Successive Oligopolies with Differentiated Firms and Endogenous Entry*

Markus Reisinger† and Monika Schnitzer‡

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Abstract

This paper develops a model of successive oligopolies with endogenous entry, allowing for varying degrees of product differentiation and entry costs in both markets. Our analysis shows that the downstream conditions dominate the overall profitability of the two-tier structure while the upstream conditions mainly affect the distribution of profits. We compare the welfare effects of upstream versus downstream deregulation policies and show that the downstream deregulation is more effective if the industry is relatively concentrated and vice versa. Furthermore, we analyze how different forms of vertical restraints shape the endogenous market structure and provide conditions under which they are welfare reducing.

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†Department of Economics, University of Munich, Kaulbachstr. 45, 80539 Munich, Germany, e-mail: markus.reisinger@lrz.uni-muenchen.de.

‡Department of Economics, University of Munich, Akademiestr. 1/III, 80799 Munich, Germany, e-mail: schnitzer@lrz.uni-muenchen.de

1 Introduction

The structure of input and output markets varies a lot between different industries. In some industries, output producers operate in a very competitive environment while input suppliers do not, in others it is the other way round.[1] Still others have oligopolistic structures with a small number of firms in both the input and output markets.[2] Yet, there is little knowledge about how these competitive structures evolve and, in particular, how the competitive environment in output markets affects competition in the input market and vice versa.

Understanding the interaction between upstream and downstream markets is important for policy issues like deregulation or the welfare effects of vertical restraints. For example, governments in all OECD countries have pursued a policy of deregulation during the last decade.[3] While there are several cases of successful deregulation,[4] reforms were less successful in other cases, arguably due to feedback effects that deregulation of one market had on the other market. An example is the German electricity market, where deregulation of the downstream market led to entry of retailing firms and initially to a fall in consumer prices. But this downstream entry was followed by a merger wave in the upstream market and a subsequent increase in consumer prices.[5]

Similarly, the welfare effects of different forms of vertical restraints can only be judged correctly if endogenous changes in the market structure stemming from these restraints are taken into account. For example, two-part tariffs are generally perceived as welfare enhancing, because they avoid double marginalization. Other restraints, like resale price maintenance (RPM), are illegal in most countries because they are commonly expected to reduce competition and to result in higher consumer prices.[6] However, the existing literature focusses on given market structures and ignores the effects these

[1] For example, in most of the semiconductor industry there are a few major players engaged in the fabrication of semiconductor devices that are suitable for the production of a particular final good. These final goods are in turn often produced by a large number of firms. In contrast, the automobile and aviation industry are examples of industries where, for many components, a large number of suppliers are confronted with only a small number of final goods producers.

[2] An example is the market for the microprocessors of personal computers. There are only few firms producing these microprocessors, mainly Intel Corporation and Advanced Micro Devices, Inc. (AMD), while the number of personal computer manufacturers is a bit larger but also small.

[3] See e.g. Nicoletti and Scarpetta (2003). The World Bank report "Doing Business 2007" documents that, from 2003 to 2007, 238 market reforms were introduced in 175 countries. 213 of these reforms facilitate business activities.

[4] For instance, after the deregulation of the natural gas market in the U.S. in the 1980s several new gas marketers entered the retail market and new pipelines were created in the upstream sector. Overall, this lead to a stabilization of the gas price in the 1990s. See OECD (2000).

[5] Deregulation in Germany started in 1998 when there were eight electricity producers which controlled most of the market. By 2002 only four them were left due to mergers between them.

[6] In Europe, RPM is illegal per se. In the U.S., it was illegal until 2007 when the Supreme Court struck down a law that would completely prohibit manufacturers to engage in this practice.

practices may have on market entry on each layer. Thus, there is a lack of a model that allows us to study how different competitive environments, pricing strategies and policy choices affect the overall market outcome when market structures upstream and downstream evolve endogenously.

In this paper, to address these kinds of questions, we provide a model of successive oligopolies with endogenous market entry. We allow for varying degrees of product differentiation and different entry costs in both markets, reflecting different competitive environments in the two markets. Thus, we can use the model to explore the endogenous two-tier market structure as a function of both product differentiation and entry costs for different forms of vertical contracts.

The idea of our theoretical approach is to use a model of circular competition with free entry in each market. This formulation of double circular competition has the advantage of allowing for a clear interpretation of the degree of differentiation and the entry costs, in addition to keep the model tractable and to allow for formulas that are relatively easy to interpret.[7] Such a model is straightforward to analyze if market demand can be assumed to be continuous. For downstream firms selling to a large number of consumers, this assumption is innocuous. However, the analysis is less straightforward for the upstream market where demand is generated by only a finite number of input buyers, as this may give rise to potential discontinuities in the demand functions. Our model provides a specification to deal with this problem. We show that if the location of buyers is uncertain, demand in the upstream market is continuous which keeps the model tractable. We discuss several scenarios for which this demand specification seems natural. We show that there exists a unique equilibrium under any form of vertical contracting. This allows us to give clear comparative static predictions, and it enables us to study the welfare implications of different contracting forms.

We start with the benchmark case of no vertical restraints. Here, we derive several results on how the competitive conditions in one market affect the adjacent market and the overall market structure. For instance, we show that the competitive conditions in the downstream market dominate the profitability of the two-tier structure while the competitive conditions in the upstream market mainly affect the distribution of profits. We also demonstrate that a more competitive upstream market induces lower prices upstream and, thereby, also lower prices downstream but, interestingly, matters are not so clear when the consequences of increased downstream competition on the upstream market are considered. Here we find that a larger number of downstream firms, first, induces fiercer

[7] In the conclusion we point out that the main insights derived from our model should also be robust to alternative specifications.

competition which puts a downward pressure on the upstream prices, while, second, more potential buyers are present, which has a positive effect on upstream profits. We show that the second effect dominates if few downstream firms are present, while the first one is more important if the number of downstream firms is large.

We can then use the model to evaluate deregulation policies. Here, among other things, we can address the question of whether it is best to foster competition downstream, despite its potentially negative effect on market entry in the upstream market, or to encourage it upstream, counting on its positive impact on downstream competition. Our analysis reveals that, if the overall number of firms, upstream and downstream, is small, i.e. if the industry is relatively concentrated, downstream deregulation is more effective. The reason is that in this case the feedback effects via the upstream market are positive while upstream deregulation has little effect on downstream prices if downstream competition is low. In contrast, upstream deregulation is preferable if the overall number of firms is large. This can provide some guidance for deregulation policies and points out which variables policy makers should pay attention to when aiming to liberalize a market.

Finally, we study the welfare implications of two important forms of vertical restraints, namely two-part tariffs and RPM. Our analysis shows that the welfare effects under endogenously evolving market structures differ substantially from those under exogenously given ones. In particular, we find that welfare under linear pricing can be higher than under two-part tariffs, even though the opposite is commonly perceived because double marginalization is avoided with two-part tariffs (see e.g. Villas-Boas (2007)). This is due to the fact that entry in the downstream market is larger under linear pricing and so downstream competition is fiercer. Moreover, we show that welfare under two-part tariffs is always larger than under RPM although the latter induces more market entry. Here the effect of double marginalization dominates.

The existing literature that deals with vertical market structures has mainly analyzed the consequences of vertical relations, like integration or vertical contracts. In this literature the basic markets are modelled in a simplified way, with an exogenous number of firms in each market (often only two) and homogeneous products in at least one market. For example, dealing with vertical integration Greenhut and Ohta (1979), Salinger (1988) and Gaudet and Long (1996) analyze markets with homogeneous goods and competition à la Cournot to assess the implications of vertical mergers. Ordover, Saloner and Salop (1990) and Chen (2001) allow for price competition in a framework with vertical integration. Assuming homogeneous products in the upstream market and restricting the number of downstream firms to two, they show that in this framework it is possible to generate asymmetric equilibria in which one firm is integrated and the other one is not.

There are several papers that deal with vertical contracting issues and their welfare implications. Some of these studies, e.g. Hart and Tirole (1990), McAfee and Schwartz (1994) or White (2007), suppose a monopolistic upstream supplier and analyze whether it is able to extract monopoly rents of competing downstream firms. Others, like Bernheim and Whinston (1998), for example, consider multiple competitive upstream firms but only one downstream firm and analyze under which conditions the downstream firm accepts an exclusive dealing contract.[8] In contrast to these papers, we allow for competition upstream and downstream and our focus is on determining the market structure endogenously to analyze feedback effects between the two markets.[9]

Some recent papers analyze a market structure similar to ours, with oligopolistic competition upstream and downstream, namely Ghosh and Morita (2007a), Hendricks and McAfee (2007) and Inderst and Valletti (2008). Ghosh and Morita (2007a) consider a model with homogeneous goods and Cournot competition upstream and downstream and allow for free entry. They show that insufficient entry can occur in one or in both markets. Hendricks and McAfee (2007) also assume Cournot competition with homogeneous goods and, in addition, allow downstream firms to exert market power in the intermediate good market. Yet, they take the number of firms as given. Due to the assumptions of homogeneous goods and Cournot competition, both papers cannot deal with the welfare implications of different forms of vertical contracts and are not able to analyze how varying degrees of substitutability affect prices and entry. Inderst and Valletti (2008) consider a model where upstream firms compete in prices and allow for different degrees of competition. To do so they consider a downstream production technology where firms use a mix of inputs and can substitute between the input goods of different upstream firms. In this set-up, they show that foreclosure incentives of an integrated firm are lower than in models with upstream Cournot competition. In contrast to our model they do not allow for more elaborate contracts than linear pricing and suppose

[8] Bernheim and Whinston (1998) also address the case with two downstream firms that operate in separate markets, which implies that they do not compete with each other. For a general contracting model with multiple upstream and downstream firms but where downstream firms exert no (direct) externalities on each other, see Prat and Rustichini (2003).

[9] Eső, Nocke and White (2008) consider a different but related question. They analyze a downstream industry in which firms compete for a scarce input good. They show that if the supply of this input good is large enough, the resulting industry structure is asymmetric with one firm being larger than the others.

that the number of firms is fixed.[10,11]

To sum up, the existing literature does not provide a framework for dealing with the question of how vertically interrelated oligopolies with free entry work and for assessing the welfare consequences of vertical contracts that affect the market structure. A contribution of this paper is to provide such a framework. We use it to study important antitrust questions like the effects of different deregulation policies and the welfare implications of two-part tariffs and RPM. However, the framework would also be suitable to study other questions, like the effects of vertical integration or other forms of vertical restraints.

The remainder of the paper is organized as follows. The next Section sets out the model. In Section 3 we solve for the subgame perfect equilibrium of the model with linear pricing. Section 4 studies the interplay of upstream and downstream market and explores the welfare effects of deregulation policies. In Section 5 we analyze the model under two forms of vertical restraints, two-part tariffs and RPM, with a focus on their welfare implications. Section 6 concludes.

2 The Model

Our goals are to formulate a model that allows for endogenous market entry upstream and downstream under varying degrees of competition and to evaluate the welfare effects of vertical contracts in this context. To do so, an important ingredient is to describe an intermediate good market where inputs are differentiated. As mentioned above, the challenge is that in this market a discrete number of buyers is present who may have heterogeneous preferences for input goods and who compete against each other on the downstream market. Existing demand models of the final consumer market are therefore not applicable. The extant literature on vertical market structures circumvents this issue by assuming homogeneous goods, which is clearly a simplified and often unrealistic scenario.[12] In the next subsection we present a tractable model that formulates such an input market with differentiated goods and that allows us to pursue the above mentioned goals. In the subsection

[10]Inderst and Valletti (2009) consider the question whether third-degree price discrimination in intermediate good markets increases consumer welfare compared to linear pricing. They analyze a homogeneous good model with two asymmetric upstream and downstream firms and Cournot competition downstream. Their finding is that linear pricing is more beneficial to consumer if there are no innovation incentives but this is reversed once taking such incentives into account.

[11]There is also a recent literature that focusses on bargaining between upstream and downstream firms, see e.g. de Fontenay and Gans (2005a) or Antelo and Bru (2006). Different to the present paper, this literature is less concerned with issues like vertical contracting, the degree of competitiveness and market entry.

[12]As explained above, the exception is Inderst and Valletti (2008) who consider a specific production function where firms can substitute between inputs from all suppliers.

thereafter we turn to several critical modeling assumptions that deserve some more discussion.

2.1 Description of the Model

Consider an industry with two successive oligopolies, an upstream and a downstream market. In the upstream market each firm produces an intermediate good at marginal cost that is normalized to zero. The upstream firms sell the intermediate goods to downstream firms, the producers of the final good. Downstream firms transform the intermediate goods into output on a one-to-one basis at zero costs.[13]

There is free entry in both markets, but all firms that enter in the upstream market must incur a fixed set-up cost of $F_u > 0$ while firms entering in the downstream market must incur a fixed set-up cost of $F_d > 0$. The number of firms that enter in each market is endogenous and we denote it by m for upstream firms and by n for downstream firms. For simplicity, we treat m and n as continuous variables. This implies that, in equilibrium, the profit of each firm net of set-up cost is zero.

In the basic model, an upstream firm i sells its intermediate good to downstream firms at a per-unit price denoted by $r_i, i \in (1, ..., m)$.[14] Similarly, we denote a downstream firm j's final good price by $p_j, j \in (1, ..., n)$.

First, we describe the downstream market. We model the downstream market in a way similar to Salop (1979). There exists a continuum of consumers of mass 1 that is uniformly distributed on a circle with unit circumference. A consumer who is located at z and purchases the good from firm j located at z_j incurs total cost of $p_j + t_d(z - z_j)^2$. We assume that the gross utility of consuming the good is sufficiently high so that all consumers buy for the range of prices considered. $t_d(z - z_j)^2$ is the disutility that a consumer incurs if she does not buy her most preferred variety. This disutility is assumed to be quadratic in the distance between the consumer and the firm.[15] The n downstream firms are equidistantly distributed. So the marginal consumer between firms j and $j + 1$ lies at

[13] The normalization of zero marginal cost in the upstream and the downstream markets is without loss of generality. All qualitative results remain unchanged, if we assume constant marginal costs of $c_u > 0$ and/or $c_d > 0$ instead.

[14] There are two reasons to analyze the case of linear pricing in more detail. First, in several vertical structures linear pricing is the prevalent practice. For example, this is the case in the UK grocery industry for goods like liquid milk, carbonated soft drinks and bakery products, as Smith and Thanassoulis (2009) and Inderst and Valletti (2009) report. Second, linear pricing is a useful benchmark against which to compare the results of more ornate schemes that we analyze in Section 5.

[15] In our case the assumption of quadratic transport costs is just a simplification to avoid the well-known problem that the profit function of firm j becomes discontinuous if its price is low enough, so that the consumers located exactly at the position of its neighboring firms prefer to buy from firm j rather than from the neighbors. All our results also hold for the case of linear transport costs under the additional assumption that t_d is sufficiently high.

distance z_m from firm j, with

$$z_m = \frac{1}{2n} + \frac{n(p_{j+1} - p_j)}{2t_d}.$$

We now turn to the upstream market. Here again, we consider a Salop circle on which the sellers, the upstream firms, are located with equal distance to each other. The buyers in the upstream market are the downstream firms. We first specify the costs a downstream firm incurs when buying its intermediate goods. Then we describe the location of downstream firms as customers in the upstream market.

When buying from upstream firm i, a downstream firm j faces per-unit cost of r_i for the intermediate good and, additionally, a fixed cost that is given by $t_u(x_j - x_i)^2$, where t_u is the transportation cost in the upstream market and $(x_j - x_i)^2$ is the shortest arc length between the location of firm j and the location of firm i in the upstream market. These fixed cost reflects how well the intermediate good of firm i fit the particular needs of the final good producer j. For instance, the characteristics of the good provided by firm i may not exactly fit the technology of firm j and so firm j must incur costs to change its production process.[16]

In contrast to the downstream market, there is only a finite number of buyers in the upstream market instead of a continuum. This can potentially lead to discontinuities in the demand curve of an upstream firm.[17] In order to deal with this problem, we suppose, first, that the location of a downstream firm in the upstream market is stochastic and uniformly distributed on the upstream circle, and, second, that, at the time when upstream firm i decides about its price, it does not observe the realizations of the downstream firms' locations. This reflects the general idea that an upstream firm often does not know if a downstream firm prefers its input or the one of its rivals. As will become clear soon, demand functions are continuous with this specification. As a consequence of it, the locations of a downstream firm in the upstream market and in the downstream market are not correlated with each other. We discuss several rationalizations for this assumption and its implications in the next subsection.

When choosing its input supplier, each downstream firm of course knows its own location in the upstream market but, as the upstream firms, it does not observe the locations of other down-

[16] We could also incorporate the location distance between firm i and j as a variable cost. But this makes the model technically more complicated without gaining new insights. The reason is that with such a formulation the travel distances enter the maximization problem of a downstream firm in a non-linear way. Instead, in our simpler specification, the travel distance only determines the choice of the input supplier and has no direct influence on the maximization problem of a downstream firm.

[17] For an in-depth discussion of this problem, see e.g. Gabszewicz and Thisse (1986).

stream firms. An obvious reason for this assumption is that a firm usually does not know the exact production technology of its rivals and so does not know which input best fits their needs.[18] As a consequence, a downstream firm does not observe the input suppliers of its rivals and, therefore, also not their input prices.

Thus, after observing the upstream price vector $\mathbf{r}$, downstream firm j forms expectations about the profit it earns when buying from supplier i at input price r_i, $E[\Pi_j^i(p_j, \mathbf{p}_{-j}, r_i, \mathbf{r}_{-i})]$.[19] Therefore, firm j buys from supplier i if[20]

$$E[\Pi_j^i] - t_u(x_j - x_i)^2 \geq E[\Pi_j^k] - t_u(x_j - x_k)^2, \qquad \forall k \neq i.^{21}$$

It follows that the probability that firm j buys its intermediate good from firm i is given by

$$q_i = \left(\frac{1}{m} + \frac{m(2E[\Pi_j^i] - E[\Pi_j^{i-1}] - E[\Pi_j^{i+1}])}{2t_u} \right). \tag{1}$$

This probability q_i decreases continuously in r_i. If upstream firm i raises r_i, $E[\Pi_j^i]$ decreases continuously while $E[\Pi_j^{i-1}]$ and $E[\Pi_j^{i+1}]$ increase continuously. Thus, there are no discontinuities in demand and best replies of upstream firms.

To make the problem interesting, we finally assume that the fixed set-up costs are low enough such that in both markets at least two firms enter. It turns out that this is fulfilled if[22]

$$F_d < \frac{1}{8}\left(t_d - \frac{t_u}{6}\right) \quad \text{and} \quad F_u < \frac{t_u}{3(t_u + 12F_d)} \sqrt[3]{48(48F_d + t_u)^2}. \tag{2}$$

We consider the following three stage game. In the first stage a large number of potential entrants exist that can enter either in the upstream or in the downstream market at the respective set-up costs F_u and F_d. After entry, both upstream and downstream firms are symmetrically distributed in their respective markets while the locations of downstream firms as customers in the upstream market are uncertain. In the second stage, upstream firms set their prices r_i.[23,24] Afterwards, the

[18] For further justifications why firms have private information about their costs, see Aghion and Schankerman (2004) or Syverson (2004).

[19] Here, and in the following, $\mathbf{r}_{-i}$ denotes the prices of all upstream firms but firm i, $\{r_1, ..., r_{i-1}, r_{i+1}, ..., r_m\}$. A similar definition applies to $\mathbf{p}_{-j}$.

[20] For the sake of notation we abbreviate $E[\Pi_j^i(p_j, \mathbf{p}_{-j}, r_i, \mathbf{r}_{-i})]$ by $E[\Pi_j^i]$.

[21] We restrict attention to those cases where the upstream market is fully covered, i.e. each downstream firm buys from one of the upstream firms, even if it is located at maximum distance to the suppliers, namely exactly between two of them. It turns out that a sufficient condition for this to hold is $t_u < (48F_d)/5$ (see footnote 33).

[22] Footnote 34 explains how these conditions are derived.

[23] For simplicity, we allow sellers to make take-it-or-leave-it offers. Each supplier's bargaining power is restricted here by the competition with its rivals. Nevertheless, this assumption is somewhat restrictive. However, our main forces at play should still be valid under a more equal distribution of bargaining power (see the discussion in the conclusion).

[24] Note that since prices are observable, there are no opportunism issues on the sellers' side as they occur in e.g. McAfee and Schwartz (1994) or de Fontenay and Gans (2005b).

locations of downstream firms in the upstream market are realized, each downstream firm learns its own location and chooses its preferred supplier of the intermediate good. In stage three, downstream firms set prices in the final good market, observe their realized quantity and order it at the upstream price of its chosen supplier.

2.2 Discussion of Assumptions

In this subsection we address the modeling features that are non-standard and, hence, deserve more discussion.

A key assumption in our demand specification is that upstream firms are uncertain about the locations of downstream firms in the intermediate good market. The idea is that the intermediate good of an upstream firm is suitable for the production of many different output goods. So each upstream firm is ex ante uncertain if a downstream firm will buy from it or from its rivals. This is a natural assumption in the beginning stages of an industry, when upstream firms do not know the particular need of firms in the adjacent market. Of course, after the first purchases a seller learns which downstream firms are closely located and may adjusts its price. Yet, there are good reasons why uncertainty in the upstream demand is a recurrent feature of the market or why sellers cannot easily react to new information and adjust their prices.

A reason for the first possibility is that in many industries new products are introduced frequently and these new products are again appropriate for many different models of output goods that are produced by downstream firms. This is especially the case in industries where products have a short life cycle and new goods are launched in short time intervals.[25] Consider, for example, the industry for microprocessors of computers. Here, a microprocessor of Intel or AMD is suitable for almost all models of personal computers and notebooks that are produced by computer manufacturers like Dell, Hewlett & Packard or Acer. Ex ante, it is not clear which microprocessor a computer manufacturer will use for the specific model and upstream firms compete for the use of their microprocessors. Once computer manufacturers have made their decision, a microprocessor producer learns which computer manufacturers prefer its input for a specific model. But whenever computer manufacturers produce new models (which nowadays is the case almost constantly) or a new generation of microprocessors is developed (which, since the 1990s, happens around every two years), each of the upstream firms

[25] As Fine (1998) notes, due to the recent acceleration of technologies and the rapid change of the market, the life cycle of products becomes shorter and the introduction of new goods gets much faster in many industries.

is again uncertain which microprocessor fits to the new personal computers or notebooks. A similar structure can be observed in most of the semiconductor industry, where semiconductor firms compete to sell their devices to downstream firms, for example in the automotive or telecommunication industry. As a consequence, in many industries with short life cycles of input or output goods there is frequent competition between suppliers, and, for each new product, suppliers are uncertain about which input good fits best the needs of a particular downstream firm, that is, in terms of our model, about the upstream location of the buyers.

An important reason why upstream sellers are unable to adjust their prices easily is that supply contracts are often of long-term nature. For example in the semiconductor industry, Flamm (1996) shows by using confidential data of European and North American firms on contracts for 64K and 256K DRAMs between 1984 and 1989 that around 50% of the contracts had a duration of at least one year. A consequence of that is that, even if an upstream supplier learns which of the potential buyers prefer its input, it cannot adjust its price immediately because of long-term contractual obligations. Thus, one can also interpret our framework as one in which repeated purchases in the downstream market take place over some time horizon but where upstream firms compete in long-term contracts at the beginning of this horizon to supply downstream firms.

An implication of our demand specification is that the location of a downstream firm in the upstream market is independent of its location in the downstream market. This reflects the fact that different locations of downstream firms in the upstream market stem from different technologies while a difference in the locations in the downstream market emerges due to production of different varieties or represents a geographic distance. An example is the market for batteries. Here, Duracell and Valance Technologies are close competitors. Yet, Valance Technologies uses a completely different technology than Duracell because, in contrast to Duracell, it engages in R&D on batteries. Therefore, the two firms need a different set of inputs and their location in the input market is far away from each other (see Bloom, Schankerman and van Reenen (2008)).[26]

Finally, one may ask whether our assumption on the locations can be rationalized in a framework where sellers and buyers can strategically choose their location in the two markets. Here we demonstrate briefly that this is indeed the case, considering an extension of the model along the following lines. Suppose that after the entry decision the entering firms play a two-stage location

[26] In general, Bloom, Schankerman and van Reenen (2008) find that the correlation between the product market closeness and the technology closeness of U.S. patent-owning firms between 1980 and 2001 is positive but only slightly so.

choice game: In the first stage, each firm decides about its location in the upstream market. In the second stage, downstream firms choose their locations in the downstream market, after observing the location choices of the upstream firms in the first stage, but not the ones of the downstream firms.[27] One can then show that our assumption on the locations constitutes a subgame perfect equilibrium of this game.[28] The reasoning is as follows: In the second stage, it is easy to show along similar lines as in Economides (1989) that downstream firms locate equidistantly given that the locations in the upstream market are the same as the ones assumed in our model. Now consider the first stage. Suppose that each downstream firm plays a mixed location strategy in which it locates at each point on the upstream circle with equal probability. Then, it is optimal for the upstream firms to locate equidistantly to each other but they are indifferent between the exact points on where to locate. Thus, they are willing to mix between all equidistant structures with equal probability. But given these strategies of the upstream firms and given the mixed location strategies of the other $n-1$ downstream firms, it is optimal for downstream firm n to play the same mixed location strategy as the other downstream firms. Therefore, our location specification can be derived endogenously as a subgame perfect equilibrium of this extended version of the model.[29]

3 Equilibrium of the Model

In this section we describe the solution of the three stage game. We solve the game by backward induction. A rigorous proof of the results can be found in Appendix A.

Downstream Market

In stage three, each downstream firm decides on its final good price, knowing m and n and the upstream price vector $\mathbf{r}$. Since upstream prices influence final good prices, downstream firms that buy from different upstream firms might set different final good prices. As described above, when setting its price p_j, downstream firm j does not observe the input prices of its neighboring firms. However, since a downstream firm observes all input prices, in equilibrium it knows the *expected*

[27] It is natural that a downstream firm does not observe the locations of the other buyers since this would involve knowledge of the production technology and the input preference of other downstream firms.

[28] The above game structure is not the only one that yields our assumption as an equilibrium outcome. For example, if one were to reverse the two stages or assume that all actions in the first stage are non observable, our location configuration would still constitute a subgame perfect equilibrium outcome.

[29] This equilibrium is not necessarily the only subgame perfect equilibrium. However, it is valid independent of the incentive of a downstream firm to choose its upstream location close to or far from its direct downstream competitors. Therefore, it does not depend on the exact parameters of the model.

input price of its rivals. Anticipating that in equilibrium all competitors with the same input price will charge the same output price, it also knows the *expected* output price.[30] As a consequence, the expected profit of firm j (disregarding fixed costs) when buying its input from upstream firm i can be written as

$$E[\Pi_j^i(p_j, r_i, p_{j-1}, p_{j+1})] = (p_j - r_i)\left(\frac{1}{n} + \frac{n(E[p_{j-1}] + E[p_{j+1}] - 2p_j)}{2t_d}\right).$$

This maximization problem is identical for all downstream firms, with the exception that they potentially face different input prices. Thus, in a symmetric equilibrium there can be at most m different output prices. Since firm j has the same expectation about the upstream location of its two neighbors, its expectation about $E[p_{j-1}]$ and $E[p_{j+1}]$ is also the same, and it is given by $\sum_{k=1}^{m} q_k p_k$, where q_k is defined in (1) and p_k is the price that a downstream firm charges when buying the input from upstream firm k. Since this holds for all downstream firms, the expected profit of a downstream firm that buys the input from upstream firm i can be written as

$$E[\Pi^i(p_i, r_i, \mathbf{p}_{-i})] = (p_i - r_i)\left(\frac{1}{n} + \frac{n\left(\sum_{k=1}^{m} q_k p_k - p_i\right)}{t_d}\right), \quad i \in \{1, ..., m\}.$$

Maximizing each profit function with respect to p_i gives a system of m equations. Solving this system for p_i yields

$$p_i = \frac{1}{2}\left(r_i + \sum_{k=1}^{m} q_k r_k + \frac{2t_d}{n^2}\right), \qquad i \in \{1, ...m\}.$$

Plugging these prices back into the respective profit function gives the expected profit of a downstream firm dependent on r_i and q_i, $i \in \{1, ..., m\}$. Since the probabilities are functions of downstream profits, we can solve for these probabilities by plugging the profits into (1). The general formula for q_i is not very enlightening, and is therefore only given in Appendix A by equation (15) for the case of $m \geq 3$ and by (16) for the case of $m = 2$. After inserting this formula back into the profit function, we can derive the output price that a downstream firm charges when it buys from upstream firm i, $p_i(r_i, \mathbf{r}_{-i})$, and the expected quantity that it buys, denoted by $E[y_i(r_i, \mathbf{r}_{-i})]$. These expressions are given by equations (17) and (19) in Appendix A for the case $m \geq 3$ and by (18) and (20) for the case that $m = 2$. Having solved the third stage of the game, we now proceed to the second stage, the upstream market.

[30] For models with a similar structure, see Raith (2003), Aghion and Schankerman (2004), or Syverson (2004).

Upstream market

Since production costs are equal to zero, the expected profit of an upstream firm i can be written as the probability that it sells to exactly one downstream firm multiplied by the expected revenue, $r_i E[y_i(r_i, \mathbf{r}_{-i})]$,[31] plus the probability that it sells to exactly two downstream firms multiplied by twice the revenue and so forth. This can be written as[32]

$$E[P_i(r_i, \mathbf{r}_{-i})] = r_i E[y_i]\left(\binom{n}{1} q_i(1-q_i)^{n-1} + 2\binom{n}{2} q_i^2(1-q_i)^{n-2} + \right.$$

$$\left. + \cdots + (n-1)\binom{n}{n-1} q_i^{n-1}(1-q_i) + nq_i^n\right), \qquad i \in \{1, ...m\}.$$

Using a modification of the Binomial Theorem, the profit function simplifies to

$$E[P_i(r_i, \mathbf{r}_{-i})] = r_i E[y_i] n q_i, \qquad i \in \{1, ...m\}. \tag{3}$$

We can now substitute the respective expressions for $E[y_i]$ and q_i that we determined in the third stage and maximize (3) with respect to r_i. Solving the system of m equations yields a unique symmetric equilibrium in which upstream firms charge a price of

$$r^\star = \frac{2t_u t_d mn}{t_u n^3(m-1) + 2m^3 t_d}. \tag{4}$$

Inserting the equilibrium upstream price into the formula for the downstream price gives

$$p^\star = \frac{t_d(2m^3 t_d + t_u n^3(3m-1))}{n^2(2m^3 t_d + t_u n^3(m-1))}. \tag{5}$$

Entry Decision

The equilibrium number of upstream and downstream firms, $m^\star$ and $n^\star$, is determined by the zero-profit conditions in the upstream and the downstream market. Inserting the equilibrium prices into the expected profit functions yields that $n^\star$ and $m^\star$ are implicitly given by[33]

$$\frac{t_d}{(n^\star)^3} - \frac{t_u}{12(m^\star)^2} - F_d = 0 \tag{6}$$

[31] Since upstream firms set prices under uncertainty of downstream firms' locations, their expected profits depend on the expected quantity that a downstream firm orders. The actual quantity can be different if downstream firms set different output prices (which, as we show below, does not occur in equilibrium).

[32] For notational simplicity, we suppress the dependence of $E[y_i]$ and q_i on r_i and $\mathbf{r}_{-i}$.

[33] The inequality in footnote 21 is derived from (6). The largest possible distance of a downstream firm to its input supplier arises when $m^\star = 2$. The distance is then 1/4. In this case, a downstream firm's profit after entering is still positive if $t_d/(n^\star)^3 - tu/16 > 0$. Now solving (6) for $n^\star$ for the case $m^\star = 2$, plugging it into the last inequality and simplifying yields $t_u < (48F_d)/5$. Naturally, for $m^\star > 2$ the constraint on t_u is less tight.

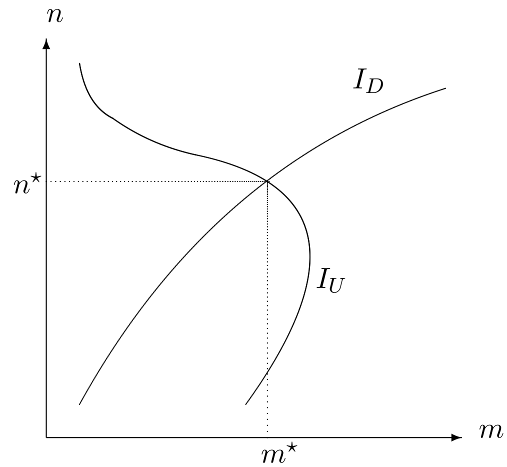

Figure 1: Equilibrium number of firms, $n^\star$ and $m^\star$

and

$$\frac{2t_u t_d n^\star}{t_u(n^\star)^3(m^\star-1)+2(m^\star)^3 t_d} - F_u = 0. \tag{7}$$

Our assumptions on F_d and F_u above guarantee that at least two firms enter in each market.[34] We now examine how the number of firms in one market affects the number of firms in the adjacent market. This is an important step to show that the equilibrium number of firms is indeed unique. Moreover, it also gives us interesting insights into the forces at play in the interaction between upstream and downstream market. For this purpose, we use the iso-profit-lines of downstream and upstream firms. For a downstream firm, the slope of its iso-profit line is given by

$$\frac{\partial n}{\partial m} = \frac{t_u(n)^4}{18t_d(m)^3} > 0\ . \tag{8}$$

Thus, the equilibrium number of downstream firms increases with the number of upstream firms. The intuition for this is simple. If the number of upstream firms increases, downstream firms benefit from lower input prices and expect to face a lower travel distance to their nearest upstream firm. As a consequence, more firms enter the downstream market. This is depicted by the iso-profit-line I_D in Figure 1.

For an upstream firm, the slope of the iso-profit-line is given by

$$\frac{\partial m}{\partial n} = \frac{2\Big((m)^3 t_d - t_u(n)^3(m-1)\Big)}{n(t_u(n)^3+6t_d(m)^2)}. \tag{9}$$

[34]The restrictions on F_d and F_u in (2) are derived from (6) and (7). Since $n^\star$ is increasing in $m^\star$ (see below), the condition on F_d assures that at least two downstream firms enter given that there are only two upstream firms. The condition on F_u is derived by solving (6) for $n^\star$, plugging this into (7), and setting $m^\star = 2$.

Here the sign is ambiguous. Inspection of (9) reveals that it is negative if $n \geq \sqrt[3]{(t_d m)/(t_u(m-1))}m$ while it is positive if the reverse holds true. The reason for this ambiguity is that a change in the number of downstream firms has two effects on the profit of an upstream firm.

Since downstream firms compete against each other, a larger number of downstream firms implies more marginal consumers in the output market. If an upstream firm lowers its price, the downstream firms that buy from this firm sell a higher quantity. But this quantity increase is the larger, the more marginal consumers exist in the downstream market. Thus, each upstream firm has a bigger incentive to lower its price and this increased competition effect lowers upstream profits.

On the other hand, with a larger number of downstream firms each upstream firm faces more potential buyers. An upstream firm then potentially cannibalizes its own demand with a price reduction when selling to more than one firm. This is due to the fact that, with some probability, its buyers are neighbors in the product market, and so a price cut does not increase demand on the margin between these two buyers. As a result of this effect, upstream firms have an incentive to raise their prices, if the number of downstream firms increases. Overall, this second effect dominates if there are few downstream firms. This is the case because the first effect of increased competition is more detrimental to profits if the number of downstream firms is large. This is depicted by the I_U-curve in Figure 1.

This non-monotonicity of the I_U-curve raises the issue of potential multiplicity of equilibria which could occur if the two functions cross more than once. However, it is easy to show that this can never be the case. Thus, the equilibrium is unique. The results of this section are summarized in the following proposition:

Proposition 1

There exists a unique symmetric subgame perfect equilibrium of the three stage game. In this equilibrium, the number of entering upstream and downstream firms, $m^\star$ and $n^\star$, is implicitly defined by

$$\frac{2t_u t_d n^\star}{t_u(n^\star)^3(m^\star-1)+2(m^\star)^3 t_d} = F_u \quad \text{and} \quad \frac{t_d}{(n^\star)^3} - \frac{t_u}{12(m^\star)^2} = F_d.$$

Upstream firms charge a price of

$$r^\star = \frac{2t_u t_d m^\star n^\star}{t_u(n^\star)^3(m^\star-1)+2(m^\star)^3 t_d}$$

and downstream firms charge a price of

$$p^\star = \frac{t_d(2(m^\star)^3 t_d + t_u(n^\star)^3(3m^\star - 1))}{(n^\star)^2(2(m^\star)^3 t_d + t_u(n^\star)^3(m^\star - 1))}.$$

Proof: See Appendix A.

4 Interplay between Upstream and Downstream Market

4.1 Feedback Effects between Upstream and Downstream Competition

In this subsection we analyze how a change in the set-up costs and in the degree of competition in each market affects the overall structure. This will give us already some insights in how a change in the competitiveness in one market carries over to the other market. Understanding this interaction is of importance to evaluate the effects of deregulation policies, as will be our aim in the next subsection.

Proposition 2

Impact of downstream market conditions :

(i) The equilibrium number of downstream firms is increasing in t_d and decreasing in F_d.

(ii) The equilibrium number of upstream firms is increasing in t_d. There exists an $\bar{F}_d$, such that the equilibrium number of upstream firms is increasing in F_d if $F_d < \bar{F}_d$, and decreasing in F_d if $F_d > \bar{F}_d$.

Impact of upstream market conditions :

(i) The equilibrium number of upstream firms is increasing in t_u and decreasing in F_u.

(ii) The equilibrium number of downstream firms is decreasing in t_u and in F_u.

Proof: See Appendix B.

The effects of t_k and F_k, $k \in \{u, d\}$, on the own market k are standard. However, the effects of a change of these variables on the adjacent market are more interesting. Proposition 2 states that a lower degree of downstream competition, i.e. an increase in t_d, leads to a rise in the number of upstream firms. The reason is that if downstream competition is less fierce, an upstream firm i can charge a higher price without losing much demand because the disadvantage for a downstream firm that buys from firm i is less severe. This results in higher upstream profits, and so leads to more entry upstream. As a consequence, the level of competition in the downstream market affects

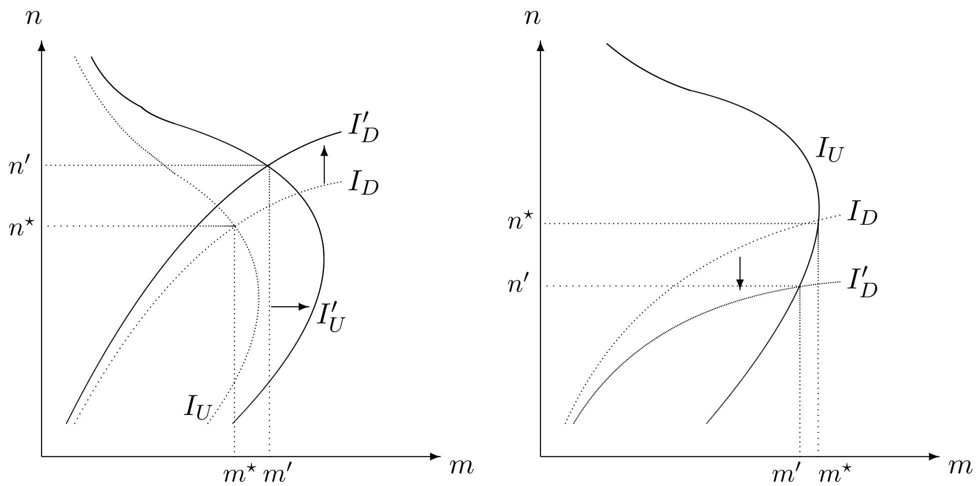

Figure 2: Increase in t_d (left) and increase in F_d (right)

both markets in a similar way because it determines the overall profitability of the two-tier market structure. In addition, it also shows that if downstream competition increases and so $p^\star$ falls, there is a countervailing effect via the upstream market because the number of upstream firms falls as well which leads to an increase in the intermediate good price that dampens the decrease in $p^\star$.

Consider next an increase in F_d, which induces a decrease in $n^\star$. As already explained in the last section, this can either have a positive or a negative impact on $m^\star$, depending on the number of downstream firms. Thus, in contrast to a change in t_d, the feedback effect on the downstream market via the upstream market resulting from a change in F_d can reinforce the direct effect on the downstream market. As shown above, this is the case if the number of downstream firms is small. Figure 2 displays the change in the market equilibrium resulting from a change in the downstream market conditions.

Finally, if the upstream market becomes more competitive (increase in t_u or F_u) this leads to a fall in the number of downstream firms because upstream prices and expected transportation costs increase. Therefore, a change in t_u affects upstream and downstream profits asymmetrically. Thus, while t_d mainly determines the aggregate profits that can be reaped in the industry, t_u affects mainly the distribution of profits between upstream and downstream markets. Figure 3 depicts the change in the market equilibrium resulting from a change in t_u and F_u.

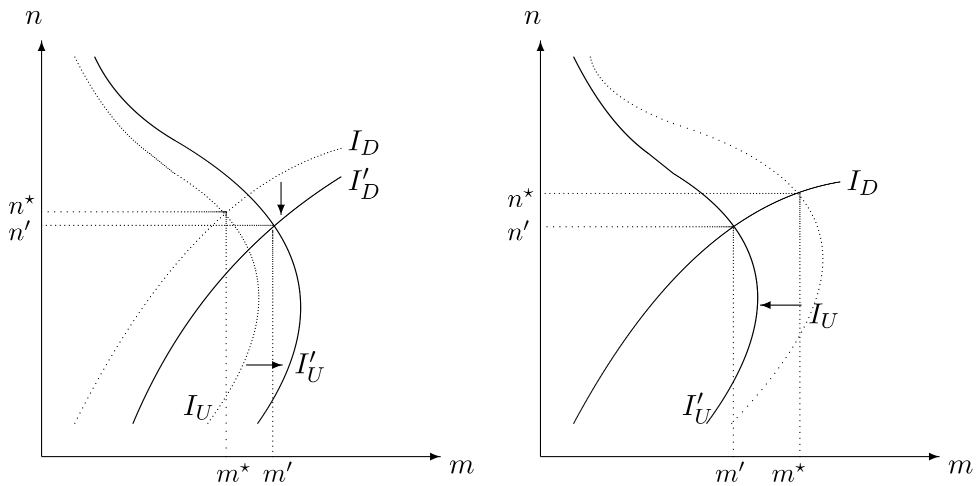

Figure 3: Increase in t_u (left) and increase in F_u (right)

4.2 Implications for Deregulation Policies

We can now use the insights gained in the last subsection to evaluate different deregulation policies. Our aim here is twofold: First, deregulation policies are often derived from models that constrain its attention to the downstream market. It is therefore of interest to find out if the implications resulting from a change in the downstream conditions are over- or undervalued when ignoring the upstream market. Second, antitrust authorities often have multiple instruments to foster competition. Thus, it is important to derive under which industry conditions upstream or downstream deregulation is more effective.

To spur competition, antitrust authorities can usually control two variables. They can decrease entry barriers, for example by reducing red tape for starting new businesses or by making access to the home market easier for foreign firms. In our model, this corresponds to a decrease in firms' set-up costs, F_u or F_d. Another possibility is to increase the degree of competition directly, e.g. by the introduction of standardization measures making it easier for consumers to compare products, or by investing in infrastructure that decreases transport or communication costs. This corresponds to a decrease in t_d or t_u.[35]

Often, policy makers are concerned only with consumer welfare instead of total welfare. Since

[35] t_d and t_u were formally introduced as preference or technology parameters. To think about a decrease in t_d or t_u as the result of deregulation, one can interpret the specification of consumer utility or downstream firms' production functions as reduced form. There, t_d reflects higher product substitutability while t_u reflects higher input substitutability. For a similar exercise and a discussion of this interpretation, see Blanchard and Giavazzi (2003).

profits are zero in equilibrium, welfare and consumer surplus coincide. Thus, our results apply to this case as well. Let a denote a consumer's gross utility from consuming the good. Then welfare can be written as

$$WF = a - p^{\star} - \frac{t_d}{12(n^{\star})^2} = a - \frac{t_d(2(m^{\star})^3 t_d + t_u(n^{\star})^3(3m^{\star}-1))}{(n^{\star})^2(2(m^{\star})^3 t_d + t_u(n^{\star})^3(m^{\star}-1))} - \frac{t_d}{12(n^{\star})^2}. \tag{10}$$

The next proposition states under which conditions the welfare effects from reducing t_d and F_d are over - or underestimated.

Proposition 3

When ignoring the interaction with the upstream market,

(i) the positive welfare effect of decreasing t_d is always overestimated.

(ii) the positive welfare effect of decreasing F_d can be either over- or underestimated. There exists an $\bar{F}_d$, such that for $F_d < \bar{F}_d$ it is overestimated, while for $F_d > \bar{F}_d$ it is underestimated.

Proof: See Appendix B.

Of course, both a decrease in t_d and a decrease in F_d have positive welfare effects. However, the proposition shows that these positive effects are mitigated because stronger competition downstream may induce upstream firms to exit the market, i.e. lowering t_d or F_d reduces $m^{\star}$ (the latter only in case of $F_d < \bar{F}_d$). Thus, the positive welfare effects of fostering downstream competition are often overestimated when ignoring the upstream market. To gain further intuition, we can explore under which conditions this overestimation is particularly high. Here, we restrict our attention to the case of a change in t_d because overestimation occurs for all parameters in this case.

Corollary 1

The magnitude of the overestimation resulting from a change t_d is increasing in t_u and decreasing in t_d and $m^{\star}$.

Proof: See Appendix B.

Thus, if competition in the upstream market is low, which means that either $m^{\star}$ is small or that t_u is high, the upstream market matters a lot for the competitive conditions in the downstream market. In this case, the mistake made when ignoring the upstream market is large. On the other hand, this mistake is also large when t_d is small. The reason is that a change in the upstream price is passed on to the downstream price almost one-to-one if downstream competition is fierce. In sum, the analysis shows that when ignoring the feedback effects via the upstream market one usually tends to overestimate the effects of downstream deregulation. Thus, policy recommendations that do not take into account previous layers in the production chain might be misguided.

The result is in line with experience from the German electricity market in which a merger wave occurred in the upstream market after deregulation of the retail market. The German electricity market was liberalized in 1998 and shortly thereafter many resellers entered the downstream market. Consumers, who beforehand could only contract with their local public firm, now often had a choice between 10 and 20 resellers. This lead to vigorous downstream competition and to a fall in retail prices by around 20%. Soon thereafter, however, electricity producers began to merge and by 2002 their total number had halved from eight to four. In 2002 the downward trend in electricity prices was reversed and prices have increased steadily since. Though the rise of oil and gas prices may have had its share in this price development, many consumer protection agencies hold the merger wave between producers responsible for this increase in retail prices.

Having determined that downstream deregulation policies might be overestimated when ignoring the upstream market, it is of utmost importance to determine under which conditions downstream deregulation is more effective that upstream deregulation. The answer seems straightforward if upstream and downstream markets are very asymmetric, one being very competitive while the other one is not. And indeed, it can be shown in our model that in such cases it is more effective to intervene in the less competitive market. The answer is no longer obvious if both markets are similarly competitive.

We first look at the case of a decrease in set-up costs and compare the welfare effect of reducing F_d with the effect of reducing F_u, i.e. $|\partial WF/\partial F_d| - |\partial WF/\partial F_u|$. We focus on the interesting cases, i.e. when market conditions upstream and downstream are similar, $t_d \approx t_u$ and $n^\star \approx m^\star$.

Proposition 4

If competitive conditions in both markets are equal, $t_d = t_u$ and $n^\star = m^\star$, then

$$\left|\frac{\partial WF}{\partial F_d}\right| - \left|\frac{\partial WF}{\partial F_u}\right| > 0$$

if and only if $m^\star < \tilde{m}$, where $\tilde{m}$ is implicitly defined by

$$429(\tilde{m})^2 - 37(\tilde{m})^3 - 555\tilde{m} - 157 = 0.$$

Proof See Appendix B.

If the competitive conditions are exactly equal, $\tilde{m}(\approx 10.62)$ is the number of firms in each market below which deregulation in the downstream market is more effective than in the upstream market. Yet, by continuity arguments the result also holds when the competitive conditions are similar in the two markets but are not exactly equal.

The intuition behind this result is the following. From Section 3 we know that if the number of downstream firms is small, entry in the downstream market induces more firms to enter upstream as well. This, in turn, induces more downstream firms to enter. Thus, the upstream market reinforces the positive effect on the downstream market if the overall number of firms is small. The opposite is true if there is a large number of downstream firms. As a consequence, the positive effect on the downstream market is diminished and so it is more effective to spur entry in the upstream market if the number of firms is large.

The implication of this result is that in relatively concentrated industries downstream liberalization is more effective than upstream liberalization and vice versa. Thus, our analysis shows that the concentration index in an industry is a useful measure on which to base the evaluation of upstream versus downstream deregulation. The advantage of this measure is that it can be observed at relatively low costs and, therefore, it can serve as an easy guidance to antitrust authorities on the question in which market intervention is more effective.

Now we turn to the case of a change in the transport costs. As in the case of set-up costs, if competitive conditions in the two markets are very asymmetric, it is easy to show that decreasing transport cost in the market with less fierce competition is more effective. Now suppose, as before, that market conditions are similar, $t_d \approx t_u$ and $n^\star \approx m^\star$.

Proposition 5

If competitive conditions in the two markets are equal,

$$\left|\frac{\partial WF}{\partial t_d}\right| - \left|\frac{\partial WF}{\partial t_u}\right| > 0$$

if and only if $m^\star < m'$, where m' is implicitly defined by

$$205(m')^2 - 37(m')^3 - 226m' + 468 = 0.$$

Proof See Appendix B.

Thus, increasing competition directly through a change in the transport costs is more effective in the downstream market if the overall number of firms is small (i.e. $m^\star < m' \approx 4.82$).[36] The result

[36] As before, due to continuity arguments, the result also holds if market conditions are not exactly equal but similar.

closely resembles that of Proposition 4 but the intuition is different. If upstream transportation costs decrease, input prices decrease and this decrease is passed on to downstream prices to some extent. However, this extent depends on the competitive conditions downstream. If the number of downstream firms is small, the downstream margin is still high and so downstream prices do not fall by much. Thus, in this case it is more effective to spur competition downstream, since this has a direct effect on downstream prices. On the other hand, with a large number of firms in both markets, a decrease in the upstream prices via a reduction of t_u is passed down to a large extent and so deregulation in the upstream market is more effective.[37]

Although the intuition behind the results of Propositions 4 and 5 is different, their implication for deregulation policy is similar, i.e. downstream deregulation is particularly effective if the concentration in an industry is relatively high. Our conclusion from this analysis is therefore that the concentration index gives useful guidance to the question in which market deregulation is more effective. This guidance is independent of the concrete deregulation policy.[38]

5 Vertical Restraints

So far we restricted the contract of an upstream firm to be linear. Yet, upstream firms often engage in different forms of vertical restraints. In this section, we analyze two prevalent forms, namely two-part tariffs and resale price maintenance (RPM). We first show that the model can be solved by the same technique as in the case of linear prices and that there is a unique equilibrium in both cases. Then, we compare the outcome of the different regimes with each other and with the outcome in the case of linear prices. This allows us to draw conclusions under which conditions different

[37] Note that $\tilde{m} > m'$. So, if market conditions are exactly equal, the superiority of downstream deregulation compared to upstream deregulation holds for more parameter constellations when deregulation is induced via reducing entry barriers than via increasing the degree of competition directly. This is the case because, for a small number of firms, decreasing F_d has a positive effect on the number of downstream and upstream firms. On the contrary, a reduction in t_d decreases both downstream and upstream prices directly but induces firms to exit the market and this has a negative effect on consumer welfare which is the larger, the lower the overall number of firms.

[38] A question we have not addressed here concerns the socially optimal number of firms. It is well known from the papers by Salop (1979) and especially Mankiw and Whinston (1986) that there is excessive entry when looking only at the downstream market because of the business-stealing effect. The result is less clear when explicitly considering the interaction between upstream and downstream market. Perhaps surprisingly, we find that in our model the result of excessive entry still holds for both markets. (The proof of this result is available in the Appendix *Additional Results - Not Submitted for Publication.*) This result can be contrasted with the findings by Ghosh and Morita (2007a) who show in a model of Cournot competition that insufficient entry may occur in both markets because the additional surplus that an entering firm generates is partly captured by firms in the adjacent market (for a similar result in a model where firms are allowed to bargain with each other, see Ghosh and Morita (2007b)). Our result shows that, although this effect is present as well, in a circular model with price competition the business-stealing effect still dominates and leads to excessive entry.

forms of vertical restraints are pro - or anticompetitive. In this respect, our analysis goes beyond previous papers that take the market structure as given. In contrast, our analysis takes into account how the number of firms changes once vertical restraints are introduced, and so it allows for a more complete picture of their advantages and drawbacks.

5.1 Two-Part Tariffs

In contrast to the case of linear prices, under a two-part tariff regime an upstream firm i now charges a per unit price r_i and a fixed fee $T_i, i \in \{1, ..., m\}$, where T_i is independent of the quantity. The game structure, the information structure concerning locations, and the time line are the same as in the case of linear prices. To make the problem interesting, we assume, as in the case of linear prices, that it is profitable for at least two firms to enter each market.[39]

The third stage of the game plays out exactly as in the case of linear prices, given the upstream price vector $\mathbf{r}$.[40] The profit function of upstream firm i can then be written as

$$E[P_i(r_i, \mathbf{r}_{-i}, T_i, \mathbf{T}_{-i})] = (r_i E[y_i] + T_i) n q_i,$$

where $E[y_i]$ and q_i are solved for in the third stage. Solving the game in the same way as in Section 3 yields the following equilibrium:

Proposition 6

There exists a unique symmetric subgame perfect equilibrium of the three stage game with two-part tariffs. In this equilibrium, the number of entering upstream and downstream firms, $m^\star_{tp}$ and $n^\star_{tp}$, is implicitly defined by

$$\frac{t_u n^\star_{tp}}{(m^\star_{tp})^3} = F_u \quad \text{and} \quad \frac{t_d}{(n^\star_{tp})^3} - \frac{13 t_u}{12 (m^\star_{tp})^2} = F_d. \tag{11}$$

Upstream firms charge a per-unit price of $r^\star_{tp}$ and a fixed fee of $T^\star_{tp}$ while downstream firms charge a price of $p^\star_{tp}$, where

$$r^\star_{tp} = 0, \qquad T^\star_{tp} = \frac{t_u}{(m^\star_{tp})^2}, \quad \text{and} \quad p^\star_{tp} = \frac{t_d}{(n^\star_{tp})^2}.$$

[39] In the following we only explain briefly how to derive the equilibrium because the proof proceeds along very similar lines as the proof of Proposition 1. A sketch of it is available in the Appendix *Additional Results - Not Submitted for Publication*.

[40] As before, we assume that it is indeed optimal for a downstream firm to buy a positive quantity, independent of its upstream location.

With two-part tariffs, the iso-profit curves of upstream and downstream firms are determined by (11). For an upstream firm the slope of its iso-profit curve is given by

$$\frac{\partial m}{\partial n} = \frac{m}{3n} > 0$$

while for a downstream firm it is given by

$$\frac{\partial n}{\partial m} = \frac{13 t_u n^4}{18 t_d m^3} > 0.$$

Both iso-profit curves are increasing, i.e. a larger number of upstream firms triggers more entry in the downstream market and vice versa. The reason for the first result is similar to that under linear prices. More upstream firms reduce the expected transport costs of downstream firms and now additionally lead to a lower fixed fee due to increased competition. On the other hand, in contrast to the case of linear prices, a larger number of downstream firms unambiguously increases the profit of an upstream firm under two-part tariffs. At a first glance, this result may look surprising. The intuition here is that upstream firms set the per-unit price equal to marginal costs and only make profits via the fixed fee. Thus, although competition downstream increases in n, this affects neither the per-unit price nor the fixed fee. As a consequence, a larger number of downstream firms only increases the probability of collecting the fixed fee since more potential buyers are present. Thus, upstream profits increase.[41]

We can now compare the equilibrium number of firms under two-part tariffs with that under linear prices.

Proposition 7

A comparison between the equilibrium market structure under two-part tariffs and under linear prices yields

$$n^{\star} > n^{\star}_{tp} \quad \textit{and} \quad m^{\star} < m^{\star}_{tp}.$$

Proof: See Appendix C.

As shown in the last section, under linear pricing the degree of downstream competition crosses over to the upstream market. Thus, both the degree of upstream and downstream competition

[41] The question may arise if downstream firms can potentially be expropriated via the fixed fee. Once a downstream firm has entered the market, its set-up cost is sunk, and so it would also accept paying a fixed fee up to its operating profit, thereby rendering entry in the first stage not optimal. Yet, this is not the case in equilibrium. In the first stage, downstream firms foresee the amount of the fixed fee in equilibrium which is bounded by upstream competition, and enter accordingly.

confine the upstream prices. Instead, under the two-part tariff regime where upstream firms get their revenue only from the fixed fee, just the degree of upstream competition matters. This can be seen from the first equation in (11).[42] As a consequence, $m^\star_{tp} > m^\star$. Since upstream firms can extract a higher revenue from downstream firms under two-part tariffs, profits of downstream firms are lower, i.e. $n^\star_{tp} < n^\star$.

We can now compare welfare under two-part tariffs with welfare under linear prices. Under two-part tariffs welfare (and consumer surplus) are given by

$$WF_{tp} = a - \frac{13t_d}{12(n^\star_{tp})^2}$$

while under linear prices they are given by

$$WF = a - \frac{t_d(26(m^\star)^3 t_d + t_u(n^\star)^3(37m^\star - 13))}{12(n^\star)^2(2(m^\star)^3 t_d + t_u(n^\star)^3(m^\star - 1))}.$$

It is often argued that two-part tariffs are welfare enhancing because they avoid the double marginalization problem, which induces higher prices and, hence, lower welfare under linear pricing.[43] Perhaps surprising, the next proposition shows that this is not necessarily the case under endogenous entry:

Proposition 8

Welfare under linear prices can either be higher or lower than under two-part tariffs. In particular, it is higher if

$$n^\star_{tp} \leq n^\star \left(\frac{\sqrt{13}\Big(\sqrt{3}t_d(t_u n^\star)^{(3/2)} + 6t_u(t_d - F_d(n^\star)^3)(n^\star\sqrt{3t_u n^\star} - 6\sqrt{t_d - F_d(n^\star)^3})\Big)}{\sqrt{3}t_d(t_u n^\star)^{(3/2)} + 6t_u(t_d - F_d(n^\star)^3)(37n^\star\sqrt{3t_u n^\star} - 78\sqrt{t_d - F_d(n^\star)^3})} \right)^{(1/2)}, \quad (12)$$

where $n^\star$ and $n^\star_{tp}$ are implicitly defined by

$$\frac{72t_u t_d(t_d - F_d(n^\star)^3)^{(3/2)}}{(n^\star)^2\Big(6t_u n^\star\sqrt{3t_u n^\star}(t_d - F_d(n^\star)^3) - 36t_u(t_d - F_d(n^\star)^3)^{(3/2)} + t_d\sqrt{3}(t_u n^\star)^{(3/2)}\Big)} = F_u$$

and

$$\frac{24\sqrt{39}(t_d - F_d(n^\star_{tp})^3)^{(3/2)}}{169\sqrt{t_u}(n^\star_{tp})^{(7/2)}} = F_u.$$

[42] To be more precise, t_d matters indirectly via $n^\star_{tp}$ but this indirect effect is also present under linear prices.

[43] Note that in our model there is no negative quantity effect arising from double marginalization. However, due to the fact that welfare and consumer surplus coincide, welfare decreases in output prices. Hence, the negative implications of double marginalization are captured in our model as well.

Proof: See Appendix C.

Compared to the case where the number of firms is exogenous, with endogenous entry there is an opposing force to the double marginalization problem. This is that more downstream firms enter which leads to increased competition downstream and to lower transport costs for consumers. The proposition shows that if the difference in the number of downstream firms is sufficiently high, the first effect dominates and welfare under linear prices is higher.[44]

It is of interest to know under which market conditions linear pricing is favored from a welfare point of view as compared to two-part tariffs. For this purpose, we look at the upstream market conditions (t_u and F_u) and at the downstream market conditions (t_d and F_d) in turn to analyze how they affect the comparison between the two regimes.

Upstream Market Conditions

We start by analyzing t_u and F_u. Prices and transport costs for downstream firms rise if t_u and F_u increase. As a consequence, the number of downstream firms decreases in both regimes. However, this decrease is larger under two-part tariffs. The reason is that the amount of the fixed fee in equilibrium is increasing in t_u and F_u (via a decrease in $m^\star_{tp}$) and downstream firms can not recoup this increase via output prices. In contrast, under linear pricing downstream firms face higher input prices but can pass them on to some extent via higher final good prices. Thus, the gap between $n^\star$ and $n^\star_{tp}$ increases in t_u and F_u. The problem of double marginalization increases as well, but overall the decrease in the number of downstream firms is more detrimental to welfare. As a consequence, welfare under linear prices is higher than under two-part tariffs if t_u or F_u is large and vice versa. It is somewhat striking that welfare under linear prices is larger under upstream market conditions at which the double marginalization problem is especially pronounced. This shows that the insights from previous analysis can be reversed when taking endogenous entry into account. The result is illustrated in Figure 4 (on the left-hand side the parameter values are $a = 0.1$, $t_d = 1$, $F_u = 0.001$ and $F_d = 0.001$[45] while on the right-hand side, they are set at $a = 0.2$, $t_u = 1$, $t_d = 1$ and $F_d = 0.001$).

[44] In a model with an upstream monopolist and a perfectly competitive downstream industry, Ordover and Panzar (1982) also show that linear prices can be welfare superior compared to two-part tariffs. Yet, in their model due to perfect downstream competition double marginalization does not arise. In contrast, we consider an oligopoly upstream and downstream with free entry and show that linear pricing can be welfare superior even if double marginalization is a central issue.

[45] One can check that with these parameters a downstream firm is active after entering as long as $t_u < 5.6$, independent of its upstream location.

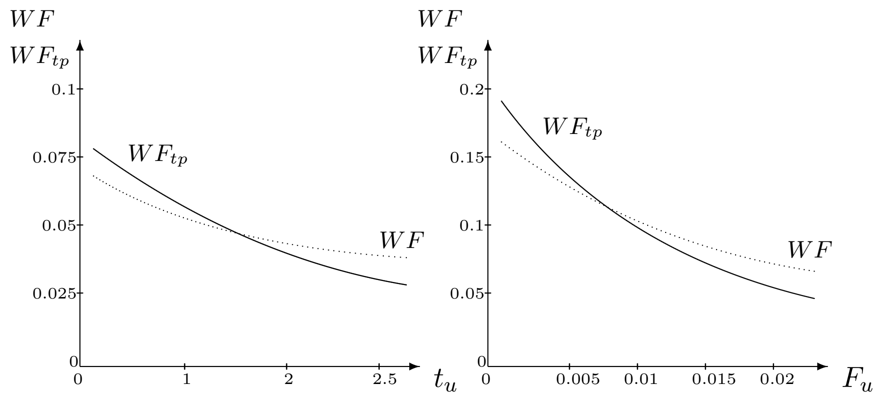

Figure 4: Change in WF and WF_{tp} with t_u (left) and with F_u (right).

Downstream Market Conditions

We now look at the effects of t_d and F_d. It is evident that a change in t_d or F_d has no direct effect on upstream prices in the two-part tariff regime (only an indirect one via a change in $n^{\star}_{tp}$). This is different in the linear pricing regime where, as we discussed in Section 4, downstream competition carries over to the upstream market. As t_d or F_d increase, double marginalization becomes more pronounced under linear prices. As t_d gets larger, the number of upstream and downstream firms increases in both regimes but this effect is of similar force. As F_d increases, $n^{\star}_{tp}$ and $m^{\star}_{tp}$ decrease under two-part tariffs, while under linear pricing $n^{\star}$ decreases and $m^{\star}$ may rise or fall. Still, the effect of pronounced double marginalization dominates the potential increase in $m^{\star}$ as F_d rises. As a consequence, if t_d or F_d is relatively small, welfare is larger in the linear pricing regime because there is almost no double marginalization and the number of downstream firms is larger, while welfare under linear pricing is smaller if t_d or F_d is relatively large. The left-hand side of Figure 5 depicts the change of WF and WF_{tp} in t_d (here $a = 0.1$, $t_u = 1$, $F_u = 0.001$ and $F_d = 0.001$) while the right-hand side depicts the change of WF and WF_{tp} in F_d (here $a = 0.1$, $t_u = 1$, $t_d = 1$ and $F_u = 0.001$).

Summing up, this section has shown that if the problem of double marginalization is especially severe, because the downstream market is not very competitive (t_d or F_d or both are large), welfare under two-part tariffs is higher than under linear pricing. On the contrary, if upstream competition is low (high t_u and/or high F_u), two-part tariffs reduce entry in the downstream market to a large extent, thereby making this market less competitive. Thus, although in this case the double marginalization problem is severe as well, two-part tariffs are likely to reduce welfare. Overall,

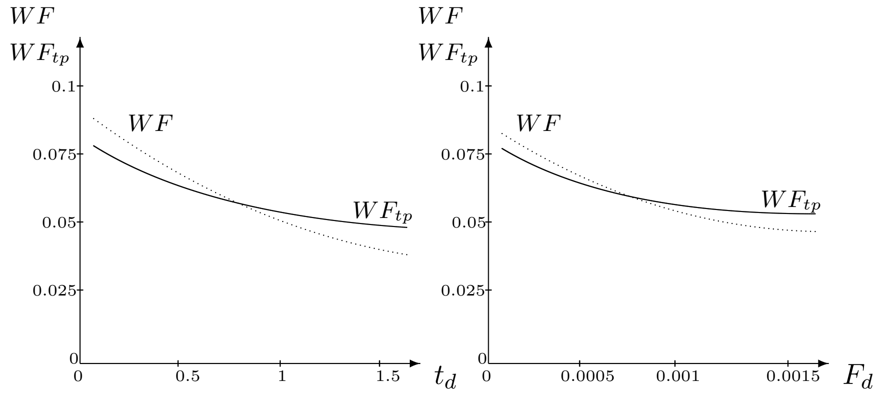

Figure 5: Change in WF and WF_{tp} with t_d (left) and with F_d (right).

the analysis shows that the effects of two-part tariffs on market entry can be large, and so welfare judgements that rely solely on double marginalization can be incorrect.

5.2 Resale Price Maintenance

Resale Price Maintenance (RPM) is the practice of upstream firms fixing the selling price of the downstream firm. As already pointed out in the introduction, RPM is prohibited in many countries. Yet, the theoretical literature does not give a clear view on RPM. In particular, as Rey and Vergé (2004) point out, it is not clear that RPM has a larger negative impact on consumer welfare than other vertical restraints, like two-part tariffs. It is thus interesting to compare these two regimes in our model of free entry and of varying degrees of competition.[46]

Under RPM, the only decision a downstream firms makes is to choose its input supplier. Upstream firms have pricing power over the input and output prices. After solving for the equilibrium, an immediate result is that the equilibrium contains double marginalization. This is intuitive: upstream firms can only make profits by charging a price above marginal costs while they can attract potential buyers only by leaving them a profit margin as well which implies that downstream prices are above upstream prices.

We can then compare the welfare effects of RPM with those under linear pricing and under two-part tariffs. Comparing RPM with linear pricing we find that welfare can be higher under either regime. The reason is that there are two opposing effects. First, the double marginalization problem is less severe under RPM. The intuition is that the final good prices are controlled by the

[46] In this section, we only discuss the results under RPM and the comparison with the other regimes. A formal derivation is available in the Appendix *Additional Results - Not Submitted for Publication.*

upstream firms who have a lower interest in maximizing downstream profits than the downstream firms themselves. On the other hand, since downstream firms have no market power under RPM, fewer of them enter. It can be shown that either of the two effects might dominate.[47]

The results concerning the comparison of RPM with two-part tariffs are more interesting. Here we find that welfare under two-part tariffs is unambiguously higher than under RPM. The basic intuition is that downstream prices are lower under two-part tariffs due to the avoidance of double marginalization. Interestingly, the number of upstream *and* downstream firms is higher under RPM. However, this effect is only of second order compared to the higher downstream prices. Loosely speaking, under RPM all competition takes place at the upstream stage and so the second layer of competition is eliminated compared to two-part tariffs. This is always detrimental to welfare.

There are two main conclusions that can be drawn from the analysis of RPM. First, compared to two-part tariffs, the effect of higher downstream prices is dominant even when taking free entry into account. Therefore, there is some justification for treating RPM as a device for reducing competition and thus prohibiting it. Second, it would not be enough to ban one instrument, e.g. two-part tariffs, if upstream firms substitute it with another, e.g. RPM, that is even more detrimental to welfare.[48]

6 Conclusion

This paper has provided a model of successive oligopolies that allows for endogenous entry and varying degrees of competition in both markets. We showed how the competitive conditions in the two markets influence each other and gave implications for deregulation policies. We also demonstrated that the welfare effects of two-part tariffs and RPM compared to linear pricing are markedly different when taking into account that using these restraints changes the equilibrium number of firms. We restricted attention to two-part tariffs and RPM because of their prominence, but it is also of interest to look at other forms of vertical restraints like fully non-linear tariffs. The model can also provide a framework to study the consequences of vertical mergers under endogenous entry. However, given the asymmetric nature of firms in this case, such an analysis becomes more intricate.

A possible limitation of our results is that we conducted our analysis in the circular competition model where aggregate demand is fixed. However, as we will briefly outline below, the main qualitative results of our model should be robust to allowing for variable demand. To see this, consider

[47] See also Chen (1999) who finds that RPM can increase or decrease welfare in a framework with price discrimination.
[48] For a further discussion of this point, see Rey and Tirole (2007).

first the implications for deregulation: In our model we find that downstream deregulation is more effective than upstream deregulation if the market is relatively concentrated and vice versa. A main reason for this result is that downstream entry can either have a positive or a negative effect on upstream entry. The first effect dominates if there are only few buyers while the second one is more important in a relatively competitive industry. Now, consider a model with variable demand. In this case, it is likely that the first effect is larger than in our model because an increase in the number of downstream firms now additionally leads to an increase in overall quantity. Nevertheless, if the number of downstream firms gets larger, competition becomes fiercer and, above some threshold, further entry will have a detrimental effect on upstream profits. Thus, the countervailing effects would be similar. It seems therefore plausible that the deregulation implications of our paper are qualitatively the same with elastic demand, though the exact threshold would probably be more in favor of downstream deregulation.

Consider next our results on vertical restraints: Here we found that two-part tariffs give upstream firms more market power to extract profits from downstream firms. Thus, fewer downstream firms enter. Therefore, although double marginalization is avoided, welfare can be lower than under linear pricing. Now if demand were elastic, the effect that less downstream firms enter would be even more detrimental to welfare because the overall quantity would be lower due to reduced downstream competition. Therefore, it is likely that linear prices are welfare superior for an even larger range of parameters than in our model.

An interesting direction for future research is to allow for bargaining between upstream and downstream firms. In our model, only upstream firms have market power over the input price. Incorporating bargaining between the two sides in our framework could give new insights into how competition and free entry shape the outside options of each firm and hence the market structure. However, since in our model upstream firms make take-it-or-leave-it offers, the resulting upstream price can be seen as an upper bound in any bargaining game. If one allows for more equal bargaining power, it is reasonable to expect that the upstream price decreases and, therefore, a smaller number of upstream and a larger number of downstream firms enter. Nevertheless, it seems plausible that our effects identified in the interaction between the two markets and our qualitative analysis concerning vertical restraints remain valid.

7 Appendix

7.1 Appendix A

Proof of Proposition 1

As described in Section 3, each downstream firm does not observe the input prices of its rivals but it can calculate their expected input prices. Downstream firms potentially buy at different input prices but are otherwise identical. So in a symmetric equilibrium there can at most be m different final good prices. Since firm j's expectation about the location of its neighbors in the upstream market is the same, we have $E[p_{j-1}] = E[p_{j+1}] = E[p_{-j}] = \sum\limits_{k=1}^{m} q_k p_k$, where p_k is the price of a downstream firm when it buys from upstream firm k. Thus, the expected profit function (disregarding fixed costs) of a downstream firm that buys from upstream firm i is given by

$$E[\Pi^i(p_i, r_i, \mathbf{p}_{-i})] = (p_i - r_i)\left(\frac{1}{n} + \frac{n\left(\sum\limits_{k=1}^{m} q_k p_k - p_i\right)}{t_d}\right), \quad i \in \{1, ..., m\}.$$

Maximizing the last equation with respect to p_i gives a reaction function of

$$p_i = \frac{t_d + n^2\left(r_i + \sum\limits_{k=1}^{m} q_k p_k\right)}{2n^2}.$$

Multiplying the last equation by q_i, summing over all m and using the fact that $\sum\limits_{k=1}^{m} q_k = 1$ yields

$$\sum_{k=1}^{m} q_k p_k = \frac{t_d}{n^2} + \sum_{k=1}^{m} q_k r_k.$$

Inserting the last expression into p_i gives

$$p_i = \frac{1}{2}\left(r_i + \sum_{k=1}^{m} q_k r_k + \frac{2t_d}{n^2}\right), \qquad i \in \{1, ...m\}. \tag{13}$$

Plugging p_i into the expected profit funtion we get, after rearranging,

$$\Pi^i(r_i, \mathbf{r}_{-i}) = \frac{1}{4t_d n^3}\left(2t_d + n^2\left(\sum_{k=1}^{m} q_k r_k - r_i\right)\right)^2, \quad i \in \{1, ..., m\}. \tag{14}$$

The next step is to determine the probabilities q_i, $i \in \{1, ..., m\}$. Here, we make some use of the formulation $a \bmod b$.[49] This allows us to write the formulas in the most general way. Since the right

[49] Recall that $a \bmod b$ is the remainder on division of a by b. Since in our case we will have $a < 2b$, $a \bmod b$ is a, whenever $a < b$, and $a \bmod b$ is $a - b$, whenever $a \geq b$.

neighbor of firm m is firm 1 and the left neighbor of firm 1 is firm m, we can express the input price of the right neighbor of any firm $i \in \{1, ..., m\}$ by $r_r \equiv r_{(i \bmod m)+1}$ and the input price of the left neighbor by $r_l \equiv r_{((i+m-2) \bmod m)+1}$. In addition, we denote by $r_{k1} \equiv r_{(k \bmod m)+1}$ the input price of the right neighbor of firm k and by $r_{k2} \equiv r_{(k+1 \bmod m)+1}$ the input price of the firm that is located two firms away to the right of firm k. The same definitions apply for the probabilities q_r, q_l, q_{k1} and q_{k2}. Inserting (14) into (1) then gives

$$q_i = \frac{1}{m} + \frac{1}{8t_u t_d n^3}\left(2\Big(2t_d + n^2\Big(\sum_{k=1}^{m} q_k r_k - r_i\Big)\Big)^2 - \Big(2t_d + n^2\Big(\sum_{k=1}^{m} q_k r_k - r_l\Big)\Big)^2 - \Big(2t_d + n^2\Big(\sum_{k=1}^{m} q_k r_k - r_r\Big)\Big)^2\right),$$

for all $i \in \{1, ..., m\}$. Solving each of the m equations for q_i, $i \in \{1, ..., m\}$, yields an inhomogeneous system of m linear equations with m unknowns. We may use the Gaussian algorithm to find q_i, $i \in \{1, ..., m\}$. Tedious but routine calculations yield the solution that can be written as

$$q_i(r_i, \mathbf{r}_{-i}) = \tag{15}$$

$$= \frac{1}{m} + \frac{n^2 m\Big(2r_i^2 - r_r^2 - r_l^2\Big)}{8t_u t_d} + \frac{n^2 m\Big(r_i(2r_i - r_r - r_l)\Big)}{4t_u t_d} - \frac{\psi\; n^2 m\Big(2r_i - r_r - r_l\Big)}{2t_u t_d n^2\Big(2t_u t_d + nm\sum_{k=1}^{m} r_k(r_k - r_{k1})\Big)},$$

where

$$\psi \equiv \Bigg\{t_u t_d\Big[2t_d + \frac{n^2(m+1)}{m} r_i + \frac{n^2}{m}\sum_{k=1,\, k\neq i}^{m} r_k\Big] + \frac{n^3 m}{8}\Bigg[6r_i^3 + 2\sum_{k=1,\, k\neq i}^{m} r_k^3 - 5r_i^2(r_r + r_l) +$$

$$+ r_i \sum_{k=1,\, k\neq j}^{m} r_k^2(2 + \min\{|k-i|, |m+i-k|\}) - 4r_i \sum_{k=i}^{i+m-3} r_{k1} r_{k2} - \sum_{k=i}^{i+m-3} r_{k1} r_{k2}(r_{k1} + r_{k2})\Bigg]\Bigg\},$$

for $m \geq 3$. For m= 2, the solution can be written as[50]

$$q_i(r_i, \mathbf{r}_{-i}) = \frac{t_u t_d n - 4t_d(r_i - r_{-i}) + n^2(r_i^2 - r_{-i}^2)^2}{2\Big(t_u t_d + n(r_i^2 + r_{-i}^2 - 2r_i r_{-i})\Big)}. \tag{16}$$

Inserting these probabilities in (13) gives downstream prices of

$$p_i(r_i, \mathbf{r}_{-i}) = \frac{\psi}{n^2\Big(2t_u t_d + nm\sum_{k=1}^{m} r_k(r_k - r_{k1})\Big)} \tag{17}$$

for $m \geq 3$, and

$$p_i(r_i, \mathbf{r}_{-i}) = \frac{4t_u t_d^2 + n^2 t_u t_d(3r_i + r_{-i}) + n^3(3r_i^3 + r_{-i}^3 + r_i r_{-i}^2 - 5r_i^2 r_2)}{4n^2\Big(t_u t_d + n(r_i^2 + r_{-i}^2 - 2r_i r_{-i})\Big)} \tag{18}$$

[50]The solution for $m = 2$ cannot be incorporated in the general formula since, in contrast to $m \geq 3$, each firm has only one direct neighbor.

for $m = 2$.

Plugging the probabilities and prices into the formula for the expected quantity that a downstream firm buys from upstream firm i, $1/n + n(\sum_{k=1}^{m} q_k p_k - p_i)/t_d$, yields

$$E[y_i(r_i, \mathbf{r}_{-i})] = \frac{1}{nt_d\Big(2t_u t_d + nm\sum_{k=1}^{m} r_k(r_k - r_{k1})\Big)} \times \Bigg[\psi - 2\frac{t_u t_d n^2(m-1)}{m} r_i + \tag{19}$$

$$+\frac{n^3 m}{8}\Big[-4r_i^3 + r_i^2(r_r + r_l) - 2r_i \sum_{k=1,\, k\neq i}^{m} r_k^2(6 - \min\{|k-i|, |m+i-k|\}) + 8r_i \sum_{k=i}^{i+m-3} r_{k1} r_{k2}\Big]\Bigg]$$

for $m \geq 3$ and

$$E[y_i(r_i, \mathbf{r}_{-i})] = \frac{4t_u t_d^2 - n^2 t_u t_d(r_i - r_{-i}) - n^3(r_i^3 - r_{-i}^3 + 3r_i r_{-i}^2 - 3r_i^2 r_{-i})}{4nt_d\Big(t_u t_d + n(r_i^2 + r_{-i}^2 - 2r_i r_{-i})\Big)} \tag{20}$$

for $m = 2$.

Having solved for the probabilities, the downstream prices and quantities, we proceed to the second stage, the upstream market.

As explained in Section 3, the profit function of an upstream firm can be written as

$$E[P_i(r_i, \mathbf{r}_{-i})] = r_i E[y_i]\Bigg(\binom{n}{1} q_i(1-q_i)^{n-1} + 2\binom{n}{2} q_i^2(1-q_i)^{n-2} +$$

$$+ \cdots + (n-1)\binom{n}{n-1} q_i^{n-1}(1-q_i) + nq_i^n\Bigg) - F_u,$$

where, for notational simplicity, the arguments of y_i and q_i are suppressed. Rearranging terms gives

$$E[P_i(r_i, \mathbf{r}_{-i})] = r_i E[y_i]\Bigg(\sum_{j=1}^{n} \binom{n}{j} j q_i^j (1-q_i)^{n-j}\Bigg) - F_u =$$

$$= r_i E[y_i] q_i n \Bigg(\sum_{j=0}^{n-1} \bigg(\frac{(n-1)!}{j!(n-1-j)!}\bigg) q_i^j (1-q_i)^{n-1-j}\Bigg) - F_u.$$

We may use a modification of the Binomial Theorem[51] to rewrite the profit function as

$$E[P_i(r_i, \mathbf{r}_{-i})] = r_i E[y_i] n q_i \Bigg(q_i + (1-q_i)\Bigg)^{n-1} = r_i E[y_i] n q_i - F_u.$$

[51] Recall that the Binomial Theorem says that $\sum_{j=0}^{n} \binom{n}{j} (z)^j (y)^{n-j} = (y+z)^n$.

We can substitute (15) and (19) in case of $m \geq 3$ or (16) and (20) in case of $m = 2$ in the last equation. Maximizing this profit function with respect to r_i gives

$$\frac{\partial E[P_i(r_i, r_{i-1}, r_{i+1})]}{\partial r_i} = E[y_i]q_i + r_i\Big(q_i\frac{\partial E[y_i]}{\partial r_i} + E[y_i]\frac{\partial q_i}{\partial r_i}\Big) = 0, \qquad i \in \{1, ..., m\}. \tag{21}$$

Solving the system of equations (21) for r_i, $i \in \{1, ..., m\}$, yields a unique symmetric solution that is given by

$$r^\star = \frac{t_u t_d n}{\sum_{j=2}^{m} \frac{t_u n^3}{2j(j-1)} + m^2 t_d} = \frac{2t_u t_d m n}{t_u n^3(m-1) + 2m^3 t_d}.$$

It is readily verified that the second-order condition is satisfied at this solution. Inserting the equilibrium upstream prices into the formula for the downstream prices gives

$$p^\star = \frac{t_d\left(m^2 t_d + t_u n^3\left(1 + \sum_{j=2}^{m}\frac{1}{2j(j-1)}\right)\right)}{n^2\left(m^2 t_d + t_u n^3\left(\sum_{j=2}^{m}\frac{1}{2j(j-1)}\right)\right)} = \frac{t_d(2m^3 t_d + t_u n^3(3m-1))}{n^2(2m^3 t_d + t_u n^3(m-1))}.$$

Having solved for the equilibrium prices in both stages, we can proceed to the first stage and determine the equilibrium number of firms in both markets. Inserting the equilibrium prices into the equations $E[\Pi_j] = 0$ and $E[P_i] = 0$ yields that $n^\star$ and $m^\star$ are implicitly defined by

$$\frac{t_d}{(n^\star)^3} - \frac{t_u}{12(m^\star)^2} - F_d = 0$$

and

$$\frac{2t_u t_d n^\star}{t_u(n^\star)^3(m^\star - 1) + 2(m^\star)^3 t_d} - F_u = 0.$$

Given our assumptions on F_d and F_u, we know that at least two firms are active in both markets.

It remains to show uniqueness of the equilibrium. To this end, we determine the iso-profit-lines of a downstream and an upstream firm. For a downstream firm it is given by

$$I_D := \frac{\partial n}{\partial m} = \frac{t_u(n)^4}{18t_d(m)^3} > 0$$

and for an upstream firm it is given by

$$I_U := \frac{\partial m}{\partial n} = \frac{2\Big((m)^3 t_d - t_u(n)^3(m-1)\Big)}{n(t_u(n)^3 + 6t_d(m)^2)}.$$

As explained in Section 3, I_D is increasing while I_U is decreasing for $n > \sqrt[3]{(t_d m)/(t_u(m-1))}m$ and increasing otherwise. Thus, a necessary condition for the two curves to cross more than once, and so for multiple equilibria to exist, is that the slope of I_U in its increasing region is smaller than the slope of I_D. In an m-n-plane the slope of I_U is

$$\frac{n(t_u(n)^3 + 6t_d(m)^2)}{2\Big((m)^3 t_d - t_u(n)^3(m-1)\Big)}.$$

It is easy to see that the numerator of the last expression is bigger than the numerator of I_D while the denominator is smaller than the denominator of I_D. It follows that the slope of I_U is strictly steeper than the one of I_D in the area where they are both increasing. As a consequence, the equilibrium is unique.

q.e.d.

7.2 Appendix B

Proof of Proposition 2

We start with the downstream market. Differentiating (6) with respect to F_d (taking into account that $n^\star$ and $m^\star$ vary with F_d) yields

$$-3\frac{t_d}{(n^\star)^4}\frac{dn^\star}{dF_d}dF_d + \frac{t_u}{6(m^\star)^3}\frac{dm^\star}{dF_d}dF_d - dF_d = 0.$$

Differentiating (7) with respect to F_d gives

$$-2\frac{t_d t_u n^\star(t_u(n^\star)^3 + 6(m^\star)^2 t_d)}{(t_u(n^\star)^3(m-1) + 2(m^\star)^3 t_d)^2}\frac{dm^\star}{dF_d}dF_d -$$

$$-\frac{4t_d t_u((t_u(n^\star)^3(m^\star-1) - (m^\star)^3 t_d)^2)}{(t_u(n^\star)^3(m^\star-1) + 2(m^\star)^3 t_d)^2}\frac{dn^\star}{dF_d}dF_d = 0.$$

Solving the last two equations for $(dm^\star)/(dF_d)$ and $(dn^\star)/(dF_d)$ gives

$$\frac{dn^\star}{dF_d} = -\frac{3(n^\star)^4(m^\star)^3\Big(t_u(n^\star)^3 + 6(m^\star)^2 t_d\Big)}{\Big(8t_u t_d(n^\star)^3(m^\star)^3 + 54t_d^2(m^\star)^5 + (n^\star)^6(m^\star-1)t_u^2\Big)} < 0$$

and

$$\frac{dm^\star}{dF_d} = \frac{6(m^\star)^3(n^\star)^3\Big(t_u(n^\star)^3(m^\star-1) - (m^\star)^3 t_d\Big)}{8t_u t_d(n^\star)^3(m^\star)^3 + 54t_d^2(m^\star)^5 + (n^\star)^6(m^\star-1)t_u^2}.$$

It is obvious that $(dm^\star)/(dF_d) \geq 0$ if and only if $n^\star \geq \sqrt[3]{(m^\star t_d)/((m^\star-1)t_u)}m^\star$. Since $(dn^\star)/(dF_d) < 0$, there exists a unique F_d denoted by $\bar{F}_d$ such that $m^\star$ is increasing (decreasing) in F_d as long as $F_d < (>)\bar{F}_d$.

Proceeding in the same way as above concerning a change in t_d gives

$$\frac{dn^\star}{dt_d} = \frac{n^\star\Big((n^\star)^6 t_u^2(m^\star-1) + 6(m^\star)^3 t_d(6(m^\star)^2 t_d + (n^\star)^3 t_u)\Big)}{2t_d\Big(8t_u t_d(n^\star)^3(m^\star)^3 + 54t_d^2(m^\star)^5 + (n^\star)^6(m^\star-1)t_u^2\Big)} > 0$$

and

$$\frac{dm^\star}{dt_d} = \frac{3(m^\star)^3\Big(2t_d(m^\star)^3 + t_u(n^\star)^3(m^\star-1)\Big)}{8t_u t_d(n^\star)^3(m^\star)^3 + 54t_d^2(m^\star)^5 + (n^\star)^6(m^\star-1)t_u^2} > 0.$$

Now we turn to the upstream market. Differentiating (6) and (7) with respect to F_u and solving the resulting equations for $dn^\star/dF_u$ and $dm^\star/dF_u$ gives

$$\frac{dn^\star}{dF_u} = -\frac{(n^\star)^3\Big(t_u(n^\star)^3(m^\star-1) + 2t_d(m^\star)^3\Big)^2}{4t_d\Big(8t_u t_d(n^\star)^3(m^\star)^3 + 54t_d^2(m^\star)^5 + (n^\star)^6(m^\star-1)t_u^2\Big)} < 0$$

and

$$\frac{dm^\star}{dF_u} = -\frac{9(m^\star)^3\Big(t_u(n^\star)^3(m^\star-1) + 2t_d(m^\star)^3\Big)^2}{2t_u n^\star\Big(8t_u t_d(n^\star)^3(m^\star)^3 + 54t_d^2(m^\star)^5 + (n^\star)^6(m^\star-1)t_u^2\Big)} < 0,$$

where the inequalities follow from the fact that $m^\star \geq 2$.

Differentiating (6) and (7) with respect to t_u and solving for $(dn^\star)/(dt_u)$ and $(dm^\star)/(dt_u)$ gives

$$\frac{dn^\star}{dt_u} = -\frac{(n^\star)^4 m^\star\Big((n^\star)^3 t_u + 2(m^\star)^2 t_d\Big)}{4\Big(8t_u t_d(n^\star)^3(m^\star)^3 + 54t_d^2(m^\star)^5 + (n^\star)^6(m^\star-1)t_u^2\Big)} < 0$$

and

$$\frac{dm^\star}{dt_u} = \frac{m^\star\Big((n^\star)^6 t_u^2(m^\star-1) + (m^\star)^3 t_d(36(m^\star)^2 t_d - t_u(n^\star)^3)\Big)}{2t_u\Big(8t_u t_d(n^\star)^3(m^\star)^3 + 54t_d^2(m^\star)^5 + (n^\star)^6(m^\star-1)t_u^2\Big)} > 0.$$

q.e.d.

Proof of Proposition 3

We first consider a change in t_d. Differentiating the welfare function (10) with respect to t_d and using $dn^\star/dt_d$ and $dm^\star/dt_d$ from Proposition 2 yields

$$\frac{dWF}{dt_d} = -\frac{t_d(m^\star)^3\Big(117t_d(m^\star)^2 + 49t_u(n^\star)^3\Big)}{6(n^\star)^2\Big(8t_u t_d(n^\star)^3(m^\star)^3 + 54t_d^2(m^\star)^5 + (n^\star)^6(m^\star-1)t_u^2\Big)}. \tag{22}$$

When ignoring the upstream market, i.e. ignoring the effect that $m^\star$ changes with t_d, we get from equation (6) that

$$\left.\frac{dn^\star}{dt_d}\right|_{\frac{dm^\star}{dt_d}=0} = \frac{n^\star}{3t_d}$$

and thus

$$\frac{dWF}{dt_d}\Big|_{\frac{dm^\star}{dt_d}=0} = -\frac{t_u(n^\star)^3(37m^\star-13)+26t_d(m^\star)^3}{36(n^\star)^2\Big(t_u(n^\star)^3(m^\star-1)+2t_d(m^\star)^3\Big)}. \tag{23}$$

Subtracting (23) from (22) gives

$$\frac{t_u n^\star\Big(4t_dt_u(n^\star)^3(m^\star)^3(7m^\star+41)+916t_d^2(m^\star)^6+t_u^2(n^\star)^6(37(m^\star)^2-50m^\star+13)\Big)}{36\Big(8t_ut_d(n^\star)^3(m^\star)^3+54t_d^2(m^\star)^5+(n^\star)^6(m^\star-1)t_u^2\Big)\Big(t_u(n^\star)^3(m^\star-1)+2t_d(m^\star)^3\Big)}>0. \tag{24}$$

Since both (22) and (23) are negative, it follows that the absolute value of $(dWF/dt_d)|_{dm^\star/dt_d=0}$ is higher than the absolute value of (dWF/dt_d). Hence, the welfare effect of changing t_d is overestimated when ignoring the upstream market.

Proceeding in the same way for a change in F_d yields

$$\frac{dWF}{dF_d}-\frac{dWF}{dF_d}\Big|_{\frac{dm^\star}{dF_d}=0} = t_u(n^\star)^4\Big(t_u(n^\star)^3(m^\star-1)-t_d(m^\star)^3\Big)\times \tag{25}$$

$$\frac{\Big(4t_dt_u(n^\star)^3(m^\star)^3(7m^\star+41)+916t_d^2(m^\star)^6+t_u^2(n^\star)^6(37(m^\star)^2-50m^\star+13)\Big)}{18\Big(8t_ut_d(n^\star)^3(m^\star)^3+54t_d^2(m^\star)^5+(n^\star)^6(m^\star-1)t_u^2\Big)\Big(t_u(n^\star)^3(m^\star-1)+2t_d(m^\star)^3\Big)}.$$

It follows that the right hand side of (25) is positive if and only if $n<\sqrt[3]{(t_dm^\star)/(t_u(m^\star-1))}m^\star$. Since both (dWF/dF_d) and $(dWF/dF_d)|_{dm^\star/dF_d=0}$ are negative, the first one is of larger magnitude than the latter if and only if $n<\sqrt[3]{(t_dm^\star)/(t_u(m^\star-1))}m^\star$. Since $n^\star$ is a decreasing function of F_d, there exists a unique $\bar{F}_d$ such that the magnitude of (dWF/dF_d) is larger than the one of $(dWF/dF_d)|_{dm^\star/dF_d=0}$, if and only if $F_d>\bar{F}_d$ and vice versa.

q.e.d.

Proof of Corollary 1

The magnitude of the overestimation is given by (24). Differentiating (24) with respect to t_u yields

$$\frac{\xi\, t_dn^\star(m^\star)^3}{6\Big(8t_ut_d(n^\star)^3(m^\star)^3+54t_d^2(m^\star)^5+(n^\star)^6(m^\star-1)t_u^2\Big)^2\Big(t_u(n^\star)^3(m^\star-1)+2t_d(m^\star)^3\Big)^2}>0,$$

where

$$\xi=\Big(16488t_d^4(m^\star)^{11}+t_d^3t_u(n^\star)^3(m^\star)^8(1008(m^\star)+5904)+t_d^2t_u^2(n^\star)^6(m^\star)^5(798(m^\star)^2+488m^\star-774)+$$

$$+t_d t_u^2 (n^\star)^9 (m^\star)^2 (558(m^\star)^3 - 1222(m^\star)^2 + 898m^\star - 234) + t_u^4 (n^\star)^{12} (57(m^\star)^3 - 163(m^\star)^2 + 155m^\star - 49)\Big) > 0.$$

The inequality follows from $m^\star \geq 2$.

Differentiating (24) with respect to $m^\star$ yields

$$-\frac{\xi\, tun^\star (m^\star)^3}{6\Big(8t_u t_d (n^\star)^3 (m^\star)^3 + 54t_d^2 (m^\star)^5 + (n^\star)^6 (m^\star - 1) t_u^2\Big)^2 \Big(t_u (n^\star)^3 (m^\star - 1) + 2t_d (m^\star)^3\Big)^2} < 0.$$

Finally, differentiating (24) with respect to t_d yields

$$-\frac{\rho\, tu\, n^\star}{6\Big(8t_u t_d (n^\star)^3 (m^\star)^3 + 54t_d^2 (m^\star)^5 + (n^\star)^6 (m^\star - 1) t_u^2\Big)^2 \Big(t_u (n^\star)^3 (m^\star - 1) + 2t_d (m^\star)^3\Big)^2} < 0,$$

with

$$\rho = 32976 t_d^5 (m^\star)^{13} + t_d^4 t_u (n^\star)^3 (m^\star)^{10} (2016(m^\star + 23004)) + t_d^3 t_u^2 (n^\star)^6 (m^\star)^7 (3768(m^\star)^2 + 1596m^\star - 1080) +$$

$$+ t_d^2 t_u^3 (n^\star)^9 (m^\star)^4 (1116(m^\star)^3 - 2069(m^\star)^2 + 11794m^\star - 585) +$$

$$t_d t_u^4 (n^\star)^{12} (m^\star)^2 (114(m^\star)^3 - 311(m^\star)^2 + 344m^\star - 147) + 4t_u^5 (n^\star)^{15} ((m^\star)^2 - 2m^\star + 1) > 0.$$

q.e.d.

Proof of Proposition 4

We are interested in the difference

$$\left|\frac{dWF}{dF_d}\right| - \left|\frac{dWF}{dF_u}\right|.$$

With a change in F_k, $k \in \{u, d\}$, the equilibrium number of firms changes and so

$$\left|\frac{dWF}{dF_k}\right| = \left|\frac{\partial WF}{\partial m^\star}\frac{dm^\star}{dF_k}\right| + \left|\frac{\partial WF}{\partial n^\star}\frac{dn^\star}{dF_k}\right|.$$

Substituting $(dm^\star)/(dF_k)$ and $(dn^\star)/(dF_k)$ from the proof of Proposition 2 in the last equation and simplifying yields

$$\left|\frac{dWF}{dF_d}\right| =$$

$$\frac{td\, n^\star (m^\star)^3 \left(37\, tu^2 n^{\star 6} m^\star - 37\, tu^2 (n^\star)^6 + 80\, (m^\star)^3 td\, tu\, (n^\star)^3\right)}{2\left(2\,(m^\star)^3 td + tu\,(n^\star)^3 m^\star - (n^\star)^3 tu\right)\left(8\,(m^\star)^3 td\, tu\,(n^\star)^3 + 54\,(m^\star)^5 td^2 + tu^2 (n^\star)^6 m^\star - tu^2 (n^\star)^6\right)} +$$

$$+\frac{td\, n^\star (m^\star)^3 \left(-78\,(n^\star)^3 td\, tu\,(m^\star)^2 + 156\,(m^\star)^5 td^2\right)}{2\left(2\,(m^\star)^3 td + tu\,(n^\star)^3 m^\star - (n^\star)^3 tu\right)\left(8\,(m^\star)^3 td\, tu\,(n^\star)^3 + 54\,(m^\star)^5 td^2 + tu^2 (n^\star)^6 m^\star - tu^2 (n^\star)^6\right)}$$

and

$$\left|\frac{dWF}{dF_u}\right| = \frac{28\,(m^\star)^4 td\, tu\,(n^\star)^3 + 37\,tu^2 (n^\star)^6 (m^\star)^2 - 50\,tu^2 (n^\star)^6 m^\star}{24(8\,(m^\star)^3 td\, tu\,(n^\star)^3 + 54\,(m^\star)^5 td^2 + tu^2 (n^\star)^6 m^\star - tu^2 (n^\star)^6}) +$$
$$+\frac{164\,(m^\star)^3 td\, tu\,(n^\star)^3 + 13\,tu^2 (n^\star)^6 + 916\,(m^\star)^6 td^2}{24(8\,(m^\star)^3 td\, tu\,(n^\star)^3 + 54\,(m^\star)^5 td^2 + tu^2 (n^\star)^6 m^\star - tu^2 (n^\star)^6}).$$

Inserting $t_d = t_u$ and $n^\star = m^\star$ in the last equations and determining the difference gives

$$\left|\frac{dWF}{dF_d}\right| - \left|\frac{dWF}{dF_u}\right| = -37\,(m^\star)^3 + 429\,(m^\star)^2 - 157 - 555\,m^\star.$$

It remains to show that there is a unique solution with $m^\star \geq 2$ to the equation

$$-37\,(m^\star)^3 + 429\,(m^\star)^2 - 157 - 555\,m^\star = 0, \tag{26}$$

and that the left-hand side of (26) is bigger than zero for all $m < \tilde{m}$. The left-hand side of (26) is a cubic equation in $m^\star$ with a negative leading term. Thus, it is negative as $m^\star$ grows large. On the other hand, it is positive at $m^\star = 2$ while it is negative again at $m^\star$ close to zero. This shows that there is indeed a unique $m^\star > 2$, denoted by $\tilde{m}$, such that (26) is fulfilled, and that for all $m^\star$ with $2 \leq m^\star < \tilde{m}$ the left hand side of (26) is positive.

q.e.d.

Proof of Proposition 5

The proof proceeds along similar lines as the proof of Proposition 4. Here we are interested in the difference

$$\left|\frac{dWF}{dt_d}\right| - \left|\frac{dWF}{dt_u}\right|,$$

where

$$\left|\frac{dWF}{dt_k}\right| = \left|\frac{\partial WF}{\partial t_k}\right| + \left|\frac{\partial WF}{\partial m^\star}\frac{dm^\star}{dt_k}\right| + \left|\frac{\partial WF}{\partial n^\star}\frac{dn^\star}{dt_k}\right|.$$

Substituting $(dm^\star)/(dt_k)$ and $(dn^\star)/(dt_k)$ from the proof of Proposition 2 in the last equation, setting t_d equal to t_u and $n^\star$ equal to $m^\star$, and simplifying yields

$$\left|\frac{dWF}{dt_d}\right| - \left|\frac{dWF}{dt_u}\right| = -37\,(m^\star)^3 + 205(m^\star)^2 - 226m^\star + 468.$$

By the same way as in the proof of Proposition 4 one can show that there exists a unique $m' > 2$ such that $|dWF/dt_d| - |dWF/dt_u| > 0$ if and only if $m < m'$.

q.e.d.

7.3 Appendix C

Proof of Proposition 7

We start with proving $m^\star < m^\star_{tp}$. The proof proceeds by way of contradiction.

First look at the equilibrium under linear prices. Solving the zero-profit condition in the downstream market for $n^\star$ yields

$$n^\star = \sqrt[3]{\frac{12t_d(m^\star)^2}{t_u + 12F_d(m^\star)^2}}.$$

Inserting this into the zero-profit condition on the upstream market gives

$$\frac{t_u\sqrt[3]{12t_d(m^\star)^2(t_u+12F_d(m^\star)^2)^2}}{(m^\star)^2(t_u(7m^\star-6)+12F_d(m^\star)^3)} = F_u. \tag{27}$$

Proceeding in the same way for the case of two-part tariffs yields

$$\frac{t_u\sqrt[3]{12t_d(m^\star_{tp})^2}}{(m^\star_{tp})^3\sqrt[3]{13t_u+12F_d(m^\star_{tp})^2}} = F_u. \tag{28}$$

Now suppose that $m^\star_{tp} = m^\star = m$. In this case, the left-hand sides of the last two equations must be the same since otherwise the profit in the upstream market in the two regimes would be different. Subtracting the left-hand side of (28) from the left-hand side of (27) and simplifying gives

$$(t_u + 12F_d m^2)^2(13t_u + 12F_d m^2)^3 m^3 - (7t_u m - 6t_u + 12F_d m^3)^3(13t_u + 12F_d m^2)^2 =$$

$$= -6t_u(13t_u + 12F_d m^2)^2[t_u^2(55m^3 - 36 + 126m - 147m^2)+$$

$$+t_u m^3 F_d(240m^2 + 216 - 504m) + F_d^2 m^6(144m - 288)] < 0,$$

where the last inequality follows from the fact that $m \geq 2$. Thus, if the number of upstream firms were the same under the two regimes, the profit under two-part tariffs would be higher. Since the upstream profit is decreasing in m, it follows that $m^\star < m^\star_{tp}$.

We now show that $n^\star > n^\star_{tp}$. Suppose to the contrary that $n^\star \leq n^\star_{tp}$. This could only be the case if the downstream profit (ignoring set-up costs) under two-part tariffs were higher than the one under linear pricing, i.e.

$$\frac{t_d}{(n^\star_{tp})^3} - \frac{13t_u}{12(m^\star_{tp})^2} \geq \frac{t_d}{(n^\star)^3} - \frac{t_u}{12(m^\star)^2}.$$

It is obvious that this can only be the case if $m^\star_{tp} \geq \sqrt{13}m^\star$. Now let us check if this can be possible. Inserting $m^\star_{tp} = \sqrt{13}m^\star$ into (28) gives

$$\frac{13^{(5/6)}t_u\sqrt[3]{2028t_d(m^\star)^2}}{2197(m^\star)^3\sqrt[3]{(t_u+12F_d(m^\star)^2)^2}} = F_u.$$

Subtracting the left-hand side of the last equation from the left-hand side of (27), the upstream profit under linear pricing, and simplifying the resulting expression reveals that the sign of this difference is given by the sign of

$$t_u(2197m^\star - 13^{(5/6)} \cdot 169^{(1/3)}(7m^\star - 6)) + F_d(m^\star)^3(26364 - 12 \cdot 13^{(5/6)} \cdot 169^{(1/3)}).$$

Since $m^\star \geq 2$, this expression is positive. But the consequence is that the upstream profit under two-part tariffs in case of $m^\star_{tp} = \sqrt{13}m^\star$ is smaller than the upstream profit under linear pricing with an equilibrium number of firms of $m^\star_{tp}$. It follows that $m^\star_{tp} < \sqrt{13}m^\star$ and so $n^\star > n^\star_{tp}$.

q.e.d.

Proof of Proposition 8

We know that the equilibrium number of downstream and upstream firms under under two-part tariffs are implicitly defined by

$$\frac{t_d}{(n^\star_{tp})^3} - \frac{13t_u}{12(m^\star_{tp})^2} = F_d \tag{29}$$

and

$$\frac{t_u n^\star_{tp}}{(m^\star_{tp})^3} = F_u. \tag{30}$$

Solving (29) for $m^\star_{tp}$ and inserting in (30) yields that $n^\star_{tp}$ is implicitly given by

$$\frac{24\sqrt{39}(t_d - F_d(n^\star_{tpt})^3)^{(3/2)}}{169\sqrt{t_u}(n^\star_{tpt})^{(7/2)}} = F_u. \tag{31}$$

Proceeding in the same way for the linear pricing regime, namely solving (6) for $m^\star$ and inserting in (7) yields that $n^\star$ is implicitly defined by

$$\frac{72t_u t_d(t_d - F_d(n^\star)^3)^{(3/2)}}{(n^\star)^2\Big(6t_u n^\star\sqrt{3t_u n^\star}(t_d - F_d(n^\star)^3) - 36t_u(t_d - F_d(n^\star)^3)^{(3/2)} + t_d\sqrt{3}(t_u n^\star)^{(3/2)}\Big)} = F_u. \tag{32}$$

Now welfare under linear prices is larger than under two-part tariffs if and only if

$$WF_{lp} = a - \frac{t_d(26(m^\star)^3 t_d + t_u(n^\star)^3(37m^\star - 13))}{12(n^\star)^2(2(m^\star)^3 t_d + t_u(n^\star)^3(m^\star - 1))} \geq a - \frac{13t_d}{12(n^\star_{tp})^2} = WF_{tp}. \tag{33}$$

Solving (6) for $m^\star$, plugging the result into (33) and solving for $n^\star_{tp}$ yields

$$n^\star\left(\frac{\sqrt{13}\Big(\sqrt{3}t_d(t_u n^\star)^{(3/2)} + 6t_u(t_d - F_d(n^\star)^3)(n^\star\sqrt{3t_u n^\star} - 6\sqrt{t_d - F_d(n^\star)^3})\Big)}{\sqrt{3}t_d(t_u n^\star)^{(3/2)} + 6t_u(t_d - F_d(n^\star)^3)(37n^\star\sqrt{3t_u n^\star} - 78\sqrt{t_d - F_d(n^\star)^3})}\right)^{(1/2)} \geq n^\star_{tp},$$

which gives us (12).

We can now easily show by means of an example that welfare under linear prices can either be higher or lower than under two-part tariffs. Suppose that $t_d = 1$, $t_u = 5$, $F_d = 0.001$ and $F_u = 0.01$. In this case, from (32) we get $n^\star = 4.971$ and from (31) we get $n^\star_{tp} = 2.8339$. Calculating the right hand side of (12) yields 3.002. Since this is larger than 2.8339, welfare under two-part pricing is lower than under linear tariffs. On the other hand, if $t_d = 1$, $t_u = 1$, $F_d = 0.001$ and $F_u = 0.0005$, we get $n^\star = 9.470$ and $n^\star_{tp} = 7.048$, while the value of the right-hand side of (12) is 6.877 which is smaller than 7.048. Thus, welfare under two-part tariffs is higher than under linear pricing.

q.e.d.

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