Subsidizing Banking Instability[*]

Hendrik Hakenes[†]

University of Bonn, Max Planck Bonn, and CEPR

Eva Schliephake[‡]

University of Bonn

March 17, 2015

Abstract:

The extent of the fragility to runs of financial institutions depends—among others—on its liquidity base: the stock of short term funds available to the bank for investment at any given time regardless of withdrawal option available to customers. A subsidy to some financial intermediaries, or a discrimination of others, distorts the distribution of the liquidity base in the banking system because it introduces a pecking order of deposit withdrawals between banks. Using the standard global games approach, we show that subsidized banks are less prone to panic runs, but the stability of the residual banks decreases. The aggregate stability of the banking system may decrease as a consequence of a subsidy.

Keywords: Bank Runs, Deposit Base, Global Games, Financial Stability

JEL-Classification: G21, G28, H23,

1   Introduction

The decision of financial investors to roll-over short term debt often inherits strategic complementarity. When investors try to coordinate their decisions based on the observed information and other investors' behavior, runs can occur based on panics in addition to fundamental information. In this paper, we discuss how governmental intervention for particular institutions or products may lead to an implicit coordination of investor decisions that establishes a pecking order in roll-over decisions. As a result, subsidized institutions gain liquidity that they can use for longer-term investments, while the liquidity base of the residual institutions is shortened. As a consequence, the latter become more fragile to panic-based runs.

Our framework can be applied to various types of institutions, which are prone to panic-based runs and where exogenous interventions distort the distribution of liquidity. Examples of such a governmental intervention are subsidized funds that attract long term savings, considerations to ring-fence retail banks' demand deposits from other activities or exclusive insurances for particular institutions. In our model, we concentrate on the example of a subsidized savings fund that lures long term savings from the residual banks and thereby erodes the deposit base of the residual commercial banks. This particular example is motivated by the European Commission's proposal to create a European savings fund that attracts private long term savings at a central European institution through a guaranteed interest rate and tax benefits.[1]

The idea to subsidize long term savings is not new. In times of low interest rate policy, households are reluctant to invest their savings on fixed terms in long term investments. However, the low interest rate policy is usually a reaction to low levels of growth and innovation. Stimulation of growth and innovation in turn requires long term investments. A way to escape this vicious circle may be a subsidy on private long term savings - so the idea.

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[1]Michel Barnier, Member of the EC in charge of Internal Market and Services, presented on 27 March 2014, a package of measures to stimulate Europe's economic growth. Among others, he presented the idea of a subsidized European savings account that collects long term savings at a central European institution to reallocate them towards the needs of small and medium sized enterprises.

One ancient example of such a subsidized savings account is the french Livret A. It was introduced 1818, after the Napoleonic wars, to restimulate private savings in France. This subsidized savings account still exists today and guarantees a fixed rent on long term savings. Moreover no taxes need to be paid for up to € 20,000 of deposited savings. The collected private savings from the Livret A are provided to the public Caisse des Dépôts and used for long term public investments.

Similarly, Canada introduced the Tax-Free Savings Account (TFSA) in 2009. In the US, the Consumer Federation of America (CFA) recently encouraged the federal government to subsidize interest rates on savings accounts to stimulate savings among the low income savers.[2]

So far, the direct and indirect effects of a subsidized savings fund on the competing banks and the stability of private banking sector is neither properly analyzed nor well understood. This paper offers a first assessment of the problem. There may exist direct competitive distortions from higher subsidized interest rates but we have a different focus.[3] We focus on the particularity of the banking sector: its intrinsic fragility due to fundamental and panic-based bank runs. We raise the question of how an asymmetry in banks' refunding opportunities affects the stability of the overall banking sector?

Our argument is the following: As argued by Diamond and Dybvig (1983) households prefer demand deposit contracts because they do not know ex-ante when they will need to consume. Demand deposits offer a way to invest in liquid assets while still earning a decent return. They are therefore superior to the market solution. Banks pool the deposits of all households and diversify idiosyncratic consumption shock risks. In each period, only a small proportion of the demand deposits are actually withdrawn to satisfy consumption needs. A considerable stock of deposits remains at the bank. This "deposit base", the proportion of deposits that are not withdrawn in each period, is used by the banks for long-term investments.

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[2]http://www.cbsnews.com/news/good-news-in-banking-traditional-savings-accounts

[3]The announcement of the European Commission to introduce a European savings account received vigorous reactions from the private banking sector. Particularly commercial banks that are specialized in collecting deposits from households to invest them in SME loans claim that they loose market share in their most important source of funding. Moreover, commercial banks argue that relationship lending is their core competence and a centralized institution as the EIB will be less efficient in the allocation of household savings to long-term investments.

In the presence of a subsidy on the long term return of a certain share of the households' savings (as the €20,000 for the Livret A), the households invest the allowed share at the subsidized bank, which pays higher rents. The residual share of households' savings is invested at the residual banks offering market rents. Households that are hit by a consumption shock now have a strict preference to withdraw their deposits first from the unsubsidized bank account and leave as much as possible at the subsidized account that offers higher long term interest rate. This pecking order in deposit withdrawals decreases the deposit base of the unsubsidized banks and increases the liquidity of the subsidized fund. This increases the stability of the subsidized fund. The lower deposit base destabilizes the residual banks, making them more fragile to panic-based bank runs. We show that the total effect on the stability of the banking system is likely to be negative.

We base our analysis on the Diamond and Dybvig (1983) model that shows that banks provide households with an insurance against idiosyncratic consumption shocks. As long as these shocks are not perfectly correlated, banks are able to provide better return-revenue combinations than the market. However, demand deposits that provide a liquidity insurance make banks vulnerable to bank runs where more than the expected fraction of depositors withdraws their deposits. Instead of the Diamond and Dybvig (1983) we could have also used other models that justify the existence of short term lending and bank crisis. Diamond and Rajan (2001) argue that the link between short term lending and bank fragility has a reverse causality. Banks that want to provide liquidity and loans to risky borrowers have to borrow short-term because the threat of a bank run from short term borrowing prevents banks from renegotiating contracting terms. Another justification of demand deposits is given Gorton and Pennacchi (1990) who argue that banks optimally offer demand deposits to uninformed agents because they are riskless such that their value does not depend on the information known only by informed agents.

In the classic Diamond and Dybvig (1983) style models bank run equilibria occur as self-fulfilling beliefs that are rather unrelated to fundamentals. The model lacks an explanation of the determinants and likelihood of each type of equilibria. In order to analyze the impact of a subsidy on the stability of the subsidized and the competing financial institutions we use a global games approach. This approach was developed by Carlsson and van Damme (1993) and allows a tighter analysis of panic-based models. Our model is based on Goldstein and Pauzner (2005) who apply the global games approach to a bank-run setting. This allows us to analyze the relationship

between the deposit base and the likelihood of inefficient bank runs.

In the model setup we focus on uninsured deposits as an extreme cause of bank fragility. Like in all run-based models, the bank fragility would disappear if a government could credibly guarantee to pay any liability at each point in time either by a perfect deposit insurance or with bail-out policies. Yet, our setup is based on a Macroeconomic shock that affects the economy as a whole and, thus, also the entire banking sector. Therefore, an interbank based liquidity insurance would not mitigate the run incentives, since all banks suffer low expected returns and therefore higher bank run probabilities. Likewise, a macro-shock also affects the tax income of a government. Therefore, also the government might not be able to rescue the entire banking sector in case of a severe recession. An anecdotal example that governments are not able to fully bail out a banking system after a macro shock can be found in the Bail-in in Cyprus that included senior unsecured debt and even deposits. Therefore, even with state guarantees and deposit insurance, we would get the same qualitative results as long as gurantees and insurance is not perfectly covering the liabilities from depositors after a macro shock. For simplicity we focus on the extreme case without any guarantee or deposit insurance. Of course the quantitative results change with the coverage of a state guarantee/deposit insurance. A partly insured banking system becomes less fragile to bank runs. The impact of an imperfect deposit insurance and state guarantees is discussed in chapter 7.

We proceed as follows. In chapter 2 we introduce our adapted version of the Diamond and Dybvig (1983) model. Our adapted version does not require the stark assumption of high relative risk aversion but still leaves a role for deposit banks, which we show in chapter 3. In chapter 4 we introduce the global game framework, and show in chapter 5 that the biased preferences of consumers to withdraw from one bank to satisfy their consumption needs destabilizes the bank considerably even when banks actually pay the same interest rates. At the same time, the probability of an inefficient bank run at the subsidized bank decreases. However, our results indicate that the aggregate probability of inefficient bank runs increases. The introduction of a subsidy influences the withdrawal preferences of depositors and, thereby, increases the instability of the overall banking sector. In chapter 6 we discuss the effect of a subsidy on the preferences to withdraw in more detail. To close the model we show that the subsidy can be refunded immediately by a flatrate tax. Therefore, the expected utility of households remains unaffected by the subsidy. Nonetheless, the pecking order in withdrawals affects the allocation of liquidity in the banking

system. In particular it reduces the deposit base of commercial banks which decreases their stability more than the gained stability from the more stable fund. The aggregate stability of the banking system decreases. This overall reduction in the banking system's stability is a general result from the asymmetric allocation of liquidity resulting from the pecking order in deposit withdrawals. Interestingly, the same results apply to a single bank that is forced to offer different conditions for separated businesses. A ring fenced subsidized bank account that is separated from the bank's residual business would therefore decrease the overall stability of the bank, which we discuss in chapter 7. We also analyze the impact of a deposit insurance. Moreover, we analyze how the relative size of the unequally treated sector may interfere with the distortion in the aggregate stability.

2   The Model Setup

Consider a economy with one good and three dates ($t = 0, 1, 2$), a continuum of consumers of mass 1, and a continuum of banks.

Consumers.   Consumers are endowed with 1 unit of the good at date 0, which they can deposit at a bank. Consumers are risk neutral and want to consume at date 1 or date 2; their utility function is

$$u(c_0, c_1, c_2) = c_1 + c_2. \tag{1}$$

Ex-ante, consumers are identical. Some consumers, called impatient consumers, receive a consumption shock in $t = 1$. The residual consumers are called patient.

In the classical Diamond and Dybvig (1983) style framework the consumption shock is binary: impatient consumers withdraw their entire deposits, while patient consumers do not consume anything in $t = 1$ but everything in $t = 2$.

A somehow more realistic assumption is that impatient consumers are heterogenous: Some consumers want to withdraw their complete deposit, some only consume a part of their deposited savings. A consumer might get the opportunity to buy a rare, expensive car that he enjoys to ride, or could become sick and spend his entire savings

on a therapy, restoring his health.[4] Other consumers may also have immediate but smaller consumption desires. They might need to fix their broken car or buy some medicine to recover. Each immediate consumption gives the particular consumer a high utility , but only up to the level needed to satisfy the consumption shock. Any deposit withdrawal beyond the consumption shock would only limit future consumption possibilities. We need this realistic assumption on heterogeneity in consumption shocks build up our argument that a pecking order in withdrawals results in an asymmetric allocation of the deposit base in the banking system.

Therefore, we deviate from Goldstein and Pauzner (2005) and Diamond and Dybvig (1983) by introducing high and low consumption shocks: A fraction η of consumers has the opportunity to consume up to $H > 1$ units and receive a utility of $\mu > 1$ (we give a stricter condition on μ below).[5] These are called H-consumers. A fraction λ of consumers has the opportunity to consume up to $L < 1$, which also produces a utility of μ.[6] They are called L-consumers. We assume that $\lambda L + \eta H < 1$. i.e., the proportion of impatient consumers is sufficiently low such that there remains a deposit base for long term investments even if all impatient consumers fully satisfied their consumption needs. The remaining fraction of consumers, $1 - \eta - \lambda$, does not experience a consumption shock. They are called patient consumers.

In contrast to Diamond and Dybvig (1983) (D&D) demand deposits are not necessary to insure consumers from consumption shocks, but in our model demand deposits minimize opportunity costs from forgone consumption possibilities. This is a stark simplification Diamond and Dybvig (1983) that is not necessary for our results but allows us to get explicit solutions for the bank run probabilities later on. Clearly, our results hold for any more general model version in the spirit of Diamond and Dybvig (1983).[7] In D&D the optimal contract results from the assumption that

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[4]This interpretation is similar to the modelling of loan demand in section 4.4 of Tirole (2006).

[5]The divisibility of the H and L is assumed for convenience and is not crucial. It allows for a continuous piecewise linear utility function for each type that has a kink at the maximum private investment opportunity for impatient consumers and is therefore quasi-concave.

[6]This assumption is sufficient but not necessary. To generate our results it is crucial that impatient consumers are heterogenous in a way that some consumers withdraw a smaller amount early than the others. Hence, the necessary assumption is $L < H$. However, the somewhat stricter assumption that $L < 1$ generates heterogeneity in the withdrawal behavior of consumers without bank deposits and is made for illustrative purposes.

[7]In particular, our arguments hold for any model setup where the optimal contract requires

consumers have a relative risk aversion that is greater than the liquidation value of the asset $RRA > 1$. This implies that the optimal contract should offer impatient consumers more than the technologies are able to produce. Our admittedly stark assumption of an arbitrary $H > 1$ reflects the very same idea that impatient consumers optimally require more than the technology is able to produce. In other words, we simplify the restrictive assumption on the curvature of the utility function in D&D to a simple restriction on the kink of the quasi-concave utility function.

Investment. There is a single risky investment technology. Per unit of investment at date 0, it returns $R > 1$ at date 2 with probability $p(\vartheta)$, otherwise it returns zero. The investment can be liquidated at date 1, in which case it returns 1. Partial liquidation is possible.

The variable ϑ is uniformly distributed on the unit interval, $\vartheta \sim [0,1]$ and represents the state of the economy. We assume that $p(\vartheta)$ increases in ϑ and $p''(\vartheta) \geq 0$. This assumption reflects an environment in which high probabilities of successful investments are relatively more likely than low success probabilities. Intuitively, we have in mind a non-crises banking system where the asset portfolio of banks generates returns with high success probability and very high probabilities of default are rare. In other words, the probability is high to experience a good state of the world, while it is rather unlikely that the economy experiences very bad states. A more detailed analysis of the distribution of $p(\vartheta)$ can be found in Appendix A.1.

Furthermore, define $\bar{p} = E[p(\vartheta)]$ the ex ante probability of long-term investment success. Assume that $\mu > \bar{p} R > 1$; the expected return at date 2 exceeds the liquidation value but is lower than the gross return from the private investment opportunity.

Information. The state of the economy ϑ is realized at date 1, but does not become public information. Instead, each consumer gets a private signal

$$x_i = \vartheta + \varepsilon_i, \tag{2}$$

where ε_i is a stochastically independent private error term that is uniformly distributed over the interval $[-\varepsilon, \varepsilon]$. Consumers observe the signal, then decide whether or not to withdraw their deposit from the bank.

$1 < c_1^* \leq c_2^* < R$.

3 The Role of Deposit Banks

Our utility structure deviates from the classic Diamond and Dybvig (1983) model. Therefore, we need to make the point that deposit banks are beneficial for consumers at all. We therefore compare the expected utility of households in the absence of financial markets and banks. We show that households are better off when they pool their savings at a bank compared to trading their claims in a financial market. Let us therefore first abstract from any private information, (i.e., discuss the extreme case of $\varepsilon \to \infty$). Households know nothing about the future performance of the investment, and hence cannot base their decisions on this information.

Autarky. Assume that all households are isolated and can neither trade their claims on investments in a market nor pool their endowments in a bank. At date 0, all households invest their unit endowment into the risky investment technology that produces an expected return of $\bar{p} R > 1$ at date 2.

At date 1 consumers learn their type and observe ϑ with some noise. If the associated expected return $p(\vartheta)R < 1$, it is optimal for all types of consumers to liquidate their investments and consume right away. If $p(\vartheta)R \geq 1$, H-consumers want to consume H but can only liquidate their entire investment, which equals 1. L-consumers want to consume L and liquidate a fraction $L < 1$ of their unit investment and retain $1 - L$ risky asset investment. Patient consumers do not liquidate their investment.

At date 2 the non-liquidated investments return R if they are successfulf and households consume. The ex ante households' expected utility under autarky is

$$u(c_0, c_1, c_2) = (\mu - \bar{p} R) (\eta + \lambda L) + \bar{p} R. \tag{3}$$

Financial Markets. Assume now, that at date 1 a market to trade claims on investments opens, whereby P units of the good at date 1 are exchanged against the promise to receive 1 unit of the good at date 2. Again, all households invest their entire endowment in the risky technology. At date 2 impatient consumers sell at the market or liquidate their investment if they can pursue their private investment.

In an arbitrage free market the equilibrium price for a claim on the risky investment at date 1 is $P = \frac{1}{\bar{p} R}$. By selling one claim at date 1, the household can obtain

$P \bar{p} R = 1$. Again, H-consumers, cannot consume H because $1 < H$. L-consumers sell a proportion of L bonds on their risky investments. They obtain L to consume and retain $1 - L$ as risky asset investment.

At date 2 risky investment returns realize. All households consume. Ex ante, the expected utility of households with a financial market is

$$u(c_0, c_1, c_2) = (\mu - \bar{p} R)(\eta + \lambda L) + \bar{p} R. \tag{4}$$

The introduction of financial markets does not affect the household's expected utility compared to autarky.

Deposit Banks. In analogy to the Diamond and Dybvig (1983) model, the households can increase their expected utility by pooling their endowments in a bank that invests on their behalf. The bank offers in return a payment $r_1 > 1$ at date 1. This allows all impatient consumers to pursue their private investments, which increases, *ex ante*, the expected utility of all households.

At date 0, the bank offers a deposit contract that promises some fixed r_1 per unit of investment at date 1, and a stochastic r_2 per unit of non-withdrawn investment at date 2. Banks are assumed to operate under perfect competition. This implies that at date 2, the bank must distribute their complete revenues. More precisely, r_2 will depend on whether the project is successful, and on the fraction of depositors that have already withdrawn at date 1. The bank invests all collected endowment in the risky technology. The optimal short term interest payment is $r_1 = H$ because it allows all impatient consumers to consume H.

At date 1 consumers learn their type. H-consumers withdraw their entire deposit and receive $r_1 \cdot 1 = H$. L-consumers withdraw only the fraction y of their deposits to consume L. The optimal fraction that L-consumers withdraw is $y = \frac{L}{H}$. Patient consumers do not withdraw their endowments and simply wait until they receive r_2 as long as $E(r_2) > H$.

At date 1, the bank liquidates a fraction $(\eta + \lambda y) r_1 = \eta H + \lambda L$ of its risky investments to satisfy the liquidity needs of impatient consumers. At date 2, in case of success, the bank can pay

$$r_2 = \frac{(1 - (\eta + \lambda y) r_1)R}{1 - (\eta + \lambda y)} = \frac{(1 - (\eta H + \lambda L)R}{1 - (\eta + \lambda \frac{L}{H})} \tag{5}$$

to patient and L-consumers in proportion to their residual deposits $(1 - y)$.

Patient consumers and L-consumers withdraw only the minimum amount at date 1 if their inter-temporal incentive constraint is satisfied. Because patient consumers are risk-neutral, and L-households are risk neutral on any payment above L, consumers are only interested in the expected payment at date 1 in the case of withdrawal. Consequently, banks in our setting do not need a pro rata rule. Such a rule would not change expected utility, or influence the consumers' strategic situation. It is sufficient to assume that the expected return for waiting to withdraw is higher than the immediate certain return from withdrawal: $\bar{p} r_2 \geq r_1$. The risk neutral incentive constraint can be summarized in the requirement

$$\bar{p} R \geq \frac{H - (\eta H + \lambda L)}{1 - (\eta H + \lambda L)}. \tag{6}$$

The incentive constraint is satisfied for high expected returns on the risky investment and relatively low levels of H and L or low proportions of impatient consumers, respectively.

The existence of a deposit bank allows depositors to obtain an expected utility of

$$u(c_0, c_1, c_2) = (\mu - \bar{p} R)(\eta H + \lambda L) + \bar{p} R. \tag{7}$$

The net gain in expected utility from a deposit bank compared to both, autarky and a financial market, is $(\mu - \bar{p} R) \eta (H - 1)$, which is strictly positive under our assumptions.

As in Diamond and Dybvig (1983) deposit banks are desirable in our adapted model because they increase social welfare. Banks allow consumers to satisfy immediate liquidity needs. However, this comes at a cost. Banks are fragile to bank runs because the contracted payment exceeds the liquidation value of all bank assets $r_1 > 1$ such that a bank will be insolvent if all depositors withdraw at the same time.

In the following, we extent our basic model to the case of private information. In particular, we assume as discussed above that each consumer gets a private signal $x_i = \vartheta + \varepsilon_i$, where the ε_i are stochastically independent private error terms that are uniformly distributed over the interval $[-\varepsilon, \varepsilon]$. We then discuss the effect of withdrawal preferences on bank stability.

4   The Equilibrium for Symmetric Banks

We first calculate the equilibrium for a representative bank. The bank offers a deposit contract that promises some fixed r_1 per unit of investment at date 1, and a stochastic r_2 per unit of non-withdrawn investment at date 2.

We have already shown that the optimal short-term gross deposit rate is $r_1 = H$. That way, H-consumers can buy their desired consumption good. L-consumers will withdraw their target consumption L for sure. Consequently, the aggregate minimum amount of withdrawals is $\eta\,H + \lambda\,L$. Because $r_1 = H$, this is equivalent to a number of $\eta + \lambda\,\frac{L}{H}$ consumers withdrawing their deposit completely. Let n define the fraction of consumers that withdraw their deposit. If some withdraw only partially, then these are counted only partially into n. Consequently, $n\,r_1 = n\,H$ is the amount of the good withdrawn from the bank at date 1. Because the fraction η withdraws the deposit completely, and the fraction λ withdraws the part L/H, the minimum n is $n_{\min} = \eta + \lambda\,\frac{L}{H}$. In addition, some depositors may withdraw strategically, depending on the private information they receive.

There are realizations of the fundamental variable that are so low (high) that no matter of what other depositors do, it is the dominant strategy of each depositor to withdraw (not withdraw) its deposits. These regions of realizations of the fundamental variable are called lower (upper) dominance region. Goldstein and Pauzner (2005) show that the equilibrium in depositor's withdrawal decision is unique if lower and upper dominance regions of arbitrary size exist.

Lower dominance region.   When fundamentals are bad, the expected return at date 2 even with minimum liquidation at date 1 can be below the certain return of date 1 withdrawal. In contrast to Goldstein and Pauzner (2005) we have two types of patient consumers to consider: patient consumers and the L-consumers that have to decide to fully or only partially withdraw their deposit. The decision to withdraw fraction $\frac{L}{H}$ is non-strategic, i.e., it is independent of the behavior of other depositors: L-consumers simply withdraw $\frac{L}{H}\,H = L$ for consumption at date 1. Only the withdrawal decision on the remaining share of their deposits $(1 - \frac{L}{H})$, depends on the behavior of other depositors. We therefore call this fraction the strategic share. We show that the decision on the strategic share is identical for patient and impatient consumers.

Consider first patient customers. We denote by $p(\underline{\vartheta})$ the realized success probability that solves $r_1 = p(\vartheta) \, r_2$. Using the equilibrium values for the repayments we get:

$$H = p(\vartheta) \left(\frac{1 - (\eta + \frac{L}{H}\lambda) \, H}{1 - (\eta + \frac{L}{H}\lambda)} \, R \right).$$

(8)

Solving for $p(\underline{\vartheta})$ yields

$$p(\underline{\vartheta}) = \frac{H - (\eta \, H + \lambda \, L)}{R \, (1 - (\eta \, H + \lambda \, L))}.$$

(9)

Consider now the lower dominance region of L-consumers. The critical success probability for the lower dominance region is implicitly defined by

$$\mu \, L + (1 - \frac{L}{H}) \, H = \mu \, L + p(\vartheta) \left(1 - \frac{L}{H} \right) \left(\frac{1 - (\eta + \frac{L}{H}\lambda) \, H}{1 - (\eta + \frac{L}{H}\lambda)} \, R \right).$$

(10)

L consumers receive utility $\mu \, L$ from their private investment opportunity no matter if they withdraw their residual deposit or not. The strategic decision to withdraw only concerns the residual deposit fraction $(1 - \frac{L}{H})$. It is straightforward to see that this strategic decision is identical to the consideration of patient consumers except for the scaling factor $(1 - \frac{L}{H})$. The critical success probability is again

$$p(\underline{\vartheta}) = \frac{H - (\eta \, H + \lambda \, L)}{R \, (1 - (\eta \, H + \lambda \, L))}.$$

(11)

To illustrate the critical success probability we take a numerical example (see Appendix A.5). In that case, we get $p(\underline{\vartheta}) = 0.35$.

For the lower dominance region to exist, there must be feasible values of ϑ for which all patient and L-depositors receive signals that clearly indicate that they are in that region. We have assumed that the noise ε of the signal x_i is uniformly distributed over the interval $[-\varepsilon, \varepsilon]$. Patient and L-consumers will therefore find it always optimal to withdraw (no matter what others do) their entire deposit if they observe a signal $x_i < \underline{\vartheta} - \varepsilon$, $\vartheta < \underline{\vartheta} - 2\varepsilon$.

Upper dominance region. The upper dominance region corresponds to realizations of the fundamental variable, for which it is never optimal for patient L-consumers to withdraw early the deposits that are not necessary for private investments. As Goldstein and Pauzner (2005) we assume that such an upper dominance region exists for a range $(\bar{\vartheta}, 1]$. Following their argumentation, we also assume that

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the liquidation value increases in very good states. Intuitively, a very high ϑ results in a certain return of R in future, which also affects the liquidation value at date 1. Assume that the certain return of R in future increases the liquidation value of the asset at date 1 R_1 to $H \leq R_1 < R$. For low and intermediate states $\vartheta \in [0, \bar{\vartheta}]$ we therefore assume the liquidation value to be equal to 1 and for $\vartheta \in (\bar{\vartheta}, 1]$ we assume it increases to $R_1 \geq H$. For extremely good states of the world not only the long term investment returns are high but also short term returns increase. The intuition is that for these very good states of the world outside investors would be willing to buy the claims of the certain asset for a price close to the This assumption makes sure that there are states of the economy where returns are so high, that banks would always be able to repay their liabilities even if a run occurs. Put differently, in very good states it would never be beneficial to run, even if all other depositors withdraw their endowments. For these fundamentals it is the dominant strategy for patient consumers to wait and withdraw only at date 2.

A similar interpretation could be, that for extremely good states of the economy, the government can collect so much taxes that it will bail out depositors at each date making it unnecessary for depositors to run. Denote $\bar{\vartheta}$ the infimum of realizations of the fundamental that result in run-proof banks. If a depositor receives a signal that indicates that all other depositors are in the upper dominance region and know that their investments are save, there is no reason to run and liquidate deposits at date 1. The upper dominance region, therefore, exists if $\bar{\vartheta} < 1 - 2\varepsilon$. In that case all depositors receive a signal indicating they are in the upper dominance region.

Intermediate region. Here, we concentrate on the intermediate region, in which the consumer's private information is crucial.

Consider a late consumer who thinks about withdrawing early. For L-consumers, who have the choice between withdrawing partially or completely, the analysis is identical (scaled down by a factor). There are two fundamentally different cases. If $n > 1/r_1 = 1/H$, the bank must liquidate its investment completely; it is insolvent. Consequently, if the depositor withdraws, he still gets $1/n$ in expected terms, leading to a utility of $1/n$. If he does not withdraw, he gets nothing, leading to an expected utility of 0. The analysis is more interesting for the case that the bank is liquid. If the depositor withdraws, he gets $r_1 = H$. If he does not withdraw, he gets his share of the final outcome. Because n depositors have already withdrawn, the bank had

to liquidate $n\, r_1 = n\, H$, the remaining $1 - n\, H$ lead to a return of $(1 - n\, H)\, R$ with probability $p(\vartheta)$. Because there are $1 - n$ consumers left who have not yet claimed a repayment, the consumer gets an expected $\frac{1 - n\, H}{1 - n}\, R$ with probability $p(\vartheta)$, leading to an expected utility of

$$p(\vartheta)\, u\!\left(\frac{1 - n\, H}{1 - n}\, R\right) = p(\vartheta)\, \frac{1 - n\, H}{1 - n}\, R. \tag{12}$$

Now in equilibrium, there is a critical level x^* such that all consumers with private signals $x_i < x^*$ withdraw, those with $x_i \geq x^*$ leave their deposit at the bank. This x^* is defined such that the critical consumers (that with private information $x_i = x^*$) is indifferent: the expected utility when withdrawing the deposit equals the expected utility when leaving the deposit at the bank. As in Goldstein and Pauzner (2005), the indifferent depositor is implicitly defined at the point where differences in the consumers' utility from withdrawing are zero:

$$0 = \int_{\eta + \lambda\, \frac{L}{H}}^{\frac{1}{H}} \left(p(\vartheta^*)\, \frac{1 - n\, H}{1 - n}\, R - H\right) dn + \int_{\frac{1}{H}}^{1} \left(0 - \frac{1}{n}\right) dn, \tag{13}$$

The first integral describes the difference in expected utility from not withdrawing and withdrawing at date 1 in case there is no bank run, i.e., $n < \frac{1}{H}$. If the depositor does not withdraw, but n deposits are withdrawn at date 1, he receives $\frac{1 - n\, H}{1 - n}\, R$ with probability $p(\vartheta^*)$. When withdrawing, he receives H. The second integral describes the difference in expected utility from not withdrawing and withdrawing the deposit in case that there is a bank run. If there is a (full) bank run $n > \frac{1}{H}$ at date 1, the patient depositor receives zero at date 2 because all assets were liquidated. If he also runs at date 1, he receives $\frac{1}{n}$

In the limit $\varepsilon \to 0$, the ϑ^* below which the bank becomes illiquid is approximately identical to the critical x^*.

Proposition 1 *The model has a unique equilibrium, in which patient consumers will run (withdraw) if they observe a signal below threshold x^*, and will not run above. In the limit $\varepsilon \to 0$, x^* is equal to ϑ^* as defined by the equation*

$$p(\vartheta^*) = \frac{1 - (\eta\, H + \lambda\, L) + \log H}{\left(1 - (\eta\, H + \lambda\, L) + (H - 1) \log\left(\frac{H - 1}{H(1 - (\eta\, H + \lambda\, L))}\right)\right) R}. \tag{14}$$

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For concreteness, take a numerical example (see Appendix A.5). In that case, we get $p(\vartheta^*) = 0.6838$. If the probability that the investment will return R is below 68.38%, consumers will panic and run on the bank. We cannot say anything about the probability of a run because we have not specified the function $p(\vartheta)$. In the simplest case, $p(\vartheta) = \vartheta$, not only ϑ is uniformly distributed but also p is. In that case, the probability of a run would be 68.38%.

5   The Equilibrium for Uneven Banks

Assume now that consumers can put a volume s into a special bank. This deposit earns a (arbitrary small) subsidy at date 2 from the state, hence long run deposit rates are slightly higher. We discuss such subsidies and their tax-refunding in detail in Section 6. The subsidized institutions are called subsidized funds, short funds. The remaining volume $1 - s$ of the endowment is deposited at ordinary commercial banks, short banks. As before, we assume that funds and banks operate under perfect competition, they do not make profits. Because consumers want to be able to consume H at date 1, the short deposit rate in a symmetric equilibrium is $r_1^f = r_1^b = H$.[8]

Because funds are subsidized, depositors in need for cash will withdraw first from a bank, then from a fund. The subsidy induces a pecking order between banks. If a consumer wants to consume, it is first the bank that must provide the consumer with the good. Formally, patient consumers do not need to consume, they withdraw only for strategic (panic) reasons that will be discussed below. H-consumers need to withdraw their complete deposit, hence they withdraw everything from both funds and bank. L-consumers need only to withdraw L.

Focusing on L-consumers there are two cases: *First*, a relatively small fund, which size s satisfies $L < (1 - s) r_1 = (1 - s) H$. Hence, withdrawing from the bank is sufficient to satisfy the desired consumption L. At date 1, L-consumers have $(1 - s) H$ at the bank and $s H$ at the fund and withdraw L from the bank and nothing from the fund. *Second*, if $L > (1 - s) H$, withdrawing only from the bank is

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[8]Note that there exists a continuum of deposit rates that could be optimal for consumers $s r_1^f + (1 - s) r_1^b = H$. We discuss those possible asymmetric equilibria further in the following chapter.

not sufficient to consume L. At date 1, the consumer withdraws $(1 - s) H$ from the bank, and the remaining $L - (1 - s) H$ from the fund. The analysis of both cases is very similar. We concentrate on the case of a small s, i.e., the first case; the second case is treated in Appendix A.3.

At date 1, patient consumers must decide whether they want to withdraw their deposits.[9] The same applies to L-consumers, who must decide whether they want to withdraw the remaining deposit. Importantly, the rational decision may differ between fund and bank. The risk in both banks is driven by the same fundamental ϑ, hence the information on both banks is the same. However, the liquidity and solvency situation differs, because more deposits are withdrawn from the bank than from the fund at date 1. We need to analyze both separately, starting with funds.

Runs on Funds. Consider again a late consumer who thinks about withdrawing early. In order to better be able to re-use former results, let us re-normalize his deposit to 1 (although, of course, he has deposited only s at the fund). The lower bound of withdrawals is $n_{\min} = \eta$. H-consumers withdraw, L-consumers cover the required amount $L \leq L(1 - s)H$ with withdrawing from the bank. Therefore, L-consumers and patient consumers withdraw only for strategic reasons from the fund. Consequently, (13) adjusts to

$$0 = \int_{\eta}^{\frac{1}{H}} \left(p(\vartheta^*) \frac{1 - n H}{1 - n} R - H \right) dn + \int_{\frac{1}{H}}^{1} \left(0 - \frac{1}{n} \right) dn. \tag{15}$$

Integrating and solving for $p(\vartheta^*)$ yields

$$p(\vartheta^*) = \frac{1 - \eta H + \log H}{\left(1 - \eta H + (H - 1) \log \frac{H - 1}{H (1 - \eta)} \right) R}. \tag{16}$$

In the numerical example, we get $p(\vartheta^*) = 0.5526$. Because $p(\vartheta)$ is increasing in ϑ, this implies that the probability of a run has dropped. In the simplest case of $p(\vartheta) = \vartheta$, the probability of a run is only 55.26%, down from 68.38%.

The deposit base of the funds increases because L-consumers withdraw their target consumption L only from the bank leaving the deposit base at the fund unaffected.

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[9]Note that also the lower dominance regions change for each type of the bank. The exact values are discussed in Appendix A.2.

The increased deposit base allows the fund to pay a higher expected return per unit of non-withdrawn investment at date 2. The prospect of higher expected date 2 return makes consumers with lower signals willing to leave their deposit at the bank. The increased deposit base shifts the critical consumer's signal down. Only consumers that receive a lower signal than the decreased critical signal are willing to run the bank. Therefore the probability of a bank run decreases when the deposit base of the bank is increased.

Proposition 2 *The bank run probability $p(\vartheta^*)$ is an increasing function of $n_{\min}$ as long as the bank is solvent $(n_{\min} \in [0, \frac{1}{H}))$.*

The proof is in the Appendix. Remember that $n_{\min}$ is the amount of deposits that is regularly withdrawn in $t = 1$ in order to satisfy consumption needs. An increase in $n_{\min}$ reduces the long term investment of the bank, and thereby, the long run return r_2 that the bank is able to offer. A decrease in the long run return increases the states that cause a bank run. Intuitively, an agent that observed a signal such that he was indifferent between running and not running, will find it optimal to run, if the long run interest rate slightly decreases. However, if $n_{\min} \geq \frac{1}{H}$ such that the bank is insolvent in $t = 2$ because it has to liquidate all its assets it is always optimal because any positive payment makes the consumer better off.

Runs on Banks. This case is slightly more complex. Each consumer deposits $(1-s)$ at a bank. The promised repayment at date 1 is $(1-s)H$. Now H-consumers withdraw the complete $(1-s)H$, counting fully into the $n_{\min}$. Patient consumers do not withdraw at all (only for strategic reasons), they do not enter into the $n_{\min}$. L-consumers withdraw L from the bank and nothing from the fund. Hence, they count into the $n_{\min}$ with a factor $\frac{L}{(1-s)H}$. For $s = 0$, we get back the factor L/H from Section 4.

Summing up, $n_{\min} = \eta + \frac{L}{(1-s)H} \lambda$. Consequently, (13) adjusts to

$$0 = \int_{\eta + \frac{L}{(1-s)H} \lambda}^{\frac{1}{H}} \left(p(\vartheta^*) \frac{1 - nH}{1 - n} R - H \right) dn + \int_{\frac{1}{H}}^{1} \left(0 - \frac{1}{n} \right) dn. \qquad (17)$$

Integrating and solving for $p(\vartheta^*)$ yields

$$p(\vartheta^*) = \frac{1 - (\eta + \frac{L}{(1-s)H} \lambda) H + \log H}{\left(1 - (\eta + \frac{L}{(1-s)H} \lambda) H + (H-1) \log \frac{H-1}{H(1-\eta-\frac{L}{(1-s)H} \lambda)} \right) R}. \qquad (18)$$

In the numerical example, we get $p(\vartheta^*) = 0.7062$.

Again, we cannot say anything about run probabilities without making an assumption about the shape of $p(\vartheta)$. Consider the simplest example $p(\vartheta) = \vartheta$ for exposition. The probability of a run is then 70.62% for this bank. Due to the introduction of subsidized banks, the old-fashioned commercial banks become more risky. Of course, we are interested in the aggregate effect.

The expected fraction of deposits lost in a run is 68.38% in the absence of funds. With funds, it is

$$0.1 \cdot 55.26\% + (1 - 0.1) \cdot 70.62\% = 69.08\%,$$

an increase by 0.7%. This implies that the introduction of funds makes the whole system more unstable, in the aggregate (at least, in this numerical example).

Proposition 3 *An uneven banking system has a higher aggregate bank-run probability than a symmetric banking system for $p''(\vartheta) \geq 0$.*

$p(\vartheta^*)$ is a convex function of $n_{\min}$. Therefore, any linear combination of minimum withdrawals results in a higher aggregate bank-run probability than in the symmetric banking sector. The detailed proof can be found in the Appendix. Unless the realization of bad states and low success probabilities of investments is relatively more likely than good states Proposition 3 holds for rather general distributions of the state variable ϑ.

6   The Effect of Subsidies

We have shown that a pecking order in the withdrawal of bank deposits has significant effects on the stability of the individual banks. Institutions from which depositors withdraw are much more prone to a panic run than institutions where depositors withdraw last. Although the latter gain stability from an increased deposit base, we have shown that in aggregate the banking sector becomes less stable and more prone to panic runs compared with symmetric banking systems.

So far we have just assumed that a subsidy can establish a seniority structure. We now analyze in more detail how a subsidy influences the decisions of depositors. To close the model we also discuss the endogenous funding of the subsidy by taxes.

The subsidy is only paid on long term deposits that were not withdrawn in $t = 1$. Therefore, a subsidy is only paid, if no bank-run occurred. If a bank experienced a bank-run, all deposits are withdrawn at $t = 1$ and no subsidy is paid in $t = 2$.

Subsidy on long-term savings. For exogenous reasons the government decides to introduce a subsidy on long-term savings. We model the subsidy based on the announcements of the European Commission, i.e., we take the *Livret A* as a role model. The stated intention of the European Commission is to incentivize long term savings by supporting the long term interest payments.

In our model the short run period reflects immediate consumption while consumption at date 2 reflects long term savings. We model the subsidy as a (possibly small) mark up δ the government adds to the long run interest rate r_2. Therefore, any deposits at the fund pay out r_1^f when withdrawn at date 1 and in case of success $r_2^f + \delta$.[10]

At date 0, due to perfect competition, both banks offer the same deposit contract $r_1^f = r_1^b$ and $r_2^f(n) = r_2^b(n)$ in case of success. Banks at date 2 must distribute their complete revenues. Therefore, no bank can offer a higher expected return at date 2 than the competitive rate. The short deposit rate offered by both banks is $r_1^f = r_1^b = H$ because banks are in perfect competition and depositors want to be able to consume H at date 1. At date 1 H-consumers withdraw their complete deposits from each bank. L-consumers need only to withdraw L and have to decide where to withdraw from. In the case where $L < (1 - s) H$ withdrawing from the bank is sufficient to satisfy consumption needs L. Knowing that the contracted repayments of each bank are equal but that for each unit he leaves at the fund the depositor receives a mark up of δ as a subsidy, he has a strong preference to withdraw first all deposits from the banks and only withdraws $L - (1 - s) H$ from the fund if $(1 - s) H$ is not sufficient to cover his consumption need.

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[10]Later on we discuss the case of a guaranteed interest rate that does not change the qualitative results. Because of the risk-neutrality of depositors the subsidy has the same effect regardless if the probability of success or the return in case of success is increased.

Taxes. In order to finance the subsidy δ paid at date 2, that government raises a flatrate tax τ from each household at date 2. The tax exactly equals the subsidy but consumers are ignorant at date 1 of their effect on the future tax and take taxes as exogenously given. We have to distinguish the two cases for L-consumers.

First, if $L \leq (1-s) H$, withdrawing from banks is sufficient to satisfy the desired consumption L. At date 2, in case of success, $(1-\eta)$ consumers withdraw their deposits from the fund and receive an overall subsidy of δ times their deposit s. The subsidy is funded by tax $\tau = s (1 - \eta) \delta$.

Ex ante, the expected utility is

$$
\begin{aligned}
u(\delta, \tau) &= (\mu - \bar{p} R) (\eta H + \lambda L) + \bar{p} R + s (1 - \eta) \delta - \tau. \\
&= (\mu - \bar{p} R) (\eta H + \lambda L) + \bar{p} R
\end{aligned}
\tag{19}
$$

Second, in case the withdrawal from the bank is not sufficient for L-consumers: $L > (1-s) H$, he also withdraws the residual required $y := \frac{L - (1-s)H}{H}$ from the fund. In that case, at date 2, $(1-\eta)$ consumers withdraw their residual deposits from the fund. Patient consumers withdraw their full deposits (also from the bank). L-consumers can only withdraw $(1-y)$ deposits from the fund. The subsidy overall amounts to $s (1 - (\eta + y\lambda) \delta$ which is funded by tax $\tau = s (1 - (\eta + y\lambda) \delta$.

Ex ante, the expected utility is

$$
\begin{aligned}
u(\delta, \tau) &= (\mu - \bar{p} R) (\eta H + \lambda L) + \bar{p} R + s (1 - (\eta + y\lambda) \delta - \tau. \\
&= (\mu - \bar{p} R) (\eta H + \lambda L) + \bar{p} R
\end{aligned}
\tag{20}
$$

In both cases, the subsidy that is fully funded by a flatrate tax paid by all consumers does not change the expected utility of consumers. However, it changes the withdrawal decisions and establishes a seniority among the banks. As shown above, this can significantly destabilized the unsubsidized bank and it may decrease the overall stability of the banking sector.

7   Discussion

Deposit insurance. A perfect deposit insurance system that credibly assures consumers to pay the contracted long term interest rate eliminates the bank run

equilibrium in the Diamond and Dybvig (1983) model. The same result holds for bank runs in global games. On the one hand, one could argue that virtually every economy with a significant banking sector provides a deposit insurance system either in the form of a deposit insurance fund or in the form of explicit or implicit government guarantees. However, not only the recent financial crisis illustrated that banks are nevertheless fragile. We, therefore, argue that even with insured deposits, depositors may have an incentive to run the bank if they get an adverse signal on the fundamental. *First*, a bad signal on the fundamental can also be indicated as information on the economic situation as a whole. If depositors expect all banks to suffer losses they may question the ability of the deposit insurance system to cover their losses. A deposit insurance fund per definition is not able to buffer systemic risk but only idiosyncratic bank defaults. Even governments may be pushed towards the border of their solvency if the whole banking system is in distress as the example of Ireland and Spain have illustrated. *Second*, even if depositors trust in the deposit insurance system's ability to meet financial obligations, they may foresee additional costs in case they are repaid in case of the insolvency of the bank. Usually the liquidation of a bank in case of insolvency is a long-term process. Even though the payments at date 2 are guaranteed to depositors they may foresee that payments could be delayed such that urgent consumption needs cannot be satisfied. This may reintroduce the incentive to withdraw their deposits immediately and reestablish a bank run equilibrium.

Assume that depositors expect to be compensated by their government at each time with a positive probability $\beta < 1$. In other words, we assume there exists a deposit insurance system but it is imperfect. The existence of such a system decreases both, the probability of fundamental bank runs (the lower dominance region) and the probability of panic-based runs.

If β is not too high (imperfect deposit insurance) our results hold.

As the deposit insurance becomes perfect, both, the fundamental and the panic-based bank run probabilities become zero. However, in contrast to Diamond and Dybvig (1983) type models, where deposit insurance is costless because it prevents the self-fulfilling banking crisis ex-ante, the deposit insurance in our model is costly, because assets are risky and the insurance has to pay depositors in the cases the bank receives nothing from its investments.

Interestingly, when a subsidy is introduced the relative gap between the stability of

a symmetric and an asymmetric banking system is higher in deposit insured banking systems. On the one hand, the bank-run probabilities of funds and banks are lower in partly insured banking systems. On the other hand, a subsidy increases the bank-run probability of banks more relative to the run probability of funds.

Public guarantees for the fund. Supposably, the implementation of a subsidized savings fund by the government would be accompanied by a public guarantee for this fund. Especially, if the fund is small relatively to the overall banking sector or the public income such a guarantee might be credible and shield the fund from any bank run making it absolutely stable. In such a case our basic results hold. The reallocation of liquidity in the banking system increases the deposit base of the fund and reduces the deposit base of the traditional banks which increases the banks's fragility to panic based bank runs. However, the full guarantee of the fund reduces the run probability to zero and creates a safe haven for deposits. In this case, the reduction of the stability of traditional banks may be dominated by the increase in stability due to the guaranteed fund. Therefore, the total effect may be positive or negative depending on the relative size of the fund and the banks.

Guaranteed interest rate. The announcement of the European Commission also indicated the idea to guarantee the long-term interest rate. Applied to our model, this could be interpreted as paying r_2 to the depositors in case of no success of the fund. With probability $1 - p$ the government would collect $\tau = s(1 - \eta)r_2$ to pay the subsidy. The expected utility remains unaffected but the prospect of a secure payment of r_2^f in comparison to a risky and therefore a lower expected payment from the commercial bank changes the preferences of patient and L-consumers. The prospect to receive a higher long-term interest rate introduces the incentive to withdraw first from the bank and only in case of residual needs from the fund. This alternative interpretation of the subsidy, therefore, does not change our qualitative results.

Investment decision. The deposit base of a bank does not only influence its fragility to bank runs but also its investment decision. For simplicity we have assumed that banks can only invest in a liquid but risky asset. We can extend our model to introduce an endogenous investment choice. Assume an investment set

identical to the classical Diamond and Dybvig (1983) model. Banks can choose between a safe and liquid investment opportunity (storage) and a risky and illiquid investment opportunity (risky investment). The risky investment is illiquid because its liquidation value ℓ at date 1 is lower than unity. In contrast to our model, liquidation of the risky asset now become costly because $\ell < 1$. In order to avoid costly liquidation, the banks now have to store some of their investment in order to be able to serve impatient consumers withdrawals. The basic mechanism is that banks can invest their deposit base into the long run productive technology and store the amount they expect depositors to withdraw at date 1. In this framework, a reduction in the deposit base also influences the investment decision of banks, i.e., it decreases the bank's ability to invest in the long run productive technology. However, the main insights of the asymmetry among banks and the result for the stability of the banking system remain unchanged.

Ring-fencing and subsidized saving products. Our model can also be used to analyze uneven seniority of investment withdrawals within an individual financial institution. Our results also hold if individual banks are forced to separate parts of their businesses and funding sources from each other by a ring-fence. One example are the considerations to separate deposit funded operations from the other activities of a bank. On the one hand, households may want to invest some of their endowment in liquid bank accounts and the residual part in less liquid investments that offer higher long term interest rates but can be liquidated only with a penalty. As long as the bank is able to cross-fund liquidity needs within the institution, the bank-run probability is similar to our example of the symmetric banking sector. However, if the bank has to ring-fence the deposit funds from its residual business, the liquidity base of the deposit funded bank is much lower than the average liquidity base of the bank. Overall the bank may became less stable because of the separation of the different liquidity sources. A similar argument can be made for subsidized saving products. Consider a government decides to allows all banks to offer a special bank account that is subsidized and refinanced by taxes. The amount collected in this bank account can be distributed to risky long-term investments by the banks. However, if the bank has liquidity needs from its daily business, it is not or only partly allowed to use the liquidity from the special account. In this case the same mechanisms that we discussed for banking systems apply for a single financial institution. Households would invest their entire savings at the one bank

but distribute as much as possible in the subsidized account and only the residual in a normal bank account at the bank. In case of a consumption shock, the short depositors would withdraw first their deposits in the normal bank account and only the residual in case of need from the subsidized account in order to maximize subsidy payments. This decreases the deposit base for the bank's normal business while increasing the deposit base of the special account. In case of ring-fencing, between the two types of accounts, the normal bank business becomes more fragile and the special account safer than the single unsubsidized bank. However, as shown above: in aggregate, the bank would become more fragile to panic-based runs even as a single institution.

Relative size of uneven banks. We discussed before that the probability of panic runs on each bank also depends of the relative size s of each bank type. In particular, a subsidized bank is safest as long as s is small such that consumers cover consumption needs fully by withdrawals from the bank (Case 1). For small values of s, the panic run probability for funds is independent of the relative size, while the panic run probability of banks increases in its relative size $1 - s$. The greatest inequality between withdrawals for each type of banks is reached when $s = \frac{H-L}{H}$ such that withdrawal from the bank just satisfies low consumption needs. In this case, L-consumers withdraw their entire deposit from the bank but nothing from the fund. In this case, the panic run probability for banks reaches it maximum because all impatient consumers withdraw their entire deposits.

For $s > \frac{H-L}{H}$ the withdrawal decision of L-consumers is described by Case 2 in Appendix A.3. In this case the fund looses parts of its deposit base because withdrawing all deposit from the bank is not sufficient. Therefore, the panic run probability of the fund increases in its relative size s while the probability of for banks remains independent at its maximum. Figure 1 summarizes this result using our numerical example. The bold line depicts the aggregate panic-run probability in the system weighted by the relative size s. If $s = 0$ no subsidized banks exist and we are in the symmetric banking sector as discussed in Chapter 4. Analogically, $s = 1$ means that all banks are equally subsidized and no banks exist. Note that this case, the aggregate bank-run probability equals the case with no subsidy. If all banks receive the same subsidy, or, analogically, if a bank can use the liquidity a subsidized fund attracts for its residual business, stability is unchanged. The aggregate probability of bank-runs increases when the deposit base of banks becomes unequal. The

Figure 1: Aggregate panic-run probabilities dependent on size of bank types
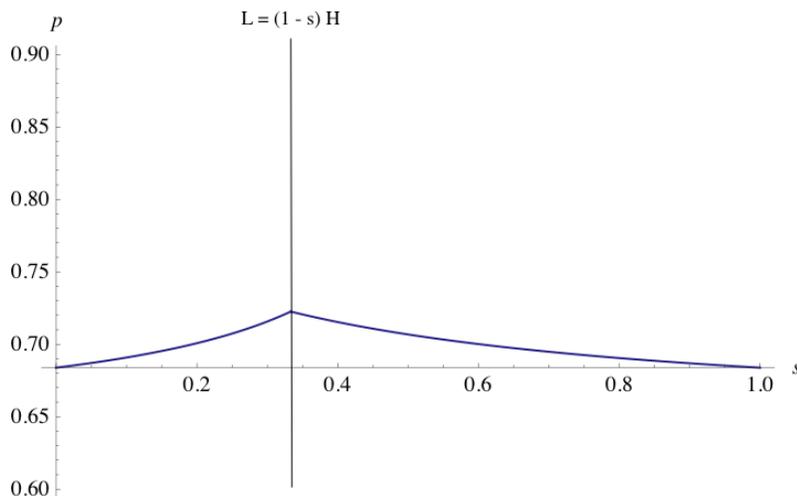
highest inequality in the deposit base is reached, when $L = (1 - s)H$. In this case, L-consumers withdraw their entire deposits from banks but nothing from funds.

Acknowledging that our model focuses only on the adverse effect of an inequality in the deposit base we are aware that we cannot fully evaluate the costs and benefits of a subsidy on long term savings. However, our analysis implies that a subsidy on limited savings collected at a separate institution destabilizes the banking system while the corner solutions of no subsidy or a subsidy for all financial institutions equally would not have this detrimental side effect.

Save heavens and contagion. The size of a bank may have a considerable impact on the cost of a failure on the economy. As discussed above, without such cost of failure any $s \in (0, 1)$ increases instability. However, we have shown that the fund becomes safer by the subsidy. If the cost of bank failure exponentially increases in the size it may become optimal to divide the banking sector into safe and risky banks. Such a division might increase the aggregate bank run probability but decrease the aggregate cost associated with the bank failure. In other words, a subsidy may lead to more bank failures but these banks would be relatively small. In such a scenario there might be an optimal size $s^* \in (0, 1)$ of safe institutions. The same arguments may apply if we consider contagion among banks and the resulting systemic risk. If there exists the risk that bank-runs spread from one bank to another bank there

are two effects. On the one hand, it would be optimal to minimize the overall probability of bank-runs. Therefore, subsidies would be harmful. On the other hand, an even banking system might be vulnerable already to little liquidity shocks that spread from one bank to the other. Therefore, the creation of safer banks could be beneficial, if those banks would be able to survive liquidity shock that would already trigger a bank panic in an even banking system.

8 Conclusion

Subsidized long-term savings provoke manifold opposition from the banking industry. Especially commercial banks complain that such a subsidy erodes their core business model. In this paper we analyze the effect of asymmetries in the withdrawal behavior of investors on the aggregate stability of the banking system. We show that the banking sector as a whole may become more prone to panic-based bank runs as a result of the asymmetric subsidy. This additional effect is special in banking because it results from the role of commercial banks as liquidity insurers. The introduction of a subsidy of long-term interest rates for certain special banks changes the preferences of depositors to withdraw their savings from banks. Households experience heterogeneous consumption shocks. Because banks pool the household savings and thereby diversify idiosyncratic risks, each bank has a steady deposit base from customers that do not withdraw at any given time. This deposit base works like a buffer against fundamental shocks. A reduction in the deposit base makes the bank more fragile to panic-based bank runs.

We show that the asymmetric subsidy on long-term savings at a certain institution (or part of the institution) changes the idiosyncratic withdrawal risk into a systemic withdrawal behavior. In case of liquidity needs, depositors strictly prefer to withdraw their deposits from the unsubsidized bank to satisfy their short term consumption need, leaving their savings at the subsidized bank to gain from the future subsidy and increase overall consumption. This significantly reduces the deposit base of the commercial banks and increases their fragility. At the same time the deposit base of the subsidized bank increases. However, we show that on aggregate, stability decreases.

A Appendix

A.1 Distribution of $p(\vartheta)$

Formally, a situation where high success probabilities are relatively higher than low success probabilities requires that $p(\vartheta)$ is a concave function in ϑ as depicted on the left hand side in Figure (2). For such a situation, the distribution function of p would is increasing. Accordingly, the density function F' is increasing and F'' positive. The inverse of an increasing convex function is concave. Since $p(\vartheta)$ is the inverse of $F(p)$, p is indeed concave in ϑ.

Figure 2: Distribution function of $p(\vartheta)$
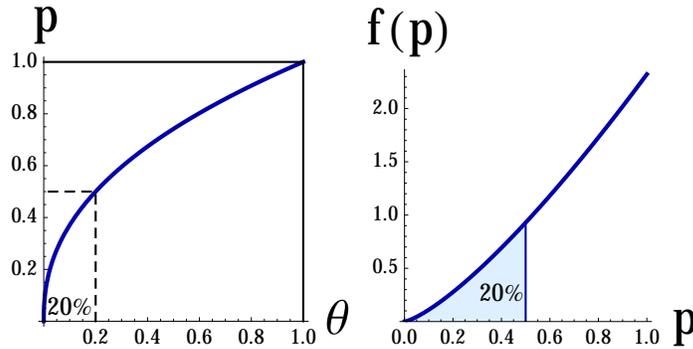
A.2 Lower dominance regions

For uneven banks the lower dominance region changes due to the changes in the deposit base.

Lower dominance region bank. Denote by $p(\underline{\vartheta}^C)$ the realized success probability that solves

$$(1-s)H = p(\vartheta)\,(1-s)\Big(\frac{1-(\eta+\lambda)\,H}{1-(\eta+\lambda)}\,R\Big) \tag{21}$$

$$p(\underline{\vartheta}^C) = \frac{H\,(1-(\eta+\lambda))}{R\,(1-(\eta+\lambda)H)}. \tag{22}$$

Using numerical example (see Appendix A.5) the critical success probability for banks is 0.375.

Lower dominance region fund. Denote by $p(\underline{\vartheta}^S)$ the realized success probability that solves

$$s\,H = p(\vartheta)\,s\left(\frac{1-\eta\,H}{1-\eta}\,R\right) \tag{23}$$

$$p(\underline{\vartheta}^S) = \frac{H\,(1-\eta)}{R(\,1-\eta\,H)}. \tag{24}$$

Using numerical example (see Appendix A.5) the critical success probability for banks is 0.321.

A.3 Case $L > (1-s)\,H$

Consider the second case, that $L > (1-s)\,H$. Withdrawing only from the bank is not sufficient to consume L. At date 1, the L-consumer therefore withdraw all deposits $(1-s)\,H$ from the bank, and the remaining $L - (1-s)\,H$ from the fund.

Runs on Funds. In addition to the proportion of $\eta\,H$-consumers that withdraw all deposits from the subsidized bank, now also $\lambda\,L$-consumers withdraw the fraction $\frac{L-(1-s)\,H}{s\,H}$ of their deposits from the fund, such that

$$s\left(\frac{L-(1-s)\,H}{s\,H}\right)H + (1-s)\,H = L. \tag{25}$$

per m Consequently, (13) adjusts to

$$0 = \int_{\eta + \frac{L-(1-s)\,H}{s\,H}\,\lambda}^{\frac{1}{H}}\left(p(\vartheta^*)\frac{1-n\,H}{1-n}\,R - H\right)dn + \int_{\frac{1}{H}}^{1}\left(0 - \frac{1}{n}\right)dn. \tag{26}$$

Integrating and solving for $p(\vartheta^*)$ yields

$$p(\vartheta^*) = \frac{1-\left(\eta + \frac{L-(1-s)\,H}{s\,H}\,\lambda\right)H + \log H}{\left(1-\left(\eta + \frac{L-(1-s)\,H}{s\,H}\,\lambda\right)H + (H-1)\log\frac{H-1}{H\left(1-\left(\eta + \frac{L-(1-s)\,H}{s\,H}\,\lambda\right)\right)}\right)R}. \tag{27}$$

Runs on Banks. In this case both impatient consumers withdraw all their deposits from the bank. Consequently, (13) adjusts to

$$0 = \int_{\eta + \lambda}^{\frac{1}{H}}\left(p(\vartheta^*)\frac{1-n\,H}{1-n}\,R - H\right)dn + \int_{\frac{1}{H}}^{1}\left(0 - \frac{1}{n}\right)dn. \tag{28}$$

Integrating and solving for $p(\vartheta^*)$ yields

$$p(\vartheta^*) = \frac{1 - (\eta + \lambda) H + \log H}{\left(1 - (\eta + \lambda) H + (H - 1) \log \frac{H-1}{H(1-(\eta+\lambda))}\right) R}. \tag{29}$$

A.4 Proofs

Proof of Proposition 1. The first part of the proposition is equivalent to Theorem 1 in Goldstein and Pauzner (2005), modelling the differences in the consumers' utility function. The second part is equivalent to Proposition 1 in Allen, Carletti, Goldstein, and Leonello (2013). In our special case, equation (5) in that paper simplifies to

$$0 = \int_{\eta + \lambda \frac{L}{H}}^{\frac{1}{H}} \left(p(\vartheta^*) \frac{1 - n H}{1 - n} R - H\right) dn + \int_{\frac{1}{H}}^{1} \left(0 - \frac{1}{n}\right) dn$$

$$0 = (p(\vartheta^*) R - 1)(1 - (\eta + \lambda) H) - \log H + p(\vartheta^*) R (H - 1) \log \frac{H - 1}{H(1 - \eta - \lambda)}.$$

Solving for $p(\vartheta^*)$ yields (14).

Proof of Proposition 2. To simplify notation define $x := n_{min}$. We can then write:

$$p(\vartheta^*) = \frac{1 - Hx + \log(H)}{R\left(1 - Hx + (H - 1) \log\left(\frac{H-1}{H(1-x)}\right)\right)} \tag{30}$$

To proof Proposition 2 we have to show that $\frac{\partial p(\vartheta^*)}{\partial x} > 0 \ \forall \ x \in [0, \frac{1}{H})$. The closed form is:

$$\frac{\partial p(\vartheta^*)}{\partial x} = \frac{(1 - x H) \log(H) + (H - 1)\left(H(1 - x) \log\left(\frac{H(1-x)}{H-1}\right) - (1 - x H)\right)}{R(1 - x)\left((H - 1) \log\left(\frac{H(1-x)}{H-1}\right) - (1 - Hx)\right)^2} \tag{31}$$

For $H > 1$ and $x \in [0, \frac{1}{H})$ it must hold that $1 - x > 0$ and $1 - x H > 0$. This implies that the denominator is unambiguously positive. The first term of the numerator is also positive in the domain. Hence, it is sufficient to show that the second term of the numerator is positive.

At the upper limit the second term in the numerator approaches zero:

$$\lim_{x \to \frac{1}{H}} \left(H(1 - x) \log \left(\frac{H(1 - x)}{H - 1} \right) - (1 - x) H \right) = 0. \tag{32}$$

The first derivative of the term is: $H \log \left(\frac{H-1}{H(1-x)} \right)$. The derivative is negative if $\left(\frac{H-1}{H(1-x)} \right) < 1$, which is the case for all $x \in [0, \frac{1}{H})$. The second term in the numerator is a decreasing function that approaches zero from above. It is therefore positive in the domain $x \in [0, \frac{1}{H})$. All terms of the first derivative are positive, such that $\frac{\partial p(\vartheta^*)}{\partial x} > 0 \ \forall \ x \in [0, \frac{1}{H})$. ∎

Proof of Proposition 3. To ease the notation we write $p(\vartheta^*, n_{\min}) := p^*(x)$, where $n_{min} := x$ as defined above. First, we have to show that $p^*(x)$ is a convex function of x in the domain $x \in [0, \frac{1}{H})$:

$$\frac{\partial^2 p^*(x)}{\partial x^2} > 0 \tag{33}$$

Because the closed form is quite complex, we consider the nominator and the denominator separately.

Consider first the denominator:

$$R(1 - x)^2 \left((1 - Hx) - (H - 1) \log \left(\frac{H(1 - x)}{H - 1} \right) \right)^3 \tag{34}$$

It approaches zero for $\lim_{x \to \frac{1}{H}}$. Its first derivative $-\frac{1 - Hx}{1 - x}$ is negative. Therefore, the denominator of $p^{*\prime\prime}(x)$ is positive.

The numerator of the $p^{*\prime\prime}(x)$ is more complicated:

$$(Hx - 1)((2Hx + H - 3) \log(H) - 3(H - 1)(Hx - 1))$$
$$- (H - 1)((Hx - 1)(H(2x - 3) + 1) + (H - 1) \log(H)) \log \left(-\frac{H - 1}{H(x - 1)} \right) \tag{35}$$

It also approaches zero for $\lim_{x \to \frac{1}{H}}$. We, thus, have to show that the numerator is a decreasing function of x, i.e., that its first derivative is negative in the domain.

This first derivative is

$$(H(4x - 3) - 1) \left(-\frac{(Hx - 1)(H - \log(H) - 1)}{x - 1} - (H - 1)H \log \left(-\frac{H - 1}{H(x - 1)} \right) \right). \tag{36}$$

It contains two coefficients:

The first coefficient is an increasing function of x, which approaches $\lim_{x \to \frac{1}{H}} (H(4x - 3) - 1) = (3 - 3H)$ which is negative for $H > 1$. It is therefore negative in the domain $x \in [0, \frac{1}{H})$. The second coefficient approaches zero at $\lim_{x \to \frac{1}{H}}$.

Its derivative $\frac{(H-1)(1-Hx+\log(H)-1)}{(1x)^2} < 0$ is negative in the domain $x \in [0, \frac{1}{H})$. The second coefficient is therefore positive. The derivative of the numerator has a negative and a positive coefficient, it is therefore negative.

The numerator is a decreasing function of x that approaches zero at the upper limit of the domain. It is therefore positive over the domain.

In summary, we find that the denominator and numerator of $p^{*\prime\prime}(x)$ are positive for $x \in [0, \frac{1}{H})$, which implies that $\frac{\partial^2 p^*(x)}{\partial x^2} > 0$. The bank-run probability is a convex function of x.

Now, consider aggregate bank-run probabilities in the different banking sectors. The bank-run probability in a symmetric banking sector can be written as

$$p^* \left(\eta + \frac{L}{H} \lambda \right). \tag{37}$$

For $s \le \frac{H-L}{H}$ the aggregate bank-run probability of an asymmetric banking sector can be written as

$$s\, p^* (\eta) + (1 - s)p^* \left(\eta + \frac{L}{(1 - s)H} \lambda \right). \tag{38}$$

For $s > \frac{H-L}{H}$ (see A.3) the aggregate bank-run probability of an asymmetric banking sector is

$$s p^* \left(\eta + \frac{L - (1 - s) H}{s\, H} \lambda \right) + (1 - s)\, p^* (\eta + \lambda). \tag{39}$$

Note that

$$\eta + \frac{L}{H} \lambda = s\,\eta + (1 - s) \left(\eta + \frac{L}{(1 - s)H} \lambda \right)$$
$$= s \left(\eta + \frac{L - (1 - s) H}{s\, H} \lambda \right) + (1 - s)\, (\eta + \lambda). \tag{40}$$

The withdrawals in uneven banking systems are a mean preserving spread of the minimum withdrawals in the symmetric banking system.

The convexity of $p^*(x)$ in $x \in [0, \frac{1}{H})$ therefore implies:

$$p^* \left(s\,\eta + \frac{L}{H}\lambda \right) < s\,p^*(\eta) + (1-s)\,p^* \left(\eta + \frac{L}{(1-s)H}\lambda \right) \tag{41}$$

for all small $\forall\, s \in \left(0, \frac{H-L}{H} \right]$ and

$$p^* \left(s\,\eta + \frac{L}{H}\lambda \right) < s\,p^* \left(\eta + \frac{L-(1-s)\,H}{s\,H}\lambda \right) + (1-s)\,p^*(\eta + \lambda) \tag{42}$$

for all high $s \in \left[\frac{H-L}{H}, 1 \right)$. The aggregate bank-run probability increases as the spread of withdrawals increases.

By assumption $p'(\vartheta) > 0$ and $p''(\vartheta) < 0$. The inverse function $\vartheta(p)$ must be convex: If $\vartheta(p)$ is the inverse function of $p(\vartheta)$ it must hold that $\vartheta(p(x)) = x$. Derivation with respect to x yields $\vartheta'(p(x))\,p'(x) = 1$ or

$$\vartheta'(p(x)) = \frac{1}{p'(x)} \tag{43}$$

Taking the second derivative with respect to x yields:

$$p'(x)^2\,\vartheta''(p(x)) + \vartheta'(p(x))\,p''(x) = 0 \tag{44}$$

Using equation (43) we get:

$$p'(x)^2\,\vartheta''(p(x)) + \frac{p''(x)}{p'(x)} = 0 \tag{45}$$

$$\vartheta''(p(x)) = -\frac{p''(x)}{p'(x)^3} = 0 \tag{46}$$

Which is negative for $p'(x) > 0$ and $p''(x) < 0$. Hence, ϑ as a function of p is convex in p. This implies that $\vartheta(p(x))$ is convex in x. Inequalities 41 and 42 hold. ∎

A.5 Definition and Parametrization of Variables

Variable	Value	Description
η	0.25	probability of becoming an H-consumer
H	1.2	early consumption of an H-consumer
λ	0.25	probability of becoming an L-consumers
L	0.8	early consumption of an L-consumer
R	4	long project return (if successful)
s	0.1	maximum deposit at funds

References

ALLEN, F., E. CARLETTI, I. GOLDSTEIN, AND A. LEONELLO (2013): "Government Guarantees and Financial Stability," Discussion paper, Wharton School, University of Pennsylvania.

CARLSSON, H., AND E. VAN DAMME (1993): "Global Games and Equilibrium Selection," *Econometrica*, 61(5), 989–1018.

DIAMOND, D. W., AND P. H. DYBVIG (1983): "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, 91(3), 401–419.

DIAMOND, D. W., AND R. G. RAJAN (2001): "Banks, short-term debt and financial crises: theory, policy implications and applications," in *Carnegie-Rochester Conference Series on Public Policy*, vol. 54, pp. 37–71. Elsevier.

GOLDSTEIN, I., AND A. PAUZNER (2005): "Demand–Deposit Contracts and the Probability of Bank Runs," *Journal of Finance*, 60(3), 1293–1327.

GORTON, G., AND G. PENNACCHI (1990): "Financial intermediaries and liquidity creation," *The Journal of Finance*, 45(1), 49–71.

TIROLE, J. (2006): *The Theory of Corporate Finance.* Princeton University Press.