Contracts as a barrier to entry when buyers are non-pivotal
PRELIMINARY

Özlem Bedre-Defolie *† Gary Biglaiser‡

August 11, 2015

Abstract

We analyze whether the use of breakup fees by an incumbent might induce an inefficient allocation of consumers and possibly foreclose efficient entry where buyers are non-pivotal (infinitesimal) and have to pay switching costs if they switch from the incumbent to an entrant. When the entrants are competitive, in the unique equilibrium the incumbent induces the efficient outcome, so there is no inefficient foreclosure. When there is a single entrant, the incumbent cannot deter the entry if it is not allowed to use a breakup fee. In the equilibrium of this case there might be too much or too little entry depending on the entrant's cost advantage versus the highest level of switching costs. When the incumbent can use a breakup fee in its long-term contract, in the unique equilibrium the incumbent forecloses the entrant by a sufficiently high breakup fee. This result does not depend on the level of switching costs or the entrant's efficiency advantage. We extend the result to a situation where consumers do not face switching costs, but they get a lower match value from the entrant's product than the incumbent's. In this case the results differ only when there is a single entrant. There are no inter temporal effects without breakup fees and if the incumbent is allowed to use breakup fees, it forecloses the entrant if and only if the entrant's cost advantage is sufficiently low compared to the highest switching cost. All results are robust to allowing the incumbent to offer a spot price.

*We would like to thank Paul Heidhues, Simon Loertscher Markus Reisinger, Andrew Rhodes and the participants of the ANR-DFG Workshop on Competition and Bargaining in Vertical Chains, in Dusseldorf, 2015, for their comments.

†European School of Management and Technology (ESMT), Berlin,ozlem.bedre@esmt.org.

‡University of North Carolina, gbiglais@email.unc.edu

1 Introduction

Breakup fees, which are also known as early termination fees (ETFs), have been widely used in consumer contracts for a variety of services including wireless telephone service, cable and satellite TV, and health club memberships. They often also apply to contracts for post-paid, fixed-term mobile and broadband services, in particular when the contract involves subsidized equipment, like a headset subsidy. The use of ETFs have drawn the attention of many regulatory agencies, due to the concern that they hurt consumers. According to the information collected by the U.S. Federal Communications Commission in 2010, the ETFs charged by wireless telephone service providers vary greatly and depend on the service plan and type of phone; there might be no breakup fee or the breakup fee may be over $300.[1] European policy makers have also been concerned about long-term contracts and ETFs included in long-term contracts for wireless phone and cable services. Since 2009 EC obliged member states to ensure that consumer contracts for electronic communication services should have initial commitment period that does not exceed 2 years under the condition that 1-year-only option must also be available.[2] On September 2013 the European Commission (EC) adopted a proposal for a regulation which (among other things) gives consumers "right to terminate any contract after 6 months without penalty with a one-month notice period; reimbursement due only for residual value of subsidized equipment/promotions, if any".[3] In March 2013 the EC "recommended that national regulators negotiate or set maximum termination fees that are reasonable and do not become a barrier to switching provider"[4].

In the above markets, where we see long-term contracts with breakup fees, consumers will have different views of the benefits of buying from an incumbent firm's competitor. One avenue for differentiation among consumers is that they will have different switching costs (like having to contact the current provider that they are breaking the contract) if they change their purchases from one firm to another. These exogenous switching costs are in addition to any breakup fees, which can be considered endogenous switching costs that a contract may include.[5] Consumers can also have heterogenous beliefs about an entrant's

[1]The FCC's 2010 survey asked cell phone users about their ETFs and found that 54% of consumers said that they would have to pay ETFs, 28% said that they would not have to pay, and 18% did not know whether they would have to pay ETFs. Of those who know the level of ETFs, 56% reported that the ETFs exceed $200. See Horrigan, J., and Satterwhite, E. (2010). Also, see https://www.fcc.gov/encyclopedia/early-termination-fees for the replies of the service providers to the queries of the FCC.

[2]The Article 30 of the Directive 2009/136/EC sets rules facilitating switching service providers.

[3]See the EC Memo, 2013, p.24.

[4]The EC Service Study, 2013, p.327-329

[5]The literature on switching costs have considered different types of endogenous switching costs, like loyalty discounts in Caminal and Matutes (1990), discounts to new customers (subscription models) as in Chen (1997), or poaching in Fudenberg and Tirole (2000). See Klemperer (1995) and Farrell and Klemperer (2007) for excellent reviews of the switching costs literature.

perceived quality relative to the incumbent firm. This can be manifested by how good a match (or mismatch) an entrant's product is for a particular consumer, or how willing they are to try a new product. The difference between these two interpretations is that if a consumer does not make a purchase from an incumbent, then she faces no switching costs when contemplating a purchase from an entrant in the future, while under the mismatch interpretation a consumer's utility from buying from an entrant will not depend on whether they made a prior purchase from the incumbent. The different interpretations will have implications for equilibrium behavior, because a consumer's outside option of signing an incumbent's contract will depend on which interpretation is more key to the market setting.

This paper analyzes whether the use of breakup fees by an incumbent might induce an inefficient allocation of consumers and possibly foreclose efficient entry where consumers face switching costs if they switch from the incumbent to an entrant supplier (which is the interpretation that we use for most of the paper) or have mismatch value from the entrant's product compared to the incumbent's. We consider a two-period model of entry. In the first period, the incumbent is the monopolist and in the second period it faces competition by more efficient entrant(s). Consumers are willing to buy one unit of the good in each period. In period 1, the incumbent offers a long-term contract consisting of a unit price for period 1, a unit price for period 2 and, when it is allowed, a breakup fee, which is paid if a consumer who signed the incumbent's contract switches to an entrant in period 2. In period 2 the incumbent may offer a spot price and the entrant(s) make their offer(s). Consumers are identical in period 1, but as in Chen (1997), differ in their realized switching costs or mismatch values at the beginning of period 2. We consider two scenarios with regards to the entrants' market power: competitive entrants and a single entrant.

By offering long-term contracts to consumers, the incumbent bundles today consumption with future consumption. However, the incumbent cannot commit not to offer a spot contract when facing a more efficient entrant in period 2. In the spot market, the incumbent has an incentive to undercut the second-period price of its long-term contract. This is similar to the durable good monopoly problem: Offering a long-term contract converts a non-durable good (consumption today) to a durable good. The use of breakup fees in the long-term contract is a possible way for the incumbent to solve the commitment problem when facing entry. We also study whether/when the incumbent uses breakup fees to inefficiently foreclose entry.

In the baseline model with competitive entrants, we find that the incumbent implements the ex-post efficient outcome by setting its second period price at its marginal cost: consumers switch to a more efficient entrant if and only if their switching cost is smaller than the entrant's cost advantage compared to the incumbent. This is true regardless of the fact that the incumbent is allowed to use a breakup fee in its long-term contract, in which case it could have foreclosed the more efficient entrants with a sufficiently high breakup fee.[6] There are two reasons for why the incumbent implements the efficient outcome: 1) Consumers' outside option of rejecting the incumbent's long-term contract is exogenous, does not depend on the incumbent's actions, since the competitive entrants price at their marginal cost; and 2) the incumbent can capture all expected surplus from ex-ante homogenous consumers via the first period unit price after leaving consumers their (exogenous) outside option. Therefore the incumbent does not find it profitable to deter entry by more efficient firms with competitive entry, even if the incumbent could have used breakup fees as a tool to foreclose.

We next consider the model with a single entrant. The main effect of allowing for entrant market power is that consumers' outside options of rejecting the incumbent's long-term contract becomes endogenous. This changes our analysis in two ways. First, the entrant's price in period 2 will depend on the incumbent's long-term contract along with any potential spot contract offered by the incumbent. Second, we need to see if the outside option for consumers can depend on whether they think other consumers will accept or reject the incumbent's long-term contract. If the incumbent is not allowed to have a breakup fee in its long-term contract, then when the entrant's efficiency advantage is large relative to the highest switching cost, the incumbent makes no sales in period 2, which is also ex-post efficient. The incumbent's profit is the single period monopoly profit less the expected second period switching costs. For lower levels of the entrant's efficiency advantage, the incumbent makes sales in period 2 with a price above marginal cost in order to induce a higher price by the entrant. This lowers the consumers' outside option in period 2 and raises the incumbent's profit. In this case, whether the equilibrium prices induce efficient switching depends on the comparison of the entrants' cost advantage with the highest level of switching costs (which increases both the dispersion and the mean of switching costs). We demonstrate conditions under which there is either too much or too little switching in equilibrium relative to the efficient level.

When the incumbent is allowed to have a breakup fee in its long-term contract, we find that it always forecloses the more efficient entrant. The incumbent makes it too costly for consumers to switch to the entrant. One benefit of foreclosing the entrant is that the incumbent does not have to compensate consumers for their expected costs of switching, since then the consumers who signed the incumbent's contract expect not to switch. On the other hand, the cost of foreclosing the entrant is that the incumbent has to compensate consumers for not buying from the more efficient entrant. Consumers' outside options to

[6] Aghion and Bolton (1987) needs entrant market power in order to have inefficient entry deterrence since the main role of breakup fees in their setup is to shift rent from the more efficient entrant to the incumbent and buyer, and this mechanism would be functional only if the entrant has positive margin from its sales (i.e., market power).

signing the incumbent's long-term contract is the net surplus of buying from the entrant which is determined by consumers' expectation about what the entrant's price will be in equilibrium. When the entrant knows that it cannot steal anyone from the incumbent's long-term contract (given that the breakup fee of that contract is very high), the entrant's weakly dominant strategy is to set its price at the incumbent's second period spot price in order to compete against the incumbent for consumers who did not sign the incumbent's long-term contract (even if the measure of these consumers is very close to zero).[7] In equilibrium, the incumbent's second period spot price is equal to the second-period price specified in the long-term contract, since the price below the long-term contract price induces losses from nearly all consumers and generates gains from measure zero of consumers. As a result, the incumbent endogenously lowers consumers' outside option of not signing the contract to zero and obtains twice the static monopoly profit from foreclosing the entrant. This result is surprising, in particular, because it does not depend on the efficiency advantage of the entrant: even when the entrant is very efficient compared to the incumbent, the entrant's equilibrium price does not depend on its marginal cost since the consumers who did not sign the incumbent's first contract are homogenous (as they do not face a switching cost), and so undifferentiated competition between the entrant and the incumbent implies that the entrant's price is equal to the incumbent's second period price in equilibrium.[8]

The above discussion was based on consumer heterogeneity due to switching costs. When the heterogeneity is due to mismatch value with the entrant's product, then consumers who do not sign the incumbent's long-term contract are not homogeneous as they were in the switching cost interpretation. When the incumbent cannot use breakup fees, there are no inter-temporal effects; the incumbent acts as static monopoly in period 1 and period 2 allocations are determined by differentiated competition between the incumbent and the entrant. Thus, this is very similar to the switching cost interpretation.

When breakup fees are feasible, the long-term contract affects the second period equilibrium and the results qualitatively differ from the analysis of switching costs. If the incumbent sets a lower breakup fee accommodating the entrant, consumers' outside options of not signing the incumbent's long-term contract are lower than the switching cost interpretation since consumers face mismatch value with the entrant's product, even when they did not sign the incumbent's long-term contract. This implies that the incumbent's profit is higher than the switching cost case if it accommodates the entrant. On the other hand if the incumbent sets a

[7] We would get the same result if there were a small measure of new consumers who entered the market in period 2.

[8] Note that switching costs do not matter for this result. If there were no switching costs, the incumbent would still be able to foreclose the efficient entrant by creating switching costs endogenously via breakup fees.

sufficiently high breakup fee making it too costly to switch to the entrant, consumers' outside option to signing the incumbent's long-term contract will be positive, and so higher than the foreclosure case with switching costs. With the mismatch interpretation, the entrant faces a downward sloping demand function for consumers that did not sign the incumbent's long-term contract. As a result, consumers always get a positive expected payoff in equilibrium and foreclosure becomes more costly for the incumbent than the switching cost case. More importantly, the incumbent's foreclosure profit is decreasing in the entrant's efficiency advantage and increasing in the highest level of mismatch value. The incumbent accommodates entry with breakup fees if and only if the entrant is sufficiently efficient relative to the incumbent. This still leads to inefficient switching by consumers.

It is well established that an incumbent might foreclose an efficient entrant by using breakup fees (liquidated damages as in Aghion and Bolton, 1987). The setup of Aghion and Bolton applies to the situation where a breakup fee is used in a contract between a seller and a single (or finitely many) buyer(s); for instance, in contracts between upstream and downstream firms. In the previously described examples, breakup fees are used in consumer contracts and one important implication is that sellers make take-it-or-leave-it contract offers to infinitesimal buyers, and so could not share the gains from trade with each individual buyer.[9] This might make it more difficult for the incumbent to deter efficient entry via breakup fees when it faces a continuum of buyers, since then the incumbent would have to compensate consumers for the expected costs of breakup fees in case they terminate their contract with the incumbent to buy from the more efficient entrant. Interestingly, we illustrate that how the incumbent's use of breakup fees lead to inefficient foreclosure in that case.

The paper proceeds as follows. In section 2, we present the basic switching cost model with competitive entry in period 2. We then allow the entrant to have market power in section 3. The model is modified to allow for consumer heterogeneity to arise from them having different values for the entrant's good instead of different switching costs. We conclude in the final section.

2 Benchmark: Competitive Entry

We consider a two-period model of entry. In the first period there is only one firm, the incumbent (I), and in the second period the incumbent faces at least two symmetric entrants,

[9] In Aghion and Bolton (1987) the incumbent and the buyer could share the gains from trade and so they both prefer to shift rent from the more efficient entrant via breakup fees even if this limits the extent of efficient entry in the case where the entrant's cost is uncertain.

each of which offers exactly the same product as the incumbent's. Let c_I and c_E denote the marginal cost of the incumbent and each entrant (E), respectively. We assume that each entrant is more efficient than the incumbent: $c_I > c_E$ and define $\Delta \equiv c_I - c_E$ as the cost difference.

There is mass 1 of consumers who are willing to buy one unit of the product in each period. Consumers have switching cost s, which is uniformly distributed over $(0, \theta]$. Assume that consumers learn their switching cost s at the beginning of period 2. Firms never observe s. The timing of the contracting is the following:

Period 1 The incumbent offers a two-period contract, (p_{I1}, p_{I2}), where p_{I1} is paid at the signature of the contract for buying one unit in period 1 and p_{I2} is paid if the consumer buys an additional unit from the incumbent in period 2. Consumers then decide whether to accept the incumbent's offer.

Period 2 Simultaneously, each entrant offers price p_E and the incumbent offers spot price p_{I2}^S. Consumers then learn their switching cost. Consumers who signed the incumbent's first period contract decides whether to stay with that contract or switch to the incumbent's spot contract or switch to the entrant's contract. Consumers who did not sign the incumbent's contract decide whether to buy the incumbent's spot offer, the entrant's offer, or nothing. A consumer who did not sign a contract faces no switching cost.

We assume that consumers' valuation from the product is sufficiently high so that the incumbent can be profitable if consumers switch to the entrants whenever it is efficient (the first two levels in the maximum function):

Assumption 1 $v > max\{c_E + \frac{\Delta c(4\theta - \Delta c)}{2\theta}, c_I + \frac{\theta}{2}, \frac{2\theta + c_E + 2c_I}{3}\}$.

Consumers' valuation should also be high enough to ensure an interior solution to the incumbent's problem when there is a single entrant in period 2 (the last level in the maximum function):[10]

2.1 Equilibrium Analysis

Consumers are ex-ante homogenous, so either all consumers sign the incumbent's long-term contract or no one signs it. Ex-post efficiency requires that consumers switch to the entrants

[10]When this assumption does not hold, there would be no effective competition between the entrant and the incumbent in period 2, i.e., the entrant would behave like a monopolist, since the incumbent's constrained optimal price would be $p_{I2} = v$ without breakup fees and the incumbent would set $p_{I2} - d = v$ with breakup fees. The entrant's price would then be $p_E = \frac{v + c_E}{2}$.

in period 2 if and only if their switching cost is lower than the entrant's cost advantage: $s < \Delta c$. When $\Delta c < \theta$ ex-post efficiency requires that $\frac{\Delta c}{\theta}$ amount of consumers switch to the entrants and $\frac{\theta - \Delta c}{\theta}$ amount of consumers buy from the incumbent in period 2. When $\Delta c > \theta$ ex-post efficiency requires that all consumers switch to the entrants. Assumption 1 ensures that in either of these cases the incumbent can profitable sell its long-term contract to all consumers in period 1.

In period 2 competitive entrants price at the marginal cost: $p_E^* = c_E$. The incumbent does not want to undercut its long-term contract price, $p_{I2}^{S*} \geq p_{I2}$, since it cannot attract any consumer to its spot contract given that competitive entrants sell at their marginal cost, which is lower than the incumbent's marginal cost. If a consumer does not sign the incumbent's contract in period 1, she gets $v - c_E$, which constitutes her outside option when she decides whether to buy the incumbent's contract in period 1. If a consumer signs the incumbent's contract in period 1, she stays with the incumbent if her switching cost is above the price difference between the incumbent and the entrant, $s > p_{I2} - c_E$, and switches to an entrant otherwise. So a consumer's expected surplus from signing the incumbent's contract is

$$EU_{signI} = 2v - p_{I1} - p_{I2}Pr(s > p_{I2} - c_E) - c_E Pr(s < p_{I2} - c_E) - \int_0^{p_{I2} - c_E} \frac{s}{\theta} ds, \quad (1)$$

which is the consumption value of two units minus the up-front fee of the incumbent's contract minus the incumbent's price times the probability of buying from the incumbent in period 2 minus the entrants' price times the probability of buying from an entrant in period 2 minus the expected switching costs. Note that the demand for entrants is always positive since we must have $p_{I2} - c_E > 0$ in equilibrium given that the incumbent is less efficient than the entrant ($p_{I2} \geq c_I > c_E$). The incumbent's second period demand is positive if and only if $p_{I2} - c_E \leq \theta$, which we will verify at the end.

All consumers sign the incumbent's contract in period 1 if and only if their expected surplus from signing it is at least equal to their outside option:

$$EU_{nosignI} \geq v - c_E. \quad (2)$$

Under the latter constraint the incumbent's expected profit over the two periods is

$$\Pi_I = p_{I1} - c_I + (p_{I2} - c_I)Pr(s > p_{I2} - c_E). \quad (3)$$

The incumbent sets (p_{I1}, p_{I2}) by maximizing its profit subject to the consumers' participation constraint, (2). At the optimal solution the incumbent sets the highest first-period price satisfying the constraint:

$$p_{I1}^* = v + (c_E - p_{I2})Pr(s > p_{I2} - c_E) - \int_0^{p_{I2}-c_E} \frac{s}{\theta} ds$$

Replacing the latter into the incumbent's profit we can rewrite it

$$\Pi_I = v - c_I - \Delta c Pr(s > p_{I2} - c_E) - \int_0^{p_{I2}-c_E} \frac{s}{\theta} ds$$

Given that s is uniformly distributed over $(0, \theta]$ the optimal period 2 price is at the marginal cost: $p_{I2}^* = c_I$. The incumbent's second period demand is positive if and only if $p_{I2}^* - c_E \leq \theta$ or $\Delta c \leq \theta$. Hence, we obtain the equilibrium of the baseline model:

Proposition 1 *If there are competitive entrants in period 2, there is a unique equilibrium where all consumers buy the incumbent's first period contract, the entrants set $p_E^* = c_E$ and the incumbent sets $p_{I2}^* = p_{I2}^{S*} = c_I$, which implement the efficient switching in period 2: consumers switch to the entrants if and only if $s \leq \Delta c$.*

- *If $\Delta c \geq \theta$, all consumers switch to the entrants in period 2 and the equilibrium profits are*

$$\Pi_I^* = v - c_I - \frac{\theta}{2}, \; \Pi_E^* = 0.$$

- *If $\Delta c \leq \theta$, the second period demands are*

$$D_{I2}^* = \frac{\theta - \Delta c}{\theta}, \; D_E^* = \frac{\Delta c}{\theta}.$$

and equilibrium profits are

$$\Pi_I^* = v - c_I - \Delta c + \frac{(\Delta c)^2}{2\theta}, \; \Pi_E^* = 0.$$

The incumbent can capture all expected consumer surplus ex-ante via upfront fees, p_{I1}, and so wants to maximize this surplus by inducing efficient switching which requires setting p_{I2} at its marginal cost. Hence, in equilibrium all consumers buy the incumbent's contract in period 1 and switch to the entrants if and only if $s \leq \Delta c$. If all consumers switch to the entrants, $\Delta c \geq \theta$, the total industry surplus is $2v - c_I - c_E - \frac{\theta}{2}$, which is the total consumption utility minus the total costs of the active firms minus the expected switching costs. However, if $\Delta c \leq \theta$, the incumbent sells to $\frac{\theta - \Delta c}{\theta}$ of consumers in period 2 and the entrant sells to $\frac{\Delta c}{\theta}$ of consumers. In this case the total industry surplus is $2(v - c_I) + \frac{\Delta c^2}{2\theta}$. The incumbent captures the total surplus after leaving consumers their outside option, $v - c_E$.

2.2 Breakup fees

As we discussed in the Introduction breakup fees are common features of long-term contracts for many services to consumers. We now analyze the equilibrium if the incumbent is allowed to use a breakup fee d in its first-period contract such that d is paid if the consumer who signed the contract buys from the entrant in period 2. There are two differences from the previous analysis when the incumbent's first-contract includes a breakup fee: 1) The switching decisions in period 2 will now depend on the breakup fee, 2) The incumbent can use the breakup fees to earn revenues from the switchers. Consider consumers who signed the incumbent's first-period contract. In period 2 these consumers switch to the entrants if and only if $s < p_{I2} - d - c_E$, in which case they have to pay the entrant's price, c_E, as well as the incumbent's breakup fee, d. If they stay with the incumbent ($s > p_{I2} - d - c_E$), they buy at p_{I2} and do not have to pay d. The expected surplus from signing the incumbent's contract is now

$$EU_{signI} = 2v - p_{I1} - p_{I2}Pr(s > p_{I2} - d - c_E) - (c_E + d)Pr(s < p_{I2} - d - c_E) - \int_0^{p_{I2}-d-c_E} \frac{s}{\theta} ds.$$

All consumers sign the incumbent's contract in period 1 if and only if $EU_{signI} \geq v - c_E$. Under this constraint the incumbent's expected profit over the two periods is

$$\Pi_I = p_{I1} - c_I + (p_{I2} - c_I)Pr(s > p_{I2} - c_E - d) + dPr(s < p_{I2} - d - c_E).$$

At the optimal solution the incumbent sets the highest first-period price satisfying the consumers' participation constraint:

$$p_{I1}^* = v + (c_E - p_{I2})Pr(s > p_{I2} - d - c_E) - dPr(s < p_{I2} - d - c_E) - \int_0^{p_{I2}-d-c_E} \frac{s}{\theta} ds.$$

Replacing the latter into the incumbent's profit, we can rewrite it as

$$\Pi_I^* = v - c_I - \Delta c Pr(s > p_{I2} - d - c_E) - \int_0^{p_{I2}-d-c_E} \frac{s}{\theta} ds$$

Observe that the incumbent's equilibrium profit depends only on the difference between the second period price and the breakup fee, $p_{I2} - d$. This is because the incumbent captures ex-ante expected surplus of consumers via p_{I1}, the breakup fee revenues are fixed transfers from consumers to the incumbent and so they are washed-out. Similar to the baseline model the incumbent can implement the efficient outcome described in Proposition 1 by setting $p_{I2} - d = c_I$. Indeed, this is the optimal thing to do if the incumbent does not foreclose the entrants. Different from the baseline model the incumbent can now foreclose the more efficient entrants by setting $p_{I2} - d < c_E$, since then the entrants cannot steal any consumer from the incumbent's first-period contract even if they sell at marginal cost. Consumers would be willing to sign such a contract if they are compensated for what they would get from not signing it, i.e., if the incumbent leaves them their outside option: $v - c_E$. But then the total industry profit would be $2(v - c_I)$ and the incumbent would capture this after leaving consumers $v - c_E$. Hence, the incumbent's profit from foreclosing the efficient entrants would be $v - c_I - \Delta c$. Comparing this with what the incumbent would earn under the efficient outcome: $\Pi_I^* = v - c_I - \frac{\theta}{2}$ if $\Delta c \geq \theta$ and $\Pi_I^* = v - c_I - \Delta c + \frac{\Delta c^2}{2\theta}$ if $\Delta c \leq \theta$, we demonstrate that the incumbent does not have an incentive to foreclose more efficient entrants even if it can do so by using breakup fees.

Corollary 1 *Suppose that the incumbent's first period contract also has a breakup fee, d, which is paid if a consumer who signed the first-period contract buys from an entrant in period 2. In equilibrium the incumbent implements the same (efficient) outcome as the one described in Proposition 1. Hence, there is no inefficient foreclosure in equilibrium.*

2.3 Constant unit prices and break fees

It is common in many markets such as wireless phone and cable to use constant monthly prices along with breakup fees. We now demonstrate that the incumbent can offer a contract that has the same unit price in each period along with a breakup fee d which is paid if the consumer who signed the contract buys from the entrant in period 2 and implement the same outcome as when the incumbent can offer different prices in each period.

Corollary 2 *Suppose that the incumbent's first period contract has a constant per period unit price p_I and a period 2 breakup fee d. In equilibrium the incumbent implements the same (efficient) outcome as the one described in Proposition 1. The equilibrium prices are $p_I^* = \frac{p_{I1}^* + p_{I2}^*}{2}$, $d^* = \frac{p_{I1}^* - p_{I2}^*}{2}$ where (p_{I1}^*, p_{I2}^*) are the equilibrium prices in the baseline model.*

3 Single Entrant

In the benchmark we assumed that entrants are perfectly competitive and so they always set their prices to their marginal costs. Now, we relax this assumption to allow for market power of entrants. To highlight the differences from the benchmark we consider the extreme case of a single entrant, which has cost $c_E < c_I$ as before. The main effect of giving the entrant market power is that consumers' outside options of rejecting the incumbent's contract are endogenized. This is for two reasons. First, the entrant's price in period 2 will depend on the incumbent's period 1 contract along with any potential spot contract offered by the incumbent. With competitive entry, the entrants priced at marginal cost and thus consumers' outside option of rejecting the incumbent's contract did not depend on the incumbents' actions. Second, we need to see if the outside option for consumers can depend on whether they think other consumers will accept or reject the incumbent's first period offer. This needs to be done to see if there are multiple equilibria which depends on the beliefs of consumers about whether other consumers will accept the incumbent's offer.

We modify the second stage of the game as the following:

Period 2 Simultaneously, the entrant offers price p_E and the incumbent has the opportunity to offer a spot price p_{I2}^S that is different than its stated second period price in the first period contract, p_{I2}. Consumers then learn their switching costs. Consumers who signed the incumbent's first period contract decide whether to stay with that contract, switch to the incumbent's spot contract, or switch to the entrant's contract. Consumers who did not sign the incumbent's contract decide whether to buy from the incumbent at the spot price or buy from the entrant.

The other assumptions and notation of the model remain the same as before.

Proposition 2 *If there is single entrant in period 2, in equilibrium all consumers buy the incumbent's first period contract.*

- *If $\Delta c \geq 2\theta$, there is a unique equilibrium where all consumers switch to the entrant. The equilibrium profits and expected utilities are*

$$\Pi_I^* = v - c_I - \frac{\theta}{2}\ ,\ \Pi_E^* = \Delta c \text{ and } U = v - c_I.$$

- *If $\Delta c \leq 2\theta$, there is a unique equilibrium where second period demands are*

$$D_{I2}^* = \frac{2\theta - \Delta c}{3\theta}, D_E^* = \frac{\theta + \Delta c}{3\theta}.$$

and equilibrium profits and utilities are

$$\begin{aligned}\Pi_I^* &= v - c_I + \frac{\theta^2 - 4\Delta c\theta + \Delta c^2}{6\theta},\\ \Pi_E^* &= \frac{(\Delta c + \theta)^2}{9\theta} \text{ and } \underline{U} \equiv v - \frac{\theta + 2c_E + c_I}{3} < v - c_I.\end{aligned}$$

To prove the result, we first assume that all consumers believe that each of their fellow consumers will accept the incumbent's first period offer and derive equilibrium payoffs. Next, we find whether this equilibrium is unique for all parameter values and beliefs of consumers about how other consumers will react to the incumbent's offer.

One key to the equilibrium, is to find the entrant's price in period 2. If no one signed the incumbent's contract, then all consumers are identical since they face no switching cost, and thus Bertrand competition would ensue where the entrant would get all consumers at a price of c_I. If the consumers do sign the incumbent's contract in period, then the entrant's price will depend on whether the incumbent is willing to offer a lower spot price in period 2 than the stated price in the period 1 contract; along the equilibrium path, the incumbent does not offer such a price. We find that there are two cases to analyze, one where a spot price offered by the incumbent would keep a positive measure of consumers, when $\Delta c \leq 2\theta$, and one where the spot price would attract no consumers, when $\Delta c \geq 2\theta$. In the latter case, a spot by the incumbent even at marginal cost, c_I, can attract no consumers because the entrant's optimal price will not exceed $c_I - \theta$. This can be seen by noting that the entrant's best response to the incumbent's second period price is

$$p_E = \frac{p_{I2} + c_E}{2},$$

and when $p_{I2} = c_I$ and the entrant sets a price of $c_I - \theta$, then $\Delta c = 2\theta$. Thus, as θ falls the entrant can set even a higher price and still attract all consumers.

By pinning down the continuation game in period 2, we go back to the period contract offered by the incumbent. Again, the analysis breaks down into two depending on how much more efficient the entrant is than the incumbent. In both settings, the incumbent (efficiently) induces all consumers to sign the first period contract and sets up the terms of the contract to make the consumers indifferent between signing the first period contract and not signing and buying the good from the entrant in period 2. The incumbent needs to take into account the consumers' expected switching costs when offering such a contract. When Δc is small, a non-signing consumer will obtain $v - p_E$, where the entrants price is greater than $c_I - \theta$. As it turns out, the incumbent's unconstrained second period price in the long-term contract is exactly the same as what its spot price would be in period 2. This is due to the fact, that the incumbent is acting as a Stackelberg leader in period 2 and just like in that model, the leader would not want to raise its output (lower its price in our model). On the other hand, when the entrant is very efficient, the incumbent finds it unprofitably to try to keep any consumers in period 2, and sets its second period price in the long term contract equal to marginal cost, c_I, since any contract with a higher period 2 price is not credible and will have the incumbent lowering its price to marginal cost. Now, the period price extracts consumers by charging v less compensating consumers for their expected switching costs, $\frac{\theta}{2}$.

To obtain uniqueness, we see if consumers have a credible outside option to reject an incumbent's offer if the incumbent does not offer them at least $v - c_I$, which is their outside option if all reject his offer, since there will be Bertrand competition with no switching costs at that point. When Δc is large, $v - c_I$ is what a consumer's expected utility is in the equilibrium we derived when all consumers expect each other to accept the incumbent's offer. When Δc is small, consumers outside option is $\underline{U}< v - c_I$. To see that it is not credible for a consumer to reject an offer of $\underline{U}$ if she thinks all other consumers will reject it. Suppose that there is such an equilibrium. If the incumbent offers $\underline{U}$ with a price that is strictly less than v, which he does in our equilibrium, than an individual consumer will deviate and accept the incumbent's contract since she gets a positive surplus in period 1 and since the incumbent will charge c_I as a spot price in period 2 to compete for the mass of consumers, and the defecting consumer will obtain the same second period utility as all consumers who rejected the incumbent's first period contract. That is, since the rejecting consumer does not change the state of the market, it is optimal to deviate from the equilibrium. Thus, we cannot have such an equilibrium. We also demonstrate that there cannot be an equilibrium where only a subset of consumers accept the incumbent's first period contract.

We have some further observations about the equilibrium. First, $p^*_{I2} > c_I$ when $\Delta c < 2\theta$. Intuitively, the incumbent sets p^*_{I2} above c_I in order to raise the entrant's equilibrium price and thus lower the consumers' outside option in period 1, i.e., to leave less rent to consumers. When there was competitive entry (Section 2), the outside option of consumers was exogenous and so the incumbent did not distort the total surplus to capture more rent from ex-ante homogenous consumers. In general, the key difference between a setup with entrant market power and a setup with perfectly competitive entrants is that in the former consumers' outside option in period 1 is endogenous: it depends on the entrant's price which depends on the initial contract of the incumbent and consumer expectations of what other consumers will do regarding the incumbent's contract.

It is interesting to see that consumers do not have a credible incentive to reject the incumbent's offer of more than $\underline{U}$, but less than $v - c_I$, which is their outside option if no consumer sign the incumbent's contract in period 1. This is because an individual consumer can be tempted to sign the incumbent's contract in period 1 if the price is less than v. The consumer will have a positive first period utility in period 1, and in the continuation game the incumbent will price down to marginal cost because it will compete for the "market" with the entrant in period 2. This is because the incumbent can offer spot contracts in period 2.

Second, efficiency requires that consumers switch to the entrant if and only if $s < \Delta c$. The previous result implies that if $\Delta c/2 > \theta$, in equilibrium all consumers switch to the entrant and this is also efficient. However, if $\Delta c/2 < \theta$, in equilibrium consumers switch to the entrant if and only if $s < p^*_{I2} - p^*_E = \frac{\theta + \Delta c}{3}$. This implies the following result:

Corollary 3 *If $\theta < \Delta c/2$, in equilibrium all consumers efficiently switch to the entrant. If $\theta \in (\Delta c/2, 2\Delta c)$ there is too little switching, and if $2\Delta c < \theta$, there is too much switching.*

To understand the intuition behind potential inefficiency in equilibrium first note that the difference between the incumbent's and the entrant's price, $p^*_{I2} - p^*_E = \frac{\theta + \Delta c}{3}$, determines the marginal consumer type in period 2 and the marginal type increases in Δc in equilibrium. At $2\Delta c = \theta$, the price difference is exactly the difference in cost and so we get efficiency. For small θ, less than $\Delta c/2$, all buyers efficiently switch. As θ grows from there up to $2\Delta c$, the price difference is less than the cost difference and hence there is too little switching. For values of θ above $2\Delta c$, the equilibrium price difference is larger than the cost difference.

Intuitively, when the switching cost heterogeneity is very small relative to the efficiency difference, the incumbent cannot compete with the entrant in period 2 and attract a positive measure of consumers even when pricing at marginal cost. As the heterogeneity in switching costs grows, the incumbent can compete with the entrant profitably in period 2. It is useful to note that the two firms' prices are strategic complements and that both are increasing in the switching cost parameter θ, with the incumbent's price increasing at twice the rate as the entrant's price. When θ is in the intermediate range, the consumer heterogeneity allows the incumbent to compete with the entrant, but the price difference is still small enough relative to the difference in firm costs such that $p^*_{I2} - p^*_E < \Delta c$ and there is too little switching. As θ grows then the heterogeneity in switching cost dominates the cost difference and the incumbent's price exceeds the entrants price by more than Δc. Thus, there will be too much switching in equilibrium.

Finally, we can show the following comparative static result:

Corollary 4 *An increase in θ i) an increase in θ will have no effect on switching behavior if $\theta < \Delta c/2$; ii) will efficiently induce more consumers to switch if $\theta \in (\Delta c/2, 2\Delta c)$; iii) will inefficiently increase more consumers to switch to the entrant if $2\Delta c < \theta$.*

3.1 Breakup fees

Now, we allow the incumbent's first-period contract to include a breakup fee d in period 1, which has to be paid if a consumer who signed the incumbent's first-period contract switches to the entrant or makes no purchase in period 2. The incumbent may also offer a spot price

p^S_{I2} in period 2. The consumers who signed the incumbent's contract in period 1 do not have to pay d if they switch to the incumbent's spot contract in period 2.

Proposition 3 *Suppose that the incumbent offers a two-period contract* (p_{I1}, p_{I2}) *and a breakup fee d in period 1 and a spot contract* p^S_{I2} *in period 2, where it faces a more efficient entrant. In the unique equilibrium all consumers buy the incumbent's first period contract, the incumbent sells to all consumers in period 2, and so the more efficient entrant is fully foreclosed. The equilibrium prices, payoffs and expected utilities are*

$$\begin{aligned} p^*_{I1} &= p^*_{I2} = p^{S*}_{I2} = p^*_E = v \\ \Pi^*_I &= 2(v - c_I),\ \Pi^*_E = 0,\ \textit{and } U^* = 0 \end{aligned}$$

We will first prove that there exists a unique equilibrium when consumers believe that all of the other consumers sign the incumbent's first period offer. Next, we show that this is the unique equilibrium for all parameter values and beliefs of consumers about how the others will react to the incumbent's offer. The key is to determine the entrant's equilibrium price, since this will determine consumers' outside option to signing the incumbent's first-period contract.

If none of the consumers signed the incumbent's first-period contract, undifferentiated Bertrand competition between the entrant and the incumbent implies that the entrant sells to all consumers at price c_I. If all consumers signed the incumbent's first-period contract, the breakup fee of the incumbent's contract affects the amount of consumers who switch from the incumbent to the entrant in period 2. If a consumer stays with the incumbent, she pays the incumbent's lowest price for a unit consumption in period 2: $min\{p_{I2}, p^S_{I2}\}$, but if she switches to the entrant she pays the entrant's price plus her switching cost and the breakup fee. Consumers could also decide not to buy in period 2 in which case they have to pay the breakup fee to the incumbent (we will verify the condition under which consumers do not want to do this in equilibrium). On the equilibrium path the incumbent sets the second period price of the long-term contract equal or above the equilibrium level of its spot contract so that it would not have an incentive to undercut itself in the continuation of the game (in the spirit of renegotiation proof contracts). This would then imply that consumers with switching costs below $p_{I2} - d - p_E$ switch to the entrant and the rest continues buying from the incumbent under the condition that buying from the incumbent gives them some non-negative surplus, $p_{I2} - d \leq v$.

The incumbent can use breakup fees as a tool to affect consumers' beliefs about what the entrant's price will be. Consider a subgame where the incumbent sets $d > p_{I2} - c_E$. Consumers then expect that the entrant cannot steal any consumer from the incumbent's long-term contract even if it sells at its marginal cost. It is then a weakly dominant strategy for the entrant to undercut the incumbent's spot price in case some consumers did not sign the incumbent's first-period contract (in the spirit of Trembling Hand Nash Equilibrium). The incumbent prefers to undercut the entrant's price in the spot market until it reaches its first period contract price. If $p_E = p_{I2}$, the incumbent does not have an incentive to undercut the entrant's price since then it would lose positive margin from a measure 1 of consumers and would gain a positive margin from consumers of measure zero. Hence, in the equilibrium of this subgame the entrant's price is equal to the incumbent's second period price. But then the incumbent could control the entrant's price perfectly via the choice of p_{I2}. It is optimal for the incumbent to reduce the consumers' expected utility from their outside option to zero by offering $p_{I2} = v$, since then consumers will expect the entrant's price to be at v and so be willing to sign the incumbent's first period contract as long as $p_{I1} \leq v$. In the equilibrium of this subgame, the incumbent earns twice its static monopoly profit by setting a sufficiently high breakup fee, $d > v - c_E$, and the prices that capture all surplus from consumers, $p_{I1} = p_{I2} = v$. We will next show that the incumbent prefers to implement this foreclosure outcome rather than any subgame equilibrium where the entrant has some positive sales in period 2.

A necessary condition to have some positive sales by the entrant is that the incumbent's first contract has a sufficiently low breakup fee: $d < p_{I2} - c_E$. But then consumers expect to switch to the entrant in period 2 with probability $p_{I2} - d - p_E$, in which case they have to pay the entrant's price as well as the breakup fee. So the expected consumer surplus from signing the incumbent's contract is

$$EU_{signI} = 2v - p_{I1} - p_{I2}Pr(s > p_{I2} - d - p_E) - (p_E + d)Pr(s < p_{I2} - d - p_E) - \int_0^{p_{I2}-d-p_E} \frac{s}{\theta} ds,$$

which is the two period consumption utility minus the period 1 price minus the expected price in period 2 minus the expected switching costs. The expected surplus from not signing the incumbent's contract is $v - p_E$, given that consumers are infinitesimal and so do not influence the equilibrium level of p_E. Consumers sign the incumbent's first period contract if and only if their expected surplus from signing is greater than their outside option: $EU_{signI} \geq v - p_E$. Under this constraint, the incumbent's profit is the profit from period 1 sales, expected profit from period 2 sales and breakup revenues to be collected from consumers who switch to the entrant:

$$\Pi_I = p_{I1} - c_I + (p_{I2} - c_I)Pr(s > p_{I2} - d - p_E) + dPr(s < p_{I2} - d - p_E).$$

It is optimal for the incumbent to set the highest first-period price satisfying the participation constraint of consumers:

$$p^*_{I1}(p_{I2}) = v + (p_E - p_{I2})Pr(s > p_{I2} - d - p_E) - dPr(s < p_{I2} - d - p_E) - \int_0^{p_{I2}-d-p_E} \frac{s}{\theta} ds.$$

After replacing the equality of the optimal upfront fee into the incumbent's profit, its profit becomes

$$\Pi_I = v - c_I + (p_E - c_I)Pr(s > p_{I2} - d - p_E) - \int_0^{p_{I2}-d-p_E} \frac{s}{\theta} ds.$$

First observe that the incumbent's profit depends only on $p_{I2} - d$, since the second period consumption behavior depends only on this difference (i.e., p_E is a function of $p_{I2} - d$ in the equilibrium of period 2), the incumbent can capture ex-ante expected consumer surplus via the first-period price and this washes out the breakup revenues and period 2 sales revenues from the incumbent's profit function: the level of breakup fee matters only via its effect on the second period consumption decisions. Hence, different levels of p_{I2} and d that lead to the same difference $p_{I2} - d$ will result in the same profit for the incumbent. Without loss of generality we could set $d = 0$ in any subgame where the entrant has positive sales in period 2. But then the equilibrium outcome without breakup fees must be the equilibrium among such subgames. In this equilibrium we have

$$p^*_{I2} - d^* = \frac{2\theta + c_E + 2c_I}{3}.$$

Assumption 1 ensures that consumers get non-negative surplus if they buy from the incumbent in period 2: $v > p^*_{I2} - d^*$. In the Appendix we show that the incumbent's profit in this solution is always lower than its profit from foreclosure. Hence, the incumbent implements the foreclosure outcome when consumers believe that the others sign the incumbent's first period contract.

Finally, we show the uniqueness of the equilibrium. Note that consumers would be better off if they could coordinate and reject the incumbent's first offer. To see this suppose that all consumers reject the incumbent's first contract. But then in period 2 the entrant wins the Bertrand competition by offering a price slightly below the incumbent's cost. Hence, consumers would get a surplus of $v - c_I$ if they could coordinate to reject the incumbent's offer. On the other hand, we show that rejecting the incumbent's offer is not credible for each consumer given that she believes the others to reject the incumbent's first offer. A consumer's expected surplus from signing the incumbent's first period contract would be $2v - p_{I1} - c_I$ since she expects c_I to be the price in period 2 equilibrium. Each consumer unilaterally prefers to sign the incumbent's first period contract as long as $p_{I1} \leq v$. If the incumbent makes the equilibrium offer of Proposition 3 with a slightly lower first-period price, $p^*_{I1} < v$, a consumer will be strictly better-off by taking this offer rather than rejecting it. Since all consumers will have this incentive, their threat of rejecting that offer is not credible. Hence, there exists no equilibrium where all consumers reject the incumbent's first-period contract. We also show in the Appendix that there exists no equilibrium where some consumers reject the incumbent's first-period offer.

This result is surprising in particular because the incumbent always forecloses the more efficient entrant and captures the total industry surplus under foreclosure regardless of the efficiency level of the entrant. It is important to note that switching costs do not matter for the equilibrium described in Proposition 3. If there were no switching costs, the incumbent would still be able to foreclose the efficient entrant by creating switching costs endogenously via breakup fees.

This result does not require ex-ante uncertainty or commitment to the first-period contract by the incumbent. In Aghion and Bolton (1987) the uncertainty on the entrants' cost level is necessary to have inefficient foreclosure in equilibrium. In our setup ex-ante uncertainty on consumers' intrinsic switching costs is not necessary to have foreclosure of the more efficient entrant in equilibrium. In Aghion and Bolton (1987) the ex-ante (before entry) commitment to the breakup fee is crucial to have inefficient foreclosure in equilibrium: If the incumbent and the buyer were allowed to renegotiate their contract after the entry decision is made, the incumbent would not be able to foreclose the entrant since then in the case of entry the coalition of the incumbent and the buyer would be better-off ex-post by setting a zero breakup fee in order to trade with the more efficient entrant. Different from Aghion and Bolton (1987) we allow consumers to buy one unit in both periods and allow the incumbent to sell a spot contract in period 2, and show that the more efficient entrant is foreclosed even if the incumbent cannot commit not to sell a spot contract in period 2.

Three things are crucial in obtaining this outcome: 1) the entrant has market power; 2) the incumbent can use a break-up fee; 3) consumers who did not sign the incumbent's contract do not have intrinsic heterogeneity (as they do not face a switching cost).

When the entrant has market power, the entrant's price depends on the incumbent's first-period contract and so consumers' outside option (i.e., surplus from not signing the incumbent's long-term contract) becomes endogenous. This is the key difference from the section where we had competitive entrants and the incumbent implemented the efficient outcome in equilibrium. However, with entrant market power, the incumbent distorts the equilibrium prices to lower consumers' outside option. If the incumbent was not allowed to use a break-up fee, the more efficient entrant always has positive sales in period 2.

The third crucial point implies undifferentiated Bertrand competition between the entrant and the incumbent, which results in the spot market equilibrium prices that are independent of the entrant's cost. In the next section we will extend our analysis by changing the interpretation of s from switching cost to mismatch value, that is, the reduction in consumers' valuation, if they buy the good from the entrant. The new interpretation implies that consumers who did not sign the incumbent's contract will have to pay s if they buy from the entrant in period 2 and will allow us to have downward sloping demand also when consumers do not sign the incumbent's first period contract.

4 Extension: Match value

Now assume that the utility from consuming the incumbent's good is v as before, but the value from consuming the entrant's good is $v - s$ such that consumers learn their draw of s at the beginning of period 2 and s is uniformly distributed over $(0, \theta]$. We interpret s as the differentiation value of the incumbent's product compared to the entrant's and assume that consumers do not know their matching value to the entrant's product before the entrant appears. We assume that the consumption value is sufficiently high to ensure that in the foreclosure scenario with breakup fees the incumbent's optimal second period price is below the consumption utility so that the entrant would not behave like a local monopoly in equilibrium:

Assumption 2 $v > 4\theta + c_E$.

In the benchmark model s was the switching cost paid by a consumer who signed the incumbent's contract in period 1 and switch to the entrant in period 2. Switching cost made the incumbent's product differentiated from the entrant's for those consumers who signed the incumbent's first period contract. In that model if a consumer did not sign the incumbent's first period contract, she would not have to pay a switching cost if she buys from the entrant in period 2, and so in that case the incumbent's and the entrant's products were homogenous. However, here we interpret s as the incumbent's differentiation value and so each consumer loses s if she buys the entrant's product rather than the incumbent's product in period 2. This is true even if the consumer did not sign the incumbent's first period contract.

Proposition 4 *In the match value interpretation, if the incumbent cannot use a breakup fee in its long-term contract, there are no inter-temporal effects. In the unique equilibrium all consumers sign the incumbent's long-term contract..*

- *If $\Delta c > 2\theta$, all consumers buy from the entrant in period 2. The equilibrium profits and consumer utility are*

$$\begin{aligned} \Pi_I^* &= v - c_I, \Pi_E^* = \Delta c - \theta, \\ U^* &= v - c_I + \frac{\theta}{2}. \end{aligned}$$

- *If $\Delta c \leq 2\theta$, some consumers switch to the entrant in period 2. The equilibrium profits and consumer utility are*

$$\begin{aligned} \Pi_I^* &= v - c_I + \frac{(2\theta - \Delta c)^2}{18\theta}, \Pi_E^* = \frac{(\theta + \Delta c)^2}{9\theta}, \\ U^* &= v - \frac{\theta + 2c_E + c_I}{3} - \frac{(\theta + \Delta c)(5\theta - \Delta c)}{18\theta}. \end{aligned}$$

We prove the result by first determining the second period equilibrium. If none of the consumers signed the incumbent's first period contract, the incumbent and the entrant are differentiated competitors in period 2 and the solution to the differentiated duopoly competition determine the equilibrium prices. Consumers with low match value relative to the price difference of the incumbent and the entrant, $s < p_{I2}^S - p_E$, would buy from the entrant and the rest would buy from the incumbent. If the entrant is very efficient ($2\theta \leq \Delta c$), the incumbent cannot compete against the entrant in period 2:

$$\begin{aligned} p_{I2}^{S*} &= c_I, p_E^* = c_I - \theta, \\ D_{I2}^{S*} &= 0, D_E^* = 1. \end{aligned} \tag{4}$$

Otherwise, the incumbent sets its best-reply price and sells to some consumers in period 2:

$$\begin{aligned} p_{I2}^{S*} &= \frac{2\theta + c_E + 2c_I}{3}, p_E^* = \frac{\theta + 2c_E + c_I}{3} \\ D_{I2}^{S*} &= \frac{2\theta - \Delta c}{3\theta}, D_E^* = \frac{\theta + \Delta c}{3\theta}, \end{aligned} \tag{5}$$

To sum up, if none of the consumers signed the incumbent's first period contract, the second period equilibrium outcome would be the same as the equilibrium of the previous section (with switching costs) where the incumbent is not allowed to use breakup fees.

If all consumers signed the incumbent's first contract, the second stage equilibrium is again the same as the analysis with switching costs, since then consumers will have to pay s if they switch to the entrant in period 2, like in the previous section. We first show the existence of an equilibrium where all consumers signed the incumbent's first contract.

Suppose that each consumer expects every other consumer to sign the incumbent's first period contract. The period 1 solution differs from the previous section's analysis, since a consumer's outside option to the incumbent's contract is now her expected surplus from buying a unit in period 2 where she buys from the more efficient entrant and gets $v - s - p_E^*$ if her s is low relative the price difference between the firms and buys from the incumbent and gets $v - p_{I2}$ if her s is high relative to the price difference (given that we have $p_{I2} = p_{I2}^{S*}$ in equilibrium):

$$\begin{aligned} EU_{nosignI} &= v - p_{I2} Prob(s > p_{I2} - p_E^*) - \int_0^{p_{I2} - p_E^*} (s + p_E^*) \frac{1}{\theta} ds, \\ &= v - p_{I2} + \frac{(p_{I2} - p_E^*)^2}{2\theta}. \end{aligned} \tag{6}$$

Consider first the case where the incumbent cannot have a breakup fee in its first contract. Observe that period 1 has no effect on period 2 competition, since a consumer's expected utility in case she signs the incumbent's first contract is equal to the first period payoff plus her outside option:

$$EU_{signI} = v - p_{I1} + v - p_{I2} Prob(s > p_{I2} - p_E^*) - \int_0^{p_{I2} - p_E} (s + p_E^*) \frac{1}{\theta} ds.$$

Hence, a consumer wants to sign the incumbent's contract if and only if $p_{I1} \leq v$. At the optimal solution the incumbent sets $p_{I1}^* = v$ (as if it was static monopoly), $p_{I2}^* = p_{I2}^{S*}$ and the second period prices are given by the equilibrium of the differentiated duopoly competition: if $2\theta \geq \Delta c$, (5); otherwise (4).

Observe that this is the unique equilibrium. If consumers believe that the other consumers reject the incumbent's first period offer, they expect the second period equilibrium prices to be the equilibrium of differentiated duopoly competition given by (5) if $2\theta \geq \Delta c$ (and by (4) otherwise), and $p_{I2}^* = p_{I2}^{S*}$. Hence, consumers' beliefs about others do not impact the outside option to signing the incumbent's contract, which is given by (6). Moreover, each consumer prefers to sign the incumbent's first contract as long as $p_{I1} \leq v$ since she expects to get exactly the same second period payoff as her outside option in case she signs the incumbent's contract.

Proposition 5 *In the match value interpretation, if the incumbent is allowed to use a breakup fee in its long-term contract, in the unique equilibrium all consumers sign the incumbent's long-term contract.*

- *If $\Delta c > 2\theta$, the incumbent allows the entrant to sell to some consumers in period 2.*

The equilibrium profits and consumer utility are

$$\begin{aligned}\Pi_I^* &= v - c_I + \frac{\theta^2 - 4\Delta c\theta + \Delta c^2}{6\theta} + \frac{\theta}{2}, \Pi_E^* = \frac{(\theta + \Delta c)^2}{9\theta}, \\ U^* &= v - \frac{\theta + 2c_E + c_I}{3} - \frac{\theta}{2}.\end{aligned}$$

- *If $\Delta c \leq 2\theta$, the incumbent forecloses the more efficient entrant. The equilibrium profits and consumer utility are*

$$\begin{aligned}\Pi_I^* &= v - c_I + 2\theta - \Delta c, \Pi_E^* = 0, \\ U^* &= v - c_I + \Delta c - 2\theta.\end{aligned}$$

If the incumbent is allowed to have a breakup fee in its first contract, a nonzero breakup fee generates inter-temporal effects. A consumer's expected utility if she does not sign the incumbent's first contract is again given by (6). Suppose that the incumbent allows the entrant to have some sales in period 2: $p_{I2} - d \geq c_E$. A consumer's expected utility if she signs the incumbent's first contract is then equal to

$$EU_{signI} = v - p_{I1} + v - p_{I2} Prob(s > p_{I2} - p_E^* - d) - \int_0^{p_{I2} - p_E^* - d} (s + p_E^* + d) \frac{1}{\theta} ds.$$

The incumbent's profit subject to the consumers' participation constraint, $EU_{signI} \geq EU_{nosignI}$, is:

$$\Pi_I^* = p_{I1} - c_I + (p_{I2} - c_I) Prob(s > p_{I2} - p_E^* - d) + d Prob(s < p_{I2} - p_E^* - d).$$

Given that $p_E^* = \frac{p_{I2} - d + c_E}{2}$ at the optimal solution the incumbent sets the same second period prices as the previous section and the highest p_{I1} satisfying the participation constraint. The second period equilibrium outcome is the same as the previous section since the way the incumbent's first-period contract affects the second period competition in the same way as the case of switching costs when all consumers sign the incumbent's first period contract. However, the equilibrium profit of the incumbent is different from the previous section, since now consumers' outside options (equilibrium utilities) have a lower value:

$$U^* = v - \frac{\theta + 2c_E + c_I}{3} - \frac{\theta}{2}$$

since consumers have to pay s whenever they buy the entrant's product, even when they did not sign the incumbent's first period contract. Hence, the incumbent's equilibrium profit if it does not foreclose the entrant is $\frac{\theta}{2}$ higher than the previous section.

We now derive the incumbent's highest profit if it forecloses the entrant to see whether, or when, we would have foreclosure in equilibrium. If the incumbent forecloses the entrant by setting $p_{I2} - d < c_E$, a consumer's expected utility from signing the incumbent's first contract is

$$EU_{signI} = 2v - p_{I1} - p_{I2},$$

since she expects to buy from the incumbent in period 2 when she signs the incumbent's first period contract. A consumer's expected utility if she does not sign the incumbent's first contract is again given by (6). Given $p_{I2} - d < c_E$, the entrant knows that it cannot attract any consumer from the incumbent's first contract and therefore competes against the incumbent for the consumers who did not sign the incumbent's contract. The incumbent's optimal spot price is $p_{I2}^S = p_{I2}$ since lowering price below p_{I2} would lead to a loss from measure 1 of consumers and some gains from measure 0 of consumers. The entrant's best-reply price is $p_E^* = \frac{p_{I2}+c_E}{2}$ if $p_{I2} \leq v$. In period 1 the incumbent sets $p_{I2}^* = 4\theta + c_E$, the highest p_{I1} satisfying consumers' participation constraint: $p_{I1}^* = v - 2\theta$, and a sufficiently high breakup fee to foreclose the entrant: $d^* > 4\theta$. The incumbent's profit from foreclosure is therefore

$$\Pi_I^* = v - c_I + 2\theta - \Delta c.$$

The incumbent's profit from foreclosure is increasing in the highest level of its differentiation value, θ, and decreasing in the efficiency gains of the entrant, Δc. This is different from the switching costs case where the incumbent's foreclosure profit did not depend on the entrant's efficiency advantage or switching costs. Moreover, the incumbent's profit without foreclosure is higher than the case with switching costs since consumers' outside option is lower given that they pay s to buy from the entrant also when they did not sign the incumbent's first-period contract. Comparing the incumbent's foreclosure profit with the highest profit without foreclosure we conclude that the incumbent prefers to foreclose the entrant if and only if $2\theta \geq \Delta c$.

Thus, since the entrant will not extract the entire surplus from consumers who do not sign the incumbent's first period contract, raising the consumers' outside options, makes it too costly for the incumbent to try to foreclose a very efficient entrant.

5 Conclusions

We have investigated the effects of breakup or early termination fees in a simple dynamic model. We found when an entrant has market power and consumer heterogeneity is due to switching costs, then the incumbent will always foreclose the entrant no matter how much more efficient the entrant is than the incumbent. On the other hand, if consumer heterogeneity is due to different values for an entrant's product, then the incumbent will not foreclose the entrant if the entrant is sufficiently more efficient relative to the incumbent.

6 Appendix

Proof of Proposition 1 If $\Delta c < \theta$, the incumbent sets $p^*_{I2} = c_I$ and induces efficient switching: Consumers switch to the entrants if and only if $s < \Delta c$. The incumbent's second period demand is $D^*_{I2} = \frac{\theta - \Delta c}{\theta}$ and the amount of consumers switching to the entrants is $D^*_E = \frac{\Delta c}{\theta}$. Replacing $p^*_{I2} = c_I$ into the equality for p^*_{I1}, (2.1), determines the optimal up-front fee of the incumbent's contract: $p^*_{I1} = v - \Delta c + \frac{(\Delta c)^2}{2\theta}$. In equilibrium the incumbent's profit is $\Pi^*_I = v - c_I - \Delta c + \frac{(\Delta c)^2}{2\theta}$ and the consumer surplus is $v - c_E$.

If $\Delta c > \theta$, the incumbent again sets $p^*_{I2} = c_I$, but sells to nobody in period 2, i.e., all consumers switch to the entrants. The incumbent's optimal first period price is then $p^*_{I1} = v - \frac{1}{2\theta}$. In equilibrium the incumbent's profit is $\Pi^*_I = v - c_I - \frac{1}{2\theta}$ and the consumer surplus is $v - c_E$.

Proof of Corollary 2 As in the base model competitive entrants set $p^*_E = c_E$ and the incumbent never sets a spot price in period 2 that will be taken by any consumers. If a consumer does not sign the incumbent's contract in period 1, she gets $v - c_E$. If a consumer signs the incumbent's contract in period 1, she expects to get

$$EU_{signI} = 2v - p_I - p_I Pr(s > p_I - c_E - d) - (c_E + d)Pr(s < p_I - c_E - d) - \int_0^{p_I - c_E - d} \frac{s}{\theta} ds. \quad (7)$$

The difference between (1) and (7) is that a consumer pays d if she breaks the contract and this affects the probability that a consumer purchases from the entrant and the payment that the consumer has to pay the incumbent for changing firms. All consumers sign the incumbent's contract in period 1 if and only if $EU_{signI} \geq v - c_E$. Under this constraint the incumbent's expected profit over the two periods is

$$\Pi = p_I - c_I + (p_I - c_I)Pr(s > p_I - c_E - d) + dPr(s < p_I - c_E - d). \quad (8)$$

The differences between (3) and (8) are the probability that a consumer will switch will now also depend on the breakup fee and the firm can use the fee as a revenue source.

First observe that the incumbent's profit expression in (2.2) will be the same as in (3), that is, the profit under the two-period contract, (p_{I1}, p_{I2}), if it sets: $p_I - d = p_{I2}$ and

$d = \frac{p_{I1} - p_{I2}}{2}$, which in turn uniquely determine the unit price of the contract with breakup fees: $p_I = \frac{p_{I1} + p_{I2}}{2}$. But then the expected surplus from buying the incumbent's contract (2.2) is equivalent to the one in (1). Hence, there is one-to-one correspondence between these two contract types. Using the equilibrium prices derived in Proposition 1 and the equivalence conditions, $p_I = \frac{p_{I1} + p_{I2}}{2}$, $d = \frac{p_{I1} - p_{I2}}{2}$, it is straightforward to show that the equilibrium prices are

- If $\Delta c \geq \theta$,

$$p_I^* = \frac{v + c_I}{2} - \frac{1}{4\theta}, d^* = \frac{v - c_I}{2} - \frac{1}{4\theta}, p_{I2}^{S*} \geq p_I^*, p_E^* = c_E.$$

- If $\Delta c \leq \theta$,

$$p_I^* = \frac{v + c_E}{2} + \frac{(\Delta c)^2}{4\theta}, d^* = \frac{v + c_E}{2} + \frac{(\Delta c)^2}{4\theta} - c_I, p_{I2}^{S*} \geq p_I^*, p_E^* = c_E.$$

Proof of Proposition 2 To derive the equilibrium, we first assume that all consumers believe that each of their fellow consumers will accept the incumbent's first period offer and derive equilibrium payoffs. Next, we find whether this equilibrium is unique for all parameter values and beliefs of consumers about how other consumers will react to the incumbent's offer.

Step 1: Equilibria where all consumers think each other consumer will accept the incumbent's offer.

Period 2:

If nobody signed I's contract in the first period, then the only equilibrium has the entrant attracting all consumers at a price of c_I. If all consumers signed I's contract in the first period and if the incumbent undercuts its long-term contract price, $p_{I2}^S < p_{I2}$, then the incumbent's spot offer dominates its long-term contract price for consumers. Consumers with switching costs lower than the difference between the incumbent's spot price and the entrant's price, $s < p_{I2}^S - p_E$, switch to the entrant and the rest buy the incumbent's spot contract. The entrant's demand is then $D_E = \frac{p_{I2}^S - p_E}{\theta}$ and the incumbent's demand for its spot contract is $D_{I2}^S = \frac{\theta - p_{I2}^S + p_E}{\theta}$. The incumbent sets p_{I2}^S, by maximizing its second period profit

$$\Pi_{I2} = (p_{I2}^S - c_I)\frac{\theta - p_{I2}^S + p_E}{\theta}$$

The best-reply of the incumbent to the entrant's price is $p_{I2}^{S*}(p_E) = \frac{\theta + p_E + c_I}{2}$. The entrant sets p_E by maximizing its profit

$$\Pi_E = (p_E - c_E)\frac{p_{I2}^S - p_E}{\theta}$$

The best-reply of the entrant to the incumbent's spot price is $p^*_E(p^S_{I2}) = \frac{p^S_{I2}+c_E}{2}$. The simultaneous solution to the best-replies determine the spot market equilibrium prices (in this sub-game where $p^S_{I2} < p_{I2}$):

$$p^{S*}_{I2} = \frac{2\theta + c_E + 2c_I}{3}, p^*_E = \frac{\theta + 2c_E + c_I}{3}. \tag{9}$$

The demand for the incumbent's spot contract and the entrant's demand are then

$$D^{S*}_{I2} = \frac{2\theta - \Delta c}{3\theta}, D^*_E = \frac{\theta + \Delta c}{3\theta}. \tag{10}$$

Observe that the incumbent's spot demand is positive if and only if the entrant's cost advantage is not too high: $\Delta c < 2\theta$.

- Suppose $\Delta c < 2\theta$
  - If $p_{I2} > \frac{2\theta+c_E+2c_I}{3}$, the previous equilibrium outcome, (9) and (10), prevails since then in period 2 the incumbent sets $p^{S*}_{I2} < p_{I2}$.
  - If $p_{I2} \leq \frac{2\theta+c_E+2c_I}{3}$, the incumbent does not undercut its long-term contract price, $p^{S*}_{I2} \geq p_{I2}$, and so the spot contract is dominated by the long-term contract of the incumbent. Consumers with switching costs $s < p_{I2} - p_E$, switch to the entrant and the rest buy from the incumbent at price p_{I2}. The entrant's demand is then $D_E = \frac{p_{I2}-p_E}{\theta}$. The entrant sets its best-reply to the incumbent's first period offer, $p^*_E(p_{I2}) = \frac{p_{I2}+c_E}{2}$, and sells to consumers of measure $D_E = \frac{p_{I2}-c_E}{2\theta}$. The remaining consumers continue buying from the incumbent at the long-term contract price: $D_{I2} = \frac{2\theta-p_{I2}+c_E}{2\theta}$.
- Suppose now that $\Delta c > 2\theta$. The entrant is very efficient and the incumbent cannot sell in the second period spot market and so it sets $p^{S*}_{I2} \geq p_{I2}$ not to compete against its long-term contract and the sub-game equilibrium outcome will be the same as the previously described one: $p_E(p_{I2}) = \frac{p_{I2}+c_E}{2}$, $D_E = \frac{p_{I2}-c_E}{2\theta}$, $D_{I2} = \frac{2\theta-p_{I2}+c_E}{2\theta}$.

Period 1:

First consider the case where $\Delta c < 2\theta$. In equilibrium, the incumbent should set $p_{I2} \leq \frac{2\theta+c_E+2c_I}{3}$ in order to induce the sub-game where in the second period the incumbent does not have an incentive to undercut its long-term contract price, $p^{S*}_{I2} \geq p_{I2}$ (in the spirit of renegotiation-proof contracts). Given this in the continuation game, the entrant sets $p^*_E(p_{I2}) = \frac{p_{I2}+c_E}{2}$. If a consumer signs the incumbent's first period contract, she expects to switch to the entrant with probability $Pr(s < p_{I2} - p^*_E(p_{I2})) = \frac{p_{I2}-c_E}{2\theta}$ and expects to continue buying from the incumbent at price p_{I2} with probability $\frac{2\theta - p_{I2} + c_E}{2\theta}$. So a consumer's expected surplus from signing the incumbent's contract is the two-period expected surplus from consumption minus the expected switching costs:

$$EU_{signI} = 2v - p_{I1} - p_{I2}\frac{2\theta - p_{I2} + c_E}{2\theta} - p_E^*(p_{I2})\frac{p_{I2} - c_E}{2\theta} - \int_0^{p_{I2} - p_E^*(p_{I2})} \frac{s}{\theta} ds.$$

If a consumer does not sign the incumbent's first-period contract, she expects that the entrant will set $p_E^*(p_{I2})$ since she expects that the other consumers will sign the incumbent's offer and her decision of not signing the incumbent's contract does not change the entrant's equilibrium price given that each consumer is infinitesimal. Observe that $p_E^*(p_{I2}) = \frac{p_{I2} + c_E}{2} < p_{I2}$ since $p_{I2} \geq c_I > c_E$. This implies that $p_E^*(p_{I2}) < p_{I2}^{S*}$, and so a consumer would prefer to buy from the entrant if she did not sign the incumbent's first contract. A consumer's surplus from not signing the incumbent's contract is therefore the net surplus of buying from the entrant in period 2: $v - p_E^*(p_{I2})$. Consumers buy the incumbent's long-term contract if and only if their expected surplus from buying is greater than their outside option: $EU_{signI} \geq v - p_E^*(p_{I2})$. Under the latter constraint the incumbent's expected profit is

$$\Pi_I = p_{I1} - c_I + (p_{I2} - c_I)\frac{2\theta - p_{I2} + c_E}{2\theta}.$$

It is optimal for the incumbent to set the highest upfront fee satisfying the participation constraint of consumers:

$$p_{I1}^*(p_{I2}) = v + (p_E^*(p_{I2}) - p_{I2})\frac{2\theta - p_{I2} + c_E}{2\theta} - \frac{(p_{I2} - p_E^*(p_{I2}))^2}{2\theta}.$$

After replacing the equality of the optimal upfront fee into the incumbent's profit, its profit becomes a function of the second period unit price:

$$\Pi_I^*(p_{I2}) = v - c_I + (p_E^*(p_{I2}) - c_I)\frac{2\theta - p_{I2} + c_E}{2\theta} - \frac{(p_{I2} - p_E^*(p_{I2}))^2}{2\theta}.$$

The incumbent's unconstrained optimal p_{I2}^* is the one that maximizes its profit:

$$p_{I2}^* = \frac{2\theta + c_E + 2c_I}{3}.$$

Observe that $p_{I2}^* = p_{I2}^{S*}$, so this satisfies the constraint ensuring that the incumbent does not want to undercut p_{I2}^* in the second period spot market. This is due to the fact, that the incumbent is acting as a Stackelberg leader in period 2 and just like in that model, the leader would not want to raise its output (lower its price in our model). The first-period unit price is then $p^*_{I1} = p^*_{I1}(p^*_{I2})$. Hence, we conclude that if $\Delta c < 2\theta$, the equilibrium prices are

$$\begin{aligned} p^*_{I1} &= v - \frac{(\Delta c + \theta)(5\theta - \Delta c)}{18\theta}, \\ p^*_{I2} &= p^{S*}_{I2} = \frac{2\theta + c_E + 2c_I}{3}, \\ p^*_E &= \frac{\theta + 2c_E + c_I}{3}. \end{aligned}$$

The incumbent sells all consumers in period 1. In period 2

$$D^*_E = \frac{2\theta - p^*_{I2} + c_E}{2\theta} = \frac{\theta + c_I - c_E}{3\theta}$$

amount of consumers switch to the entrant and the rest,

$$D^*_{I2} = \frac{2\theta - p^*_{I2} + c_E}{2\theta} = \frac{2\theta + c_E - c_I}{3\theta},$$

continue buying from the incumbent at p^*_{I2}. The resulting equilibrium profits are

$$\begin{aligned} \Pi^*_I &= v - c_I + \frac{\theta^2 - 4\Delta c\theta + \Delta c^2}{6\theta}, \\ \Pi^*_E &= \frac{(\Delta c + \theta)^2}{9\theta}. \end{aligned}$$

On the other hand, if $\Delta c > 2\theta$, the entrant is very efficient compared to the incumbent and so the incumbent cannot compete against the entrant in period 2; it does not find it profitable to keep any consumers when the entrant's costs are at least twice the highest possible consumer switching cost. As a result, in equilibrium, the incumbent sets $p^*_{I1} = v - \frac{\theta}{2}$, $p^*_{I2} = p^{S*}_{I2} = c_I$, the entrant sets $p_E = c_I$, all consumers buy the incumbent's long-term contract in period 1 and switch to the entrant in period 2. The resulting profits will be $\Pi^*_I = v - c_I - \frac{\theta}{2}, \Pi^*_E = \Delta c$. Observe that here the incumbent has to compensate consumers for the expected switching costs, $\frac{\theta}{2}$.

Step 2. Uniqueness. To see whether the equilibrium is unique, we need to see if consumers have a credible outside option to all reject the incumbent's offer and get a payoff of $v - c_I$, which is the unique equilibrium utility if all reject the offer.

A consumer's expected equilibrium payoff that we computed in step 1 was

$$\underline{U} \equiv v - \frac{\theta + 2c_E + c_I}{3}.$$

If this value is less than $v - c_I$, then consumers do not have a credible threat to reject the incumbent's offer of $\underline{U}$ from step 1. Thus, we have a unique equilibrium for $\Delta c > \theta/2$.

Step 3. Now, we argue that the equilibrium is unique for $\Delta c \leq \theta/2$. The difference between the argument for this case and for when $\Delta c > \theta/2$, is that $\underline{U} < v - c_I$ in this case. The distinction is that now the consumers outside option if they all say no to the incumbent's offer is higher than the equilibrium utility that the incumbent offered when all consumers believe that all other consumers will accept the offer.

So, we need to check if the consumers can credibly reject an offer that gives them more than $\underline{U}$ but less than $v - c_I$. Suppose that consumers coordinate to reject any offer from the incumbent in period 1 that gives them less expected utility than $\overline{U} \in (\underline{U}\ , v - c_I]$. Suppose that the incumbent makes the offer of $\underline{U}$, which will include a price of $p_{I1} = p^*_{I1} < v$. It cannot be an equilibrium that all consumers will reject the offer. A consumer could deviate, accept the offer, and improve her payoff. To see this, the continuation equilibrium has the incumbent and the entrant both offering a price of c_I, no matter what an individual consumer does since she does not affect the state next period. The deviating consumer improves her payoff by deviating and accepting the contract, since $v - p^*_{I1} + v - c_I > v - c_I$. Since all consumers will have this incentive, their threat of rejecting $\underline{U}$ is not credible.

Finally, can an equilibrium where only some consumers accept the incumbent's offer exist? If a measure α consumers were to accept an incumbent's contract in period 2, then for spot prices that give the incumbent positive second period sales and the entrant's price are

$$p^{S*}_{I2} = \frac{\theta(1+\alpha)/\alpha + c_E + 2c_I}{3}, p^*_E = \frac{\theta(2-\alpha)/\alpha + 2c_E + c_I}{3}.$$

We should note that for small α no consumers will buy from the incumbent in period 2. The consumers who reject the offer have a utility of

$$U(\alpha) = v - \frac{\theta(2-\alpha)/\alpha + 2c_E + c_I}{3}.$$

$U(\alpha) <\underline{U}$ for any $\alpha < 1$. But, this contradicts the fact that consumers will reject offers that give them some utility $U \in (\underline{U}, v - c_I]$.

Proof of Proposition 3 Similar to the equilibrium analysis without breakup fees, we first assume that all consumers believe that each of the other consumers will accept the incumbent's first period offer and derive equilibrium payoffs. Next, we find whether this equilibrium is unique for all parameter values and beliefs of consumers about how other consumers will react to the incumbent's offer.

Step 1:

Assume that all consumers believe that each of their fellow consumers will accept the incumbent's first period offer. We first solve the second stage equilibrium given the first-period contract of the incumbent. The key is to find the entrant's equilibrium price, since this will determine consumers' outside option to signing the incumbent's first-period contract. If none of the consumers signed the incumbent's first-period contract, undifferentiated Bertrand competition between the entrant and the incumbent implies that the entrant sells to all consumers at a price c_I. If all consumers signed the incumbent's first-period contract, the breakup fee of the incumbent's first contract affects the amount of consumers who switch from the incumbent to the entrant in period 2. If a consumer stays with the incumbent, she pays the incumbent's lowest price for a unit consumption in period 2: $min\{p_{I2}, p^S_{I2}\}$, but if she switches to the entrant she pays the entrant's price plus her switching cost and the breakup fee. Consumers could also decide not to buy in period 2 in which case they have to pay the breakup fee to the incumbent (we will verify the condition under which consumers do not want to do this in equilibrium). Hence, the measure of consumers who switch from the incumbent to the entrant is given by the probability that $s < min\{p_{I2}, p^S_{I2}\} - d - p_E$, which determines the entrant's demand,

$$D_E(p_E) = \frac{min\{p_{I2}, p^S_{I2}\} - d - p_E}{\theta},$$

The entrant sets p_E by maximizing its profit:

$$\Pi_E = (p_E - c_E)D_E(p_E).$$

The entrant's equilibrium reaction to the incumbent's prices is the solution to the first-order condition:

$$p^*_E(p^S_{I2}, p_{I2}, d) = \frac{min\{p_{I2}, p^S_{I2}\} - d + c_E}{2},$$

if this price is at least at its marginal cost, $min\{p_{I2}, p^S_{I2}\} - d \geq c_E$, and not all consumers switch to the entrant, $min\{p_{I2}, p^S_{I2}\} - d \leq 2\theta + c_E$.

If $min\{p_{I2}, p^S_{I2}\} - d \leq c_E$, the entrant cannot steal any consumer from the incumbent, even if it sells at its marginal cost. Then the entrant's best-reply to $min\{p_{I2}, p^S_{I2}\} - d < c_E$ is

$$p^*_E(p^S_{I2}, p_{I2}, d) = p^S_{I2},$$

in order to sell to all consumers who may not have bought the incumbent's first-period contract.

If $min\{p_{I2}, p^S_{I2}\} - d \geq 2\theta + c_E$, the entrant then charges the highest price convincing all consumers to switch to it:

$$p_E^*(p_{I2}^S, p_{I2}, d) = min\{p_{I2}, p_{I2}^S\} - d - \theta.$$

In the spot market, the incumbent has two options. Either it does not undercut its long-term contract price, $p_{I2}^S \geq p_{I2}$, in which case $min\{p_{I2}, p_{I2}^S\} = p_{I2}$ (this will be the case on the equilibrium path). Or it undercuts its long-term price, $p_{I2}^S < p_{I2}$, in which case $min\{p_{I2}, p_{I2}^S\} = p_{I2}^S$ and the incumbent's demand for its spot contract would be $D_{I2}^S = 1 - D_E = \frac{\theta - p_{I2}^S + p_E + d}{\theta}$. The incumbent would then set p_{I2}^S by maximizing its second period profit,

$$\Pi_{I2} = (p_{I2}^S - c_I)\frac{\theta - p_{I2}^S + p_E + d}{\theta} + d\frac{p_{I2}^S - p_E - d}{\theta},$$

which is the net profit from sales in the spot market plus the breakup fees collected from the consumers who switch from the incumbent's long-term contract to the entrant. Compared to the analysis without breakup fees, increasing the spot price has an additional benefit if $d > 0$: more breakup fee revenues are generated from consumers who switch to the entrant (we will show below that on the equilibrium path the incumbent would never set $d < 0$). The incumbent's equilibrium reaction to the entrant's price is the price that maximizes its profit:

$$p_{I2}^{S*}(p_E) = d + \frac{\theta + p_E + c_I}{2}.$$

if this price is at least at its marginal cost, $p_E \geq c_I - \theta - 2d$, and at this price its demand is positive, $p_E > c_I - \theta$. The incumbent's equilibrium price is greater than

$$p_E + \theta + d,$$

if $c_I - \theta - d \leq p_E \leq c_I - \theta$, since then the lowest price inducing all consumers to switch to the entrant, $p_E + \theta + d$ is above the incumbent's cost and any price above $p_E + \theta + d$ would lead to the same profit for the incumbent: this would make all consumers switch and the incumbent would gain d from each consumer. The incumbent's spot demand would be positive at any spot price below $p_E + \theta + d$ and the incumbent would have an incentive to raise its price given that $p_E + \theta + d \leq d + \frac{\theta + p_E + c_I}{2}$ (since $p_E \leq c_I - \theta$). In this case, all consumers switch to the entrant. Observe that this case exists only if $d > 0$.

The incumbent's equilibrium price is c_I,if $p_E \leq c_I - \theta - d$, since then the incumbent's spot demand is zero even if the incumbent's spot price is c_I, i.e., all consumers switch to the entrant. The incumbent's equilibrium price is c_I, if $c_I - \theta - d \leq p_E \leq c_I - \theta - 2d$, since the incumbent's spot demand would be positive at any spot price below $p_E + \theta + d$ and the incumbent would have an incentive to lower its price until its marginal cost given that $p_E + \theta + d \geq c_I > d + \frac{\theta + p_E + c_I}{2}$ (since $p_E > c_I - \theta$). In this case all consumers switch to the entrant. Observe that this case exists only if $d < 0$).

We identify two cases:

- If $\Delta c \leq 2\theta$,
  - In a subgame where $d \geq \frac{\Delta c - 2\theta}{3}$ and $p^S_{I2} < p_{I2}$, the best-reply prices of the firms are:
  $$p^*_E(p^S_{I2}, d) = \frac{p^S_{I2} - d + c_E}{2}, p^{S*}_{I2}(p_E) = d + \frac{\theta + p_E + c_I}{2}.$$
  The spot market equilibrium prices are given by the simultaneous solution to the best-replies. The equilibrium prices and demands are then:[11]
  $$\begin{aligned} p^{S*}_{I2} &= d + \frac{2\theta + c_E + 2c_I}{3}, p^*_E = \frac{\theta + 2c_E + c_I}{3}, \\ D^{S*}_{I2} &= \frac{2\theta + c_E - c_I}{3\theta}, D^*_E = \frac{\theta + c_I - c_E}{3\theta}. \end{aligned} \tag{11}$$
  where both the incumbent and the entrant would sell some positive amount in the second period (given $\Delta c \leq 2\theta$).
  - In a subgame where $d \leq \frac{\Delta c - 2\theta}{3}$, $p^S_{I2} < p_{I2}$, we have $d + \frac{\theta + c_E + c_I}{2} < c_I$ (since $\frac{\Delta c - \theta}{2} > \frac{\Delta c - 2\theta}{3}$). The spot market equilibrium prices and demands are:
  $$\begin{aligned} p^{S*}_{I2} &= c_I, p^*_E = c_I - \theta, \\ D^{S*}_{I2} &= 0, D^*_E = 1 \end{aligned} \tag{12}$$
  where the entrant sells to all consumers in period 2 and the incumbent's spot demand is zero. Observe that in this subgame the incumbent offers switching subsidies, $d < 0$, which cannot be on the equilibrium path since the incumbent's 2nd period profit would be negative and the incumbent would then have a profitable deviation: It could raise d above zero and lower p_{I1} by the same amount to compensate consumers for the increase in d so that they still prefer to sign the incumbent's contract in equilibrium. This would not change consumers' behavior, but increase the incumbent's profit.

[11] Note that the equilibrium demands and profits of this sub-game are the same as the equilibrium of the sub-game without breakup fees when $p_{I2} > \frac{2\theta + c_E + 2c_I}{3}$ (see the proof of Proposition 2). This is not surprising since what matters for the second period consumption decisions is the difference between the incumbent's price and the breakup fee, $p^S_{I2} - d$, not the individual level of the breakup fee.

- If $\Delta c \geq 2\theta$, we have $c_E < c_I - \theta$ (since $\Delta c - \theta > \Delta c - 2\theta \geq 0$).

  - In a subgame where $d \geq \frac{\Delta c - 2\theta}{3}$ and $p_{I2}^S < p_{I2}$, the incumbent's demand would be zero if the incumbent set $p_{I2}^S = d + \frac{\theta + p_E + c_I}{2}$. The equilibrium prices and demands of this subgame are:

$$\begin{aligned} p_{I2}^{S*} &= c_I + d, p_E^* = c_I - \theta, \\ D_{I2}^{S*} &= 0, D_E^* = 1. \end{aligned} \tag{13}$$

    The incumbent does not have an incentive to lower its price below $c_I + d$ given $p_E = c_I - \theta$. To see this, suppose that the incumbent sets $p_{I2}^S = c_I + d - \epsilon$. This would convince the consumers at the margin, those with the highest switching cost, say measure η of consumers, to stay with the incumbent rather than switching to the entrant. Hence, the incumbent would gain $(d - \epsilon)\eta$ from this deviation. However, the incumbent would lose $d\eta$ breakup revenues from these consumers. As a result, this deviation is unprofitable. Clearly, the incumbent does not have an incentive to increase its price.

  - In a subgame where $d \leq \frac{\Delta c - 2\theta}{3}$ and $p_{I2}^S < p_{I2}$, we have $d + \frac{\theta + c_E + c_I}{2} < c_I$ (since $\frac{\Delta c - \theta}{2} > \frac{\Delta c - 2\theta}{3}$). Moreover, we have $d < \Delta c - \theta$, and so $c_E < c_I - \theta - d$. These imply that the following equilibrium prices and demands:

$$\begin{aligned} p_{I2}^{S*} &= c_I, p_E^* = c_I - \theta - d, \\ D_{I2}^{S*} &= 0, D_E^* = 1. \end{aligned} \tag{14}$$

  First, observe that as in either of the previous subgames, the incumbent would never set $d < 0$. Given $d \geq 0$ and that all consumers switch to the entrant in period 2, the incumbent prefers the subgames where the entrant's equilibrium price is higher, since this implies a lower outside option for consumers in period 1 (the incumbent is indifferent between any nonnegative d, since it must lower p_{I1} by the same increase in d to induce consumers to sign its contract in period 1).

Period 1:

In equilibrium, the incumbent sets the second period price of the contract to induce a sub-game where it does not want to undercut it in the spot market. As we have shown above, in equilibrium the incumbent sets $d \geq \frac{\Delta c - 2\theta}{3}$. If $\Delta c \leq 2\theta$, the renegotiation-proof contract requires $p_{I2} - d \leq \frac{2\theta + c_E + 2c_I}{3}$. If $\Delta c \geq 2\theta$, renegotiation-proof contract requires $p_{I2} - d \leq c_I$.

The incumbent has two options: Foreclosing the entrant by setting a sufficiently high d or accommodating entry.

1. Foreclose the entrant: Suppose that the incumbent sets $p_{I2} - d < c_E$.

   Given $p_{I2} - d < c_E$ the entrant cannot steal any consumer from the incumbent's long-term contract even if it sells at its marginal cost. The entrant's best-reply to $p_{I2} - d < c_E$ and $p^S_{I2} = p_{I2}$ is $p^*_E = p_{I2}$ in order to sell to any consumers who did not buy the incumbent's first-period contract. Given $p_E = p_{I2}$, the incumbent does not have an incentive to undercut the entrant's price since then it would lose positive margin from a measure 1 of consumers and would gain a positive margin from consumers measure zero of consumers. Hence, in an equilibrium of this sub-game the incumbent sets $p^{S*}_{I2} = p_{I2}$ and the entrant sets $p^*_E = p_{I2}$. In this equilibrium, all consumers who signed the incumbent's first-period contract continue buying from the incumbent at p_{I2} and the consumers who did not sign the incumbent's first period contract (if any) buy from the entrant at price equal to p_{I2}. Indeed this is the unique equilibrium of this sub-game. To see this consider an equilibrium where $p^{S*}_{I2} > p_{I2}$. The entrant's best-reply to $p_{I2} - d < c_E$ and $p^S_{I2} > p_{I2}$ would be $p^*_E = p^S_{I2}$ in order to sell to all consumers who did not buy the incumbent's first-period contract. Given $p_E = p^S_{I2}$, the incumbent has an incentive to undercut the entrant's price down to $p^S_{I2} = p_{I2}$ (the incumbent does not have an incentive to go below p_{I2}. Hence, there cannot be an equilibrium where $p^S_{I2} > p_{I2}$ or where $p^S_{I2} < p_{I2}$. We conclude that in equilibrium of this sub-game $p^*_E = p^{S*}_{I2} = p_{I2}$, all consumers who signed the incumbent's long-term contract continue buying from the incumbent at price p_{I2} and the ones who did not sign the incumbent's contract (if any) buy from the entrant at p_{I2}.

   A consumer's expected surplus from signing the incumbent's contract is the two-period expected surplus from consumption $EU_{signI} = 2v - p_{I1} - p_{I2}$. A consumer's surplus from not signing the incumbent's contract is the net surplus of buying from the entrant in period 2: $v - p_{I2}$. Consumers buy the incumbent's first period contract if and only if their expected surplus from buying is greater than their outside option: $v - p_{I1} \geq 0$. Under this constraint, the incumbent's profit is $\Pi_I = p_{I1} - c_I + p_{I2} - c_I$. It is then optimal for the incumbent to set the highest prices satisfying the participation constraint of consumers: $p^*_{I1} = p^*_{I2} = v$, and sufficiently high breakup fee: $d^* > v - c_E$. As a result, the incumbent would earn twice static monopoly profit: $\Pi_I = 2(v - c_I)$. To sum up, at the optimal solution of this option all consumers buy the incumbent's first-period contract at $p^*_{I1} = v$, continue buying from the incumbent at $p^*_{I2} = v$ in period 2, and so the entrant is fully foreclosed.

2. Accommodate the entrant: Suppose that the incumbent sets $p_{I2} - d \geq c_E$

   - If $\Delta c \leq 2\theta$, the renegotiation-proof contract requires $p_{I2} - d \leq \frac{2\theta + c_E + 2c_I}{3}$. In the continuation of the game consumers with switching costs $s < p_{I2} - d - p_E$ switch to the entrant and the rest buy from the incumbent at price p_{I2}. The entrant's demand is then $D_E = \frac{p_{I2} - d - p_E}{\theta}$. The entrant's best-reply to the incumbent's first period offer is,[12]

   $$p_E^*(p_{I2}, d) = \frac{p_{I2} - d + c_E}{2}$$

   and the entrant's demand is then $D_E^*(p_{I2}, d) = \frac{p_{I2} - d - c_E}{2\theta}$. The remaining consumers, $D_{I2}^*(p_{I2}, d) = \frac{2\theta - p_{I2} + d + c_E}{2\theta}$, continue buying from the incumbent at the long-term contract price.

   A consumer's expected surplus from signing the incumbent's contract is then

   $$\begin{aligned} EU_{signI} &= 2v - p_{I1} - p_{I2}\frac{2\theta - p_{I2} + d + c_E}{2\theta} - (p_E^*(p_{I2}, d) + d)\frac{p_{I2} - d - c_E}{2\theta} \\ &\quad - \int_0^{p_{I2} - d - p_E^*(p_{I2}, d)} \frac{s}{\theta} ds, \end{aligned}$$

   which is the two-period expected surplus from consumption minus the expected switching costs. A consumer's surplus from not signing the incumbent's contract is the net surplus of buying from the entrant in period 2: $v - p_E^*(p_{I2}, d)$. Consumers buy the incumbent's first period contract if and only if their expected surplus from buying is greater than their outside option:

   $$EU_{signI} \geq v - p_E^*(p_{I2}, d).$$

   Under this constraint, the incumbent's profit is

   $$\Pi_I = p_{I1} - c_I + (p_{I2} - c_I)\frac{2\theta - p_{I2} + d + c_E}{2\theta} + d\frac{p_{I2} - d - c_E}{2\theta}.$$

   It is optimal for the incumbent to set the highest upfront fee satisfying the participation constraint of consumers, (**??**):

   $$\begin{aligned} p_{I1}^*(p_{I2}) &= v + (p_E^*(p_{I2}, d) - p_{I2})\frac{2\theta - p_{I2} + d + c_E}{2\theta} - d\frac{p_{I2} - d - c_E}{2\theta} \\ &\quad - \frac{(p_{I2} - d - p_E^*(p_{I2}, d))^2}{2\theta}. \end{aligned}$$

[12] Observe that $c_E < \frac{2\theta + c_E + 2c_I}{3}$ and so the set where $c_E < p_{I2} - d \leq \frac{2\theta + c_E + 2c_I}{3}$ is not empty.

After replacing the equality of the optimal upfront fee into the incumbent's profit, its profit becomes

$$\Pi_I(p_{I2}, d) = v - c_I + (p_E^*(p_{I2}, d) - c_I)\frac{2\theta - p_{I2} + d + c_E}{2\theta} - \frac{(p_{I2} - d - p_E^*(p_{I2}, d))^2}{2\theta}.$$

First observe that the incumbent's profit depends only on $p_{I2} - d$, but not the individual levels of p_{I2} and d. After replacing the equality for $p_E^*(p_{I2}, d) = \frac{p_{I2}-d+c_E}{2}$, the first-order condition with respect to p_{I2} and the first-order condition with respect to d give us the same equation:

$$p_{I2}^* - d^* = \frac{2\theta + c_E + 2c_I}{3}. \tag{15}$$

Since consumers get non-negative surplus from buying from the incumbent in period 2: $v > \frac{2\theta+c_E+2c_I}{3}$ (by Assumption 1). Hence, the incumbent is indifferent between different levels of p_{I2} and d as long as condition (15) holds. Observe that $p_{I2}^* - d^* \geq c_E$ and so satisfies the initial constraint of this subgame. Moreover, these prices are the same as the period 2 prices when the incumbent does not use breakup fee (see Proposition 2). Hence, the resulting equilibrium demands would be

$$D_{I2}^{S*} = \frac{2\theta + c_E - c_I}{3\theta}, D_E^* = \frac{\theta + c_I - c_E}{3\theta}. \tag{16}$$

The incumbent's equilibrium profit would be

$$\Pi_I^* = v - c_I + \frac{\theta^2 - 4\Delta c\theta + \Delta c^2}{6\theta}. \tag{17}$$

- If $\Delta c \geq 2\theta$, renegotiation-proof contract requires $p_{I2} - d \leq c_I$. In the continuation game, $p_E^* = c_I - \theta$. If a consumer buys from the incumbent in period 1, she expects to switch to the entrant in period 2 and pay switching cost $\frac{\theta}{2}$. So consumers' expected surplus from signing the incumbent's contract is

$$EU_{signI} = 2v - p_{I1} - c_I + \theta - \frac{\theta}{2},$$

and the expected surplus from not signing the incumbent's contract is $v - c_I + \theta$. The incumbent would then set the highest first period price satisfying the participation constraint of consumers: $p_{I1}^* = v - \frac{\theta}{2}$. The incumbent's profit from this option is $\Pi_I^* = v - c_I - \frac{\theta}{2}$

Hence, we conclude that if $\Delta c \geq 2\theta$, the incumbent prefers to foreclose the entrant, since then its profit from foreclosure, $2(v - c_I)$ is greater than accommodation, $v - c_I - \frac{\theta}{2}$. If $\Delta c \leq 2\theta$, the incumbent prefers to foreclose the entrant if and only if

$$v - c_I > \frac{\theta^2 - 4\Delta c\theta + \Delta c^2}{6\theta}. \tag{18}$$

Observe that for $\Delta c \leq 2\theta$ the right hand-side of inequality (18) is a decreasing function of Δc and minimized at $\Delta c = 2\theta$. This implies that

$$\frac{\theta^2 - 4\Delta c\theta + \Delta c^2}{6\theta} \leq \frac{\theta}{6} \tag{19}$$

When c_E goes to c_I, Assumption 1 implies that $v - c_I > \frac{2\theta}{3}$ and so $v - c_I > \frac{\theta}{6}$. But then (19) implies that foreclosure is more profitable for $\Delta c \leq 2\theta$. This completes the proof when consumers believe that all of the other consumers sign the incumbent's contract in period 1.

Step 2: Uniqueness of the equilibrium outcome.

We first check whether the consumers could gain by coordinating their behavior and rejecting the incumbent's offer in the first period. If all consumers believe that the others do not sign the incumbent's first-period contract, they expect in period 2 the incumbent and the entrant compete a la undifferentiated Bertrand, $p_{I2}^{S*} = p_E^* = c_I$, and the entrant sells to all consumers. If a consumer does not sign the incumbent's contract, she would expect to get $v - c_I$. Given that in the previously described equilibrium outcome consumers get zero payoff, they would definitely be better off if they could coordinate and reject the incumbent's first period contract in period 1, since then they would get $v - c_I$ rather than 0. Now, we check whether each consumer has unilateral incentive to reject the incumbent's offer given that she expects all of the other consumers to reject it.

If a consumer signs the incumbent's contract, she would expect to stay with the incumbent with probability 1, since $s > p_{I2}^* - d - c_I$ given that $p_{I2}^* = c_I$ and $d \geq 0$. Hence, a consumer's expected surplus from signing the incumbent's first period contract would be

$$EU_{signI} = 2v - p_{I1} - c_I.$$

We conclude that each consumer unilaterally prefers to sign the incumbent's first period contract as long as $p_{I1} \leq v$. Hence, if the incumbent makes the previously described equilibrium offer by slightly lowering the first-period price, $p_{I1}^* < v$, a consumer is strictly better-off by taking this offer rather than rejecting it. Since all consumers will have this incentive, their threat of rejecting that offer is not credible. Hence, there exists no equilibrium where all consumers reject the incumbent's first-period contract.

Finally, we check whether there exists an equilibrium where only some consumers accept the incumbent's first period offer. Suppose that in period 1 the incumbent offers $p_{I1} = v - \epsilon$, $d > v - c_E$, $p_{I2} = v$ and consumers believe that a measure $0 < \alpha < 1$ of consumers accepted the incumbent's contract in period 1. All consumers will then expect that in period 2 the entrant cannot attract the consumers from the entrant's first period contract and so compete for the consumers who did not sign the entrant's contract by slightly lowering the entrant's spot contract price. The incumbent will lower its spot contract down to its first-period contract price, $p_{I2} = v$. Suppose that the entrant sets $p_E = v$. If the incumbent lowers its spot contract below v by δ, it will lose $\delta\alpha$ revenue from people who signed its contract and gain $(v - \delta - c_I)(1 - \alpha)$ from people who did not sign the incumbent's contract. Hence if α goes to zero the incumbent sets its spot contract price at its marginal cost and has zero sales in period 2. If α goes to 1, the incumbent does not have an incentive to undercut its first-period contract price, $p^{S*}_{I2} = v$. In general for any $0 < \alpha < 1$, the incumbent has an incentive to undercut its first period contract price since when $\delta \to 0$ we have

$$\delta\alpha < (v - \delta - c_I)(1 - \alpha).$$

But then we would have $p^{S*}_{I2} = p^*_E = c_I$. A consumer's outside option to signing the incumbent's contract is $v - c_I$. A consumer's expected surplus from signing the incumbent's first period contract would be

$$EU_{signI} = 2v - p_{I1} - c_I.$$

We conclude that each consumer unilaterally prefers to sign the incumbent's first period contract since $p_{I1} < v$. This completes the proof of uniqueness.

Proof of Proposition 4 If none of consumers signed the incumbent's first period contract, the incumbent and the entrant are differentiated competitors in period 2 and the solution to the differentiated duopoly competition determine the equilibrium prices. The entrant's demand would then be $D_E = Prob(s < p^S_{I2} - p_E)$ and the incumbent's demand would be $D_{I2} = Prob(s > p^S_{I2} - p_E)$. The entrant's best-reply to the incumbent's spot price would be $p^*_E = \frac{p^S_{I2}+c_E}{2}$ if $c_E \leq p^S_{I2} \leq \min\{2\theta + c_E, v\}$. If $c_E > p^S_{I2}$, the entrant would not be able to compete against the incumbent and would set $p^*_E = c_E$.[13] If $p^S_{I2} > v$, the entrant would be local monopoly with price $p^*_E = \frac{v+c_E}{2}$ and demand $D^*_E = \frac{v-c_E}{2\theta}$. If $2\theta + c_E < p^S_{I2} \leq v$, the entrant sells all consumers by setting $p^*_E = p^S_{I2} - \theta$. Similarly the incumbent's best-reply to the entrant's price would be $p^{S*}_{I2} = \frac{\theta+p_E+c_I}{2}$ if $p^S_{I2} \leq \min\{\theta + c_I, v\}$. If $p_E > v$, the incumbent would be local monopoly with price $p^{S*}_{I2} = \frac{v+c_E}{2}$ and demand $D^{S*}_{I2} = \frac{v-c_E}{2\theta}$. If $\theta + c_I < p_E \leq v$, the incumbent sells all consumers by setting $p^{S*}_{I2} = p_E$. The interior solution to the duopoly

[13]This cannot happen in equilibrium since the incumbent is less efficient than the entrant.

competition is then

$$p_{I2}^{S*} = \frac{2\theta + c_E + 2c_I}{3}, p_E^* = \frac{\theta + 2c_E + c_I}{3}$$
$$D_{I2}^{S*} = \frac{2\theta - \Delta c}{3\theta}, D_E^* = \frac{\theta + \Delta c}{3\theta} \tag{20}$$

which would be the equilibrium if $2\theta \geq \Delta c$, which is required to have non-zero demand for the incumbent, and $v \geq \frac{2\theta+c_E+2c_I}{3}$, which is required to have active competition between the incumbent and entrant. The latter inequality is satisfied due to Assumption 1. If $2\theta \leq \Delta c$, the equilibrium prices would be $p_{I2}^{S*} = c_I$ and $p_E^* = c_I - \theta$, and the entrant serves all consumers.

The change in the model would not affect the solution of the second stage equilibrium if all consumers signed the incumbent's first contract. We now look for an equilibrium where all consumers signed the incumbent's first contract. Suppose that each consumer expects every other consumer to sign the incumbent's first period contract. In equilibrium we must have $p_{I2} \leq p_{I2}^{S*}$ where p_{I2}^{S*} refers to the equilibrium spot price of the incumbent in a sub-game where $p_{I2}^S < p_{I2}$. As we show in the proof of Proposition 2, p_{I2}^{S*} corresponds to the spot price given in (20) when the incumbent's first contract cannot have a breakup fee. As we show in the proof of Proposition 3, $p_{I2}^{S*} - d$ corresponds to the spot price given in (20) when the incumbent is allowed to use a breakup fee in its first contract. In the model without breakup fees given $p_{I2} \leq p_{I2}^{S*}$, the entrant sets $p_E^*(p_{I2}) = \frac{p_{I2}+c_E}{2}$. In the model with breakup fee, given $p_{I2} \leq p_{I2}^{S*}$ and $p_{I2} - d \geq c_E$, the entrant sets $p_E^*(p_{I2}) = \frac{p_{I2}-d+c_E}{2}$.

The solution of period 1 differs from the previous section analysis since a consumer's outside option to the incumbent's contract is now her expected surplus from buying a unit in period 2 where she buys from the more efficient entrant and gets $v - s - p_E^*$ if her s is low relative the price difference between the firms and buys from the incumbent and gets $v - p_{I2}$ if her s is high relative to the price difference (given that we have $p_{I2} = p_{I2}^{S*}$ in equilibrium):

$$\begin{aligned} EU_{nosignI} &= v - p_{I2}Prob(s > p_{I2} - p_E^*) - \int_0^{p_{I2}-p_E^*} (s + p_E^*)\frac{1}{\theta}ds, \\ &= v - p_{I2} + \frac{(p_{I2} - p_E^*)^2}{2\theta}. \end{aligned} \tag{21}$$

Consider first the case where the incumbent cannot have a breakup fee in its first contract. Observe that period 1 has no effect on period 2 competition, since a consumer's expected utility in case she signs the incumbent's first contract is equal to

$$EU_{signI} = v - p_{I1} + v - p_{I2}Prob(s > p_{I2} - p_E^*) - \int_0^{p_{I2}-p_E} (s + p_E^*)\frac{1}{\theta}ds.$$

Hence, a consumer wants to sign the incumbent's contract if and only if $p_{I1} \leq v$. At the optimal solution the incumbent sets $p_{I1}^* = v$ (as if it was static monopoly) and the second period prices are given by the equilibrium of the differentiated duopoly competition.

Observe that this is the unique equilibrium. If consumers believe that the other consumers reject the incumbent's first period offer, they expect the second period equilibrium prices to be the equilibrium of differentiated duopoly competition given by (20), and $p_{I2}^* = p_{I2}^{S*}$. Hence, consumers' beliefs about others do not impact the outside option to signing the incumbent's contract, which is given by (21). Moreover, each consumer prefers to sign the incumbent's first contract as long as $p_{I1} \leq v$ since she can guarantee she expects to get exactly the same second period payoff as her outside option in case she signs the incumbent's contract.

Proof of Proposition 5 If the incumbent is allowed to have a breakup fee in its first contract, a nonzero breakup fee generates inter-temporal effects. A consumer's expected utility if she does not sign the incumbent's first contract is given by (21). Suppose that the incumbent allows the entrant to have some sales in period 2: $p_{I2} - d \geq c_E$. A consumer's expected utility if she signs the incumbent's first contract is then equal to

$$EU_{signI} = v - p_{I1} + v - p_{I2}Prob(s > p_{I2} - p_E^* - d) - \int_0^{p_{I2}-p_E^*-d} (s + p_E^* + d)\frac{1}{\theta}ds.$$

The incumbent's profit subject to the consumers' participation constraint, $EU_{signI} \geq EU_{nosignI}$, is:

$$\Pi_I^* = p_{I1} - c_I + (p_{I2} - c_I)Prob(s > p_{I2} - p_E^* - d) + dProb(s < p_{I2} - p_E^* - d).$$

At the optimal solution the incumbent sets the highest p_{I1} satisfying the participation constraint:

$$p_{I1}^* = v + p_{I2} - \frac{(p_{I2} - p_E^*)^2}{2\theta} - p_{I2}Prob(s > p_{I2} - p_E^* - d) - \int_0^{p_{I2}-p_E^*-d} (s + p_E^* + d)\frac{1}{\theta}ds.$$

Replacing the latter into the incumbent's profit we write it as

$$\Pi_I^* = v - c_I + p_{I2} - \frac{(p_{I2} - p_E^*)^2}{2\theta} - c_I Prob(s > p_{I2} - p_E^* - d) - \int_0^{p_{I2}-p_E^*-d} (s + p_E^*)\frac{1}{\theta}ds.$$

Given that $p_E^* = \frac{p_{I2}-d+c_E}{2}$ the first-order condition with respect to p_{I2} and with respect to d are respectively

$$\begin{aligned} 2p_{I2} &= d + c_I + c_E + 2\theta, \\ 2d &= p_{I2} - c_I. \end{aligned}$$

The solution to the latter equalities give us the equilibrium prices:

$$\begin{aligned} p_{I2}^* &= \frac{c_I + 2c_E + 4\theta}{3}, d^* = \frac{2\theta - \Delta c}{3} \\ p_E^* &= \frac{\theta + 2c_E + c_I}{3} \end{aligned}$$

We verify that the initial condition ensuring some positive sales to the entrant is satisfied at these prices: $p_{I2}^* - d^* = \frac{c_E + 2c_I + 2\theta}{3} > c_E$ given that $c_I > c_E$ and $\theta > 0$. Observe that the second period demands and the entrant price are the same as the previous section where s referred to switching cost. The equilibrium profit of the incumbent is different from the previous section since now consumers' outside option (equilibrium utility) has a lower value:

$$U^* = v - \frac{\theta + 2c_E + c_I}{3} - \frac{\theta}{2}$$

since consumers have to pay s whenever they buy the entrant's product, regardless of the fact that they did not sign the incumbent's first contract. Hence, the incumbent's equilibrium profit if it does not foreclose the entrant is $\frac{\theta}{2}$ higher than the previous section:

$$\Pi_I^* = v - c_I + \frac{\theta^2 - 4\Delta c\theta + \Delta c^2}{6\theta} + \frac{\theta}{2}.$$

We now derive the incumbent's highest profit if it forecloses the entrant to see whether/when we would have foreclosure in equilibrium. If the incumbent forecloses the entrant by setting $p_{I2} - d < c_E$, consumer's expected utility from signing the incumbent's first contract is

$$EU_{signI} = 2v - p_{I1} - p_{I2},$$

since she expects to buy from the incumbent in period 2 when she signs the incumbent's first period contract. A consumer's expected utility if she does not sign the incumbent's first contract is again given by (21). Given $p_{I2} - d < c_E$, the entrant knows that it cannot attract any consumer from the incumbent's first contract and therefore competes against the incumbent for the consumers who did not sign the incumbent's contract. The entrant's best-reply price is therefore $p_E^* = \frac{p_{I2}^S + c_E}{2}$ under the condition that $p_{I2}^S \leq v$. The incumbent's optimal spot price is $p_{I2}^S = p_{I2}$ since lowering price below p_{I2} would lead to a loss of margin from measure 1 of consumers and gains from measure 0 of consumers. Hence, in equilibrium of the second period we have $p_E^* = p_{I2}^{S*} = p_{I2}$. In period 1 the incumbent sets the highest p_{I1}

satisfying consumers' participation constraint:

$$p_{I1}^* = v - \frac{(p_{I2} - p_E^*)^2}{2\theta}.$$

Replacing the latter into the incumbent's profit we get:

$$\Pi_I^* = v - 2c_I + p_{I2} - \frac{(p_{I2} - p_E^*)^2}{2\theta}. \tag{22}$$

Given that $p_E^* = \frac{p_{I2}+c_E}{2}$ the first-order condition with respect to p_{I2} determines the equilibrium price:

$$p_{I2}^* = 4\theta + c_E.$$

and the incumbent sets a sufficiently high breakup fee to foreclose the entrant: $d^* > 4\theta$. The incumbent's profit from foreclosure is

$$\Pi_I^* = v - c_I + 2\theta - \Delta c. \tag{23}$$

Comparing the highest profit with foreclosure (23) and the highest profit without foreclosure (22) we conclude that the incumbent prefers to foreclose the entrant if and only if $2\theta \geq \Delta c$.

References

[1] Aghion, P., and Bolton, P. (1987). Contracts as a barrier to entry. The American Economic Review, 388-401.

[2] Caminal, R., and Matutes, C. (1990). Endogenous switching costs in a duopoly model. International Journal of Industrial Organization, 8(3), 353-373.

[3] Chen, Y. (1997). Paying customers to switch. Journal of Economics and Management Strategy, 6(4), 877-897.

[4] The European Commission Memo, 11 September 2013, MEMO-13-779.

[5] The EC internet service study (2013). Consumer market study on the functioning of the market for internet access and provision from a consumer perspective, CIVIC Consulting .

[6] Farrell, J., and Klemperer, P. (2007). Coordination and lock-in: Competition with switching costs and network effects. Handbook of industrial Organization, 3, 1967-2072.

[7] Horrigan, J., and Satterwhite, E. (2010). Americans? perspectives on early termination fees and bill shock. The Federal Communications Commission's survey report.

[8] Fudenberg, D., and Tirole, J. (2000). Customer poaching and brand switching. RAND Journal of Economics, 634-657.

[9] Klemperer, P. (1995). Competition when consumers have switching costs: An overview with applications to industrial organization, macroeconomics, and international trade. The Review of Economic Studies, 62(4), 515-539.