Speculative Partnership Dissolution with Auctions*

Ludwig Ensthaler[†] Thomas Giebe[‡] Jianpei Li[§]

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Abstract

The literature on partnership dissolution generally takes the dissolution decision as given and examines whether dissolution is efficient, i.e., whether the asset is allocated to the partner with the highest valuation. A well-known result is that $k+1$-price auctions dissolve a partnership efficiently when the share structure is sufficiently close to equal. In this paper, we endogenize the dissolution decision in an equal-share partnership and show that, if $k+1$-price auctions are used, inefficient dissolution occurs for speculative reasons. We show that specifying a reserve price for the dissolution auction improves efficiency, while adding a veto right does not help.

JEL classifications:

Keywords: Partnership dissolution, $k+1$-price auction, Efficiency.

Very preliminary and incomplete!

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[†]Humboldt University at Berlin, Institute of Economic Theory I, Spandauer Str. 1, 10178 Berlin, Germany, and Graduate Center of Economic and Social Research DIW Berlin, `lensthaler@diw.de.`

[‡]Humboldt University at Berlin, Institute of Economic Theory I, Spandauer Str. 1, 10178 Berlin, Germany, `thomas.giebe@wiwi.hu-berlin.de`.

[§]University of International Business and Economics, East Huixin Street 10, Beijing 100029, China; `lijianpei2007@gmail.com`.

1 Introduction

In this paper, we examine the efficiency properties of a widely used partnership dissolution mechanism: $k+1$-price auctions.[1] In these auctions, both partners simultaneously submit sealed bids and the partner who submitted the larger bid buys the other partner's share of the asset. The transaction price for the whole asset is a convex combination of the low and high bid: $kb_L + (1-k)b_H$ with $k \in [0,1]$. For $k = 0$ the auction is a winner's bid auction (WBA), while for $k = 0$ it is a loser's bid auction (similar to first- and second-price auctions). In an equal share partnership, which we consider here, the winner pays one half of the transaction price to the loser since he already owns the other half.

Our main departure from the existing literature is that we endogenize the dissolution decision. The existing literature typically assumes that dissolution decisions are exogenously given and examines the efficiency of the outcome, i.e., whether the party with the highest valuation obtains the entire asset.[2] In their seminal contribution, Cramton, Gibbons, and Klemperer (1987) show that $k+1$-price auctions implement ex post efficiency for equal-share partnerships.[3] In this paper, we first show that if their model is slightly modified, such that the dissolution decision is endogenized, ex post efficiency is no longer achievable. Second, we show that a veto right that allows one partner to veto the dissolution cannot restore efficiency. Third, we show that if the dissolution auction is augmented by an appropriate reserve price (or, minimum transaction price), ex post efficiency can be restored. This suggests that the existing partnership dissolution literature, by ignoring the impact of strategic dissolution decisions, only gives a partial picture of the efficiency problem.

When it comes to dissolving a partnership, more often than not, the decision to dissolve is not given but is made strategically by the partners, for various reasons. For example, one partner might think that he can run the business more efficiently on his own. Another reason could be that one partner speculates that the other partner has a high valuation for the asset and thus would be willing to pay a high price in order to become the single owner. The incentives for strategically proposing a dissolution depend on the dissolution mechanism in place. When $k+1$-price auctions are used, a partner (i) might call for dissolution even if it is inefficient for him to become the single owner

[1] In the literature, this auction is also referred to as k-double auction, for example in Kittsteiner (2003).

[2] An exception to this is Li and Wolfstetter (2010).

[3] Efficiency can be implemented by $k+1$-price auctions as long as the shares are sufficiently close to equal.

of the asset, i.e., when his valuation for the ownership of the entire partnership is below the partnership's continuation value. If the other partner's (j) value is also below the partnership's continuation value, the partnership will be dissolved inefficiently. However, such a call for dissolution might change the perception of partner j regarding i's valuation and thus induce partner j to bid more aggressively at the bidding stage if j indeed has a high value and would like to buy out partner i. This will result in a higher selling price for partner i if he indeed turns out to be a seller of shares. The inefficiency arising from the partners' speculative behavior is an important issue that has been ignored in the existing literature, and we make an effort to fill this gap.

[**The remaining part of the introduction will be rewritten....**] In the standard literature on mechanism design a mechanism is said to be interim individually rational if an agent, knowing only his own valuation, expects no loss from participating in the mechanism, i.e. if he can expect to not end up with less than he had before. What is the appropriate notion of IR in our setting?

Usually, partners agree on the dissolution procedure when forming the partnership. In fact, legal advisors can be held accountable if they do not propose to include a dissolution mechanism in the partnership agreement. Once a dissolution mechanism is included in the partnership agreement, that agreement is binding.

When forming a partnership, partners usually neither know the value of the partnership at a certain point of time in the future, nor do they know their valuation for the whole partnership at that future point in time. To stay in the standard framework of mechanism design one should think of the signing of the partnership agreement as the ex-ante stage. Therefore, ex-ante rationality assumes that partners initially assume to profit from the joint venture. This includes not just the dissolution mechanism but many other things such as funding, salaries etc. Our analysis starts at the interim stage when both players have learned about the continuation value of the partnership and their respective private value of the entire asset should they become the single owner.

The paper is organized as following.... All the proofs and technical details are in the appendix.

1.1 Related Literature

Further reference McAfee (1992). **To be written up ......**

2 The Model

Two risk-neutral partners, $i \in \{1, 2\}$, own equal shares of some indivisible asset. The continuation value of the partnership is commonly known to be $C \in (0, 1]$.[4] Each partner i has a private valuation v_i for single ownership of the assets. It is common knowledge that private valuations are drawn independently from distribution F with support $[0, 1]$ and positive continuous density f.

A partnership contract specifies that if the partners decide to break up, a $k + 1$-price auction will be used to determine the transaction price at which one partner buys out the other partner. There, the high bid wins. The winner pays one half of the transaction price to the loser and becomes the single owner of the entire asset. The transaction price is $kb_L + (1-k)b_W$ where $k \in [0, 1]$ and b_L (resp. b_W) is the losing (resp. winning) bid. With $k = 1$ (resp. $k = 0$), the auction is the loser's bid auction (resp. winner's bid auction).

The partnership contract may in addition specify that 1) whenever a partner calls for dissolution, the partnership will be dissolved; or 2) when a partner calls for dissolution, the other partner has the right to veto the proposal in which case the partnership continues; or 3) whenever a partner calls for dissolution, the partnership will be dissolved, but the transaction price can not be lower than a certain reserve price. In the following, our main analysis is conducted on the basis of the first case. In Sections 4 and 5, we analyze the latter two cases.

A partner's utility from sole ownership is $u_i(v, m) = v + m$ where v is the partner's private valuation of the assets if he has sole ownership and m is the monetary transaction he receives.

Time structure of the game is as follows:

1. ("proposal stage".) Partners 1 and 2 learn their private values, v_1 and v_2, for single ownership of the partnership's asset. They simultaneously choose an action, either D or N, where D means "calling for a dissolution" and N means "not calling for a dissolution". The action profile $(a_1, a_2) \in \{D, N\} \times \{D, N\}$ determines how the game continues. Profile (N, N) implies that no one has called for a dissolution, and hence the partnership continues and each player's payoff is $C/2$. Otherwise, the

[4] We focus on the interesting cases with $C \in (0, 1]$ where inefficient dissolution might occur. If $C = 0$, dissolution is always efficient, and if $C \geq 1$, dissolution is always inefficient. In those cases, inefficiencies can be avoided with a simple clause in the partnership contract that prevents or triggers dissolution conditional on these continuation values.

game proceeds to the next stage.

2. ("dissolution stage".) The partnership is dissolved using a $k+1$-price auction. There, the high bidder buys the loser's share at a price equal to one half of the transaction price $kb_L + (1-k)b_W$, where b_W and b_L are the winner's and the loser's bids respectively.

We evaluate the equilibrium outcome of the game in terms of ex post efficiency, straightforwardly defined as follows.

Definition 1. *Dissolution of the partnership is ex post efficient if 1) the asset is allocated to the partner who has the highest value, and, 2) the continuation value of the partnership is not higher than that value.*

This paper is mainly concerned with the issue of ex post efficiency. Therefore, we neither make an attempt to solve for all equilibria of the $k+1$-price auction, nor do we look for an optimal dissolution mechanism. Instead, we concentrate a class of equilibria that, intuitively, contain all equilibria that possibly implement ex post efficiency, given that $k+1$ price auctions are the stipulated dissolution mechanism.

3 Speculative Dissolution with $k+1$-price Auction

In this section we consider a partnership contract stipulating that the partnership is dissolved whenever a partner calls for dissolution. Since valuations for single ownership are private information, ex post efficiency requires that partner i calls for dissolution if and only if $v_i \geq C$.[5]

Definition 2 (Cutoff Equilibrium)**.** *Partner i plays a* cutoff strategy *if he proposes dissolution if and only if his private value is above a cutoff value, $v_i \geq \bar{v}$. A* cutoff equilibrium *is a pure-strategy equilibrium where the partners play cutoff strategies.*

We are mainly concerned whether the partnership is dissolved ex post efficiently if $k+1$-price auctions are used. Therefore, we focus on the existence of a symmetric cutoff equilibrium, i.e., both partners play D at the proposal stage if and only if $v_i \geq \bar{v}$, $i = \{1,2\}$ where $\bar{v}$ is the cutoff value. Ex post efficiency requires that $\bar{v} = C$ in equilibrium.

[5] In the nongeneric case $v_i = C$, any of i's actions is consistent with efficiency.

We adopt the following tie-breaking rule at the dissolution stage: if both partners have proposed dissolution at the proposal stage, each partner wins the auction with equal probability. If only one partner has proposed dissolution, the partner who has proposed dissolution wins.[6]

For the class of cutoff equilibria with cutoff value $\bar{v} \in [0, 1]$, the only consistent belief system is that after observing that partner j has played D at the proposal stage, partner i's updated belief is that j's value must be distributed according to the c.d.f. F conditional on $v_j \geq \bar{v}$. We denote this conditional c.d.f. by $G(x) := F(x|x \geq \bar{v})$, and the belief of partner i by $\mu_i(x \mid a_j) := \Pr\{V_j \leq x \mid a_j\}$, $i, j \in \{1, 2\}$, $j \neq i$. Thus, partner i's prior belief is $\mu_i(x) = F(x)$ and the posterior belief, after observing partner j's action choice $a_j \in \{D, N\}$, is

$$\mu_i(x|a_j) = \begin{cases} G(x) & \text{if } a_j = D \\ \frac{F(x)}{F(\bar{v})} & \text{if } a_j = N \end{cases}, \text{ where } G(x) := \begin{cases} 0 & \text{if } x < 0 \\ \frac{F(x)-F(\bar{v})}{1-F(\bar{v})} & \text{if } x \in [\bar{v}, 1] \\ 1 & \text{if } x > 1. \end{cases} \tag{1}$$

Denote $g := G'$.

We proceed by stating the main result of this section and then delivering its proof in a series of lemmas.

Theorem 1 (Impossibility)**.** *Ex post efficiency is unattainable with $k+1$-price auctions, i.e., for any continuation value $C \in (0, 1]$, there exist (generic) sets of valuations for which the partnership cannot be dissolved efficiently in equilibrium.*

We prove Theorem 1 step by step in Lemmas 1-5. We first solve for the equilibrium of the dissolution stage on the equilibrium path, i.e., given that both players have chosen D if and only if their values are in the range $[\bar{v}, 1]$ at the proposal stage and given the belief system (1). The game proceeds to that stage only if at least one player has called for dissolution, D.

Lemma 1 (Equilibrium at the dissolution stage)**.** *Suppose at the proposal stage, partner $i \in \{1, 2\}$ has played D if and only if $v_i \in [\bar{v}, 1]$. Depending on the proposal stage action*

[6] This tie-breaking rule in favor of the partner who has proposed dissolution is only chosen for simplicity of the analysis. The result does not change if one assumes a tie-breaking rule where each player wins the auction with equal probability in every case.

profiles, equilibrium bids at the dissolution stage are:

$$(a_1, a_2) = (D, D): \quad b_i = \beta(v_i) = v_i - \int_{G^{-1}(k)}^{v_i} \frac{(G(t)-k)^2}{(G(v_i)-k)^2} dt, \; i \in \{1,2\}, \quad (2)$$

$$(a_1, a_2) \in \{(D, N), (N, D)\}: \quad b_i = \bar{v}, \; i \in \{1,2\}. \quad (3)$$

Equilibrium expected payoffs at the dissolution stage are

$$(a_1, a_2) = (D, D): \quad U(v_i) = \frac{1}{2} + \frac{1}{2}\int_{\bar{v}}^{1} G^2(t)dt - \int_{v_i}^{1} G(t)dt, \; i \in \{1,2\} \quad (4)$$

$$(a_1, a_2) \in \{(D, N), (N, D)\}: \quad U_i = v_i - \frac{\bar{v}}{2}, \; U_j = \frac{\bar{v}}{2}, \; v_i \in [\bar{v}, 1], \; v_j \in [0, \bar{v}), \; i, j \in \{1,2\}. \quad (5)$$

The next lemma shows that, besides the equilibria stated in Lemma 1, there are no other equilibria at the dissolution stage that potentially achieve efficient allocation.

Lemma 2 (Uniqueness of Dissolution Stage Equilibrium)**.** *Given that partner $i \in \{1,2\}$ plays D at the proposal stage if and only if $v_i \in [\bar{v}, 1]$, the equilibrium at the dissolution stage, characterized in Lemma 1, is unique.*

Having characterized the unique equilibrium at the dissolution stage, we now derive necessary and sufficient conditions for the existence of a cutoff equilibrium, i.e., an equilibrium where a partner plays D at the proposal stage if and only if his valuation is in the range $[\bar{v}, 1]$.

Lemma 3 (Equilibrium at the Proposal Stage)**.** *A cutoff equilibrium requires that partner $i \in \{1,2\}$ proposes dissolution if and only if $v_i \in [\bar{v}, 1]$. This is equivalent to the following two conditions*

$$C \geq \bar{v} + \int_{\bar{v}}^{1} \frac{(1-F(t))^2}{F(\bar{v})(1-F(\bar{v}))} dt, \quad \forall v_i \in [0, \bar{v}), \quad (6)$$

$$C \leq 2v_i - \bar{v} - \int_{\bar{v}}^{v_i} 2\frac{1-F(t)}{F(\bar{v})} dt + \int_{\bar{v}}^{1} \frac{(1-F(t))^2}{F(\bar{v})(1-F(\bar{v}))} dt, \quad \forall v_i \in [\bar{v}, 1]. \quad (7)$$

Having characterized the properties of a cutoff value equilibrium we now analyze whether such a cutoff equilibrium exits and when it exists, whether it is unique. Define v^m implicitly by $F(v^m) = 1/2$.

Lemma 4 (Existence and Uniqueness)**.** *A cutoff equilibrium exists if and only if $C \in [v^m + 4\int_{v^m}^{1} (1-F(t))^2 \, dt, 1]$ and whenever it exists, the equilibrium is unique. There, the cutoff value $\bar{v}$ is determined by*

$$C = \bar{v} + \int_{\bar{v}}^{1} \frac{(1-F(t))^2}{F(\bar{v})(1-F(\bar{v}))} dt. \quad (8)$$

Proof of Lemma 4. First, in a cutoff equilibrium, condition (6) must hold for all $v_i \in [0, \bar{v})$ while condition (7) needs to hold for all $v_i \in [\bar{v}, 1]$. Therefore, (7) also needs to hold for $v_i = \bar{v}$. Inserting $v_i = \bar{v}$ into (6) and (7), we see that the two conditions can hold simultaneously if and only if (8) holds.

Second, since (7) also needs to hold for any $v_i > \bar{v}$, taking the above into account, we need that the RHS of (7) increases in v_i for $v_i \in (\bar{v}, 1]$. The derivative of the RHS of (7) with respect to v_i is $1 - \frac{1-F(v_i)}{F(\bar{v})}$, which simplifies to $\frac{F(\bar{v})+F(v_i)-1}{F(\bar{v})}$. This is positive if and only if $\bar{v} \geq v^m$. (Suppose $\bar{v} < v^m$ where $F(\bar{v}) < \frac{1}{2}$. Then there exists $\varepsilon > 0$ and $v_i = \bar{v} + \varepsilon$ such that $F(\bar{v}) + F(v_i) - 1 < 0$.) Hence, any $\bar{v} < v^m$ cannot satisfy (7) for all $v_i \geq \bar{v}$. Thus, $\bar{v} \geq v^m$ is a necessary condition for the existence of a cutoff equilibrium.

Third, taking the derivative of the RHS of (8) with respect to $\bar{v}$, we get:

$$\begin{aligned} 1 &- \frac{f(\bar{v})(1-2F(\bar{v}))}{F^2(\bar{v})(1-F(\bar{v}))^2} \int_{\bar{v}}^{1} (1-F(t))^2 dt - \frac{1-F(\bar{v})}{F(\bar{v})} \\ &= \frac{2F(\bar{v})-1}{F(\bar{v})} + \frac{f(\bar{v})(2F(\bar{v})-1)}{F^2(\bar{v})(1-F(\bar{v}))^2} \int_{\bar{v}}^{1} (1-F(t))^2 dt. \end{aligned} \tag{9}$$

(9) is negative if $\bar{v} < v^m$ and positive if $\bar{v} > v^m$ and which implies that the RHS of (8) is decreasing in $\bar{v}$ for $\bar{v} < v^m$ and increasing in $\bar{v}$ at $\bar{v} > v^m$. Therefore, the RHS reaches its minimum value $v^m + 4\int_{v^m}^{1} (1-F(t))^2\, dt$ at $\bar{v} = v^m$. For $\bar{v} > v^m$, the RHS increases, reaching its maximum value 1 at $\bar{v} = 1$. We have already learnt that $\bar{v} \geq v^m$ is a necessary condition for the existence of a cutoff equilibrium. Therefore, (8) can ~~only~~ be satisfied if and only if $\bar{v} \geq v^m$.

Finally, when $\bar{v} \geq v^m$, the domain of the RHS of (8) is $[v^m + 4\int_{v^m}^{1} (1-F(t))^2\, dt, 1]$. Therefore, we conclude that the cutoff equilibrium exists and is unique if and only if $C \in [v^m + 4\int_{v^m}^{1} (1-F(t))^2\, dt, 1]$.

□

If the cutoff equilibrium is efficient, then, the cutoff value $\bar{v}$ has to be equal to the continuation value C. The next lemma concludes the proof of Theorem 1.

Lemma 5 (Inefficiency)**.** *In any cutoff equilibrium, the cutoff value, $\bar{v}$, is strictly below the continuation value of the partnership, C, implying that dissolution is inefficient with positive probability.*

Proof of Lemma 5. By Lemma 4, the complete range of continuation values for which a (unique) cutoff equilibrium exists is $C \in [v^m + 4\int_{v^m}^{1} (1-F(t))^2\, dt, 1]$. However, the equilibrium cutoff value determined by (8) implies $\bar{v} < C$. Whenever $C \geq \max\{v_1, v_2\} \geq$

$\bar{v}$, while it is efficient for the partnership to continue, it is nevertheless dissolved in the unique cutoff equilibrium of the $k+1$-price auction. □

The inefficiency result is intuitive. Ex post efficiency requires that both partners play a cutoff strategy with cutoff value $\bar{v} = C$. Suppose partner 2 plays that strategy and partner 1's value is "slightly" below C. Partner 1's payoff from playing the same strategy is $C/2$. If partner 1 deviates and proposes dissolution instead, then his payoff depends on partner 2's value. If $v_2 \geq C$, then 2, believing that $v_1 \geq C$ as well, will bid $b_2 = \beta(v_2) \geq C$ and, thus, increase partner 1's payoff above $C/2$. If $v_2 < C$ and partner 2 bids $b_2 = C$ in the auction, partner 1 wins the auction with payoff $v_2 - C/2$ which is only "slightly" below $C/2$. For v_1 sufficiently close to C, partner 1's gain from a higher price when he sells strictly dominates the loss from inefficiently breaking up the partnership. Therefore it is beneficial for partner 1 to deviate rather than to follow the cutoff strategy with cutoff value C. In equilibrium, both partners propose dissolution more often than is efficient, thus rendering this kind of speculation unprofitable.

In our setting, since each partner has private information about the value of his single ownership, neither of the partners knows whether dissolution is efficient or not. Hence, ex post efficiency can not be obtained even though the partners have equal shares, contrasting the result in Cramton, Gibbons, and Klemperer (1987) that $k+1$-price auction implements efficient dissolution when share structure is (or close to) equal.

Example 1. *Suppose v_i is uniformly distributed on $[0,1]$ for $i = \{1,2\}$. Then $F(t) = t$ and $G(t) = \frac{t-\bar{v}}{1-\bar{v}}$, and $g(t) = \frac{1}{1-\bar{v}}$. Conditions (6) and (7) become*

$$C \geq \frac{1}{3\bar{v}}\left(1 + 4\bar{v}^2 - 2\bar{v}\right) \quad \forall\, v_i < \bar{v} \tag{10}$$

$$C \leq \frac{1}{3\bar{v}}\left(1 + 4\bar{v} - 6v_i - 5\bar{v}^2 + 3v_i^2 + 6v_i\bar{v}\right), \quad \forall\, v_i \geq \bar{v} \tag{11}$$

Equation (8) becomes a quadratic equation in $\bar{v}$,

$$4\bar{v}^2 - 2\bar{v} - 3C\bar{v} + 1 = 0$$

Its solutions are

$$\bar{v}_1 = \frac{2 + 3C - \sqrt{(2+3C)^2 - 16}}{8}, \quad \textit{and} \quad \bar{v}_2 = \frac{2 + 3C + \sqrt{(2+3C)^2 - 16}}{8}.$$

Since $\bar{v}_2$ is the only solution that satisfies (10) for every $v_i < \bar{v}$ and (11) for all $v_i \geq \bar{v}$, $\bar{v}_2$ is the unique cutoff value in a cutoff equilibrium. This implies that when $C \in [\frac{2}{3}, 1]$, a partner proposes dissolution if and only if his private value is above $\bar{v}_2$.

Further, it is straightforward to confirm that $\bar{v}_2 \leq C$. This implies that, if $\max\{v_1, v_2\} \in [\bar{v}_2, C]$*, though it is efficient to continue the partnership, one (or both) of the partners will propose dissolution and the partnership will be broken up inefficiently—in equilibrium, excessive dissolution occurs in comparison to the efficiency benchmark.*

4 Does Veto Right Restore Efficiency?

In previous section, we assumed that the partners are forced to participate in the prescribed dissolution mechanism whenever one of the partners proposes dissolution. In this section, we discuss whether by giving the partners veto right restores efficiency. The game structure is as follows:

1. Partners decide whether they would like to propose a dissolution;
2. If both partners have proposed dissolution, the game proceeds to the next stage. If one partner has proposed dissolution, the other partner decides whether to veto. If he vetoes, the partnership continues. If not, the game proceeds to the next stage.
3. If both partners have proposed dissolution or if one partner has proposed dissolution while the other one has not vetoed, the partnership is dissolved using the $k+1$-price auction.

Proposition 2. *Adding the right to veto does not restore efficiency.*

Proof. Since implementation of efficiency requires for the existence of a cutoff equilibrium, we focus on this class of equilibria where the cutoff is denoted as $\hat{v}$ and examine whether such an equilibrium exists. After characterizing the equilibrium, we will check whether the right to veto can make $\hat{v}$ equal to C.

If both partners have proposed dissolution, no veto occurs, and the partnership is dissolved using the $k+1$-price auction. If neither of the partners has proposed, the right to veto has no impact either. Veto right only has an impact in an asymmetric scenario where one partner has a value above $\hat{v}$ and the other partner has a value below $\hat{v}$, and in that case the low value partner can veto if the high value partner has proposed a dissolution.

Suppose partner i has received a high value, i.e. $v \geq \hat{v}$. Suppose the other partner, j, follows the proposed strategy of proposing if he has a value above $\hat{v}$ and not proposing if his value is below $\hat{v}$. When partner i did not propose in the first stage, not vetoing signals that he has a high value. This will make partner j to bid as if he is facing a high type at the bidding stage. Hence, as a high value type, not proposing and not vetoing does not improve partner i's payoff in comparison to proposing. If he has not proposed, partner i can only change his payoff by vetoing if partner j has proposed. The condition that prevents a high-value type from deviating to not proposing is:

$$U(v)(1-F(\hat{v}))+\frac{C}{2}F(\hat{v})\geq\frac{C}{2}(1-F(\hat{v}))+\frac{1}{2}CF(\hat{v}) \tag{12}$$

Since this condition has to hold for all $v\in[\hat{v},1]$, it has to hold for $v=\hat{v}$. Note that

$$U(\hat{v})=\frac{\hat{v}}{2}+\frac{1}{2(1-F(\hat{v}))^2}\int_{\hat{v}}^{1}(1-F(t))^2dt \tag{13}$$

Therefore, we have:

$$C\leq\hat{v}+\frac{1}{(1-F(\hat{v}))^2}\int_{\hat{v}}^{1}(1-F(t))^2dt \tag{14}$$

On the other hand, for a partner with low value, not proposing dissolution and veto if the other proposes must dominate proposing dissolution. This requires

$$\frac{C}{2}\geq U(\hat{v})(1-F(\hat{v}))+\frac{C}{2}F(\hat{v}) \tag{15}$$

which is equivalent to

$$C\geq 2U(\hat{v})=\hat{v}+\frac{1}{(1-F(\hat{v}))^2}\int_{\hat{v}}^{1}(1-F(t))^2dt \tag{16}$$

The only C that satisfies both (14) and (16) is that

$$C=\hat{v}+\frac{1}{(1-F(\hat{v}))^2}\int_{\hat{v}}^{1}(1-F(t))^2dt \tag{17}$$

Since the second term on the RHS is strictly positive, it is impossible for $\hat{v}=C$ to hold. Therefore, adding the right to veto does not restore efficiency. □

(**Add more discussion about the comparison of $\bar{v}$ and $\hat{v}$ and explain intuitively why veto right does not restore efficiency.**)

5 WBA with Reserve Price

Now suppose ex ante the partners stipulate a minimum transaction price, $r=r(C)$, in the partnership contract for the case of dissolution. This price is conditional on the, yet unknown, continuation value. This price is binding whenever the transaction price determined in the dissolution auction is lower than r. For simplicity of exposition, we only consider the winner's bid auction in this section, i.e., $k = 0$ and, thus, the transaction price becomes $\max\{b_H, r\}$. At the interim stage, when partners decide whether to propose dissolution, the continuation value and, thus, the minimum price r is known.

Without loss of generality, we restrict the partners' bids in the auction to $b_i \geq r$. The tie-breaking rule is the same as in the previous sections, i.e., if only one partner has proposed dissolution, this partner wins the auction if both partners submit the same bid.

Since we are interested in the question whether ex post efficiency can be restored with an appropriate reserve price, we will, again, focus on the existence of cutoff equilibria, with cutoff value $\tilde{v} \in (0, 1)$. A reserve price is only binding if $\tilde{v} \leq r$.[7] In a cutoff equilibrium with $\tilde{v}$, partner i proposes dissolution at the proposal stage if and only if his valuation, v_i, is above the cutoff, $\tilde{v}$. The belief system that is consistent with these equilibria is similar to the one in Section 3: After observing that the other partner played D at the proposal stage, one's belief is updated such that the other partner's value must be distributed according to the c.d.f. F conditional on the value being above the cutoff $\tilde{v}$. We denote this conditional c.d.f. by $K(x) := F(x|x \geq \tilde{v})$. We denote the belief of partner i by $\mu_i(x) := \Pr\{V_j \leq x\}$, $i, j \in \{1, 2\}$, $j \neq i$. The prior is $\mu_i(x) = F(x)$ and the posterior belief, after observing the other partner's action choice $a_j \in \{D, N\}$, is

$$\mu_i(x|a_j) = \begin{cases} K(x) & \text{if } a_j = D \\ \frac{F(x)}{F(\tilde{v})} & \text{if } a_j = N \end{cases}, \text{ where } K(x) := \begin{cases} 0 & \text{if } x < 0 \\ \frac{F(x)-F(\tilde{v})}{1-F(\tilde{v})} & \text{if } x \in [\tilde{v}, 1] \\ 1 & \text{if } x > 1. \end{cases} \tag{18}$$

Also denote $k := K'$.

As in Section 3 we start with our main result and then prove it in a series of lemmas.

Proposition 3. *Adding a reserve improves efficiency and reduces opportunistic dissolution.*

In Lemma 6, we state the equilibria of the dissolution subgames on the equilibrium path, depending on the outcome of the proposal stage. We assume a binding reserve price, $r > \tilde{v}$.

Lemma 6 (Dissolution Stage)**.** *Suppose at the proposal stage partner i plays D if and only if $v_i \in [\tilde{v}, 1]$, $i \in \{1, 2\}$. Given the updated beliefs,* (18)*, partners' equilibrium bids at*

[7] If the equilibrium is such that $\tilde{v} > r$, the outcome is exactly the same as in Section 3.

the auction stage are, depending on the stage-1 action profiles:

$$(D,D): \quad b_i = \begin{cases} r & \text{if } v_i \in [\tilde{v}, r] \\ \beta(v_i) & \text{if } v_i \in (r,1], \end{cases} \quad \text{where } \beta(v) := v - \int_r^v \frac{K^2(s)}{K^2(v)} ds, \tag{19}$$

$$(D,N),(N,D): \quad b_i = r. \tag{20}$$

Proof of Lemma 6. Throughout the proof we assume that partner j plays as proposed in the lemma, and we confirm that partner i has no incentive to deviate from the equilibrium candidate.

1. Consider the profile (D,D) at the proposal stage. Updated beliefs are $\mu_i(x|D) = \mu_j(x|D) = K(x)$. Since both partners have proposed dissolution, the tie-breaking rule selects each partner with equal probability. For partner i's deviation incentives we consider two cases: **(1a)** $v_i \in [\tilde{v}, r]$ and **(1b)** $v_i \in (r,1]$.

(1a) Suppose $v_i \in [\tilde{v}, r]$. First, suppose i follows the candidate strategy, $b_i = r$. With probability $K(r)$, $v_j \in [\tilde{v}, r]$ and $b_j = r$. Partner i wins with fifty per cent probability, yielding an expected payoff of $.5(v_i - r/2) + .5(r/2) = v_i/2$. With probability $1 - K(r)$, $v_j \in (r,1]$ and bids $\beta(v_2)$. Then partner i is the loser with expected payoff $.5E[\beta(V_j)|V_j \in (r,1]]$. Thus, i's expected payoff is

$$K(r)\frac{v_i}{2} + \frac{1}{2}\int_r^1 \beta(s)k(s)ds. \tag{21}$$

Now suppose partner i deviates to a larger bid, $b_i' > r$.[8] With probability $K(r)$, $v_j \in [\tilde{v}, r]$ and $b_j = r$, partner i wins with payoff $v_i - b_i'/2$ which is less than before since $v_i - b_i'/2 < v_i - r/2 \leq v_i - v_i/2 = v_i/2$. With probability $1 - K(r)$, $v_j \in (r,1]$. Partner i's previous payoff (from losing for sure by bidding $b_i = r$) was $\beta(v_2)/2 > r/2$.[9] Now he gets the same payoff as before if $\beta(v_j) > b_i'$ but if $\beta(v_j) < b_i'$, partner i wins with payoff $v_i - b_i'/2$ which is lower than before since $v_i - b_i'/2 < v_i - r/2 \leq r/2$. Thus, deviation is not profitable.

(1b) Suppose $v_i \in (r,1]$. Since partner j follows the candidate strategy, $b_j \in [r, \beta(1)]$. Partner i chooses $b_i \in [r, \beta(1)]$: Bidding lower than r is not feasible and bidding higher than $\beta(1)$ is dominated since bidding $\beta(1)$ already wins for sure while higher bids only increase the payment and reduce partner i's

[8] **Note that deviation to lower bids, $b_i' < r$, is ruled out by the reserve price.**

[9] Do we have to show that $\beta(v) > r$ for $v > r$? **Note: This is quite obvious, right?**

payoff. Thus, ~~as a best response,~~[10] partner i might consider either bidding his minimum feasible bid $b_i = r$ or bidding $\beta(x)$ with $x \in (r, 1]$. In the following we first show that $x = v_i$ uniquely maximizes i's expected payoff among all $x \in (r, 1]$, then we show that bidding $b_i = r$ is dominated by bidding $\beta(v_i)$ for partner i.

With probability $K(x)$, $v_j \in [\tilde{v}, x]$, and partner i, by bidding $b_i = \beta(x) > \beta(v_j)$, wins with payoff $v_i - \beta(x)/2$. With probability $1 - K(x)$, $v_j \in (x, 1]$ and partner i loses with expected payoff $.5E[\beta(V_j)|V_j \in (x, 1]]$. Thus, partner i's expected payoff from bidding $\beta(x)$, $x \in (r, 1]$, is

$$u_i(x, v_i) = K(x)\left(v_i - \frac{\beta(x)}{2}\right) + \frac{1}{2}\int_x^1 \beta(s)k(s)ds$$

Using (59) in Lemma 11, $u_i(x, v)$ can be rewritten as

$$\begin{aligned} u_i(x, v_i) &= K(x)v_i - \frac{K(x)}{2}\left(x - \int_r^x \frac{K^2(s)}{K^2(x)}ds\right) \\ &\quad + \frac{1}{2}\left(1 - K(x)x - 2\int_x^1 K(s)ds + \int_r^1 K^2(s)ds - \int_r^x \frac{K^2(s)}{K(x)}ds\right) \\ &= \frac{1}{2} + K(x)(v_i - x) - \int_x^1 K(s)ds + \frac{1}{2}\int_r^1 K^2(s)ds. \end{aligned}$$

Note that

$$u_i(v_i, v_i) - u_i(x, v_i) = K(x)(x - v_i) + \int_x^{v_i} K(s)ds.$$

This is positive for all $x \in (r, 1]$, $x \neq v_i$. Thus among all $x \in (r, 1]$, $x = v_i$ uniquely maximizes partner i's payoff. It remains to be shown that for partner i, bidding $\beta(v_i)$ is strictly better than bidding $b_i = r$. By bidding $b_i = r$, partner i's expected profit is equal to (21) as shown in the case (1a). It is easily verified that $u_i(v_i, v_i)$ exceeds (21) by

$$\int_r^{v_i} K(s)ds - \frac{K(r)(v_i - r)}{2} = \int_r^{v_i}\left(K(s) - \frac{K(r)}{2}\right)ds,$$

which is positive since K is nondecreasing and $v_i > r$.

2. Suppose the actions $a_i = D$ and $a_j = N$ have been played at the proposal stage. At the dissolution stage, the updated believes are that $v_i \in [\tilde{v}, 1]$ and $v_j \in [0, \tilde{v})$.

[10] **[T: There is only one best response, but several responses to consider in order to find it.]**

Partner i is favored by the tie-breaking rule and $b_j = r$. Since partner i has to place a bid not lower than r, partner i always wins with payoff $v_i - b_i/2$, which is maximized at $b_i = r$.

3. Finally, suppose the actions $a_i = N$ and $a_j = D$ have been played at the proposal stage. At the dissolution stage, the updated believes are that $v_i \in [0, \tilde{v})$ and $v_j \in [\tilde{v}, 1]$. Partner j is favored by the tie-breaking rule and $b_j = r$. Following the candidate $b_i = r$ makes partner i the sure loser with payoff $r/2$. Bidding $b_i > r$ makes partner i the sure winner, with payoff $v_i - b_i/2$. Since $v_i < r < \tilde{v}$, this payoff is bounded from above by $v_i - r/2 < r/2$ and, thus, bidding above $b_i = r$ does not pay.

□

We summarize the partners' payoffs from the dissolution stage in the next lemma.

Lemma 7. *Equilibrium payoffs of partners, $i, j \in \{1, 2\}$, at the dissolution stage, depending on the action profiles chosen at the proposal stage, are:*

$$(D,D): \quad U_i(v_i) = \begin{cases} \frac{1}{2} + \frac{1}{2}\int_r^1 K^2(t)dt - \int_{v_i}^1 K(t)dt & \text{if } v_i \in (r, 1], \\ \frac{1}{2}K(r)v_i + \frac{1}{2}\int_r^1 \beta(s)k(s)ds & \text{if } v_i \in [\tilde{v}, r). \end{cases} \tag{22}$$

$$(D,N),(N,D): \quad U_i = v_i - \frac{r}{2}, \; U_j = \frac{r}{2}, \; v_i \in [\tilde{v}, 1], \; v_j \in [0, \tilde{v}), \tag{23}$$

Having characterized the equilibrium play at the dissolution stage, in the next lemma, we spell out the conditions that are necessary and sufficient for the existence of a cutoff equilibrium with $\tilde{v}$.

Lemma 8 (Proposal Stage)**.** *The necessary and sufficient conditions for the existence of the cutoff equilibrium with cutoff value $\tilde{v}$ are, for partner $i \in \{1, 2\}$,*

$$\begin{aligned} F(\tilde{v})\frac{C}{2} + (1 - F(\tilde{v}))\frac{r}{2} \geq{}& F(\tilde{v})\left(v_i - \frac{r}{2}\right) \\ &+ (1 - F(\tilde{v}))\left(K(r)\frac{v_i}{2} + \frac{1}{2}\int_r^1 \beta(s)k(s)ds\right), \; \forall v_i \in [0, \tilde{v}] \end{aligned} \tag{24}$$

$$\begin{aligned} F(\tilde{v})\left(v_i - \frac{r}{2}\right) &+ (1 - F(\tilde{v}))\left(K(r)\frac{v_i}{2} + \frac{1}{2}\int_r^1 \beta(s)k(s)ds\right) \\ &\geq F(\tilde{v})\frac{C}{2} + (1 - F(\tilde{v}))\frac{r}{2}, \; \forall v_i \in [\tilde{v}, r] \end{aligned} \tag{25}$$

$$\begin{aligned} F(\tilde{v})\left(v_i - \frac{r}{2}\right) &+ (1 - F(\tilde{v}))\left(\frac{1}{2} - \int_{v_i}^1 K(s)ds + \frac{1}{2}\int_r^1 K^2(s)ds\right) \\ &\geq F(\tilde{v})\frac{C}{2} + (1 - F(\tilde{v}))\left(v_i - \frac{r}{2}\right), \; \forall v_i \in (r, 1]. \end{aligned} \tag{26}$$

Proof of Lemma 8. Throughout the proof we suppose that partner j plays the equilibrium candidate, i.e., proposing dissolution if and only if his value is at least $\tilde{v}$, and examine partner i's incentive of proposing vs. not proposing. The existence of a cutoff-equilibrium requires that partner i proposes dissolution if and only if $v_i \geq \tilde{v}$. We partition the range of partner i's valuations into low **(L)**, $v_i \in [0, \tilde{v})$, medium **(M)**, $v_i \in [\tilde{v}, r]$, and high **(H)**, $v_i \in (r, 1]$ and consider the three cases in sequence.

1. Suppose partner i's value is low with $v_i \in [0, \tilde{v})$. With probability $F(\tilde{v})$, partner j's value is in $[0, \tilde{v})$ and plays N. With probability, $1 - F(\tilde{v})$, partner j's value is in $[\tilde{v}, 1]$ and plays D. Denote partner i's profits at the dissolution stage by $\pi_{iL}^{(a_i, a_j)}$, with $a_i, a_j \in \{D, N\}$. Existence of a cutoff equilibrium with $\tilde{v}$ requires that for partner i with $v_i \in [0, \tilde{v})$, it is better to take action N instead of D. This implies:

$$F(\tilde{v})\pi_{iL}^{(N,N)} + (1 - F(\tilde{v}))\pi_{iL}^{(N,D)} \geq F(\tilde{v})\pi_{iL}^{(D,N)} + (1 - F(\tilde{v}))\pi_{iL}^{(D,D)} \tag{27}$$

   On the LHS of inequality (27), $\pi_{iL}^{(N,N)} = C/2$ since the partnership continues after action profile (N, N). Further, we have $\pi_{iL}^{(N,D)} = r/2$ by Lemma 7. In the following we derive $\pi_{iL}^{(D,N)}$ and $\pi_{iL}^{(D,D)}$ on the RHS of inequality (27).

   Consider $\pi_{iL}^{(D,N)}$. Since only partner i has proposed, the updated belief is that partner j has a low value and bids $b_j = r$ at the dissolution stage. Partner i is favored by the tie-breaking rule. Partner i wins with any of the feasible bids $b_i \geq r$. His payoff, $v_i - b_i/2$, decreases in b_i. Thus, the best bid for him is $b_i = r$ and his payoff is $\pi_{iL}^{(D,N)} = v_i - r/2$.

   Now consider $\pi_{iL}^{(D,D)}$. Since both partners have proposed, the updated belief is that partner j has a value from the interval $[\tilde{v}, 1]$. The tie-breaking rule selects each partner as the winner with equal probability in the event of a tie. With probability $K(r)$, partner j's value $v_j \in [\tilde{v}, r]$ and partner j bids $b_j = r$. By bidding $b_i = r$, partner i wins with fifty per cent probability, yielding himself an expected payoff of $.5(v_i - r/2) + .5(r/2) = v_i/2$. With probability $1 - K(r)$, partner j's value is $v_j \in (r, 1]$ and partner j bids $\beta(v_j)$. In that case, by bidding $b_i = r$, partner i is the loser with expected payoff $.5E[\beta(V_j)|V_j \in (r, 1]]$. Thus, partner i's expected payoff from bidding $b_i = r$ is given by (21)

$$K(r)\frac{v_i}{2} + \frac{1}{2}\int_r^1 \beta(s)k(s)ds.$$

Now suppose partner i places a larger bid than r, $b_i > r$. With probability $K(r)$, $v_j \in [\tilde{v}, r]$, and partner i wins with payoff $v_i - b_i/2$ which is less than his payoff from bidding r, since $v_i - b_i/2 < v_i - r/2 \leq v_i - v_i/2 = v_i/2$. With probability $1 - K(r)$, $v_j \in (r, 1]$. Partner i's previous payoff (from losing) was $\beta(v_i)/2 > r/2$. He gets the same payoff as his payoff from bidding r if $\beta(v_j) > b_i$ but if $\beta(v_j) < b_i$, partner i now wins with payoff $v_i - b_i/2$ which is lower than his payoff from bidding r since $v_i - b_i/2 < v_i - r/2 < r/2$. Thus, $b_i = r$ is the optimal bid and, $\pi_{iL}^{(D,D)}$ is equal to (21).

Inserting $\pi_{iL}^{(N,N)}$, $\pi_{iL}^{(N,D)}$, $\pi_{iL}^{(D,N)}$ and $\pi_{iL}^{(D,D)}$ into (27), we get

$$F(\tilde{v})\frac{C}{2} + (1 - F(\tilde{v}))\frac{r}{2} \geq F(\tilde{v})\left(v_i - \frac{r}{2}\right) + (1 - F(\tilde{v}))\left(K(r)\frac{v_i}{2} + \frac{1}{2}\int_r^1 \beta(s)k(s)ds\right)$$

Therefore, the existence of cutoff equilibrium requires that for $v_i \in [0, \tilde{v})$, condition (24) holds.

2. Suppose partner i's value falls into the intermediate range with $v_i \in [\tilde{v}, r]$. Given that partner j proposes dissolution if and only if $v_j \geq \tilde{v}$, the existence of cutoff equilibrium with $\tilde{v}$ requires that partner i prefers D to N in terms of expected payoff from the dissolution stage. This is equivalent to the condition

$$F(\tilde{v})\pi_{iM}^{(D,N)} + (1 - F(\tilde{v}))\pi_{iM}^{(D,D)} \geq F(\tilde{v})\pi_{iM}^{(N,N)} + (1 - F(\tilde{v}))\pi_{iM}^{(N,D)} \tag{28}$$

The LHS of inequality (28) is partner i's payoff if he indeed proposes dissolution. As derived in Lemma 6, $\pi_{iM}^{(D,N)} = v_i - r/2$ since both partners bid $b_i = r$ at the dissolution stage and partner i wins by the tie-breaking rule. Moreover, $\pi_{iM}^{(D,D)}$ is given by (21).

The RHS of inequality (28) is partner i's payoff if he does not propose dissolution. Clearly, $\pi_{iM}^{(N,N)} = C/2$ since the partnership continues after action profile (N, N). Finally, we show that $\pi_{iM}^{(N,D)} = r/2$. There, partner j bids $b_j = r$ and is favored by the tie-breaking rule. If partner i makes the lowest feasible bid, $b_i = r$ and loses with payoff $r/2$. Bidding higher, $b_i > r$, does not pay since then the payoff is lower: $v_i - b_i/2 < v_i - r/2 \leq r - r/2 = r/2$.

Inserting $\pi_{iM}^{(D,N)}$, $\pi_{iM}^{(D,D)}$, $\pi_{iM}^{(N,N)}$, and $\pi_{iM}^{(N,D)}$ into (28), we get the condition that

$$F(\tilde{v})\left(v_i - \frac{r}{2}\right) + (1 - F(\tilde{v}))\left(K(r)\frac{v_i}{2} + \frac{1}{2}\int_r^1 \beta(s)k(s)ds\right) \geq F(\tilde{v})\frac{C}{2} + (1 - F(\tilde{v}))\frac{r}{2}$$

must hold for all $v_i \in [\tilde{v}, r]$.

3. Suppose partner i's value is high with $v_i \in (r, 1]$. Given that partner j proposes dissolution if and only if $v_j \geq \tilde{v}$, the existence of a cutoff equilibrium with $\tilde{v}$ requires that partner i prefers D to N in terms of expected payoff from the dissolution stage. This is equivalent to the condition

$$F(\tilde{v})\pi_{iH}^{(D,N)} + (1 - F(\tilde{v}))\pi_{iH}^{(D,D)} \geq F(\tilde{v})\pi_{iH}^{(N,N)} + (1 - F(\tilde{v}))\pi_{iH}^{(N,D)} \tag{29}$$

The LHS of inequality (29) is partner i's payoff if he plays D. From Lemma 7, $\pi_{iH}^{(D,N)} = v_i - r/2$ since both partners bid $b_i = r$ and partner i wins by the tie-breaking rule. Moreover, $\pi_{iH}^{(D,D)}$ is given by $U_i(v_i, v_i)$:

$$\pi_{iH}^{(D,D)} = \frac{1}{2} - \int_{v_i}^{1} K(s)ds + \frac{1}{2}\int_{r}^{1} K^2(s)ds. \tag{30}$$

The RHS of inequality (29) is partner i's payoff if he plays N. Clearly, $\pi_{1H}^{(N,N)} = C/2$ since the partnership continues in that case. Finally, it remains to determine $\pi_{iH}^{(N,D)}$. Partner j bids $b_j = r$ and is favored by the tie-breaking rule. By bidding $b_i = r$, partner i gets a payoff $r/2$ since he loses the auction. Bidding higher, say, $b_1 = r + \varepsilon$ for some $\varepsilon > 0$, makes partner i the sure winner with payoff $v_i - (r + \varepsilon)/2$. Since $v_i > r$, there exist sufficiently small $\varepsilon > 0$ such that the payoff from winning, $v_i - (r + \varepsilon)/2$, exceeds the payoff from losing, $r/2$. Since condition (29) needs to hold for every $\varepsilon > 0$ for the existence of the cutoff equilibrium, we use $\pi_{iH}^{(N,D)} = v_i - r/2$, i.e., the lowest upper bound of partner i's profit. Inserting $\pi_{iH}^{(D,N)}$, $\pi_{iH}^{(D,D)}$, $\pi_{iH}^{(N,N)}$, and $\pi_{iH}^{(N,D)}$ into (29), we get that

$$F(\tilde{v})\left(v_i - \frac{r}{2}\right) + (1 - F(\tilde{v}))\left(\frac{1}{2} - \int_{v_i}^{1} K(s)ds + \frac{1}{2}\int_{r}^{1} K^2(s)ds\right) \geq F(\tilde{v})\frac{C}{2} + (1 - F(\tilde{v}))\left(v_i - \frac{r}{2}\right)$$

has to hold for any $v_i \in (r, 1]$.

Thus, we have developed the three conditions, i.e., (24), (25), and (26), that have to be satisfied for the existence of any cutoff equilibrium with $\tilde{v}$. □

Our next task is to examine whether there exists a r such that $\tilde{v} = C$.

Lemma 9. *For the existence of a cutoff equilibrium with $\tilde{v} = C$, r has to be such that:*

$$F(r)(r - C) = \int_{r}^{1} \frac{(1 - F(s))^2}{1 - F(C)} ds \tag{31}$$

subject to the constraint that for any $v_i \in (r,1]$, it holds

$$F(C)\left(v_i - \frac{r}{2}\right) + (1-F(C))\left(\frac{1}{2} - \int_{v_i}^{1} \frac{F(s)-F(C)}{1-F(C)}ds + \frac{1}{2}\int_r^1 \left(\frac{F(s)-F(C)}{1-F(C)}\right)^2 ds\right)$$
$$\geq F(C)\frac{C}{2} + (1-F(C))\left(v_i - \frac{r}{2}\right) \tag{32}$$

Proof. First note that (24) and (25) can only hold simultaneously if

$$F(\tilde{v})\frac{C}{2} + (1-F(\tilde{v}))\frac{r}{2} = F(\tilde{v})\left(\tilde{v} - \frac{r}{2}\right)$$
$$+ (1-F(\tilde{v}))\left(K(r)\frac{\tilde{v}}{2} + \frac{1}{2}\int_r^1 \beta(s)k(s)ds\right) \tag{33}$$

When (33) holds, then (24) also holds for any other $v_i \in [0,\tilde{v})$ since RHS of (24) is increasing in v_i. Similarly, (25) holds for any $v_i \in [\tilde{v}, r]$ since LHS of (25) is increasing in v_i. Replacing $K(x)$ by $\frac{F(x)-F(\tilde{v})}{1-F(\tilde{v})}$ and using (62), condition (33) can be simplified as follows

$$F(\tilde{v})\frac{C}{2} + (1-F(\tilde{v}))\frac{r}{2} = F(\tilde{v})\left(\tilde{v} - \frac{r}{2}\right)$$
$$+ (1-F(\tilde{v}))\left(\frac{\tilde{v}}{2}\frac{F(r)-F(\tilde{v})}{1-F(\tilde{v})} + \frac{1}{2}\left(\frac{1-F(r)}{1-F(\tilde{v})}r + \int_r^1 \left(\frac{1-F(s)}{1-F(\tilde{v})}\right)^2 ds\right)\right)$$
$$\iff F(\tilde{v})(C-\tilde{v}) + F(r)(r-\tilde{v}) = \int_r^1 \frac{1-F(s)^2}{1-F(\tilde{v})}ds \tag{34}$$

Recall that efficiency requires that a partner calls for a dissolution if and only if his private signal is above C, ie. $\tilde{v} = C$. Reserve r has to satisfy (34) for efficiency to obtain given $\tilde{v} = C$. Hence, a feasible r that achieves efficiency must satisfy (31). From (31), we note that a feasible $r > C$. Further, note that for r close to C, we have RHS>LHS and for $r = 1$ we have LHS>RHS. Moreover, LHS is increasing in r and RHS is decreasing in r. Thus there is a unique r in the range $r \in (C, 1)$ for which (31) holds. □

Lemma 10. *If $C \geq v^m$, adding reserve price restores ex post efficiency.*

Proof. Given r that satisfies (31), the existence of an efficient equilibrium is equivalent to the condition that (32) holds for any $v_i \in (r,1]$.

Rewrite (32), we get

$$(2F(C)-1)\left(v_i - \frac{r}{2}\right) - \int_{v_i}^1 (F(s)-F(C))ds$$
$$\geq F(C)\frac{C}{2} - \frac{1}{2}(1-F(C)) - \frac{1}{2}\int_r^1 \frac{(F(s)-F(C))^2}{1-F(C)}ds \tag{35}$$

First note that the derivative of LHS of (35) with respect to v_i is given by $F(C)+F(v_i)-1$. From Lemma 9, we know that a feasible $r > C$. Therefore, for any $C \geq C^m$ and $v_i \geq r$, we have $F(C) \geq \frac{1}{2}$ and $F(v_i) > \frac{1}{2}$. Therefore, LHS of (35) is a strictly increasing function of v_i for $C \geq v^m$.

Second, note that for $v_i = r$, (35) can be rewritten as

$$F(C)(C-r) \leq \int_r^1 \frac{(1-F(s))^2}{1-F(C)} ds$$

Since $C < r$, LHS is negative while RHS is positive. Therefore, (35) holds for $v_i = r$. This implies that (35) holds for any $v_i > r$ since LHS of (35) is monotonically increasing in v_i while RHS of (35) is constant with respect to v_i. This concludes our proof that if $C \geq C^m$, adding reserve price indeed restores ex post efficiency. □

From the intuition developed in Section 3, the inefficiency is due to a low-value partner's incentive to call for dissolution, hoping that the other partner has a high value and will thus buy the low-value partner's share at a high price. A reserve price should reduce such incentive by punishing the low-value partner in case the speculation breaks up the partnership inefficiently: whenever the low-value partner proposes and wins, he has to pay at least r for the other partner's share. Thus, the punishment is more severe with larger r, suggesting that his might, in principle, remove the inefficiency.

6 Conclusion

The literature on partnership dissolution generally takes the dissolution decision as exogenously given and examines whether dissolution can be efficient, i.e., whether the asset is allocated to the partner who has the highest value for it. There, an influential result is that $k+1$-price auction dissolves a partnership efficiently when the share structure is equal. In this paper, we endogenize the dissolution decision in an equal-share partnership and show that dissolution occurs for speculative reasons when continuation is efficient in $k+1$-price auction. Adding the veto right does not achieve efficiency, while specifying a reserve price in the partnership contract restores efficiency.

7 Appendix

Proof of Lemma 1. As stated in the lemma, we suppose that partner $i \in \{1,2\}$ has played D at the proposal stage if and only if $v_i \in [\bar{v}, 1]$. We have to distinguish three cases, according to the action profiles chosen at the proposal stage: $(a_1, a_2) \in \{(D, D), (D, N), (N, D)\}$. **In the first case, the two players are symmetric at the dissolution stage. In the latter two cases, the proof that $b_2 = \bar{v}$ is a best response to $b_1 = \bar{v}$ in the case of $(a_1, a_2) = (D, N)$ is exactly the same as proving $b_1 = \bar{v}$ is a best response to $b_2 = \bar{v}$ in the case of $(a_1, a_2) = (N, D)$.** ~~By symmetry of the players,~~ **Therefore, in the entire proof** it suffices to consider the deviation incentives of partner 1, taking as given that partner 2 plays according to the equilibrium candidate.

1. $(a_1, a_2) = (D, D)$: Updated beliefs are $\mu_1(x|D) = \mu_2(x|D) = G(x)$. For partner 1, bidding outside ~~of~~ the range $b_1 \in [\beta(\bar{v}), \beta(1)]$ is strictly dominated: The bid $\beta(\bar{v})$ dominates all lower bids since all those bids make partner 1 the sure loser but lower bids reduce the transaction price and, thus, partner 1's payoff. Bids above $\beta(1)$ are dominated since partner 1 is already the sure winner and bidding higher increases the price he has to pay.

   Partner 1's expected payoff at the dissolution stage, when his value is v_1 and he bids $\beta(x)$, $x \in [\bar{v}, 1]$, is

$$
\begin{aligned}
U_1(x, v_1) &= G(x)\left(v_1 - \frac{1}{2}\left(kE\left[\beta(V_2)|V_2 < x\right] + (1-k)\beta(x)\right)\right) \\
&\quad + (1 - G(x))\frac{1}{2}\left(k\beta(x) + (1-k)E\left[\beta(V_2)|V_2 > x\right]\right) \\
&= G(x)v_1 - \frac{k}{2}\int_{\bar{v}}^{x} \beta(s)g(s)ds - \frac{G(x)}{2}(1-k)\beta(x) + \frac{1-G(x)}{2}k\beta(x) + \frac{1-k}{2}\int_{x}^{1}\beta(s)g(s)ds \\
&= G(x)v_1 - \frac{k}{2}\int_{\bar{v}}^{x} \beta(s)g(s)ds - \frac{1}{2}\left(G(x) - k\right)\beta(x) + \frac{1-k}{2}\int_{x}^{1}\beta(s)g(s)ds.
\end{aligned}
\tag{36}
$$

   (**Note: something wrong with the tex file, number (41) refers to an empty line.**) The derivative of $U_1(x, v_1)$ with respect to x is given by

$$
\partial_x U_1(x, v_1) = g(x)v_1 - \beta(x)g(x) - \frac{1}{2}(G(x) - k)\beta'(x). \tag{37}
$$

   The proposed bid function, (2), can be rewritten as

$$
\beta'(x) = \frac{2g(x)}{G(x) - k}(x - \beta(x)). \tag{38}
$$

   Replacing $\beta'(x)$ in (37) with (38), we get $\partial_x U_1(x, v_1) = g(x)(v_1 - x)$.[11] This implies that $\partial_x U_1(x, v_1) > 0$ for $x < v_1$ and $\partial_x U_1(x, v_1) < 0$ for $x > v_1$. Hence, $U_1(x, v_1)$ is uniquely maximized at $x = v_1$.

[11] For the case $G(x) = k$, $\partial_x U_1(x, v_1) = g(x)v_1 - \frac{1}{2}\beta(x)g(x)$ and $\beta(x) = x$. Thus, $\partial_x U_1(x, v_1) = g(x)(v_1 - x)$, too.

Given that both partners bid according to (2), partner i's expected equilibrium payoff at the dissolution stage, $U(v) := U_i(v, v)$, is

$$U(v) = G(v)v - \frac{k}{2}\int_{\bar{v}}^{v} \beta(s)g(s)ds - \frac{1}{2}\left(G(v) - k\right)\beta(v) + \frac{1-k}{2}\int_{v}^{1} \beta(s)g(s)ds. \quad (39)$$

Rearranging (39), we get

$$U(v) = G(v)v - \frac{1}{2}\left(G(v) - k\right)\beta(v) + \frac{1}{2}\int_{v}^{1} \beta(s)g(s)ds - \frac{k}{2}\int_{\bar{v}}^{1} \beta(s)g(s)ds. \quad (40)$$

Denote parts of (40) as follows:

$$A := \left(G(v) - k\right)\beta(v), \qquad B := \int_{v}^{1} \beta(s)g(s)ds, \qquad C := \int_{\bar{v}}^{1} \beta(s)g(s)ds. \quad (41)$$

Inserting (2), we get

$$A = G(v)v - kv - \frac{1}{G(v)-k}\int_{G^{-1}(k)}^{v} (G(t)-k)^2 dt, \quad (42)$$

$$\begin{aligned}
B &= \int_{v}^{1}\left(s - \int_{G^{-1}(k)}^{s}\left(\frac{G(t)-k}{G(s)-k}\right)^2 dt\right) g(s)ds \\
&= \int_{v}^{1} sg(s)ds - \int_{v}^{1}\int_{G^{-1}(k)}^{s}(G(t)-k)^2 dt \frac{g(s)}{(G(s)-k)^2}ds \\
&= \int_{v}^{1} sg(s)ds + \int_{v}^{1}\int_{G^{-1}(k)}^{s}(G(t)-k)^2 dt d\frac{1}{(G(s)-k)} \\
&= 1 - vG(v) - \int_{v}^{1} G(s)ds + \frac{1}{1-k}\int_{G^{-1}(k)}^{1}(G(t)-k)^2 dt \\
&\quad - \frac{1}{G(v)-k}\int_{G^{-1}(k)}^{v}(G(t)-k)^2 dt - \int_{v}^{1}(G(s)-k)ds \\
&= 1 - vG(v) - 2\int_{v}^{1} G(s)ds + k(1-v) + \frac{1}{1-k}\int_{G^{-1}(k)}^{1}(G(t)-k)^2 dt \\
&\quad - \frac{1}{G(v)-k}\int_{G^{-1}(k)}^{v}(G(t)-k)^2 dt,
\end{aligned} \quad (43)$$

$$\begin{aligned}
C &= \int_{\bar{v}}^{1}\left(s - \int_{G^{-1}(k)}^{s}\left(\frac{G(t)-k}{G(s)-k}\right)^2 dt\right) g(s)ds \\
&= \int_{\bar{v}}^{1} sg(s)ds + \int_{\bar{v}}^{1}\int_{G^{-1}(k)}^{s}(G(t)-k)^2 dt d\frac{1}{G(s)-k} \\
&= 1 - \int_{\bar{v}}^{1} G(s)ds + \frac{1}{1-k}\int_{G^{-1}(k)}^{1}(G(t)-k)^2 dt \\
&\quad - \frac{1}{k}\int_{\bar{v}}^{G^{-1}(k)}(G(t)-k)^2 dt - \int_{\bar{v}}^{1}(G(s)-k)ds \\
&= 1 - 2\int_{\bar{v}}^{1} G(s)ds + k(1-\bar{v}) + \frac{1}{1-k}\int_{G^{-1}(k)}^{1}(G(t)-k)^2 dt \\
&\quad - \frac{1}{k}\int_{\bar{v}}^{G^{-1}(k)}(G(t)-k)^2 dt.
\end{aligned} \quad (44)$$

Inserting A, B and C back into (40) and simplifying, we get (4).

2. $(a_1, a_2) = (D, N)$. **Given that partner 2 bids $b_2 = \bar{v}$ and partner 1 is favored by the tie-breaking rule,** partner 1 wins the auction with any bid $b_1 \in [\bar{v}, 1]$ and loses with $b_1 < \bar{v}$. The payoff **of partner 1** from winning is $u_w = v_1 - 0.5(k\bar{v} + (1-k)b_1)$ and his payoff from losing is $u_l = 0.5(kb_1 + (1-k)\bar{v})$.

   For $k \in (0, 1)$, u_w decreases in b_1. Its maximum value equals to $v_1 - 0.5\bar{v}$ at $b_1 = \bar{v}$. u_l increases in b_1 and is less than $0.5\bar{v}$ as losing means $b_1 < \bar{v}$. Since $v_1 \geq \bar{v}$, $u_w > u_l$. Winning dominates losing for partner 1. Therefore, partner 1's best response to $b_2 = \bar{v}$ is indeed $b_1 = \bar{v}$.

   If $k = 1$, $u_w = v_1 - 0.5\bar{v}$ is independent of b_1 given that $b_1 \geq \bar{v}$. $u_l = 0.5b_1$ increases in b_1 and is less than $0.5\bar{v}$ as losing means $b_1 < \bar{v}$. Since $v_1 \geq \bar{v}$, $u_w > u_l$ and winning dominates losing. For partner 1, any bid with $b_1 \geq \bar{v}$ is a best response to $b_2 = \bar{v}$ and hence bidding $b_1 = \bar{v}$ is also a best response for partner 1.

   If $k = 0$, $u_w = v_1 - 0.5b_1$ decreases in b_1, and its maximum value $v_1 - 0.5\bar{v}$ is reached at $b_1 = \bar{v}$. $u_l = 0.5\bar{v}$ and is independent of b_1 given that $b_1 < \bar{v}$. Since $v_1 \geq \bar{v}$, winning dominates losing and bidding $b_1 = \bar{v}$ is a unique best response for partner 1 to $b_2 = \bar{v}$.

   For $k \in [0, 1]$, in equilibrium, partner 1 wins by the tie-breaking rule and obtains a payoff equal to $v_1 - \bar{v}/2$. Partner 2's payoff is $\bar{v}/2$.

3. $(a_1, a_2) = (N, D)$: **Given that partner 2 bids $b_2 = \bar{v}$ and is favored by the tie-breaking rule,** partner 1 wins in the auction with bids $b_1 \in (\bar{v}, 1]$, with payoff $u_v = v_1 - 0.5(k\bar{v} + (1-k)b_1)$, and loses in the auction with bids $b_1 \in [0, \bar{v}]$, with payoff $u_l = 0.5(kb_1 + (1-k)\bar{v})$. Again we differentiate $k \in (0, 1)$, $k = 0$, and $k = 1$.

   If $k \in (0, 1)$, u_v is decreasing in b_1 and since $b_1 > \bar{v}$ in case of winning, $u_v \leq v_1 - \bar{v}/2$. u_l increases in b_1 and takes its maximum value $0.5\bar{v}$ at $b_1 = \bar{v}$. Since $v_1 < \bar{v}$, $u_w < u_l$ and losing dominates winning for partner 1. Hence, bidding $b_1 = \bar{v}$ is indeed a best response to $b_2 = \bar{v}$.

   If $k = 1$, $u_v = v_1 - 0.5\bar{v}$ is independent of b_1. The losing payoff $u_l = 0.5b_1$ takes its maximum value $0.5\bar{v}$ at $b_1 = \bar{v}$. Since $v_1 < \bar{v}$, $u_w < u_l$ and losing dominates winning for partner 1. Hence, bidding $b_1 = \bar{v}$ is indeed a best response to $b_2 = \bar{v}$.

If $k = 0$, $u_v = v_1 - 0.5b_1$ decreases in b_1. Since $b_1 > \bar{v}$ in case of winning, $u_v < v_1 - 0.5\bar{v}$. $u_l = 0.5\bar{v}$ is independent of b_1 as long as $b_1 \leq \bar{v}$. Since $v_1 < \bar{v}$, $u_w < u_l$ and losing dominates winning for partner 1. Hence, any $b_1 \leq \bar{v}$ is a best response to $b_2 = \bar{v}$, and bidding $b_1 = \bar{v}$ is indeed a best response.

For $k \in [0, 1]$, in equilibrium, partner 2 wins by the tie-breaking rule and obtains a payoff $v_2 - \bar{v}/2$. Partner 1 obtains a payoff $\bar{v}/2$.

□

Proof of Lemma 2. For the case (D, D), the uniqueness of the equilibrium, **equation** (2) **in Lemma 1**, has been established by Kittsteiner (2003).

[J: Shall we replace the notation of β_1 and β_2 in the following two paragraphs by b_1 and b_2? We use the latter in the proofs of other lemmas. Even in the later part of this proof, we use b instead of β.]

Now consider the case (N, D). This implies $v_1 \in [0, \bar{v})$ and $v_2 \in [\bar{v}, 1]$. ~~and, w.l.o.g.,~~ Ex post efficiency requires that partner 2 always wins **in the auction**, **ie.** $\beta_1(v_1) \leq \beta_2(v_2)$, where partner 2 wins by the tie-breaking rule if $\beta_1(v_1) = \beta_2(v_2)$. **Suppose such an efficient equilibrium indeed exists. We differentiate three cases: $k \in (0, 1)$, $k = 0$, and $k = 1$. In each case, we first show that in an efficient equilibrium, the partners necessarily submit the same bids. Then we show that the bid equals to $\bar{v}$.**

Suppose $k \in (0, 1)$.~~Now,~~ **For partner 2,** $\beta_2 > \beta_1^{\sup} := \sup_{v_1} \beta_1(v_1)$ is strictly dominated **by** $\beta_2 = \beta_1^{\mathbf{sup}}$ since partner 2's payoff, $v_2 - (k\beta_1 + (1-k)\beta_2)/2$ is strictly decreasing in β_2 and he is favored by the tie-breaking rule. Thus, $\beta_2 = \beta_1^{\sup}$ is the unique best response **to partner 1's strategy $\beta(v_1)$ in an efficient equilibrium**.

Analogously, **in an efficient equilibrium**, partner 1's payoff, $(k\beta_1 + (1-k)\beta_2)/2$, is strictly increasing in β_1. Thus, $\beta_1 = \beta_1^{\sup}$ is the unique optimal bid, **independent of** β_2. Thus, in **an** ~~any (potentially)~~ efficient equilibrium, both partners must make the same bid~~, say,~~ **which we denote as** b^*, and partner 2 wins by the tie-breaking rule. The payoffs are $v_2 - b^*/2$ for the winning partner 2 and $b^*/2$ for the losing partner 1.

Next, we argue that $b^* = \bar{v}$. First, suppose per absurdum that $b^* = \min\{\bar{v} + s, 1\}$ for some $s > 0$ and restrict attention to $v_2 \in [\bar{v}, \min\{\bar{v} + s, 1\})$. (**Note: I can not see why we need a half open interval.**) If both bid $b^* = \min\{\bar{v} + s, 1\}$, then partner 2's payoff is $v_2 - \min\{\bar{v} + s, 1\}/2 < v_2/2$. If, however, partner 2 deviates to $b_2' = v_2$ then his payoff is $(kv_2 + (1-k)\min\{\bar{v} + s, 1\})/2 > v_2/2$, a profitable deviation. Thus, we can rule out $b^* > \bar{v}$. Second, suppose per absurdum that $b^* = \max\{\bar{v} - s, 0\}$ for some $s > 0$ and restrict attention to $v_1 \in (\max\{\bar{v} - s, 0\}, \bar{v}]$. (**Note: I can not see why we need a half open interval instead of a closed one.**) If both bid $b^* = \max\{\bar{v} - s, 0\}$, then partner 1's payoff is $\max\{\bar{v} - s, 0\}/2 < v_1/2$. If, however, partner 1 deviates to $b_1' = v_1$ then his payoff is $v_1 - (k \max\{\bar{v} - s, 0\} + (1-k)v_1)/2 > v_1/2$, a profitable deviation. Thus, we can rule out $b^* < \bar{v}$. Finally, for $b_1^* = b_2^* = \bar{v}$ there is no such **profitable deviation.** ~~incentive.~~ [**Note: In this paragraph I have replaced $\bar{v} + s$ by $\min\{\bar{v} + s, 1\}$ as a partner will never submit a bid above 1, and $\bar{v} - s$ by $\max\{\bar{v} - s, 0\}$.**]

Now suppose $k = 0$ (WBA)~~, and, as above and w.l.o.g., we only consider the case (N, D)~~. ~~As above, by efficiency, we must have $\beta_1(v_1) \leq \beta_2(v_2)$ which implies that partner 2 always wins.~~ **In an efficient equilibrium where partner 2 wins for any $v_2 \in [\bar{v}, 1]$, the payoff of partner 2,** $v_2 - \beta_2(v_2)/2$, is strictly decreasing in his bid. Thus, **given partner 1's strategy $\beta_1(v_1)$, partner 2's** optimal bid ~~is a constant,~~ **equals to** $b_2^* = \beta_1^{\sup} := \sup_{v_1} \beta_1(v_1)$. Now we show that $\beta_1^{\sup} = \bar{v}$. Suppose $b_2^* = \beta_1^{\sup} = \bar{v} - s$ for some $s > 0$. Then partner 1 prefers winning if $v_1 - (\bar{v} - s + \varepsilon)/2 > (\bar{v} - s)/2$ for some $\varepsilon > 0$ which, since $v_1 < \bar{v}$, is equivalent to $v_1 \in (\bar{v} - s + \varepsilon/2, \bar{v})$. In that case, partner 1 would bid more than $b_2^* = \beta_1^{\sup}$, a contradiction. Thus, $b_2^* < \bar{v}$ can be ruled out. Similarly, $b_2^* = \beta_1^{\sup} = \bar{v} + s$ for some $s > 0$ can be ruled out since bidding $b_2 = v_2$ is a more profitable bid for partner 2 if $v_2 \in [\bar{v}, \bar{v} + s)$. Finally, for $b_2^* = \bar{v}$ there is no such incentive.

Now we show that $b_1^* = \bar{v}$ as well. In order to do that we suppose per absurdum that player 1 bids such that there is positive probability mass on bids below $\bar{v}$. We show that player 2 can then profitably deviate from the bid $b_2^* = \bar{v}$.

We consider the type $v_2 = v_t := \bar{v} + t$ with $t > 0$. First note that the status quo profit of this player 2 type from bidding $b_2^* = \bar{v}$ is $v_t - \bar{v}/2 = \bar{v}/2 + t$. We want to show that the deviation profit is above that.

Now suppose this player 2 type bids a constant bid of $b_2 = \bar{v} - s$ for some $s > 0$. Then his expected profit is

$$p_{\bar{v}} \frac{\bar{v}}{2} + p_W \left(v_t - \frac{\bar{v} - s}{2} \right) + (1 - p_W) \left(\frac{1}{2} E[\beta_1(V_1) | V_1 \in [0, \bar{v}), \beta_1 > \bar{v} - s] \right), \quad (45)$$

where $p_{\bar{v}} \geq 0$ is the probability that player 1 bids $b_1 = \bar{v}$ and p_W is player 2's probability of winning in all other cases (note that p_W is a function of s).

Now insert $v_t = \bar{v} + t$ and note that player 1's expected bid above satisfies $\beta_1 > \bar{v} - s$

since in the relevant events player 1 is the winner. Thus, player 2's expected profit exceeds

$$p_{\bar{v}}\frac{\bar{v}}{2} + p_W\left(\bar{v} + t - \frac{\bar{v} - s}{2}\right) + (1 - p_W)\left(\frac{\bar{v} - s}{2}\right) \tag{46}$$

This can be written as

$$\left(\frac{\bar{v}}{2} + t\right) + \left(-t(1 - p_W) + p_{\bar{v}}\frac{\bar{v}}{2} + \left(p_W - \frac{1}{2}\right)s\right). \tag{47}$$

The first parenthesis is the status quo profit. We want to know if the second parenthesis is positive. It is positive if

$$t < \frac{1}{1 - p_W}\left(p_{\bar{v}}\frac{\bar{v}}{2} + \left(p_W - \frac{1}{2}\right)s\right) \tag{48}$$

Obviously, for $p_W > \frac{1}{2}$ the RHS is positive (even if $p_{\bar{v}} = 0$). Thus, the types $v_2 \in [\bar{v}, \bar{v} + t]$ can profitably deviate. Now choose $s > 0$ such that $p_W > 1/2$.

Now suppose $k = 1$ (LBA), and, as above and w.l.o.g., we only consider the case (N, D). As above, by efficiency, we must have $\beta_1(v_1) \leq \beta_2(v_2)$ which implies that partner 1 is always the loser. His payoff, $\beta_1(v_1)/2$, is strictly increasing in his bid. Thus, his optimal bid is a constant, $b_1^* = \beta_2^{\inf} := \inf_{v_2} \beta_2(v_2)$. Now we show that $b_1^* = \beta_2^{\inf} = \bar{v}$. Suppose $b_1^* = \beta_2^{\inf} = \bar{v} - s$ for some $s > 0$. Then if $v_1 \in (\bar{v} - s, \bar{v})$ then bidding $b_1 = v_1$ is more profitable for partner 1 regardless of whether he wins or loses with that bid. Thus, $b_1^* < \bar{v}$ can be ruled out.

Similarly, $b_1^* = \beta_2^{\inf} = \bar{v} + s$ for some $s > 0$ can be ruled out since if $v_2 \in [\bar{v}, \bar{v} + s)$ then partner 2 would prefer bidding $b_2 = v_2$ regardless of whether he wins or loses with that bid. Finally, for $b_1^* = \bar{v}$ there is no such incentive.

Now, similarly to the case $k = 0$ one can show that there exists $t > 0$ such that for all $v_1 \in [\bar{v} - t, \bar{v}]$ there is an $s > 0$ such that bidding $b_1 = \bar{v} + s$ is more profitable in expectation than $b_1^* = \bar{v}$ whenever partner 2 bids more than $b_2 = \bar{v}$ with positive probability.

□

Proof of Lemma 3. By symmetry of the game, it is w.l.o.g. to restrict attention to player 1's incentives to deviate while taking as given that player 2 plays the proposed equilibrium **strategy of calling for a dissolution if and only if** $v_2 \geq \bar{v}$.

We distinguish two cases, (**L**), if $v_1 \in [0, \bar{v})$, and (**H**), if $v_1 \in [\bar{v}, 1]$. A partner's **expected** payoff at the dissolution stage depends on his private valuation and the proposal stage action profile. We denote partner 1's **expected** profit at the dissolution stage by $\pi_{1k}^{(a_1, a_2)}$, where $k \in \{L, H\}$, $a_i \in \{D, N\}$, $i \in \{1, 2\}$.

(L) Suppose partner 1's value is low, $v_1 \in [0, \bar{v})$. If he proposes dissolution, D, then his payoff depends on whether partner 2 also proposes (which happens with probability $1-F(\bar{v})$) or not (with probability $F(\bar{v})$). Partner 1's corresponding payoffs are $\pi_{1L}^{(D,D)}$ and $\pi_{1L}^{(D,N)}$, respectively. If, however, **partner** 1 does not propose (as prescribed by the candidate), then his corresponding payoffs are either $\pi_{1L}^{(N,D)}$ or $\pi_{1L}^{(N,N)}$. Since partner 1's value is low, according to the candidate, partner 1 must prefer N over D, which leads to the condition

$$F(\bar{v})\pi_{1L}^{(N,N)} + (1 - F(\bar{v}))\pi_{1L}^{(N,D)} \geq F(\bar{v})\pi_{1L}^{(D,N)} + (1 - F(\bar{v}))\pi_{1L}^{(D,D)} \qquad (49)$$

~~where~~ The LHS of (49) is partner 1's payoff from playing ~~corresponds to~~ the equilibrium candidate. **There,** $\pi_{1L}^{(N,N)} = C/2$, since the partnership continues **after** (N, N). ~~Second,~~ $\pi_{1L}^{(N,D)} = \bar{v}/2$ by Lemma 1.

The RHS of (49) **is partner 1's payoff from unilateral deviation.** ~~The Third,~~ We determine $\pi_{1L}^{(D,N)}$ **first and then** $\pi_{1L}^{(D,D)}$. By Lemma 1, partner 2 bids $b_2 = \bar{v}$ and partner 1 is favored by the tie-breaking rule. Thus, partner 1 wins with bids $b_1 \in [\bar{v}, 1]$, with payoff $v_1 - .5(k\bar{v} + (1 - k)b_1)$. This is maximized at $b_1 = \bar{v}$ with payoff $v_1 - \bar{v}/2$ which is less than $\bar{v}/2$. Partner 1 loses with bids $b_1 \in [0, \bar{v})$, with payoff $.5(kb_1 + (1 - k)\bar{v})$. This is increasing in b_1 and, since $b_1 < \bar{v}$, bounded from above by $\bar{v}/2$. There exist sufficiently small $\varepsilon > 0$ such that the losing payoff from bidding $b_1 = \bar{v} - \varepsilon$ is larger than the winning payoff. Since condition (49) needs to hold for every $\varepsilon > 0$, we **have** ~~use~~ $\pi_{1L}^{(D,N)} = \bar{v}/2$ ~~in (49)~~.

It still remains to determine $\pi_{1L}^{(D,D)}$. There, partner 2 bids according to (2). We show that losing for sure is more profitable for partner 1 than winning. His payoff from winning, $v_1 - 0.5(kb_2 + (1 - k)b_1)$, is decreasing in both partners' bids. Thus, if partner 1 wins, ~~then~~ his payoff cannot exceed

$$v_1 - \frac{1}{2}(k\beta(\bar{v}) + (1 - k)\beta(\bar{v})) = v_1 - \frac{\beta(\bar{v})}{2}.$$

If partner 1's **payoff from losing,** ~~loses,~~ $0.5(kb_1 + (1 - k)\beta(v_2))$, ~~then his payoff~~ is increasing in both bids. The bid $b_1 = \beta(\bar{v})$ is, obviously, the most profitable bid partner 1 can make while making sure that he loses. His payoff is

$$\frac{1}{2}(k\beta(\bar{v}) + (1 - k)\beta(\bar{v})) = \frac{\beta(\bar{v})}{2}.$$

Losing is more profitable if

$$\frac{\beta(\bar{v})}{2} > v_1 - \frac{\beta(\bar{v})}{2} \iff \beta(\bar{v}) > v_1.$$

~~It is easily verified that~~ **From** (2)**,** $\beta(\bar{v}) \geq \bar{v}$. ~~Since,~~ By assumption, $\bar{v} > v_1$. Therefore, losing is more profitable than winning **for partner 1**. Thus, $\pi_{1L}^{(D,D)}$ **equals to** ~~is~~ partner 1's expected profit if he bids $\beta(\bar{v})$ and partner 2 bids $\beta(v_2)$. That profit can be computed as

$$\begin{aligned}\pi_{1L}^{(D,D)} &= \frac{1}{2}\left(k\beta(\bar{v}) + (1-k)E\left[\beta(V_2)|V_2 \in [\bar{v},1]\right]\right) \\ &= \frac{1}{2}\left(k\bar{v} - \frac{1}{k}\int_{G^{-1}(k)}^{\bar{v}}(G(t)-k)^2 dt\right) \\ &\quad + \frac{(1-k)}{2}\int_{\bar{v}}^{1}\left(s - \int_{G^{-1}(k)}^{s}\left(\frac{G(t)-k}{G(s)-k}\right)^2 dt\right) g(s)ds \qquad (50)\end{aligned}$$

Note that

$$\int_{\bar{v}}^{1} sg(s)ds = 1 - \int_{\bar{v}}^{1} G(s)ds. \qquad (51)$$

Also

$$\begin{aligned}&\int_{\bar{v}}^{1}\int_{G^{-1}(k)}^{s}\left(\frac{G(t)-k}{G(s)-k}\right)^2 dtg(s)ds \\ &= -\int_{\bar{v}}^{1}\int_{G^{-1}(k)}^{s}(G(t)-k)^2 dtd\frac{1}{G(s)-k} \\ &= -\frac{1}{1-k}\int_{G^{-1}(k)}^{1}(G(t)-k)^2 dt + \frac{1}{k}\int_{\bar{v}}^{G^{-1}(k)}(G(t)-k)^2 dt + \int_{\bar{v}}^{1}(G(t)-k)dt.\end{aligned} \qquad (52)$$

Inserting (51) and (52) into (50) and simplifying, we get

$$\pi_{1L}^{(D,D)} = \frac{1}{2} - \int_{\bar{v}}^{1} G(t)dt + \frac{1}{2}\int_{\bar{v}}^{1} G^2(t)dt. \qquad (53)$$

Comparing (53) ~~the above~~ with (4), we **get** ~~note that~~ $\pi_{1L}^{(D,D)} = U(\bar{v})$.

Using $\pi_{1L}^{(a_i,a_j)}$ to rewrite condition (49), we get

$$F(\bar{v})\frac{C}{2} + (1-F(\bar{v}))\frac{\bar{v}}{2} \geq F(\bar{v})\frac{\bar{v}}{2} + (1-F(\bar{v}))U(\bar{v}). \qquad (54)$$

(**H**) Suppose partner 1's value is high, $v_1 \in [\bar{v}, 1]$. Partner 2 does not propose dissolution with probability $F(\bar{v})$ and proposes dissolution with probability $1 - F(\bar{v})$. The equilibrium requires that partner 1's payoff from D is at least as high as the payoff from playing N. Therefore, we have

$$F(\bar{v})\pi_{1H}^{(D,N)} + (1-F(\bar{v}))\pi_{1H}^{(D,D)} \geq F(\bar{v})\pi_{1H}^{(N,N)} + (1-F(\bar{v}))\pi_{1H}^{(N,D)} \qquad (55)$$

The LHS of (55) represents partner 1's expected payoff if he proposes dissolution as prescribed by the equilibrium candidate. There, by Lemma 1, $\pi_{1H}^{(D,N)} = v_1 - \bar{v}/2$ and $\pi_{1H}^{(D,D)}$ is given by (4).

The RHS of (55) **is partner 1's expected payoff if he makes unilateral deviation.** There, $\pi_{1H}^{(N,N)} = C/2$ since the partnership continues after action profile (N, N). It remains to determine $\pi_{1H}^{(N,D)}$. **Following action profile** (D, N)**,** partner 2 bids $b_2 = \bar{v}$ and is favored by the tie-breaking rule. Thus, partner 1 wins with bids $b_1 > \bar{v}$ and loses with $b_1 \in [0, \bar{v}]$. Partner 1's winning payoff, $v_1 - 0.5(k\bar{v} + (1-k)b_1)$, is **nonincreasing** ~~decreasing~~ in b_1, and is bounded from above by $v_1 - \bar{v}/2$.[12] **Partner 1's** payoff from losing, $0.5(kb_1 + (1-k)\bar{v})$, is **nondecreasing** ~~increasing~~ in b_1 and, therefore, it is maximized at $b_1 = \bar{v}$ with **value** ~~payoff~~ equal to $\bar{v}/2$. Since $v_1 \geq \bar{v}$ we find that for $v_1 = \bar{v}$, losing is (marginally) more profitable with payoff $\bar{v}/2$ which, in that case, can be written as $v_1 - \bar{v}/2$. Otherwise, for any $v_1 > \bar{v}$, there exists sufficiently small $\varepsilon > 0$ such that winning with a bid of $b_1 = \bar{v} + \varepsilon$ is more profitable than losing, with payoff marginally below $v_1 - \bar{v}/2$ but above $\bar{v}/2$. Since condition (55) must hold for every $\varepsilon > 0$, we come to the conclusion that $\pi_{1H}^{(N,D)} = v_1 - \bar{v}/2$.

Inserting $\pi_{1H}^{(a_i,a_j)}$ into condition (55) and simplifying, we get

$$F(\bar{v})\left(v_1 - \frac{\bar{v}}{2}\right) + (1 - F(\bar{v}))U(v_1) \geq F(\bar{v})\frac{C}{2} + (1 - F(\bar{v}))\left(v_1 - \frac{\bar{v}}{2}\right). \tag{56}$$

Since $G(v_1) = \frac{F(v_1) - F(\bar{v})}{1 - F(\bar{v})}$ (for $v_1 \in [0, 1]$), $U(\bar{v})$ is equal to

$$U(\bar{v}) = \frac{\bar{v}}{2} + \frac{1}{2(1 - F(\bar{v}))^2}\int_{\bar{v}}^{1}(1 - F(t))^2 dt. \tag{57}$$

Using (57) to simplify and rearrange (54), **and noting that** (54) **has to hold for partner 2 when we rotate the role of the two partners,** we get (6).

$U(v)$ can be written as:

$$\begin{aligned} U(v_1) &= \frac{1}{2} + \frac{1}{2}\int_{\bar{v}}^{1}\left(\frac{F(t) - F(\bar{v})}{1 - F(\bar{v})}\right)^2 dt - \int_{v_1}^{1}\frac{F(t) - F(\bar{v})}{1 - F(\bar{v})}dt \\ &= v_1 - \frac{\bar{v}}{2} + \frac{1}{2(1 - F(\bar{v}))^2}\int_{\bar{v}}^{1}(1 - F(t))^2 dt - \frac{1}{1 - F(\bar{v})}\int_{\bar{v}}^{v_i}(1 - F(t))dt \end{aligned} \tag{58}$$

Using (58) to simplify and rearrange (56), **and noting that** (56) **has to hold for partner 2 when we rotate the role of the two partners,** we get (7).

□

[12]**When $k = 1$, the winning payoff is always $v_1 - 0.5\bar{v}$, independent of b_1.**

The following is an auxiliary result that we use for the proofs in the reserve price section.

Lemma 11. *The following identities are used in several proofs. The function* β *is defined in* (19)*.*

$$\int_x^1 \beta(s)k(s)ds = (1-K(x))x + \int_x^1 (1-K(s))^2 ds - \frac{1-K(x)}{K(x)} \int_r^x K^2(s)ds, \tag{59}$$

$$= \frac{1}{2}\left(\frac{1-F(x)}{1-F(\tilde{v})}x + \int_x^1 \left(\frac{1-F(s)}{1-F(\tilde{v})}\right)^2 ds - \frac{1-F(x)}{F(x)-F(\tilde{v})}\int_r^x \left(\frac{F(s)-F(\tilde{v})}{1-F(\tilde{v})}\right)^2 ds\right), \tag{60}$$

$$\int_r^1 \beta(s)k(s)ds = (1-K(r))r + \int_r^1 (1-K(s))^2 ds \tag{61}$$

$$= \frac{1-F(r)}{1-F(\tilde{v})}r + \int_r^1 \left(\frac{1-F(s)}{1-F(\tilde{v})}\right)^2 ds. \tag{62}$$

Proof of Lemma 11.

$$\begin{aligned}
\int_x^1 \beta(s)k(s)ds &= \int_x^1 \left(s - \int_r^s \frac{K^2(t)}{K^2(s)}dt\right) k(s)ds \\
&= \int_x^1 sk(s)ds - \int_x^1 \int_r^s K^2(t)dt \left(-\frac{1}{K(s)}\right)' ds \\
&= 1 - K(x)x - \int_x^1 K(s)ds + \int_r^x \left[\frac{1}{K(s)}\right]_x^1 K^2(t)dt + \int_x^1 \left[\frac{1}{K(s)}\right]_t^1 K^2(t)dt \\
&= 1 - K(x)x - \int_x^1 K(s)ds + \int_r^1 K^2(t)dt - \int_r^x \frac{K^2(s)}{K(x)}ds - \int_x^1 K(t)dt \\
&= 1 - K(x)x - 2\int_x^1 K(s)ds + \int_r^1 K^2(s)ds - \int_r^x \frac{K^2(s)}{K(x)}ds \\
&= (1-K(x))x + \int_x^1 (1-K(s))^2 ds - \frac{1-K(x)}{K(x)}\int_r^x K^2(s)ds
\end{aligned}$$

Making use of the relation between $K(x)$ and $F(x)$, we get the following:

$$\begin{aligned}
&= \left(1 - \frac{F(x)-F(\tilde{v})}{1-F(\tilde{v})}\right)x + \int_x^1 \left(1 - \frac{F(s)-F(\tilde{v})}{1-F(\tilde{v})}\right)^2 ds - \frac{\left(1-\frac{F(x)-F(\tilde{v})}{1-F(\tilde{v})}\right)}{\left(\frac{F(x)-F(\tilde{v})}{1-F(\tilde{v})}\right)} \int_r^x \left(\frac{F(s)-F(\tilde{v})}{1-F(\tilde{v})}\right)^2 ds \\
&= \frac{1-F(x)}{1-F(\tilde{v})}x + \int_x^1 \left(\frac{1-F(s)}{1-F(\tilde{v})}\right)^2 ds - \frac{1-F(x)}{F(x)-F(\tilde{v})}\int_r^x \left(\frac{F(s)-F(\tilde{v})}{1-F(\tilde{v})}\right)^2 ds
\end{aligned}$$

Equation (61) follows directly from (59) (and (62) from (60)) after inserting $x = r$. □

References

Cramton, P., R. Gibbons, and P. Klemperer (1987): "Dissolving a Partnership Efficiently," *Econometrica*, 55, 615–32.

Kittsteiner, T. (2003): "Partnerships and Double Auctions with Interdependent Valuations," *Games and Economic Behavior*, 44(1), 54–76.

Li, J., and E. Wolfstetter (2010): "Partnership Dissolution, Complementarity, and Investment Incentives," *Oxford Economic Papers*, 62, 529–552.

McAfee, R. P. (1992): "Amicable Divorce: Dissolving a Partnership with Simple Mechanisms," *Journal of Economic Theory*, 56, 266–293.