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Is Substitutability the New Efficiency?

Endogenous Investment in the Elasticity of Substitution between Clean and Dirty Energy^{*}

Fabian Stöckl[†] 

August 6, 2020

Abstract

When analyzing potential ways to counter climate change, standard models of green growth abstract from investment in substitutability between “clean” and “dirty” energy inputs. Instead, they rely on the assumption that efficiency with respect to fossil fuels can be increased perpetually. However, this is not in line with observed firm investment behavior and the limits to efficiency imposed by thermodynamic laws. In this paper, I develop a growth model that explicitly accounts for endogenous investment to increase input substitutability, in addition to investment in efficiency. The model predicts that, for a growing economy, there is always investment in both substitutability and efficiency, even without a carbon cap and with non-infinite fossil fuel prices. Most importantly, in the long-run, with sufficient investment in substitutability, fossil fuels become inessential for production. Moreover, the model predicts a declining income share of fossil fuels, an outcome not featured by standard models based on purely efficiency-enhancing technological progress. Overall, the model generates an endogenous path of transition from an economy characterized by a low elasticity of substitution to one characterized by a high elasticity. In doing so, it still nests the results derived from a purely efficiency-based directed technical change framework as a special case. In addition, this paper analyzes the scope for policy intervention, showing that even a temporary subsidy/tax can trigger a full transformation toward green growth.

JEL classification: Q20, Q30, O30, O40

Keywords: elasticity of substitution, endogenous (sigma-augmenting) technological change, growth, investment incentives, climate policy, decarbonization

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1 Introduction

Global warming is a major challenge facing the world today. To limit further increases in the global mean temperature, anthropogenic CO₂ emissions need to be reduced drastically and economic growth must be decoupled from the use of fossil fuels (IPCC, 2014). Given that energy is an essential input in the production of many goods and services, there are only two ways to decrease CO₂ emissions in a growing economy. Either there is investment in efficiency in the use of dirty fossil fuels or there is investment in better substitutability such that clean renewable energy inputs can eventually fully replace dirty, non-renewable fossil fuels.

Existing economic research mainly focuses on efficiency-enhancing technological progress and neglects the possibility to invest in better substitutability between clean and dirty energy inputs.¹ This is at odds with the observed investment behavior of firms and governments alike, both of which invest in technologies that enable and facilitate the replacement of fossil fuels with clean alternatives (Lazkano et al., 2017; Mattauch et al., 2015). Moreover, the idea of perpetually increasing efficiency in the use of fossil fuels conflicts with the second thermodynamic law (Meran, 2019). Thus, the missing possibility to invest in better substitutability constitutes a substantial research gap. This is especially surprising as the general idea of substitutability-increasing investment, at least in the context of capital and labor, is already mentioned by Hicks (1932). In an environmental context, it is proposed to be put at the top of the research agenda by Bretschger (2005).

My study aims to close this gap by analyzing the implications of introducing the possibility to invest in better substitutability between clean and dirty energy inputs on growth dynamics. Specifically, I analyze (i) what the incentives to invest in better substitutability are and which factors they depend on; (ii) how these incentives interact with efficiency-enhancing technological progress; and (iii) whether and under what conditions these incentives trigger investment in better substitutability, thereby inducing a green growth path.

To answer these questions, I develop a model that allows for endogenous investment in better substitution possibilities such that clean and dirty inputs can

¹For standard neoclassical growth models also employing, in addition to capital and labor, an energy composite (see Groth (2007) and Smulders et al. (2014) for overviews), the assumption of a low elasticity of input substitution is required by the second thermodynamic law as a necessary constraint (Dasgupta and Heal, 1979; Meran, 2019). This, however, is unrelated to substitution between clean and dirty inputs within the energy composite. For empirical evidence on capital-energy substitution see, e.g., Kemfert (1998) and van der Werf (2008).

turn from complements to substitutes, rendering fossil fuels eventually inessential for production. The option to invest in efficiency-enhancing technological progress with respect to dirty inputs is still maintained and, additionally, extended by the possibility to invest in the efficiency of clean inputs. In other words, I conceptually distinguish between technological progress that increases efficiency within the existing production structure and technological progress that changes the production structure itself by increasing the elasticity of substitution. For example, I distinguish, between reducing the gasoline consumption of cars with a standard combustion engine (efficiency) and the emergence of an alternative clean technology providing almost the same service, like battery electric vehicles (substitutability).²

My model demonstrates that the decision to invest in a higher elasticity between clean and dirty inputs is driven by its dampening effect on the diminishment of their marginal returns. That is, with a higher elasticity, more of the relatively cheaper input factor can be employed while suffering less from a decrease in its marginal productivity. The magnitude of this investment incentive depends positively on overall production (output effect) and the relative costs of the two inputs (ratio effect). Furthermore, I can show that, for a growing economy, investing in better substitutability always becomes profitable at some point in time. Once profitable, it alternates with investment in efficiency-enhancing technological progress. As a consequence of improved substitutability, fossil fuels gradually lose importance in production and eventually become inessential. This is also reflected by a decreasing income share of dirty production processes, an important result (see IEA, 2020) that cannot be reproduced in efficiency based models, which usually generate a balanced growth path. Thus, with the possibility to invest in better substitution, a full transition toward clean production may be the outcome of optimal investment behavior of producers. Moreover, during growth phases with no investment in substitutability, the model reproduces the standard result of a balanced growth path. Importantly, all these results do not hinge on the existence of a cap on CO₂ emissions or infinitely high prices of, and taxes on, fossil fuels. Rather, a relative cost advantage of clean production processes suffices. Results are also robust to different specifications of the research process. Furthermore, the determinants of investment in substitutability identified in my model are in line with empirical findings for, e.g., the electricity sector (Lazkano et al., 2017). Analyzing the scope for policy intervention, I

²The development of a (perfect) substitute for a specific task results in a small increase in the degree of substitutability on the aggregate level, which comprises a multitude of different tasks.

can show that even an only temporary subsidy/tax can trigger a transformation toward green growth in the long-run. Finally, derived investment and growth dynamics are illustrated in a numerical simulation exercise. This simulation also reveals that, against widespread fear, investment in better substitutability between clean and dirty energy inputs does not slow down growth. Quite the contrary, such environmental policy can accelerate economic growth, even in the transition phase.

This study contributes to the very small but important literature that analyzes investment in better substitution possibilities (Growiec and Schumacher, 2008; Fenichel and Zhao, 2015; Kemnitz and Knobloch, 2020).³ Most importantly, this paper not only analyzes the effects of investment for a growing economy but, to the best of my knowledge, is also the first providing an analytical framework to study both the incentives to invest in a higher elasticity of substitution as well as how these interact with the incentives to invest in higher efficiency.⁴

In general, my research contributes to three strands of the existing literature. First, concerning the distinction between clean and dirty production inputs and the focus on technological innovation, this paper is related to models employing Acemoglu’s (1998; 2002) directed technical change framework to green growth (e.g., Di Maria and Valente, 2008; Acemoglu et al., 2012; Greaker et al., 2018; Hart, 2019). However, while the degree of substitutability remains an exogenous variable in these models, I endogenize the evolution of the elasticity of substitution as the result of optimal investment decisions of a profit-maximizing representative producer. Still, my framework nests the general results of models like Acemoglu et al. (2012) as a special case of a (temporarily) constant elasticity of substitution. Importantly, model predictions differ in one central respect: With the possibility to invest in better substitutability, a temporary policy intervention can be enough to trigger a full and permanent decarbonization of the economy even in the case of an initially low elasticity of substitution. Thus, this paper suggests a much more optimistic outlook.

³Growiec and Schumacher (2008) numerically investigate the effect of an exogenously increasing elasticity on the optimal depletion rate of fossil fuels. Similarly, Fenichel and Zhao (2015) numerically identify trajectories for natural capital extraction in a model with investment in substitutability and a depletable but recovering resource. Kemnitz and Knobloch (2020) investigate endogenous investment in capital-labor substitutability, but neglect investment in factor-efficiency.

⁴For a discussion on the currently observable degree of substitutability between clean and dirty inputs on the aggregate level, see Papageorgiou et al. (2017), Malikov et al. (2018), and Pottier et al. (2014). See also Pelli (2012) and Stöckl and Zerrahn (2020) for the electricity sector.

Second, my model relates to several approaches in which the elasticity of substitution cannot be increased explicitly through investment but rather changes as a by-product of, e.g., growth and structural change.⁵ Foremost, this includes the literature on variable elasticity of substitution (VES) production functions (Lu and Fletcher, 1968; Sato and Hoffman, 1968; Revankar, 1971; Kadiyala, 1972) in which substitutability is directly linked to the ratio of, e.g., capital and energy inputs (Lazkano and Pham, 2016). Alternatively, in a multi-sectoral framework comprising any two inputs, the aggregate elasticity of substitution may rise as the result of sectoral change (Jones, 1965; Miyagiwa and Papageorgiou, 2007; Xue and Yip, 2012). In the context of green growth, this channel is studied in Bretschger and Smulders (2012), where production endogenously reallocates toward sectors with a higher elasticity of substitution. However, in their model, all sector-specific elasticities are themselves constant and predetermined.

Third, highlighting the interplay between output and the endogenous emergence of a green growth path, this paper also links to the literature on a possible (CO₂-specific) environmental Kuznets curve (Grossman and Krueger, 1991; Copeland and Taylor, 2004; Stern, 2017). Like in Tahvonen and Salo (2001), in my model, the transition toward clean inputs may occur even in the absence of policy intervention; i.e., for a business-as-usual or laissez-faire scenario. Hence, the most important question is not whether or not the transformation occurs but rather whether it happens early and fast enough to avoid a climate disaster.

Moreover, the framework presented in this paper can easily be applied to the analysis of substitution between any other two input factors. For instance, endogenous changes in capital-labor substitutability have so far been neglected in the literature. It might, however, play an important role in explaining the decline of the global labor income share observed since the 1980s (Elsby et al., 2013; Karabarbounis and Neiman, 2014).

The rest of this paper is structured as follows: Section 2 introduces the model and studies the general incentives to invest in better substitutability. In Section 3, the interplay between investment in substitutability and efficiency is analyzed, and investment dynamics for a growing economy are derived. Moreover, alternative specifications of the research process are discussed. Following, Section 4 discusses the scope for policy intervention and sketches possible implications for the design of optimal environmental policy. An illustrative numerical simulation is presented in Section 5. Section 6 concludes.

⁵See Growiec and Mućk (2019) for an overview of different approaches. See also Knoblich and Stöckl (2020) for a discussion of various concepts of the elasticity of substitution and potential determinants of the latter.

2 The Model

The basic model consists of a representative producer providing both clean (CO₂-free) and dirty (CO₂-emitting) energy service intermediates. These are then combined to an energy service composite for final good production.⁶ Moreover, at least in 2020, clean and dirty inputs are only imperfect substitutes for each other such that both are necessary for the provision of the composite energy service. Thus, it is assumed that the initial elasticity of substitution is below unity.⁷

Following Acemoglu et al. (2012), the use of fossil fuels in dirty production is not modeled explicitly. Rather, capital is the only physical input in the production of both clean and dirty intermediates. This assumption is intuitive for the production of clean intermediates, but needs further explanation in the case of dirty intermediates, which is heavily based on fossil fuel input. In that case, the costs of usually capital-intensive production of fossil fuels, like drilling or mining, are implicitly added to the costs of investment in production capacities for dirty intermediates. Moreover, CO₂ emissions are assumed to be proportionate to the use of dirty energy service intermediates. Finally, following Papageorgiou et al. (2017), labor is assumed to play only a minor role in the production of energy service intermediates and, thus, is not considered explicitly.

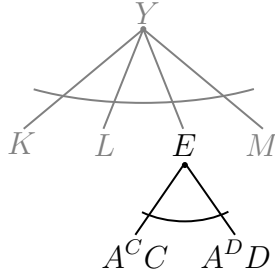
Figure 1 illustrates the use of clean and dirty energy service intermediates in providing the energy service composite, E , and the use of the latter in final good production, Y , where C and D denote clean and dirty energy service intermediates themselves, while A^C and A^D capture efficiency in their use.⁸

Yet the theoretical analysis presented in the following focuses on the production of the energy service composite from clean and dirty intermediates, abstracting from its further use in final good production. This clear focus keeps the model tractable and allows for an intuitive analysis of the economic forces at work. Results and insights remain valid in more complex structures as long as only demand for the energy service composite is affected.

⁶Energy services capture the provision of energy-based functionalities like heat, electricity, and kinetic energy, rather than physical energy inputs themselves. Theoretically, the concept of energy services is flexible enough to also account for production processes where emissions are rather a by-product, e.g., agriculture or cement and steel production.

⁷As Pelli (2012) argues, if the elasticity of substitution between clean and dirty inputs is below unity for just one production process, and if this process is itself essential for the energy service composite, then also on the aggregate level the elasticity of substitution between clean and dirty inputs is necessarily below unity.

⁸ A^C and A^D can alternatively be interpreted as efficiency in the production of energy service intermediates.



Note: Arcs indicate input substitution possibilities.
Top-Level-Inputs: capital (K), labor (L), energy (E), materials (M)

Figure 1: Production of the Energy Composite (Within Final Good Production).

At the beginning of every time period, the representative producer faces two investment possibilities. First, investment in research increasing the efficiency in the use of or the substitutability between clean and dirty energy service intermediates. Second, investment in clean and dirty intermediate production capacities. The price-taking producer always distributes investment such that production costs are minimized for an exogenously given demand of the energy service composite. When making investment decisions, the producer is myopic, reacting only to price signals of the current period.⁹ That is, intertemporal externalities of investment decisions, e.g., depletion of resources or of a carbon budget, are only accounted for if reflected by prices. Thus, they are, in general, not internalized in the cost minimization problem of the myopic producer.¹⁰ Rather, all incentives to invest are based on their immediate effect on production costs for the current period. Repeated for consecutive periods, the resulting path of production and investment decisions is Pareto-efficient.

2.1 The General Model Framework

In every period, a representative producer employs clean and dirty energy service intermediates to provide an energy service composite, E_t , according to the following modified constant elasticity of substitution (CES) production function:

$$E_t = \left(\alpha (A_t^C C_t)^{\rho_t} + (1 - \alpha) (A_t^D D_t)^{\rho_t} \right)^{\frac{1}{\rho_t}}, \quad (1)$$

⁹This is in contrast to a social planner foreseeing potential future limitations to the use of fossil fuels and, thus, investing in substitutability to render dirty processes inessential for the production of the energy service composite before, e.g., a carbon cap, becomes binding.

¹⁰Given the currently known stock of resources, depletion indeed appears to remain a secondary issue compared to CO₂ emissions (Heede and Oreskes, 2016). Similarly, to date, there is no effective global carbon cap. Thus, assuming that externalities are not yet captured by prices appears to be a pessimistic, yet, realistic assumption.

where $\alpha \in (0, 1)$ is a share parameter, clean and dirty energy service intermediates are denoted by $C_t, D_t > 0$, respectively, and efficiency in their use is captured by $A_t^j \geq 1$ with $j \in \{C, D\}$. Henceforth, they are referred to as clean and dirty *intermediates* (C_t, D_t), *efficiency* (A_t^C, A_t^D), and *inputs* ($A_t^C C_t, A_t^D D_t$). Finally, the elasticity of substitution, $\sigma_t \in (-1, \infty)$, i.e., the measure of the degree of substitutability between clean and dirty inputs, is expressed in terms of the substitution parameter, $\rho_t = \frac{\sigma_t - 1}{\sigma_t} \in (-\infty, 1)$. Most importantly, the elasticity of substitution can now increase over time through purposeful investment. This distinguishes the above modified production function from standard CES representations.¹¹

In every period, the representative producer can invest in clean and dirty intermediate production capacities, M_t^j . Without loss of generality, I assume that one unit of production capacity provides exactly one unit of the respective intermediate:

$$j_t = G(M_t^j) = M_t^j. \quad (2)$$

This allows for using capacities and intermediates interchangeably. The supply of clean and dirty intermediates then increases linearly with investment in the respective production capacities. The productivity of a marginal investment in capacities is constant and given by:

$$\frac{\partial j_t}{\partial I_t^j} = \frac{1}{\phi^j}, \quad (3)$$

such that the corresponding law of motion is:

$$\frac{dj_t}{dt} = \frac{I_t^j}{\phi^j} - \delta^j j_t, \quad (4)$$

where I_t^j is intermediate-specific investment, and ϕ^j is a time-invariant parameter capturing the costs to build one unit of production capacity.¹² The depreciation rate of production capacities is given by δ^j . Alternatively, there is the possibility to invest in research increasing either efficiency in the use of intermediates or input substitutability. However, in contrast to the case of production capacities for

¹¹For the sake of a parsimonious notation and without loss of generality, I abstract from an explicit normalization of the CES production function as proposed by de La Grandville (1989) and Klump and de La Grandville (2000); rather I use the implicitly normalized form as presented in Equation (1). A more detailed discussion of the normalization point and its importance in quantitative analyses is presented in Appendix A.2.

¹²To simplify the following analysis, costs are assumed to be constant over time. The effect of changes in costs, e.g., due to a tax/subsidy, is discussed in Section 4.

intermediates, the productivity of a marginal investment in research potentially depends on the current technological state and, thus, also on past innovation:

$$\frac{\partial A_t^j}{\partial I_t^{A^j}} = \frac{1}{\phi^{A^j}} (A_t^j)^{\gamma_{A_j}}, \quad (5)$$

$$\frac{\partial \rho_t}{\partial I_t^\rho} = \frac{1}{\phi^\rho} \left(\frac{\rho_0 - \rho_{max}}{\rho_t - \rho_{max}} \right)^{\gamma_\rho}, \quad (6)$$

with the corresponding laws of motion given by:

$$\frac{dA_t^j}{dt} = \frac{I_t^{A^j}}{\phi^{A^j}} (A_t^j)^{\gamma_{A_j}}, \quad (7)$$

$$\frac{d\rho_t}{dt} = \frac{I_t^\rho}{\phi^\rho} \left(\frac{\rho_0 - \rho_{max}}{\rho_t - \rho_{max}} \right)^{\gamma_\rho}, \quad (8)$$

where ϕ^{A^j} and ϕ^ρ capture the costs of research increasing efficiency and substitutability, respectively. $I_t^{A^j}$ and I_t^ρ denote research-specific investment, while γ_{A_j} and γ_ρ control whether past innovation makes further improvements easier ($\gamma > 0$) or more difficult ($\gamma < 0$).¹³ The initial elasticity of substitution is denoted by ρ_0 while maximum achievable substitutability is captured by $\rho_{max} \leq 1$.¹⁴ Moreover, unlike for production capacities for intermediates, there is no depreciation of efficiency and substitutability. Total investment, I_t , is given exogenously and allocated such that the following budget constraint holds:

$$I_t = I_t^C + I_t^D + I_t^{A^C} + I_t^{A^D} + I_t^\rho. \quad (9)$$

Equation (9) implicitly embodies a *lab equipment* specification (Rivera-Batiz and Romer, 1991) assuming that investment in research employs the same input as investment in production capacities.¹⁵ This allows for a direct comparison of the effect of different types of investment on the production of the energy composite, i.e., of the respective investment incentives. These incentives are given by the marginal products of investment weighted by the respective costs of

¹³As common in the growth literature, all technological progress is assumed to be disembodied, i.e., efficiency and substitutability improvements apply to intermediates from both existing and new production capacities (Solow, 1962). For substitutability increases to only affect new capacities, the normalization point would have to increase along with the ratio of clean to dirty inputs (cf. Antony, 2010).

¹⁴Note that, for $\gamma < 0$, $\lim_{\rho_t \rightarrow \rho_{max}} \frac{\partial \rho_t}{\partial I_t^\rho} = 0$ such that ρ_t is bound from above.

¹⁵For instance, assuming investment to be in terms of the final good implies that investment comprises of the same inputs as used in final good production. Yet there would be no direct rivalry between final good production and research for, e.g., labor input (cf. Romer, 1990; Acemoglu, 1998, 2002).

capacity building and research, henceforth denoted *profitabilities*. The specific profitabilities of a marginal investment at a point in time are equal to:

$$\frac{\partial E_t}{\partial I_t^j} = \frac{\partial E_t}{\partial j_t} \frac{\partial j_t}{\partial I_t^j} = \frac{1}{\phi^j} \frac{\partial E_t}{\partial j_t}, \quad (10a)$$

$$\frac{\partial E_t}{\partial I_t^{A^j}} = \frac{\partial E_t}{\partial A_t^j} \frac{\partial A_t^j}{\partial I_t^{A^j}} = \frac{1}{\phi^{A^j}} (A_t^j)^{\gamma_{A^j}} \frac{\partial E_t}{\partial A_t^j}, \quad (10b)$$

$$\frac{\partial E_t}{\partial I_t^\rho} = \frac{\partial E_t}{\partial \rho_t} \frac{\partial \rho_t}{\partial I_t^\rho} = \frac{1}{\phi^\rho} \left(\frac{\rho_0 - \rho_{max}}{\rho_t - \rho_{max}} \right)^{\gamma_\rho} \frac{\partial E_t}{\partial \rho_t}. \quad (10c)$$

Independent of the size of the investment budget, optimality, i.e., cost minimization, requires that, for any initial conditions, the first marginal unit of investment is, at least partially, directed toward the most profitable option (see Appendix A.1). Thus, to answer the central question of whether investment in substitutability will ever become the (temporarily) most profitable option, it is sufficient to show whether it ever becomes optimal to (partially) direct the first marginal unit of investment toward substitutability. Therefore, the analysis of the incentive to invest in substitutability as well as its interaction with other types of investment can be fully based on a comparison of relative profitabilities at time t , as given by Equations (10a)-(10c). The budget constraint, Equation (9), can be neglected for the analysis of general dynamics.¹⁶ Without loss of generality, this simplifies the analysis substantially and allows for general insights otherwise obstructed by mathematical complexity.

2.2 The Incentive to Invest in Substitutability

As shown, investment incentives are driven by the immediate effect on production costs. Therefore, the greater the marginal effect on output, the greater the investment incentive. Consequently, this subsection studies the properties of the partial derivative of the CES production function with respect to the elasticity of substitution and its interaction with changes in the quantity and ratio of inputs. Thereby, I focus on values of the elasticity of substitution below unity,

¹⁶Qualitatively, this does not affect investment patterns. Quantitative differences between cases with a marginal and with a non-marginal investment budget, including the point in time when investment in substitutability becomes profitable for the first time, are discussed in Appendix A.1.

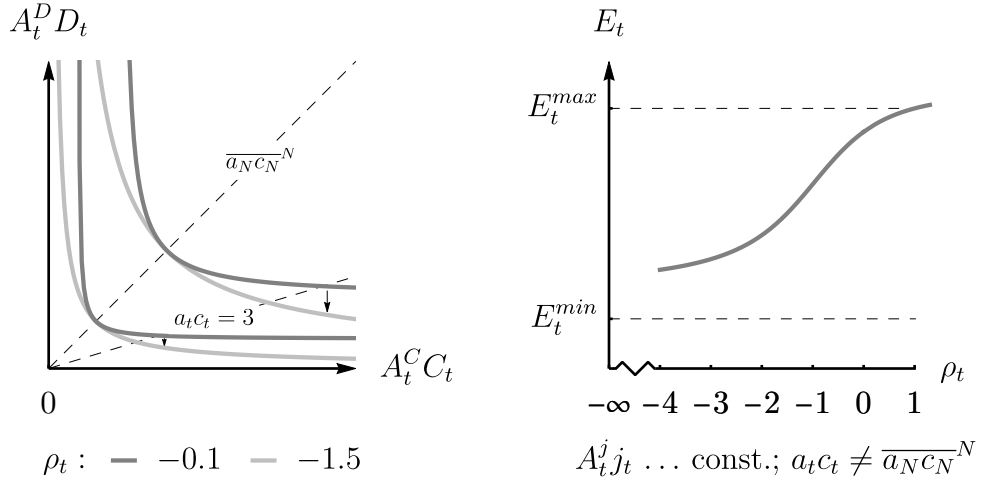
i.e., $\rho_t < 0$. For $\rho_t > 0$, dirty inputs are inessential for production and, whether it is used or not, in general only depends on its relative price (see Section 3.2 for a discussion of $\rho_t > 0$). The proofs of the findings presented in this and the following (sub)sections are provided in Appendix A.3, if not stated otherwise.

The and immediate effect of a marginal increase in the elasticity of substitution on production is always non-negative and, for $\rho_t \in (-\infty, 1)$, given by:

$$\frac{\partial E_t}{\partial \rho_t} = \underbrace{E_t}_{OE_t > 0} \underbrace{\left(\frac{\alpha(a_t c_t)^{\rho_t} \log[a_t c_t]}{(\alpha(a_t c_t)^{\rho_t} + (1 - \alpha)) \rho_t} - \frac{\log[\alpha(a_t c_t)^{\rho_t} + (1 - \alpha)]}{\rho_t^2} \right)}_{RE_t \geq 0} \geq 0, \quad (11)$$

where $a_t = \frac{A_t^C}{A_t^D}$ and $c_t = \frac{C_t}{D_t}$. A direct, analytical proof of $\frac{\partial E_t}{\partial \rho_t} \geq 0$ is presented in de La Grandville (2016, pp. 111-113). Moreover, $\frac{\partial E_t}{\partial \rho_t} = 0$ holds if, and only if, the input ratio is equal to that defined by the normalization point, $a_t c_t = \overline{a_N c_N}^N$.¹⁷

As revealed by Equation (11), for any given level of substitutability, the size of the effect of a marginal increase in the elasticity of substitution is dependent on the ratio of clean to dirty inputs, $a_t c_t = \frac{A_t^C C_t}{A_t^D D_t}$, and increases linearly (with slope one) in output, E_t . Accordingly, the overall impact can be disentangled into an output dependent part, OE_t (*output effect*), and an input ratio-dependent part, RE_t (*ratio effect*). The effect of an increase in the elasticity of substitution and its disentanglement are illustrated in Figure 2a:



(a) The Effect of an Increase in the Elasticity of Substitution on Production Isoquants for Different Levels of Output.

(b) The Effect of Increasing Substitutability on Output for Fixed Input Levels.

Figure 2: The Effect of an Increase in the Elasticity of Substitution.

¹⁷For notational simplicity and without loss of generality, $\overline{a_N c_N}^N = 1$. See Appendix A.2 for a discussion.

Holding inputs constant, the positive relationship between output and the elasticity of substitution has one and only one inflection point in ρ_t and is convex before and concave thereafter.¹⁸ Thus, as illustrated in Figure 2b, the second derivative of the CES production function with respect to the elasticity of substitution can either be positive or negative:

$$\frac{\partial^2 E_t}{\partial \rho_t^2} \gtrless 0. \quad (12)$$

The effect of an increase in clean inputs on the effect of a marginal increase in the elasticity of substitution is given by:

$$\frac{\partial^2 E_t}{\partial \rho_t \partial (A_t^C C_t)} = \underbrace{\frac{\partial E_t}{\partial (A_t^C C_t)}}_{\Delta_t^{OE} \geq 0} \underbrace{\left(\frac{1}{E_t} \frac{\partial E_t}{\partial \rho_t} \right)}_{\geq 0 (RE_t)} + \underbrace{\left(\frac{\partial}{\partial (A_t^C C_t)} \left(\frac{1}{E_t} \frac{\partial E_t}{\partial \rho_t} \right) \right)}_{\Delta_t^{RE} \leq 0} \underbrace{E_t}_{> 0 (OE_t)}. \quad (13)$$

Again, this effect can be disentangled based on whether it has an impact on the output effect or on the ratio effect. Δ_t^{OE} measures by how much a rise of clean inputs increases the production to which, in turn, an improvement of substitutability applies. As the marginal return to an increase in clean inputs is always positive, also Δ_t^{OE} is always positive. Δ_t^{RE} captures how the effect of an increase in the elasticity of substitution changes if one moves along the production isoquant to a higher ratio of clean to dirty inputs. In general, the direction of Δ_t^{RE} is not clear *a priori*. However, from a green growth perspective, the interesting case is the one where production is increasingly based on the use of clean energy inputs such that $a_t c_t > \overline{a_N c_N}^N$. In that case, the sign of Δ_t^{RE} is always positive:

$$\Delta_t^{RE} > 0 \quad \text{for } a_t c_t > \overline{a_N c_N}^N. \quad (14)$$

Thus, for $a_t c_t > \overline{a_N c_N}^N$, Δ_t^{OE} and Δ_t^{RE} work in the same direction, and the overall effect is unambiguously positive. However, the marginal impact of a rise in clean inputs along a given E_t on the ratio effect decreases and finally vanishes as $a_t c_t$ moves away from $\overline{a_N c_N}^N$ (see Figure 2a):

$$\lim_{(A_t^C C_t) \rightarrow \infty} \left(\Delta_t^{RE} \Big|_{E_t = \overline{E}} \right) = 0 \quad \text{for } \rho_t < 0. \quad (15)$$

¹⁸This is first conjectured in de La Grandville and Solow (2006) and formally proven by Nam and Mach (2008).

Moreover, along a production isoquant, the ratio effect and, thus, also the effect of an increase in the elasticity of substitution on output, are bounded from above by:

$$\lim_{A_t^C C_t \rightarrow \infty} (RE_t|_{E_t=\bar{E}}) = \underbrace{-\frac{\log[1-\alpha]}{\rho_t^2}}_{>0} < \infty \quad \text{for } \rho_t < 0, \quad (16)$$

and, therefore:

$$\lim_{A_t^C C_t \rightarrow \infty} \left(\frac{\partial E_t}{\partial \rho_t} \Big|_{E_t=\bar{E}} \right) = \bar{E} \cdot RE_t = \underbrace{-\frac{\log[1-\alpha]\bar{E}}{\rho_t^2}}_{>0} < \infty \quad \text{for } \rho_t < 0, \quad (17)$$

where $E_t = \bar{E}$ pins down the production isoquant along which $a_t c_t$ is varied. This is an important result as it highlights that an increasing share of clean inputs in production alone may not be enough to trigger investment in better substitutability, i.e., to make it the most profitable option. Rather, output works as a multiplier pointing to an important market size effect.

Finally, it is, *a priori*, not apparent whether an increase in the elasticity of substitution rather favors investment in clean or in dirty inputs. Yet, for $a_t c_t > \bar{a}_{NCN}^N$, the effect on marginal products is unambiguously in favor of clean inputs:

$$\frac{\partial}{\partial \rho_t} \left(\frac{\frac{\partial E_t}{\partial(A_t^C C_t)}}{\frac{\partial E_t}{\partial(A_t^D D_t)}} \right) = \frac{\alpha}{1-\alpha} (a_t c_t)^{-2\rho_t} \log[a_t c_t] > 0 \quad \text{for } a_t c_t > \bar{a}_{NCN}^N = 1. \quad (18)$$

Thus, an increase in the elasticity of substitution not only increases the marginal product of investment in clean inputs, which follows from symmetry of the cross second derivative given by Equation (13), but also favors further investment in clean inputs relatively more than investment in dirty ones.

3 Investment Dynamics in a Growing Economy

3.1 Investment Patterns Toward Green Growth ($\rho < 0$)

In this subsection, I analyze how the profitabilities of the different investment possibilities change with economic growth and how they interact with each other. That is, investment patterns are derived, and predictions with respect to possible growth paths are discussed. To keep the analysis concise, growth in final good

production and its impact on the demand for the energy service composite, E_t , are not modeled explicitly. Instead, demand for E_t is assumed to exogenously increase over time (see Figure 1). Thus, the only objective of the representative producer is to provide the energy composite at the lowest possible costs. Moreover, for the baseline analysis, it is assumed that past innovation has no effect on current innovation such that $\gamma_\rho = \gamma_{A^j} = 0$. Additionally, without loss of generality, potential depreciation of production capacities for intermediates is neglected, i.e., $\delta^j = 0$. Finally, the analysis is separated into three consecutive phases, each of which may, in principle, be the starting point of an analysis.

Phase 1 - Business-as-Usual Growth

Phase 1 is supposed to resemble the production of the energy service composite during most of modern growth before there are first efforts to push the use of clean energy service intermediates. In this phase, dirty intermediates, based on fossil fuels, are the main input in production, while clean intermediates play only a minor role. Moreover, investment in better substitutability is assumed to not yet be profitable during this phase.

Cost minimization in the production of E_t then requires that for every demand for the energy composite, the following first-order conditions hold:

$$\frac{\frac{1}{\phi^C} \frac{\partial E_t}{\partial C_t}}{\frac{1}{\phi^{AC}} \frac{\partial E_t}{\partial A_t^C}} \stackrel{!}{=} 1, \quad \frac{\frac{1}{\phi^D} \frac{\partial E_t}{\partial D_t}}{\frac{1}{\phi^{AD}} \frac{\partial E_t}{\partial A_t^D}} \stackrel{!}{=} 1, \quad \frac{\frac{1}{\phi^C} \frac{\partial E_t}{\partial C_t}}{\frac{1}{\phi^D} \frac{\partial E_t}{\partial D_t}} \stackrel{!}{=} 1. \quad (19)$$

These three conditions imply that, in optimum, all investment possibilities, except investment in better substitutability, have the same marginal profitability. Together, they determine the cost-minimizing ratio of inputs, $\overline{a_t c_t}^*$:

$$\overline{a_t c_t}^* = \left(\frac{\alpha}{1 - \alpha} \right)^{\frac{2}{1 - 2\rho_t}} (r_\phi r_{\phi^A})^{\frac{-1}{1 - 2\rho_t}}, \quad (20)$$

where $r_\phi = \frac{\phi^C}{\phi^D}$ and $r_{\phi^A} = \frac{\phi^{AC}}{\phi^{AD}}$ denote cost ratios. The optimal ratio of clean to dirty inputs during this early phase can be interpreted as the “natural” normalization point ratio of inputs, $\overline{a_N c_N}^N$ (see also Appendix A.2). This implies that $\overline{a_t c_t}^* = \overline{a_t c_t}^N$ such that investment in better substitutability has no effect on output and is, therefore, not profitable. Furthermore, there is both convergence to and stability at $\overline{a_t c_t}^*$ (see Appendix A.3). Thus, for constant prices and a constant elasticity of substitution, the production of the energy composite from clean and dirty inputs follows a balanced growth path, with the growth rate of

inputs equal to that of the exogenous demand for E_t .¹⁹ Finally, along $\overline{a_t c_t}^*$, the marginal profitabilities of investment in clean and dirty intermediates as well as in their corresponding efficiencies are given by:

$$\left. \frac{1}{\phi^j} \frac{\partial E_t}{\partial j_t} \right|_{a_t c_t = \overline{a_t c_t}^*} = \left. \frac{1}{\phi^{A^j}} \frac{\partial E_t}{\partial A_t^j} \right|_{a_t c_t = \overline{a_t c_t}^*} = \underbrace{\alpha \left(\frac{e_t}{a_t c_t} \right)^{\frac{1-2\rho_t}{2}} \left(\phi^C \phi^{A^C} \right)^{-\frac{1}{2}}}_{\text{const. } |_{a_t c_t = \overline{a_t c_t}^*}} \sqrt{E_t}, \quad (21)$$

where $e_t = (A_t^D D_t)^{-1} E_t$. As illustrated in Figure 3, along $\overline{a_t c_t}^*$, the profitabilities of marginal investments all grow at the same rate, but grow sub-linearly in E_t .

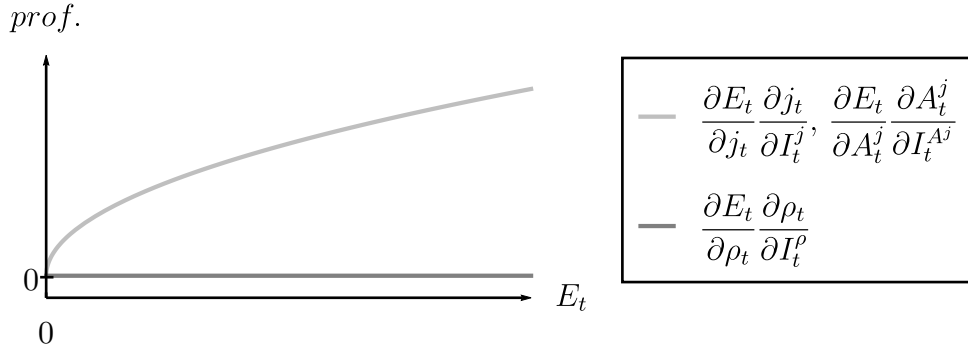


Figure 3: Marginal Profitability (*prof.*) of Investment in Intermediates, Efficiencies, and Substitutability Along $a_t c_t = \overline{a_t c_t}^* = \overline{a_N c_N}^N$ - Comparison.

Phase 2 - More Clean Inputs and First Investment in Substitutability

At the beginning of Phase 2, clean energy inputs receive an exogenous push, for instance, by falling prices of clean inputs ($\phi^C, \phi^{A^C} \downarrow$). As a result, the cost-minimizing input ratio increases ($\overline{a_t c_t}^* \uparrow$), and investment is redirected toward clean inputs. Moreover, while $\overline{a_t c_t}^* = \overline{a_N c_N}^N$ held during Phase 1, now $\overline{a_t c_t}^* > \overline{a_N c_N}^N$ becomes true.²⁰ This is because cost advantages make it optimal to use more clean inputs despite their marginal product decreasing rapidly outside of $\overline{a_t c_t}^* = \overline{a_N c_N}^N$. Thus, for $\overline{a_t c_t}^* > \overline{a_N c_N}^N$, increasing substitutability between clean and dirty inputs now has a positive effect on output. Along

¹⁹Note, the growth rate of inputs is equal to that of E_t , but those of intermediates and efficiency are lower: $g_t^j = g_t^{A^j} = ((1 + g_t^E)^{0.5}) - 1$, where g_t^E is the growth rate of demand for the energy composite and g_t^j and $g_t^{A^j}$ are the growth rates of intermediates and efficiency, respectively.

²⁰ $\left(\frac{\alpha}{1-\alpha} \right)^2 (r_\phi r_{\phi^A})^{-1} > \overline{a_N c_N}^N$ guarantees $\overline{a_t c_t}^* > \overline{a_N c_N}^N$. That is, the more important dirty inputs are for production (low α), the cheaper clean intermediates or respective research must be (low $\frac{1}{r_\phi} \frac{1}{r_{\phi^A}}$). Moreover, if $\overline{a_t c_t}^* > \overline{a_N c_N}^N$ for any degree of substitutability, then this condition is fulfilled for all values ρ_t .

$\overline{a_t c_t}^*$, the development of the profitability of a marginal investment in better substitutability is given by:

$$\frac{\partial E_t}{\partial I_t^p} = \frac{1}{\phi^\rho} \underbrace{E_t}_{OE_t > 0} \underbrace{\left(\frac{\alpha(a_t c_t)^{\rho_t} \log[a_t c_t]}{(\alpha(a_t c_t)^{\rho_t} + (1 - \alpha)) \rho_t} - \frac{\log[\alpha(a_t c_t)^{\rho_t} + (1 - \alpha)]}{\rho_t^2} \right)}_{\substack{RE_t \geq 0 \\ \text{const. } |_{a_t c_t = \overline{a_t c_t}^*}}}, \quad (22)$$

where RE_t is constant along any $\overline{a_t c_t}^*$. Thus, exclusively driven by the output effect, OE_t , the profitability of investment in better substitutability grows linearly in E_t .²¹ This is illustrated in Figure 4.

Finally, as also illustrated in Figure 4, along $\overline{a_t c_t}^*$, the profitability of investment in better substitutability, from some point on, grows faster than that of further investment in clean and dirty intermediates and efficiency. Thus, even if initially less profitable, investment in substitutability always becomes the most profitable option at some point in time such that investment is targeted toward research increasing ρ_t . This is the moment when the elasticity of substitution endogenously starts to rise.

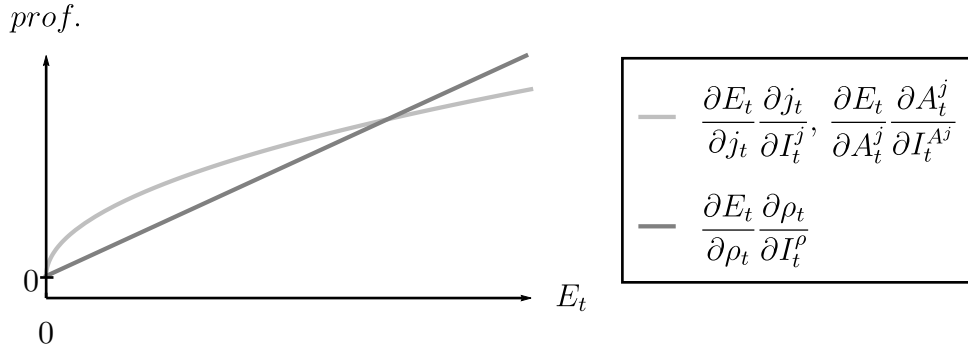


Figure 4: Marginal Profitability ($prof.$) of Investment in Intermediates, Efficiencies, and Substitutability Along $a_t c_t = \overline{a_t c_t}^* > \overline{a_N c_N}^N$ - Comparison.

Phase 3 - After the First Investment in Substitutability

Once that investment in the elasticity of substitution has become the most profitable option and, thus, ρ_t has increased, there are three effects to be considered for the further analysis of investment dynamics:

²¹During the transition period from $\overline{a_t c_t}^* = \overline{a_N c_N}^N$ to $\overline{a_t c_t}^* > \overline{a_N c_N}^N$, the ratio effect, RE_t , becomes non-zero and increases (see Equation (14)).

First, given $\overline{a_t c_t}^* > \overline{a_N c_N}^N$, an increase in the elasticity of substitution always increases the optimal ratio of inputs in favor of the clean ones:

$$\frac{\partial \overline{a_t c_t}^*}{\partial \rho_t} > 0 \quad \text{for } \overline{a_t c_t}^* > \overline{a_N c_N}^N. \quad (23)$$

Intuitively speaking, an increase in the elasticity of substitution makes inputs more interchangeable and, thus, allows for exploiting the relative cost advantage of clean energy inputs to a larger extent without rapidly suffering from a declining marginal product. *Ceteris paribus*, there is again convergence toward the new, higher $\overline{a_t c_t}^*$. Once reached, the same (investment) dynamics as for Phase 2 apply.

Second, for every (fixed) $E_t = \overline{E}$, an increase in the elasticity of substitution also increases the absolute profitability of investment in intermediates and efficiency in the new optimal input ratio:²²

$$\frac{\partial}{\partial \rho_t} \left(\frac{1}{\phi^C} \frac{\partial E_t}{\partial C_t} \Big|_{a_t c_t = \overline{a_t c_t}^* \text{ \& } E_t = \overline{E}} \right) \geq 0. \quad (24)$$

Along $a_t c_t = \overline{a_t c_t}^*$, the absolute profitability additionally increases due to the positive effect of rising output, as shown in Equation (21). Together, this guarantees that after an increase in the elasticity of substitution, the marginal profitability of investment in clean inputs is higher both in the new $\overline{a_t c_t}^*$ and during the transition to it.

Third, as shown in Equation (12), the effect of an increasing elasticity on the marginal product of a further increase of it can either be positive or negative. This property directly translates to the profitability of investment in the elasticity of substitution:²³

$$\frac{1}{\phi^\rho} \frac{\partial^2 E_t}{\partial \rho_t^2} \gtrless 0. \quad (25)$$

Overall, the effect of improving substitutability on the profitability of investment in clean inputs is always positive, whereas the effect on the profitability of further investment in substitutability can be either positive or negative. Thus, whether a marginal investment in substitutability rather fosters further investment in substitutability or in clean intermediates and efficiency depends on the specific parameters and the state of the economy at time t .

For a sequence of marginal investments, three scenarios, and combinations thereof, are possible. For the sake of a clear exposition, the starting point for

²²Since, along $\overline{a_t c_t}^*$, the profitability of all investment possibilities must be equal, it is sufficient to analyze only one type of investment, here investment in C_t .

²³All other properties, e.g., the convex-concave shape, are preserved as well.

these scenarios is the end of Phase 2, i.e., when all investment possibilities are equally profitable and there is an increase in the elasticity of substitution for the first time.

Possible Scenarios:

- #1 There is first only investment in clean inputs until the new, higher optimal input ratio, $\overline{a_t c_t}^*$, is reached. During this transition, both the ratio effect (RE_t) and the output effect (OE_t) increase the marginal profitability of further investment in the elasticity of substitution. However, further investment in substitutability only becomes the most profitable option again after $\overline{a_t c_t}^*$ is reached, and there has been a phase of balanced growth along the new $\overline{a_t c_t}^*$ (OE_t increases).
- #2 Initially, there is only investment in clean inputs but, before the new, higher optimal input ratio, $\overline{a_t c_t}^*$, is reached, the increasing ratio effect (RE_t) and output effect (OE_t) already trigger further investment in better substitutability. In a way, $a_t c_t$ is chasing $\overline{a_t c_t}^*$.
- #3 An increase in the elasticity of substitution immediately makes further investment in better substitutability the most profitable option.²⁴

Most importantly, in all three scenarios, there is recurrent, endogenous investment in better substitutability such that eventually dirty inputs inevitably become inessential for the production of the energy composite. This is the moment when green growth becomes possible.²⁵

3.2 Investment Patterns During Green Growth ($\rho \geq 0$)

The above analysis for $\rho_t < 0$ also applies to $\rho_t = 0$. Thus, also for the Cobb-Douglas case, there will necessarily be investment in better substitutability at some point in time. Consequently, the degree of substitutability eventually becomes $\rho_t > 0$. Moreover, starting in $\rho_t = 0$, an increase in substitutability again favors further investment in clean inputs rather than investment in dirty ones. Moreover, unlike for $\rho_t < 0$, for $\rho_t > 0$, investment in clean inputs makes further

²⁴That is, the effect captured by Equation (25) dominates that of an increase in substitutability on the profitability of investment in inputs as captured by Equation (13), taking into account symmetry of cross second derivatives, together with (3) and (5).

²⁵Note that Scenarios #1 and #2 build on the assumption that there can only be one type of investment at the same time. However, if simultaneous investment is possible, there will always also be investment in inputs whenever there is investment in substitutability, unless Scenario #3 applies.

investment in clean inputs more profitable than investment in dirty ones (see Appendix A.3). Therefore, if there is investment in inputs, it will be investment in clean inputs. In other words, there is a “lock-in” in clean inputs. Thus, like in Acemoglu et al. (2012), for $\rho_t > 0$, there is a continuous rise in $a_t c_t$ even if the elasticity of substitution is constant. Most importantly, for $\rho_t > 0$, green growth is not only possible but also chosen as the cost-minimizing way to produce the energy composite.²⁶ Moreover, as the relative profitability of investment in clean inputs increases steadily for $\rho_t > 0$, possibly existent subsidies on clean inputs can be phased out while green growth continues. The same applies to taxes on dirty inputs.

Although there is only green growth for $\rho_t > 0$, there may well be further investment in substitutability.²⁷ The following implications of $\rho_t > 0$ for the two equations governing investment in inputs and in substitutability, i.e., Equations (21) and (22), have to be considered:

Equation (22) remains unchanged. However, even for a constant ρ_t , $a_t c_t$ now continuously increases in E_t such that $\Delta_t^{RE} > 0$. Thus, the profitability of investment in better substitutability now increases more than linearly (convex) in E_t . In contrast, Equation (21) needs to be changed as, for $\rho_t > 0$, $a_t c_t$ no longer converges to a constant $\overline{a_t c_t^*}$, and now reads:

$$\frac{1}{\phi^j} \frac{\partial E_t}{\partial j_t} = \frac{1}{\phi^{A^j}} \frac{\partial E_t}{\partial A_t^j} = \alpha \left(\phi^C \phi^{A^C} \right)^{-\frac{1}{2}} E_t^{1-\rho_t} \left[\frac{E_t^{\rho_t} - (1-\alpha)(A_t^D D_t)^{\rho_t}}{\alpha} \right]^{\frac{2\rho_t-1}{2\rho_t}}, \quad (26)$$

where $A_t^D D_t$ is constant, and investment in capacity and in efficiency within the clean intermediate are assumed to be equally profitable. Although it is not feasible to analytically identify a clear convex/concave relationship between E_t and the profitability of investment in clean inputs, numerical simulations and limit considerations hint again at a generally concave relationship. Therefore, with the profitability of investment in substitutability increasing more than linearly, and that of investment in clean capacities and efficiency most likely increasing sub-linearly, substitutability will necessarily also rise over time for $\rho_t > 0$. This is also confirmed by the numerical simulation Section 5.

²⁶With depreciation of clean and dirty intermediate production capacities (at the same rate), there is not only green growth but also a full decarbonization of the stock of existing production capacities (see Section 5).

²⁷As long as there is a positive stock of dirty intermediate production capacities, there is a positive effect of higher substitutability on production (see Section 2.2).

3.3 Alternative Specifications of the Research Process

So far, the productivity of investment in research has been assumed to be independent of past innovation, i.e., $\gamma_\rho = \gamma_{Aj} = 0$. In this subsection, this assumption is relaxed. For $\gamma < 0$, past innovation makes further improvements easier, whereas, for $\gamma > 0$, past innovation makes further technological progress more difficult.²⁸

3.3.1 Efficiency-Enhancing Technological Progress

Only $\gamma_{Aj} < 0$ is considered for efficiency-enhancing technological progress.²⁹ First, because, for dirty inputs, it is unlikely that efficiency improvements become easier over time given thermodynamic limitations. Second, because efficiency improvements are also limited for clean inputs. For instance, the maximum theoretical conversion rate of sunlight to electricity is at about 86.8% (Chambadal-Novikov efficiency - Chambadal, 1957; Novikov, 1958), and much lower under real-life conditions. Finally, for simplicity, dependence of research productivity on past innovation is assumed to be identical for clean and dirty inputs, i.e., $\gamma_{Ac} = \gamma_{Ad} = \gamma < 0$.³⁰

As shown in Appendix A.3, for $\rho_t \leq 0$, there is again convergence to a cost-minimizing input ratio $\overline{a_t c_t}^*$, which is now dependent on γ .³¹

$$\overline{a_t c_t}^* = (r_{\phi j})^{\frac{\gamma}{1+\rho_t(-2+\gamma)-\gamma}} \left(r_{\phi j} r_{\phi A j} \right)^{\frac{-1}{1+\rho_t(-2+\gamma)-\gamma}} \left(\frac{\alpha}{1-\alpha} \right)^{\frac{2-\gamma}{1+\rho_t(-2+\gamma)-\gamma}}, \quad (27)$$

where $\overline{a_t c_t}^* > \overline{a_N c_N}^N$ holds whenever $(r_{\phi j})^\gamma \left(r_{\phi j} r_{\phi A j} \right)^{-1} \left(\frac{\alpha}{1-\alpha} \right)^{2-\gamma} > \overline{a_N c_N}^N$. Equation (27) also shows that $\overline{a_t c_t}^*$ is independent of output, E_t . However, unlike for $\gamma = 0$, the ratio of intermediates to the respective level of efficiency (j_t/A_t^j) now increases in A_t^j and, thus, in output:

$$\frac{\partial}{\partial A_t^j} \left(\frac{j_t}{A_t^j} \Big|_{a_t c_t = \overline{a_t c_t}^*} \right) = -\gamma \frac{\phi^{A_j}}{\phi^j} (A_t^j)^{-\gamma-1} > 0 \quad \text{for } \gamma < 0. \quad (28)$$

²⁸These two cases are sometimes also referred to as “fishing out” (Groth, 2007, p. 131) and “standing on the shoulders of giants” (see Caballero and Jaffe (1993) and the references therein).

²⁹For the case with $\gamma_{Aj} > 0$, more specifically, for $\gamma_{Aj} > 1$, a lock-in in either clean or dirty inputs is possible even under $\rho_t < 0$. A proof is available upon request.

³⁰The case $0 > \gamma_{Ad} \neq \gamma_{Ac} < 0$ is briefly discussed in Appendix A.3 - *Claim 13*.

³¹The same optimality conditions as for the case with $\gamma = 0$ apply (see Equation (19)).

Still, along $\overline{a_t c_t}^*$, the productivities of marginal investments in clean and dirty intermediates and efficiency must all be equal and, for $\gamma < 0$, are given by:

$$\left. \frac{\partial E_t}{\partial I_t^j} \right|_{a_t c_t = \overline{a_t c_t}^*} = \left. \frac{\partial E_t}{\partial I_t^{A^j}} \right|_{a_t c_t = \overline{a_t c_t}^*} = \underbrace{\frac{\alpha}{\phi^C} \left(\frac{e_t}{a_t c_t} \right)^{\frac{(1-\rho_t)(2-\gamma)-1}{2-\gamma}} \left(\frac{\phi^C}{\phi^{A^C}} \right)^{\frac{1}{2-\gamma}}}_{\text{const. } |_{a_t c_t = \overline{a_t c_t}^*}} E_t^{\frac{1}{2-\gamma}}. \quad (29)$$

Importantly, Equation (29) shows that, along $\overline{a_t c_t}^*$, the productivities of investments in inputs again grow sub-linearly in output. Moreover, the profitability of a marginal investment in substitutability is not directly affected by γ and, thus, remains linear (see Equation (22)).³² Summarized, qualitatively, the analysis of investment dynamics throughout the different phases of growth remains unchanged compared to the case with $\gamma = 0$. Most importantly, again, there is recurrent investment in better substitutability such that dirty inputs become inessential for the production of the energy composite at some point in time. Finally, also for $\rho_t > 0$, the analysis remains unchanged.³³

3.3.2 Substitutability-Increasing Technological Progress

Bloom et al. (2020) present evidence from various industries pointing in favor of the “fishing out” case for total factor productivity. Thus, only $\gamma_\rho < 0$ is considered here, which can be discussed separately from $\gamma_{AC} = \gamma_{AD} = \gamma < 0$. Importantly, γ_ρ only affects investment dynamics in Phase 3, i.e., when there is already investment in substitutability. Specifically, $\gamma_\rho < 0$ increases the costs of further investment in substitutability. Thus, for each of the three scenarios in Phase 3, this implies that, *ceteris paribus*, there is a lower incentive to invest in substitutability and ρ_t increases more slowly compared to the case with $\gamma_\rho = 0$.

4 Government Intervention and Optimal Policy

The analysis in the previous sections shows that $\overline{a_t c_t}^* > \overline{a_N c_N}^N$ is a sufficient condition to trigger an endogenous transformation toward green growth. This transformation is driven by increases in the elasticity of substitution, which gradually raises the cost-minimizing share of clean inputs in production and, eventually, renders dirty inputs inessential. In this section, I analyze how a tax on dirty

³²Also the second derivative of output with respect to the elasticity of substitution is not affected by γ other than through $\overline{a_t c_t}^*$, and, thus, remains unchanged.

³³As for $\rho_t \leq 0$, the ratio of intermediates to the respective level of efficiency (j_t/A_t^j) also increases for $\rho_t > 0$, if $\gamma < 0$.

intermediate production capacities as well as subsidies for research increasing efficiency or substitutability affect both the onset and the speed of the transformation process. Moreover, it is discussed how taxes and subsidies can be used to implement the socially optimal trajectory of investments.

4.1 Taxes and Research Subsidies

Subsidies reduce the costs of research, ϕ^{AC} and ϕ^ρ , by the amount of τ_t^{AC} and τ_t^ρ , whereas a tax on dirty inputs increases the costs of building new intermediate production capacities, ϕ^D , by the amount τ_t^D .³⁴

A Subsidy for Research on Clean Efficiency (τ_t^{AC}):

A subsidy for research targeted at increasing efficiency in the use of clean intermediates has two effects. First, as can be seen from Equation (27), for $\rho_t < 0$, a subsidy shifts the cost-minimizing ratio of clean to dirty inputs in favor of clean ones, i.e., $\overline{a_t c_t^*}$ rises. This increases the ratio effect, RE_t such that the profitability of investment in substitutability increases more rapidly in the production of the energy composite, E_t . Second, along with $\overline{a_t c_t^*}$, the absolute profitability of investment in clean (and dirty) inputs also increases in τ_t^{AC} for every level of production of the energy composite (see Appendix A.3). Thus, as illustrated in Figure 5, a subsidy for clean research makes both investment in clean inputs and investment in substitutability more profitable. Yet it is, *a priori*, not clear whether it prepones or postpones investment in substitutability.³⁵ The identification and analysis of qualification constraints for either case are beyond the scope of this paper and, therefore, left for future research.

A Tax on New Production Capacities for Dirty Intermediates (τ_t^D):

On the one hand, a tax on building new intermediate production capacities also raises $\overline{a_t c_t^*}$, thereby increasing the profitability of investment in substitutability. On the other hand, such a tax reduces the absolute profitability of a marginal investment in clean (and dirty) intermediates for every level of production of the

³⁴Moreover, although only presented for the case with $\gamma = 0$ here, the analysis of taxes and subsidies also applies to $\gamma < 0$ (see Appendix A.3).

³⁵Numerical simulations indicate that there is always a preponing effect. Moreover, even if the onset of investment in substitutability was postponed, production along $\overline{a_t c_t^*}$ is always cleaner in the presence of a subsidy. However, a subsidy also reduces the production costs of the energy composite, thereby increasing demand for it. Therefore, the overall effect of a subsidy on CO₂ emissions over time is again not clear.

energy composite (see Appendix A.3). Thus, as can be seen from Figure 5, the onset of investment in substitutability is unambiguously preponed in this case.³⁶

A Subsidy on Substitutability-Increasing Research (τ_t^ρ):

This kind of subsidy lowers the costs of investment in research targeted at increasing the substitutability between clean and dirty inputs. As a result, the profitability of investment in substitutability increases more rapidly in the production of the energy composite, E_t (see Equation (22)). By contrast, the profitability of investment in intermediates and efficiencies is not affected. Thus, *ceteris paribus*, a research subsidy always induces an earlier onset of investment in substitutability, as illustrated in Figure 5.

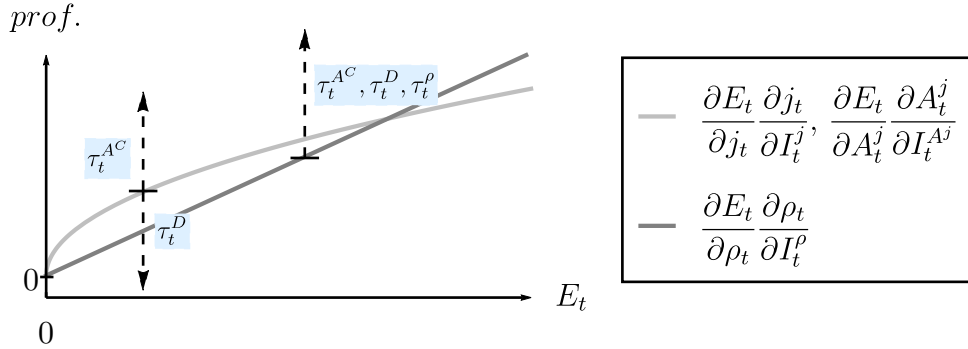


Figure 5: The Effects of τ_t^{AC} , τ_t^D , and τ_t^ρ on the Marginal Profitability (*prof.*) of Investment in Intermediates, Efficiencies, and Substitutability Along $a_t c_t = \overline{a_t c_t}^* > \overline{a_N c_N}^N$ - Comparison.

In general, temporary taxes and subsidies do not change the long-run cost-minimizing ratio of clean to dirty inputs, $\overline{a_t c_t}^*$. That is, the use of inputs converges back to the ratio that is cost-minimizing in the case without taxes and subsidies, as soon as these are withdrawn. However, if, as a consequence of temporary policy intervention, the degree of substitutability increases, then $\overline{a_t c_t}^*$ also increases, as shown in Equation (23). Importantly, this effect is not (completely) reverted if taxes and subsidies are withdrawn again, such that there is a long-run impact on the cost-optimal utilization of clean and dirty inputs. This implies that an only temporary subsidy or tax can be sufficient to render dirty

³⁶However, in the current framework, a tax only affects the profitability of investment in new production capacities. That is, the tax does not cover existing production capacities of dirty intermediates, and its effect on investment dynamics is less pronounced compared to more direct approaches increasing the price of fossil-fuel input or of carbon emissions.

inputs inessential for production ($\rho_t > 0$) and to guarantee green growth in the long-run.

Overall, subsidies and taxes can work as an accelerator for the transformation process. In particular, a temporary subsidy for research that improves substitution possibilities is likely to be an important instrument to push the elasticity of substitution above the threshold level of unity such that green growth sets in before a climate disaster happens.

4.2 Optimal Policy

For a sequence of investment decisions of a myopic representative producer of the energy composite, policy intervention must always correct for at least two major externalities: First, the environmental damage caused by the use of dirty, CO₂ emitting energy services. Second, intertemporally non-optimal (too low and/or too late) investments in substitutability and efficiency due to the fact that myopic producers do not take into account the effect of these investments on future productivity. Without policy intervention, the resulting investment path is not only suboptimal in terms of intertemporal welfare maximization but, due to the neglected immediate effect of the environmental externality, also potentially Pareto-suboptimal within each period.

Unlike in standard growth models, because of the non-linearities that come along with the introduction of the possibility of investment in substitutability, there exists, in general, no longer a balanced growth path as long as the elasticity of substitution is increasing. Rather, growth rates and investment patterns vary substantially over time, at least in the transition period characterized by an increasing elasticity of substitution, as can be seen in Section 5. Still, based on the above analysis of possible policy interventions, it is clear that every possible investment trajectory can be implemented with a combination of research subsidies and a tax. In general, taxes and subsidies are required to vary over time.³⁷ Thus, with the right (temporary) environmental policy, the “market solution” based on a myopic representative producer coincides with that of a forward-looking social planner that explicitly accounts for externalities, e.g., those of CO₂ emissions. However, a full quantitative analysis of what optimal policy may look like is beyond the scope of this paper and, thus, left for future research.³⁸

³⁷At any point in time, policy intervention also needs to account for changes in investment patterns induced by exogenous factors, e.g., increasing costs of dirty intermediate production capacities due to fading resources or decreasing costs due to new explorations.

³⁸Importantly, in general, there exists no analytical solution for the equations of motion of optimal investment.

5 Numerical Simulations

The following numerical simulation exercise illustrates the dynamics of investment in substitutability, efficiency, and intermediate production capacities for a growing economy. The primary goal is to highlight the central investment dynamics and its impact on key variables of economic growth, especially the effect of an improving substitutability between clean and dirty inputs.

5.1 Setup

Importantly, in contrast to the above analysis, the simulation is based on a sequence of optimal investments for a non-marginal budget. Thereby, I abandon the simplifying assumption of only marginal investments directed toward the most profitable option as used for the sake of a clear argument in the theoretical analysis above (see also the discussion in Appendix A.1).

For the simulation, I employ a simple multi-level Solow-style growth model (Solow, 1956) with the structure depicted in Figure 6. Specifically, in a top-level CES production function, the energy composite, E_t , and capital, K_t , are combined to a final good, Y_t (see van der Werf, 2008). Labor input is neglected in final good production to keep the model simple and to avoid additional complexity due to labor-augmenting technological progress. The energy composite is provided with the structure presented in the theoretical analysis presented above. At the beginning of every time period, a constant fraction s of last period's final good production, Y_t , is invested in the capital stock in final good production, K_t , or in the production of the energy composite, E_t .³⁹

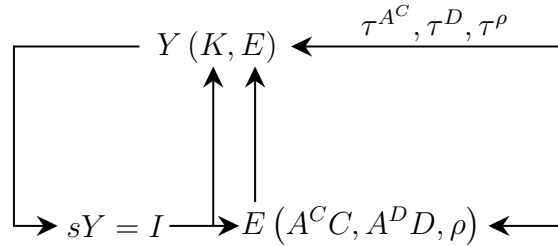


Figure 6: Production Structure for the Numerical Simulation.

³⁹Note that the budget constraint, Equation (9), now additionally needs to account for investment in capital, I_t^K .

Functional forms for the production of the energy composite, E_t , are generally as presented in Section 2. However, to correct for increasing returns to scale, the square root is applied to Equation (1) such that it now reads:⁴⁰

$$E_t = \sqrt{(\alpha (A_t^C C_t)^{\rho_t} + (1 - \alpha) (A_t^D D_t)^{\rho_t})^{\frac{1}{\rho_t}}}.$$

For the provision of the final good at the top-level, the following CES production function is used:

$$Y_t = (\beta K_t^\nu + (1 - \beta) E_t^\nu)^{\frac{1}{\nu}}, \quad (30)$$

where $\beta \in (0, 1)$ is the share parameter. The time-invariant elasticity of substitution, $\kappa \in (-1, \infty)$, between capital and the energy composite is expressed in terms of the substitution parameter, $\nu = \frac{\kappa - 1}{\kappa} \in (-\infty, 1)$. There is no factor-augmenting technological progress for capital.⁴¹ With respect to the energy composite, efficiency increases are already accounted for in its production by A_t^j .

As shown above, for $\rho_t < 0$, i.e., if both clean and dirty inputs are essential for the production of the energy composite, the only requirement for investment in substitutability to become profitable at some point in time is: $\overline{a_t c_t^*} > \overline{a_N c_N^N}$.⁴² Moreover, $\nu < 0$ is assumed to guarantee essentiality of both capital and energy. Baseline parameter choices and starting values are presented in Table A.4.1 in Appendix A.4.

5.2 Simulation Results

Simulation results for one scenario with and one without the possibility to invest in better substitutability are presented in Figure 7. Profitabilities of the respective investment possibilities are depicted in the bottom row of Figure 7. All results are in line with theoretical predictions.

Simulation 1 - With the Possibility to Invest in Substitutability

First, and importantly, as shown by Figure 7, from some point on, there is repeated investment in substitutability. Therefore, the elasticity of substitution

⁴⁰Increasing returns to scale are due to the fact that efficiency is treated as a separate input factor. Without correction, growth rates would explode. The correction only changes the relative spending between capital and energy but otherwise preserves the pattern of investment within the energy aggregate.

⁴¹This assumption is in line with Uzawa's (1961) famous theorem on balanced growth (see also Acemoglu (2003)).

⁴²Technically, the condition is: $\overline{a_t c_t^*} \neq \overline{a_N c_N^N}$. However, the case $\overline{a_t c_t^*} < \overline{a_N c_N^N}$ is treated as an artifact of the symmetry of the CES production function and, thus, ignored throughout the paper (see also Appendix A.2).

converges to its upper bound ($\rho_{max} = 1$), yet, with increases slowing down due to further increases becoming more difficult ($\gamma_\rho < 0$).

Before the onset of investment in substitutability, economic development is structurally similar to a balanced growth path with investment in both clean and dirty inputs such that a constant ratio ($a_t c_t = \overline{a_t c_t^*}$) is maintained. The growth rate of output, g_t^Y , decreases over time but converges to a positive value. This decrease in the growth rate is caused by the increasing difficulty to further improve efficiency ($\gamma_{A^j} < 0$).⁴³ As soon as ρ_t starts to increase, and, thus, already before $\rho_t > 0$, there is no further investment in dirty inputs, and the respective intermediate production capacities begin to fall because of depreciation.⁴⁴ As dirty technology, A_t^D , cannot depreciate, it remains constant from this point on. This halt of investment in dirty inputs is also reflected by an ever-increasing ratio of clean to dirty inputs, reaching infinity as soon as dirty intermediate production capacities have depreciated completely. Moreover, the income share, IS_t^C , of clean inputs in energy production increases with ρ_t and converges to unity.⁴⁵ Importantly, and in contrast to models with a constant elasticity, the model presented here can reconcile an increasing income share with a combination of an increasing ratio of clean to dirty inputs ($a_t c_t \uparrow$) and a low (but rising) elasticity of substitution ($\rho_t < 0 \uparrow$).⁴⁶ Furthermore, as increases in efficiency become more difficult over time, the costs of producing the energy composite increase in E_t . In contrast, there is no such effect for investment in the capital stock in final good production. Consequently, investment in E_t becomes relatively more expensive compared to investment in K_t such that the ratio E_t/K_t continually decreases. This pattern is only interrupted during the transition phase. In this phase, investment in better substitutability increases the productivity in the production of the energy composite, thereby lowering the costs of E_t (see Equation (13)). The same reasoning also applies to the hump with respect to growth in final output, g_t^Y . Thus, against common fear, investment in research that facilitates substituting clean for dirty energy inputs accelerates economic growth rather than slowing it down. Moreover, since CO₂ emissions are assumed to be propor-

⁴³For $\gamma_{A^j} = 0$, the growth rate is constant whenever there is not yet or no further investment in substitutability, thus characterizing a balanced growth path.

⁴⁴This is the pattern described by “Scenario #2” in Section 3.1.

⁴⁵The income share is calculated based on the assumption that factor enumeration is equal to the respective marginal product.

⁴⁶As can be seen from Equations (23) and (24), for $\rho_t \leq 0$, a rise in the elasticity of substitution increases $\overline{a_t c_t^*}$ while the ratio of investment profitabilities for clean and dirty inputs remains equal to unity. For $\rho_t > 0$, $a_t c_t$ continues to rise, while, additionally, the ratio of investment profitabilities for clean and dirty inputs increases in favor of the clean ones (see also Section 3.2).

tional to dirty intermediate production capacities, emissions per period decrease with the stock of dirty intermediate production capacities and, therefore, the increase in accumulated emissions, $\sum_{t=0}^t \text{CO}_2$, eventually comes to a halt. Finally, once there is no more investment in the elasticity of substitution, there is a return to a pattern structurally resembling balanced growth. Yet, importantly, now, growth is entirely based on clean inputs.

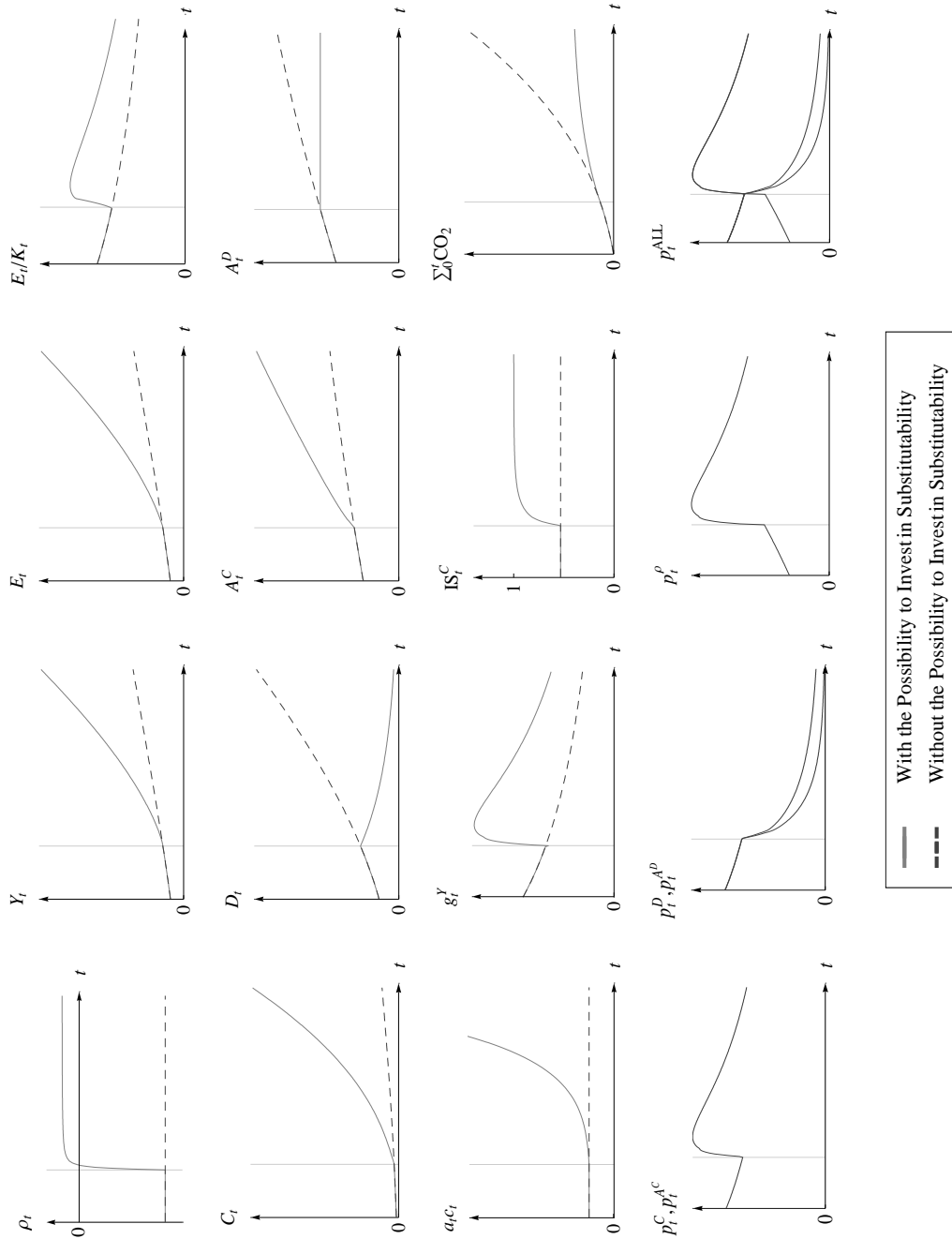


Figure 7: (Investment) Dynamics for a Non-Marginal Investment Budget.

Simulation 2 - Without the Possibility to Invest in Substitutability

Without the possibility to invest in better substitutability, dirty inputs remain essential for the production of the energy composite. Thus, along with dirty intermediate production capacities, emissions also increase exponentially. Moreover, rather than increasing to infinity, the ratio of clean to dirty inputs remains constant over time. Similarly, the income share of dirty inputs is constant and does not decrease. Finally, without investment in better substitutability, it is not possible to better exploit the cost advantage of clean inputs, thereby reducing the costs of producing the energy composite. Consequently, the ratio E_t/K_t is always lower compared to the case with investment in substitutability. The lack of this option to lower the costs of producing the energy composite is also reflected by a lower growth rate of output $,g_t^Y$.

6 Conclusion

Standard models of green growth build on perpetually increasing efficiency in the use of fossil fuels and neglect the possibility of an improving substitutability between clean and dirty energy inputs. However, this approach conflicts with thermodynamic laws and does not reflect the observed firm investment behavior. In this paper, I develop and analyze a growth model that explicitly accounts for endogenous investment to increase input substitutability, in addition to investment in efficiency. Importantly, investment in substitutability allows to render dirty, CO₂-emitting fossil fuels inessential for production in the long-run. Therefore, green growth no longer solely relies on perpetually increasing efficiency.

The new modeling approach yields four main insights. First, the higher the elasticity of substitution, the easier it is to replace the relatively more expensive energy input with the cheaper one. Thus, there is always an incentive to invest in better substitutability. This incentive increases linearly in output. Second, for a growing economy, investment in better substitutability always becomes profitable at some point in time. In parallel to investment in substitutability, investment in input efficiency continues. With ongoing investment in substitutability, dirty inputs eventually become inessential for production. Third, at the latest when clean and dirty inputs turn from complements to substitutes, there is a complete shift toward clean inputs with no further investment in dirty inputs. Fourth, temporary policy interventions directly or indirectly promoting investment in better substitutability can trigger a full transformation toward green growth.

Importantly, all these results do not hinge on a carbon cap or infinitely high prices of, or taxes on, fossil fuels.

The possibility to invest in better substitutability between clean and dirty inputs has a major impact on growth dynamics, as a simple simulation exercise shows. First, during the phase of an increasing elasticity, the growth rate of output receives a positive push. Additionally, at any point in time, the growth rate of output depends, *ceteris paribus*, positively on the elasticity of substitution. This result challenges the widespread fear of strict environmental policy being potentially growth-dampening, especially with respect to the transition period. Moreover, along with the increase in the degree of substitutability, the income share of clean inputs also rises and converges to unity. Finally, driven by the crucial dependence of the profitability of investment in substitutability on output, the development of CO₂ emissions follows a convex-concave trajectory with emissions increasing exponentially until investment in substitutability sets in. This finding suggests the potential existence of an environmental Kuznets curve.

While this paper provides a sound theoretical framework to analyze the dynamics during the transition toward green growth, a thorough quantitative assessment is beyond the scope of this paper. In particular, it is an important task for future research to assess under what conditions a transformation toward green growth happens fast enough to avoid a climate disaster and what optimal policy should look like.

Overall, this paper provides a novel approach to rationalize an endogenous transition from a world based on fossil fuels and characterized by a low elasticity of substitution toward a world that builds on green growth and exhibits a high degree of substitutability. Hence, to some extent, this paper also fills the gap between the two extreme cases presented in Acemoglu et al. (2012).

A Appendix

A.1 Marginal vs. Non-Marginal Investments

The analysis of investment in intermediate production capacities, efficiency, and substitutability is based on a comparison of the profitabilities of a marginal investment for any initial conditions. This approach relies on the claim that if there is investment in an option for a marginal budget, then there is also investment in that option for a non-marginal budget. This claim can be shown to be true as follows:

First, for a constant ρ_t , i.e., whenever there is no investment in the elasticity of substitution, E_t is strictly concave in investment in inputs. Thus, integrating over a sequence of marginal investments directed toward the most profitable option results in the same solution that is obtained for a non-marginal investment budget. This optimal solution is characterized by $a_t c_t = \overline{a_t c_t^*}$ as defined in Section 3.

Second, the optimization problem for the case with only marginal investments can be seen as an additionally constrained variant of that with a non-marginal investment budget. Then, every solution to the optimization problem with only marginal investments is also feasible for the problem with a non-marginal investment budget. Most importantly, this guarantees that the positive effect of an increase in the elasticity of substitution on output for the non-marginal case is always equal to or higher than in the case with only marginal investments. Intuitively speaking, for a non-marginal budget, there is additionally the effect of a higher elasticity of substitution on how profitable the remaining budget can be spent on clean and dirty inputs.

Third, if an investment possibility has the highest cost-weighted marginal product, i.e., profitability as defined in Section 2, it is guaranteed that at least some of the investment budget is spent on it. This is a direct application of the Karush-Kuhn-Tucker-conditions of the corresponding optimization problem with non-negativity constraints.

In general, during transition, i.e., when substitutability increases, the timing of investment in substitutability, efficiency, and inputs, in general, does not coincide for the two cases.⁴⁷ However, for a growing economy with increasing energy demand but no further investment in the elasticity of substitution, the two cases

⁴⁷For instance, as an extreme case, for a sufficiently large investment budget, it may be optimal to increase the elasticity of substitution directly to a value above unity before there is any further investment in inputs.

always converge to the same outcome in terms of ratios, growth rates, and shares in the long-run.

Figure A.1.1 illustrates differences in investment dynamics between the case with a sequence of marginal investments in the most profitable option and the case with a sequence of non-marginal investment budgets.⁴⁸ Importantly, it shows the earlier onset of investment in better substitutability for a non-marginal investment budget due to the additional investment incentive. However, it also shows that in both cases, there is a halt to investment in dirty inputs once the elasticity of substitution exceeds unity. Therefore, the income share of clean inputs, IS_t^C , in the production of the energy composite converges to unity for both cases.

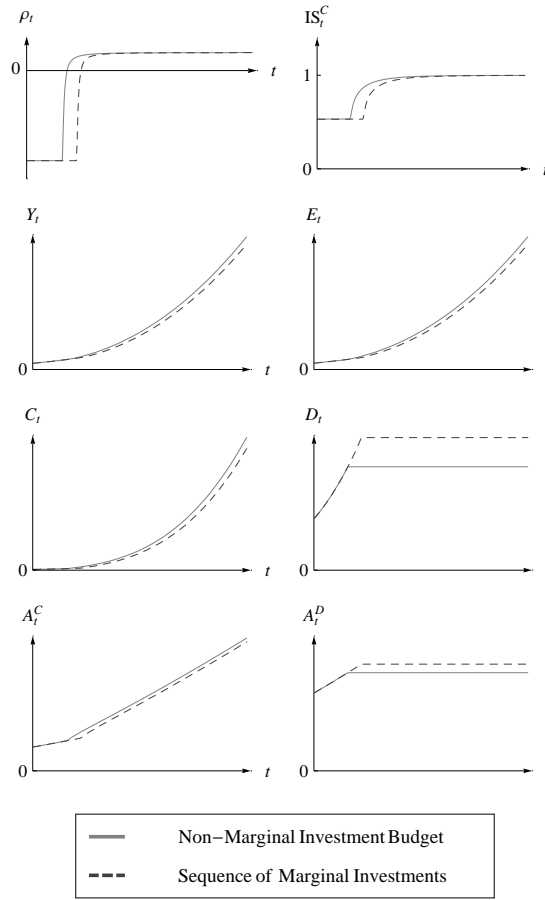


Figure A.1.1: Dynamics for an Adjusted Sequence of Marginal Investments and a Sequence of Investment for a Non-Marginal Investment Budget - Comparison.

⁴⁸Simulations are based on the same parameters and starting values used in Section 5. However, to facilitate comparison, without depreciation of intermediate production capacities ($\delta = 0$). The sequence of (almost) marginal investments in the option with the highest profitability (see Sections 2 and 3) is adjusted such that only those simulation points are considered where the sum of marginal investments is equal to savings, sY_t , for the simulation with a non-marginal investment budget.

A.2 Normalization

The elasticity of substitution is a measure of relative change. Thus, in order to pin down a specific CES function, it is necessary to define a reference point, called normalization point (see de La Grandville, 1989; Klump and de La Grandville, 2000).⁴⁹ More intuitively, the normalization point can be understood as defining the efficient ratio of inputs if there were no substitution possibilities; i.e., if the CES takes the form of a Leontief production function ($\rho_t = -\infty$). In other words, in this normalization point ratio, inputs are used in such proportions that there is no relatively abundant input that is used as an imperfect substitute for the relatively scarce one. In a way, the normalization point defines the “natural” input ratio. Therefore, in (and only) in the normalization point ratio, an increase in the elasticity of substitution has no positive effect on output. Outside of the normalization point ratio, better substitutability lowers the productivity losses when the abundant input is used as an imperfect substitute for the scarce one and, thus, has a positive effect on output. More technically, an increase in the elasticity of substitution mitigates the diminishment of marginal returns to inputs. Thus, the normalization point ratio only defines the efficient ratio in terms of the production function. Whenever price advantages for either input factor outweigh the diminishment of marginal returns, the cost-optimal ratio of production factors differs from that in the normalization point, i.e., in general $\overline{a_t c_t^*} \neq \overline{a_t c_t^N}$.

While the choice of the normalization point has no qualitative effect on the question of whether there is investment in substitutability or not, the quantitative impact on results may be substantial. Unfortunately, a proper choice of the normalization point (ratio) can be very difficult for numerical analysis at the aggregate level.⁵⁰ In particular, in an energy context, there is no “natural” or “straightforward” candidate for the normalization point ratio. However, as fossil fuels are currently the standard input for almost all energy-based production processes, the normalization point ratio of clean to dirty inputs, $\overline{a_N c_N^N}$, is likely to be very low. As a first approximation to the normalization point ratio, one could take the currently observable structure of the production of the energy composite. Alternatively, the observable structure some quarter of a century ago

⁴⁹As long as the elasticity of substitution is held constant, the choice of a specific point along the production isoquant as normalization point has no effect on the economic outcome and, thus, is often set to unity for both inputs for the sake of simple notation. This results in the standard Arrow et al. (1961) representation of the CES production function. See Klump et al. (2012) for an introduction to the normalization of production functions.

⁵⁰See Temple (2012) and Cantore and Levine (2012) for a discussion of problems that can arise when choosing a normalization point.

might be used to correct for recent climate policy favoring clean inputs, thereby pushing $a_t c_t$ above $\overline{a_N c_N}^N$. Yet both candidate ratios are still rather cost-optimal market outcomes than parameters of the production function.⁵¹

Finally, note that investment in better substitutability does “not always influence the substitutability in both directions” (Mattauch et al., 2015, p. 58). The main aim is to replace fossil fuels with clean alternatives. Fossil fuels, in turn, are likely to be a perfect substitute for renewables. Thus, the branch of the production isoquant exhibiting input ratios lower than defined by the normalization point should rather be interpreted as an artifact of the symmetry of the CES production function and, thus, be ignored.

A.3 Proofs and Derivations

General Remarks:

The explicitly normalized version of the CES production function presented in Equation (1) is given by:

$$E_t = E_N \left(\alpha_N \left(\frac{A_t^C C_t}{A_N^C C_N} \right)^{\rho_t} + (1 - \alpha_N) \left(\frac{A_t^D D_t}{A_N^D D_N} \right)^{\rho_t} \right)^{\frac{1}{\rho_t}}, \quad (\text{A.3.1})$$

with $\frac{A_N^C C_N}{A_N^D D_N} = \overline{a_N c_N}^N > 0$ as the normalization point ratio of inputs. $E_N > 0$ is a scaling parameter and $\alpha_N \in (0, 1)$ is the share parameter, which is identical to the factor income share of clean inputs in the normalization point ratio. For $A_N^C = C_N = A_N^D = D_N = 1$ ($\rightarrow \overline{a_N c_N}^N = 1$) and $E_N = 1$, the explicitly normalized production function collapses to the (implicitly normalized) one used in this paper:

$$E_t = \left(\alpha (A_t^C C_t)^{\rho_t} + (1 - \alpha) (A_t^D D_t)^{\rho_t} \right)^{\frac{1}{\rho_t}}. \quad (1) \text{ revisited}$$

Without loss of generality, but simplifying notation a lot, all proofs are given for the implicitly normalized form. However, whenever there needs to be a case distinction based on whether $a_t c_t \gtrless \overline{a_N c_N}^N$, this distinction is expressed relative to $\overline{a_N c_N}^N$ rather than to the ratio in the implicit normalization point where it takes the value “1.”

Moreover, for all proofs, the extreme cases $\rho_t = -\infty$ (Leontief production function) and $\rho_t = 1$ (linear production function) are excluded. Also extreme

⁵¹See Kemnitz and Knobloch (2020) for a novel approach in which the normalization point gradually changes with investment in substitutability.

values like j_t , A_t^j , $E_t = \{0, \infty\}$ are not considered except for limit considerations. Moreover, unless stated otherwise, also $\rho_t = 0$ (Cobb-Douglas production function) is excluded. Finally, if not stated otherwise, proofs hold for both $\rho_t < 0$ and $\rho_t > 0$ as well as for all $\gamma \leq 0$.

Proofs and Derivations:

Claim 1. Both inputs are necessary for production if $\rho_t < 0$ (essentiality).

Proof.

$E_t(0, A_t^D D_t)$ is not defined. However, taking the limit of E_t for $A_t^D D_t \rightarrow 0$ gives:

$$\begin{aligned} \lim_{A_t^D D_t \rightarrow 0} E_t(\cdot) &= \lim_{A_t^D D_t \rightarrow 0} A_t^D D_t \cdot \lim_{A_t^D D_t \rightarrow 0} (\alpha(a_t c_t)^{\rho_t} + (1 - \alpha))^{\frac{1}{\rho_t}} \\ &= 0 \cdot (1 - \alpha)^{\frac{1}{\rho_t}} \\ &= 0. \end{aligned}$$

The proof is analog for $E_t(A_t^C C_t, 0)$. □

Claim 2. Either input on its own is sufficient for production if $\rho_t > 0$ (inessentiality).

Proof.

$$E_t(A_t^C C_t, 0) = \alpha^{\frac{1}{\rho_t}}(A_t^C C_t) \geq 0$$

and

$$E_t(0, A_t^D D_t) = (1 - \alpha)^{\frac{1}{\rho_t}}(A_t^D D_t) \geq 0.$$

□

Claim 3. $OE_t > 0$ - Equation (11).

Proof.

By definition: $OE_t = E_t$. Thus, $OE_t > 0$ whenever $E_t > 0$. □

Claim 4. $RE_t \geq 0$ - Equation (11).

Proof.

As shown above: $OE_t > 0$. Moreover, $\frac{\partial E_t}{\partial \rho_t} \geq 0$ (see Section 2.2). Since $\underbrace{\frac{\partial E_t}{\partial \rho_t}}_{\geq 0} = \underbrace{OE_t}_{>0} \cdot RE_t \geq 0$, it follows that $RE_t \geq 0$. As $\frac{\partial E_t}{\partial \rho_t} = 0$ is true if and only if $a_t c_t = \overline{a_N c_N^N}$, and since $E_t > 0$, it follows that also $RE_t = 0$ holds if and only if $a_t c_t = \overline{a_N c_N^N}$. \square

Claim 5. $\Delta_t^{OE} \geq 0$ - Equation (13).

Proof.

By definition, $\Delta_t^{OE} = \frac{\partial E_t}{\partial(A_t^C C_t)} \left(\frac{1}{E_t} \frac{\partial E_t}{\partial \rho_t} \right)$. Moreover, $0 < \frac{1}{E_t} < \infty$ as well as $\frac{\partial E_t}{\partial \rho_t} \geq 0$. Additionally, the properties of the CES function guarantee $\frac{\partial E_t}{\partial(A_t^C C_t)} > 0$. Thus, it follows that $\Delta_t^{OE} \geq 0$. \square

Claim 6. $\Delta_t^{RE} \leq 0$, if $a_t c_t \leq \overline{a_0 c_0^N}$ - Equations (13) and (14).

Proof.

By definition: $\Delta_t^{RE} = \left(\frac{\partial}{\partial(A_t^C C_t)} \left(\frac{1}{E_t} \frac{\partial E_t}{\partial \rho_t} \right) \right) E_t$. As $E_t > 0$, the sign of Δ_t^{RE} only depends on $\frac{\partial}{\partial(A_t^C C_t)} \left(\frac{1}{E_t} \frac{\partial E_t}{\partial \rho_t} \right)$:

$$\frac{\partial}{\partial(A_t^C C_t)} \left(\frac{1}{E_t} \frac{\partial E_t}{\partial \rho_t} \right) = \frac{\overbrace{(A_t^C C_t)^{\rho_t-1} (A_t^D D_t)^{\rho_t} (1-\alpha) \alpha \log \left[\frac{a_t c_t}{\overline{a_N c_N^N}} \right]}^{>0}}{\underbrace{\left(-((1-\alpha)(A_t^C C_t) + \alpha(A_t^D D_t)) \right)^2}_{>0}},$$

with the denominator always positive due to quadrature. Moreover, most of the numerator is a product of numbers which are required to be positive. Thus, the sign of Δ_t^{RE} eventually only depends on the logarithm of the quotient of the input ratio and the normalization point ratio (here: $\overline{a_N c_N^N} = 1$) such that the following case distinction applies:

$$\Delta_t^{RE} = \left(\frac{\partial}{\partial(A_t^C C_t)} \left(\frac{1}{E_t} \frac{\partial E_t}{\partial \rho_t} \right) \right) E_t \begin{cases} > 0, & \text{if } a_t c_t > \overline{a_N c_N^N} = 1. \\ = 0, & \text{if } a_t c_t = \overline{a_N c_N^N} = 1. \\ < 0, & \text{if } a_t c_t < \overline{a_N c_N^N} = 1. \end{cases}$$

\square

Claim 7. See Equation (15).

Proof.

Case 1: $\rho_t < 0$

$$\begin{aligned}
\lim_{A_t^C C_t \rightarrow \infty} \left(\Delta_t^{RE} \Big|_{E_t = \bar{E}} \right) &= \lim_{A_t^C C_t \rightarrow \infty} \left(\left(\frac{\partial}{\partial(A_t^C C_t)} \left(\frac{1}{E_t} \frac{\partial E_t}{\partial \rho_t} \right) \right) E_t \Big|_{E_t = \bar{E}} \right) \\
&= \lim_{A_t^C C_t \rightarrow \infty} \underbrace{(a_t c_t)^{\rho_t - 1}}_{\equiv \Gamma_1} \underbrace{\log[a_t c_t]}_{\equiv \Gamma_2} \underbrace{e_t^{1 - 2\rho_t}}_{\equiv \Gamma_3} \underbrace{\alpha(\alpha - 1)\bar{E}^{-1}}_{\equiv \Phi \text{ const.}} \\
&= \Phi \cdot \lim_{A_t^C C_t \rightarrow \infty} \Gamma_1 \cdot \lim_{A_t^C C_t \rightarrow \infty} \Gamma_2 \cdot \lim_{A_t^C C_t \rightarrow \infty} \Gamma_3 \\
&= \Phi \cdot 0 \cdot \infty \cdot (1 - \alpha)^{\frac{1 - 2\rho_t}{\rho_t}},
\end{aligned}$$

Applying L'Hospital's rule, the following can be shown:

$$\lim_{A_t^C C_t \rightarrow \infty} \Gamma_1 \cdot \lim_{A_t^C C_t \rightarrow \infty} \Gamma_2 = 0,$$

such that:

$$\lim_{A_t^C C_t \rightarrow \infty} \Delta_t^{RE} \Big|_{E_t = \bar{E}} = 0.$$

Case 2: $\rho_t > 0$ (not used in this paper)

Proof available upon request. □

Claim 8. See Equation (16).

Proof.

Case 1: $\rho_t < 0$

$$\begin{aligned}
\lim_{A_t^C C_t \rightarrow \infty} (RE_t|_{E_t=\bar{E}}) &= \lim_{A_t^C C_t \rightarrow \infty} \left(\left(\frac{1}{E_t} \frac{\partial E_t}{\partial \rho_t} \right) \Big|_{E_t=\bar{E}} \right) \\
&= \lim_{A_t^C C_t \rightarrow \infty} \underbrace{\frac{\alpha (a_t c_t)^{\rho_t} \log[a_t c_t]}{(\alpha (a_t c_t)^{\rho_t} + (1 - \alpha)) \rho_t}}_{\rightarrow (1-\alpha)\rho_t} \\
&\quad - \frac{\overbrace{\log [\alpha (a_t c_t)^{\rho_t} + (1 - \alpha)]}^{\Gamma_1 \rightarrow \log[1-\alpha]}}{\rho_t^2} \\
&= \frac{\alpha}{(1 - \alpha)\rho_t} \cdot \lim_{A_t^C C_t \rightarrow \infty} \Gamma_1 \cdot \lim_{A_t^C C_t \rightarrow \infty} \Gamma_2 - \frac{\log[1 - \alpha]}{\rho_t^2} \\
&= \Phi \cdot 0 \cdot \infty - \frac{\log[1 - \alpha]}{\rho_t^2} \quad \text{for } \rho_t < 0.
\end{aligned}$$

Applying L'Hospital's rule, it can be shown that:

$$\lim_{A_t^C C_t \rightarrow \infty} \Gamma_1 \cdot \lim_{A_t^C C_t \rightarrow \infty} \Gamma_2 = 0,$$

such that:

$$\lim_{A_t^C C_t \rightarrow \infty} RE_t|_{E_t=\bar{E}} = -\frac{\log[1 - \alpha]}{\rho_t^2}.$$

Case 2: $\rho_t > 0$ (not used in this paper)

Proof available upon request. □

Claim 9. See Equation (18).

Proof.

$$\frac{\partial}{\partial \rho_t} \left(\frac{\frac{\partial E_t}{\partial(A_t^C C_t)}}{\frac{\partial E_t}{\partial(A_t^D D_t)}} \right) = \underbrace{\frac{\alpha}{1 - \alpha} (a_t c_t)^{\rho_t - 1}}_{>0} \log \left[\frac{a_t c_t}{a_N c_N^N} \right].$$

Most of the derivative is a product of numbers which are required to be positive. Thus, the sign of the derivative only depends on the logarithm of the

quotient of the input ratio and the normalization point ratio (here: $\overline{a_N c_N^N} = 1$) such that the following case distinction applies:

$$\frac{\partial}{\partial \rho_t} \left(\frac{\frac{\partial E_t}{\partial(A_t^C C_t)}}{\frac{\partial E_t}{\partial(A_t^D D_t)}} \right) \begin{cases} < 0, & \text{if } a_t c_t < \overline{a_N c_N^N} = 1. \\ = 0, & \text{if } a_t c_t = \overline{a_N c_N^N} = 1. \\ > 0, & \text{if } a_t c_t > \overline{a_N c_N^N} = 1. \end{cases}$$

□

Claim 10. There is convergence to and stability at a constant $\overline{a_t c_t^*}$ for $\rho_t \leq 0$ but not for $\rho_t > 0$.

Proof.

Case 1: $\rho_t \leq 0$

The following three properties guarantee convergence to $\overline{a_t c_t^*}$ for $\rho_t \leq 0$:

First, within either type of input, there is always convergence toward the cost-optimal ratio of intermediates and efficiency, independent of the other input:

$$\frac{\partial}{\partial j_t} \left(\frac{\frac{\partial E_t}{\partial I_t^j}}{\frac{\partial E_t}{\partial I_t^{A^j}}} \right) < 0 \text{ such that } \frac{\frac{\partial E_t}{\partial I_t^j}}{\frac{\partial E_t}{\partial I_t^{A^j}}} \begin{cases} > 1, & \text{if } (j_t/A_t^j) < (j_t/A_t^j)^*, \\ = 1, & \text{if } (j_t/A_t^j) = (j_t/A_t^j)^*, \\ < 1, & \text{if } (j_t/A_t^j) > (j_t/A_t^j)^*, \end{cases}$$

where $(j_t/A_t^j)^*$ is determined by the optimality condition $\frac{\partial E_t}{\partial I_t^j} = \frac{\partial E_t}{\partial I_t^{A^j}}$.

Second, investment in one input leaves the relative profitabilities of investments in intermediates and efficiency of the other input unchanged:

$$\frac{\partial}{\partial(j_t A_t^j)} \left(\frac{\frac{\partial E_t}{\partial I_t^i}}{\frac{\partial E_t}{\partial I_t^{A^i}}} \right) = 0 \quad \text{with } i \neq j \in \{C, D\}.$$

Third, investment in an input increases not only the profitability of investment in the complementary part within the same input, but favors investment in either of the two components of the other input even more:

$$\frac{\partial}{\partial j_t} \left(\frac{\frac{\partial E_t}{\partial I_t^i}}{\frac{\partial E_t}{\partial I_t^{Aj}}} \right), \frac{\partial}{\partial j_t} \left(\frac{\frac{\partial E_t}{\partial I_t^i}}{\frac{\partial E_t}{\partial I_t^{Aj}}} \right) > 0 \text{ such that:}$$

$$\frac{\frac{\partial E_t}{\partial I_t^i}}{\frac{\partial E_t}{\partial I_t^{Aj}}}, \frac{\frac{\partial E_t}{\partial I_t^i}}{\frac{\partial E_t}{\partial I_t^{Aj}}} \begin{cases} > 1, & \text{if } \frac{j_t A_t^j}{i_t A_t^i} > \frac{j_t A_t^j}{i_t A_t^i}^*, \\ = 1, & \text{if } \frac{j_t A_t^j}{i_t A_t^i} = \frac{j_t A_t^j}{i_t A_t^i}^*, \\ < 1, & \text{if } \frac{j_t A_t^j}{i_t A_t^i} < \frac{j_t A_t^j}{i_t A_t^i}^*, \end{cases}$$

and

$$\frac{\partial}{\partial A_t^j} \left(\frac{\frac{\partial E_t}{\partial I_t^i}}{\frac{\partial E_t}{\partial I_t^j}} \right), \frac{\partial}{\partial A_t^j} \left(\frac{\frac{\partial E_t}{\partial I_t^i}}{\frac{\partial E_t}{\partial I_t^j}} \right) > 0 \text{ such that:}$$

$$\frac{\frac{\partial E_t}{\partial I_t^i}}{\frac{\partial E_t}{\partial I_t^j}}, \frac{\frac{\partial E_t}{\partial I_t^i}}{\frac{\partial E_t}{\partial I_t^j}} \begin{cases} > 1, & \text{if } \frac{j_t A_t^j}{i_t A_t^i} > \frac{j_t A_t^j}{i_t A_t^i}^*, \\ = 1, & \text{if } \frac{j_t A_t^j}{i_t A_t^i} = \frac{j_t A_t^j}{i_t A_t^i}^*, \\ < 1, & \text{if } \frac{j_t A_t^j}{i_t A_t^i} < \frac{j_t A_t^j}{i_t A_t^i}^*, \end{cases}$$

given that $(j_t/A_t^j) = (j_t/A_t^j)^*$. If $(j_t/A_t^j) \neq (j_t/A_t^j)^*$, the within-input ratio might have to converge to $(j_t/A_t^j)^*$ first for the above relations to hold.

Case 2: $\rho_t > 0$

For $\rho_t > 0$, there is a “lock-in” in investment in either of the two inputs. That is, while there is continuous investment in intermediates and efficiency of one input, the level of the other input remains constant. Thus, there is no more constant $\overline{a_t c_t}^*$ and $a_t c_t$ depends on the initial level of the input for which there is no “lock-in” and on output E_t . This is a direct consequence of the following relations:

First, again, within either type of input, there is always convergence toward the cost-optimal ratio of intermediates and efficiency, independent of the other input:

$$\frac{\partial}{\partial j_t} \left(\frac{\frac{\partial E_t}{\partial I_t^i}}{\frac{\partial E_t}{\partial I_t^{Aj}}} \right) < 0 \text{ such that } \frac{\frac{\partial E_t}{\partial I_t^i}}{\frac{\partial E_t}{\partial I_t^{Aj}}} \begin{cases} > 1, & \text{if } (j_t/A_t^j) < (j_t/A_t^j)^*, \\ = 1, & \text{if } (j_t/A_t^j) = (j_t/A_t^j)^*, \\ < 1, & \text{if } (j_t/A_t^j) > (j_t/A_t^j)^*, \end{cases}$$

where $(j_t/A_t^j)^*$ is determined by the optimality condition $\frac{\partial E_t}{\partial I_t^j} = \frac{\partial E_t}{\partial I_t^{A^j}}$.

Second, as above, investment in one input leaves the relative profitabilities of investments in intermediates and efficiency of the other input unchanged:

$$\frac{\partial}{\partial(j_t A_t^j)} \left(\frac{\frac{\partial E_t}{\partial I_t^i}}{\frac{\partial E_t}{\partial I_t^{A^i}}} \right) = 0 \quad \text{with } i \neq j \in \{C, D\}.$$

Third, unlike with $\rho \leq 0$, investment in an input favors investment in the complementary part within the same input rather than investment in the other input:

$$\frac{\partial}{\partial j_t} \left(\frac{\frac{\partial E_t}{\partial I_t^i}}{\frac{\partial E_t}{\partial I_t^{A^j}}} \right), \frac{\partial}{\partial j_t} \left(\frac{\frac{\partial E_t}{\partial I_t^{A^i}}}{\frac{\partial E_t}{\partial I_t^{A^j}}} \right), \frac{\partial}{\partial A_t^j} \left(\frac{\frac{\partial E_t}{\partial I_t^{A^i}}}{\frac{\partial E_t}{\partial I_t^j}} \right), \frac{\partial}{\partial A_t^j} \left(\frac{\frac{\partial E_t}{\partial I_t^i}}{\frac{\partial E_t}{\partial I_t^j}} \right) < 0.$$

□

Claim 11. See Equation (23) (for $\rho_t < 0$).

Proof.

$$\frac{\partial \overline{a_t c_t}^*}{\partial \rho_t} = \underbrace{\frac{\left(\left(\frac{\alpha}{1-\alpha} \right)^{2-\gamma} (r_{\phi^i})^\gamma (r_{\phi^i} r_{\phi^A})^{-1} \right)^{\frac{1}{1+\rho_t(-2+\gamma)-\gamma}}}{(2-\gamma)^{-1}(1+\rho_t(-2+\gamma)-\gamma)^2}}_{>0} \log \left[\underbrace{\left(\frac{\alpha}{1-\alpha} \right)^{2-\gamma}}_{\equiv \Omega} \underbrace{\frac{(r_{\phi^i})^\gamma}{(r_{\phi^i} r_{\phi^A})}}_{\text{const.}} \right]$$

Thus, $\frac{\partial \overline{a_t c_t}^*}{\partial \rho_t} > 0$ if $\Omega > 1$, which is the case if $\overline{a_t c_t}^* > \overline{a_N c_N}^N$ (see Footnote 20 for $\gamma = 0$ and Section 3.3 for $\gamma < 0$). □

Claim 12. See Equation (24) (for $\rho_t < 0$).

Proof.

The profitability of a marginal investment in clean inputs can be written as:

$$\frac{\partial E_t}{\partial I_t^C} = \frac{1}{\phi^C} e_t^{1-\rho_t} \alpha (a_t c_t)^{\rho_t-1} A_t^C,$$

where $e_t = (A_t^D D_t)^{-1} E_t$. Moreover, for any fixed $E_t = \overline{E}$:

$$\frac{\overline{E}}{e_t} = A_t^D D_t.$$

Finally, in $\overline{a_t c_t}^*$:

$$D_t = \frac{\phi^{A^D}}{\phi^D} (A_t^D)^{1-\gamma} \quad \text{and} \quad A_t^D = \frac{\phi^D}{\phi^C} \left(\frac{\alpha}{1-\alpha} \right) (a_t c_t)^{\rho_t-1} A_t^C.$$

Altogether, this yields:

$$\begin{aligned} \left. \frac{\partial E_t}{\partial I_t^C} \right|_{a_t c_t = \overline{a_t c_t}^* \text{ \& } E_t = \overline{E}} &= e_t^{\frac{(1-\rho_t)(2-\gamma)-1}{2-\gamma}} \underbrace{\alpha (\overline{E})^{\frac{1}{2-\gamma}} \left(\frac{\phi^D}{\phi^{A^D}} \right)^{\frac{1}{2-\gamma}} \left(\frac{1}{\phi^D} \right) \left(\frac{1-\alpha}{\alpha} \right)}_{\equiv \Upsilon \text{ const.}} \\ &= e_t^{\frac{(1-\rho_t)(2-\gamma)-1}{2-\gamma}} \Upsilon. \end{aligned}$$

This can be rewritten to:

$$\left. \frac{\partial E_t}{\partial I_t^C} \right|_{a_t c_t = \overline{a_t c_t}^* \text{ \& } E_t = \overline{E}} = \left(\alpha \left((r_{\phi^j})^{\frac{\gamma}{2-\gamma}} (r_{\phi^j} r_{\phi^{A^j}})^{\frac{-1}{2-\gamma}} \left(\frac{\alpha}{1-\alpha} \right) \right)^{\varrho_t} + (1-\alpha) \right)^{\frac{1}{\varrho_t}} \Upsilon,$$

where $\varrho_t = \frac{(2-\gamma)\rho_t}{1-\gamma-2\rho_t+\gamma\rho_t}$ with $\frac{\partial \varrho_t}{\partial \rho_t} > 0$. This equation has the same structure as a CES production function (a general mean function) and, thus, the same proofs as for $\frac{\partial E_t}{\partial \rho_t} \geq 0$ (see Section 2.2) can be used to show:

$$\frac{\partial}{\partial \rho_t} \left(\left. \frac{\partial E_t}{\partial I_t^C} \right|_{a_t c_t = \overline{a_t c_t}^* \text{ \& } E_t = \overline{E}} \right) \geq 0.$$

□

Claim 13. For $0 > \gamma_{A^D} \neq \gamma_{A^C} < 0$, the optimal ratio of inputs, $\overline{a_t c_t}^*$, is not constant anymore but depends on output, E_t (for $\rho_t \leq 0$).

Proof.

For $0 > \gamma_{A^D} \neq \gamma_{A^C} < 0$, the optimal ratio of inputs, $\overline{a_t c_t}^*$, is given by:

$$\overline{a_t c_t}^* = (A_t^C)^{\gamma_{A^C} - \gamma_{A^D}},$$

which implies that $\overline{a_t c_t}^*$ increases in A_t^C if $\gamma_{A^C} > \gamma_{A^D}$, i.e., if the profitability of investment in efficiency in the use of the clean intermediate suffers relatively less

from “fishing out” (γ_{AC} is less negative than γ_{AD}), and *vice versa*. Moreover, in optimum, E_t can be written as:

$$E_t = \left(\alpha \left((A_t^C)^{2-\gamma_{AC}} \right)^{\rho_t} + (1-\alpha) \left((A_t^C)^{\frac{(1-\gamma_{AC})(2-\gamma_{AD})}{1-\gamma_{AD}}} \right)^{\rho_t} \right)^{\frac{1}{\rho_t}},$$

with all exponents positive such that $\frac{\partial E_t}{\partial A_t^C} > 0$ and, thus, as long as both derivatives are defined, $\left(\frac{\partial E_t}{\partial A_t^C} \right)^{-1} = \frac{\partial A_t^C}{\partial E_t} > 0$. Together, this implies:

$$\frac{\partial \overline{a_t c_t^*}}{\partial A_t^C} \frac{\partial A_t^C}{\partial E_t} = \frac{\partial \overline{a_t c_t^*}}{\partial E_t} \begin{cases} > 0, & \text{if } \gamma_{AC} > \gamma_{AD}. \\ < 0, & \text{if } \gamma_{AC} < \gamma_{AD}. \end{cases}$$

This non-constant optimal ratio of inputs impedes a straightforward analysis as possible for $\gamma_{AC} = \gamma_{AD} = \gamma$ and is beyond the scope of this paper. \square

Claim 14. $\frac{\partial}{\partial \tau_t^{AC}} \left(\frac{\partial E_t}{\partial I_t^j} \Big|_{a_t c_t = \overline{a_t c_t^*} \text{ \& } E_t = \overline{E}} \right) = \frac{\partial}{\partial \tau_t^{AC}} \left(\frac{\partial E_t}{\partial I_t^{Aj}} \Big|_{a_t c_t = \overline{a_t c_t^*} \text{ \& } E_t = \overline{E}} \right) > 0$ (for $\rho_t \leq 0$).

Proof.

In cost minimum and along $\overline{a_t c_t^*}$, the profitabilities of all investment possibilities are equal and, for fixed output, $E_t = \overline{E}$, can be written as:

$$\frac{\partial E_t}{\partial I_t^D} \Big|_{a_t c_t = \overline{a_t c_t^*} \text{ \& } E_t = \overline{E}} = \overline{E}_t^{1-\rho_t} (1-\alpha) (A_t^D D_t)^{\frac{(\rho_t-1)(2-\gamma)+1}{2-\gamma}} \frac{1}{\phi^D} \left(\frac{\phi^D}{\phi^{AD}} \right)^{\frac{1}{2-\gamma}}.$$

Moreover, along $E_t = \overline{E}$, for convex functions, like the CES production function:

$$\frac{\partial A_t^D D_t}{\partial A_t^C C_t} \Big|_{E_t = \overline{E}} < 0.$$

Finally, from Equation (27) it follows:

$$\frac{\partial \overline{a_t c_t^*}}{\partial \phi^{AC}} < 0 \rightarrow \frac{\partial A_t^D D_t}{\partial \phi^{AC}} \Big|_{E_t = \overline{E}} > 0.$$

Together this implies:

$$\frac{\partial}{\partial \tau_t^{AC}} \left(\frac{\partial E_t}{\partial I_t^j} \Big|_{a_t c_t = \overline{a_t c_t^*} \text{ \& } E_t = \overline{E}} \right) = \frac{\partial}{\partial \tau_t^{AC}} \left(\frac{\partial E_t}{\partial I_t^{Aj}} \Big|_{a_t c_t = \overline{a_t c_t^*} \text{ \& } E_t = \overline{E}} \right) =$$

$$\begin{aligned}
& \underbrace{\overline{E}_t^{1-\rho_t} (1-\alpha) \frac{1}{\phi^D} \left(\frac{\phi^D}{\phi^{A^D}} \right)^{\frac{1}{2-\gamma}}}_{\text{const.}} \underbrace{\frac{(\rho_t - 1)(2 - \gamma) + 1}{2 - \gamma}}_{<0} \\
& \underbrace{(A_t^D D_t)^{\frac{(\rho_t - 1)(2 - \gamma) + 1}{2 - \gamma} - 1}}_{>0} \underbrace{\frac{\partial A_t^D D_t}{\partial \phi^{A^C}} \Big|_{E_t = \overline{E}}}_{>0} \underbrace{\frac{\partial \phi^{A^C}}{\partial \tau_t^{A^C}}}_{<0} > 0.
\end{aligned}$$

The proof for $\tau_t^{A^D}$ is symmetric. \square

A.4 Parameters and Starting Values used for the Simulation in Section 5

Table A.4.1: Parameters and Starting Values

Par./Var.	Value	Remark
s	0.1	savings rate
β	0.7	low initial income share of energy to capital
ν	-3	guarantees essentiality of the energy composite
ϕ^K	1	costs of capital
K_0	~ 6.6	derived starting value: $\frac{\partial E_t}{\partial I_t^K} = \frac{\partial E_t}{\partial I_t^C}$
α	0.3	high initial income share of dirty inputs
ρ_0	-5	initial essentiality of both clean and dirty energy inputs
C_0	6	exogenously set starting value
D_0	~ 5.3	derived starting value: $\frac{\partial E_t}{\partial I_t^D} = \frac{\partial E_t}{\partial I_t^C}$
A_0^C	~ 1.8	derived starting value: $\frac{\partial E_t}{\partial I_t^{A^C}} = \frac{\partial E_t}{\partial I_t^C}$
A_0^D	~ 1.7	derived starting value: $\frac{\partial E_t}{\partial I_t^{A^D}} = \frac{\partial E_t}{\partial I_t^C}$
ϕ^C	0.5	costs of clean intermediate production capacities
ϕ^D	2	costs of dirty intermediate production capacities
ϕ^{A^C}	0.5	costs of research on clean inputs
ϕ^{A^D}	2	costs of research on dirty inputs
ϕ^ρ	10	costs of research on substitutability
$\tau_t^{A^C}$	0	subsidy for research on clean inputs
τ_t^D	0	tax on building dirty capacities
τ_t^ρ	0	subsidy for research on substitutability
$\gamma_{A^C} = \gamma_{A^D} = \gamma$	-2	degree of difficulty increase in research on efficiency
γ_ρ	-1.1	degree of difficulty increase in research on substitutability
$\delta^C = \delta^D = \delta$	1%/t	depreciation rate of intermediate production capacities
$\overline{a_0 c_0}^*$	~ 1.2	derived starting value ($\overline{a_N c_N}^N = 1$)
E_0	~ 3.1	derived starting value
Y_0	~ 4.3	derived starting value

Note: Starting values of variables are indicated by $t = 0$.

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