Decomposing Violence:  
Political Murder in Colombia, 1946-1999

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Abstract: We apply the Hodrick-Prescott and Beveridge-Nelson business-cycle decomposition methods to a time-series of homicides in Colombia (1946-1999). Separating out permanent from cyclical murder, we hypothesize that the cyclical part coincides with politically motivated unrest in the country. For 1946 to 1991, the results show an almost perfect match between the political events in the country and the computed cyclical murder component. After 1991, the nature of politically-motivated murder changed as revolutionary and para-military groups became closely affiliated with the drug-trade and generalized banditry, so that murder attributable to them becomes part of the permanent murder component.

Keywords: Colombia, homicide, Beveridge-Nelson, Hodrick-Prescott, business cycle, decomposition, time-series

JEL classification codes: C22, D63, D74, H56, K42, N46, O54

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Introduction

The countries of Latin America record, by far, the highest homicide rates in the world, averaging 20 to 30 murders per 100,000 people, i.e., two to three times as many as in the next most violent regions of the world (see, e.g., Guerrero, 1998; and, especially, Londoño, 1998, p. 72). And within Latin America, Colombia is known for its extremely high levels of homicidal violence, resulting in one of the highest murder rates in the world (see figure 1). According to Colombian National Police statistics, homicides increased from around 5,000 per year in the 1950s and 1960s to about 10,000 per year by 1980 and to about 25,000 per year by 1990. A further surge to nearly 30,000 murders per year was seen in the early 1990s. This has moderated somewhat but, in absolute numbers, still hovers between 20,000 to 25,000 per year (dotted line with circle markers, left scale, in figure 1).

When adjusted for population growth, i.e., computing homicides per 100,000 people in the population, a somewhat different pattern emerges (solid line with square markers, right scale, in figure 1). A rapid per capita murder increase occurred from 1946 to the late 1950s, followed by a ten-year period of sustained murder reductions. This downward trend reversed in the 1970s and then shows exactly the same trend as for the absolute numbers. The country’s murder rate varies substantially not only over

Figure 1: Homicides in Colombia, 1946-1999

Thom: total homicides
Thomp: total homicides per 100,000 persons

Source: Revista Criminalidad (various issues), based on Colombian National Police records, population data from Departamento Administrativo Nacional de Estadistica (via www.dane.gov.co)
time, but also from region to region (e.g., Dinar and Keck, 1997, pp. 9-10; Guerrero, 1998, pp. 96-97; Londoño, 1998, p. 76), with rates as low as 16/100,000 that would be regarded as not too far off “normal” for the world to rates as high as 900/100,000
(Guerrero, 1998, p. 97).

Frightful as the absolute and population-adjusted numbers are, they underestimate the truth. Following a survey, Rubio (1998a, p. 606) writes that even for murder, “more than half of the households victimized stated that they had ‘not done anything,’ and only 38 percent reported that they had made a formal complaint” to the authorities. Incredibly, by comparing separate statistical reporting by the police and the justice agencies Rubio finds wide disparities for more than a quarter of Colombia’s municipalities. The disparities are largest in municipalities characterized by the presence of any armed force (military, para-military, drug-gangs, guerrillas; Rubio, 1998a, p. 607). Apparently, victims fear reprisals.

Without doubt, Colombia’s murderous violence is related to two of its most salient features, the drug trade and the political violence, both of which have marred the country for decades. But less well-known and appreciated is that these two factors account only for a small portion of all murders in the country (Guerrero, 1998, p. 98). For murder, the primary risk factors are alcohol consumption, possession of firearms, and weekends. For example, a quarter of all murders take place on Sundays, more than half on Friday, Saturday, and Sunday, with disproportionate increases on holidays. Most murders are non-political, take place at night, in urban areas, are committed by poor people on poor people, and alcohol is frequently found in the victims (Londoño, 1998, especially p. 75; Guerrero, 1998), although Guerrero observes that while alcohol consumption might explain the high levels it cannot explain the drastic increase in violence in Colombia in the 1980s and 1990s (1998, p. 98).

Also contrary to popular perception, several studies have failed to establish links between murderous violence and poverty rates, unemployment rates, urbanization rates, or rates of economic growth (Londoño, 1998, p. 74; Guerrero, 1998, p. 97). Indeed, Rubio (1997) and others have made persuasive arguments according to which the educated and uneducated classes both engage in criminal and violent activity for the simple reason that crime pays well. Income and education are no longer linked, but income and crime are (Rubio, 1997, p. 812). Average annual incomes from crime have been variously estimated at up to $70,000 per person, a huge premium over Colombia’s per capita 1995 GDP of around $1,800 (Bejarano, 1997, p. 12). The break-down of the Colombian justice system further encourages criminal and violent behavior, as the probability of being caught, tried, and convicted is becoming smaller over time. By 1994, convictions rates had dropped to below four percent (Rubio,
1998a, p. 606), and sentences rarely exceeded six months of jail time (Rubio, 1998b, p. 91).

There is wide-spread agreement among analysts of all stripes that Colombia’s violence is costly, both at the microeconomic level (e.g., Dinar and Keck, 1997) and at the macroeconomic level – estimated at up to 15 percent of GDP (Bejarano, 1997, p. 10) – and there is good evidence that major perpetrators of violence – the military and paramilitary forces, the drug-traders, and the various guerrilla groups – act in semi-collusive fashion to keep the spoils of war going (Richani, 1997), evidence almost perfectly in line with the theory suggested by Brito and Intriligator (1992).

While on-going research will have to identify and disentangle the various causes and possible intervention mechanisms of extreme, generalized violence in Colombia, our concern in this paper is much more narrow and limited. Rather than to specify an underlying structural model – which we do in forthcoming research – we focus here on a pure time-series study to separate out politically motivated murders from all other murders. On the hypothesis that political unrest and politically motivated murder are cyclical, we apply business-cycle decomposition methods to the murder time-series, and then compare the estimated cyclical or “transitory” component to a narrative account of cycles of political violence in Colombia. We find a good overlap between our estimates and the narrative: when the political context suggests much unrest, the cyclical component of murders increases, and vice versa. The trend or “permanent” component is interpreted as that part of the murder time-series that would have occurred without political violence. Once the time-series is separated into politically motivated and non-political murder, and estimates of the respective numbers are made, further research can build underlying structural models for each part of the series.

The paper proceeds as follows. We begin with the technical aspect, so that the next section presents a discussion of the data and the decomposition methods employed. This is followed by the interpretation of our findings, i.e., the matching of the cyclical component to the political narrative. The final section concludes.

Data and methods

The Colombian National Police has a record of crime statistics that reaches back to 1946 (various issues of Revista Criminalidad). It distinguishes among various types of crime (see text box 1). Item number 13 (crimes against life and person) contains some 20 categories, such as murder, abortion, and personal injury. Apart from abortions, car-accident related deaths, and so on, the murder categories are primarily murder (homicidio) and aggravated murder (homicidio agravado). Since 1993, there
is also an attempt to separate out further types of murder, e.g., murder with terrorist intent (*homicidio con fin terrorista*) and death associated with the exercise of official police duties (*homicidio con función, razón cargo o ejercicio de sus funciones*). For our analysis we use the number of murders for these four categories combined, i.e., we collapse all these categories of murder into a single group.¹

As is well known, time series can be broken into constituent components. The seasonal component does not apply to our case since we have annual data. The long-run trend component is often simply modeled as a linear or non-linear trend line over time, and the cyclical component is the remaining variation around this trend. We apply the Hodrick-Prescott (1997) and the Beveridge-Nelson (1981) decomposition techniques.

### Hodrick-Prescott

To estimate the trend line, a variety of smoothing techniques are available, from simple time-trend lines to various forms of moving averages, to diverse exponential smoothing methods. Exponential smoothing is a method that assigns an exponentially increased weight the more recent a particular observation is to the current-time data point. The analyst sets the smoothing parameter arbitrarily. Smaller parameters produce smoother trend series; larger parameters track the original series better because the observations closest in time to the point being estimated are weighted most heavily. Analysts will use a variety of smoothing parameters and then choose the one yielding the best (smallest) of various forecast error values. Econometric software can automatically search for and estimate this “best” parameter. One or two-parameter double exponential smoothing is used for data with trend (to adjust the slope of the forecast). Without going into any detail here, none of these methods are appropriate for our purpose.

For business cycle research, a popular method to smooth a time series and produce its long-run trend component is the Hodrick-Prescott filter (1997).
Technically, consider

\[
(1) \sum_{t=1}^{T} (y_t - s_t)^2 + \lambda \sum_{t=2}^{T-1} ((s_{t+1} - s_t) - (s_t - s_{t-1}))^2,
\]

where \( y \) is the original series, \( t \) is time (\( t = 1, ..., T \)), and \( s \) is the smoothed series. The filter produces the smoothed series by minimizing equation (1) whose first term denotes the squared difference between an original and its smoothed value at time \( t \) (i.e., the variance) and whose second term defines upper and lower bounds of \( s \), where \( \lambda = 0 \) returns the series of squared differences and \( \lambda \neq 0 \) produces a linear smoothed series. Since we deal with annual data, we use the recommended default value of \( \lambda = 100.2 \). E-Views 4 contains a Hodrick-Prescott routine that produces the relevant smoothed trend estimates which, along with the original data and the Beveridge-Nelson results, are displayed in figure 2.3 (The data are also recorded in appendix 1.) Deviations from the smoothed trend constitute the cyclical component (figure 3).

Our hypothesis is that, in figure 3, an upward movement of the cyclical component of the homicide time-series corresponds to time-periods with marked political unrest whereas a downward movement of the cyclical component denotes relative political calm in Colombia. We offer a discussion in the “Interpretation” section.

**Beveridge and Nelson**

In 1981, Beveridge and Nelson (BN) introduced a new business cycle decomposition technique. Their objective was to produce a better dating technique, i.e., a technique with superior turning-point performance. Since the estimation of data points in time \( t \) completely relies on past values, \( t-1, t-2, ..., t-n \), BN describe their method as a “real time” technique: plug new data points into the model as data become available, and it will tell you whether the business cycle is turning. The technique requires to first fit an ARIMA model on the first-differenced natural logs of the dependent variable. Since in the spirit of BN we do not propose any structural model explaining the data movement, nor propose to engage in any forecasting of the data points, we conducted an unabashed best-fit search which resulted in an ARIMA(0,1,13) model with moving average terms at lags 1, 5, and 13. The results are:
where $d(l\text{thompc})$ is the first difference of the natural log of total homicides per 100,000 persons in Colombia, 1946 to 1999. Estimating this model minimized the Akaike Information Criterion (AIC) as well as the Schwartz Criterion (SC). We ran the model with RATS 4 as well as with E-Views 4 and, except for rounding, obtained the same estimates.\(^4\)

Once a model that best fits (or reproduces) the original data is estimated, the permanent and cyclical components can be extracted from the estimated data points. The resulting data are displayed in figures 2 and 3 (and reported in appendix 1).\(^5\)

**Interpretation of results**

It would appear that the Hodrick-Prescott (HP) and Beveridge-Nelson (BN) methods yield substantially different permanent trend information (figure 2). But both result in comparable estimates of the cyclical movement (not in magnitude but in turning points), at least for the early and the late years of the series. From 1965 to 1980, the HP method tracks the actual data almost without deviation because there is nothing to be smoothed in the original data series. (HP is a smoothing technique, after all.) In contrast, the BN method produces cyclical
information throughout the entire time-period. We therefore focus here on interpreting the political events of Colombia in light of the BN cyclical component we extracted from the data series. The chronology and event description is taken from Bushnell (1993).  

It appears, in figure 3, that there are three major periods in the cyclical component of political violence in Colombia. The first occurs from 1946 to 1958, with a short-term decline in the rate of increase from 1952 to 1957. The two peaks – in 1952 and 1958 – represent, respectively, 56 and 53 percent of all murders. In other words, our method suggests that politically-motivated murder accounts for more than half of all (reported) murders in those years. The second period occurs from 1958 to the late 1970s. One could also argue that the second period lasts from 1965 to 1985, a twenty-year period during which the estimated cyclical (i.e., political) violence is negative, i.e., lies below the permanent trend line. But since we cannot be sure about the actual number of murders, it is best to focus on turning-points and movements, rather than magnitudes, of the series. The third period occurs as from the late 1970s when an upsurge in political violence is observed, until the series peaks in 1991. Thereafter the cycle declines once more.

*The first period: 1946 to 1958*

Between 1930 and 1946, the Liberal Party was in continuous power, indeed so much so that Colombians refer to this time period simply as the “Liberal Republic.” But with the election of 1946, the Conservative Party’s Mariano Ospina Perez assumed the country’s presidency in August 1946. Then, on 9 April 1948, Jorge Eliecer Gaitán, a
charismatic, leftist Liberal Party presidential candidate was murdered by unknown assailants. This murder initiated a violent outburst in Botogá – the Bogotazo – which began the period commonly known as La Violencia, hence the upsurge in political violence tracked in figure 3. This period lasted until 1957, a period where political violence primarily, but not only, emerges from the confrontation between the adherents to the two major political parties. Following Gaitán’s murder, the Liberals achieved victory in congressional elections in June 1949, but presidential victory is obtained by the Conservative Party’s Laureano Gómez in November 1949. Meanwhile, political violence with leftist guerrillas – who view both the Liberal and the Conservative Party as establishment parties of land-owners and other vested economic interests – continued from 1950 to 1953 to such an extent that the military took power in 1953 under General Gustavo Rojas Pinilla. This resulted in an initial decrease in the pace of violence (see the decline in figure 3 in 1953 and 1954) but picked up in the latter years of Rojas Pinilla’s tenure. Put differently, murderous violence first declined as killings originated by guerrillas were stopped but then increased as the military began to kill increasing numbers of the guerrillas (which was to become, in 1963, the Fuerzas Armadas Revolucionarias de Colombia, or FARC). Rojas Pinilla’s initial success led the national assembly to elect him to a full four-year presidential term in 1954. But by May 1957 Rojas Pinillas’ inability to actually put an end to La Violencia led to a nation-wide general strike, a military junta took control, and Rojas Pinilla went into exile.

The second period: 1958-1978

The military junta yielded to a bipartisan coalition, called the National Front, which remained in control until 1978. The National Front essentially was an agreement between the Liberal and Conservative Parties to trade presidential terms and to adhere to a quota system in the assignment of ministerial and other government posts. This should reduce the amount of political murder due to the clash between the two parties, as indeed it did (see the cyclical decline in figure 3). The first National Front president was Alberto Lleras Camargo (Liberal, who also briefly held the presidency in 1945-1946). President from 1958 to 1962, he was followed by Conservative Party member Guillermo León Valencia (1962-1966). The third president (1966-1970) was Liberal Carlos Lleras Restrepo (not related to the first Lleras), during whose term murder rates fell to 20/100,000, the lowest since the Bogotazo and almost “normal” by other countries standards. The fourth National Front president was Conservative Misael Pastrana Borrero (1970-1974), father of Colombia’s current president, Andrés
Pastrana (1998-2002). The end of the end of *La Violencia* occurred somewhere between the early to mid-1970s. From 1974-1978, the president was Liberal Alfonso López Michelsen. During his tenure, Colombia experienced considerable economic growth, driven by a commodity export boom (coffee, coal, oil, and marijuana; not yet cocaine). It is also now generally accepted and acknowledged that Michelsen’s administration suffered from wide-spread corruption and involvement with the emerging drug-trade.

The various old and new guerrilla movements which had been active for several decades were now formalized. The Fuerzas Armadas Revolucionarias de Colombia (FARC) was formed in 1963, the Ejército de Liberación Nacional (ELN) in 1965, the Ejército Popular de Liberación (EPL) in 1967, and the M-19 (Movimiento 19 de Abril) in 1970.

**The third period: 1978 to 1999**

The twenty-year period of the National Front ends – as does the relative political calm – when another Liberal wins the presidential election (Julio César Turbay Ayala; 1978-1982). His term of office sees an upsurge in revolutionary activity and Colombia’s economic boom also ends. For example, in a dramatic move that made the world’s news headlines, the M-19 guerrilla group seized in December 1979 the Embassy of the Dominican Republic in Bogotá and held thirteen ambassadors hostage who had met at the embassy. The hostages included the US envoy. Both the HP and the BN cyclical violence series show an upward movement of political murder for the Turbay years (figure 3). The next election is won by Conservative Belisario Betancur Cuartas (1982-1986). He begins peace negotiations between government and guerrillas (except with the relatively small ELN). The cyclical component of our series declines in 1982 and 1983, holding even in 1984 (HP shows a further decline in 1984). But the peace negotiations ultimately did not result in guerrilla demobilization and weapons surrender. Instead, in 1985, three presidential candidates were assassinated; moreover, in a spectacular move, the M-19 occupied the Colombian Justice Palace (the seat of Colombia’s Supreme Court) and many judges were murdered. Betancur broke off peace negotiations, and our series shows an up-tick in political murder. Indeed, the period of the greatest amount of political murder is now ushered in. Interestingly, this is also the time-period during which the *slope* of the BN-*permanent* violence series begins to steepen, suggesting that as from the early 1980s one or more additional, *non-political* components come into play that pushed up Colombia’s murder rate to among the highest in the world.
Liberal Virgilio Barco Vargas, an MIT-trained civil engineer, was elected president (1986-1990). Even though a former M-19 member, Bernardo Jaramillo Ossa, founded a new political party, the Unión Patriótica, there is now open violent conflict among government, various guerrilla and paramilitary groups, and–increasingly–drug traffickers as cocaine replaces marijuana and the dollar-volume at stake becomes ever larger.

It is now commonplace knowledge that guerrillas need to finance themselves and that one prime method of finance is through the exploitation of a country’s natural resources (diamonds in Angola; drugs in Colombia; etc.). Similarly, governments exploit natural resources to raise funds as well (oil in Angola; and oil in Colombia). Thus, we suggest, what once may have been genuine political grievance giving rise to guerrilla activity in the countryside was transformed into greed to sustain an agenda that, in the post-cold war world, is no longer viable. If we are correct, we should see a switch in the “character” of murder in Colombia. That switch appears to have occurred as of 1991 when the cyclical component of our series declines sharply (figure 3). The Liberal César Augusto Gaviria Trujillo (1990-1994) became president and pushed the legislature to formally adopted a new constitution in 1991 (to replace the constitution of 1886). But there is no particular reason why a mere change in the country’s constitution should mark the beginning of the pronounced fall in the cyclical (i.e., political) murder series. Instead, we suggest that the increasing dollar-value of the drug-traffic is the primary additional component in Colombia’s story, pushing up the slope of the permanent murder series and strongly entangling politics and economic vested interests. Indeed, “popular” knowledge in Colombia asserts across the spectrum of opinion that the administration of Liberal Ernesto Samper Pizano (1994-1998) was “controlled” by the predominant drug cartels (Medellín y Cali), with increasing participation in that trade by the main guerrilla groups. There is, therefore, no particular “need” for political violence which thus switches to become non-political permanent violence. Indeed, another Liberal president, Andrés Pastrana Arango (1998-2002), went so far as to grant in 1999 the largest rebel group, the FARC, a demilitarized zone the size of Switzerland (42,000 km²) and placed it under the FARC’s administrative control. This arrangement ended in February 2002 as it became clear that the FARC had no particular intention to arrange a peace that would leave it stripped of resources.

Colombia’s future

As a collusive arrangement to jointly exploit the country’s resources (á la the National
Front of 1958 to 1978) appears not possible among the major vested interests – the government and its military units, the paramilitary units, the drug-producers, and the various guerrilla groups – there remains, in our opinion, only the possibility of a renewed upsurge in political violence and murder. All sides to the conflict still appear to believe that each can triumph over the others, for an ever larger share of the spoils. As further data points become available – for 2000, 2001, 2002, and so on – we would therefore expect an upturn in the BN-cyclical component of Colombia’s murder time-series.

Conclusion

The cyclical (or transitory) component of Colombia’s homicide time-series is associated and strongly coincides with the country’s political events from 1946 to 1991. As from 1991, the character of violence appears to have changed. According to the news media, it would appear that there has been an upsurge in political violence in the 1990s, but the HP and BN decomposition methods we employed both suggest that while permanent violence continued to grow, cyclical violence declined, and drastically so. We hypothesize and offer for further discussion the suggestion that the character of Colombia’s “political” violence has changed, i.e., that even though it is cloaked in terms of revolutionary and counter-revolutionary language, the observed violence is in fact closely linked to economics, the economics of the drug-traffic in particular. Revolutionaries and counter-revolutionaries, in other words, may have become bandits who, mafia-like, defend their respective territories and interests with murder. If we are correct, the current war in Colombia then is essentially an economic war, much like those we have observed in Africa in the 1990s (e.g., Sierra Leone, Liberia, Angola), a war over access to and exploitation of natural resources. This war is unlikely to cease unless the major source of funding – US drug-purchases – ceases.

In future research, we plan to construct structural models to explain the movement of the permanent and cyclical murder series in Colombia but also to repeat the decomposition exercise with sub-national data (i.e., by administrative departamento). Provided that reliable figures are available, we believe that an application of the decomposition method is also possible and worthwhile for countries other than Colombia.

Notes

We thank the Margaret Ann Isely Foundation, Eugenia Almand, and Barbara Bertram
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1. Data by province are available, and we are engaged in further research to collect and analyze these at the provincial level.

2. See Maravall and del Río (2001) for a discussion of the default values.

3. In E-Views, double-click on the original data series icon, click on Procs, select Hodrick-Prescott Filter, and type in the desired smoothing parameter, \(\lambda\), or accept the default value \(\lambda = 100\).

4. E-Views and RATS use different implementations of the Schwartz Criterion (SC). E-Views works off the log likelihood function, whereas RATS works off the sum of squared residuals. We confirmed both formulas by hand-computing the respective criterion to check the computer printouts. Running a variety of ARIMA models, the model with moving average model with lags at 1, 5, and 13 returns the lowest SC in E-Views and RATS, respectively. (RATS does not report the AIC.)

5. Neither E-Views nor RATS contains a Beveridge-Nelson routine. Using the original BN (1981) method, the extraction of permanent and cyclical components from the original series is computationally very intense. Cuddington and Winters (1987), Miller (1988), and Newbold (1990) provide computationally easier methods. In his dissertation, Cárdenas (1991) provided an exceptionally easy and conceptually appealing way to compute the components, and that is the method we apply. Since we failed to locate a published account of Cárdenas’ method, we provide the mathematical details in appendix 2. To ensure correctness we computed the permanent and cyclical components by BN’s original method, by the Cuddington/Winters method, and by the Cárdenas method, using the first actual observation as the initial value. All three methods then turned up identical estimates.

6. A more dramatic, sometimes personal account is Giraldo (1996). It takes particular aim at the Colombian military and paramilitary forces’ contribution to the violence.

7. On “greed and grievance” see, e.g., Paul Collier, 1999 and 2000.
## Appendix 1: data table

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<td>29.03</td>
</tr>
</tbody>
</table>
Appendix 2: The Beveridge-Nelson decomposition of a time-series

Let $w_t$ be the stationary first differences of a non-stationary series $z_t$

1. $w_t = z_t - z_{t-1}$

Wold’s (1938) decomposition theorem states that

2. $w_t = \mu + \lambda_0 e_t + \lambda_1 e_{t-1} + \ldots$, where $\lambda_0 \equiv 1$

and $\lambda_t$ are constants and the $e$’s are uncorrelated errors. Beveridge and Nelson [BN] (1981) relate each $z_t$ to its own future values or “forecast profile,” where the profile in
time \( t \) is interpreted as \( z_t \)’s “permanent” component, \( \bar{z}_t \). Thus, the estimated \( z_t \) forecast \( k \) periods ahead becomes the expected value of \( z_{t+k} \), conditional on \( z_t \)’s past values

\[
\hat{z}_t (k) = E (z_{t+k} | \ldots, z_{t-1}, z_t)
\]

which, since \( w \) accumulates past \( z \)’s, may be written as

\[
\hat{z}_t (k) = z_t + E (w_{t+1} | \ldots, w_1, w_{t+k} | \ldots, w_{t-1}, w_t)
\]

\[
= z_t + \hat{w}_t (1) + \ldots + \hat{w}_t (k)
\]

But from (2), each estimated \( w \), say \( w_{t+i} \), is

\[
\hat{w}_t (i) = \mu + \lambda_{t-1} \varepsilon_{t-1} + \lambda_{t-2} \varepsilon_{t-2} + \ldots
\]

\[
= \mu + \sum_{j=i}^{\infty} \lambda_j \varepsilon_{t+i-j}
\]

Substituting (5) recursively into (4) and approximating to an infinite time horizon, we obtain

\[
\hat{z}_t (k) \approx k \mu + z_t + \left( \sum_{1}^{\infty} \lambda_i \right) \varepsilon_t + \left( \sum_{2}^{\infty} \lambda_i \right) \varepsilon_{t-1} + \ldots
\]

(6)

\[
\hat{z}_t (k) - k \mu \approx z_t + \left( \sum_{1}^{\infty} \lambda_i \right) \varepsilon_t + \left( \sum_{2}^{\infty} \lambda_i \right) \varepsilon_{t-1} + \ldots
\]

Beveridge and Nelson (1981) suggest that it is natural to interpret the LHS of (6) as \( z_t \)’s “permanent” component, denoted as \( \bar{z}_t \). The cyclical component, \( c_t \), then is

\[
\bar{z}_t - z_t = z_t = \left( \sum_{1}^{\infty} \lambda_i \right) \varepsilon_t + \left( \sum_{2}^{\infty} \lambda_i \right) \varepsilon_{t-1} + \ldots
\]

(7)

The unknown \( \mu \) and \( \lambda_i \)’s in (5) must be estimated. Beveridge and Nelson suggest an ARIMA procedure of order \( (p,1,q) \) with drift \( \mu \).
\( w_t = \mu + \frac{\left(1 - \theta_1 L^1 - \ldots - \theta_q L^q\right)}{\left(1 - \phi_1 L^1 - \ldots - \phi_p L^p\right)} \epsilon_t = \mu + \frac{\theta(L)}{\phi(L)} \epsilon_t. \)

Cuddington and Winters [1987, p. 127, equation (7)] realized that in the steady state, i.e., \( L=1 \), (8) reduces to

\[
\tilde{z}_t - \tilde{z}_{t-1} = \mu + \frac{(1 - \theta_1 - \ldots - \theta_q)}{(1 - \phi_1 - \ldots - \phi_p)} \epsilon_t = \mu + \frac{\theta(1)}{\phi(1)} \epsilon_t,
\]

where \( \mu \) will be the estimated mean, \( \Theta \), the estimated moving-average terms, and \( \Phi \), the estimated autoregressive terms. Now iterate (9) recursively, i.e., replace \( t \) by \( (t-1) \) and \( (t-1) \) by \( (t-2) \), etc. Then we get

\[
\tilde{z}_t - \tilde{z}_{t-1} = \mu + \frac{\theta(1)}{\phi(1)} \epsilon_t
\]

\[
\tilde{z}_{t-1} - \tilde{z}_{t-2} = \mu + \frac{\theta(1)}{\phi(1)} \epsilon_{t-1}
\]

\[
\tilde{z}_{t-2} - \tilde{z}_{t-3} = \mu + \frac{\theta(1)}{\phi(1)} \epsilon_{t-2}
\]

\[
\tilde{z}_1 - \tilde{z}_0 = \mu + \frac{\theta(1)}{\phi(1)} \epsilon_1
\]

Adding these equations, the terms on the LHS cancel out except for \( \tilde{z}_t \) and \( \tilde{z}_0 \), and on the RHS \( \mu \) is added “t” times and the fraction in the second term on the RHS is a constant to be multiplied by the sum of error terms. Thus, we obtain

\[
\tilde{z}_t = \tilde{z}_0 + \mu t + \frac{\theta(1)}{\phi(1)} \sum_{i=1}^{t} \epsilon_i
\]
This is, except for notation, Newbold’s equation [Newbold, 1990, p. 457, equation (6)] and the problem reduces to finding an initial value for $z_0$.

Mauricio Cárdenas (1991), in his unpublished dissertation, suggests that $z_0 = z_i$, i.e., the very first data point of the original series. This makes intuitive sense. Since a forecast profile conditional on past values cannot be computed in the absence of past values, $z_0 = z_0$ by definition. Formally, Cárdenas suggests the following (we changed the notation to conforms to ours here), where $z_t$ refers to the original data series,

\begin{equation}
(12) \quad z_t - z_{t-1} = \mu + \sum_{i=1}^{p} \phi_i \Delta z_{t-i} + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t .
\end{equation}

In words, the series of the first differences of $z_t$ equals the series mean, adjusted for autoregressive and moving-average terms. Bring the autoregressive term to the LHS to get

\begin{equation}
(13) \quad (z_t - z_{t-1}) - \left( \sum_{i=1}^{p} \phi_i \Delta z_{t-i} \right) = \mu + \sum_{j=1}^{q} \theta_j \varepsilon_{t-j} + \varepsilon_t ,
\end{equation}

and expand both summation terms

\begin{equation}
(14) \quad (1 - \phi_1 L - \phi_2 L^2 - \ldots - \phi_p L^p)(z_t - z_{t-1}) = \mu + \left( 1 + \theta_1 L + \ldots + \theta_q L^q \right) \varepsilon_t .
\end{equation}

Rearrange (14) to obtain

\begin{equation}
(15) \quad z_t - z_{t-1} = \frac{\mu}{\phi(L)} + \frac{\theta(L)}{\phi(L)} \varepsilon_t .
\end{equation}

Now, recursively replace $t$ with $(t-1)$, and $(t-1)$ with $(t-2)$, etc.
\begin{equation}
\begin{aligned}
    z_t - z_{t-1} &= \frac{\mu}{\phi(L)} + \frac{\theta(L)}{\phi(L)} \epsilon_t \\
    z_{t-1} - z_{t-2} &= \frac{\mu}{\phi(L)} + \frac{\theta(L)}{\phi(L)} \epsilon_{t-1}
\end{aligned}
\end{equation}

(16)

\begin{equation}
\begin{aligned}
    z_1 - z_0 &= \frac{\mu}{\phi(L)} + \frac{\theta(L)}{\phi(L)} \epsilon_1
\end{aligned}
\end{equation}

which, when added together “t” times, yield

\begin{equation}
\begin{aligned}
    z_t - z_0 &= \frac{\mu}{\phi(L)} t + \frac{\theta(L)}{\phi(L)} \sum_{i=1}^{t} \epsilon_i
\end{aligned}
\end{equation}

(17)

Rearranged, write

\begin{equation}
\begin{aligned}
    z_t &= z_0 + \frac{\mu}{\phi(L)} t + \frac{\theta(L)}{\phi(L)} \sum_{i=1}^{t} \epsilon_i
\end{aligned}
\end{equation}

(18)

In the steady state, i.e., \(L=1\), equation (18) readily yields the permanent component of \(z_t\),

\begin{equation}
\begin{aligned}
    z_t^\perp &= z_0 + \frac{\mu_t}{\phi(1)} + \frac{\theta(1)}{\phi(1)} \sum_{i=1}^{t} \epsilon_i
\end{aligned}
\end{equation}

(19)

which, except for notation, is Cárdenas’ final formula [1991, p. 27, equation (15)]. In practice, it is easy to set up a spreadsheet for (19) by replacing \(\theta(1)\) and \(\phi(1)\) with \((1 - \sum \theta_i)\) and \((1 - \sum \phi_i)\), i.e., with the estimated ARIMA coefficients. In our
particular case, where the ARIMA did not result in any AR terms at all, the denominators fell out of (19) altogether, making the computation of the permanent, and hence cyclical, components, of the Colombian murder series even easier.

List of references


