Udo Ebert

Recursively aggregable inequality measures: An extension of Gini's mean difference and the Gini coefficient

Address: Department of Economics, University of Oldenburg, D-26111 Oldenburg, Germany

e-mail: ebert@uni-oldenburg.de

Extended Abstract

In the eighties of the last century a number of papers were published which clarified the structure of (additively) decomposable inequality measures. If a given population is split into any two mutually exclusive and exhausting subgroups, overall income inequality can be decomposed into a within-group and a between-group inequality term. The first one is a weighted sum of the subgroup income inequality values. The between-group term measures the inequality between both subgroups by considering a smoothed income distribution for each subgroup – which is usually generated by replacing the actual incomes by the respective mean income. Thus the decomposition process is a top-down approach. We know the overall inequality and decompose it into several (meaningful) components.

In this paper we present a new type of aggregation property and use a bottom-up approach. Starting from a given population (size) we enlarge this population by exactly one individual, i.e. the original income distribution is augmented by the new individual's income. Then also two terms are taken into account: Overall inequality is constructed from the inequality within the original population and from a second term which describes the 'additional' inequality generated by the new individual('s income). The representation of this term is based on a naïve idea: When a new individual is added to society, we should compare her income with the given income distribution. Therefore we take into account the additional income inequality between this individual and each individual of the original population (the sum of these inequality values). That is, this term is based on the pairwise comparison of incomes. Overall inequality has to be a weighted sum of both terms. The condition we impose is not a decomposition, but basically an aggregation property (two groups are aggregated). Beginning the process with an inequality measure for a group of two individuals we are then able to derive an inequality measure for a population of any size by using this aggregation property repeatedly. The proceeding is recursive and the aggregation process is additive.

The paper is organized as follows. Section 2 introduces the framework, the notation used and the basic set of properties. Normalization requires that there is no inequality if all incomes are identical. Symmetry postulates that the identity of individuals is irrelevant. The principle of population makes the inequality measures for different population sizes consistent. Then the aggregation property is proposed and further motivated. In section 3 the implications of these properties are derived. Given the aggregation axiom the properties are not independent: The principle of population implies symmetry. They jointly already determine the structure of an inequality measure. The total measure is based on pairwise comparisons of income: Symmetry implies that for a given population size overall inequality is proportional to the sum of the levels of income inequality between all (different) pairs of individuals. The population principle in addition imposes a condition on the factors of proportionality for different population sizes. The results hold for an arbitrary inequality measure defined for two individuals.

In section 5 at first a simple class of measures for two individuals is characterized. It is essentially given by the absolute income differential (or a power of it). The properties used are the basic ones employed for inequality measurement. Then this measure is combined with the aggregation property, the principle of population and the principle of progressive transfers which requires that a rank-preserving transfer of income from a richer individual to a poorer one decreases inequality. We obtain the characterization of a one parameter family of absolute measures. The measures are equal to the average of the (power of the) income differentials between all pairs of individuals. These measures form a one parameter (the exponent) extension of Gini's (absolute) mean difference and have – to the best knowledge of this author – not yet been characterized in the literature. These measures are absolute and homogeneous of a given degree. Therefore in section 6 the axiomatization is extended to the corresponding family of relative measures: The families consist of compromise measures. This property can be exploited: We then get a one parameter extension of the Gini coefficient. In a second step this family is also characterized by a variant of the aggregation property which takes into account mean incomes. This result demonstrates that the Gini coefficient also satisfies a simple aggregation property. Section 7 concludes.

Keywords: Inequality measures, aggregation, Gini's mean difference, Gini coefficient, extension

JEL-codes: D63, D31, C43