Scale Economies and Returns to Scale in Non-Parametric Cost and Production Models: Exploring the Impact of Convexity

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Outline of the presentation

1. Goals & Motivations
2. Technology and Cost Functions
- Definitions and Duality
- Non-Parametric Convex & Non-Convex Specifications
- Criticisms of the Convexity Assumption

3. Empirical Methodology
- Sections of Cost Functions
- Characterising Returns to Scale & Economies of Scale
- Efficiency Decomposition with Technical & Scale Efficiency

4. Description of the Samples

5. Empirical Results
- Cost Frontier Estimates & Cost Function Sections in the Output
- Returns to Scale & Economies of Scale Results
- Basic Efficiency Decomposition

6. Conclusions

1. Introduction
Basic motivation

- Conventional empirical production analysis maintains convexity: Implicit or explicit assumption that non-convexities do not impact empirical results.

- Some reasons for non-convexities in technology:

1. Indivisibilities
2. Economies of scale
3. Economies of specialization (e.g., nonrival inputs in new growth theory)
4. Externalities

- ***Theoretical results pointing to impact of convexity:***

1. Jacobsen (1970), Shephard (1970, 1974): cost function is non-decreasing and convex (non-convex) in outputs when technology is convex (non-convex).
2. Briec et al. (2004): cost function on convex technology $\leq$ cost function on non-convex technology.

- Convexity can then only be maintained if there is well-established empirical evidence that its impact on most or some specific applications is negligible.

1. Introduction
Empirical evidence revealing impact of convexity (1)

Traditional analysis:

- Non-convexities in electricity generation due to minimum up and down time constraints, multi-fuel effects, etc. lead to non-convex and non-differentiable variable costs (Bjørndal & Jörnsten (2008), Park et al. (2010)).

- Costs in car manufacturing are non-convex due to changes in the number of shifts and in the shutting down of plants for some time (e.g., Copeland & Hall (2011)).

Frontier analysis:

- Cummins & Zi (1998) and Grifell-Tatjé & Kerstens (2008) offer cost frontier estimates and cost efficiency ratios for USA life insurance and Spanish electricity distribution respectively that are different from convex results.

- For oil field petroleum data, Kerstens & Managi (2012) report substantial differences in Luenberger productivity indicator between convex and non-convex technologies and only find both convergence for latter technology.

1. Introduction
Empirical evidence revealing impact of convexity (2)

Frontier analysis (cont.):

Wheelock & Wilson (2009), J. Business & Economic Statistics.
5 inputs & 5 outputs; 11993, 9585 & 6075 banks for 1985, 1994 & 2004
All observations in each year are on the NC-frontier. Only 7.9 to 8.8 % are on C frontier. Thus, all inefficiency is due solely to the convexity assumption.

Engineering production function literature:

Many operations management problems in industry and distribution involve indivisibilities and require integer optimisation.

Similar to arguments of engineering production function literature: most production processes yield neo-classical technologies only under strict conditions (see Wibe (1984)).

Conclusion:

Despite this limited amount of evidence, the assumption of convexity in our view ideally requires testing.

1. Introduction

3 goals & Use of non-parametric convex and non-convex cost functions

We explore differences in:

- Cost frontier estimates based on convex and non-convex cost functions (incl. illustrations with sections relating costs to outputs for specific units).
- Characterization of economies of scale and returns to scale for convex and non-convex cost functions and technologies.
- Technical and scale efficiencies based on convex and non-convex technology and cost function estimations.

Use of non-parametric convex and non-convex cost functions:

- There are hardly any alternative semi-parametric or parametric specifications that easily allow for testing convexity.
- This non-parametric approach coincides with the non-parametric nature of the axioms under scrutiny.
  Fuss, McFadden and Mundlak (1978: 223):
  "Given the qualitative, non-parametric nature of the fundamental axioms, this suggests … that the more relevant tests will be non-parametric, rather than based on parametric functional forms, even very general ones."

2. Technology and Cost Functions
Basic definitions

- Efficiency is measured using : - deterministic,
  - nonparametric technologies.

- Production technologies are based on K observations using a vector of inputs x to produce a vector of outputs y.

- Technology is represented by its production possibility set :
  $$T = \{(x,y): x \text{ can produce } y\}.$$

- Input set $L(y)$ denotes all input vectors x producing the output vector y:
  $$L(y) = \{x: (x,y) \in T\}.$$

- A convenient characterisation of technology is the input distance function:

$$D_i(x, y) = \max\{\lambda : \lambda \geq 0,\ x/\lambda \in L(y)\}.$$

- Radial input efficiency measure ($DF_i(x,y)$) is the inverse of the input distance function.

2. Technology and Cost Functions
Cost function & duality

- Cost function:

$$C(y,w) = \min \{wx \mid x \in L(y)\}.$$

- Duality relations link primal and dual formulations of technology: it allows a well-behaved technology to be reconstructed from the observations on cost minimizing producer behavior, and the reverse.

- Duality between input distance function and cost function:

$$D_i(x,y) = \min_w \{wx : C(y,w) \geq 1\},\ x \in L(y)$$
$$C(y,w) = \min_x \{wx : D_i(x,y) \geq 1\},\ p > 0.$$

- Traditional duality relation is established under the convexity hypothesis. Briec et al (2004) establish a local duality result between non-convex technologies obeying different scaling laws and the corresponding non-convex cost functions.

2. Technology and Cost Functions

Non-parametric convex and non-convex specifications of technology and cost functions

Unified algebraic representation of convex and non-convex technologies under different returns to scale assumptions (Briec et al (2004)):

$$T^{\Lambda,\Gamma} = \left\{ (x, y): \quad x \geq \sum_{k=1}^{K} x_k \delta\, z_k,\ y \leq \sum_{k=1}^{K} y_k \delta\, z_k,\ z_k \in \Lambda, \delta \in \Gamma \right\},$$

where (i) $\Gamma \equiv \Gamma^{CRS} = \{\delta: \quad \delta \geq 0\}$;

(ii) $\Gamma \equiv \Gamma^{VRS} = \{\delta: \quad \delta = 1\}$;

(iii) $\Gamma \equiv \Gamma^{NIRS} = \{\delta: \quad 0 \leq \delta \leq 1\}$;

(iv) $\Gamma \equiv \Gamma^{NDRS} = \{\delta: \quad \delta \geq 1\}$; and

where (i) $\Lambda \equiv \Lambda^{C} = \left\{ \sum_{k=1}^{K} z_k = 1 \text{ and } z_k \geq 0 \right\}$, and

(ii) $\Lambda \equiv \Lambda^{NC} = \left\{ \sum_{k=1}^{K} z_k = 1 \text{ and } z_k \in \{0,1\} \right\}$.

2. Technology and Cost Functions

Non-Parametric Convex and Non-Convex Specifications of Technology and Cost Functions (2)

Note:

(i) activity vector (z) operates subject to a non-convexity or convexity constraint,

(ii) scaling parameter (δ) allows for a particular scaling of observations spanning the frontier:

- δ free under constant returns to scale (CRS),
- $\delta = 1$ under variable returns to scale (VRS),
- $\delta \leq 1$ under non-increasing returns to scale (NIRS)
- $\delta \geq 1$ non-decreasing returns to scale (NDRS).

Computational Issues:

Computing radial input efficiency :

- relative to convex technologies: NLP, or LP.
- relative to non-convex technologies: NLMIP, MIP, LP, or enumeration.

Computing cost function:

- relative to convex technologies: LP
- relative to non-convex technologies: LP, or enumeration.

2. Technology and Cost Functions
Results on impact convexity on cost function (1)

Briec et al (2004) prove:

• Costs evaluated on non-convex technologies are higher or equal to costs evaluated on convex technologies:

$$C^{NC}(y,w) \quad \geq \quad C^{C}(y,w).$$

• In the case of (i) CRS and (ii) a single output:

$$C^{NC}(y,w) \quad = \quad C^{C}(y,w).$$

Note: The above convex technologies are similarly the most conservative, inner bound approximations of technology satisfying (A.1) to (A.5).

Source: Briec et al (2004) , p. 171.

Proposition 4: Let the non-convex cost function $C^{NC,\Gamma}(p,y) = \min\{p \cdot x : x \in L^{NC,\Gamma}(y)\}$ and let the convex cost function $C^{C,\Gamma}(p,y) = \min\{p \cdot x : x \in L^{C,\Gamma}(y)\}$. Then, we have the following properties:

(1) In general: $C^{C,\Gamma}(p,y) \leq C^{NC,\Gamma}(p,y)$.
(2) In the case of $\Gamma = CRS$ and a single output: $C^{C,\Gamma}(p,y) = C^{NC,\Gamma}(p,y)$.

2. Technology and Cost Functions
Results on impact convexity on cost function (3)

This relation reflects the property that cost functions are non-decreasing in outputs and convex (non-convex) in the outputs depending on whether the technology is convex (non-convex) (see Jacobsen (1970): Proposition 5.2) or Shephard (1970, 1974).

Source: Jacobsen (1970): Proposition 5.2 (Q9) on p. 765.

PROPOSITION 5.2: (Q.9): *If P has a convex graph then Q is convex in u;*
(Q.10): *if P is positively homogeneous of degree k $(\neq 0)$, then Q is positively homogeneous of degree $1/k$ in u;*
(Q.11): *if P has a closed graph,[7] then for all $p > 0$, Q is l.s.c. in u.*

Source: Shephard (1974): Proposition 5.2 on p. 15.

Q.10 $Q(u,p)$ is a convex function of u on R^m_+ if the graph of $u \to L(u)$ is convex.

2. Technology and Cost Functions
Results on impact convexity on cost function (5)

Advanced micro-economic textbooks ignore this issue when discussing duality (e.g., Varian (1992: p. 84)).

This proposition shows that the cost function for the technology $V(y)$ is the same as the cost function for its convexification $V^*(y)$. In this sense, the assumption of convex input requirement sets is not very restrictive from an economic point of view.

Let us summarize the discussion to date:

(1) Given a cost function we can define an input requirement set $V^*(y)$.

(2) If the original technology is convex and monotonic, the constructed technology will be identical with the original technology.

(3) If the original technology is nonconvex or nonmonotonic, the constructed input requirement will be a convexified, monotonized version of the original set, and, most importantly, the constructed technology will have the same cost function as the original technology.

We can summarize the above three points succinctly with the fundamental principle of duality in production: *the cost function of a firm summarizes all of the economically relevant aspects of its technology.*

Duality cost function & input distance f. established under convexity of input sets.
Empirical methodologies impose convexity on technology.
While this difference is known to matter for the cost function, textbooks ignore this issue.

2. Technology and Cost Functions
Results on impact convexity on cost function (5)

Advanced micro-economic textbooks ignore this issue when discussing the properties of the cost function (e.g., Jehle and Reny (2011: p. 138)).

THEOREM 3.2 **Properties of the Cost Function**

If f is continuous and strictly increasing, then $c(\mathbf{w}, y)$ is

1. *Zero when $y = 0$,*
2. *Continuous on its domain,*
3. *For all $\mathbf{w} \gg 0$, strictly increasing and unbounded above in y,*
4. *Increasing in $\mathbf{w}$,*
5. *Homogeneous of degree one in $\mathbf{w}$,*
6. *Concave in $\mathbf{w}$.*

Moreover, if f is strictly quasiconcave we have

7. *Shephard's lemma: $c(\mathbf{w}, y)$ is differentiable in $\mathbf{w}$ at $(\mathbf{w}^0, y^0)$ whenever $\mathbf{w}^0 \gg 0$, and*

$$\frac{\partial c(\mathbf{w}^0, y^0)}{\partial w_i} = x_i(\mathbf{w}^0, y^0), \qquad i = 1, \ldots, n.$$

If this issue is not mentioned in these books, then it must not be important, no?
The answer is an empirical issue: we simply do not know whether it matters.

2. Technology and Cost Functions

Results on impact convexity on cost function (6)

Burgeoning literature uses these non-convex models in comparative perspective:
•Cummins & Zi (1998) and Grifell-Tatjé & Kerstens (2008) offer cost frontier estimates and cost efficiency ratios for USA life insurance and Spanish electricity distribution respectively that are different from convex results.

•For oil field petroleum data, Kerstens & Managi (2012) report substantial differences in Luenberger productivity indicator between convex and non-convex technologies and only find both convergence for latter technology.

Sometimes these non-convex models are employed on their own:
•Alam and Sickles (2000) examine time series of technical efficiency in the USA airline industry for convergence.

•Balaguer-Coll et al. (2007) analyse Spanish local government efficiency from a production as well as a cost viewpoint.

2. Technology and Cost Functions
Criticism of the convexity assumption (1)

Critique of convexity assumption can consider a variety of arguments:

• Convexity justified because of time divisibility of technologies:
This ignores setup costs (some of which may be sunk) that make switching between the underlying activities costly.

• Convexity is not a primitive axiom, but implied by additivity and divisibility.
(i) Perfect divisibility of inputs and/or outputs is a most debatable assumption. Most operations management problems in industry and distribution involve indivisibilities and input fixities resulting in integer, possibly non-linear optimization problems.

In general, all production processes have some lower limit below which a process cannot possibly be scaled down realistically.

Thus: Divisibility is questionable (see Scarf (1994) or Winter (2008))

2. Technology and Cost Functions
Criticism of the convexity assumption (2)

(ii) Additivity is essential to define free entry, but presupposes spatial separation and non-interaction which are both debatable (see Winter (2008)).
Since additivity relates to the aggregation of results of activities occurring in geographically distinct places, transportation and coordination costs must be small to be safely ignored. When activities are close for transportation costs to be negligible, then the risk of production externalities looms when activities get “too close” to create interactions.

(iii) Additivity and divisibility do not only imply convexity, but also CRS.
The CRS assumption is at odds with indivisibilities and the lower bounds on the scaling of almost all production processes (see Scarf (1994: 114-115) for a sharp critique).

> Both linear programming and the Walrasian model of equilibrium make the fundamental assumption that the production possibility set displays constant or decreasing returns to scale; that there are no economies associated with production at a high scale. I find this an absurd assumption, contradicted by the most casual of observations. Taken literally, the assumption of constant
>
> returns to scale in production implies that if technical knowledge were universally available we could all trade only in factors of production, and assemble in our own backyards all of the manufactured goods whose services we would like to consume. If I want an automobile at a specified future date, I would purchase steel, glass, rubber, electrical wiring and tools, hire labor of a variety of skills on a part-time basis, and simply make the automobile myself. I would

3. Empirical Methodology
Sections of cost functions

Reconstruction and visualization of production frontiers is discussed in some articles (e.g., Hackman et al. (1994) or Hackman (2008)).

Some articles exploit the fact that that non-parametric technologies are convex polyhedra to enumerate facets. A 2-dimensional projection is then defined relative to a particular point in the technology.

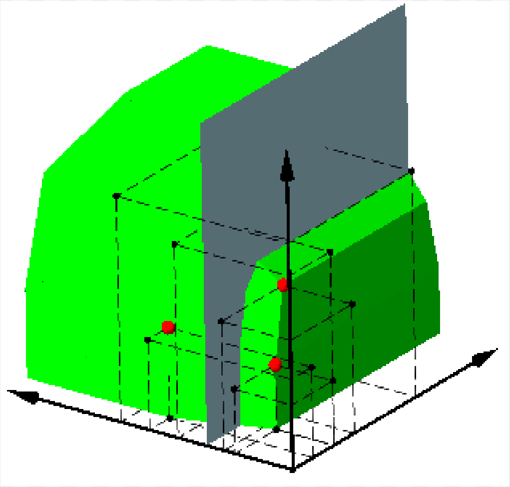
Krivonozhko et al. (2004) present a family of parametric optimization methods to construct an intersection of the multidimensional frontier with a 2-dimensional plane determined by any pair of given directions.

Here, for a given observation a section of a cost function along one particular output dimension is computed using parametric programming: grid of 1000 points within the empirical range of the sample for the output selected.

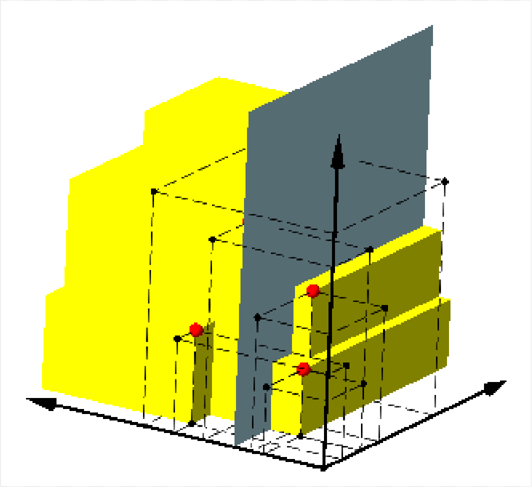
3. Empirical Methodology

Sections of cost functions: Example in 2 inputs and 1 output

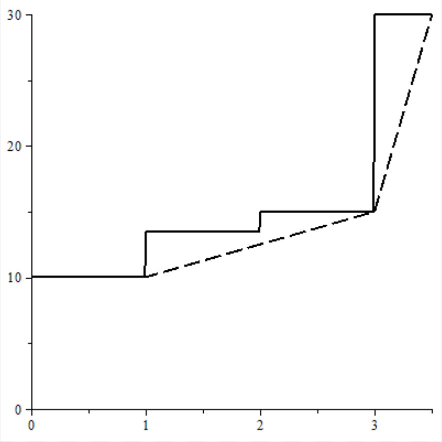
Convex Technology with Planar Section



Non-Convex Technology with Planar Section



Convex and Non-Convex Cost Function in Single Output



3. Empirical Methodology
Characterising returns to scale

Kerstens and Vanden Eeckaut (1999) generalise a goodness-of-fit method to suit all (including non-convex) technologies.

Definition 1: *Using $DF_i(x,y)$ and conditional on the optimal projection point, technology is locally characterised by:*

$CRS \Leftrightarrow DF_i(x,y \mid C) = \max\{ DF_i(x,y \mid C), DF_i(x,y \mid NIRS), DF_i(x,y \mid NDRS) \}$;
$IRS \Leftrightarrow DF_i(x,y \mid NDRS) = \max\{ DF_i(x,y \mid C), DF_i(x,y \mid NIRS), DF_i(x,y \mid NDRS) \}$; or
$DRS \Leftrightarrow DF_i(x,y \mid NIRS) = \max\{ DF_i(x,y \mid C), DF_i(x,y \mid NIRS), DF_i(x,y \mid NDRS) \}$.

3. Empirical Methodology
Characterising economies of scale

Goodness-of-fit method based on the inclusion of different overall efficiency components estimated relative to different return to scale assumptions can be used.

Definition 2: *Using* $C(y,w\,|\,.)$ *and conditional on the optimal projection point, the cost function is locally characterised by:*

$CRS \Leftrightarrow C(y,w\,|\,C) = \max\{\, C(y,w\,|\,C),\, C(y,w\,|\,NIRS),\, C(y,w\,|\,NDRS)\,\}$;

$IRS \Leftrightarrow C(y,w\,|\,NDRS) = \max\{\, C(y,w\,|\,C),\, C(y,w\,|\,NIRS),\, C(y,w\,|\,NDRS)\,\}$; or

$DRS \Leftrightarrow C(y,w\,|\,NIRS) = \max\{\, C(y,w\,|\,C),\, C(y,w\,|\,NIRS),\, C(y,w\,|\,NDRS)\,\}$.

3. Empirical Methodology
Efficiency decomposition with technical & scale efficiency

Definition 3: *Under the assumptions on the input set L(y), the following input-oriented efficiency notions can be distinguished:*

1. Technical Efficiency is the quantity: $TE_i(x,y) = DF_i(x,y \mid V)$.

2. Overall Technical Efficiency is the quantity: $OTE_i(x,y) = DF_i\,(x,y \mid C)$.

3. Scale Efficiency is the quantity: $SCE_i(x,y) = DF_i\,(x,y \mid C)/DF_i(x,y \mid V)$.

4. Economic Efficiency for given scale is the quantity: $OE_i(x,y,w \mid V) = C(y,w \mid V)/wx$.

5. Overall Economic Efficiency is the quantity: $OE_i(x,y,w \mid C) = C(y,w \mid C)/wx$.

6. Cost-based Scale Efficiency is the quantity:

$$CSCE_i\,(x,y,w) = \frac{C(y,w|\,C)\big/wx}{C(y,w|V)\big/wx} = \frac{OE_i\,(x,y,w|C)}{OE_i\,(x,y,w|V)}.$$

Note: gap between the above technical efficiency notions and their corresponding overall efficiency notions results in the definitions of allocative efficiency components.

3. Empirical Methodology

Efficiency decomposition with technical & scale efficiency

Link primal and dual approaches to scale efficiency via allocative efficiency components:

$$CSCE_i(x,y,w) = \left[\frac{DF_i(x,y|C,S)}{DF_i(x,y|V,S)}\right] \cdot \left[\frac{AE_i(x,y,w|C)}{AE_i(x,y,w|V)}\right]$$

$$= SCE_i(x,y) \cdot \left[\frac{AE_i(x,y,w|C)}{AE_i(x,y,w|V)}\right]$$

Note: Second ratio can be smaller, equal or larger than unity, hence both scale efficiency notions cannot be related to one another ($\lesseqgtr$).

3. Empirical Methodology
Efficiency decomposition with technical & scale efficiency

Relations between the decompositions in Definition 3 relative to convex and non-convex technologies are trivially defined:

Proposition 1: *Relations between convex and non-convex decomposition components are:*

1. $$OTE_i^C(x,y) \le OTE_i^{NC}(x,y)$$
2. $$TE_i^C(x,y) \le TE_i^{NC}(x,y)$$
3. $$C^C(y,w|C) \le C^{NC}(y,w|C)$$
4. $$C^C(y,w|V) \le C^{NC}(y,w|V)$$
5. $$OE_i^C(x,y,w|C) \le OE_i^{NC}(x,y,w|C)$$
6. $$OE_i^C(x,y,w|V) \le OE_i^{NC}(x,y,w|V)$$

3. Empirical Methodology
Efficiency decomposition with technical & scale efficiency

Convexity can be tested for any comparison between convex and non-convex technologies and cost functions imposing a similar returns to scale hypothesis: differences in these components are completely attributable to convexity and offer goodness-of-fit test.

Definition 4: *The convex and non-convex efficiency components based upon constant returns to scale technologies and cost functions respectively can be related by:*

1. $$CRTE_i\left(x,y\right)=OTE_i^{C}\left(x,y\right)/OTE_i^{NC}\left(x,y\right)$$

2. $$CRCE_i\left(x,y,w\right)=OE_i^{C}\left(x,y,w\middle|C\right)/OE_i^{NC}\left(x,y,w\middle|C\right)$$

Note: Both these measures are ≤ 1.

4. Description of the Samples
Two secondary data sets

Use 2 secondary data sets in empirical analysis:

1. Chilean hydro-electric power generation plants

16 Chilean hydro-electric power generation plants observed on a monthly basis for several years (Atkinson & Dorfman (2009)).
Focus on single year 1997: ignore technical change & specify inter-temporal frontier.
Total: 192 observations.
One output: electricity generated.
Prices and quantities of 3 inputs: (i) labour, (ii) capital, & (iii) water.

2. Unbalanced panel of 3 years of French fruit producers

Based on annual accounting data collected in a survey (Ivaldi et al. (1996)).
Short panel justifies use of intertemporal frontier (ignore technical change).
Total: 405 observations
Two outputs: (i) production of apples, and (ii) aggregate of alternative products.
Prices and quantities of 3 inputs: (i) capital (including land), (ii) labour, & (iii) materials.

5. Empirical Results
Cost frontier estimates: Descriptive statistics

Table 1: Non-Convex and Convex Cost Frontier Values: Descriptive Statistics

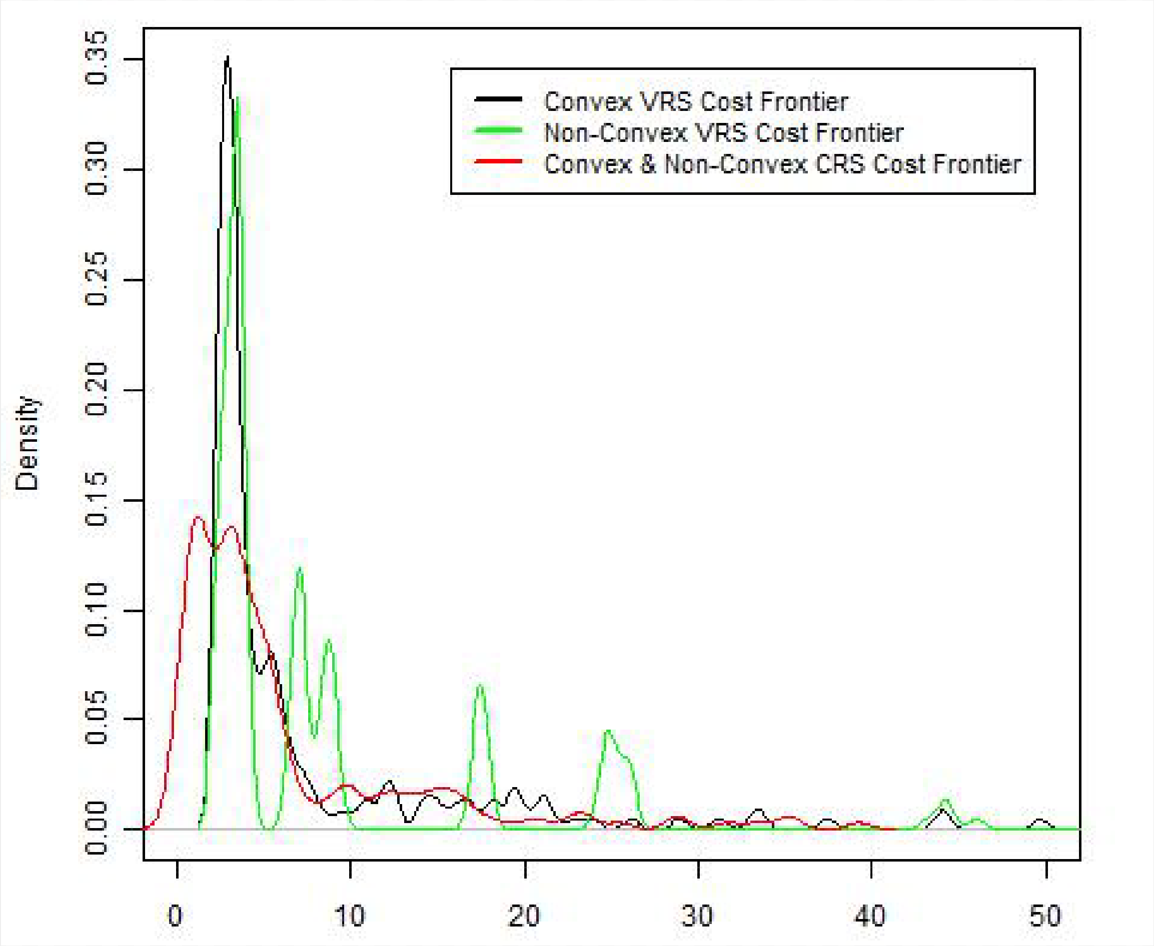
Chilian Hydro-power Plants				
	Non-Convex Cost Frontier		Convex Cost Frontier	
	VRS	*CRS*	*VRS*	*CRS*
Average	10.6663	6.2228	8.4522	6.2228
Stand.Dev.	13.6422	7.4285	11.0802	7.4285
Minimum	2.4912	0.0442	2.4912	0.0442
Maximum	65.8023	39.2475	65.8023	39.2475
French Fruit Producers				
	Non-Convex Cost Frontier		Convex Cost Frontier	
	VRS	*CRS*	*VRS*	*CRS*
Average	1160.91	683.06	718.84	511.51
Stand.Dev.	1730.08	880.89	1124.45	758.76
Minimum	150.11	13.15	150.11	8.51
Maximum	13448.39	6754.19	11815.72	6095.27

Conclusions:

- NC cost frontier estimates are on average higher than their C counterparts.
- VRS cost frontier estimates are again higher than the CRS ones.
- For hydro-power plants: NC and C results are identical for CRS, since single output.
- Li-test statistics (not visible): all distributions C/NC are different.

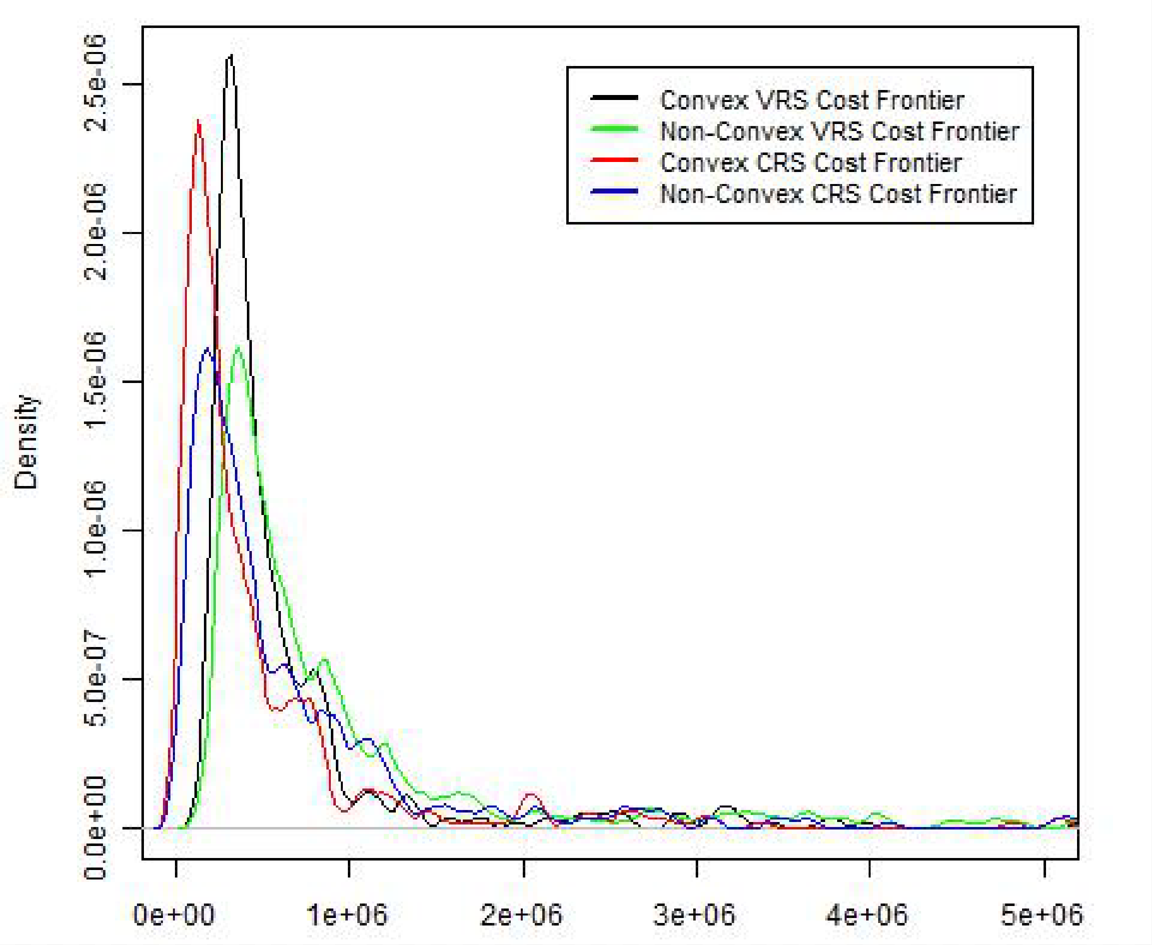
5. Empirical Results
Cost frontier estimates: Densities (1)

Figure 1: Kernel Density Estimates of Cost Frontiers for Chilean Hydro-power Plants



5. Empirical Results
Cost frontier estimates: Densities (2)

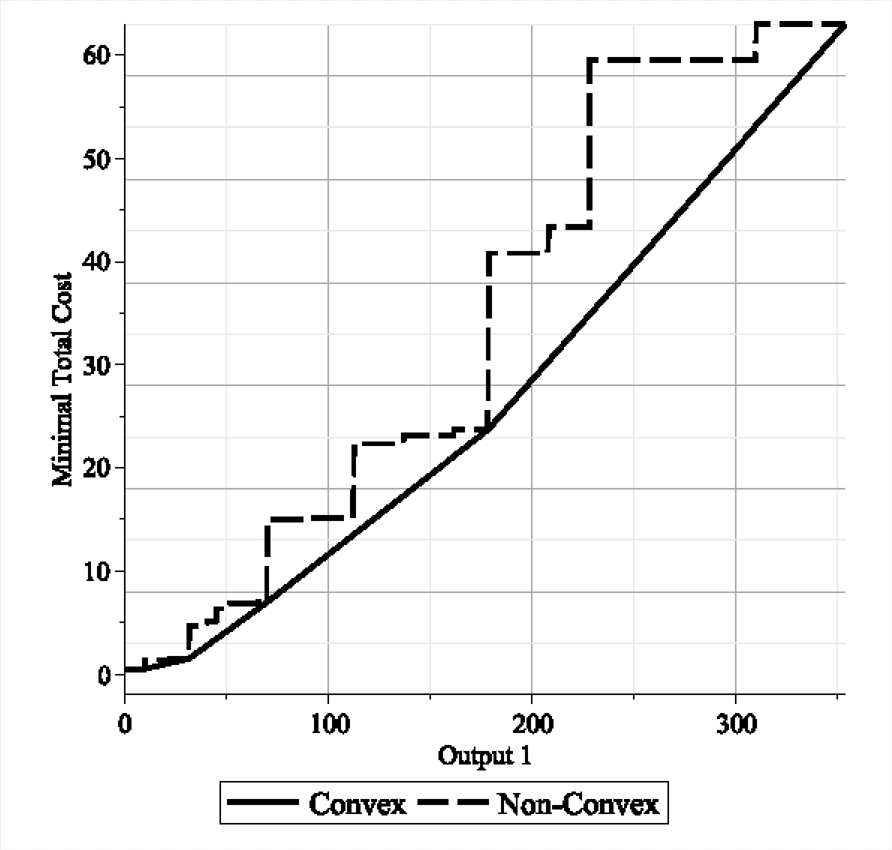
Figure 2: Kernel Density Estimates of Cost Frontiers for French Fruit Producers



5. Empirical Results

Sections of the Cost Function in the Output

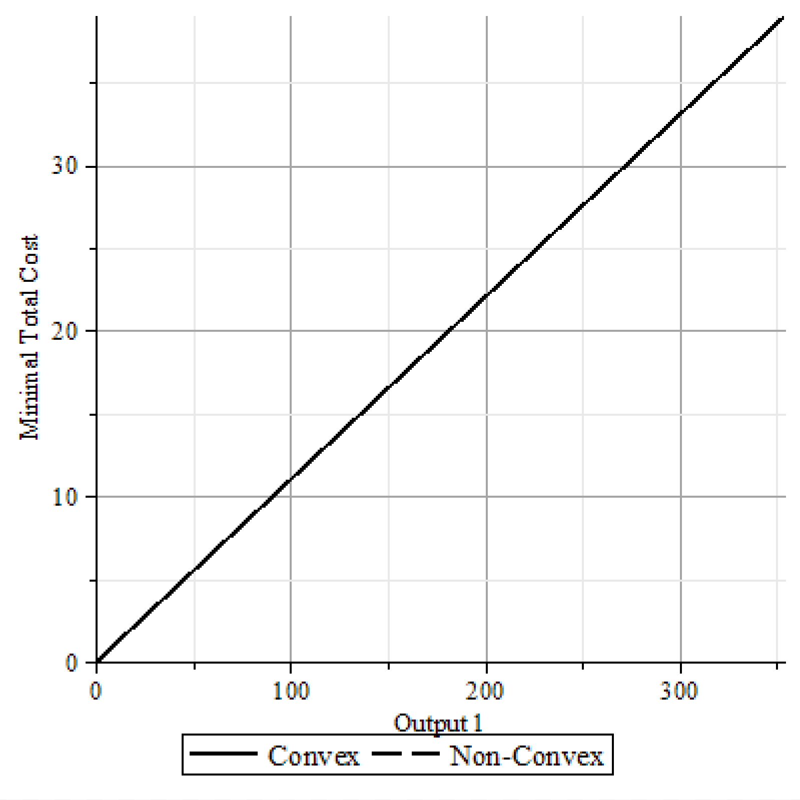
Figure 3: VRS Cost Function in the Single Output for Hydro-power Plant 5



5. Empirical Results

Sections of the Cost Function in the Output

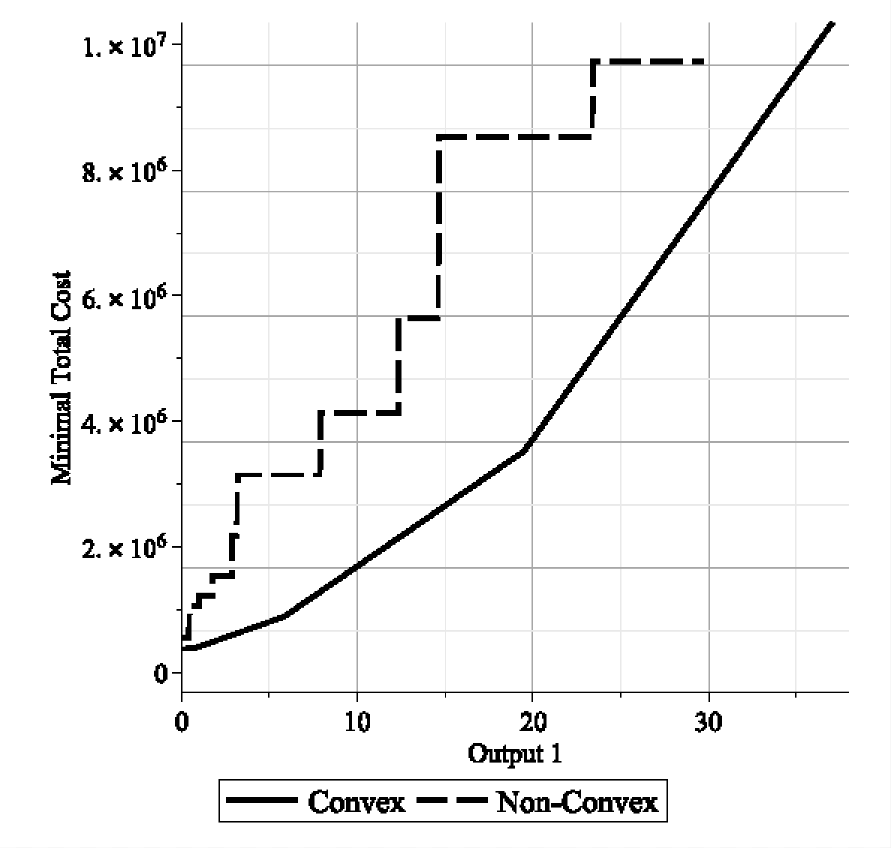
Figure 4: CRS Cost Function in the Single Output for Hydro-power Plant 5



5. Empirical Results

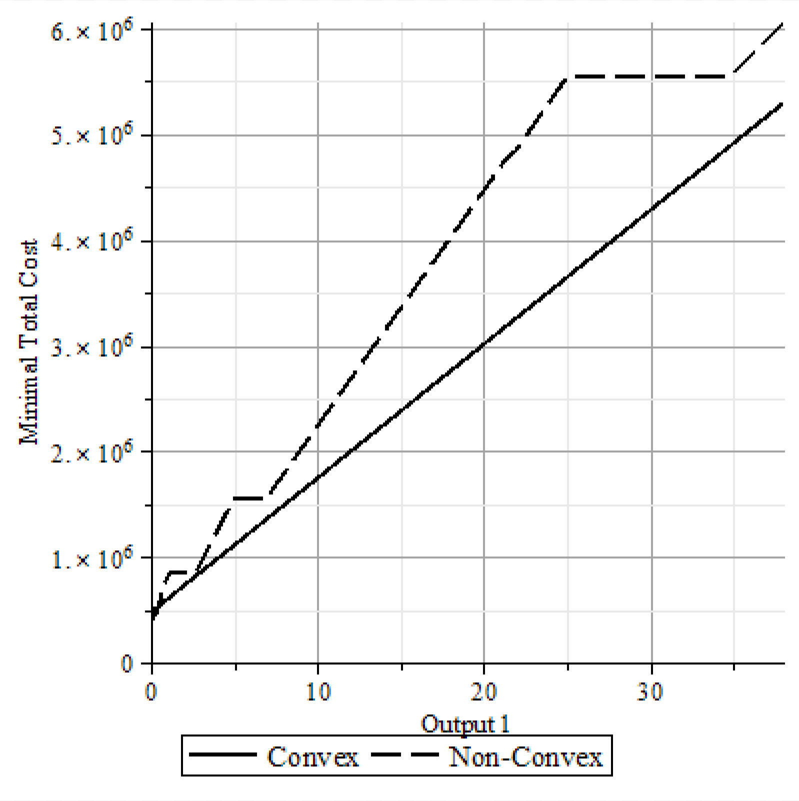
Sections of the Cost Function in the Output

Figure 5: VRS Cost Function in Output 1 for Fruit Producer 19



5. Empirical Results
Sections of the Cost Function in the Output

Figure 6: CRS Cost Function in Output 1 for Fruit Producer 19



5. Empirical Results

Returns to scale & Economies of scale results

Table 2: Returns to Scale and Economies of Scale Results

Chilean Hydro-power Plants (%)			
Production	*IRS*	*CRS*	*DRS*
Non-convex	70.31	16.67	13.02
Convex	76.04	2.60	21.35
Cost	*IRS*	*CRS*	*DRS*
Non-convex	51.56	0.52	47.92
Convex	68.23	9.38	22.40
French Fruit Producers (%)			
Production	*IRS*	*CRS*	*DRS*
Non-convex	74.07	12.84	13.09
Convex	90.37	1.73	7.90
Cost	*IRS*	*CRS*	*DRS*
Non-convex	73.83	1.98	24.20
Convex	93.33	0.25	6.42

Conclusions:

• Majority of observations operate under IRS.

Qualification: NC cost approach for the hydro-power plants indicates about an equal amount of IRS and DRS.

• NC cost approach reveals a larger share of observations subject to DRS compared to the production-based analysis.

• More CRS under NC.

Exception: cost approach for hydro-power plants.

5. Empirical Results

Returns & Economies of scale: Conflicting information

Per data set and per production and cost method, Table 3 reports the % observations for which the returns and economies to scale classification coincides/diverges.

Focus on part of results in Table 3 (Table suppressed):
Switch from IRS (economies to scale) to DRS (diseconomies to scale), or the reverse.
•Hydro-power plants: 7.81% (production) - 21.88% (cost)
•Fruit producers: 6.91% (production) - 17.78% (cost).

Per data set and per convexity or non-convexity assumption, Table 4 reports the % observations for which the returns and economies to scale classification coincides/diverges.

Focus on part of results in Table 4 (Table suppressed):
Switch from IRS (economies to scale) to DRS (diseconomies to scale), or the reverse.
•Hydro-power plants: 2.08% (convex) - 27.60% (non-convex)
•Fruit producers: 6.17% (non-convex) - 6.42% (convex).

5. Empirical Results

Basic Efficiency Decompositions

Table 5: Non-Convex and Convex Decompositions: Descriptive Statistics

Chilean Hydro-power Plants							
	Non-Convex Decomposition				Convex Decomposition		
	$TE_i(.)$[¶]	$SCE_i(.)$	$OTE_i(.)$	$CRTE_i(.)$	$TE_i(.)$	$SCE_i(.)$	$OTE_i(.)$
Average*	0.9391	0.7452	0.6998	0.9027	0.7410	0.8525	0.6317
Stand.Dev.	0.1382	0.2049	0.2333	0.0809	0.2035	0.1568	0.2297
Minimum	0.3154	0.1663	0.0647	0.6869	0.1325	0.1868	0.0647
% Effic. Obs.	80.21	22.40	22.40	11.98	16.67	6.77	6.77
	$OE_i(. \mid V)$	$CSCE_i(.)$	$OE_i(. \mid C)$	$CRCE_i(.)$	$OE_i(. \mid V)$	$CSCE_i(.)$	$OE_i(. \mid C)$
Average*	0.4360	0.5108	0.2227	1.0000	0.3587	0.6210	0.2227
Stand.Dev.	0.3581	0.2155	0.2433	0.0000	0.3075	0.2565	0.2433
Minimum	0.0549	0.0177	0.0010	1.0000	0.0549	0.0177	0.0010
% Effic. Obs.	13.02	0.52	0.52	100.00	2.60	0.52	0.52
French Fruit Producers							
	Non-Convex Decomposition				Convex Decomposition		
	$TE_i(.)$	$SCE_i(.)$	$OTE_i(.)$	$CRTE_i(.)$	$TE_i(.)$	$SCE_i(.)$	$OTE_i(.)$
Average*	0.8210	0.6087	0.4997	0.6200	0.5721	0.5416	0.3098
Stand.Dev.	0.1904	0.2379	0.2804	0.1545	0.1933	0.2589	0.2194
Minimum	0.3590	0.0789	0.0486	0.3713	0.1868	0.0728	0.0481
% Effic. Obs.	45.68	12.84	12.84	2.72	5.43	2.22	2.22
	$OE_i(. \mid V)$	$CSCE_i(.)$	$OE_i(. \mid C)$	$CRCE_i(.)$	$OE_i(. \mid V)$	$CSCE_i(.)$	$OE_i(. \mid C)$
Average*	0.5754	0.5483	0.3155	0.6830	0.3939	0.5470	0.2154
Stand.Dev.	0.2476	0.2049	0.2186	0.1399	0.1898	0.2435	0.1614
Minimum	0.1337	0.0619	0.0393	0.5205	0.1039	0.0567	0.0364
% Effic. Obs.	15.31	1.98	1.98	13.58	1.73	0.49	0.49

6. Conclusions

- What has been achieved?
  - First to empirically illustrate the differences in distributions between convex and non-convex cost frontier estimates.
  - Sections of cost in function of a single output illustrated the differences for individual observations.
  - Characterization of both economies of scale and returns to scale for individual observations turns out to be seriously conditioned by convexity.
  - Differences in the relative importance of the sources of poor performance.
    Substantially less inefficiency under non-convexity.
    More observations are efficient under non-convexity.
- General perspectives:
  - Be cautious with the use of convex technologies and cost functions.
  - Quantify incidence of convexity as 1st step to statistical testing.

Dilemma:

- ***If you do not like large inefficiencies, then accept non-convexity.***
- ***If you do not like non-convexity, then accept large inefficiencies.***

The End

Thanks for your attention

Any questions???