i}\cdot\left(\sigma_{s_t}^{j}\right)^{2}+\omega^{i}\left(\mu_{s_t}^{j}-\mathbb{E}_{s_t}\tilde{R}_{s_t}^{i'}\right)>1-\gamma^{i}-\omega^{i}\mathbb{E}_{s_t}\tilde{R}_{s_t}^{i'}$$
$$\gamma^{i}<\frac{\mu_{s_t}^{j}(\omega^{i}+1)}{1+\mu_{s_t}^{j}+\left(\sigma_{s_t}^{j}\right)^{2}}$$

We assume that the above condition for both banks' risk-aversion always holds, so there is positive investment at every point in time.

We consider two situations. In the first, banks do not have any strategic considerations, i.e. the relative performance coefficient for both is zero. They choose the project to invest in by comparing the expected utility coming from the two exclusive investments. In section 2.1, we show that banks will choose to invest in the riskier project only after a number of good realizations have occurred up to that point and expectations have been revised upwards. The minimum number of "up" movements depends on the risk-aversion of the bank and relative ranking of projects' payoffs. Once we allow for relative performance influencing their utility, banks act in a strategic way. Again they choose to invest in the project that maximizes their payoff, given the action of the competing bank, resulting in a Nash equilibrium, since there are strategic consideration due to the relative performance effect. The strategy set of both banks is $F=(L,H)$, since they can either invest in the safer or the riskier project. The equilibrium actions are those such that no bank has an incentive to deviate, given the optimal action of the other bank. We show in section 2.2 that, given a relative performance coefficient, banks will switch to the riskier project once expectations are revised upwards and that the number of good realizations necessary for such a switch are lower than in the case when banks do not compete with each other. Thus, relative performance and competition for higher expected returns exacerbate the problem of switching to riskier investments.

2.1 Banks do not compete for higher expected returns

We start with an assumption of a zero value for the relative performance coefficient of both banks. The investment decision of the competing bank will not be taken into consideration when bank i decides in which project to invest. Thus, it will prefer to invest in the risker project in s_t if its expected utility at s_t is higher compared to that coming for the safer investment, i.e.

$$\mathbb{E}_{s_t}\left(X^L\right) - \gamma^i\left(\mathbb{E}_{s_t}\left(X^L\right)\right)^2 < \mathbb{E}_{s_t}\left(X^H\right) - \gamma^i\left(\mathbb{E}_{s_t}\left(X^H\right)\right)^2$$

$$\pi_{s_t}X_u^L + (1-\pi_{s_t})X_d^L - \pi_{s_t}X_u^H - (1-\pi_{s_t})X_d^H - \gamma^i\left(\pi_{s_t}\left(X_u^L\right)^2 + (1-\pi_{s_t})\left(X_d^L\right)^2 - \pi_{s_t}\left(X_u^H\right)^2 - (1-\pi_{s_t})\left(X_d^H\right)^2\right) < 0$$

$$\Rightarrow \pi_{s_t} > \frac{X_d^L - X_d^H - \gamma^i\left(X_d^L\right)^2 + \gamma^i\left(X_d^H\right)^2}{X_u^H - X_u^L + X_d^L - X_d^H + \gamma^i\left(X_u^L\right)^2 - \gamma^i\left(X_u^H\right)^2 + \gamma^i\left(X_d^H\right)^2 - \gamma^i\left(X_d^L\right)^2} = \bar{A}^i \tag{2}$$

Given that the probability at t of the good state of the world occurring at t+1 is higher than $\bar{A}$, then bank i will choose to invest in the riskier project. The same holds for the other bank. Note than both the numerator and the denominator of the above expression are positive, since γ^i is a positive number close to zero (usually between 0.05 and 0.2). Thus, $\bar{A}^i < 1$. Using expressions 1 and 2, we can calculate the minimum number of good realizations needed to make banks invest in the riskier project.

Proposition 1: ***Given that banks do not compete with each other (i.e.*** $\omega_i = 0\ \forall i \in I$***), bank*** i ***will choose to invest in the riskier project if the number of good realization before the time of the decision is*** n^* ***with*** $\pi_{s_t}(n^*) > \bar{A}^i$***, where t is the date that the decision is taken,*** $\bar{A}^i$ ***is given by expression (2) and*** $\pi_{s_t}(n)$ ***by expression (1).***

2.2 Banks compete for higher expected returns

When banks compete with each other for higher returns, they do not evaluate their investment decisions solely on grounds of their individual return and risk profile, but also take into consideration their profitability in comparison with the profitability of their competitors. Thus, strategic considerations become important and one has to resort to a game-theoretic approach. We first list the payoffs to bank i from choosing one project or the other given the investment decision of bank i'.

Due to the linear modelling of the relative performance criterium, the N.E. are easily computed.[4] . For any choice of project j by bank i', bank i will choose to invest in the safer project and will not have an incentive to deviate if:

$$\left(1+\mu^L_{s_t}\right)-\gamma^i\left(1+\mu^L_{s_t}\right)^2-\gamma^i(\sigma^L_{s_t})^2+\omega^i\left(\mu^L_{s_t}-\mathbb{E}_{s_t}\tilde{R}^{i'}_{s_t}\right)\geq\left(1+\mu^H_{s_t}\right)-\gamma^i\left(1+\mu^H_{s_t}\right)^2-\gamma^i(\sigma^H_{s_t})^2+\omega^i\left(\mu^H_{s_t}-\mathbb{E}_{s_t}\tilde{R}^{i'}_{s_t}\right)$$

$$\omega^i\leq\bar{\omega}^i=-1+2\gamma^i+\gamma^i\left(\mu^H_{s_t}+\mu^L_{s_t}+\frac{(\sigma^H_{s_t})^2-(\sigma^L_{s_t})^2}{\mu^H_{s_t}-\mu^L_{s_t}}\right) \tag{3}$$

Proposition 2: ***For a given risk-aversion and distribution of returns, there exists an $\bar{\omega}^i$ after which bank i chooses to invest in the riskier project***

Proof. The result derives directly from expression 3. □

Proposition 3: ***For a given distribution of returns, the more risk averse a bank is the higher its relative performance coefficient has to be for it to invest in the riskier project.***

Proof. The derivative of $\bar{\omega}^i$ in expression (3) with respect to the risk aversion coefficient, γ^i, is positive. Hence, the more risk averse banks are the higher $\bar{\omega}^i$ is. This means that if banks are very risk averse the competition among them for higher returns has to be very high for them to invest in the riskier project. □

Proposition 4: ***For a given risk-aversion, when expectations about project returns become more optimistic, it is more profitable for a bank to deviate to the riskier project.***

Proof. Assume that at time t-1, $\bar{\omega}^i$ is marginally higher than the relative performance coefficient ω^i. We need to show that if expectations become more optimistic, i.e. the good state of the world is realized at time t, then $\bar{\omega}^i$ decreases for the investment decision at that point in time. This means that banks that invested in the safer project at t-1 will choose to invest in the risker project after they have revised their expectations upwards. We need to calculate the derivative of $\bar{\omega}^i$ with respect to

[4] Alternatively, banks could have enjoyed only benefits from achieving higher expected returns compared to their competitors. This situtation is modeled by introducting a max operator instead of a linear term for relative performance, as is done in Bhattacharya *et al.* (2007). Under a nonlinear operator multiple equilibra exist; Two pure and a mixed strategy one. In a pure strategy equibrium, one bank invests in the safer and the other in the riskier project, when ω is larger than the same $\bar{\omega}$ calculated in proposition 2. However, both banks switch to the riskier project once expectations are optimistic enough (propostion 1). Given that our focus in section 3, where we analyse the leverage cycle, is on boosted beliefs, we choose to model relative performance using a linear term for ease of calculation.

the probability of the good state occurring. For ease of notation, we will denote this probability by π.

Expanding the expression for $\bar{\omega}^i$ we get:

$$
\begin{aligned}
&-1+2\gamma^i+\gamma^i\left(\mu^H_{s_t}+\mu^L_{s_t}+\frac{(\sigma^H_{s_t})^2-(\sigma^L_{s_t})^2}{\mu^H_{s_t}-\mu^L_{s_t}}\right)\Rightarrow\\
&-1+2\gamma^i+\gamma^i\left(\pi X^H_u+(1-\pi)X^H_d+\pi X^L_u+(1-\pi)X^L_d\right)+\\
&+\gamma^i\frac{\pi\left(X^H_u-\pi X^H_u-(1-\pi)X^H_d\right)^2+(1-\pi)\left(X^H_d-\pi X^H_u-(1-\pi)X^H_d\right)^2}{\pi X^H_u+(1-\pi)X^H_d-\pi X^L_u-(1-\pi)X^L_d}-\\
&-\gamma^i\frac{\pi\left(X^L_u-\pi X^L_u-(1-\pi)X^L_d\right)^2+(1-\pi)\left(X^L_d-\pi X^L_u-(1-\pi)X^L_d\right)^2}{\pi X^H_u+(1-\pi)X^H_d-\pi X^L_u-(1-\pi)X^L_d}
\end{aligned}
$$

The derivative of the above expression with respect to π is:

$$
\gamma^i\frac{\left(X^H_u-X^L_u\right)\left(X^H_d-X^L_d\right)\left(X^H_u-X^H_d+X^L_u-X^L_d\right)}{\left(\pi X^H_u-(-1+\pi)X^H_d-\pi X^L_u+(-1+\pi)X^L_d\right)^2}
$$

Given that $X^H_u>X^L_u>X^L_d>X^H_d>0$ the derivative is negative, which means that $\bar{\omega}^i$ decreases when expectations become more optimistic. □

The next step is to examine whether relative performance and competition for higher expected returns result in banks choosing the riskier project even if expectations are revised upwards less than in the case that there were no strategic considerations. When $\omega^i>0$ expression 2 becomes:

$$
\pi_{s_t}>\frac{(1+\omega^i)\left(X^L_d-X^H_d\right)-\gamma^i\left(X^L_d\right)^2+\gamma^i\left(X^H_d\right)^2}{(1+\omega^i)\left(X^H_u-X^L_u+X^L_d-X^H_d\right)+\gamma^i\left(X^L_u\right)^2-\gamma^i\left(X^H_u\right)^2+\gamma^i\left(X^H_d\right)^2-\gamma^i\left(X^L_d\right)^2}=\bar{A}^i_\omega \quad (4)
$$

Proposition 5: ***Given that banks compete with each other, bank** i **will choose to invest in the riskier project after a lower number of good realizations compared to the case when there is no competition, i.e.** $n^*_{\omega^i}<n^*$ **where** $n^*_{\omega^i}$ **is the minimum number of good realizations to deviate to the riskier project when** $\omega^i>0$.*

Proof. Following the proof of proposition 1 bank i will invest in the riskier project after $n^*_{\omega^i}$ good realisations, where $\pi_{s_t}\left(n^*_{\omega^i}\right)>\bar{A}^i$, $\bar{A}^i_\omega$ is given by expression (4) and $\pi_{s_t}(n)$ by expression (1). For n_{ω^i} to be less than n, $\bar{A}^i_\omega$ has to be less than $\bar{A}^i$. We can just calculate the derivative of $\bar{A}^i_\omega$ with respect to ω^i. After some algebra we find that $\frac{\partial A^i_\omega}{\partial \omega^i} < 0$. $\square$

One would expect that relative performance considerations would typically mitigate the effects of increasing optimism in equilibrium, since competitors would invest in different assets depending on their attitude towards risk. This was the case in Bhattacharya *et al.* (2007). However, in our model the impact of increasing optimism dominates the tendency to switch to different asset classes from your competitors. Thus, instead of choosing mutually exclusive investments, the competiting institutions form identical portfolios and, consequently, exacerbate leverage and resulting default, as it is shown in the following section.

3 A Model with access to the Credit Markets

In this section, we enhance the baseline model to include access to credit markets for fund raising, and introduce budget constraints at each point in time. We will consider three time periods, $t \in \{0,1,2\}$, to capture the dynamic structure and portfolio reallocation. As before, one of the two states of the world (up or down) can realize at t=1 and t=2. The amount of funds available for investment by banks is equal to the equity capital, plus the funds borrowed from credit markets, plus the profits from the previous period's investment than are not distributed as profits and consumed. We relax the assumption that the projects are mutually exclusive and that bank can only invest in one unit, and we consider a general portfolio problem under which banks decide how much of the available funds to invest in the safer project and how much in the riskier one. We denote by $w^i_{s_t,j}$ the portfolio holding of bank i in project $j \in \{L,H\}$ at $s_t \in \{0,u,d,uu,ud,du,dd\}$. For example, the riskier project's holdings in the second period after a good state realization in the intermediate period are denoted by $w^i_{u,H}$ The interest rate for borrowing from the credit market is denoted by r_{s_t}, $s_t \in \{0,u,d\}$.

We allow for default in the credit market. The amount repaid is an endogenous decision by bank i, which weighs the benefits from defaulting against a deadweight loss. The latter is assumed to be a linear function of the amount that the bank chooses not to deliver. Denoting by $1 - v^i_{s_t}$, $\mathrm{s_t} \in \{\mathrm{u,d,uu,ud,du,dd}\}$ the percentage default, the deadweight loss is equal to $\lambda(1 - v^i_{s_t})(1 + r_{s_{t-1}})$, where λ is the default penalty and $r_{s_{t-1}}$ the interest rate set at the node preceding state s_t (for $s_t \in \{uu, ud\}$, $s_{t-1} = u$, for $s_t \in \{du, dd\}$, $s_{t-1} = d$ and for $s_t \in \{u, d\}$, $s_{t-1} = 0$). We assume risk-neutral creditors. Thus, the interest rate will be inversely related to their expectation about future percentage delivery. The amount of funds that bank i chooses to borrow is denoted by $w^i_{s_t}$, $s_t \in \{0, u, d\}$ and its initial capital by $\bar{w}^i_0$. We show that when expectations become more optimistic, i.e. state 1 realizes in the intermediate period, then banks reallocate their portfolio towards riskier projects. What matters in this setting is the relative weight between the riskier and the safer project in the portfolio.

Allowing for default and variable portfolio weights, we can examine the interaction between Minsky's financial instability hypothesis and the leverage cycle. Once expectations become more optimistic, banks will reallocate their portfolios towards the riskier asset. In order to fund their position, they will increase their leverage, since they cannot go short in the safer asset. This allows us to analyse the effect of expectations on leverage and subsequently default. Once uncertainty is resolved, banks need to repay their loans and they are confronted with the decision to default. If realisations turn out to be bad after a period of previously good news, they will default more on their loans, since they would have invested more in the riskier asset. This is the core of argument, which follows Minsky's intuition. One might have expected that creditors would reduce their credit extension and leverage would go down, since loss given default would be higher. However, this is not the case since the probability of a good outcome has increased and consequently the interest rate creditors charge is lower. This allows banks to increase their leverage after a period of good realizations and invest more in the riskier project.

The leverage cycle has been studied by Geanakoplos (2003, 2010) and Fostel and Geanakoplos (2008) via the modelling of collateralised loans. Herein, we follow an alternative approach to modelling default using non-pecuniary default penalties.[5] We do so for analytical simplicity and because our focus is not on margins, spirals and fire-sales, which have been extensively studied in the literature (for example, Brunnermeier and Pedersen (2009), Gromb and Vayanos (2002)). It is rather expected default and loss given default that we are interested in, which are examined in a much

[5] Shubik and Wilson (1977) and Dubey, Geanakoplos and Shubik (2005) are the seminal papers.

more intuitive way through the use of non-pecuniary default penalties. The aforementioned papers argue that crashes are initiated after an intermediate period of bad news (Geanakoplos, Fostel and Geanakoplos) or an income shock (Brunnermeier and Pedersen). The propagation mechanism is tighter margins for collateralised loans and lower leverage ex-ante. On the contrary, we believe (and show) that crashes are much more severe after an intermediate period of good news, which makes expectations more optimistic.

Sections 3.1 and 3.2 describe the model here, and section 3.3 defines the equilibrium. Due to the dynamic nature and the complexity that default introduces, we resort to simulations to show our results. In order to do so we choose (the few) exogenous parameters carefully to get realistic results for the equilibrium variables. Section 3.4 lists the choice of the exogenous parameters and discusses the resulting equilibrium. We have also performed comparative statics with respect to the most important exogenous parameters, which in our view are the probability of a good realization after a period of good news and the relative performance coefficient. The dynamics of the endogenous variables are qualitatively equivalent to those of the baseline model presented in section 2. However, our richer framework allows us to examine the interaction between leverage and expected loss given default.

3.1 Bank i's Optimization Problem

Banks want to maximize their expected utility over time. At t=0 and t=1 they decide how much to leverage up and how much to spend on the two projects. We assume that the utility at t=1 comes only from the relative performance. Under this assumption banks will not distribute (in essence consume) any profits, but will rather retain them and reinvest them. At t=2, the economy comes to an end, when banks then enjoy the utility coming from their profits and their relative performance. Both at t=1 and at t=2 they choose how much to repay on the loan they undertook in the previous period. Given default, they will have to suffer a non-pecuniary penalty. This penalty is modeled as a negative linear term in the utility function. Banks are penalized proportionally to the amount of loan they default on and the marginal penalty for each unit of default is equal to λ_{s_t}.[6] Bank $i \in I$

[6] We initially assume constant λ for all $s_t\{u,d,uu,ud,du,dd\}$. We relax this assumption later when we talk about regulation to minimize the effects of the leverage cycle.

want to maximize its life time expected utility.

$$\max_{w^i_{s_t,j},v^i_{s_{t+1}},\Pi^i_{s_{t+1}},T^i_u,T^i_d}\sum_{t=0}^{1}\mathbb{E}_0\tilde{U}^i_{s_{t+1}}-\lambda_{s_{t+1}}\mathbb{E}_0\max\left[(1-\tilde{v}^i_{s_{t+1}})\tilde{w}_{s_t}(1+\tilde{r}_{s_t}),0\right]=$$

$$=\max\mathbb{E}_0\left[\tilde{U}^i_{s_1}+\mathbb{E}_{s_1}\left[\tilde{U}^i_{s_2}\right]\right]-\lambda_{s_{t+1}}\mathbb{E}_0\left[\max\left[(1-\tilde{v}^i_{s_1})w_0(1+r_0)\right]\right]-\lambda_{s_{t+1}}\mathbb{E}_0\left[\mathbb{E}_{s_1}\left[\max\left[(1-\tilde{v}^i_{s_2})\tilde{w}_{s_1}(1+\tilde{r}_{s_1})\right]\right]\right]$$

under the following budget constraint at each point in time[7]:

$w^i_{0,L}+w^i_{0,H}\leq\bar{w}^i_0+w^i_0\quad(\psi^i_0)$

i.e. investment in the safer and the riskier assets $\leq$ initial capital + leverage at t=0

$w^i_{s_t,L}+w^i_{s_t,H}\leq T^i_{s_t}+w^i_{s_t}\quad(\psi^i_{s_t})\quad\forall\ \mathrm{s_t}\in\{\mathrm{u},\mathrm{d}\}$

i.e. investment in the safer and the riskier projects $\leq$ reinvested profits + leverage in $s_t\in\{u,d\}$

$\Pi^i_{s_t}+T^i_{s_t}\leq w^i_{0,L}X^L_g+w^i_{0,H}X^H_g-w^i_0v^i_{s_t}(1+r_0)\quad(\phi_{s_t})\quad\forall\ \mathrm{s_t}\in\{\mathrm{u},\mathrm{d}\}$

i.e. distributed + retained profits $\leq$ safer and riskier investments' payoff - loan repayment in $s_t\in\{u,d\}$

$\Pi^i_{s_t}\leq w^i_{s_{t-1},L}X^L_g+w^i_{s_{t-1},H}X^H_g-w^i_{s_{t-1}}v^i_{s_t}(1+r_{s_{t-1}})\quad(\phi_{s_t})\quad\forall\ \mathrm{s_t}\in\{\mathrm{uu},\mathrm{ud},\mathrm{du},\mathrm{dd}\}$

i.e. distributed profits $\leq$ safer and riskier investments' payoff - loan repayment in $s_t\in\{uu,ud,du,dd\}$

<u>where:</u>

$\tilde{U}^i_{s_t}=\omega^i\left(\tilde{R}^i_{s_t}-\tilde{R}^{i'}_{s_t}\right)\quad\forall\ \mathrm{s_t}\in\{\mathrm{u},\mathrm{d}\}$

i.e. utility at t=1 depends only on relative performance

$\tilde{U}^i_{s_t}=\tilde{\Pi}^i_{s_t}-\gamma^i\cdot\left(\tilde{\Pi}^i_{s_t}\right)^2+\omega^i\left(\tilde{R}^i_{s_t}-\tilde{R}^{i'}_{s_t}\right)\quad\forall\ \mathrm{s_t}\in\{\mathrm{uu},\mathrm{ud},\mathrm{du},\mathrm{dd}\}$

i.e. utility at t=2 is equal to the sum of the utility coming from profits and relative performance

$$R^i_{s_t}=\frac{w^i_{0,L}X^L_g+w^i_{0,H}X^H_g-w^i_0v^i_{s_t}(1+r_0)}{\bar{w}^i_0}\quad\forall\ \mathrm{s_t}\in\{\mathrm{u},\mathrm{d}\}$$

i.e. the return in $s_t\in\{u,d\}$ is the investments' payoff minus loan repayment over the initial capital

$$R^i_{s_t}=\frac{w^i_{s_{t-1},L}X^L_g+w^i_{s_{t-1},H}X^H_g-w^i_{s_{t-1}}v^i_{s_t}(1+r_{s_{t-1}})}{T^i_{s_{t-1}}}\quad\forall\ \mathrm{s_t}\in\{\mathrm{uu},\mathrm{ud},\mathrm{du},\mathrm{dd}\}$$

i.e. the return in $s_t\in\{uu,ud,du,dd\}$ is the investments' payoff minus loan repayment over reinvested capital

Note that due to the specification of $\tilde{U}^i_{s_1}$, Π^i_u and Π^i_d are zero in equilibrium

[7] Lagrange multipliers for each budget constraint are in brackets

3.2 Creditors' Optimization Problem

Assume a continuum of creditors $c \in C = [0,1]$ who want to maximize expected utility as well. At t=0 and t=1, their are endowed with capital and they face the decision how much to lend and how much to consume. For simplicity we assume that they are risk-neutral. Their consumption in state $s_t \in \{0,u,d,uu,ud,du,dd\}$ is denoted by $c^c_{s_t}$, $s_t \in \{0,u,d,uu,dd,du,dd\}$, whereas their credit extension by $w^c_{s_t}$ and the capital endowment by $\bar{w}^c_{s_t}$, $s_t \in \{0,u,d\}$. They face the following optimisation problem:

$$\max_{c^c_{s_t}, w^c_{s_t}} \sum_s \mathbb{E}_0 \tilde{c}^c_{s_t}$$

$$= c^c_0 + \pi_0 c^c_u + (1-\pi_0)c^c_d + \pi_0\pi_u c^c_{uu} + \pi_0(1-\pi_u)c^c_{ud} + (1-\pi_0)\pi_d c^c_{du} + (1-\pi_0)(1-\pi_d)c^c_{dd}$$

$$s.t.\ c^c_0 \leq \bar{w}^c_0 - w^c_0$$

i.e. consumption $\leq$ initial endowment - credit extension at t=0

$$c^c_{s_t} \leq \bar{w}^c_{s_t} + v^i_{s_t}(1+r_0)w^c_0 - w^c_{s_t} \quad \forall\ \mathrm{s_t} \in \{\mathrm{u},\mathrm{d}\}$$

consumption $\leq$ endowment + loan repayment - credit extension in $s_t \in \{u,d\}$

$$c^c_{s_t} \leq v^i_{s_t}(1+r_{s_{t-1}})w^c_{s_{t-1}} \quad \forall\ \mathrm{s_t} \in \{\mathrm{uu},\mathrm{ud},\mathrm{du},\mathrm{dd}\}$$

consumption $\leq$ loan repayment in $s_t \in \{uu,ud,du,dd\}$

Optimizing with respect to credit extension, we get the following expression that connects the interest rate with the expected delivery on the loan.

$$\mathbb{E}_{s_t}\left[v^i_{s_{t+1}}\right] \cdot (1+r_{s_t}) = 1,\ \mathrm{s_t} \in \{0,\mathrm{u},\mathrm{d}\} \tag{5}$$

For example, $1+r_u = \dfrac{1}{\pi_u v^i_{uu} + (1-\pi_u)v^i_{ud}}$. One can observe the reverse relationship between the interest rate and expected percentage delivery. When the latter increases, the interest rate charged falls. This provides some intuition for the seemingly counterintuitive result that when expectations are optimistic, banks increase their leverage due to lower interest rates, though at the end their percentage repayment is lower and default is higher. The result obtains from the fact that the probability that a good state realizes is higher, since expectations are optimistic. Thus, overall expected delivery is higher, though loss given default is higher as well.

3.3 Equilibrium

Equilibrium is reached when creditors and banks optimize given their constraints and the credit and projects' markets clear. Interest rates are determined endogenously by equation 5 and are taken as fixed by agents. Credit market clear when supply of credit $\int_0^1 w_{s_t}^c dc$ is equal to the aggregate demand $\sum_i w_{s_t}^i$. Condition 5 is a necessary one for credit markets to clear. The above modelling has assumed a perfectly elastic supply of projects. Equilibrium purchases are determined by banks' demand at a given price of 1 for each project. The analysis of equilibrium and our main result that leverage, investment in the riskier project and realised default increase when expectations become more optimistic would not have changed had we assumed an upward sloping supply curve. One can find endowments of projects that support the price of 1 in equilibrium.

The variables determined in equilibrium and taken by agents as fixed are, thus, given by $\eta = \{r_0, r_u, r_d\}$. The choices by agents i and c are given by $\Box^i = \{\ w_{s_t,j}^i, v_{s_{t+1}}^i, \Pi_{s_{t+1}}^i, T_u^i, T_d^i\}$, $s_t \in \{0, u, d\}$ and $\Box^c = \{c_{s_t}^c, w_0^c, w_u^c, w_d^c\}$, $s_t \in \{0, u, d, uu, ud, du, dd\}$, respectively.

We say that $(\eta, (\Box^i)_{i\in I}), (\Box^c)_{c\in C}$ is an equilibrium of the economy $\mathbb{E} = \left(\left(c^i, \omega^i, \bar{w}^i\right)_{i\in I}, (\bar{w}^c)_{c\in C}; \lambda, \theta_1, \theta_2 \right)$ if and only if:

1. $(\Box^i) \in Argmax_{\Box^i \in B(\eta)}\ \ \mathbb{E}_0 \tilde{U}^i$
2. $(\Box^c) \in Argmax_{\Box^c \in B(\eta)}\ \ \mathbb{E}_0 \tilde{U}^c$
3. $\mathbb{E}_{s_t}\left[v_{s_{t+1}}^i\right] \cdot (1 + r_{s_t}) = 1$, $s_t \in \{0, u, d\}$
4. $\int_0^1 w_{s_t}^c dc = \sum_i w_{s_t}^i$, $s_t \in \{0, u, d\}$
5. Total demand for project j, $\sum_i w_{s_t,j}$, $s_t \in \{0, u, d\}$, is equal to the supply of projects, which is trivially satisfied due to perfectly elastic supply
6. Creditors expectations are rational, i.e. they anticipate correctly the delivery $v_{s_t}^i$, $s_t \in \{u, d, uu, ud, du, dd\}$ by bank $i \in I$

Conditions 1 and 2 says that all agents optimize; 3 and 4 says that credit markets clear; 5 says that projects' markets clear, and 6 that creditors are correct about their expectations of loan delivery or default.

3.4 Quantitative analysis

Given the complexity arising from short-sales constraints, i.e. that banks can only go long on the projects, relative performance and default, we were unable to get a closed form solution and resorted to a numerical calibration. Moreover, learning adds additional complexity to the problem. The choice of exogenous parameters is in line with our assumptions about the evolution of beliefs and the riskiness of the two projects, as outlined in section 2. We have also made sure that resulting interest rates for credit extension and expected default levels are reasonable. In particular, we assume that agents have a initial belief that the good state will realize in the intermediate period with probability $\pi_0 = 0.82$. Given a good realization at t=1, the (subjective) probability of a good outcome increases to $\pi_u = 0.87$, while it falls to $\pi_d = 0.59$ after a bad realization.[8] The safer project's payoff in the good state is $X_g^L = 1.37$, while the riskier project pay out $X_g^H = 1.89$. In the bad state their payoffs are $X_b^L = 0.78$ and $X_b^H = 0.19$ respectively. We have assumed that banks are symmetric, thus they have the same risk-aversion and relative performance coefficient. Finally, the default penalty is constant at every point in time. Table 1 presents the choice of the exogenous variable.

Table 1: Exogenous variables

Probability of good outcome at $t = 0$	π_0=0.82	Relative performance coefficient	ω^i=0.1
Probability of good outcome at $s_t = u$	π_u=0.87	Risk-aversion coefficient	γ^i=0.035
Probability of good outcome at $s_t = d$	π_d=0.59	Default penalty	λ=0.94
Safer's project payoff in good state	X_g^L=1.37	Initial banking capital	$\bar{w}_0^i$=1.5
Safer's project payoff in bad state	X_b^L=0.78		
Riskier's project payoff in good state	X_g^H=1.89		
Riskier's project payoff in good state	X_b^H=0.19		

We present the equilibrium values for the endogenous variables in table 8 in the Appendix and proceed with discussing the most important ones, which capture the interaction between Minsky's hypothesis and the leverage cycle.

[8]The values that support these probabilities are $\theta_1 = 0.898$, $\theta_2 = 0.286$ and the prior $Pr(\theta = \theta_1) = 0.879$.

3.4.1 Minsky and the leverage cycle

The main result we presented in section 2 that banks reallocate their portfolios towards the riskier asset once expectations become more optimistic holds in the more complicated version of the model as well. However, we are now able to examine the effects on leverage, interest rates and most importantly default, which is (or should be) at the heart of any financial instability analysis. We follow the Goodhart-Tsomocos definition of financial instability of states that are characterized by high default and low banking profits-welfare (see Goodhart, Sunirand and Tsomocos (2006)).

In the initial period banks choose not to invest any capital in the risky project. The same holds for the intermediate period when a bad state realizes. However, once expectations are updated upwards (the economy moves to the good state in the intermediate period) the bank starts investing in the riskier project. Actually, its portfolio weight on it is almost twice the weight on the safer project (Table 2).

Table 2: Portfolio weight of the riskier project

Portfolio weight on the risky project at $t = 0$	$w^i_{0,H}$=0
Portfolio weight on the risky project after bad news	$w^i_{d,H}$=0
Portfolio weight on the risky project after good news	$w^i_{u,H}$=65.97%
Risky-to-safe project ratio of weights at t=0	$w^i_{0,H}/w^i_{0,L}$=0
Risky-to-safe asset ratio of weights after bad news	$w^i_{d,H}/w^i_{d,L}$=0
Risky-to-safe asset ratio of weights after good news	$w^i_{u,H}/w^i_{u,L}$=1.94

The increased holdings of the riskier asset in state $s_t = u$ are mainly financed by an increase in leverage. Holdings of the safer project marginally decrease. In particular, as shown in table 3, borrowing almost increases by a factor of three once good news materialize. Although the good state realized in the intermediate period and banks portfolio investment yielded high profits, they choose to borrow more and switch to riskier investments. An increase in borrowing is facilitated by a decrease in the interest rate charged, which falls by 10.36%. Although the bank leverages up and undertakes riskier projects, the interest rate falls, since expectations about a good outcome are more optimistic. Expected percentage default goes down, but inevitably loss given default is much higher once the bad state realises at t=2. Percentage default and loss given default are higher when prosperity prevailed in the past than in the case that a bad outcome materialized. In particular, we find that percentage default in state $s_t = ud$ is 64.78% compared to 24.29% in state $s_t = dd$ and 52.08% in state $s_t = d$. After a round of bad news leverage goes down, since the prospects of the economy have deteriorated. This is captured in a higher interest rate charged. Banks are inclined to default as a percentage less in state $s_t = dd$ than in state $s_t = d$. The interest rate at $s_t = d$ is already higher due to bad expectations, thus by defaulting less they are facing a lower cost of leveraging. However, to do so they invest only in the safer asset. Naturally, loss given default is much lower. The most important result of our analysis is that loss given default in state $s_t = ud$ is substantially higher than in any other state as shown in table 3. Optimism allowed banks to leverage up with low interest rates and undertake much riskier projects, which eventually can result in a catastrophic scenario. This is not the case when bad news realize in the intermediate period and expectations are not boosted upwards.

Table 3: Interest rates, leverage and default

Increase in leverage after good news	182%	Interest rate change after good news	-10.36%
Decrease in leverage after bad news	14.51%	Interest rate change after bad news	9.14%
Expected default at $s_t = 0$	9.18%	Realized default at $s_t = d$	52.08%
Expected default at $s_t = u$	8.24%	Realized default at $s_t = ud$	64.78%
Expected default at $s_t = d$	9.86%	Realized default at $s_t = d$	24.29%
Loss given default at $s_t = d$	2.33	Loss given default at $s_t = ud$	8.10
Loss given default at $s_t = dd$	0.94		

3.5 Comparative Statics

We proceed with presenting the effects of the most important, to our view, shocks. These are an increase in the probability of a good outcome and an increase in the relative performance coefficient (in accordance to section 2.2) after a good realization in the intermediate period. We have performed a number of other comparative statics, such as a decrease in risk-aversion and an increase in the initial banking capital. The results are not presented herein but are available upon request.

3.5.1 Increase in the probability of a good outcome

The equilibrium is very sensitive to changes in π_u. We consider a 1% shock in θ_1, which is the upper limit for the perceived probability of the good state occuring (see section 2). This shock will affect all probabilities due to Bayesian updating. In particular, π_0 and π_u go up by 0.96% and 1.01%

respectively, while π_d decreases by 1.67% in relative terms. The reason that π_u increases more than π_0 is that in state u agents are more optimistic; thus a higher upper bound will affect the probability more. On the other hand, agents become more pessimistic in state d after the realization of bad news, thus the probability of a good outcome decreases. The prospects for the economy are better in state $s_t = u$, thus banks shift their portfolio towards the riskier project even more. Portfolio holdings of the riskier asset increase by 7.68% compared to the initial equilibrium. What is striking is that the ratio of the riskier portfolio holdings over leverage goes up by 18.83%. Naturally, this would push default up. Expected default in $s_t = u$ increases by 3.30%. The increase in optimism is not enough to outweigh the default stemming from the fact that banks have increased their riskier holdings. This is captured by percentage default, which goes up by 11.04%. Also, loss given default increases by 0.91%. Since expected default increases, creditors acting rationally charge a higher interest rate, which makes banks reduce their loans they take and shift funds from the safer investment to finance the riskier one. Thus, the increase in loss given default is smaller than the increase in percentage default due to the reduction in borrowing.

Naturally, the improved economic outlook at t=0 induces banks to invest more in the safer asset by increasing their leverage. What allows banks to borrow more is the lower interest rate due to better expectations. Increased holdings should result in higher percentage default, since leverage went up. However, this is not the case. The reason is that the prospects of the economy in state d have deteriorated. The perceived probability of a good state realizing in the final period then goes down by 1.67%. Banks, when comparing the benefits from defaulting and investing more in that state versus repaying and reducing the size of their portfolio, choose the latter. Thus, they decrease their investment in the safer asset in state d by -9.51%. Naturally, loss given default in state dd goes down substantially (a 25.28% decrease). Table 4 presents the changes in the equilibrium variables (comprehensive comparative statics results for all endogenous variabels are presented in table 10 in the Appendix).

The leverage cycle interpretation of our model described in section 3.4.1 within the new equilibrium still holds, although leverage in $s_t = u$ across equilibria fell. Still, banks increase their leverage after good news realization in the intermediate period. They still invest nothing in the riskier asset in

Table 4: Change in equilibrium variables after a 1% shock in θ_1

Equilibrium variable	Change	Equilibrium Variable	Change
Holdings of riskier project in $s_t = u$	7.68%	Holdings of safer project in $s_t = u$	-35.14%
Riskier holdings over leverage in $s_t = u$	18.83%	Leverage in $s_t = u$	-9.38%
Risky portfolio weight after good news	15.65%		
Interest rate in $s_t = 0$	-5.44%	Interest rate in $s_t = u$	3.58%
Interest rate in $s_t = d$	-8.63%		
Expected default in $s_t = 0$	-5.62%	Expected default in $s_t = u$	3.30%
Expected default in $s_t = d$	-7.88%		
Percentage default in $s_t = d$	-1.19%	Percentage default in $s_t = ud$	11.04%
Percentage default in $s_t = dd$	-10.08%		
Loss given default in $s_t = d$	2.23%	Loss given default in $s_t = ud$	0.91%
Loss given default in $s_t = dd$	-25.28%		

the initial period and shift to the riskier investment after expectations are updated upwards. Finally, percentage default and loss given default are higher in $s_t = ud$ compared to $s_t = d$ and $s_t = dd$ respectively. We present this in tables 5, which are the equivalent of tables 2 and 3 for the new equilibrium.

Table 5: Portfolio weight of the riskier project

Portfolio weight on the risky project at $t = 0$	$w^i_{0,H}$=0
Portfolio weight on the risky project after bad news	$w^i_{d,H}$=0
Portfolio weight on the risky project after good news	$w^i_{u,H}$=76.29%
Risky-to-safe project ratio of weights at t=0	$w^i_{0,H}/w^i_{0,L}$=0
Risky-to-safe asset ratio of weights after bad news	$w^i_{d,H}/w^i_{d,L}$=0
Risky-to-safe asset ratio of weights after good news	$w^i_{u,H}/w^i_{u,L}$=3.22

Table 6: Interest rates, leverage and default

Increase in leverage after good news	146%	Interest rate change after good news	-1.81%
Decrease in leverage after bad news	31.09%	Interest rate change after bad news	5.45%
Expected default at $s_t = 0$	8.66%	Realized default at $s_t = d$	51.46%
Expected default at $s_t = u$	8.51%	Realized default at $s_t = ud$	71.93%
Expected default at $s_t = d$	9.08%	Realized default at $s_t = d$	21.84%
Loss given default at $s_t = d$	2.38	Loss given default at $s_t = ud$	8.17
Loss given default at $s_t = dd$	0.70		

3.5.2 Increase in the relative performance coefficient

Increasing the relative performance coefficient is equivalent to assuming that banks compete more strongly for relatively higher expected returns on their portfolios. To perform a meaningful comparative static, we first elaborate on proposition 5, which says that banks switch to the riskier project faster when their relative performance coefficient is higher. The baseline model does not provide a straightforward implication for the change in portfolio weights due to its simplicity. In this section, we extended the model to account for access to the credit markets and generalized portfolio holdings, which go beyond the case of just two mutually exclusive investments. In our simulation, banks start investing in the riskier project after a round of good news. When banks become more competitive they face a trade off. On one hand, they desire higher expected returns and want to invest more in the riskier asset once expectations allow it. On the other hand, this gives rise to higher default, thus banks would be penalized with a higher interest rate resulting in lower borrowing.

The boosted expectations in state u support positive investment in the riskier project for values of ω around 0.1 in accordance with proposition 1. A meaningful comparative static would be to shock the relative performance coefficient after expectations have been updated upwards and examine the change in the riskier portfolio holdings. As expected, an increase in the relative performance coefficient results in a increase in risk-taking behaviour in state u. Changes in the equilibrium variables are presented in table 7 (comprehensive comparative statics results for all endogenous variables are present in table 9 in the Appendix).

Table 7: Change in equilibrium variables under 1% shock in ω^i in state u

Equilibrium variable	Change
Holdings of riskier project in $s_t = u$	0.15%
Holdings of safer project in $s_t = u$	-1.33%
Riskier holdings over leverage in $s_t = u$	0.70%
Leverage in $s_t = u$	-0.54%
Interest rate in $s_t = u$	0.39%
Expected default in $s_t = u$	0.36%
Loss given default in $s_t = ud$	-0.15%

Investment in the riskier project increases by 0.15% with an increase of 1% in the relative performance coefficient. The trend is the same for higher levels of the coefficient. This small change can be explained by looking at the change in the interest rate charged at $s_t = u$ and in the level of leverage. The former increases by 0.39%, which forces banks to decrease their borrowing. Given that they have increased their holdings of the riskier asset, expected default goes up due to creditors having rational expectations. Consequently, they will charge a higher interest rate and banks will be able to obtain less borrowing. Regardless of the small quantative impact on portfolio holdings, the channel through which more intense competion among banks for higher returns leads to higher expected default exists. Expected default in $s_t = u$ increases by 0.36% as does percentage default. Although expected default goes up, loss given default decreases by 0.15% due to a decrease in leverage.

The relative performance channel seems to have shut down in state $s_t = u$, since higher risk-taking is fully anticipated by creditors. Aggregate holdings of the riskier project increase marginally and loss given default decreases due to lower borrowing. Yet, risk-taking and expected default increase for higher levels of competition, consistent with proposition 5. In the equilibrium we examined, risk-taking behaviour and investment in the riskier project was already at high levels once expectations were boosted up. An alternative exercise would be to start with an equilibrium corresponding to a much lower value for the relative performance coefficient keeping other exogenous variables constant. Consistent with our thesis, we find that aggregate riskier holdings are much lower for a significantly lower coefficient (figure 1).

Moreover, risk-taking increases as the relative performance coefficient becomes higher. A measure of risk taking could be the volatility of the bank's portfolio, which increases for higher levels of ω. As expected, percentage default after the realization of bad news is higher as well (figure 2). However, we have chosen to construct another measure to capture risk taking behaviour, which is presented in figure 1. It is the difference between riskier and safer holdings in aggregate terms per unit of leverage. We elaborate more on its advantages over other measures of riskiness, such as implied volatility, in section 5, when we discuss the empirical implications of our model.

4 Policy Responses

The main driving force behind over-leveraging and increased risk taking is the optimism that comes after the realization of good news. The expectations formation mechanism is exogenous in our model and is implemented through Bayesian updating. Agents have incomplete information about the real world probability of a good state occurring and they try to infer it by observing past realizations. They are Bayesian learners. There is also no additional asymmetry of information. Every agent knows and observes the same things. Thus, regulation cannot control optimism in the markets. Agents are rational and no institution has more information than them. Regulation cannot affect optimism, but it can control its consequences. Simplified as it is, our model can be used to evaluate regulatory policies to control the leverage cycle and mitigate excess risk-taking and default. The first type of policy is to enforce more severe default penalties for banks, while the second to control their leverage ratios in the good state of the world. We discuss this in turn. We use the equilibrium calculate in section 3.4.

4.1 Stricter Default Penalties

The fundamental reason that allows banks to invest largely on the riskier project after expectations have been updated upwards is that interest rates go down. Although loss given default is higher in bad outcomes, optimism dominates making expected default lower. Thus, banks are able to leverage up to invest in the risker project without having to substantially reduce their investment in the safer one. It looks like that banks are not penalized ex-ante with a higher interest rate due to optimistic expectations. The only penalty is due to default. Policy cannot set the credit spreads, since they are market based and depend on expectations, which follow an exogenous process. But, what policy can affect is the propensity to default, which drives the credit spread, by changing the default penalty or introducing other requirements, such as capital adequacy ratios or leverage restrictions.[9]

To visualize the effects of stricter regulation on leverage and risk-taking, we increase the default penalty when agents become more optimistic, i.e. state u.

As expected, loss given default in state ud goes down (figure 3). Banks find it too costly to default. On the other hand, expectations of a good realization are boosted and banks have an incentive to increase their holdings of projects that seemed riskier until then. Weighing the future benefits of this together with a higher cost of default in state ud, they decide to reduce their borrowing in state

[9] The default penalty can be seen to be determined by both market forces, such as loss of reputation, and regulatory sanctions. Policy can affect the second component.

u. Since their, available for investment, funds decrease, they find it more profitable to default more on the loan undertaken in the initial period. Percentage default goes up, as does the interest rate at t=0, and this results in lower borrowing in the initial period as well (figure 3).

The last has adverse consequences for percentage default when the very bad state dd realizes. Given that banks reduced their leverage in the initial period and the payoff from their investment is now lower in state d, their available funds in the latter state reduce a lot. As a result their prospensity to default in state dd increases, which results in higher percentage default and a higher interest rate charged in state d. Due to a higher interest rate banks reduce debt and loss given defaults goes down. Increasing the default penalty once expectations have been updated upwards is effective to reduce excessive default in state ud. But this comes at the cost of increasing percentage default in all other states of the world. Banking profits do not change much apart from state ud, where the reduction is substantial (figure 4). This may seem insignificant within our model, but considerations for recovery after a bad shock following good realizations come into the picture.

Our main consideration is that policy cannot affect default penalties in the straightforward way described here. As mentioned, they rely a lot on market reaction and market discipline. Moreover and most importantly, default penalties are really hard to quantify. Their presence is implicit and necessary, but their level is not directly observable.[10] This is partially the reason that policy has concentrated of different types of regulation, such as capital requirements. We turn to these policy measures in the following section.

4.2 Leverage Requirements

An alternative suggestion would be to restrict leverage once the good state of the world realizes in the intermediate period. A leverage requirement can take the form of a maximum ratio of borrowing over the total investment in projects. We show that such a requirement achieves the exact opposite

[10] Given limited liability, direct ways for the legal system or regulation to affect these penalties are lengthy bankruptcy processes or time-consuming investigations to discover fraudulent behaviour, in which case default penalties are higher due to legal sactions. Regulation can affect this component of default penalties and make default more costly during good times, but the overall change and the feedback on market discipline is hard to quantify. An alternative way to impose penalties in a more quantifiable way is through renumeration reforms involving deferred payments, which should allow clawback in the case of bad outcomes and default.

result than hoped. That is, it results in increased loss given default in state *ud* instead of bringing it down (figure 5). The intuition is simple. Banks will divert their own funds away from the safer asset and put them to the riskier one. Although borrowing goes down, they will invest even more in the riskier asset to compensate for the loss in gearing, since expectation are optimistic. Geanakoplos (2010) proposes a similar policy response to control the leverage cycle. In particular, he argues that the regulator should control haircuts in the upturn, where they are naturally low, in order to allow less room for banks to leverage up and invest in risky projects. But, banks can again reduce their leverage, divert their own resources away from the safer projects and invest more in the riskier ones, while at the same time meeting a higher haircut requirement.

It is exactly such a situation that capital requirements in the Basel Accords try to avoid by imposing requirements on equity capital in relation to risk-weighted assets. The idea is that the capital requirement becomes tighter if banks invest in riskier asset. Both Basel I and II regulation have the feature that equity capital is higher after good realizations due to the higher returns from previous investments. However, when expectations become more optimistic, risk-weights based on internal models decrease and, hence, capital requirements are less likely to bind. This is true for both safer and riskier assets. For example, in our numerical simulation above, the standard deviations of prospective returns are 3.86% and 32.07% in state u, while they are 8.39% and 69.69% in state d, for the safer and riskier asset respectively. Thus, capital requirements of this kind would not restrict leverage to the desired extent once expectations become more optimistic. In the following section, we propose a requirement that could be successful in mitigating the consequences of risk-taking behaviour for bank fragility, but it may involve other costs.

4.3 An alternative requirement

Our analysis highlights that the adverse consequences of the leverage cycle depend on financial institutions shifting their portfolios towards previously riskier projects due to the fact that beliefs have been updated upwards. Since leverage also goes up, this results in a more fragile financial system. In the previous sections, we showed that requirements on leverage when market expectations are high, exacerbate the problem instead of mitigating it. The reason is that financial instituitions will divert funds from safer to riskier projects in the presence of borrowing constraints, i.e. perform the deleveraging process through internal portfolio risk shifting.

Not only should we cosider the priors in calculating risk weights, but a more conservative measure would be to consider the standard deviations of the bad state *d* to have maximum financial stability in the spirit of Kashyap *et al.* (2011) and Admati *et al.* (2010). This could amount to increasing capital requirement or decreasing leverage to a substantial level to equate the loss given default in states *d* and *ud* following the rational that the social cost of default is higher than what is taken into consideration by banks. However, this could have negative effects on the quantity of investment.

A weaker version of the totally safely oriented leverage could restrict relative portfolio holdings, as there is still a point to be made on the effect that expectations, updating and optimism have on the internal risk weights. Thus, a non-risk weight regulation based on leverage could target the difference between riskier and safer holdings per unit of leverage results in higher financial stability. The intuition is straigthforward. As already mentioned, it is the shift towards riskier projects in combination with high leverage that creates the problem, which is something that leverage requirements by themselves cannot handle. Also, restricting leverage in good times can be harmful for the economy and banking profits. It is leverage that goes directly to risky investment which is the appropriate variable to control. We choose to create a requirement that is based on the difference between riskier and safer holdings, as financial institutions can relax their constraint by investing more in safer projects, which can be desirable for the economic activity in good times.[11]

Our comparative statics exercise suggests that loss given default goes down with stricter requirements of this type (figure 6). Also, percentage default goes down in every state of the world.

[11] Our proposal relies on risk being priced efficiently. In our model, rational expectations guarantee this. Moreover, there is a need for a measure to rank projects according to their riskiness. This is straightforward in our model as well, since the payoffs of the projects do not change and their distribution is known. Restricting leverage is inadequate to reduce risk-shifting. Basel II capital requirements should perform even worse, since risk-weights go down during good times. Our proposal results in lower risk-taking accompanied by higher leverage and higher investment in safer projects. However, relative risk is frequently not accurately measured. In particular the top tranches of CDOs and MBS were given far too high a rating before the 2007 financial crisis and hence European banks, which were subject to an RWA, but not a leverage ratio, tended to expand their leverage enormously on the basis of supposedly risk-free assets, which actually were not so. To account for such circumstances, our proposal could be accompanied by a leverage requirement in order not only to mitigate risk-shifting, but also reduce investments in projects that are mistakenly perceived to be safer. Nevertheless, efficient pricing and competitive markets should imply that higher yields, which characterised the aforementioned financial products, are compensation for risk, thus a relative ranking may still be possible assuming that existing models can capture risk in the cross section.

Finally, the results verify the intuition that what decreases are riskier holding, while safer ones and borrowing increases to contribute to higher economic activity and a more stable finacial system.

5 Empirical Implications

A number of indicators have been constructed to capture the business cycle. The main concern is whether they have predictive ability at each point in time. The consensus of industry and academic commentators indicates that business and financial cycles do not necessarily move together. Recessions have often been preceded by a tightening in lending standards (Lown and Morgan (2006)). Asea and Blomberg (1998) find empirical evidence for procyclicality in credit and conclude that cycles in bank lending standards are important in explaining aggregate economic activity. A measure of cyclicality in credit is the index on Bank Lending Practices produced by the Federal Reserve System's Senior Loan Officer Opinion Survey.[12] Changes in the Fed Lending Standards Index Granger-causes changes in output, loans, and the federal funds rate, but the macroeconomic variables are not successful in explaining variation in the Lending Standards Index (Lown and Morgan (2006)). It is then crucial to find variables to predict changes in the Lending Standards Index (or another indicator chosen to capture the credit cycle). [13]

According to the arguments in this paper, after periods of prolonged prosperity and optimism about future prospects, financial institutions invest in riskier assets, which can make the economic system more vulnerable in the case that default materializes. Minsky's analysis can be used to identify

[12]Various other indicators, such as credit growth to GDP or housing prices growth, have been proposed in the literature. For a broad overview that differentiates between (1) system-wide and (2) bank-specific variables, see Drehmann *et al.* (2010). According to Demirguc-Kunt and Detragiache (2005), credit booms are the best single-variable leading indicator of banking distress, and combinations of credit and asset price deviations from long-term trends are considered to be even better (Borio and Lowe (2002), Borio and Drehmann(2009)). Barrell *et al.* (2010) find that countercyclical macroprudential policy is best calibrated on house prices and current accounts. Other indicators to detect the point in the cycle are suggested, for example, by Kaminsky and Reinhart (1999), Reinhart and Rogoff (2008) and Alessi and Detken (2009). The (negative) skewness is also proposed as a measure of crash risk, for example, by Bates (1991) and Chen *et al.* (2001). Finally, Repullo *et al.* (2010) propose GDP growth as a good indicator.

[13]Gorton and He (2008) construct a Performance Difference Index based on two pieces of information, the number of loans made in the period by each rival and the default performance of each rivals' loan portfolio. If rival banks have performed better in the past, then a bank would increase its monitoring effort to identify better projects in order to compete with its rivals for higher performance. This results in tighter credit standards and a credit crunch. They show that their Performance Difference Index can predict changes in the Lending Standards Index. In our framework, it is banks ex-ante decision to invest in riskier projects due to expected relative performance considerations that should affect future credit standard once the true state of nature realizes. Thus, our framework can also account for cases that banks do not cut down on credit after good portfolio realizations, since then they would also update their expectations upwards.

the point in the leverage cycle. In particular, we constructed a theoretical model to highlight the variables that can be used to construct such an index. In our framework, as expectations become more optimistic due to good realization, banks start investing in riskier projects and increasing their leverage. Although the loss given default increases under a riskier portfolio composition, expected default and credit spreads reduce, since the expectation effect dominates. This suggest that not only credit growth, but also portfolio switches to riskier projects should be used to identify the point in the leverage cycle in combination with lower (ex-ante) risk premia.

An important element of the identification strategy is expectations formation. The effectiveness of capturing the time-varying transition probabilities between good and bad regimes should be the main objective in model selection for empirical work. One of the conjectures that can be tested is that the riskiness of the financial system increases as people become more optimistic. Another is the empirical testing of the relative performance effect. That is, when banks compete more with each other for higher expected returns, then they switch to riskier assets only after a few realizations of good news. This will provide some evidence that not only credit, but also the structure of the financial sector and the level of competitive behavior, determine the leverage cycle.

Measuring the riskiness of banking portfolios or of the financial sector as a whole over the leverage cycle is not an easy task. As highlighted in this paper, although banks engage in more risky behaviour after a period of good realizations, this results from expectations becoming more optimistic. Commonly used measures to capture risk building up, such as the volatility of banking assets or credit spreads, fail to do so due to the fact that they are biased by optimistic expectations. It is evident that market volatility as measured by the VIX index was below its long-term trend before the financial crisis. The same holds for the TED spread, i.e. the difference between the interest rates on interbank loans and short-term US government debt (figure 7).

The index we propose is the difference between riskier and safer portfolio holdings per unit of leverage. Once expectations become optimistic riskier projects are perceived less risky. But the same holds for safer ones, which are assessed as even more safe. Although absolute riskiness goes down for both types, their ranking is preserved. Consider for example risk weighted assets (RWAs) as defined by the Basel Accord II, under which risk weights follow an Internal Rate approach and change over the cycle. As mentioned, the literature on procyclicality has shown that all risk weights go down in good times as empirical data also suggest (figure 8).[14] Thus, RWA do not increase as much as they should when banks shift their portfolio towards projects previously regarded as risky. This issue may disappear once we consider the difference between projects with a higher and lower risk-weight, since their relative ranking should remain. Finally, we normalize by leverage, because it is default on debt that causes a financial crisis, tightening in credit and forced liquidations that lead to fire sales externalities. In the following diagram (fig. 9), we simulate our model for different levels of optimism and show how the proposed index can predict risk-taking and financial instability where the aforementioned and commonly used volatility measure fails. As a proxy for VIX we calculate the volatility of banking portfolios.

6 Conclusions

We argue that the perceived risk profile of investment opportunities changes over time. Financial agents are Bayesian learners and update their beliefs about future good realisations by observing the sequence of past ones. After a prolonged period of good news, expectations are boosted and financial instituions find it profitable to shift their portfolios towards projects that are on average riskier, but promise higher expected returns. Creditors are willing to provide them with funds, since their expectations have improved as well. As a result, leverage increases, risk premia go down and banking portfolios consist of relatively riskier projects. When bad news realise, default is higher and the consequences for financial stability are more severe.

We also explore the effect of higher competition among financial institutions for higher expected returns. We show that they will shift their portfolios towards riskier assets faster, compared to the case that there are no strategic considerations. This channel is mitigated by higher borrowing costs, since creditors rationally expect this behaviour.

[14] We calculate the average risk weights as the ratio of the aggregate risk weights assets over the aggregate assets of a panel of 33 international big banks. The panel includes the National Bank of Australia, ANZ, Macquarie, Dexia, China Merchants Bank, BNP Paribus, Credit Agricole, Societe Generale, Natixis, Deutsche Bank, Commerzbank, Unicredit, Monte dei Paschi, ING, Santander, BBVA, Nordea, SEB, Svenska Handelsbanken, UBS, Credit Suisse, Royal Bank of Scotland, Barclays, HSBC, Lloyds, Standard Charted, JP Morgan Chase, Citigroup, Bank of America, Wells Fargo, Bank of NY Mellon, State Street and PNC.

Moreover, we consider the effectiveness of different policy responses. Making default most costly can stabilise the leverage cycle. This is subject to two considerations; how rigid regulatory penalties are and whether policy can affect market based penalties, such as loss of reputation. An alternative is to restrict leverage or equivalently regulate a minimum level of haircuts in good times. We show that such a response does not yield the desired outcome, since banks will divert funds form safer to riskier projects. Credit markets will allow this, since expectations are overall optimistic. We propose a different regulatory response, which is to restrict the difference between riskier and safer portfolio holdings per unit of leverage during good times. This measure has the advantage that it is not biased by optimistic expectations. Although the overall riskiness of both riskier and safer assets is perceived as decreasing, their relative riskiness on average should be preserved.

Finally, we use our theory to identify a potential reason for the failure of measures, such as VIX or the TED spread, to predict the catastrophic events after the August of 2007. These measures are sensitive to prevailing expectations and are biased when financial participants become more optimistic.

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Appendix

Table 8: Initial equilbrium variables

Interest rate in $s_t = 0$	r_0=10.01%	Interest rate in $s_t = u$	r_u=8.98%
Interest rate in $s_t = d$	r_d=10.93%	Profits (reinvested) in $s_t = u$	T_u=3.15
Profits (reinvested) in $s_t = d$	T_d=2.20	Profits (distributed) in $s_t = uu$	Π_{uu}=12.55
Profits (distributed) in $s_t = ud$	Π_{ud}=1.31	Profits (distributed) in $s_t = du$	Π_{du}=3.92
Profits (distributed) in $s_t = dd$	Π_{dd}=1.51	Investment in safer asset in $s_t = 0$	$w_{0,L}$=5.56
Investment in riskier asset in $s_t = 0$	$w_{0,H}$=0	Loan amount in $s_t = 0$	w_0=4.06
Investment in safer asset in $s_t = u$	$w_{u,L}$=4.98	Investment in riskier asset in $s_t = u$	$w_{u,H}$=9.65
Loan amount in $s_t = u$	w_u=11.47	Investment in safer asset in $s_t = d$	$w_{d,L}$=5.67
Investment in riskier asset in $s_t = d$	$w_{d,H}$=0	Loan amount in $s_t = d$	w_d=3.47
Percentage delivery in state $s_t = u$	v_u=100%	Percentage delivery in state $s_t = d$	v_d=48.92%
Percentage delivery in state $s_t = uu$	v_{uu}=100%	Percentage delivery in state $s_t = ud$	v_{ud}=35.22%
Percentage delivery in state $s_t = du$	v_{du}=100%	Percentage delivery in state $s_t = dd$	v_{dd}=75.71%

Table 9: Shock 1% in the relative performance coefficient

Equilibrium variable	Change	Equilibrium Variable	Change
Interest rate in $s_t = 0$	-	Interest rate in $s_t = u$	+
Interest rate in $s_t = d$	-	Profits (reinvested) in $s_t = u$	+
Profits (reinvested) in $s_t = d$	+	Profits (distributed) in $s_t = uu$	+
Profits (distributed) in $s_t = ud$	+	Profits (distributed) in $s_t = du$	+
Profits (distributed) in $s_t = dd$	-	Investment in safer asset in $s_t = 0$	+
Investment in riskier asset in $s_t = 0$	same	Loan amount in $s_t = 0$	+
Investment in safer asset in $s_t = u$	-	Investment in riskier asset in $s_t = u$	+
Loan amount in $s_t = u$	-	Investment in safer asset in $s_t = d$	+
Investment in riskier asset in $s_t = d$	same	Loan amount in $s_t = d$	+
Percentage delivery in state $s_t = u$	same	Percentage delivery in state $s_t = d$	+
Percentage delivery in state $s_t = uu$	same	Percentage delivery in state $s_t = ud$	-
Percentage delivery in state $s_t = du$	same	Percentage delivery in state $s_t = dd$	+

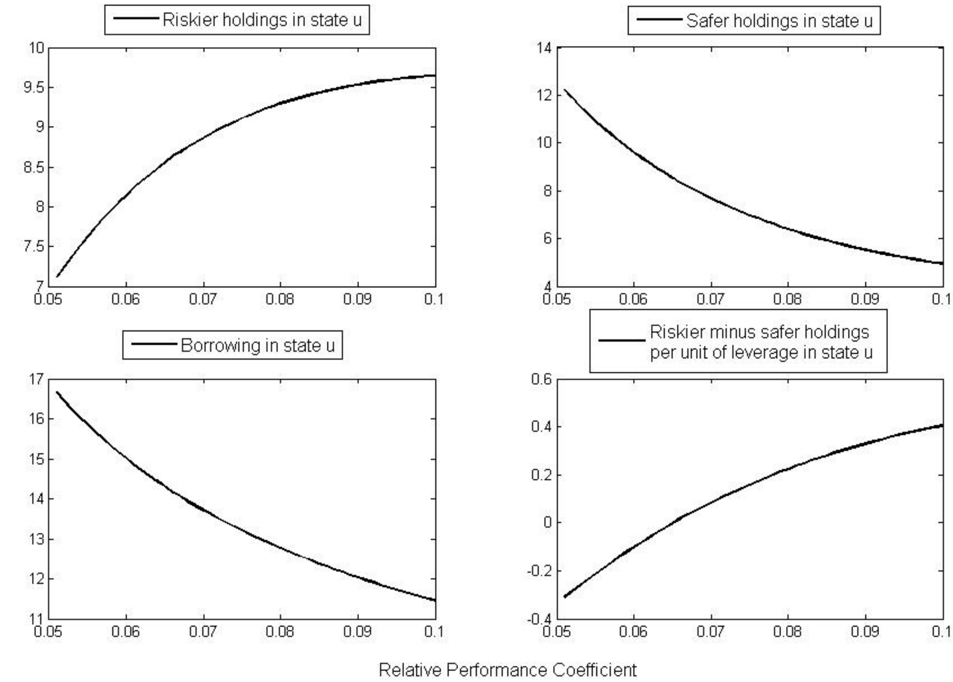

Figure 1: Portfolio holdings under low and high ω's in state u

Table 10: Shock 0.1% in π_u

Equilibrium variable	Change	Equilibrium Variable	Change
Interest rate in $s_t = 0$	-	Interest rate in $s_t = u$	+
Interest rate in $s_t = d$	-	Profits (reinvested) in $s_t = u$	+
Profits (reinvested) in $s_t = d$	+	Profits (distributed) in $s_t = uu$	+
Profits (distributed) in $s_t = ud$	-	Profits (distributed) in $s_t = du$	-
Profits (distributed) in $s_t = dd$	-	Investment in safer asset in $s_t = 0$	+
Investment in riskier asset in $s_t = 0$	same	Loan amount in $s_t = 0$	+
Investment in safer asset in $s_t = u$	-	Investment in riskier asset in $s_t = u$	+
Loan amount in $s_t = u$	-	Investment in safer asset in $s_t = d$	-
Investment in riskier asset in $s_t = d$	same	Loan amount in $s_t = d$	-
Percentage delivery in state $s_t = u$	same	Percentage delivery in state $s_t = d$	+
Percentage delivery in state $s_t = uu$	same	Percentage delivery in state $s_t = ud$	-
Percentage delivery in state $s_t = du$	same	Percentage delivery in state $s_t = dd$	+

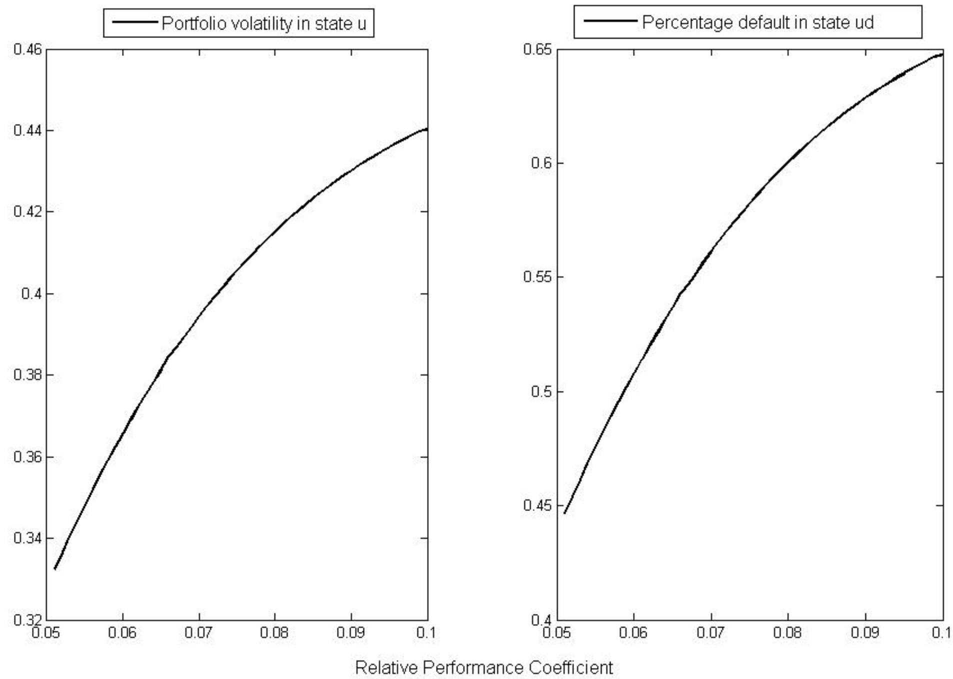

Figure 2: Risk-taking and default for low and high relative performance coefficient

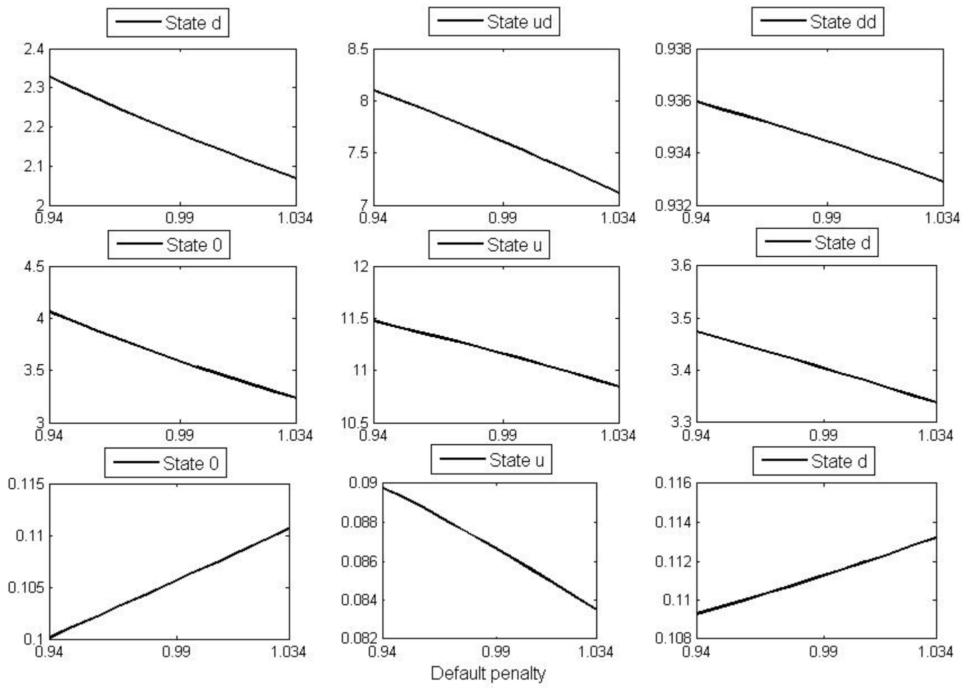

Figure 3: Loss given default (top), borrowing (middle) and interest rates (bottom) under various default penalties in state *ud*

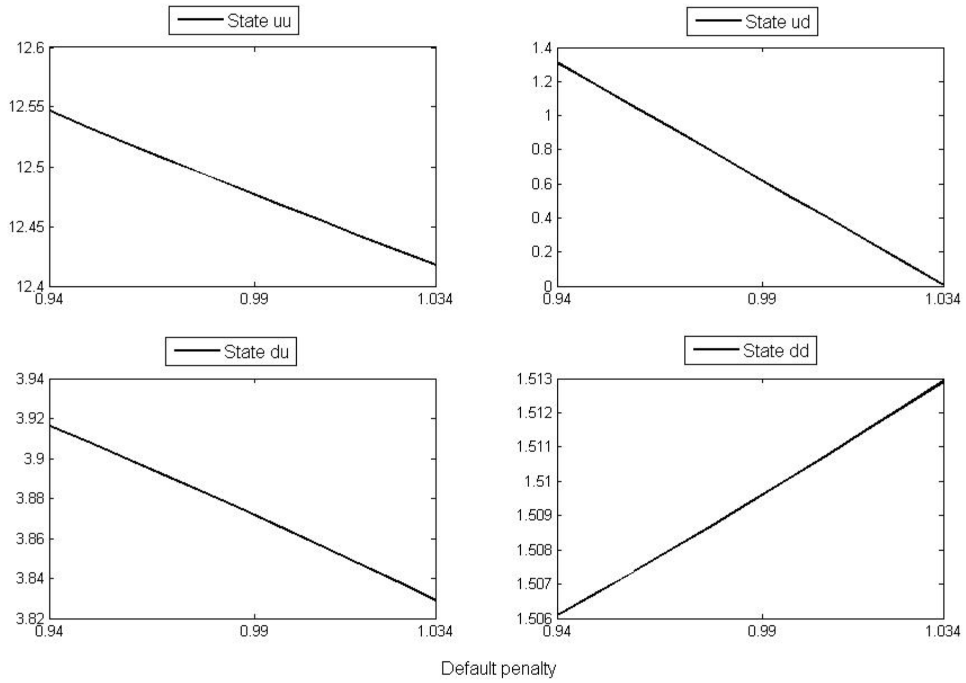

Figure 4: Banking profits under various default penalties in state *ud*

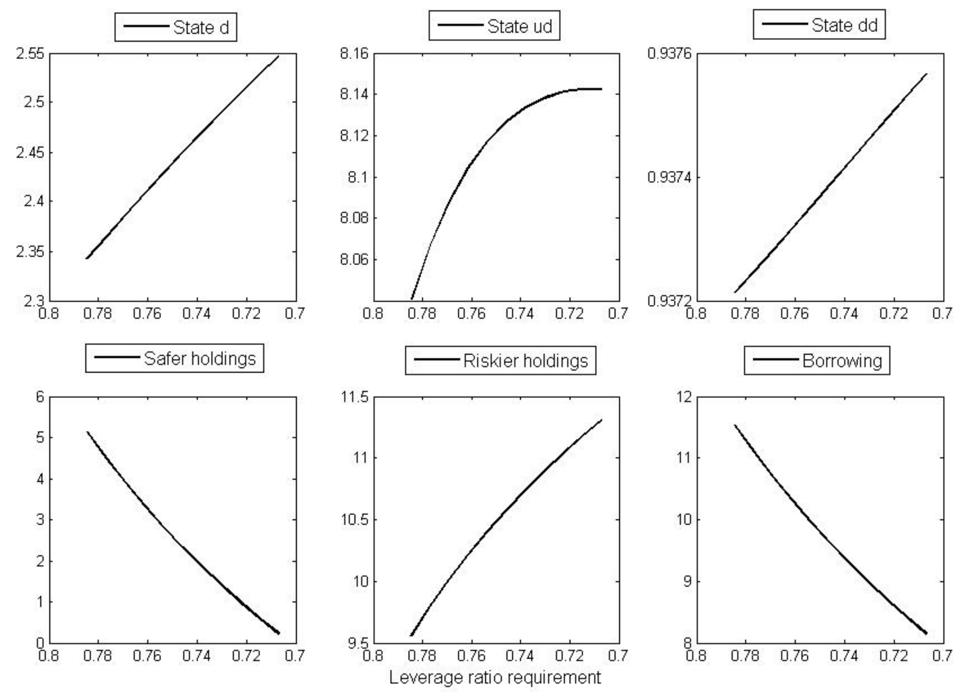

Figure 5: Loss given default (top) and portfolio holdings (bottom) under various leverage requirement in state *u*

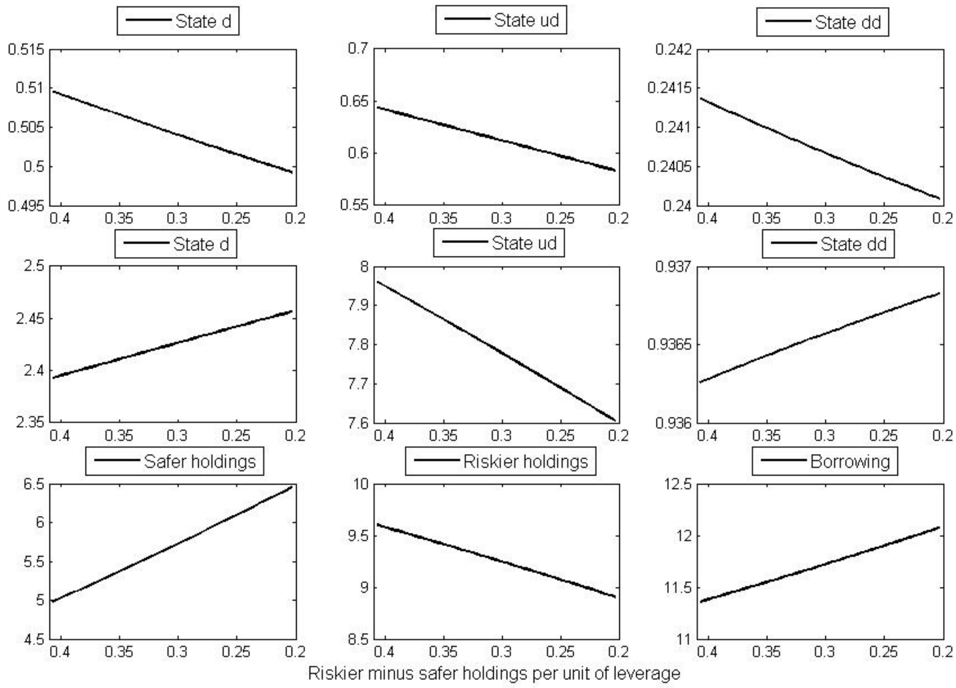

Figure 6: Percentage default (top), loss given default (middle) and portfolio holdings in state u

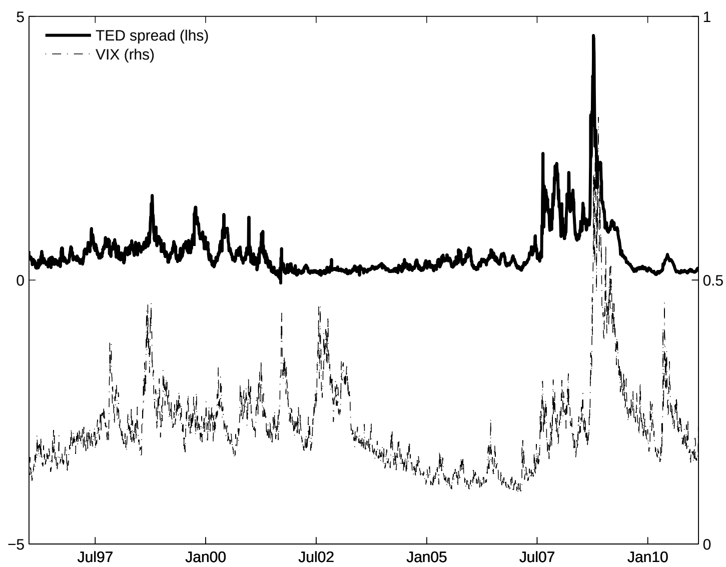

Figure 7: VIX and TED spread evolution over time

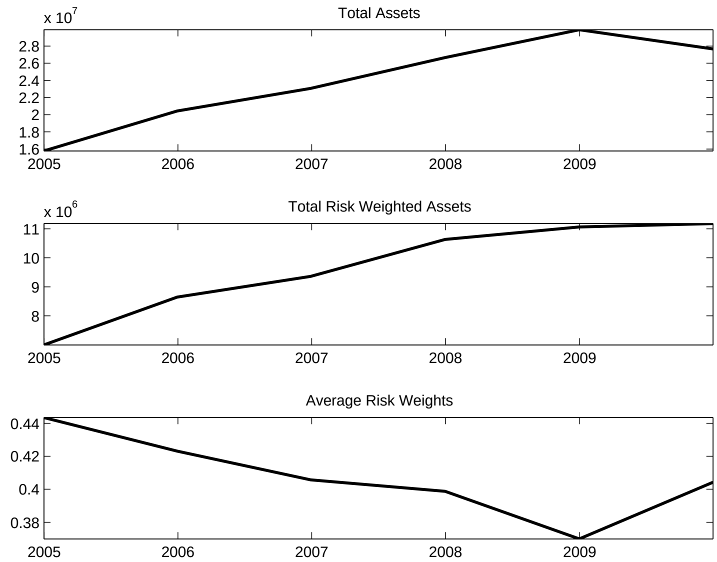

Figure 8: Average risk weights evolution over time

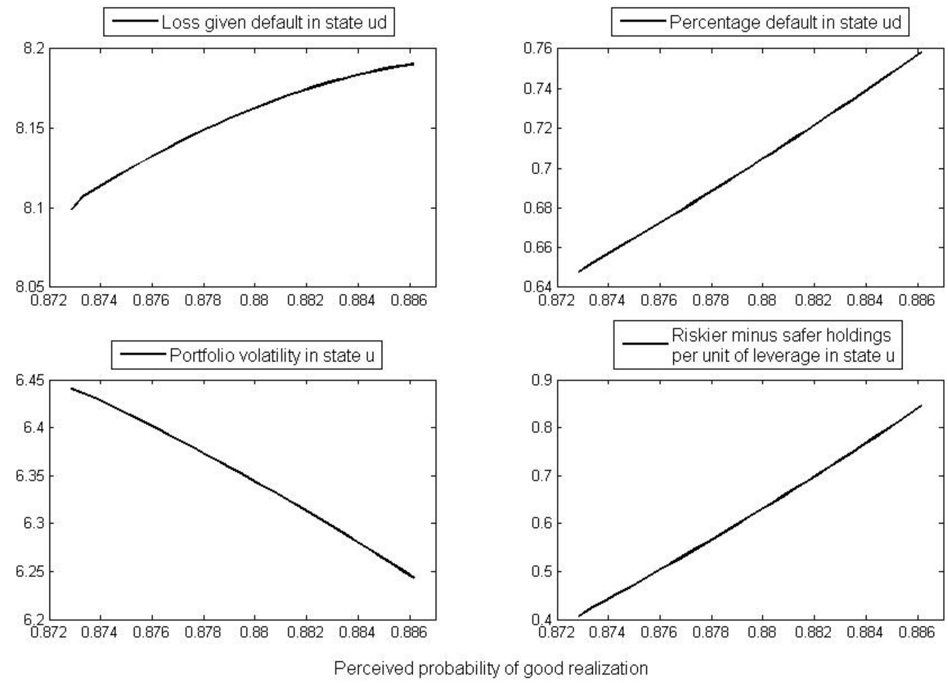

Figure 9: Optimism and riskiness