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**Employment Effects of a
Sectoral Minimum Wage in Germany**

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Employment effects of a sectoral minimum wage in Germany

Semi-parametric estimations from cross-sectional data

Kai-Uwe Müller*

Abstract

In this paper employment effects of a sectoral minimum wage in the German construction sector are estimated from a single cross-sectional wage distribution using parametric and semi-parametric models. Parametric functional form assumptions seem too restrictive and lead to implausible results. We suggest semi-parametric censored quantile regression models to relax these assumptions and find that employment levels would be 4-5% higher without the minimum wage in the East German construction sector. That the effect for the West is clearly smaller (only 1-2%) is theoretically plausible, since the level of the minimum wage in East Germany was set much higher in relation to the wage distribution. There is heterogeneity hidden in the mean effect: employment losses are mostly borne by young construction workers, employees not covered by collective bargaining agreements and individuals working in small establishments. Although the paper confirms previous findings of a negative employment effect for East Germany, its magnitude is substantially larger than previously estimated.

KEYWORDS: minimum wage, wage distribution, employment effects, labor demand.

JEL classification: J23, J31, J38.

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1 Motivation

The debate about the introduction of a federal minimum wage in Germany has been going on for some time and figures to remain on the political agenda for the foreseeable future (Müller, 2009). The most controversial issue is the likely effect of a statutory minimum on employment. Nearly all of the existing papers are ex ante simulations based on wage and employment data as well as aggregated labor demand elasticities.¹ Those studies provided rather rough estimates of average effects based on restrictive assumptions. The lone ex post analysis for the German construction sector was conducted by König and Möller (2008) and found negative employment effects for East Germany and insignificant estimates for the West. The study provoked a lot of controversy about the validity of the findings since precise information on working hours is missing in the underlying data set.

Empirical studies on minimum wages are commonly based either on time series or panel data. Alternatively, quasi-experimental settings (regional or sectoral variation in minimum wage levels) are utilized (Neumark and Wascher, 2007). In this paper we follow a different approach. The employment effects of the sectoral minimum wage are estimated with structural labor demand models which are based on a single cross-sectional distribution of individual hourly wages. The original labor demand model was derived by Meyer & Wise (Meyer and Wise, 1983a,b). Dickens, Machin & Manning later discussed and extended the approach and applied it to data from the UK (Dickens, Machin, and Manning, 1998). In a nutshell, the basic idea of these models is to parameterize the observable (censored) wage distribution under an existing minimum wage with covariates at hand and certain distributional assumptions. Then this distribution is compared with an estimated counterfactual distribution which is not subject to a minimum wage. Employment effects are simulated from differences between the observed and counterfactual distributions. Critical assumptions concern the functional form of the (underlying) distribution and the selection of a censoring point for the estimation. Since individual wage information on the whole distribution is used, wage and employment effects can be modeled conditional on the distance to the minimum. Moreover, potential spill-over

¹Recent ex ante evaluation studies include Müller (2009), Bauer, Kluve, Schaffner, and Schmidt (2009), Kalina and Weinkopf (2009), Knabe and Schöb (2008), Müller and Steiner (2008), Ragnitz and Thum (2007), Ragnitz and Thum (2008).

effects on wages above the level of the minimum wage can be (at least informally) tested for in the extended model version of Dickens et al.

Currently there is no federal minimum wage in Germany; only certain industries (e.g. the construction sector, postal and cleaning services) are regulated by specific minima. We can therefore estimate the described models with the data at hand only for the construction sector, more precisely for blue-collar workers in the main construction trade, since a sectoral minimum has already been established in 1997. Besides adding to the empirical literature on minimum wages in the German economy, especially to the hotly debated question about employment effects of the minimum wage in the construction sector, the paper contributes to the theoretical and methodological literature on estimating the employment effects of minimum wages from single cross sections. The restrictive functional form assumptions of the original models are relaxed. Other than estimating the models with maximum likelihood we employ a semi-parametric approach and estimate the observed wage distribution with a series of censored quantile regressions that do not rely on symmetry and normality of the residual distribution. We exploit a unique matched employer-employee data set named 'Gehalts- und Lohnstrukturerhebung 2001' which has reliable and precise information about hourly wages and is large enough to conduct semi-parametric estimations on the sectoral level.

We find theoretically consistent patterns of employment effects for the parametric and the semi-parametric models. The sectoral minimum wage led to negative employment effects in East Germany. For the West German main construction trade where the minimum wage hardly bit we find only small negative effects. We also reveal robustness issues of the parametric models: they prove to be sensitive with respect to the choice of a censoring point and yield implausibly large employment effects. We get more reasonable magnitudes with the semi-parametric estimator. Negative employment effects of the minimum wage vary between 4 and 5% in the East German and 1-2% in the West German construction sector. Employment losses are mostly borne by young construction workers, employees not covered by collective bargaining agreements and individuals working in small establishments.

The paper proceeds as follows. After outlining the parametric models of Meyer & Wise and Dickens et al. we show how semi-parametric censored quantile regression estimators can be applied to estimate the underlying structural labor demand model

without strong functional form assumptions. We then discuss in what way the sectoral minimum wage in the German construction sector creates a quasi-experimental situation that is comparable to the studies of reference and enables us to identify the employment effect of the sectoral minimum. Then the data set is introduced, the estimation sample is outlined and the chosen variables are defined. In the empirical section we first present estimates of the parametric Meyer & Wise and Dickens et al. models. The robustness of these findings is tested with regard to the choice of different censoring points. We then calculate the employment effects on the basis of semi-parametric censored quantile regression models and break them down by observable characteristics. By comparing the results for the main construction trade with estimates for other sub-sectors (building installations and other building industries) and groups (white-collar workers) where the minimum wage was not binding we discuss the plausibility of the findings and potential substitution effects. The last section summarizes the findings and concludes.

2 Theoretical and econometric framework

This section first outlines the parametric models of Meyer & Wise and Dickens et al. After that we show how the functional form assumptions of those parametric estimators can be relaxed with semi-parametric censored quantile regression models. Finally we discuss the application for the German construction sector.

2.1 The Meyer & Wise approach

Meyer and Wise (1983a,b) developed an approach to estimate wage and employment effects of the minimum wage from individual cross-sectional data when a federal minimum is already in existence. The one-equation version of their model starts from an ‘underlying wage distribution’ without a minimum wage which could be written as a latent variable w_i^* : $f(w_i^*)$. For a given minimum wage M , Meyer & Wise assume that because of non-coverage and non-compliance some workers with underlying wages $w_i^* < M$ remain employed at wages $w_i < M$ with probability P_1 . Moreover, they assume that a fraction of persons with $w_i^* < M$ are now paid at $w_i = M$ with probability P_2 . Therefore the probability of people with $w_i^* < M$ to

Fig. 1: Underlying and observed wage distribution

Sources: own illustration

be without work after the introduction of a minimum wage is $1 - P_1 - P_2 = 1 - P$ with $P = P_1 + P_2$. Probabilities P_1 and P_2 are constant for all $w_i^* < M$, i.e. they do not depend on the individual wage. Note that the model assumes that there is no unemployment without the minimum wage. Meyer & Wise explicitly rule out spill-over effects of the minimum on individuals with $w_i^* \geq M$. The underlying (latent) distribution is specified as follows:

$$w_i^* = X_i\beta + \epsilon_i \quad \epsilon_i \sim N(0, \sigma^2) \quad (1)$$

where X_i is a matrix containing individual and regional attributes and ϵ_i is a normally distributed error term with variance σ^2 . The underlying distribution $f(w)$ and the observed wage distribution $f_1(w)$ are exemplarily displayed in Fig. 1 for hourly wages between zero and 20€/hour with the minimum wage being fixed at 7.50€/hour. The solid line marks the underlying, the dashed line the observed wage distribution. In this illustration some individuals earn wages below the minimum wage (non-coverage or non-compliance). Several workers with an underlying wage below M get paid exactly the minimum wage which induces the spike in the wage distribution. There are no spill-over effects in the distribution above the minimum wage level M .

For $f_1(w)$ being the likelihood of observed wage rates, w^* or, e.g., $\log w^*$ normally distributed, and Φ the standardized normal distribution, Meyer & Wise write the

likelihood of observed hourly wages w as:

$$f_1(w) = \begin{cases} \frac{f(w) \cdot P_1}{D} & \text{if } w_i < M \\ \frac{\Phi[(M - X_i\beta)/\sigma] \cdot P_2}{D} & \text{if } w_i = M \\ \frac{f(w)}{D} & \text{if } w_i > M \end{cases} \quad (2)$$

where $D = 1 - Pr[w_i^* < M](1 - P_1 - P_2) = 1 - \Phi[M - X_i\beta/\sigma] \cdot (1 - P)$ which is the probability that an individual who is employed without the minimum is also employed after its introduction. The distribution $f_1(w)$ is the conditional distribution of observed hourly wages in terms of the underlying distribution - given that wages are observed. The first part of the likelihood with $w_i < M$ is observed with probability P_1 times the likelihood for $w_i^* = w_i$. The second part of the likelihood for observed wages $w_i = M$ is given by probability P_2 times the likelihood that $w_i^* < M$ is raised by the minimum to $w_i = M$. The third part refers to observed wages above the minimum and is equal to the underlying distribution except for the fact that the share of people with $w_i > M$ might be higher than the share with $w_i^* > M$ which is expressed in the denominator. Meyer & Wise use an interval around M as in their data the pile-up of hourly wages varies around the nominal minimum due to measurement error and potential spill-over effects.

Note that this specification is quite similar to a standard Tobit model with censoring at M . In addition to common censored data there is also the case where wages below the ‘censoring point’ are observed which is mirrored in the first term of the likelihood as well as the denominator of all terms in the likelihood function. For N persons with observed wage rates, among them N_1 with hourly wages below, N_2 at, and N_3 above M the full log-likelihood is given as follows:

$$\begin{aligned} \log L &= \sum_{i=1}^{N_1} \ln f_1(w_i) + \sum_{i=1}^{N_2} \ln f_1(w_i) + \sum_{i=1}^{N_3} \ln f_1(w_i) \\ &= \sum_{i=1}^{N_1} \frac{f(w) \cdot (P_1)}{D_i} + \sum_{i=1}^{N_2} \frac{\Phi[(M - X_i\beta)/\sigma] \cdot (P_2)}{D_i} + \sum_{i=1}^{N_3} \frac{f(w)}{D_i} \end{aligned} \quad (3)$$

The parameters in β as well as P_1 and P_2 are estimated by maximizing (3) for the sample of observed people in employment. The employment effects are calculated by way of simulation. Intuitively, the number of employed people below M without a minimum wage which is predicted on the basis of the underlying distribution is compared with the number of observed people with $w_i < M$. To be more precise, the employment effects of the minimum wage are simulated using the estimated parameters for (3). Remember that conditional on X_i D_i is the individual’s probability to

be still employed under the minimum given that he or she would be in employment under a minimum wage at $w_i < M$. Conversely the inverse $1/D_i$ is the expectation that a person would be in employment without a minimum wage.² For a sample of N persons the total expected number of employed people without the minimum amounts to

$$T = \sum_{i=1}^N \frac{1}{D_i} \quad (4)$$

The percent increase in employment is therefore $(T - N)/N$. Meyer & Wise focus their analysis on youth employment and find that about 30%-50% of the youths that would normally be employed in the absence of a minimum wage are without work because of the minimum. This figure represents 7% of all young men. They acknowledge that their estimation does depend upon a number of assumptions they have to make about the censoring point or the distribution of the error term, but claim that robustness tests show that their results are not overly sensitive to these assumptions. These critical points were carried forward by Dickens, Machin, and Manning (1998). We will discuss their model in the following sub-section.

2.2 Critique by Dickens, Machin, and Manning

Dickens, Machin, and Manning (1998) apply the Meyer & Wise approach to UK data to test its robustness with regard to the selection of the censoring point as well as the functional form assumption. They start from a simple version of the Meyer & Wise model with $P_1 = 0$ which means that people are not employed at $w_i < M$ with a minimum wage being in effect. Therefore only the probability $P_2 = P$ of remaining employed at $w_i = M$ under the minimum wage is part of the model. In order to point out the critical assumptions, Dickens et al. start from the following reformulation of the Meyer & Wise model: In the absence of a minimum wage employment L_0 is reached with the distribution of wages given by $f(w; \theta)$ with θ being a set of parameters to be estimated. When a minimum is introduced the density function changes to $f_1(w; \theta)$ which leads to employment L_1 .

While f_1 can be estimated from observed wages, one has to assume that there is a wage w_1 above which wages are not affected by the minimum in order to infer on the underlying distribution f and L_0 . Dickens et al. point out that Meyer & Wise

²A brief derivation for this relationship is given in the Appendix.

assume w_1 to be very close to the minimum wage. They show that the choice of w_1 will be crucial for the estimated employment effect if spill-over effects are present. Under the assumptions made the distribution of observed wages and the underlying wage distribution are related as follows:

$$\begin{aligned} f_1(w; \theta) &= \frac{L_0}{L_1} f(w; \theta) \\ &= \gamma f(w; \theta) \quad \text{for } w > w_1 \end{aligned} \quad (5)$$

The ratio γ of employment without and with the minimum serves as a measure of the employment effect. Equation (5) states that for wages above the censoring point w_1 the observed and the underlying distribution are equal up to the scaling factor γ . This holds because of the assumption that wages are not affected by the minimum above w_1 .³ Since they assume that employment above w_1 remains constant under the minimum it holds that

$$\begin{aligned} L_1(1 - F_1(w_1; \theta)) &= L_0(1 - F(w_1; \theta)) \\ F_1(w_1; \theta) &= 1 - \gamma(1 - F(w_1; \theta)) \end{aligned} \quad (6)$$

Specifying a tobit model for the wage equation with the censoring point at w_1 and plugging in (5) and (6) the log-likelihood becomes:

$$\begin{aligned} \log L &= \sum_{i=1}^j \log f_1(w_i; \theta) + (L_1 - j) \cdot \log F_1(w_1; \theta) \\ &= \sum_{i=1}^j \log f(w_i; \theta) + j \cdot \log \gamma + (L_1 - j) \cdot \log[1 - \gamma \cdot (1 - F(w_1; \theta))] \end{aligned} \quad (7)$$

Note that this tobit model is estimated only on those people who are observed to be employed with $N = L_1$ as the total number of observations. Moreover, j denotes the number of persons with $w_i \geq w_1$ and $L_1 - j$ comprises those who are below the truncation point. As in the Meyer & Wise model there is no unemployment without a minimum wage. Parameters γ and θ are estimated by maximizing (7) which yields the following Maximum Likelihood estimator of γ :

$$\gamma_{MLE} = \frac{j}{L_1 \cdot [1 - F(w_1; \theta)]} \quad (8)$$

The intuitive interpretation is that employment will decrease (increase) under the minimum wage if the observed fraction of workers below w_1 is smaller (larger) than it is predicted on the basis of the distribution of those paid above w_1 . Inserting

³Explanations of the assumptions and the derivation of the concentrated likelihood is given in the Appendix.

Fig. 2: Choosing different censoring points

Sources: own illustration

this estimator in (7) yields the concentrated likelihood which is equal to a likelihood from a sample of workers with observations truncated at w_1 :

$$\log L = \sum_{i=1}^j \log f(w_i; \theta) - j \cdot \log[(1 - F(w_1; \theta))] + \text{constant} \quad (9)$$

Having estimated θ from the truncated regression model in (9) γ can be obtained from (8). Differently from the Meyer & Wise model the truncated regression can in principle be estimated for many different truncation points w_1 . Dickens et al. experiment with two different functional forms for $F(w_1; \theta)$: first, they assume a log-normal wage distribution as have Meyer & Wise. Second, they specify the Singh-Maddala distribution ($F(w_1; \theta) = 1 - [1 + w_1/\theta_1]^{\theta_2}$) ^{θ_3} with $\theta_1, \theta_2, \theta_3 > 0$).

Dickens et al. apply this model to UK Wage Council data between 1987-90 for the retail and wholesale sector and estimate it separately for men and women. They show that estimates of the employment effect are sensitive with respect to two critical assumptions. First, choosing different *censoring points* (at the 10th, 20th, 30th and 40th decile) yields vastly different results. It is obvious that setting w_1 too high results in inefficient estimates of γ whereas setting it too low may yield inconsistent estimates. The latter might happen if the minimum affected higher parts of the wage distribution above the chosen censoring point ('spillover effects'). This clearly violates the assumption of (6) as is demonstrated in Fig. 2. Again the solid line marks the underlying and the dashed line the observed wage distribution. If the first censoring point at 7.50€/hour was chosen, estimates would be inconsistent as

the observed distribution is influenced by the spill-over effects of the minimum. The second cut-off point at 9.50€/hour does not suffer from this problem. Dickens et al. thus emphasize that the Meyer & Wise model could yield inconsistent estimates when spill-over effects of the minimum wage to higher parts of the wage distribution occur. Since Meyer & Wise only consider w_1 values close to the minimum, spill-over effects are rather likely.⁴ We will test the robustness of our results specifying a range of different censoring points.

Second, Dickens et al. also show that with their data the choice of the *functional form* is crucial for the estimated employment effects. Intuitively, as soon as one assumes a symmetric distribution and then infers from the right part of a left-truncated observed wage distribution (e.g. the log-normal distribution) to an underlying distribution, estimates will become inconsistent if the underlying distribution is indeed asymmetric. In this instance results are driven by the non-truncated part of the distribution which might be fundamentally different from the truncated part which occurs regularly with income data. Dickens et al. reject the symmetry assumption for their data and find markedly different results for the asymmetric Singh-Maddala compared to the symmetric log-normal distribution. Therefore we specify semi-parametric models that relax the functional form assumptions.

The main result of Dickens et al. is the potential sensitivity of the Meyer & Wise approach to its critical assumptions. Dickens et al.’s findings markedly differ from those of Meyer & Wise with their own employment effects being implausibly large for certain specifications. We will subsequently address both critical points. In the following section we relax the functional form assumption by specifying semi-parametric models for a truncated wage distribution. In section 2.4 we discuss the selection of the censoring point for the German case.

2.3 Semi-parametric estimators

The major criticism put forward by Dickens et al. which has already been addressed by Meyer & Wise refers to the functional form assumptions (e.g. $\epsilon \sim N(0, \sigma)$) needed to infer from the observed on the underlying wage distribution and thereby simulat-

⁴One other difference between the Meyer & Wise approach and Dickens et al. is that the former estimate the probability P to remain employed after the minimum whereas the latter specify the employment ratio $\gamma = L_0/L_1$. Dickens et al. discuss how both measures are related ($P = (\gamma^{-1} - 1) \cdot F(W_1; \theta)^{-1}$) and what the advantage is to estimate γ rather than P .

ing the employment effects of the minimum wage. If parametric assumptions are not met by the data, the maximum likelihood estimator is inconsistent which means that the underlying distribution is not correctly estimated. Meyer and Wise (1983a) use a Box-Cox transformation whereas Dickens, Machin, and Manning (1998) reject the (log-)normality assumption on statistical grounds and specify an alternative model on the basis of the Singh-Maddala distribution.

We will take another route here by specifying semi-parametric models for censored (or truncated) distributions that relax the functional form assumptions. A number of those models have been suggested over the last few years (Chay and Powell, 2001). The main idea of these models is to parameterize the regression function as usual without putting any parametric restrictions on the error term. In the following we focus on the *censored quantile regression* (CQR) which is described extensively by Buchinsky (1994). Our estimation framework for the employment effects remains similar to the parametric version. Employment effects are simulated based on the comparison of the observed with the estimated underlying wage distribution. We start again from a latent underlying distribution for w_i^* as in equation (1) above which is now modeled specifically for different quantiles τ :

$$w_i^{\star(\tau)} = X_i\beta^{(\tau)} + \epsilon_i^{(\tau)} \quad (10)$$

The first step is thus to estimate a series of censored quantile regressions for a number of quantiles of this distribution. Then the conditional (underlying) distribution is estimated for different quantiles using the parameters from the first step regressions and the employment effects are simulated by comparing the underlying with the observed distribution.

The idea of the quantile regression model is to model the τ -th sample quantile $Q^{(\tau)}$ of the distribution of wages w conditional on a set of X -variables.⁵ A quantile is defined as the inverse of the cumulative distribution function at τ : $Q^{(\tau)} = F^{-1}(\tau)$. Since the distribution of w is censored in our application we have to use CQR models where it is of relevance if the τ -th quantile lies below or above the censoring point. As long as the quantile is in the uncensored region of the distribution (less than a share of τ observations are below the cut-off point in case of left-censored data as in our case) it is unaffected by the censoring. If the quantile lies in the censored

⁵ $Q^{(\tau)}$ means a share of τ observations are smaller than Q with $0 \leq \tau \leq 1$.

region, it is equal to the censoring point (see Fig. 3). $Q^{(\tau)}(X_i, \beta_\tau) = \max[w_1, X_i\beta_\tau]$ then denotes the conditional quantile function of the observed wages w_i censored at w_1 and depends on the regressors X_i and the parameter vector β_τ . The CQR model implies the following assumptions:

$$\begin{aligned} Q^{(\tau)}(\epsilon_i^{(\tau)}|X_i) &= 0 \\ Q^{(\tau)}(w_i^*|X_i) &= X_i\beta_\tau \end{aligned} \tag{11}$$

It is assumed that the conditional quantile of the error term is zero. In that case the conditional quantile of the true underlying variable w_i^* would be equal to $X_i\beta_\tau$. The CQR neither requires additional distributional assumptions about $\epsilon_i^{(\tau)}$ nor homoscedasticity (since β_τ are allowed to vary with τ). The CQR estimator thus handles nonnormal, heteroscedastic and asymmetric errors which is important for the analysis of empirical wage distributions. The parameter vector β_τ is estimated by minimizing the weighted sum of the absolute deviations of w_i from $\max[w_1, X_i\beta_\tau]$ over all β_τ in the following objective function:

$$Q^{(\tau)}(\beta_\tau) = \sum_{w_i > X\beta_\tau} \tau |w_i - \max[w_1, X_i\beta_\tau]| + \sum_{w_i < X\beta_\tau} (1 - \tau) |w_i - \max[w_1, X\beta_\tau]| \tag{12}$$

There is no closed-form solution to this problem. The model is estimated by solving the linear programming representation of the maximization problem. For the QCR Buchinsky suggested the Iterative Linear Programming Algorithm (ILPA) which alternates between two steps (Buchinsky, 1994): In the first step, the model applies the least absolute deviations estimator to all observations. In the second step, the data set is re-censored by excluding all those values for which the CQR-estimated values are smaller than the censoring point. Then, step one is repeated with the re-censored data. The steps are repeated until convergence is achieved and further re-censoring is no longer needed. The final CQR-estimation thus does not use all observations.⁶ Alternative semi-parametric approaches would be censored least squares estimators, namely the symmetrically censored least squares (SCLS) or the identically censored least squares (ICLS) estimator (Powell, 1986; Honore and Powell, 1994).⁷

⁶If a certain number of observations is censored and the respective quantile lies in the censored region, the CQR-estimator will not converge. In this case we have to approximate this quantile by a higher quantile where convergence is achieved.

⁷SCLS is based on symmetric trimming combined with standard OLS analysis. ICLS does not rely on symmetry; the wage distribution is re-censored for pairs of observations such that the densities have the same shape for each pair. Both approaches iterate between the estimation of the regression function and a re-censoring step until convergence is achieved.

Fig. 3: Density of $w^* = X\beta + \epsilon_i$ for $X\beta > w_1$ and $X\beta < w_1$

Sources: own illustration.

We estimate a series of CQR models to calculate the underlying distribution conditional on the observed explanatory variables in a very flexible form at different quantiles. Similar procedures that estimate conditional distributions are, e.g., discussed in the literature on decomposition of distributions to analyze sources of wage inequality (Gosling, Machin, and Meghir, 2000; Melly, 2005, 2006) or the estimation of unconditional distributions and quantile treatment effects (Firpo, Fortin, and Lemieux, 2009; Firpo, 2007). In fact most of those papers also estimate CQR models since wage or income distributions are censored, mostly at the top. In our application the censoring is from below and the censored part of the distribution is approximated by the lowest estimable quantile. In this version we estimate the model at the following quantiles: 0.02 (0.02) 0.3; from there on we use a coarser grid: 0.3 (0.1) 0.9. We use Buchinsky's ILPA implemented in Stata (Jolliffe, Krushelnysky, and Semykina, 2000) in a slightly modified version. Having estimated the underlying distribution the employment effects are simulated analogous to equation (4) or equation (8). We compare the probability mass below the cut-off point with its counterpart under the density of the observed wages. The simplest way to do this is to compare the number of predicted observations below the chosen cut-off point with the number of observed observations:

$$\Delta E = (\hat{N}_{below} - N_{below})/L_1 \quad (13)$$

This difference standardized by the observed employment level is the percentage change of employment that would result if the minimum wage was not in effect.

2.4 Application for the German construction sector

The Meyer & Wise approach like the modified version suggested by Dickens et al. cannot be applied straightforwardly to the German economy, because no federal minimum wage currently exists. Over the last ten plus years sectoral minimum wages have been implemented in several industries (e.g. the construction sector and, more recently, also in postal and cleaning services). In the construction sector a sectoral minimum wage was introduced in 1997 and since then amended repeatedly (Rattenhuber, 2010). The legislation covered only blue-collar workers (so-called 'gewerbliche Arbeitnehmer') in large parts of the main construction trade ('Bauhauptgewerbe'). Minimum wage levels were set differently for West (9.80€) and East Germany (8.63€). This situation is an ideal test case for the above-mentioned labor demand models for the following reasons:

1. The sectoral minimum wage created a situation like in the studies of reference mentioned above. We have only a single cross-section of wage data. The minimum is binding for a sizeable proportion of the individuals observed, but not all of them. Therefore we can estimate the parametric models with the German data and see if their findings can be replicated for the sectoral minimum wage in the German construction sector.
2. Although the fixed minimum wage levels differed between the East and West, we show that the minimum was more relevant in East compared to West Germany. According to theory the effects should hence be much more pronounced in the East. We can test this hypothesis by estimating separate models for East and West Germany with the West serving as a quasi-control group.
3. The sectoral minimum wage was mainly implemented for the greater part of the main construction trade ('Bauhauptgewerbe') and only for blue-collar workers. Barring substitution or complementaries employment effects of the minimum should not be detectable for white-collar workers or in other branches of the construction sector. We will estimate similar models for those sub-samples to test the robustness of our findings and discuss potential substitution effects.
4. So far employment effects in the German construction sector have been tough to analyze with standard ex post evaluation methods and existing data. The

above-mentioned study by König and Möller (2008) is one exception. We contribute to this policy question and will relate our results to previous findings.

We come back to those points in the discussion of the results. All findings should be taken with a grain of salt for the following reasons. First, we conduct a partial analysis of employment effects in the construction sector which is different from other studies where the minimum wage covered not only one sector but the whole economy. The labor demand models utilized here cannot explicitly analyze substitutional or complementary employment effects with other sectors and between covered blue and non-covered white-collar workers within the main construction trade. We will use separate estimations for non-covered sub-groups of construction workers to discuss such effects, though. Second, capital-labor-substitution may to occur to some degree in the construction sector as the price of labor increases under the sectoral minimum wage which we do not estimate here. Third, we do not explicitly consider the output price elasticity for the construction sector. Increasing the price of one production factor likely reduces the demand for construction tasks to some degree. All those points suggest that we rather underestimate the employment effects of the sectoral minimum wage.

3 Data, sample, variables

The empirical analysis is based on data from the 'Gehalts- und Lohnstrukturerhebung 2001' (Salary and Wage Survey, GLS). In this version of the paper we use data from the scientific use file for the year 2001. The GLS is a linked employer-employee data set provided by the German Federal Statistical Office (Hafner, 2006). The 2001 wave does not include employees in firms with less than 10 employees and several sectors of the economy (e.g. agriculture, public services, health care and social services).⁸ The large sample size (about 1 million observations in total) enables precise estimations for sub-groups of employees. This is indispensable especially for the semi-parametric estimators and for sub-samples like the German construction sector. Another great advantage of the GLS data is that the hourly

⁸We will supplement the analysis with data from the latest wave of 2006 as soon as the scientific use file will become available. The latest version of this dataset goes under the name 'Verdienststrukturerhebung 2006' (VSE) (Bundesamt, 2009).

wage measures are more reliable than in household surveys like, e.g., the German Socioeconomic Panel (SOEP), since the information comes directly from the firm and is based on the employment contract. Measurement errors due to incomplete memory of the respondent, discrepancies between reported working hours and wage income are therefore less of a problem. On the other hand several drawbacks of the data have to be acknowledged. Firms with less than or equal to 10 employees and certain sectors (agriculture, the public sector and household services) are not included in the sample. Both gaps lead to a systematic under-representation of certain individuals. Marginally employed, e.g., work more often in small firms (Müller, 2009). Furthermore, it lacks information on the household context (family status, children, etc.). That the GLS is not a panel data set is not important here.

The *sample* is restricted to the to main construction trade ('Bauhauptgewerbe') of the German construction sector in 2001 where a sectoral minimum wage was in place that was binding for a sizeable proportion of workers. In order to get a more homogeneous sample estimations are further constrained to male blue-collar workers and employees who are not in vocational training since the minimum only covers blue-collar workers and males clearly dominate this industry. Note that in this sample there is non-coverage (some sectors in the main construction trade were exempted) and there might also be some non-compliance which does not affect the results. In 2001 coverage amounted to about 39% in East and 35% of all workers in the West German construction sector. Rattenhuber (2010) provides detailed information on the minimum wage for the German construction sector. We will show below that for the majority of employees in the West this minimum was not binding. All models are therefore estimated separately for the West and East.

The *hourly wage measure* is based on reported gross income from work in the month of the survey. Any payments for additional (overtime) work in the observed month are subtracted from this amount. Hourly wages are calculated by dividing this number by reported monthly working hours also diminished by overtime if applicable. Wages used in the analysis thus refer to regular payments and actual working hours as opposed to contractual wages and hours. Given the reliability of the GLS data we are confident that this gives a precise wage measure which can be related to the legal minimum wage levels.

The selection of explanatory variables is constrained by the GLS data set. The

specification of our models is theoretically motivated by the standard Mincerian wage equation which explains earned wages on the basis of human capital (Mincer, 1974). We therefore include polynomials for *age* and the *level of education* which should approximate human capital accumulation. We also distinguish different types of *employment contracts* (full-time, part-time, and marginal employment). Furthermore, as the literature on internal labor markets suggests, additional years of *tenure* in a firm lead to an increase in wages (Abraham and Medoff, 1981). As we have this information in the data we include tenure in our wage regressions. We have no information on the entire labor market career of the individuals, though, and cannot account for the potential depreciation of human capital over past periods of unemployment or inactivity. In addition to observable individual and job characteristics some factors on the labor demand side are also important for the wage. We therefore add firm attributes to our model. To be more precise, those characteristics are measured at the establishment level. We include dummy variables for the *establishment size* and the *industry* where the individual works as different forms pay different wages for equally skilled and productive people. We also have information on the type of *collective bargaining* agreement (sectoral, firm, or no agreement) which varies widely between East and West Germany. Lastly we control for the influence of the *public sector* in the firm.

The descriptive statistics of the log wage and all explanatory variables used are reported in Tab. 1. They reveal first the differences in the average wage level between West and East Germany. Second, an important institutional discrepancy which is crucial for the bargained wages as well as the agreed minimum wage levels concerns the degree of unionization (Rattenhuber, 2010). In West Germany almost 80% of all individuals in the sample work under a collective bargaining agreement (CBA) whereas this share is only about half that size in the East. Since firm CBAs did not play a significant role in the German construction sector at that time the majority of East German workers is not directly covered by any CBA.

With respect to other individual characteristics construction workers are slightly older in the West compared to the East. Their average tenure in the job is 35 weeks longer and the share of people with a higher school degree is slightly higher. Concerning firm characteristics the public sector has a slightly larger influence in East German firms whereas establishment sizes are rather similar between the West

Tab. 1: Descriptive statistics: wages & explanatory variables

	East Germany		West Germany	
	mean	[s.d.]	mean	[s.d.]
Log wage	2.3338	[0.1722]	2.6405	[0.1755]
Age	39.1049	[9.5005]	40.5143	[10.7692]
Tenure	81.5588	[95.0811]	115.2525	[112.7403]
Dummy 'Abitur'	0.0036	[0.0600]	0.0044	[0.0665]
Dummy no CBA	0.6188	[0.4858]	0.2008	[0.4006]
Dummy firm CBA	0.0233	[0.1509]	0.0110	[0.1044]
Dummy sector CBA	0.3579	[0.4795]	0.7882	[0.4086]
Dummy no public	0.9509	[0.2161]	0.9746	[0.1574]
Dummy limited public	0.0236	[0.1518]	0.0148	[0.1207]
Dummy high public	0.0255	[0.1577]	0.0106	[0.1026]
Dummy establishment size 10 – 20	0.1615	[0.3680]	0.1553	[0.3622]
Dummy establishment size 20 – 50	0.2209	[0.4149]	0.2514	[0.4338]
Dummy establishment size 50 – 100	0.2630	[0.4403]	0.2114	[0.4083]
Dummy establishment size 100 – 250	0.2372	[0.4254]	0.2500	[0.4330]
Dummy establishment size 250 – 500	0.0824	[0.2750]	0.0835	[0.2767]
Dummy establishment size > 500	0.0350	[0.1837]	0.0484	[0.2147]
Observations	3,604		10,343	

Source: GLS; own calculations.

and East German construction firms. The sample comprises about 3,600 East German construction workers whereas the sample size for the West is more than 10,300 employees.

4 Results

All wage regressions are estimated on log hourly wages to reduce the asymmetry in the distributions. We do not report the results for the regression coefficients of the explanatory variables in the paper.⁹ Except for space restrictions parameter estimates for the explanatory variables are not the focus of our analysis, since we are mainly interested in the (underlying) conditional distribution. Direction and size of the coefficients are in line with theoretical expectations.¹⁰ First, we present descriptive graphical evidence for East and West Germany. Second, we discuss the parametric estimates from the Models of Meyer & Wise as well as Dickens et al. Third, we present semi-parametric censored quantile regression results and relate

⁹Quite a large number of different models for sub-groups and several censoring points was estimated, especially for the censored quantile regressions. Complete estimation results are available from the author upon request.

¹⁰See Tab. 7 and 8 in the appendix for exemplary regression results.

them to the previous evidence. Finally, employment effects are differentiated by individual and establishment characteristics, robustness checks are carried out and the results are related to previous findings.

4.1 Descriptive evidence

After the minimum in the German construction sector was introduced in 1997 it was amended several times. We apply data gathered in September 2001 and use therefore minimum wage levels set in September 2001 at 8.63 €/hour in East and 9.80 €/hour in West Germany. The histograms of Fig. 4 show the empirical distributions of log hourly wages for the sample of the construction workers in 2001 separately for East (left panel) and West Germany (right panel). The same graphs for log hourly wages are reported in Fig. 5 in the Appendix. The log-transformation which is used in all subsequent estimations enhances symmetry and reduces the spread in the distributions. The respective minimum wage levels are also included in every chart. For both regions the expected pattern for the wage distribution under a minimum wage arises (compare with Fig. 1 above). A clear spike of wages at the legal minimum wage level is visible, although much more pronounced in East Germany. One also observes hourly wages below the defined minimum indicating non-coverage (and potentially some non-compliance). There is slight descriptive evidence for some spill-over effects directly above the minimum wage; this cannot be tested formally, though.

Fig. 4: Hourly wages, main construction trade, East & West Germany

Note: Normal distribution in graphs for comparison. Sources: GLS 2001; own calculations.

The main result of the descriptive analysis is that the minimum wage in the

construction sector was much more binding in East compared to West Germany since the defined nominal minimum wage level is clearly closer to the median of the wage distribution. This is important for the interpretation of the model estimates. The chosen minimum wage level for East Germany was economically more relevant for East German firms. According to this descriptive evidence we would expect negative employment effects for the East German construction sector whereas the results should be much less clear for West Germany. Therefore the West may indeed serve as a control group. The differential effects should be mirrored in the estimates of the employment effects that are carried out separately for the East and the West.

4.2 Parametric estimates

We will first present estimation results for the two parametric models of Meyer & Wise and Dickens et al. (see Tab. 2). As mentioned above the models are estimated separately for East and West Germany. The figures in Tab. 2 refer to a percentage change in employment that would result if there was no minimum wage at all, i.e. positive values indicate negative employment effects of the minimum and vice versa. Bootstrapped 95-percent-confidence-bands are reported in parentheses. We also analyze the sensitivity of the models with respect to the choice of different censoring points by estimating all models for the cut-off points given in the first column. The dashed horizontal lines in the table mark the minima set for the East and West; they separate cut-off points we chose below and above the legally set minimum wage levels.

The findings for *East Germany* are consistent with our theoretical expectations. We estimate negative employment effects for the East German construction sector in 2001 in the interpretable range of cut-off points for both parametric models. Yet those findings are sensitive to different model assumptions as well as the selection of the cut-off points. The interpretation of results crucially depends on those assumptions. Starting with the Meyer & Wise model it has to be made clear that estimates are only interpretable around the set minimum wage level of 8.63€/hour, since the spike at the minimum (see Fig. 4 above) is explicitly modeled. We use an interval of 0.20€/hour above the stated censoring point; therefore only the reported cut-off points 8.50€/hour and 8.60€/hour include the observed spike. We find negative

Tab. 2: Employment change without minimum wage in %: parametric models

Cut-off (€/h)	East Germany			
	Meyer & Wise		Dickens et al.	
8.5	15.21	[7.41 ; 23.02]	28.90	[28.64 ; 29.15]
8.6	9.49	[3.70 ; 15.28]	36.30	[36.03 ; 36.57]
8.7	-0.65	[-1.22 ; -0.08]	19.21	[18.62 ; 19.79]
8.8	-1.39	[-1.80 ; -0.98]	20.82	[20.05 ; 21.57]
8.9	-1.80	[-2.23 ; -1.36]	22.27	[21.26 ; 23.26]
9.0	-2.10	[-2.59 ; -1.60]	20.95	[19.48 ; 22.36]
9.1	-2.52	[-3.13 ; -1.91]	23.71	[21.74 ; 25.58]
9.2	-3.20	[-3.87 ; -2.53]	27.41	[24.74 ; 29.91]
9.3	-3.36	[-4.15 ; -2.57]	20.03	[14.83 ; 24.64]
9.4	-4.35	[-5.23 ; -3.46]	25.98	[18.16 ; 32.43]
9.5	-5.59	[-6.58 ; -4.59]	29.71	[22.29 ; 35.84]
Cut-off (€/h)	West Germany			
	Meyer & Wise		Dickens et al.	
9.5	0.03	[-0.01 ; 0.07]	-1.68	[-1.72 ; -1.63]
9.6	0.03	[0.00 ; 0.06]	-1.72	[-1.77 ; -1.67]
9.7	0.02	[-0.01 ; 0.05]	-1.83	[-1.88 ; -1.78]
9.8	0.01	[-0.01 ; 0.04]	-2.19	[-2.23 ; -2.14]
9.9	0.02	[-0.01 ; 0.06]	-2.64	[-2.69 ; -2.59]
10.0	0.02	[-0.02 ; 0.06]	-2.97	[-3.02 ; -2.92]
10.1	0.02	[-0.02 ; 0.06]	-3.03	[-3.08 ; -2.98]
10.2	0.03	[-0.02 ; 0.07]	-3.30	[-3.35 ; -3.25]
10.3	0.02	[-0.02 ; 0.07]	-3.65	[-3.70 ; -3.60]
10.4	0.03	[-0.03 ; 0.08]	-3.84	[-3.89 ; -3.79]
10.5	0.05	[-0.02 ; 0.11]	-4.15	[-4.20 ; -4.10]

Notes: All models estimated for varying censoring points according to 1st column. Bootstrapped 95%-confidence bands in parentheses.

Source: GLS; own calculations.

employment effects for those two cut-offs. According to these estimates employment would be 10-15% higher without the minimum wage which is a rather large effect. All other estimates should return inconsistent estimates since the theoretical spike is specified above the observed spike in the distribution. Those estimates are actually negative indicating theoretically implausible positive employment effects. This shows that the Meyer & Wise approach can only be estimated with a censoring point near the set minimum wage level which mirrors the critique of Dickens et al. Meyer & Wise's model overly hinges on a narrow region of censoring points which in addition makes this model potentially vulnerable to spill-over effects.

Therefore Dickens et al. constructed their model such that the sensitivity with

respect to the choice of different censoring points is indirectly testable. Note that, since opposite to Meyer & Wise their model is based on a truncated regression, only those estimates based on cut-off points above the legal minimum wage level (8.70 €/hour in our application for the East) are consistent. Moreover, the robustness of findings for different cut-off points serves as an informal test for the influence of spill-over effects. We find negative employment effects for the interpretable range of estimates. Employment would be about 20% higher without a sectoral minimum wage which is even higher than for the Meyer & Wise model and hardly convincing. The estimates are of similar size between cut-off points of 8.7 €/hour-9.0 €/hour. For censoring points further above the distribution (where decreasingly less information of the observed distribution is used to estimate the underlying distribution) the effects become even larger. It is noteworthy that Dickens, Machin, and Manning (1998) report even higher estimates for the employment losses in their paper.

The parametric results for *West Germany* also mostly fit our hypotheses as we find no or only very small positive employment effects of the sectoral minimum wage in the construction sector. For the Meyer & Wise model the estimates are zero which would confirm the hypothesis that the minimum was hardly binding in the West and therefore should have only minor implications for employment. The Dickens et al. model yields even slight positive effects. This is not very plausible and rather hints to slightly inconsistent estimates for the West.

Overall the findings of the parametric models replicate the result patterns of the studies of reference and are qualitatively consistent with our theoretical expectations. We do find negative employment effects for the East German construction sector whereas estimates tend to zero for the West. On the other hand the problems of the parametric approaches become obvious. We could show that the models are sensitive with respect to the choice of a cut-off point. Moreover, the size of the employment effects raise the suspicion that the parametric assumptions (i.e. normality of error terms) are too restrictive and lead to inconsistent estimates. It is theoretically rather inconceivable that the still moderate sectoral minimum wage would lead to employment losses of 10-20% in the short term. It seems that the Dickens et al. model is more vulnerable with respect to violations of these assumptions as it relies on a smaller part of the observed distribution compared to Meyer & Wise's approach. We therefore turn now to the semi-parametric models.

4.3 Semi-parametric estimates

Do the findings change if we relax the functional form assumption? The results of the semi-parametric estimators are displayed in Tab. 3. The models are again estimated separately for East and West Germany and a range of cut-off choices. The figures in the table also refer to a percentage change in employment which would result if there was no minimum wage with positive numbers indicating negative employment effects and vice versa. Bootstrapped 95-percent-confidence-bands are given in parentheses and the dashed horizontal lines mark the minima set for the East and West. As in the Dickens et al. model the estimates are consistent starting with the cut-off point of 8.70 €/hour. Different cut-offs above this threshold serve as an informal test for the existence of spill-over effects with the caveat being that identification rests on an increasingly smaller part of the distribution as in the parametric models with higher censoring points.

Tab. 3: Employment change without minimum wage in %: semi-parametric models

Cut-off (€/h)	East Germany		Cut-off (€/h)	West Germany	
8.5	6.85	[4.10 ; 9.60]	9.5	0.79	[-0.22 ; 1.80]
8.6	10.90	[9.03 ; 12.78]	9.6	1.13	[0.38 ; 1.88]
8.7	5.99	[4.33 ; 7.66]	9.7	1.42	[0.72 ; 2.12]
8.8	5.63	[3.49 ; 7.77]	9.8	1.57	[0.93 ; 2.21]
8.9	4.91	[2.35 ; 7.48]	9.9	1.40	[0.78 ; 2.03]
9.0	4.05	[1.72 ; 6.39]	10.0	1.44	[0.78 ; 2.11]
9.1	4.77	[2.63 ; 6.91]	10.1	2.28	[1.62 ; 2.94]
9.2	4.08	[1.93 ; 6.23]	10.2	1.46	[1.04 ; 1.88]
9.3	1.89	[-0.21 ; 3.98]	10.3	1.37	[0.97 ; 1.77]
9.4	1.61	[-7.67 ; 10.89]	10.4	0.59	[0.42 ; 0.76]
9.5	0.89	[-6.10 ; 7.88]	10.5	0.47	[0.34 ; 0.61]

Notes: All models estimated for varying censoring points according to 1st column.

Bootstrapped 95%-confidence bands in parentheses.

Source: GLS; own calculations.

Overall the semi-parametric estimates are qualitatively consistent with the parametric model results and theoretical expectations. We find again negative employment effects for the East German construction sector whereas estimates are only slightly negative for West Germany. Regarding the size of the effect we estimate that employment would be about 4-5% higher in the East German construction sector if there was no minimum wage. This seems to be a more reasonable magnitude

compared to the 10-20% range for the parametric models and suggests that functional form assumptions might indeed have biased those results. According to the Cqreg estimates employment in the West German construction sector would have been 1-2% higher without the minimum wage. So we also find minor employment losses for West Germany induced by the minimum. The censored quantile regression model seems to work better when it is based on a smaller part of the observable distribution compared to the parametric models.

The semi-parametric estimates are relatively robust with respect to the choice of a cut-off point up to 9.2€/hour in the East and 10.3€/hour. Nevertheless there is some evidence for spill-over effects for East Germany since estimates directly above the minimum wage levels are markedly larger. Around 9.0€/hour they are reduced to about 4% which is consistent with the descriptive findings of Fig. 4 above. The density above the spike at the set minimum wage level of 8.63€/hour is clearly higher than around 9.0€/hour. Although the difference is not very large this would suggest that employment effects of the minimum wage in the East German construction are between 4 and 5%. A look at Fig. 4 may also explain that estimates differ for higher censoring points. At a cut-off point of 9.5€/hour for East Germany the model is based essentially only on half of the observable distribution.

Since the estimations are based on individual data we are able to break down the average employment effects by *individual and firm characteristics* (see Tab. 4). The detailed analysis helps to uncover heterogeneity in the overall effects of the minimum wage. Note that we still work with the estimated underlying distributions from the pooled models of all construction workers in the respective samples for the East and the West. The employment effects are calculated as described above by comparing the observed and underlying distribution, but now separately for different sub-groups of individuals. We chose our preferred cut-off points of 9.0€/hour for East and 10.0€/hour for West Germany which lie not directly above the legally set minima to reduce the bias of potential spill-over effects. The first line in the table represents the aggregate estimate and corresponds to Tab. 3.

Several clear patterns emerge from Tab. 4. Young construction workers' employment chances are worst hit by the minimum wage in the main construction trade. We find that employment of workers between 18 and 25 years of age would be about 27% higher without a minimum wage in East Germany. For the age group

Tab. 4: Employment change without minimum wage in % by sub-groups: semi-parametric models

Characteristics		East Germany		West Germany	
All		4.05	[1.72 ; 6.39]	1.44	[0.73 ; 2.15]
Age	18 – 25 years	27.40	[21.25 ; 33.54]	10.46	[7.14 ; 13.78]
	26 – 30 years	17.66	[11.98 ; 23.35]	5.46	[3.42 ; 7.50]
	31 – 35 years	6.98	[3.04 ; 10.93]	0.84	[-0.24 ; 1.93]
	36 – 40 years	-0.15	[-3.78 ; 3.49]	0.00	[-1.16 ; 1.16]
	41 – 45 years	-5.76	[-9.30 ; -2.23]	-0.07	[-1.20 ; 1.06]
	46 – 50 years	-4.48	[-7.30 ; -1.66]	-0.57	[-1.60 ; 0.47]
	51 – 55 years	0.68	[-3.99 ; 5.36]	0.00	[-0.71 ; 0.71]
	56 – 65 years	5.11	[-1.88 ; 12.10]	-1.05	[-1.88 ; -0.21]
Qualif.	primary school no voc. educ.	9.60	[6.47 ; 12.72]	11.36	[8.33 ; 14.40]
	prim. school and voc. educ.	-5.12	[-7.34 ; -2.89]	-1.25	[-1.66 ; -0.84]
	secondary school	-2.38	[-5.44 ; 0.68]	13.16	[5.44 ; 20.88]
CBA	no agreement	-2.38	[-8.69 ; 3.93]	1.73	[0.41 ; 3.04]
	sectoral agreement	4.30	[1.88 ; 6.71]	1.14	[0.51 ; 1.76]
	firm agreement	4.18	[0.50 ; 7.85]	2.56	[0.79 ; 4.32]
Size	10 – 20 employees	12.37	[6.95 ; 17.80]	3.55	[1.56 ; 5.54]
	20 – 50 employees	14.20	[8.92 ; 19.47]	3.12	[1.68 ; 4.55]
	50 – 100 employees	1.37	[-1.57 ; 4.32]	0.37	[-0.51 ; 1.25]
	100 – 250 employees	-3.86	[-7.26 ; -0.46]	-0.08	[-0.82 ; 0.66]
	250 – 500 employees	-5.39	[-7.96 ; -2.81]	0.23	[-0.84 ; 1.30]
	> 500 employees	-2.38	[-5.11 ; 0.35]	0.60	[-1.21 ; 2.41]

Notes: The models are estimated for cut-off points of €9.0/h and €10.0/h for the East and West respectively. Bootstrapped 95%-confidence bands in parentheses.

Source: GLS; own calculations.

26-30 this figure is still more than 17% whereas the average effect is about 4%. The two youngest age groups in West Germany also exhibit negative employment effects which are also 6 and 3 times higher compared with the modest average effect. On the contrary employment effects are slightly positive for the age groups between 36 and 50 years which might indicate some substitution of older for younger workers within the main construction trade. We altogether replicate previous findings that younger employees with usually below-average wages suffer most from a statutory minimum wage. Results concerning qualification levels are of limited meaning since qualification for blue-collar construction workers does not vary much. Most of them possess a primary school education and some vocational degree. Therefore the effect for this group is close to the average estimate for East and West Germany.

Of more interest are the effects by type of collective bargaining agreement (CBA). Tab. 4 shows that employees which are not covered by any form of CBA are most

adversely affected by the legal minimum wage. This can be explained by the wage premium that covered employees receive. The statutory minimum wage is more often binding for workers with labor contracts not covered by collective bargaining. One of the main objectives for this sectoral minimum wage was to avoid wage dumping outside of collective agreements. Finally there are large differences with respect to the establishment size. Employment effects are about three times more negative for establishment sizes between 10 and 50 employees compared with the mean effect. This holds equally for East and West Germany. The minimum wage is thus more relevant for small firms confirming results from previous studies (Müller, 2009). Remember that establishments below 10 employees are de facto not included in this sample. This means that the overall employment effects in the construction sector were in all likelihood worse. Overall there is considerable heterogeneity in the employment effects of the sectoral minimum wage in the German construction sector. Employment losses are mostly borne by young construction workers, employees which are not covered by CBAs and individuals working in small establishments.

4.4 Discussion of results

How do our estimates relate to previous findings? We reproduce some of the result patterns that are reported in the studies of reference. Although Meyer & Wise's model is only consistent for a very small range of cut-off points, it apparently gives more reasonable estimates than the Dickens et al. model among the parametric approaches. The reason could be that the Dickens et al. model utilizes a smaller amount of information from the observable distribution by choosing higher censoring points. Although for both models the parametric assumption for the error term leads to rather high estimates of the employment effect, this seems to be more of a problem for the Dickens et al. model. Interestingly, the authors report even higher negative employment effects in their paper. We seemingly re-enact this problem with our data. The semi-parametric estimator yields more reasonable effects and we argue that it helps to model the underlying distribution more adequately.

We used the West German case as a quasi-control group for the employment effect of the sectoral minimum wage finding that employment effects are substantially different between East and West Germany as the theoretical considerations and descriptive results suggest. As indicated above the way the sectoral minimum wage

was introduced allows to test the robustness of the results and to gain some evidence on potential substitution effects within the industry. Three sub-samples can be distinguished within the construction sector that were not covered by the minimum wage: white-collar workers within the main construction trade, blue-collar workers in building installations (without electricians) and blue-collar workers in other construction industries. The minimum in the main construction trade may have influenced wage negotiations and triggered the adaption of employment in other sub-sectors. In addition some of the volume of work done in the main construction trade could have been shifted to other sub-sectors to avoid higher wage costs induced by the minimum wage. Therefore labor might have been substituted between sub-sectors. All robustness analyses remain within the construction sector to hold other (macro) variables as equal as possible.

Descriptive evidence for the wage distribution of *white-collar workers* in the main construction trade can be found in Fig. 6 in the Appendix. It is obvious that white-collar workers have higher wages than blue-collar workers (compare with Fig. 5). Therefore the minimum wage is farther to the left of the distribution and would have been hardly binding. There is no graphical evidence that white-collar were affected by the minimum for blue-collar workers. The distributional graphs for *building installations* and *other building sector* industries are depicted in Fig. 7 and Fig. 8 in the Appendix. Minimum wage levels are clearly higher up the distribution for both East and West Germany without any visible effect on the shape of the distribution. Descriptive findings suggest that we should not expect sizeable effects for any 'control group' from the model estimates.

Semi-parametrically estimated employment effects for the three sub-groups are reported in Tab. 5 with figures again referring to the percentage change in employment without a minimum wage and dashed horizontal lines marking the level of minimum wages. For white-collar workers in the main construction trade we estimate that employment levels would be slightly higher (between 0.5 and 1.5%) without a minimum wage in East and West Germany. Complementarities with or spill-over effects from blue-collar workers would be plausible explanations. Considering the small size of the effects, interpretation should not be overstretched in that direction. Compared with the main construction trade in East Germany the effects for white-collar workers are rather small.

Tab. 5: Employment change without minimum wage in %: robustness of semi-parametric models

Cut-off (€/h)	White-collar workers main construction trade	East Germany	
		Building installation	Other building sector
8.5	-0.35	-2.47	-5.40
8.6	-0.17	0.69	-3.92
8.7	0.35	-2.12	-1.94
8.8	0.35	-2.02	-0.36
8.9	0.35	-2.71	-3.36
9.0	0.69	-1.38	-0.82
9.1	1.39	-2.71	-1.38
9.2	1.39	-2.17	-5.15
9.3	2.25	-3.75	-6.27
9.4	1.91	-2.61	1.53
9.5	1.39	-16.17	-0.15
Cut-off (€/h)	White-collar workers main construction trade	West Germany	
		Building installation	Other building sector
9.5	1.33	0.03	0.90
9.6	1.25	0.57	0.86
9.7	1.14	0.98	0.88
9.8	1.22	0.03	-0.04
9.9	1.22	0.11	-0.11
10.0	1.25	-0.98	-0.32
10.1	1.22	-0.41	-0.30
10.2	1.18	-0.05	-0.22
10.3	1.25	-1.22	-1.06
10.4	1.37	-0.54	-0.75
10.5	1.45	-1.93	-0.75

Notes: All models estimated for varying censoring points according to 1st column.
Source: GLS; own calculations.

For the other control groups the estimates are different for the East and the West. For West Germany we find that employment effects are effectively zero. Similar to the main construction trade the minimum does not affect employment in the West. The East German estimates are not very robust for different censoring points. As seen in Fig. 7 and 8 the minimum wage level is close to the middle of the distribution which complicates identification of the semi-parametric estimator. The estimation of the underlying distribution only rests on the right part of the observable wage distribution and is thus based on merely half of all available observations. This problem is slightly worse for other building sector industries as wages

are a tad lower there. The estimates suggest positive employment effects of the minimum wage for those sub-sectors. One could argue that labor-labor substitution might have occurred between the main construction trade and other construction industries. Again we would be careful with definitive conclusions as the estimates are not reliable enough. Altogether the results for different control groups emphasize that the negative employment effects found for the main construction trade in East Germany are unique for all construction industries and can thus in all likelihood be linked with the sectoral minimum wage there.

Another robustness issue concerns institutional features of the German economy. We argued elsewhere (Müller and Steiner, 2009) that the German tax-and-transfer system constitutes an *implicit minimum wage* which is defined by the level of social assistance (nowadays called unemployment benefit (UB) II) for those who are able and willing to work. Individual labor supply decisions and thus the observed wage distribution are therefore not only influenced by the statutory minimum but also by the implicit minimum wage. Whether the implicit is below the sectoral minimum wage in the main construction trade - and is thus binding and relevant for the labor supply decision - depends on individual and household characteristics. The labor demand models of Meyer & Wise and Dickens et al. abstract from those considerations: any person whose productivity is below the minimum wage and who has become unemployed would work if there was no minimum wage. This is not necessarily true as, for example, married individuals with high-income spouses will face a combination of high marginal tax rates, and high opportunity costs of working. Those people will not be on the labor market if their productivity is below their implicit minimum regardless of a statutory minimum wage. The observed wage distribution is therefore not only affected by the sectoral minimum, but also by individual reservation wages which are themselves determined by a number of factors (gender, human capital, children, marital status, unobservable individual time preferences, etc.).

We are not able to integrate the institutional and household features in the labor demand models because the data set lacks necessary individual and household information. All we can do is to indirectly test the robustness of our estimates with respect to implicit minimum wages. The main problem for the validity of the results arises from the following scenario: imagine we estimate a negative employment effect

of the sectoral minimum wage based on an underlying distribution like in Fig. 1 which is not bounded to the left. This assumes that workers would accept hourly wages close to zero without a statutory minimum. If implicit minima are below the legal minimum wage, not every wage below the sectoral minimum will be realized depending on individual reservation wages. A first measure of pre-caution is to exclude wages below 3€/hour right away from our sample as noted above. Second, as a robustness check predicted underlying wages below this threshold are excluded from all simulations of the employment effects as those wages in all likelihood would not exist in the absence of the sectoral minimum. All results reported in this paper do not change when this is done; the underlying wages which are estimated based on observable characteristics are always above this threshold. This may not fully dispel the concern about this problem as implicit minimum wages can of course be higher than 3€/hour. We are confident that results would not change substantially if we could deal with the problem explicitly.

Are the findings in line with previous research for the German construction sector? To put the differences which are presented in Tab. 6 in perspective one has to keep in mind that König and Möller (2008) use a different methodology and data base. They estimate a difference-in-difference framework on an administrative data set which lacks crucial information about hours worked. Their construction of the treatment and control group rests on the imputation of working hours on the basis of a probability model.

Tab. 6: Comparison of effects (employment change without minimum wage in %)

	East Germany		West Germany	
Koenig & Moeller - treated	4.10	[2.20 ; 6.00]	-2.20	[-3.40 ; -1.00]
Koenig & Moeller - overall	0.45	[0.24 ; 0.66]	-0.09	[-0.14 ; 0.00]
Cqreg model	4.05	[1.72 ; 6.39]	1.44	[0.78 ; 2.11]

Notes: Bootstrapped 95%-confidence bands in parentheses. For König and Möller (2008) first line refers to translated figures from estimated re-employment probabilities for treated compared to non-treated. Second line translates effect size to all employed.

Source: König and Möller (2008); GLS; own calculations.

Qualitatively we almost reproduce their findings, i.e. negative employment effects for East Germany and close to zero effects for the West. The first discrepancy

is that our semi-parametric specification yields also slightly negative effects for West Germany whereas König and Möller (2008) even report positive employment effects of the minimum in some of their specifications. The second and more important point is that the negative effects for East Germany are markedly higher in comparison with König and Möller (2008) (see Tab. 6). If their employment effects are translated to the whole construction sector, they become smaller than 0.5%. We estimate an effect of 4-5%. This contributes empirical evidence to the economic policy debate about the effects of a sectoral minimum wage in the German construction sector. We have to draw a more pessimistic picture of the employment effects of the minimum wage in the construction sector than hitherto presumed.

5 Conclusion

In this paper we applied different parametric and semi-parametric approaches to estimate the employment effects of a sectoral minimum wage in the German construction sector from a single cross-sectional wage distribution in 2001. The pattern of the employment effects is consistent throughout different models with clearly negative effects for East Germany and only slightly negative effects for West Germany. This result confirms our theoretical expectations which were based on the economic influence of differential minimum wage levels that were set much higher in the East German construction sector.

Concerning the size of the effect the results for the parametric models range between 10-20% and are thus implausibly high. We conclude that parametric functional form assumptions are overly restrictive for the observed wage distributions and drive those estimates. These results confirm previous findings and reservations about this approach in the literature. We therefore suggest an innovative way to relax the parametric assumptions by estimating a series of semi-parametric censored quantile regression models. We find smaller and more reasonable estimates with this approach. According to the semi-parametric estimates employment levels would be 4-5% higher without the sectoral minimum wage in East Germany. Moreover, we also estimate slightly negative effects for the West of about 1-2%. We conclude that this model is a meaningful extension to existing approaches that allows to estimate underlying wage distributions more adequately.

Since the models are estimated on individual data employment effects can be decomposed according to individual and firm characteristics. We uncover considerable heterogeneity in the effects of the sectoral minimum wage in the German construction sector. Employment losses are de facto borne by young construction workers, those employees not covered by any collective bargaining agreement and individuals working in small establishments. This dimension is often neglected in public debates about minimum wages.

The paper also contributes to the policy question about the employment effects of the sectoral minimum wage in the German main construction trade and more generally about the effects of a minimum wage in the German economy. We confirm previous findings for Germany and reiterate the negative employment effect of the sectoral minimum wage in East Germany. Especially the differences in the levels of the minimum and ensuing employment effects between West and East Germany should be taken into account for future amendments of the minimum wage in the construction sector. The results are also relevant for the ongoing debate about additional sectoral minima in Germany.

The scope of results is obviously limited by the fact that we neither explicitly estimate substitution effects with other sectors nor account for capital-labor substitution and overall output adjustments in the construction sector. Nevertheless the results proved plausible in the light of findings for several robustness checks. We do not get similar negative effects for any of the groups not covered by the minimum wage. There is some evidence for labor-labor substitution with other construction industries, but this should not be overstated as these estimates are based on a comparably small share of observations.

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Appendix

Additional figures

Fig. 5: Log hourly wages, main construction trade, East & West Germany

Note: Normal distribution in graphs for comparison. Sources: GLS 2001; own calculations.

Fig. 6: Log hourly wages, main construction trade, white-collar workers, East & West Germany

Note: Normal distribution in graphs for comparison. Sources: GLS 2001; own calculations.

Fig. 7: Log hourly wages, building installation, East & West Germany

Note: Normal distribution in graphs for comparison. Sources: GLS 2001; own calculations.

Fig. 8: Log hourly wages, other building sector, East & West Germany

Note: Normal distribution in graphs for comparison. Sources: GLS 2001; own calculations.

Additional tables

Tab. 7: Estimation results: East Germany

	Meyer &Wise		Dickens et al.		Cqreg	
Age	0.263	[0.027]	0.035	[0.004]	0.018	[0.003]
Age squared	-0.003	[0.000]	0.000	[0.000]	0.000	[0.000]
Tenure (months)	0.004	[0.000]	0.000	[0.000]	0.000	[0.000]
Education high	1.496	[0.573]	0.137	[0.062]	0.137	[0.065]
No collective agreement	-0.876	[0.084]	-0.099	[0.011]	-0.072	[0.007]
Firm collective agreement	-0.084	[0.243]	-0.025	[0.027]	-0.057	[0.017]
No public influence	0.245	[0.234]	0.011	[0.027]	0.012	[0.016]
Limited public influence	-0.477	[0.325]	-0.025	[0.047]	-0.021	[0.022]
Firm size: below 21	-1.647	[0.223]	-0.195	[0.027]	-0.119	[0.022]
Firm size: below 21-50	-1.720	[0.217]	-0.197	[0.025]	-0.117	[0.020]
Firm size: below 51-100	-1.374	[0.208]	-0.132	[0.023]	-0.081	[0.019]
Firm size: below 101-250	-0.681	[0.205]	-0.049	[0.022]	-0.037	[0.020]
Firm size: below 251-500	0.136	[0.224]	0.023	[0.023]	0.026	[0.022]
Constant	5.639	[0.611]	1.650	[0.097]	2.031	[0.063]
p1	0.208	[0.020]				
p2	0.320	[0.029]				
sigma	0.675	[0.017]	1.650	[0.005]		
Observations	3,604		3,052		3,517	
Log-likelihood	-7,242		2,264			

Notes: All models estimated with specific censoring point. Standard errors in parentheses. Cqreg model for 0.5 quantile. Sample size changes as not all observations are used for estimation due to censoring.

Source: GLS; own calculations.

Tab. 8: Estimation results: West Germany

	Meyer & Wise		Dickens et al.		Cqreg	
Age	0.218	[0.015]	0.015	[0.001]	0.015	[0.001]
Age squared	-0.002	[0.000]	0.000	[0.000]	0.000	[0.000]
Tenure (months)	0.006	[0.000]	0.000	[0.000]	0.000	[0.000]
Education high	0.292	[0.348]	0.021	[0.023]	0.027	[0.029]
No collective agreement	-0.708	[0.061]	-0.033	[0.004]	-0.051	[0.004]
Firm collective agreement	-3.130	[0.230]	-0.178	[0.018]	-0.210	[0.022]
No public influence	-0.397	[0.227]	-0.027	[0.014]	-0.036	[0.015]
Limited public influence	0.702	[0.294]	0.031	[0.019]	0.024	[0.021]
Firm size: below 21	-0.319	[0.122]	-0.027	[0.008]	-0.018	[0.011]
Firm size: below 21-50	-0.581	[0.116]	-0.041	[0.008]	-0.034	[0.011]
Firm size: below 51-100	-0.319	[0.117]	-0.017	[0.008]	-0.021	[0.011]
Firm size: below 101-250	-0.065	[0.115]	0.000	[0.007]	-0.004	[0.011]
Firm size: below 251-500	-0.344	[0.133]	-0.018	[0.009]	-0.023	[0.012]
Constant	9.692	[0.399]	2.337	[0.026]	2.354	[0.024]
p1	0.595	[0.049]				
p2	0.231	[0.026]				
sigma	0.847	[0.008]	0.147	[0.001]		
Observations	10,343		10,000		10,123	
Log-likelihood	-23,429		5,422			

Notes: All models estimated with specific censoring point. Standard errors in parentheses. Cqreg model for 0.5 quantile. Sample size changes as not all observations are used for estimation due to censoring.

Source: GLS; own calculations.

Simulated employment effects in Meyer & Wise model

D_i can be interpreted as the probability that an individual remains employed (with a wage either below or at the minimum wage) after the introduction of the minimum wage given that he had been employed without the minimum (M) and earned a wage below the minimum. Note that in Meyer & Wise's neoclassical labor market model there is no unemployment without a minimum wage. Moreover, they assume that an individual's wage and employment probability are not affected by the minimum when his or her underlying hourly wage (without a minimum) is above M . D_i can thus be written as follows:

$$\begin{aligned} D_i &= 1 - Pr[w_i^* < M](1 - P_1 - P_2) \\ &= Pr[Emp|(M \cap Emp|NM \cap w_i^* < M)] \end{aligned} \quad (14)$$

P_1 marks the probability that someone who earns a wage below M remains employed at this wage after the minimum is introduced. P_2 is the probability for individuals with $w_i^* < M$ to remain employed under the minimum with a hourly wage of M . Therefore $1 - P_1 - P_2$ marks the probability of becoming unemployed under the minimum wage. $Pr[w_i^* < M]$ is the probability of having an underlying wage below the minimum wage level. In the second line of (14) the expression is written as conditional probability: $Pr[Emp]$ is the probability of being employed as opposed to being unemployed ($Pr[Unemp]$). M denotes the event where a minimum wage is put in place whereas NM denotes the contrary situation without a minimum wage.

The claim is that the inverse of D_i is the expected number of individuals that would be employed at $w_i < M$ if there was no minimum wage:

$$\frac{1}{D_i} = E[Emp|NM \cap w_i^* < M] \quad (15)$$

From the definition of conditional probabilities it follows that D_i can be written as probability of being employed given a minimum is put in place, the underlying wage is below the minimum and the individual would be employed without the minimum:

$$\begin{aligned} D_i &= Pr[Emp|(M \cap Emp|NM \cap w_i^* < M)] \\ &= \frac{Pr[Emp \cap M \cap Emp|NM \cap w_i^* < M]}{Pr[M \cap Emp|NM \cap w_i^* < M]} \end{aligned} \quad (16)$$

The inverse of D_i is therefore:

$$\frac{1}{D_i} = \frac{Pr[M \cap Emp|NM \cap w_i^* < M]}{Pr[Emp \cap M \cap Emp|NM \cap w_i^* < M]} \quad (17)$$

Because of the above-mentioned assumptions about the labor market in the Meyer & Wise model, an individual can either remain employed or become unemployed when the minimum wage is introduced. Therefore the probability $Pr[M]$ is given by sum $Pr[M] = Pr[Emp \cap M] + Pr[Unemp \cap M]$. Hence the enumerator of (17) can be written as follows:

$$\frac{1}{D_i} = \frac{Pr[Emp \cap M \cap Emp|NM \cap w_i^* < M] + Pr[Unemp \cap M \cap Emp|NM \cap w_i^* < M]}{Pr[Emp \cap M \cap Emp|NM \cap w_i^* < M]} \quad (18)$$

Again stressing the assumptions of the Meyer & Wise model it also holds that $Pr[Emp \cap NM] = Pr[Emp \cap M] + Pr[Unemp \cap M]$, since there is no unemployment without the minimum wage. The same holds for the joint probabilities in the enumerator of (18) as the other events ($Emp|NM$ and $w_i^* < M$) are independent of M or NM . Therefore the inverse of D_i can be re-written with the following probabilities

$$\begin{aligned} \frac{1}{D_i} &= \frac{Pr[Emp \cap NM \cap Emp|NM \cap w_i^* < M]}{Pr[Emp \cap M \cap Emp|NM \cap w_i^* < M]} \\ &= E[Emp|NM \cap w_i^* < M] \end{aligned} \quad (19)$$

which equals the expected number of persons that would be employed without the minimum. The inverse is the expected number of individuals that would work without a minimum wage because $Pr[Emp \cap NM] \geq Pr[Emp \cap M]$. Both probabilities would be equal if the minimum wage caused no unemployment ($Pr[Unemp \cap M] = 0$). To illustrate the argument consider a simple example: Assume that the probability in the numerator, i.e. the probability of being employed without the minimum wage and a wage below M , would be equal to $1/2$, and the probability in the denominator, i.e. the probability of remaining employed under the minimum, would be $1/4$. Then the inverse of D_i would yield 2. That means that one would expect for each individual who is employed under the minimum wage with an underlying wage below M 2 individuals to work without the minimum because the probability is twice as high.

Assumptions and derivation of the concentrated likelihood function in the Dickens et al. model

The key assumption in the Dickens et al. model is that above the cut-off point of w_1 wages and employment are not affected by the minimum wage M which is set somewhere below w_1 . Therefore the observed wage distribution $f_1(w)$ and the underlying distribution $f(w)$ are identical above w_1 . Since both $f_1(w)$ and $f(w)$ are densities and integrate to one it must hold that above w_1 they are equal up to a scaling factor γ which is the assumption described in (5) above:

$$f_1(w; \theta) = \gamma f(w; \theta) \quad \text{for } w > w_1 \quad (20)$$

Depending on how employment changes due to the minimum γ is below or above one. For $\gamma < 1$ there is *relatively* more probability mass to the left of w_1 in $f(w; \theta)$ compared with $f_1(w; \theta)$ as some individuals become unemployed. For $\gamma > 1$ more people are employed with a wage below w_1 under the minimum wage compared to the counterfactual without a minimum. In that case more probability mass to the left of w_1 would be in $f_1(w; \theta)$ compared to $f(w; \theta)$. This scenario where the minimum wage creates additional jobs is not captured in Meyer & Wise's model. The scaling factor is determined by the employment change under the minimum wage which is given by the relation of total employment without the minimum wage L_0 and under the minimum wage L_1 : $\gamma = L_0/L_1$.

By the same logic the number of employed individuals above w_1 is identical above and below the minimum wage. This is expressed in (6) above:

$$\begin{aligned} L_1(1 - F_1(w_1; \theta)) &= L_0(1 - F(w_1; \theta)) \\ F_1(w_1; \theta) &= 1 - \gamma(1 - F(w_1; \theta)) \end{aligned} \quad (21)$$

The derivation of the concentrated likelihood function is straightforward. It starts from a Tobit model for observed wages w_i with the censoring point $w_1 \geq M$, j observations above and $L_1 - j$ observations below w_1 :

$$\begin{aligned} \log L &= \sum_{i=1}^j \log f_1(w_i; \theta) + (L_1 - j) \cdot \log F_1(w_1; \theta) \\ &= \sum_{i=1}^j \log f(w_i; \theta) + j \cdot \log \gamma + (L_1 - j) \cdot \log[1 - \gamma \cdot (1 - F(w_1; \theta))] \end{aligned} \quad (22)$$

The maximization of (22) with respect to γ yields:

$$\begin{aligned}
\frac{\partial \ln L}{\partial \gamma} &= \frac{j}{\gamma} + \frac{(L_1 - j)(-1 - F(w_1; \theta))}{1 - \gamma(1 - F(w_1; \theta))} = 0 \\
0 &= j - j\gamma(1 - F(w_1; \theta)) + \gamma(L_1 - j)(-1 + F(w_1; \theta)) \\
j &= \gamma[j(1 - F(w_1; \theta)) + (L_1 - j)(-1 + F(w_1; \theta))] \\
\gamma &= \frac{j}{j - jF(w_1; \theta) + L_1 - L_1F(w_1; \theta) - j - jF(w_1; \theta)} \\
\gamma &= \frac{j}{L_1(1 - F(w_1; \theta))}
\end{aligned} \tag{23}$$

When this estimator is inserted back into in (22) one can derive the concentrated likelihood which boils down to the likelihood of a truncated regression model for a sample of workers with observations truncated at w_1 :

$$\begin{aligned}
\log L &= \sum_{i=1}^j \log f(w_i; \theta) + j \cdot \log \gamma + (L_1 - j) \cdot \log[1 - \gamma \cdot (1 - F(w_1; \theta))] \\
&= \sum_{i=1}^j \log f(w_i; \theta) + j \cdot \log \left[\frac{j}{L_1(1 - F(w_1; \theta))} \right] + (L_1 - j) \cdot \log \left[1 - \frac{j(1 - F(w_1; \theta))}{L_1(1 - F(w_1; \theta))} \right] \\
&= \sum_{i=1}^j \log f(w_i; \theta) + j \cdot [\log(j) - (\log(L_1) + \log(1 - F(w_1; \theta)))] + (L_1 - j) \cdot \log \left[1 - \frac{j}{L_1} \right] \\
&= \sum_{i=1}^j \log f(w_i; \theta) - j \cdot \log[(1 - F(w_1; \theta))] + j \log(j) - j \log(L_1) + (L_1 - j) \cdot \log \left[1 - \frac{j}{L_1} \right] \\
&= \sum_{i=1}^j \log f(w_i; \theta) - j \cdot \log[(1 - F(w_1; \theta))] + \text{constant}
\end{aligned} \tag{24}$$

Therefore in the Dickens et al. framework a truncated regression model is estimated. All parameters of interest can then be derived as outlined. Note that this simplification to a concentrated likelihood does only work without parameterizing the distribution with respect to individual characteristics. If there are covariates the derivation is less elegant; the basic principle remains the same, though.