

Constructing Turning Point Chronologies with Markov–Switching Vector Autoregressive Models: the Euro–Zone Business Cycle

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Abstract

In this paper we advocate a parametric approach to the construction of turning point chronologies for the euro-zone business cycle. In generalization of Hamilton (1989), the Markov-switching vector autoregressive (MS-VAR) model is utilized for the analysis of the business cycle, providing the mechanism for identifying peaks and troughs of the business cycle. Building upon ideas developed in Krolzig (1997a) and Artis, Krolzig and Toro (2003), the approach for the constructing the turning point chronology consists of (i) modelling the euro-zone business cycle as a single common factor generated by a hidden Markov chain, (ii) fitting a congruent statistical model to the data, (iii) deriving the conditional probabilities of the regimes ‘*expansion*’ and ‘*recession*’ from the estimated model, (iv) classifying each point in time to the regime with the highest probability, and (v) dating the turning points of the business cycle. The MS-VAR also provides measures of uncertainty associated with the turning point chronology, facilitates real-time detection of business cycle transitions, and offers a well-developed theory for the prediction of the business cycle. Examining the properties of the proposed dating procedure, we show that aggregation and model misspecification can severely hamper the detection of business cycle turning points. In the empirical part, the MS-VAR approach is applied to three multi-country data sets consisting of real GDP and industrial production growth rates of 12 euro-zone countries (in total) over the period from 1973 to 2002. Although the empirical models are found to be sensitive to data quality (with seasonal adjustment, outlier correction and smoothing being all influential), the estimated models were found useful for the assessment of business cycle synchronization among the EMU member states, and for the construction of a turning point chronology of the euro-zone business cycle.

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1 Introduction

The creation of the euro zone with begin of Stage Three of the Economic and Monetary Union (EMU) on 1 January 1999 has raised several important issues. Among them, one of paramount relevance concerns the economic convergence of the EMU member states. A strong indicator for convergence is the existence of a common euro-zone business cycle. Furthermore, business cycle synchronization constitutes a pre-condition for symmetry of the effects of monetary policy in the euro zone. The presence of a common euro-zone business cycle implies the coincidence of business cycle turning points in the countries of the euro-zone. The detection of its turning points of the common cycle is, therefore, crucial for monetary policy making in the union.

There is a huge literature evaluating the degree of business cycle synchronization in the EMU. Among others, Artis *et al.* (2003) conclude that *'the conception of a common or "European" business cycle is an intelligible one'*. Building upon that result, this paper deals with the construction of a turning point chronology for the euro-zone business cycle. A parametric approach is advocated for doing so, which consists of first fitting a congruent statistical model to the data, and then utilizing the estimated coefficients for the classification of observations as 'recession' or 'expansion'; the implied dating of the turning points finally constitutes the business cycle chronology. The approach is based on the Markov-switching time series model innovated by Hamilton (1989) in his analysis of the US business cycle and generalized in Krolzig (1997b) to a Markov-switching vector autoregressive model, which will be used for the modelling and dating of the euro-zone business cycle. In the framework considered here, the euro-zone business cycle is defined as the common factor of the macroeconomic fluctuations in the countries of the euro-zone, and it is assumed that the transition of its phases are generated by an unobservable stochastic process, which is a hidden Markov chain.

The paper proceeds as follows. First, in § 2 we discuss the Markov-switching time series model and its application to the modelling of the business cycle. Commencing from the univariate Hamilton (1989) model of the US Business Cycle, the Markov-switching vector autoregressive (VAR) model of the business cycle is presented. Then, § 3 considers the construction of turning point chronologies with Markov-switching models. We present the regime inference in Markov-switching VAR models and discuss how the derived regime probabilities can in turn be used to date the turning points of the business cycle, finally producing the turning point chronology. We also point out three distinctive advantages of the MS-VAR approach: the provision of an uncertainty measure as a by-product of the regime classification, the possibility of real-time detecting of turning points, and a well-developed theory of the prediction of the business cycle. In § 4 the properties of the proposed regime classification rule are evaluated for univariate and multivariate Markov-switching models. We analyze the effects of business cycle synchronization and regime-dependent heteroscedasticity on the classification procedure, and show that aggregation obstructs the detection of turning points, when the variable are driven by a common business cycle. Section 5 presents an empirical analysis of the euro-zone business cycle. Starting with a review of the literature featuring Markov-switching studies of the European business cycle, we discuss the empirical modelling approach used in this paper, and present results for three Markov-switching VAR models of the euro-zone business cycle using quarterly OECD GDP data (1973-2002), OECD industrial production data (1974-2002), and Eurostat GDP data (1980-2002), respectively. After having established evidence for business cycle synchronization in the euro area, the estimated models are used for the construction of turning point chronologies of the euro-zone business cycle. Finally § 6 concludes.

2 Business Cycle Modelling with Markov-switching Vector Autoregressions

Recent theoretical and empirical business cycle research has revived interest in the co-movement of macroeconomic time series and the regime-switching nature of macroeconomic activity. The general idea behind regime-switching models of the business cycle is that the parameters of a time series model of some macroeconomic variables depend upon a stochastic, unobservable regime variable $s_t \in \{1, \dots, M\}$ which represents the state of business cycle. The number of regimes, M , is often assumed to be two reflecting economic expansions and contractions.

The Markov-switching autoregressive time series model has emerged as a leading approach for the detection and dating of business cycle turning points. Since Hamilton's (1989) application of this technique to measure the US business cycle, there has been a number of subsequent extensions and refinements (see Krolzig, 1997b, for an overview). We first present the original contribution of Hamilton, then consider the multivariate generalization, and finally discuss its application to the business cycle.

2.1 Hamilton's Model of the US Business Cycle

In the original contribution of Hamilton (1989), contractions and expansions are modelled as switching regimes of the stochastic process generating the growth rate of real output Δy_t :

$$\Delta y_t - \mu(s_t) = \alpha_1 (\Delta y_{t-1} - \mu(s_{t-1})) + \dots + \alpha_4 (\Delta y_{t-4} - \mu(s_{t-4})) + u_t. \quad (1)$$

In (1), the two regimes are associated with different conditional distributions of the growth rate of real output, where the mean growth rate μ depends on the state or 'regime', s_t . For a meaningful business cycle model, the mean growth rate should be negative in the first regime, $\mu_1 < 0$ ('*recession*'), and positive in the second, $\mu_2 > 0$ ('*expansion*'). The variance of the disturbance term, $u_t \sim \text{NID}(0, \sigma^2)$, is assumed to be the same in both regimes.

The stochastic process generating the unobservable regimes is an ergodic Markov chain defined by the constant transition probabilities:

$$\begin{aligned} p_{12} &= \Pr(\text{recession in } t \mid \text{expansion in } t-1), \\ p_{21} &= \Pr(\text{expansion in } t \mid \text{recession in } t-1), \end{aligned}$$

The transition probabilities have to be estimated together with the parameters of equation (1).

By inferring the probabilities of the unobserved regimes conditional on the available information set, it is possible to reconstruct the regimes and to date the turning points of the business cycle. Hamilton was able to show that his model is able to track the NBER reference cycle very closely. This suggests that the Markov-switching approach should also be useful for the modelling and dating of the euro-zone business cycle.

In the literature (see the review in § 5.1), univariate Markov-switching models like the one in (1) have been applied to aggregated euro-zone data (with mixing results). However, it is important to note that, by definition, univariate Markov-switching models as proposed by Hamilton (1989) are only able to capture some of the stylized facts of the business cycle. They can represent the non-linearity or asymmetry stressed in some part of the literature, but obviously, they are unable to reflect the idea of co-movement among time economic series. This is particularly important in the context of the euro-zone since there is a genuine interest in assessing the synchronization of the business cycles among the euro-zone countries.

In this paper, we think of the euro-zone business cycle as a common cyclical feature of the macroeconomic dynamics in the EMU member states. It is a common factor, which shares all the properties of the regime variable in (1) by being generated by a hidden Markov chain. We shall therefore utilize multivariate generalizations of Hamilton's approach for the modelling of the euro-zone business cycle using real GDP and industrial production time series for the countries of the euro-zone (to be presented in § 5). Identifying the transitions of the regime variable will give us the turning points of the common cycle component of the EMU member states leading towards a chronology of the euro-zone business cycle. Next, we introduce the statistical model: the *Markov-switching Vector Autoregression*. Its interpretation as a business cycle model follows in § 2.3).

2.2 Markov-switching Vector Autoregressive Models

Like other regime-switching models, the Markov-switching vector autoregressive (MS-VAR) model is a vector autoregressive process of the *observed* time series vector $\mathbf{y}_t = (y_{1t}, \dots, y_{Kt})'$, whose parameters are, at least partly, unconditionally time-varying but constant when conditioned on an *unobservable* discrete regime variable $s_t \in \{1, \dots, M\}$:

$$\mathbf{y}_t - \boldsymbol{\mu}(s_t) = \mathbf{A}_1(s_t) (\mathbf{y}_{t-1} - \boldsymbol{\mu}(s_{t-1})) + \dots + \mathbf{A}_p(s_t) (\mathbf{y}_{t-p} - \boldsymbol{\mu}(s_{t-p})) + \mathbf{u}_t, \quad (2)$$

where $\mathbf{u}_t$ is a Gaussian error term conditioned on s_t : $\mathbf{u}_t|s_t \sim \text{NID}(\mathbf{0}, \boldsymbol{\Sigma}(s_t))$. A p -th order, M -state Markov-switching vector autoregressive model of a K dimensional time series vector is denoted MS(M)–VAR(p). The parameter matrix shift functions $\boldsymbol{\mu}(s_t)$, $\mathbf{A}_1(s_t), \dots, \mathbf{A}_p(s_t)$ and $\boldsymbol{\Sigma}(s_t)$ describe the dependence of the VAR parameters $\boldsymbol{\mu}$, $\mathbf{A}_1, \dots, \mathbf{A}_p$ and $\boldsymbol{\Sigma}$ on the regime variable s_t as in:

$$\boldsymbol{\mu}(s_t) = \begin{cases} \boldsymbol{\mu}_1 = (\mu_{11}, \dots, \mu_{K1})' & \text{if } s_t = 1, \\ \vdots & \\ \boldsymbol{\mu}_M = (\mu_{1M}, \dots, \mu_{KM})' & \text{if } s_t = M. \end{cases} \quad (3)$$

The decisive characteristic of a Markov-switching model is that the unobservable realizations of the regime $s_t \in \{1, \dots, M\}$ are generated by a discrete time, discrete state Markov stochastic process, which is defined by its transition probabilities:

$$p_{ij} = \Pr(s_{t+1} = j | s_t = i), \quad \sum_{j=1}^M p_{ij} = 1 \quad \text{for all } i, j \in \{1, \dots, M\}. \quad (4)$$

It is usually assumed that the Markov process is irreducible and ergodic.

In the model (2) there is an immediate one-time jump in the process mean after a change in the regime. Frequently, it is more plausible to assume that the mean smoothly approaches a new level after the transition from one state to another. In such a situation, the following model with a regime-dependent intercept term $\boldsymbol{\nu}(s_t)$ could be used:

$$\mathbf{y}_t = \boldsymbol{\nu}(s_t) + \mathbf{A}_1(s_t) \mathbf{y}_{t-1} + \dots + \mathbf{A}_p(s_t) \mathbf{y}_{t-p} + \mathbf{u}_t. \quad (5)$$

In contrast to linear time-invariant VAR models, the mean adjusted form (2) and the intercept form (5) are not equivalent. They imply different dynamic adjustments of the observed variables after a change in regime. While a permanent regime shift in the mean $\boldsymbol{\mu}(s_t)$ causes an immediate jump of the observed time series vector onto its new level, the dynamic response to a once-and-for-all regime shift in the intercept term $\boldsymbol{\nu}(s_t)$ is identical to an equivalent shock in the white noise series $\mathbf{u}_t$.

The MS-VAR model allows for a great variety of specifications. In principle, it would be possible to alternate the specification of the regime generating process (i) by making all parameters time-varying and (ii) by introducing separate regimes for each equation (or even for each parameter). But, this is not a practicable solution as the number of parameters of the Markov chain grows quadratic in the number of regimes and coincidentally shrinks the number of observations usable for the estimation of the regime-dependent parameters. For these reasons, we prefer to impose the assumption of a common regime variable generating simultaneous regime shifts in all equations of the system, *i.e.* identical break-points for all variables and parameters, and also to restrict the number of regime-dependent parameters.

In empirical research, only some parameters will be conditioned on the state of the Markov chain while the other parameters will be regime invariant. In order to establish a unique notation for each model, we specify with the general $MS(M)$ term the regime-dependent parameters:

- M Markov-switching *mean*,
- I Markov-switching *intercept*,
- A Markov-switching *autoregressive parameters*,
- H Markov-switching *heteroscedasticity*.

The MS-VAR model provides a very flexible framework which allows for heteroscedasticity, occasional shifts, reversing trends, and forecasts performed in a non-linear manner. The model can be easily extended for the presence of cointegration (see Krolzig, 1996, for the statistical analysis of these systems and Krolzig, 2001a, for applications to the business cycle). In business cycle analysis, the focus is on models where the mean (MSM(M)-VAR(p) models) or the intercept term (MSI(M)-VAR(p) models) are subject to occasional discrete shifts creating episodes of economic expansion versus contraction or stagnation. Regime-dependent covariance structures are often required to model changes in the correlation structure or the considered as additional features.

2.3 MS-VAR Models of the Business Cycle

We start by presenting a simple, multivariate extension of Hamilton's original model in (1). Since we are interested in modelling the business cycle, only shifts in the vector of mean growth rates $\boldsymbol{\mu}(s_t)$ are considered:

$$\Delta \mathbf{y}_t - \boldsymbol{\mu}(s_t) = A_1 (\Delta \mathbf{y}_{t-1} - \boldsymbol{\mu}(s_{t-1})) + \dots + A_p (\Delta \mathbf{y}_{t-p} - \boldsymbol{\mu}(s_{t-p})) + \mathbf{u}_t, \quad (6)$$

where the error term $\mathbf{u}_t$ is again Gaussian: $\mathbf{u}_t|s_t \sim \text{NID}(\mathbf{0}, \boldsymbol{\Sigma})$. The MSM(M)-VAR(p) characterizes business cycles as common regime shifts in the stochastic process of some macroeconomic time series. In the context of this paper, those are the growth rates of macroeconomic activity in the euro-zone countries.

MS-VAR models are by no means restricted to the presence of two regimes as assumed by Hamilton (1989). In practice, two-regime models exhibiting episodes of '*recession*' and '*expansion*' may not congruent since, for example, structural change may require to allow for multiple expansionary regimes such as '*high growth*' and '*low growth*' (see, *inter alia*, Krolzig, 2001a). Also, note that without further restrictions on the matrix of transition probabilities, for $M > 2$, the model does not impose a sequential ordering of the regimes (*e.g.*, a rigid phase structure of business cycle). In (6) each regime, say $s_t = m$, is associated with a particular mean vector $\boldsymbol{\mu}_m = (\mu_{1m}, \dots, \mu_{Km})'$ such that $E[\Delta \mathbf{y}_t | s_t = m] = \boldsymbol{\mu}_m$. As a normalization, the regimes are ordered with increasing mean, *i.e.* $\|\boldsymbol{\mu}_1\| < \|\boldsymbol{\mu}_2\|$. Identification requires that for all $m \in \{1, \dots, M\}$ there exist at least one $k \in \{1, \dots, K\}$ such that $\mu_{km} \neq \mu_{1i}$ for all $i \neq m$. Assume, for instance, that the mean growth rates are identical for all countries, $\mu_{km} = \mu_m$ for all k and m . Then, in the case of $M = 2$: $\mu_1 < 0 < \mu_2$ implies that latter regime represents the

expansionary phase and the former the recessionary phase of the *classical* business cycle. Suppose now that $0 < \mu_1 < \mu_2$, then the two regimes correspond to the phases of the *growth* cycle. When the growth rates vary across countries, the distinction between classical and growth cycles can be more difficult. In general, we will require that at least some country-specific mean growth rates are negative during recessions. A stronger condition may demand that a well defined index or aggregate of $\mathbf{y}_t$ contracts during recessions: $E[y_t^a | s_t = 1] = \mathbf{a}'\boldsymbol{\mu}_1 < 0$, where $y_t^a = \mathbf{a}'\mathbf{y}_t$ with $\mathbf{a} > 0$ and $\mathbf{1}'\mathbf{a} = 1$. Similar criteria can be applied to distinguish the phases of the growth cycle, i.e. $\mathbf{a}'\boldsymbol{\mu}_2 > \mathbf{a}'\boldsymbol{\mu}_1 > 0$.

Despite its simplicity, the model is able to capture the non-linear, regime-switching, and the common factor structures of the business cycle simultaneously. The design of the model allows the identification and analysis of the latent euro-zone cycle, while restricting the propagation of country-specific shocks to the vector autoregression. In other words, the hidden Markov process is not informative about the propagation of business cycles across countries. Modelling the ‘*business cycle*’ as a common factor implies that the countries share the same turning points. However, this does not presuppose that all countries are synchronized with the common cycle, i.e. it is feasible that some countries – but not all – are unaffected by shifts in regime: $\mu_{km} = \mu_k$ for all m . Clearly, the model would not be identified if this were true for all k . A more flexible approach can be found in Phillips (1991) and Hamilton and Lin (1996), where some variables might precede others in their cycle or different variables follow completely different Markov processes. Given the relatively large number of countries considered and our special interest in the phenomenon of a *common* euro-zone cycle, introducing separate Markov processes for each country is not very appealing within the modelling strategy outlined here. In other instances, however, it may be appropriate to allow more than one state variable, such that each variable may respond to a specific state variable.¹ It is worth pointing out that if (6) were formulated as an MSI(M)–VAR(p) model, regime shifts would cause lagged responses of the variables in the system, so that some countries would react faster to business cycle transitions than others.

In many regards, Markov-switching VAR models can be considered as rather straightforward multivariate generalizations of the univariate Markov-switching model originally proposed by Hamilton (1989) for the US business cycle. This also helps to explain some of their distinctive attributes: they are foremost statistical models designed to learn the features of the business cycle from the data. As statistical representations of the business cycle, they require minimal theoretical prior information about the relevant macroeconomic structures. The common factor hypothesis is such an identifying assumption: one regime variable drives the business cycle transitions in the variables of the system. Given the minimal input of identifying assumptions regarding the regime generating process, the interpretation of its realization as the ‘*business cycle*’ must be based on similarities with a reference cycle. This may be called an ‘*ex-post identification of the business cycle*’. The other important assumption made in Markov-switching models says that the latent regime variable is generated by a Markov chain. This allows the ‘*reconstruction*’ of the evolution of the regime (e.g., as the time path of the ‘*probability of being in a recession*’) by solving a *signal-extraction problem*. If the business cycle is a common feature of the vector of economic time series, then modelling the system does not only reflect the basic definition of the business cycle, but also enhances the extraction of the common ‘business cycle’ component from the variables. In the next two sections, we explore this issue further by discussing the regime inference in MS-VAR models, its exploitation for the construction of a turning point chronology as well as the properties of resulting regime classification.

¹Considering two-country two-regime models of monthly growth rates of industrial production in the US, the UK, Germany and Japan, Phillips (1991) assumes separate Markov chains for the business cycle of each country. Interestingly, in none of the estimated models, the null hypothesis of perfectly correlated regime shifts could be rejected which supports the presumptions made here.

3 Constructing the Turning Point Chronology

For an estimated MS-VAR model, the construction of a business cycle chronology consists of three major steps:

- i. *Regime inference*: identification of the unobserved regime variable s_t from the information set $\mathbf{Y}_T$
 - Filtering: $\Pr(s_t|\mathbf{Y}_t)$;
 - Smoothing: $\Pr(s_t|\mathbf{Y}_T)$;
 - Predicting: $\Pr(s_{t+h}|\mathbf{Y}_T)$.
- ii. *Construction of the business cycle chronology*
 - Regime classification using a classification rule based on $\Pr(s_t|\mathbf{Y}_T)$: $\hat{s}_t$;
 - Dating the turning points.
- iii. *Control of the regime classification*
 - Uncertainty: $\sum_{s_t \neq \hat{s}_t} \Pr(s_t|\mathbf{Y}_T)$;
 - Reliability: $\|\Pr(s_t|\mathbf{Y}_T) - \Pr(s_t|\mathbf{Y}_t)\|$;
 - Robustness;
 - Parameter constancy.
- iv. *Implications for real-time turning point detection.*

3.1 Regime Inference in MS-VAR Models

The framework for the statistical analysis of MS-VAR models is the state-space form, where the VAR of $\mathbf{y}_t$ conditional on the state variable s_t in equation (6) forms the measurement equation, and the hidden Markov chain in equation (4) is the transition equation of the state variable (see Krolzig, 1997b, ch.2, for more details). In the following, we only give a brief introduction to the calculation of the filtered and smoothed regime probabilities, which are essential for the dating of the business cycle with MS-VAR models.

3.1.1 Filtering

The filtering algorithm is usually associated with Hamilton (1989), but it can be traced back to earlier statistics literature including Baum and Petrie (1966) and Lindgren (1978). The aim of the filter is to infer the probability distribution of the unobserved regime variable s_t given the currently available information set $\mathbf{Y}_t$.

Let $p(\mathbf{y}_t|s_t, \mathbf{Y}_{t-1})$ denote the probability density of observing $\mathbf{y}_t$ conditional on the regime variable s_t , and $\Pr(s_t|\mathbf{Y}_{t-1})$ the transition probability from regime s_{t-1} to s_t . By invoking the law of Bayes, the (posterior) probability $\Pr(s_t|\mathbf{Y}_t)$ of the regime variable s_t conditional on all available information at time t is given by:

$$\Pr(s_t|\mathbf{Y}_t) = \Pr(s_t|\mathbf{y}_t, \mathbf{Y}_{t-1}) = \frac{p(\mathbf{y}_t|s_t, \mathbf{Y}_{t-1}) \Pr(s_t|\mathbf{Y}_{t-1})}{p(\mathbf{y}_t|\mathbf{Y}_{t-1})}, \quad (7)$$

where the predicted regime probability $\Pr(s_t|\mathbf{Y}_{t-1})$ is the *prior* probability of regime s_t given the information set of the previous period and $p(\mathbf{y}_t|\mathbf{Y}_{t-1})$ is the *marginal* density of $\mathbf{y}_t$ conditional on the information set $\mathbf{Y}_{t-1}$. The predicted regime probabilities can be calculated as:

$$\Pr(s_t = j|\mathbf{Y}_{t-1}) = \sum_{i=1}^M \sum_{j=1}^M p_{ij} \Pr(s_{t-1} = i|\mathbf{Y}_{t-1}),$$

which in turn are used in the derivation of the predicted density of $\mathbf{y}_t$:

$$p(\mathbf{y}_t | \mathbf{Y}_{t-1}) = \sum_{j=1}^M p(\mathbf{y}_t, s_t = j | \mathbf{Y}_{t-1}) = \sum_{s_t} p(\mathbf{y}_t | s_t, \mathbf{Y}_{t-1}) \Pr(s_t | \mathbf{Y}_{t-1}). \quad (8)$$

Using (7) the filtered regime probabilities for a sample $\mathbf{Y}_T = (\mathbf{y}_T, \dots, \mathbf{y}_1)$ can be calculated by a forward recursion for $t = 1, \dots, T$ initialized by some estimate of the initial value s_0 of the regime variable. Suppose, for example, $p = 0$ and $M = 2$ where one regime represents ‘recession’ and the other ‘expansion’, then the filter recursion in (7) can be simplified to the following odds ratio of the regimes:

$$\frac{\Pr(\text{‘recession’ at time } t | \mathbf{Y}_t)}{\Pr(\text{‘expansion’ at time } t | \mathbf{Y}_t)} = \frac{p(\mathbf{y}_t | \text{‘recession’}) \Pr(\text{‘recession’ at time } t | \mathbf{Y}_{t-1})}{p(\mathbf{y}_t | \text{‘expansion’}) \Pr(\text{‘expansion’ at time } t | \mathbf{Y}_{t-1})}.$$

The probability of a recession given all available information at time t depends (i) on the relative likelihood of observing $\mathbf{y}_t$ in a recession versus in an expansion and (ii) on the relative likelihood of a recession given the history of $\mathbf{y}_t$, namely the information set $\mathbf{Y}_{t-1}$ available in the previous period.

3.1.2 Smoothing

The filter recursions deliver estimates for s_t , with $t = 1, \dots, T$, based on the available information up to time point t . This is a limited information technique, as we have observations up to $t = T$. Thus, the regime inference can be improved further by using future observations of $\mathbf{y}_t$, in which case the resulting regime probabilities, $\Pr(s_t | \mathbf{Y}_s)$ with $s > t$, are called ‘smoothed’. The smoothing algorithm gives the best estimate of the unobservable state at any point within the sample.

In the following, we discuss how the sample information $\mathbf{Y}_{t+1:T} = (\mathbf{y}'_{t+1}, \dots, \mathbf{y}'_T)'$ neglected so far is incorporated into the full-sample inference about the unobserved regime s_t . The smoothing algorithm proposed by Kim (1994) may be interpreted as a backward filter that starts at the end point, $t = T$, of the previously applied filter. The full-sample smoothed inferences $\Pr(s_t | \mathbf{Y}_T)$ are found by starting from the last output of the filter $\Pr(s_T | \mathbf{Y}_T)$, and then by iterating backward from $t = T - 1$ to $t = 1$.

The algorithm exploits the identity:

$$\begin{aligned} \Pr(s_t | \mathbf{Y}_T) &= \sum_{s_{t+1}} \Pr(s_t, s_{t+1} | \mathbf{Y}_T) \\ &= \sum_{s_{t+1}} \Pr(s_t | s_{t+1}, \mathbf{Y}_T) \Pr(s_{t+1} | \mathbf{Y}_T). \end{aligned} \quad (9)$$

For pure VAR models with Markovian parameter shifts, the probability laws for y_t and s_{t+1} depend only on the current state s_t , but not on its history. Thus, we have:

$$\begin{aligned} \Pr(s_t | s_{t+1}, \mathbf{Y}_T) &\equiv \Pr(s_t | s_{t+1}, \mathbf{Y}_t, \mathbf{Y}_{t+1:T}) \\ &= \frac{p(\mathbf{Y}_{t+1:T} | s_t, s_{t+1}, \mathbf{Y}_t) \Pr(s_t | s_{t+1}, \mathbf{Y}_t)}{p(\mathbf{Y}_{t+1:T} | s_{t+1}, \mathbf{Y}_t)} \\ &= \Pr(s_t | s_{t+1}, \mathbf{Y}_t). \end{aligned}$$

It is, therefore, possible to calculate the smoothed probabilities $\Pr(s_t | \mathbf{Y}_T)$ by getting the last term from the previous iteration of the smoothing algorithm, $\Pr(s_{t+1} | \mathbf{Y}_T)$, while it can be shown that the first term can be derived from the filtered probabilities $\Pr(s_t | \mathbf{Y}_t)$:

$$\begin{aligned} \Pr(s_t | s_{t+1}, \mathbf{Y}_t) &= \frac{\Pr(s_{t+1} | s_t, \mathbf{Y}_t) \Pr(s_t | \mathbf{Y}_t)}{\Pr(s_{t+1} | \mathbf{Y}_t)} \\ &= \frac{\Pr(s_{t+1} | s_t) \Pr(s_t | \mathbf{Y}_t)}{\Pr(s_{t+1} | \mathbf{Y}_t)}. \end{aligned} \quad (10)$$

The recursion is initialized with the final filtered probability vector $\Pr(s_t|\mathbf{Y}_T)$:

$$\Pr(s_t|\mathbf{Y}_T) = \sum_{s_{t+1}=1}^M \frac{\Pr(s_{t+1}|s_t)\Pr(s_t|\mathbf{Y}_t)}{\Pr(s_{t+1}|\mathbf{Y}_t)}\Pr(s_{t+1}|\mathbf{Y}_T). \quad (11)$$

Recursion (11) describes how the additional information $\mathbf{Y}_{t+1:T}$ is used in an efficient way to improve the inference on the unobserved state s_t :

$$\frac{\Pr(s_t|\mathbf{Y}_T)}{\Pr(s_t|\mathbf{Y}_t)} = \sum_{s_{t+1}=1}^M \Pr(s_{t+1}|s_t) \frac{\Pr(s_{t+1}|\mathbf{Y}_T)}{\Pr(s_{t+1}|\mathbf{Y}_t)}. \quad (12)$$

If at time $t + 1$ the full information inference, $\Pr(s_{t+1}|\mathbf{Y}_T)$, coincides with its prediction, $\Pr(s_{t+1}|\mathbf{Y}_t)$, then knowing the future of $\mathbf{y}_t$ does not help to improve the regime inference at time t . Hence the filtering solution $\Pr(s_t|\mathbf{Y}_t)$ is not updated: $\Pr(s_t|\mathbf{Y}_T) = \Pr(s_t|\mathbf{Y}_t)$.

In the case of two regimes, say ‘recession’ ($s_t = 1$) and ‘expansion’ ($s_t = 2$), we have that:

$$\begin{aligned} & \frac{\Pr(\text{‘recession’ at time } t|\mathbf{Y}_T)}{\Pr(\text{‘recession’ at time } t|\mathbf{Y}_t)} \\ &= p_{11} \frac{\Pr(\text{‘recession’ at time } t+1|\mathbf{Y}_T)}{\Pr(\text{‘recession’ at time } t+1|\mathbf{Y}_t)} + p_{12} \frac{\Pr(\text{‘expansion’ at time } t+1|\mathbf{Y}_T)}{\Pr(\text{‘expansion’ at time } t+1|\mathbf{Y}_t)} \\ &= p_{11} \frac{\Pr(\text{‘recession’ at time } t+1|\mathbf{Y}_T)}{\Pr(\text{‘recession’ at time } t+1|\mathbf{Y}_t)} + (1 - p_{11}) \frac{1 - \Pr(\text{‘recession’ at time } t+1|\mathbf{Y}_T)}{1 - \Pr(\text{‘recession’ at time } t+1|\mathbf{Y}_t)}. \end{aligned} \quad (13)$$

It can be shown that iff $p_{11} > p_{21}$, i.e., whenever the probability of staying in a recession is greater than the probability of moving from an expansion to a recession, $\Pr(\text{‘recession’ at time } t+1|\mathbf{Y}_t)$ is a monotonically increasing function in $\Pr(\text{‘recession’ at time } t+1|\mathbf{Y}_t)$. In other words, if new information will make the researcher believe that the probability of a recession in $t + 1$ is greater than expected at time t , she will update her beliefs about the past by increasing the probability that the economy has already been in a recession at time t . Finally, note that the difference between the two measures indicates the amount of revisions in the regime classification, when new observations become available.

3.2 Constructing the Turning Point Chronology

3.2.1 Classification of the Regimes

In Markov-switching models, the classification of the regimes and the dating of the business cycle amounts to assigning every observation $\mathbf{y}_t$ to a regime $\hat{s}_t \in \{1, \dots, M\}$. The rule that is applied here maps $\mathbf{y}_t$ to the regime with the highest smoothed probability:

$$\hat{s}_t = \arg \max_{1, \dots, M} \Pr(s_t | \mathbf{Y}_T) \quad (14)$$

At each point in time, the smoothed regime probabilities are calculated as described in §3.1, i.e. the inference is made using the whole set of data points, and each observation is matched with the most likely regime.

In the simplest case of two regimes, the classification rule proposed rule in (14) simplifies to assigning the observation to the first regime if $\Pr(s_t = 1 | \mathbf{Y}_T) > 0.5$ and to the second if $\Pr(s_t = 1 | \mathbf{Y}_T) < 0.5$, which was proposed in Hamilton (1989). For the purpose of business cycle analysis, this rule might also be applied to multi-regime models ($M > 2$) when, for instance, regime 1 captures ‘recessions’ and the remaining regimes ($s_t = 2, \dots, M$) can be aggregated to the superstate ‘expansion’ with each regime representing different phases of an expansion.

3.2.2 Dating the Business Cycle Turning Points

The regime classification procedure allows dating the turning points of the business cycle by identifying the transitions of the regime variable. Suppose again that there are two-regimes: ‘recession’ ($s_t = 1$) and ‘expansion’ ($s_t = 2$). Then, the peak date denotes the period t just before the beginning of a recession, i.e. $\Pr(\text{‘recession’ at time } t \mid \mathbf{Y}_T) < 0.5$ and $\Pr(\text{‘recession’ at time } t + 1 \mid \mathbf{Y}_T) > 0.5$; the trough is the last period of the recession:

$$\begin{aligned} \text{Peak at } t = \tau &\iff \hat{s}_\tau = 2 \text{ and } \hat{s}_{\tau+1} = 1 \\ \text{Trough at } t = \tau &\iff \hat{s}_\tau = 1 \text{ and } \hat{s}_{\tau+1} = 2. \end{aligned}$$

It would also be feasible to impose the additional condition that each recession and expansion should prevail for at least two quarters, which would add a certain smoothness to the regime classification. A chronology of the turning points of the euro-zone business cycle will be discussed in § 5.5 (see tables 9 and 10).

3.3 Quantifying the Classification Uncertainty

An important advantage of the MS-VAR model is its ability not only to classify observations, but also to quantify the uncertainty associated with the produced business cycle chronology. If we attach the observation at time t to the regime $\hat{s}_t$ according to rule (14), then an appropriate measurement of the classification uncertainty is given by:

$$\frac{M}{M-1} \sum_{i \neq \hat{s}_t} \Pr(s_t = i \mid \mathbf{Y}_T),$$

where $1 - 1/M$ is the maximal uncertainty when all regimes $m = 1, \dots, M$ are possible with the same probability $1/M$. Hence, the proposed measurement is bounded between 0 and 1. Obviously, we get for $M = 2$, that the probability of a wrong classification, which is given by the complementary probability, is normalized to $2 \Pr(s_t \neq \hat{s}_t \mid \mathbf{Y}_T)$.

A different measure of uncertainty is given by the difference of the filtered and smoothed probabilities of a recession, which reports the update of the regime inference as new information becomes available. This is crucial for the reliability of real-time detection of business cycle transitions with MS-VAR models, which we discuss next.

3.4 Real-Time Detecting of Turning Points

The proposed approach supports the identification of business cycle turning points without a lag. The filtered regime probability $\Pr(s_T \mid \mathbf{Y}_T)$, which coincides with the smoothed probability of s_T for the sample $\mathbf{Y}_T$, constitutes the optimal regime inference for given data set. When additional data become available, the regime inference will be updated by the smoothed regime probabilities, $\Pr(s_T \mid \mathbf{Y}_{T+j})$ with $j > 0$. In other words, we can interpret the new datings as a ‘real-time’ classification of the euro-zone business cycle, which only uses the information up to turning point.²

To gauge the reliability of the filtered regime probabilities in identifying new turning points, the resulting regime classification could be compared to the business cycle chronologies derived from the smoothed regime probabilities, which use the full-sample information.

²We still employ the full-sample parameter estimates.

3.5 Predicting the Business Cycle

Detecting recent regime shifts is essential to predict MS-VAR processes. A useful framework for the prediction of future regimes is the VAR(1) representation of the hidden Markov chain, where the discrete state variable s_t is substituted by the unobservable state vector ξ_t consisting of M binary indicator variables:

$$\xi_t = \begin{bmatrix} I(s_t = 1) & \cdots & I(s_t = M) \end{bmatrix}'.$$

Let ξ_t be an $(M \times 1)$ vector whose i th element is unity when $s_t = i$, and zero otherwise. Then, the hidden Markov chain can be written as:

$$\xi_{t+1} = \mathbf{F}\xi_t + v_{t+1}, \quad (15)$$

where v_{t+1} is a martingale difference sequence and $\mathbf{F} = \mathbf{P}'$ is called the ‘transition matrix’. This representation allows the application of the well-established theory of forecasting linear systems to the problem of calculating future regime probabilities (see Krolzig, 1997b). If the Markov chain is ergodic, *i.e.* there does not exist an absorbing state, the process in (15) is stationary and will revert to its mean $E[\xi_t] = \bar{\xi}$, where $\bar{\xi}$ is the vector of ergodic regime probabilities.

Since s_t is unobservable, so is ξ_t . Therefore we have to utilize the filter in (7) again, and work with the vector of filtered regime probabilities $\hat{\xi}_{t|t}$ instead:

$$\hat{\xi}_{t|t} = \begin{bmatrix} \Pr(s_t = 1|\mathbf{Y}_t) & \cdots & \Pr(s_t = M|\mathbf{Y}_t) \end{bmatrix}'.$$

By using the linearity of the transition equation (15), the h -step prediction of ξ_{t+h} follows as:

$$\hat{\xi}_{t+h|t} = \mathbf{F}^h \hat{\xi}_{t|t}. \quad (16)$$

The predictive value of the regime inference, at the time when the prediction is made, depends positively on the persistence of the regime generating process. Under ergodicity, the regime probabilities converge to the unconditional ones:

$$\lim_{h \rightarrow \infty} \mathbf{F}^h \hat{\xi}_{t|t} = \bar{\xi}.$$

For a two regime model, these ergodic probabilities are given by:

$$\bar{\xi}_1 = \frac{p_{21}}{p_{12} + p_{21}} \text{ and } \bar{\xi}_2 = \frac{p_{12}}{p_{12} + p_{21}}.$$

While the future distribution of the regimes is well defined, the model is less suited to forecast future turning points. In two regime model of § 2.1, ‘peaks’ (transitions from regime 2 to 1) and ‘troughs’ (transitions from regime 1 to 2) will on average occur with same probability:

$$\Pr(s_t = 1, s_{t+1} = 2) = \bar{\xi}_1 p_{12} = \frac{p_{21} p_{12}}{p_{12} + p_{21}} = \bar{\xi}_2 p_{21} = \Pr(s_t = 2, s_{t+1} = 1).$$

A related question is when the next turning point is likely to occur. Therefore it is useful to know the average duration h of ‘expansions’ and ‘contractions’, which is given by:

$$E[h_1] = \frac{1}{1 - p_{12}} \text{ and } E[h_2] = \frac{1}{1 - p_{21}}.$$

Due to the stationary VAR(1) representation of an ergodic Markov chain, MS-VAR processes have short memory. The longer the forecast horizon, the better the linear approximation of the optimal predictor and the greater the potential unpredictability of (detrended) economic time series (see Krolzig, 2003b, for further details on predicting MS-VAR processes). Finally, it is worth noting that the MS-VAR’s ability of forecasting business cycle transitions can be enhanced further by making the transition probabilities time-varying (see, *inter alia*, Diebold, Lee and Weinbach, 1994).

4 Properties of the Regime Classification Rule

In the following, we analyze the properties of the regime classification rule derived in the previous section. We are particularly interested in the dependence of the regime classification on (i) our beliefs regarding the state of the economy in the previous period and (ii) the current realization of the variables of interest. For sake of simplicity, we restrict our analysis to the ‘*real-time*’ classification problem (filtering) and ignore the effects of future observations. We also abstract from estimation uncertainty by analyzing the situation when the true data generating process is an MS-VAR and the parameters are known. First, we consider a univariate Markov-switching model, and then the system case.

4.1 Univariate Markov-Switching Models

4.1.1 Analytics

The model we analyze here is kept general by defining it in terms of the conditional density of the variable of interest, y_t , in the two regimes, which in the context of business cycle analysis are supposed to represent recessions and expansions.

Regime 1 (*Recession*):

$$p(y_t | \mathbf{Y}_{t-1}, s_t = 1) = \frac{1}{\sigma_1} \phi\left(\frac{y_t - \mu_{1t}}{\sigma_1}\right) = f_1(y_t | \mathbf{Y}_{t-1}).$$

Regime 2 (*Expansion*):

$$p(y_t | \mathbf{Y}_{t-1}, s_t = 2) = \frac{1}{\sigma_2} \phi\left(\frac{y_t - \mu_{2t}}{\sigma_2}\right) = f_2(y_t | \mathbf{Y}_{t-1}).$$

where $\mu_{2t} > \mu_{1t}$ with μ_{2t} being positive, and μ_{1t} being negative if regime 1 is supposed to represent contractions in economic activity rather than low growth episodes; $\phi(\cdot)$ is the standard normal pdf.

As discussed in § 3.2, the observation of y_t will be classified as belonging to the regime 1, iff:

$$\frac{\Pr(s_t = 1 | \mathbf{Y}_t)}{\Pr(s_t = 2 | \mathbf{Y}_t)} = \frac{f_1(y_t | \mathbf{Y}_{t-1}) \Pr(s_t = 1 | \mathbf{Y}_{t-1})}{f_2(y_t | \mathbf{Y}_{t-1}) \Pr(s_t = 2 | \mathbf{Y}_{t-1})} > 1.$$

Under the assumed normality of $f_1(y_t | \mathbf{Y}_{t-1})$ and $f_2(y_t | \mathbf{Y}_{t-1})$, it can be shown that a linear discrimination rule emerges when the two regimes are homoscedastic, $\sigma_1^2 = \sigma_2^2 = \sigma^2$:

$$y_t < \frac{1}{2}(\mu_{2t} + \mu_{1t}) + \frac{\sigma^2}{\mu_{2t} - \mu_{1t}} \ln \pi, \quad (17)$$

where $\pi = (\Pr(s_t = 1 | \mathbf{Y}_{t-1}) / [1 - \Pr(s_t = 1 | \mathbf{Y}_{t-1})])$ is the logit of the predicted probability of a recession formed at time $t - 1$. Whenever y_t is less than the specified threshold, y_t is classified as belonging to the regime 1. The lower y_t , the higher the chance that the economy is in the state of a recession. The threshold depends on the parameters of the model and the predicted regime probabilities.

Suppose now that the two regimes are heteroscedastic: $\sigma_1^2 \neq \sigma_2^2$. Then, the condition for a recession is quadratic in y_t :

$$(\omega - 1)y_t^2 - 2(\omega\mu_{2t} - \mu_{1t})y_t > -(\omega\mu_{2t}^2 - \mu_{1t}^2) + \sigma_1^2(\ln \omega - 2 \ln \pi), \quad (18)$$

where $\omega = \sigma_1^2 / \sigma_2^2$. If $\omega < 1$, the recessions are declared if y_t is in the interior region (y_t^-, y_t^+) with the threshold values:

$$y_t^\pm = \frac{\omega\mu_2 - \mu_1}{\omega - 1} \pm \frac{\sqrt{\omega}}{\omega - 1} \sqrt{(\omega - 1)\sigma^2 \ln \omega - 2(\omega - 1)\sigma^2 \ln \pi + (\mu_2 - \mu_1)^2}$$

Otherwise, recessions are found in the outer regions of $\mathbb{R}$. This highlights the danger of making both the first and second conditional moment of y_t regime dependent. Great care is required to ensure that the regime classification is economically meaningful (see Example 4 for a bivariate illustration).

4.1.2 Example 1: Hamilton Model

We evaluate the properties of the regime classification procedure in a simplified variant of Hamilton's business cycle model discussed in § 2.1:

Regime 1 (*Recession*):

$$(y_t \mid \mathbf{Y}_{t-1}, s_t = 1) \sim \mathcal{N}(-0.4, 0.5^2).$$

Regime 2 (*Expansion*):

$$(y_t \mid \mathbf{Y}_{t-1}, s_t = 2) \sim \mathcal{N}(+1.2, 0.5^2).$$

Markov chain (*Transition probabilities*):

$$p_{11} = \Pr(s_t = 1 \mid s_{t-1} = 1) = 0.7,$$

$$p_{22} = \Pr(s_t = 2 \mid s_{t-1} = 2) = 0.9.$$

Expansions are more persistent than recessions. For the transition probabilities given, the ergodic probability of a recession is 0.25.

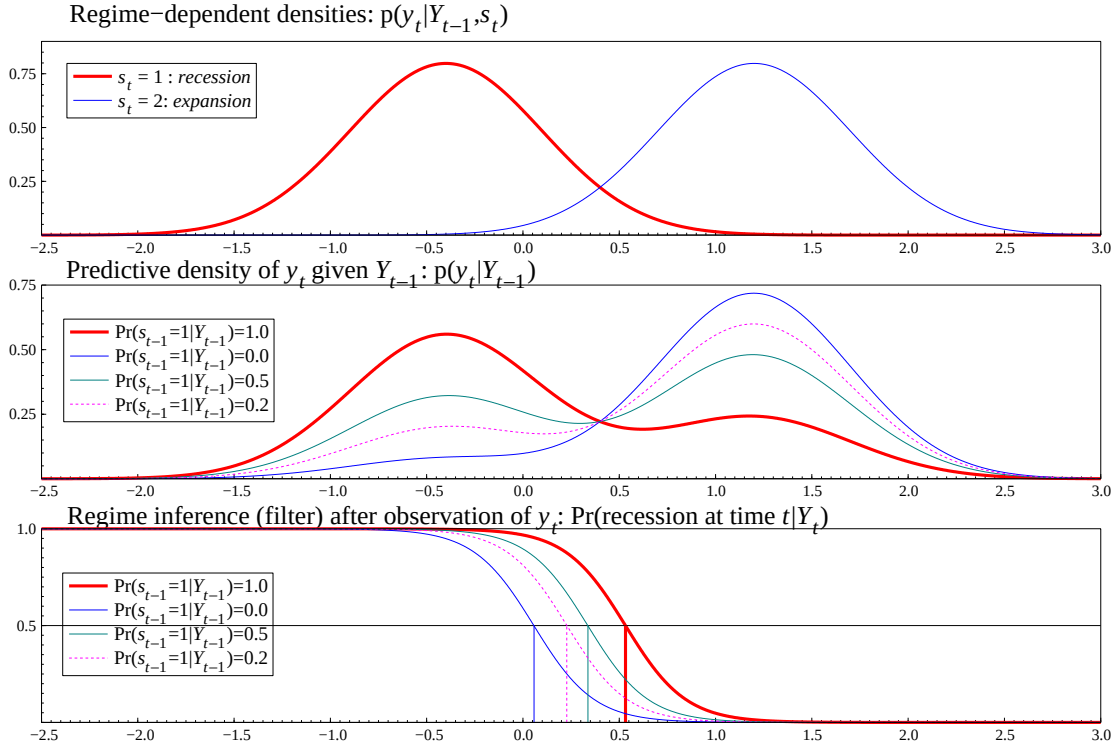


Figure 1 Regime inference in univariate Markov-switching models.

The mechanics of the result in (17) are illustrated in Figure 1. The first panel shows the conditional density of the variable of interest, y_t , in recessions ($s_t = 1$) and in expansions ($s_t = 2$). Suppose now that the regime probabilities at time $t - 1$ are known. Then the predicted regime probabilities,

$\Pr(s_t|Y_{t-1})$, result as follows:

$$\begin{aligned}\Pr(s_t = 1 | \mathbf{Y}_{t-1}) &= 0.7 \Pr(s_{t-1} = 1 | \mathbf{Y}_{t-1}) + 0.1 \Pr(s_{t-1} = 2 | \mathbf{Y}_{t-1}), \\ \Pr(s_t = 2 | \mathbf{Y}_{t-1}) &= 0.3 \Pr(s_{t-1} = 1 | \mathbf{Y}_{t-1}) + 0.9 \Pr(s_{t-1} = 2 | \mathbf{Y}_{t-1}),\end{aligned}$$

where we used that $\Pr(s_t = j | \mathbf{Y}_{t-1}) = \sum_{i=1}^M p_{ij} \Pr(s_{t-1} = i | \mathbf{Y}_{t-1})$.

The predicted density of y_t is given by equation (8) as:

$$p(y_t | \mathbf{Y}_{t-1}) = \sum_{j=1}^M \Pr(s_t = j | \mathbf{Y}_{t-1}) f_j(y_t | \mathbf{Y}_{t-1}),$$

which is a mixture of normals. In the second panel of Figure 1, the predicted density is plotted for different values of the filtered probability of a recession in the previous period, $\Pr(s_{t-1} = 1 | \mathbf{Y}_{t-1}) = w_{1t}$. Those determine the weights of the two Gaussian densities of y_t conditioned on s_t :

$$p(y_t | \mathbf{Y}_{t-1}) = [0.1 + 0.6w_{1t}] f_1(y_t | \mathbf{Y}_{t-1}) + [0.9 - 0.6w_{1t}] f_2(y_t | \mathbf{Y}_{t-1}).$$

The third panel shows the resulting filtered probability of a recession ($s_t = 1$) given the observed value of y_t according to (7). Finally notice how the classification rule proposed in (14) implies a unique threshold y^* such that for $y_t < y^*$ ($y_t \geq y^*$), the econometrician would conclude that the economy is in the state of a recession (expansion).

4.2 Regime Classification in an MS-VAR Model

4.2.1 Analytics

We now extend the analysis of the regime classification in Markov-switching models, applied so far to univariate models, to the vector case. Consider the following two-regime MS-VAR with:

Regime 1 (*Recession*):

$$(\mathbf{y}_t | \mathbf{Y}_{t-1}, s_t = 1) \sim \mathcal{N}(\boldsymbol{\mu}_{1t}, \boldsymbol{\Sigma}_1);$$

Regime 2 (*Expansion*):

$$(\mathbf{y}_t | \mathbf{Y}_{t-1}, s_t = 2) \sim \mathcal{N}(\boldsymbol{\mu}_{2t}, \boldsymbol{\Sigma}_2),$$

where $\boldsymbol{\mu}_{2t} > \boldsymbol{\mu}_{1t}$ is the conditional mean of $\mathbf{y}_t$.

As in § 4.1.1, the observation of $\mathbf{y}_t = (y_{1t}, \dots, y_{Kt})'$ will be classified as belonging to the regime 1 iff the likelihood of regime 1 is greater than the likelihood of regime 2:

$$\frac{\Pr(s_t = 1 | \mathbf{Y}_t)}{\Pr(s_t = 2 | \mathbf{Y}_t)} = \frac{f_1(\mathbf{y}_t | \mathbf{Y}_{t-1}) \Pr(s_t = 1 | \mathbf{Y}_{t-1})}{f_2(\mathbf{y}_t | \mathbf{Y}_{t-1}) \Pr(s_t = 2 | \mathbf{Y}_{t-1})} > 1.$$

Under normality of $f_1(\mathbf{y}_t | \mathbf{Y}_{t-1})$ and $f_2(\mathbf{y}_t | \mathbf{Y}_{t-1})$, parametric conditions can be derived for the values of $\mathbf{y}_t$ leading to the conclusion that the economy is stuck in a recession. If the two regimes are homoscedastic, i.e. $\boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_2$, it can be shown that again a linear discrimination rule results (see Krolzig, 2003a, for details):

$$(\boldsymbol{\mu}_{2t} - \boldsymbol{\mu}_{1t})' \boldsymbol{\Sigma}^{-1} \mathbf{y}_t < \frac{1}{2} (\boldsymbol{\mu}_{2t}' \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}_{2t} - \boldsymbol{\mu}_{1t}' \boldsymbol{\Sigma}^{-1} \boldsymbol{\mu}_{1t}) + \ln \pi, \quad (19)$$

where $\pi = \Pr(s_t = 1 | \mathbf{Y}_{t-1}) / [1 - \Pr(s_t = 1 | \mathbf{Y}_{t-1})]$. For bivariate processes, $K = 2$, with a diagonal variance-covariance matrix, i.e. $\sigma_{kl} = 0$ for $k \neq l$, the rule can be further simplified to:

$$\begin{aligned} \mathbf{b}'_t \mathbf{y}_t &< c_t, \\ \text{with: } \mathbf{b}_t &= \left[\sigma_1^{-2} (\mu_{1.2t} - \mu_{1.1t}), \sigma_2^{-2} (\mu_{2.2t} - \mu_{2.1t}) \right]', \\ c_t &= \frac{1}{2} \left(\frac{\mu_{1.2t}^2 - \mu_{1.1t}^2}{\sigma_1^2} + \frac{\mu_{2.2t}^2 - \mu_{2.1t}^2}{\sigma_2^2} \right) + \ln \left(\frac{\Pr(s_t = 1 | \mathbf{Y}_{t-1})}{1 - \Pr(s_t = 1 | \mathbf{Y}_{t-1})} \right). \end{aligned} \quad (20)$$

The linearity of the divide between recessions and expansions can be easily spotted in the examples given in Figures 4 and 7, which will be discussed next. For a heteroscedastic error process, $\Sigma_1 \neq \Sigma_2$, the condition for a recession is quadratic in $\mathbf{y}_t$; this will be investigated further in the fourth example (see, also, Figure 10).

It is worth remembering that the regime classification rule (19) for MS-VAR models is independent of the weight of any country in the index. Indeed, scaling one of the countries would result in exactly the same regime classification. However, note that larger countries tend to be associated with less noisy time series than smaller countries. Fluctuations in the former countries will deliver clearer signals and, therefore, will usually have a stronger impact on the regime probabilities. Furthermore, the stronger the country is synchronized with the common business cycle and the more pronounced the cycle affects the country's economy; the higher is the weight of the country in the discriminator function.

This has serious implications for business cycle analysis with aggregated data (see Krolzig, 2003a). The question is whether the aggregation of the country data $\mathbf{y}_t = (y_{1t}, \dots, y_{Kt})'$ to an index y_t^a affects the efficiency of the detection of business cycle turning points, when $\mathbf{y}_t$ is generated by an MS-VAR as defined in (6). In other words, can the aggregate y_t^a be a sufficient statistic for $\hat{s}_t$? It can be shown that the answer to this question depends on:

$$\Pr(s_t | y_t^a) = \Pr(s_t | \mathbf{Y}_t), \quad (21)$$

which is investigated in Krolzig (2003a). In the case of a heteroscedastic error term, any linear aggregation of the data $y_t^a = \mathbf{a}' \mathbf{y}_t$ must clearly be inefficient.

4.2.2 Example 2: Bivariate Markov-Switching Model

The regime classification in MS-VAR models is illustrated by three variations of a bivariate MS-VAR process, which generalizes the univariate business cycle model considered earlier. We can think here of two countries driven by the same business cycle, such that y_{kt} denotes the growth rate of output in country $k \in \{1, 2\}$. We assume that the distribution of $\mathbf{y}_t = (y_{1t}, y_{2t})'$ is Gaussian in both regimes, and that output contracts by 0.4 percent per quarter in *recessions* and grows by 1.2 percent in *expansions*. The standard deviation of output growth is 0.5 percent in both regimes.

Regime 1 (*Recession*):

$$(\mathbf{y}_t | s_t = 1) \sim \mathcal{N} \left(\begin{bmatrix} -0.4 \\ -0.4 \end{bmatrix}, \begin{bmatrix} 0.5^2 & 0 \\ 0 & 0.5^2 \end{bmatrix} \right).$$

Regime 2 (*Expansion*):

$$(\mathbf{y}_t | s_t = 2) \sim \mathcal{N} \left(\begin{bmatrix} 1.2 \\ 1.2 \end{bmatrix}, \begin{bmatrix} 0.5^2 & 0 \\ 0 & 0.5^2 \end{bmatrix} \right).$$

The hidden *Markov chain* generating the regimes is chosen to reflect the properties of the US business cycle as found in Hamilton (1989):

$$\begin{aligned} p_{11} &= \Pr(s_t = 1 \mid s_{t-1} = 1) = 0.7, \\ p_{22} &= \Pr(s_t = 2 \mid s_{t-1} = 2) = 0.9. \end{aligned}$$

Figure 2 plots the probability density function (pdf) of $\mathbf{y}_t$ in *recessions*, $s_t = 1 : f_1(\mathbf{y}_t | \mathbf{Y}_{t-1})$, and in *expansions*, $s_t = 2 : f_2(\mathbf{y}_t | \mathbf{Y}_{t-1})$. Given the state of the business cycle, the density of y_{1t} and y_{2t} is simply Gaussian with mean vector $\boldsymbol{\mu}_1$ or $\boldsymbol{\mu}_2$, and variance matrix $\boldsymbol{\Sigma}$.

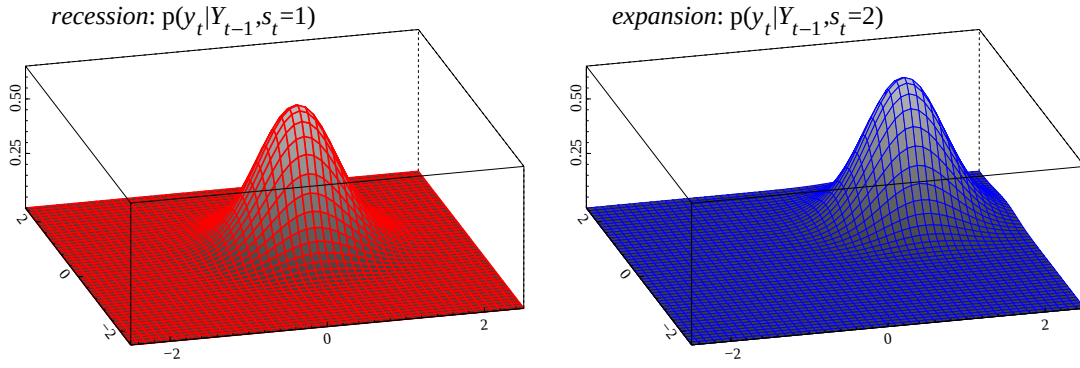


Figure 2 Probability density of $\mathbf{y}_t$ in *recessions* and *expansions*.

Since the regime itself is unobservable, our knowledge of the distribution of $\mathbf{y}_t$ given the information available at time $t - 1$ is in fact much more limited. The predicted density of $\mathbf{y}_t$ plotted in Figure 3 is a mixture of normals:

$$p(\mathbf{y}_t \mid \mathbf{Y}_{t-1}) = w_{1t} f_1(\mathbf{y}_t | \mathbf{Y}_{t-1}) + (1 - w_{1t}) f_2(\mathbf{y}_t | \mathbf{Y}_{t-1}),$$

where w_{1t} denotes the weight attached to conditional pdf of regime 1 at time t . The weight w_{1t} of regime 1 is given by its predicted regime probability, $\Pr(s_t = 1 \mid \mathbf{Y}_{t-1})$, which is determined by the regime inference at time and the structure of the matrix of transition probabilities:

$$\begin{aligned} \Pr(s_t = 1 \mid \mathbf{Y}_{t-1}) &= p_{11} \Pr(s_{t-1} = 1 \mid \mathbf{Y}_{t-1}) + p_{21} \Pr(s_{t-1} = 2 \mid \mathbf{Y}_{t-1}) \\ &= p_{21} + (p_{11} - p_{21}) \Pr(s_{t-1} = 1 \mid \mathbf{Y}_{t-1}). \end{aligned}$$

Using (7), we can use this information to calculate the filtered probability of the regimes – shown in Figure 4 for regime 2:

$$\Pr(s_t = 2 | \mathbf{y}_t, \mathbf{Y}_{t-1}) = \frac{(1 - w_{1t}) f_2(\mathbf{y}_t | \mathbf{Y}_{t-1})}{w_{1t} f_1(\mathbf{y}_t | \mathbf{Y}_{t-1}) + (1 - w_{1t}) f_2(\mathbf{y}_t | \mathbf{Y}_{t-1})}.$$

The $\Pr(s_t = 2 | \mathbf{y}_t, \mathbf{Y}_{t-1}) = 0.5$ isolate, which gives the classification into recessions and contractions, is a linear function of y_{1t} and y_{2t} .

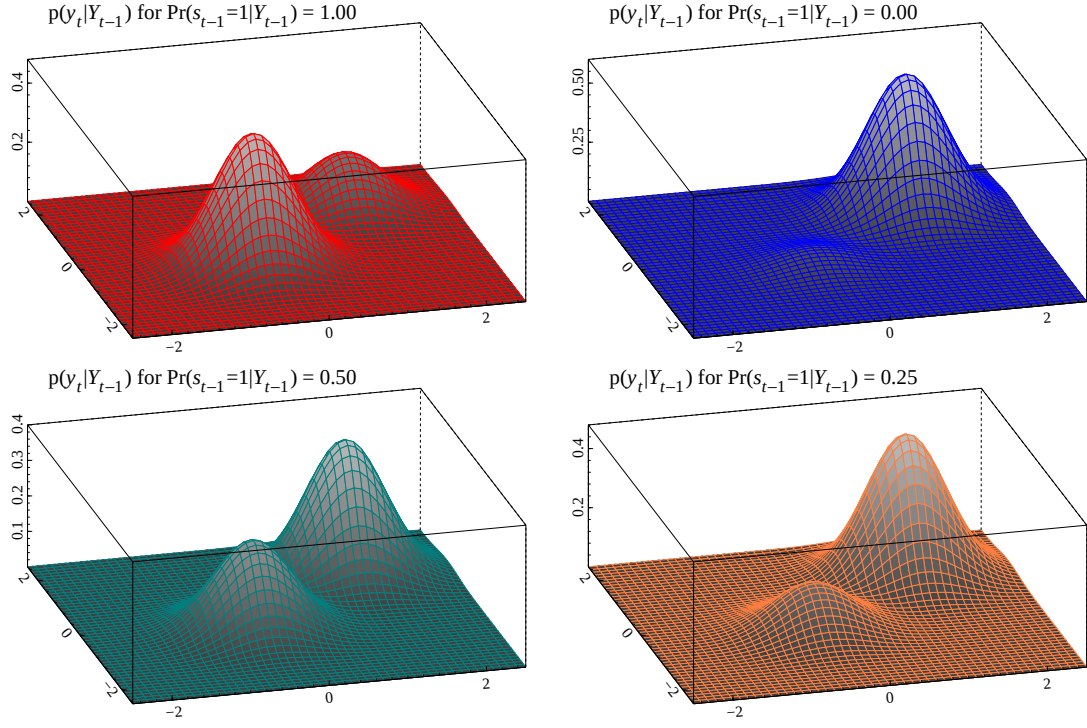


Figure 3 Predicted density of $\mathbf{y}_t$ given $\mathbf{Y}_{t-1}$ and the probability of *recessions* at time $t - 1$.

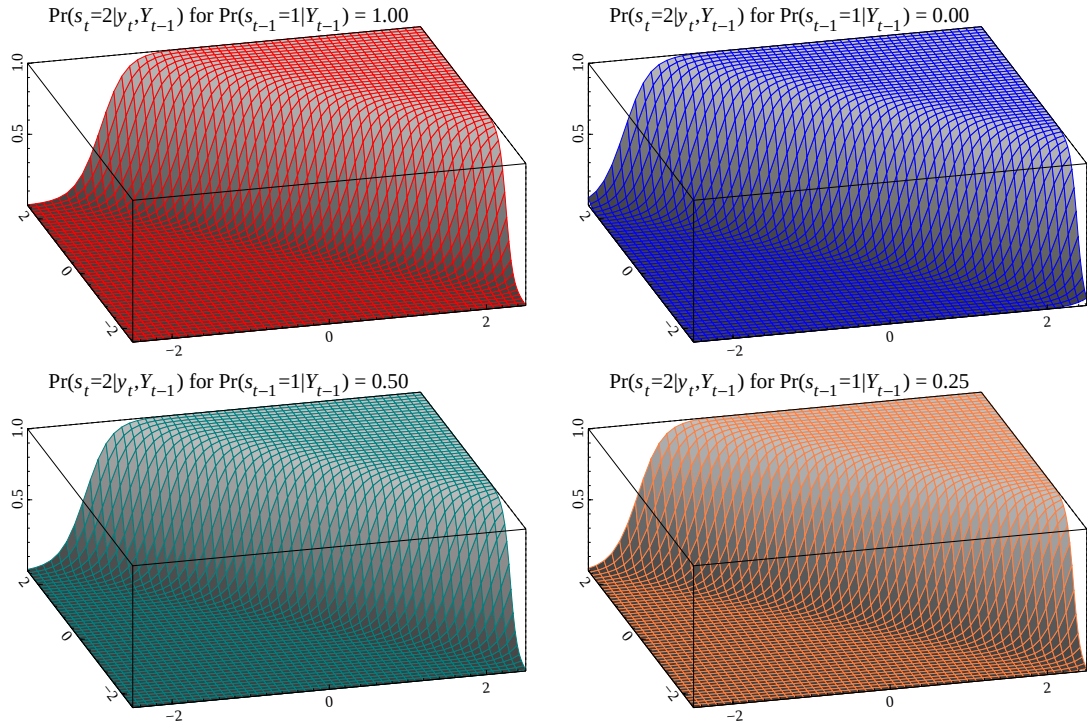


Figure 4 Filtered probability of regime 2 (*expansion*) inferred from the observation of $\mathbf{y}_t$.

4.2.3 Example 3: Lack of Business Cycle Synchronization

Here, we alternate Example 2 by limiting the business cycle to the first variable:

$$\mu_1 = \begin{bmatrix} -0.4 \\ 1.2 \end{bmatrix}, \quad \mu_2 = \begin{bmatrix} 1.2 \\ 1.2 \end{bmatrix}.$$

The same calculations as in the previous example apply. Since the mean growth rate of the second variable is unaffected by shifts in the regime variable s_t , we have that the unconditional pdf of y_{1t} and y_{2t} shown in Figure 5, just shifts in the horizontal direction whenever there is a transition in the state of the business cycle. The resulting predicted density depicted in Figure 6 is clearly seen to be bimodal with both modes lying on a horizontal line.

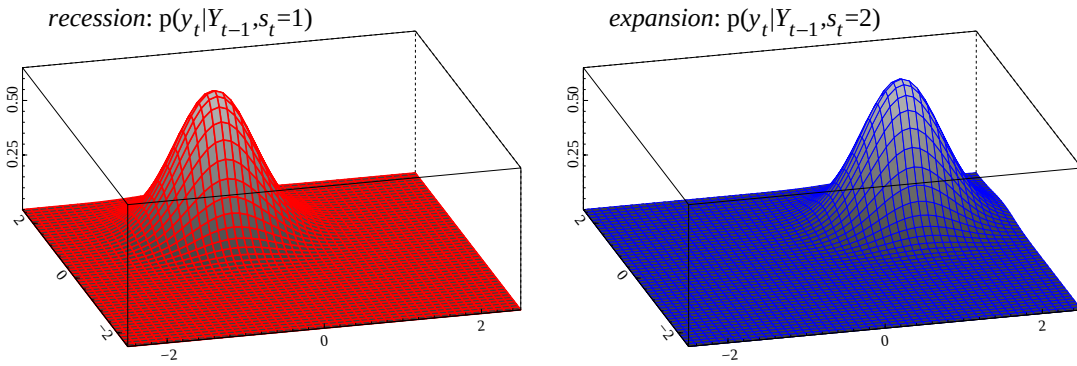


Figure 5 Probability density of y_t in *recessions* and *expansions* for $\mu_{1,1} = \mu_{1,2}$.

Figure 7 again plots the filtered probability of an *expansion* (regime 2) at time t conditional on the probability of a *recession* (regime 1) at time $t - 1$ and the realizations of y_{1t} and y_{2t} . We are again interested in what we can learn about the state of business cycle from the observed values of y_{1t} and y_{2t} . It turns out that y_{2t} does not contribute to the understanding of the underlying state of the economy. The reason for this is that since y_{2t} does not depend on s_t , observing y_{2t} can not reveal any information about the current realization of s_t :

$$\Pr(s_t | y_{1t}, y_{2t}, \mathbf{Y}_{t-1}) = \Pr(s_t | y_{1t}, \mathbf{Y}_{t-1}).$$

Hence, modelling y_{2t} does not contribute to the solution of the signal extraction problem. The regime inference just depends on y_{1t} in the same way as in the simplified Hamilton model considered in Example 1.

4.2.4 Example 4: Regime-Dependent Heteroscedasticity

This final example highlights the potential dangers in interpreting Markov-switching models as representations of the business cycle when they are characterized by time-varying volatility (regime-dependent variances).³ The MS-VAR here differs from the model in the second example by allowing for regime-dependent heteroscedasticity:

$$\Sigma_1 = \begin{bmatrix} 0.5^2 & 0 \\ 0 & 0.5^2 \end{bmatrix}, \quad \Sigma_2 = \begin{bmatrix} 0.8^2 & 0 \\ 0 & 0.8^2 \end{bmatrix}.$$

³Similar problems can result from making the VAR coefficients regime dependent.

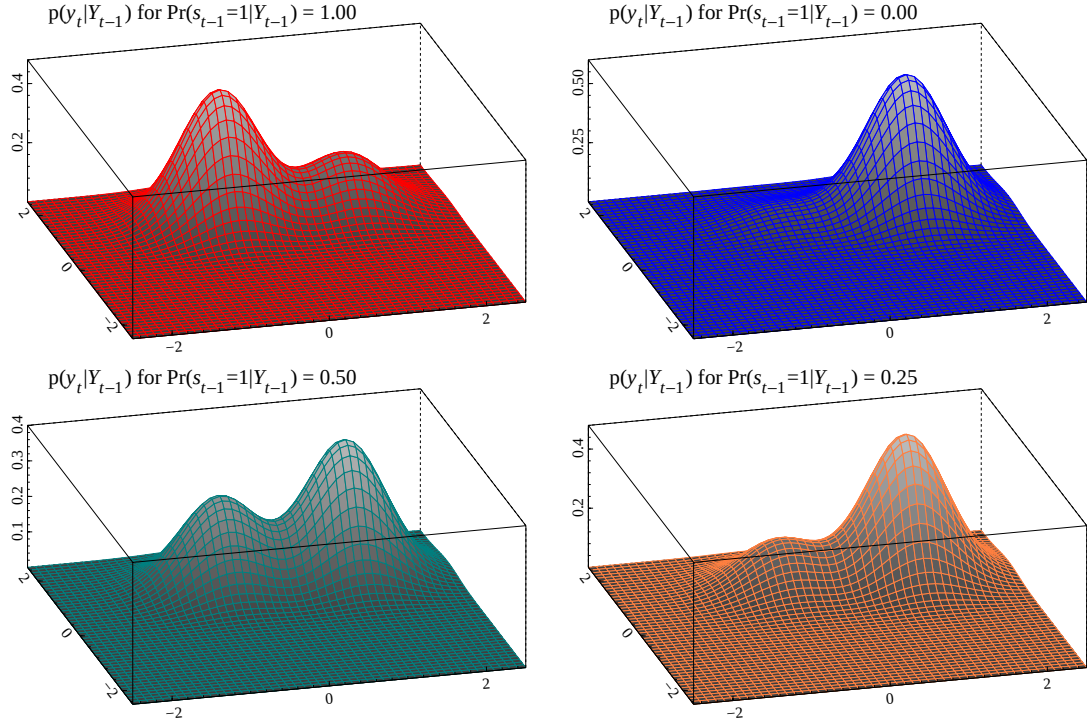


Figure 6 Predicted density of y_t given Y_{t-1} and the probability of *recessions* at time $t - 1$ for $\mu_{1.1} = \mu_{1.2}$.

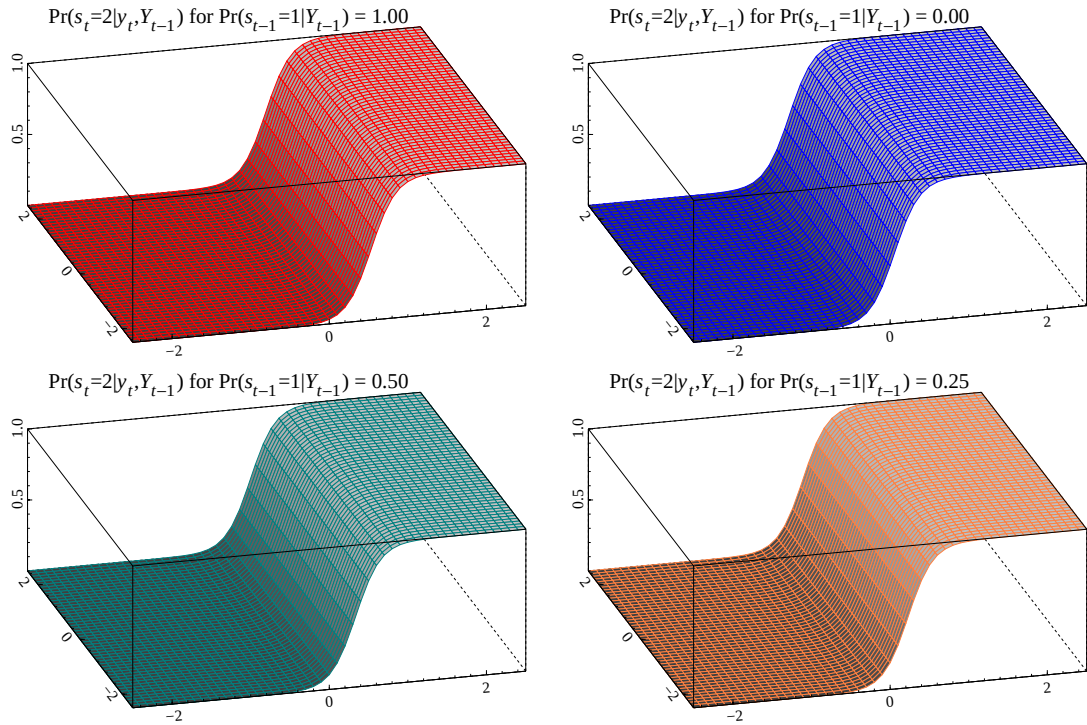


Figure 7 Filtered probability of regime 2 (*expansion*) inferred from the observation of y_t for $\mu_{1.1} = \mu_{1.2}$.

So the expansionary regime is characterized by a higher variance. The mean vectors are unchanged:

$$\boldsymbol{\mu}_1 = \begin{bmatrix} -0.4 \\ 1.2 \end{bmatrix}, \quad \boldsymbol{\mu}_2 = \begin{bmatrix} -0.4 \\ 1.2 \end{bmatrix}.$$

The conditional density of y_{1t} and y_{2t} in the two regimes is depicted in Figure 8. If the regime were known, the conditional variance of y_t would be higher in recessions than in expansions. This is, however, not so trivial when we have to rely on statistical inference regarding the unobserved regime variable.

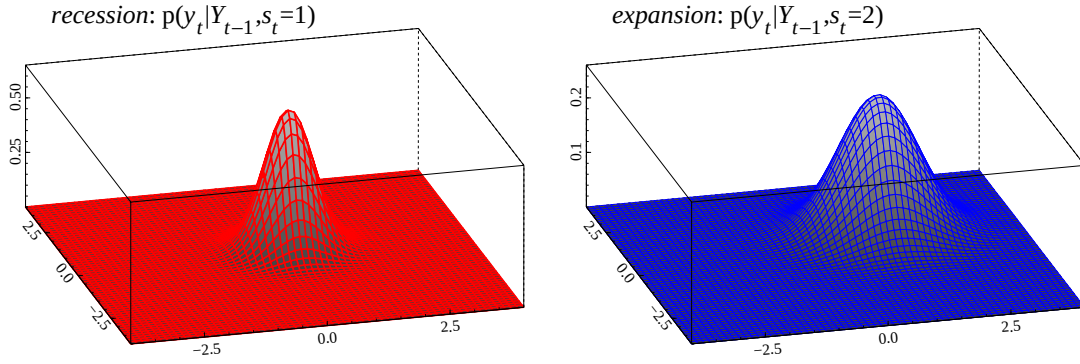


Figure 8 Probability density of y_t in *recessions* and *expansions* for $\Sigma_2 > \Sigma_1$.

The predicted density of y_t given its history and the probability of regime 1 at time $t - 1$ is shown in Figure 9. The pdf is bimodal, with the weight of the two conditional densities being determined by $w_{1t} = \Pr(s_t = 1 \mid Y_{t-1})$. So the conditional variance of y_{1t} and y_{2t} is given by:

$$\begin{aligned} \text{Var}[y_{kt} \mid \mathbf{Y}_{t-1}] &= \mathbb{E}_{s_t} \{ \text{Var}[y_{kt} \mid s_t, \mathbf{Y}_{t-1}] \} + \text{Var}_{s_t} \{ \mathbb{E}[y_{kt} \mid s_t, \mathbf{Y}_{t-1}] \} \\ &= w_{1t}\sigma_1^2 + (1-w_{1t})\sigma_2^2 + w_{1t}(\mu_1 - \mathbb{E}[y_{kt} \mid \mathbf{Y}_{t-1}])^2 + (1-w_{1t})(\mu_2 - \mathbb{E}[y_{kt} \mid \mathbf{Y}_{t-1}])^2 \\ &= \sigma_2^2 + (\sigma_1^2 - \sigma_2^2)w_{1t} + (\mu_1 - \mu_2)^2(1 - w_{1t})w_{1t} \\ &= 0.64 - 2.56w_{1t} + 2.17w_{1t}^2. \end{aligned}$$

It is highest for $\Pr(s_t = 1 \mid \mathbf{Y}_{t-1}) \approx 0.424$, or in other words when the available information at time $t - 1$ is inconclusive regarding the current state of the economy: $\Pr(s_{t-1} = 1 \mid \mathbf{Y}_{t-1}) \approx 0.54$. This is expressed in Figure 9 in the widespread distribution of probability mass for $\Pr(s_{t-1} = 1 \mid \mathbf{Y}_{t-1}) = 0.5$.

The signal extraction problem has become more complicated when compared to the previous examples. Due to the regime-dependent heteroscedasticity, regime 1 exhibits not only lower mean growth rates than regime 2, but also a lower variance. In general, negative growth rates signal *recessions* (regime 1). However, if the observed growth rates of y_{1t} and y_{2t} become increasingly low (*i.e.* negative), the filter reinterprets the situation: while y_t appears to be an outlier from the perspective of regime 1, it is still quite likely under regime 2 due to its greater variance. This in turn makes regime 2 look more likely to prevail. The outcome of the filter is recorded in Figure 10, which displays the probability of regime 2. It is worth remembering that great care is needed when interpreting the business cycle features of an MS-VAR model with switching variance or switching coefficients.

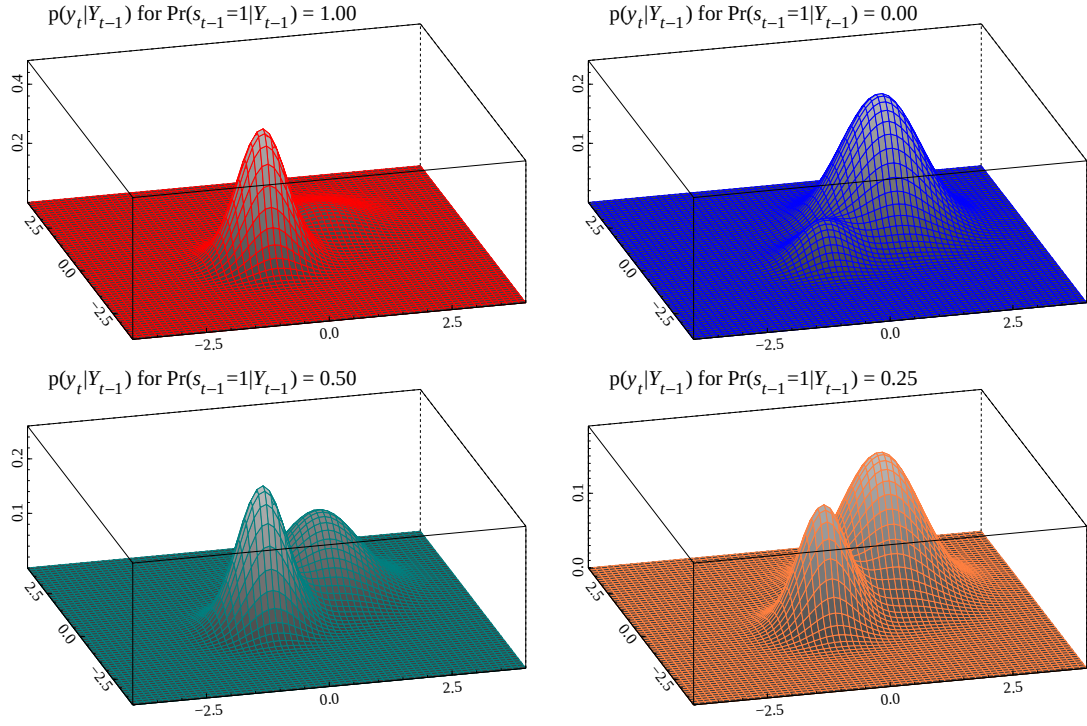


Figure 9 Predicted density of y_t given $\mathbf{Y}_{t-1}$ and the probability of *recessions* at time $t - 1$ for $\Sigma_2 > \Sigma_1$.

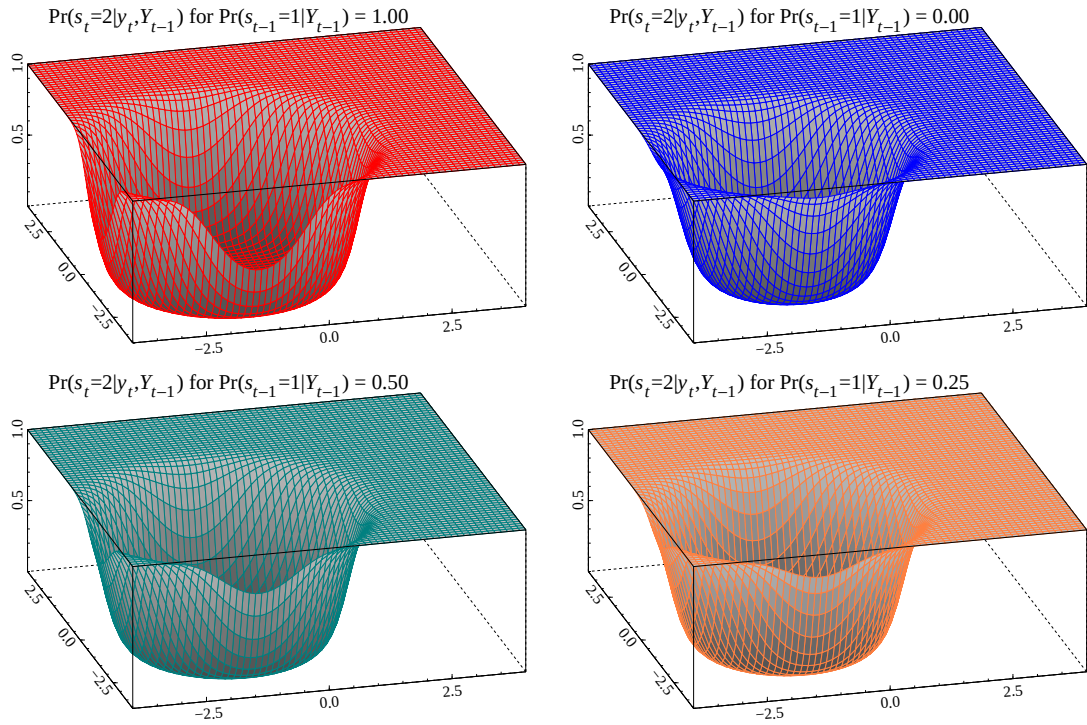


Figure 10 Filtered probability of regime 2 (*expansion*) inferred from the observation of y_t for $\Sigma_2 > \Sigma_1$.

4.3 Summary

The results derived here have decisive implications for the detection of business cycle turning points:

- i. Importance of parsimony and meaningfulness of the regimes.
- ii. General inefficiency of aggregation.
- iii. The stronger the country is synchronized with the common business cycle and the more pronounced the cycle affects the country's economy, the higher is the weight of the country in the regime classification.
- iv. Regime classification is independent of the size of the countries or their weight in an index or aggregate (such as euro-zone GDP).

The importance of these issues will become evident in the following empirical analysis of the euro-zone business cycle

5 The Euro-Zone Business Cycle

Throughout in this paper, we emphasized the importance of modelling systems for the construction of turning point chronologies of the euro-zone business cycle. In § 2 we argued that the euro-zone business cycle constitutes a common feature of the macroeconomic dynamics in the EMU member states. In contrast to the analyses of aggregated euro-zone data, this notion is reflected in Markov-switching vector autoregressive models of macroeconomic activity in euro-zone countries. The results derived in § 4 provided further reasons for analyzing systems instead of aggregates, foremost the missing invariance of the regime classification under aggregation. In this section we will first discuss various empirical Markov-switching models of the European (EU/EMU) business cycle proposed in the literature. We then present the empirical modelling approach, which will be applied to three different statistical measures of macroeconomic activity in EMU member states. Finally, we present the empirical results of this paper derived from the estimated Markov-switching vector autoregressive models of the euro-zone business cycle.

5.1 Markov-Switching Studies of the European Business Cycle

Recent years have seen an increasing attempt to investigate international business cycles with modern non-linear time series models. First contributions to the analysis of international business cycles with Markov-switching models were made by Phillips (1991) and Filardo and Gordon (1994). With the Treaty of Maastricht, the focus shifted towards the analysis of business cycle synchronization in the European Union, and later in the euro zone. Table 1 gives an overview on the recent literature on Markov-switching models of the European Business Cycle.

Artis *et al.* (2003) investigated the existence and identification of a European Business Cycle by analyzing monthly industrial production data of nine EU countries from 1970M01 to 1996M12. Two important issues arose in their investigation: (i) the convergence process of Southern Europe and (ii) the secular decline of the mean growth rates in the post-Bretton Woods era. Analyzing GDP data for six EU countries from 1970Q3 to 1995Q4, Krolzig (2001a) concludes that two-regime models representing contractions and expansions are inconsistent with these two stylized facts of the post-war economic history of Western Europe. Krolzig and Toro (2000) compare the 'classical' approach proposed in Burns and Mitchell (1946) of dating and analyzing the business cycle with the Markov-switching approach as its 'modern' alternative. By using the model's regime probabilities as an optimal statistical inference of the turning point of the European business cycle, they demonstrate the capacity

Table 1 MS-Studies of the European Business Cycle.

Model		Data	M	Sample period	DE	FR	IT	ES	NL	BE	AT	FI	PT	GR	IE	LU	UK
MSIH(3)-VAR(4)	(22)	GDP (OECD)	3	1973Q3 – 2002Q1	+	+	+	+	+	–	+	–	–	–	–	–	–
MSM(2)-VAR(2)	(23)	IIP (OECD)	2	1974Q1 – 2002Q1	+	+	–	+	–	+	+	–	+	+	–	+	–
MSI(2)-VAR(2)	(24)	GDP (Eurostat)	2	1980Q3 – 2002Q1	+	+	+	+	+	+	–	+	–	–	–	–	–
Krolzig (2001b)	(K2)	GDP	2	1980Q2 – 2000Q4	+	+	+	+	+	+	+	+	–	–	–	–	–
Peersman and Smets (2001)	(PS)	IIP	2	1978Q4 – 1998Q4	+	+	+	+	+	+	+	–	–	–	–	–	–
Artis <i>et al.</i> (2003)	(AKT)	IIP	3	1970M1 – 1996M12	+	+	+	+	+	+	+	–	+	–	–	–	+
Krolzig and Toro (2000), Krolzig (2001a)	(KT)	GDP	3	1970Q3 – 1995Q4	+	+	+	+	–	–	+	–	–	–	–	–	+
Krolzig (2001b)	(K1)	agg. GDP	2	1980Q2 – 2000Q4													
Anas and Ferrara (2002)*	(AF1)	agg. GDP	2	1980Q1 – 2002Q2													
Anas and Ferrara (2002)	(AF2)	agg. IIP	3	1971M1 – 2002M8													

Notes:: * Anas and Ferrara (2002) also consider a three-regime model, which is less robust.

M number of regimes + country is included.

IIP index of industrial production – country is excluded.

GDP gross domestic product (real)

of the MS-VAR approach to generate the stylized facts of the *classical* cycle in Europe. These studies have in common the presence of a third regime, which however is rarely found after 1980. We can therefore presume that for data beyond that period two-regime models provide an adequate description of the business cycle. Consequently, Krolzig (2001b) have fitted two-regime Markov-switching models to aggregated euro-zone real GDP data and disaggregated single-country euro-zone for the 1980Q2 to 2000Q4 period. Recessions in the euro-zone are found from 1980Q1 to 1981Q1 and 1992Q3 to 1993Q2. Also, Peersman and Smets (2001) find that a two-regime MS-VAR model delivers reasonable results for quarter-to-quarter growth rates of detrended industrial production in seven euro-area countries of the period 1978Q4 – 1998Q4.

In Figure 11, the (smoothed) probability of a recession in the studies of Artis *et al.* (2003), Krolzig and Toro (2000), and Krolzig (2001b) is plotted. When taking account of the different sample periods involved, the regime classifications consistently support the presence of three major recessions in the periods 1973/74, 1980-82 and 1991 to 1994, though there is increased uncertainty regarding the recession in the 1990s. Anas and Ferrara (2002) extend the univariate analysis of Krolzig (2001b) up to 2002Q2. Using aggregated euro-zone GDP data, they discover a more recent recession starting in 2001Q1, with somehow different results when analyzing euro-zone IIP data. In the following, we reconsider the issue by analyzing three new multivariate data sets. An important question we will investigate is whether the turning point chronology remains robust when more recent economic data are considered.

5.2 The Empirical Modelling Approach

In this paper, we follow the modelling approach taken in Krolzig (1997a), featuring a data-driven model specification as the basis for the derivation of economically meaningful results. For the dating of the European (EU) business cycle, Artis *et al.* (2003) proposed the following empirical modelling procedure:

i. Data transformation

- Seasonal adjustment (here: X-12-ARIMA);
- Outlier correction (here: *PcGets*);
- Smoothing (*e.g.*, centered MA of monthly data);
- Differencing.

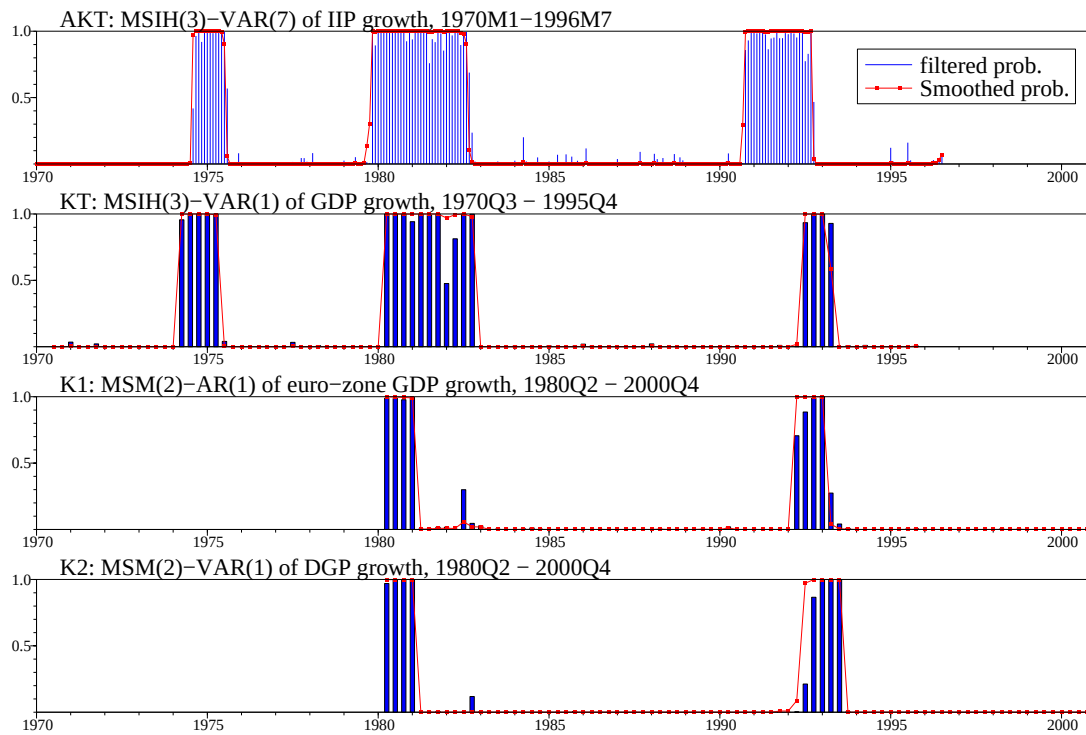


Figure 11 Classifications of the euro-zone Business Cycle in the literature: the probability of a recession.

ii. *Preliminary data analysis*

- Co-switching analysis (using univariate Markov-switching models);
- Co-integration analysis (using linear VAR models).

iii. *Markov-switching vector autoregressions*

- Maximum Likelihood estimation;
- Specification analysis.

iv. *Constructing the turning point chronology*

- Regime inference: filtering and smoothing;
- Business cycle chronology: regime classification and turning point dating;
- Control of the regime classification;
- Turning point detection.

It is beyond the scope of this paper to discuss all the issues involved. The construction of the turning point chronology has been presented in the previous sections of this paper. A general overview and discussion of the econometric issues can be found in Krolzig (1997b). Readers, who might want to get some impression involved in practical use of these models, are referred to Artis *et al.* (2003).

Here we shall focus on the Maximum Likelihood (ML) estimation of the parameters of the MS-VAR model.⁴ For the sample $\mathbf{Y}_T = (\mathbf{y}'_T, \dots, \mathbf{y}'_1)'$ and initial values $(\mathbf{y}_0, \dots, \mathbf{y}_{1-p})'$, the likelihood function

⁴We assume throughout that the data generating process is stationary, *i.e.* the Markov chain is ergodic and the roots of the characteristic VAR polynomial are outside the unit circle (see Krolzig, 1996, for Markov-switching vector equilibrium correction models for cointegrated variables).

is given by:

$$L(\lambda) = \prod_{t=1}^T \sum_{i=1}^M f(\mathbf{y}_t \mid \mathbf{Y}_{t-1}; \boldsymbol{\theta}_m, \boldsymbol{\theta}_c) \Pr(s_t = i \mid \mathbf{Y}_{t-1}; \lambda),$$

where $f(\mathbf{y}_t \mid \mathbf{Y}_{t-1}; \boldsymbol{\theta}_m, \boldsymbol{\theta}_c)$ is the pdf of $\mathbf{y}_t$ in regime m , $f_m(\mathbf{y}_t \mid \mathbf{Y}_{t-1})$. The parameter vector $\lambda = (\boldsymbol{\theta}_1, \dots, \boldsymbol{\theta}_M; \boldsymbol{\theta}_c; \{p_{ij}\}; s_0)'$ includes:

- $\{p_{ij}\}$ set of free transition probabilities,
- $\boldsymbol{\theta}_m$ vector of regime-dependent VAR parameters,
- $\boldsymbol{\theta}_c$ vector of regime-invariant (common) parameters,
- s_0 initial state of the regime variable.

Note that in MSM-VAR models (with switching means), the problem becomes more involved since $\mathbf{y}_t$ does not only depend on s_t , but also on $s_{t-1}, \dots, s_{t-p}$. Due to the structure of the likelihood function, the first order conditions of the ML estimator are highly non-linear. So any estimation procedure is faced with the typical problems of non-linear optimization.

The Expectation-Maximization (EM) algorithm considered by Hamilton (1990) is a very robust technique to derive the ML estimates. It consists of zigzag procedure iterating between an *Expectation step*, which runs the regime inference given the parameters of the previous iteration, and a *Maximization step*, which in turn uses the resulting smoothed regime probabilities to perform the parameter estimation:

- i. Transition probabilities:

$$\tilde{p}_{ij}^{(n)} = \frac{\sum_{t=1}^{T-1} \Pr(s_t = i, s_{t+1} = j \mid \mathbf{Y}_T; \tilde{\lambda}^{(n-1)})}{\sum_{t=1}^{T-1} \Pr(s_t = i \mid \mathbf{Y}_T; \tilde{\lambda}^{(n-1)})}.$$

- ii. VAR parameters:

$$\begin{aligned} \tilde{\boldsymbol{\theta}}_i^{(n)} &= \arg \max_{\boldsymbol{\theta}_i} \sum_{t=1}^T \ln f(\mathbf{y}_t \mid \mathbf{Y}_{t-1}; \boldsymbol{\theta}_m, \boldsymbol{\theta}_c) \Pr(s_t = i \mid \mathbf{Y}_T; \tilde{\lambda}^{(n-1)}); \\ \tilde{\boldsymbol{\theta}}_c^{(n)} &= \arg \max_{\boldsymbol{\theta}_c} \sum_{t=1}^T \sum_{i=1}^M \ln f(\mathbf{y}_t \mid \mathbf{Y}_{t-1}; \boldsymbol{\theta}_m, \boldsymbol{\theta}_c) \Pr(s_t = i \mid \mathbf{Y}_T; \tilde{\lambda}^{(n-1)}). \end{aligned}$$

- iii. Initial State:

- (a) ergodic regime probability: $\tilde{p}_{i,0}^{(n)} = \Pr(s_t = i \mid \lambda^{(n-1)})$;
- (b) ML estimate: $\tilde{s}_0^{(n)} = \arg \max_{s_0 \in \{1, \dots, M\}} L(\boldsymbol{\theta}_1^{(n-1)}, \dots, \boldsymbol{\theta}_M^{(n-1)}; \boldsymbol{\theta}_c^{(n-1)}; \{p_{ij}^{(n-1)}\}; s_0)$;
- (c) backcasting: $\tilde{p}_{i,0}^{(n)} = \Pr(s_0 = i \mid \mathbf{Y}_T; \tilde{\lambda}^{(n-1)})$.

It can be shown that the estimates in (i) and (ii) converge to the ML estimator of the parameters of the MS-VAR. All the computations reported in this paper were carried with the MSVAR class for Ox (see Krolzig, 1998, and Doornik, 1999).

5.3 Markov-Switching Models of the Euro-Zone Business Cycle

5.3.1 A Three-Regime Model of Real GDP Growth (1973-2002)

We start by applying the MSVAR approach to the disaggregated real GDP growth in the euro-zone. The quarterly, seasonally adjusted data from 1973Q3 to 2002Q1 are drawn from the OECD Main Economic Indicators data base. Since cointegration analysis gave no clear indication of the presence of a

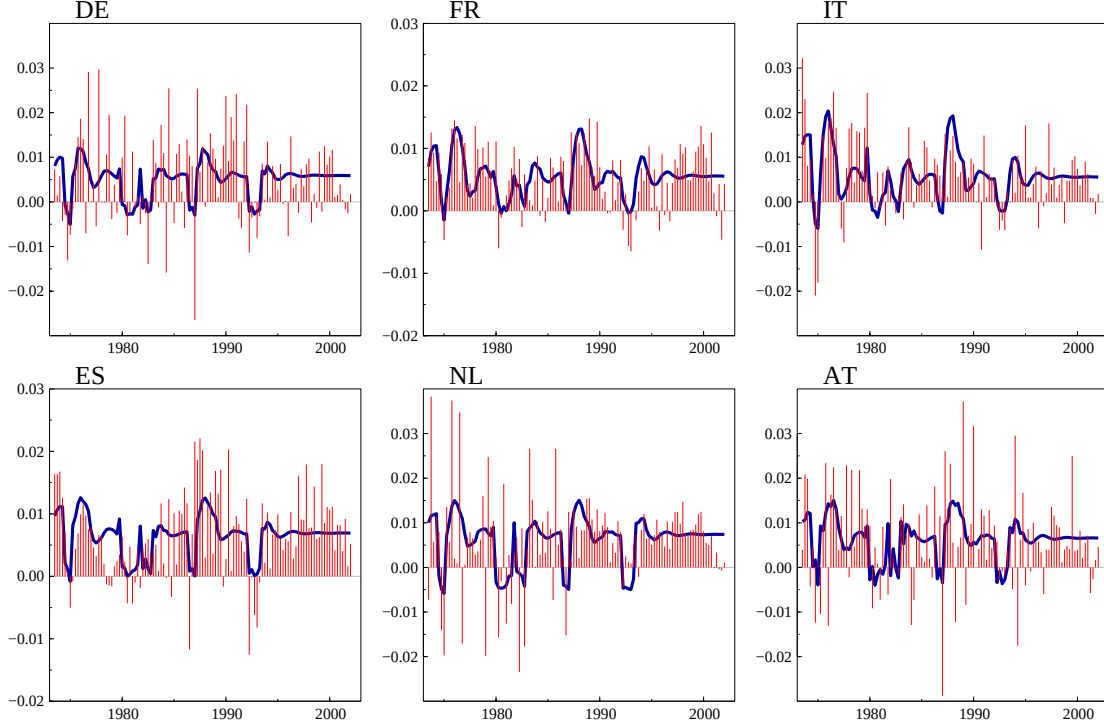


Figure 12 GDP growth data (OECD) and their common cycle.

cointegrating vector, all variables are modelled in first differences. Thus, $\Delta \mathbf{y}_t$ is the vector of GDP growth rates of Germany (DE), Finland (FI), France (FR), Italy (IT), Spain (ES), the Netherlands (NL) and Austria (AT). Given the findings in the literature — particularly the observation made in Krolzig and Toro (2000) and Krolzig (2001a) that the catching-up process of the Mediterranean countries in the 1970s constitutes a third regime, which is fundamentally different from the traditional business cycle concept — we consider a three-regime MS-VAR model with regime-dependent intercepts and variance:

$$\Delta \mathbf{y}_t = \boldsymbol{\nu}(s_t) + \mathbf{A}_1 \Delta \mathbf{y}_{t-1} + \mathbf{A}_4 \Delta \mathbf{y}_{t-4} + \mathbf{u}_t, \quad \mathbf{u}_t | s_t \sim \text{NID}(\mathbf{0}, \boldsymbol{\Sigma}(s_t)). \quad (22)$$

According to the mnemonic proposed in §2.2, the model is denoted MSIH(3)-VAR(4). The maximum lag-order is four; the autoregressive parameters at lag 2 and 3 were found to be insignificant and have been dropped from the model. Graphs of the time series reporting real GDP growth for the six euro-zone countries under consideration can be found in Figure 12. Even after the outlier correction with *PcGets* (see Hendry and Krolzig, 2001), there are still some possible outliers remaining, which could affect the ability of the MS-VAR to extract the common business cycle component from the variables.

The estimated parameters are presented in Table 2. Recessions are more pronounced in Germany, the Netherlands and Austria. Interestingly, these countries had *de facto* a monetary union for almost the entire sample period, with the Austrian and Dutch central bank shadowing the Bundesbank's monetary policy and the exchange rates moving in the narrowest of bands. One might wonder whether this indicates an increase in business cycle symmetry in monetary unions. All countries exhibit higher growth rates in the third regime, which can be thought of as a 'high-growth regime'. For most countries it is also associated with an increased volatility of growth. This, however, is not the case for Germany where the standard errors appear to be regime-invariant.

Table 2 ML estimate of the MSIH(3)-VAR(4) of real GDP growth (1973Q3 – 2002Q1).

	DE	FR	IT	ES	NL	AT
<i>Mean growth rate (intercept)</i>						
Recession	-0.122	0.148	0.142	0.065	-0.265	-0.098
Expansion	0.666	0.491	0.617	0.697	0.906	0.871
High growth	0.861	0.616	1.354	0.947	1.117	1.159
<i>Short-run dynamics</i>						
DE	0.053	-0.092	0.147	-0.094	-0.220	-0.112
FR	0.044	-0.195	-0.368	0.141	-0.354	0.805
ES	0.251	0.557	-0.019	0.243	0.399	0.639
IT	-0.276	0.031	0.124	-0.014	-0.057	-0.916
NL	-0.041	-0.074	-0.020	-0.070	-0.284	-0.114
AT	-0.076	-0.078	-0.002	-0.155	0.282	-0.353
<i>Standard errors</i>						
Recession	0.832	0.329	0.588	0.671	0.983	0.612
Expansion	0.851	0.434	0.592	0.539	0.630	0.820
High growth	0.765	0.503	0.817	0.604	1.395	1.386

The coefficients of the short-run dynamics report $\hat{A}_1 + \hat{A}_4$.

The matrix of estimated transition probabilities is given by:

$$[\tilde{p}_{ij}] = \begin{bmatrix} 0.768 & 0.136 & 0.096 \\ 0.042 & 0.943 & 0.016 \\ 0.096 & 0.101 & 0.803 \end{bmatrix},$$

where $p_{ij} = \Pr(s_t = j | s_{t-1} = i)$. From the estimated transition probabilities, the measures of the persistence of the regimes can be deduced, which are summarized in Table 3. The expected number of quarters a recession prevails (duration) and the unconditional (ergodic) probability of recessions. Whereas recessions (regime 1) have an expected duration of 4 quarters, regimes 2 and 3 prevails 17 and 5 quarters, respectively. The unconditional probability of a recession is 18%.

Table 3 Regime properties of the MSIH(3)-VAR(4) of real GDP growth (1973Q3 – 2002Q1).

	observations	ergodic probability	expected duration
Regime 1	22.0	0.181	4.32
Regime 2	73.0	0.678	17.43
Regime 3	20.0	0.141	5.07

The contribution of the common business cycle to the process of economic growth in the six euro-zone countries is depicted in Figure 12. Major differences in the mean growth rate across regimes are evident. But there are also differences with regard to the countries. For example, some countries recover faster from the 1992/93 recession than others. Particularly, the German economy appears to be falling behind its historical record of economic growth in expansions. Overall, the euro-zone cycle is most closely matched by the Dutch growth rates.

5.3.2 An MS-VAR of Growth in Industrial Production (1974-2002)

Next we check if we can confirm our previous findings when macroeconomic activity is measured by indices of industrial production (IIP) for eight EMU member states: Germany (DE), France (FR), Spain (ES), Belgium (BE), Austria (AT), Portugal (PT), Greece (GR) and Luxembourg (LU). The sample period again is 1974Q1 to 2002Q1. The data drawn from the OECD database are plotted in Figure 13.

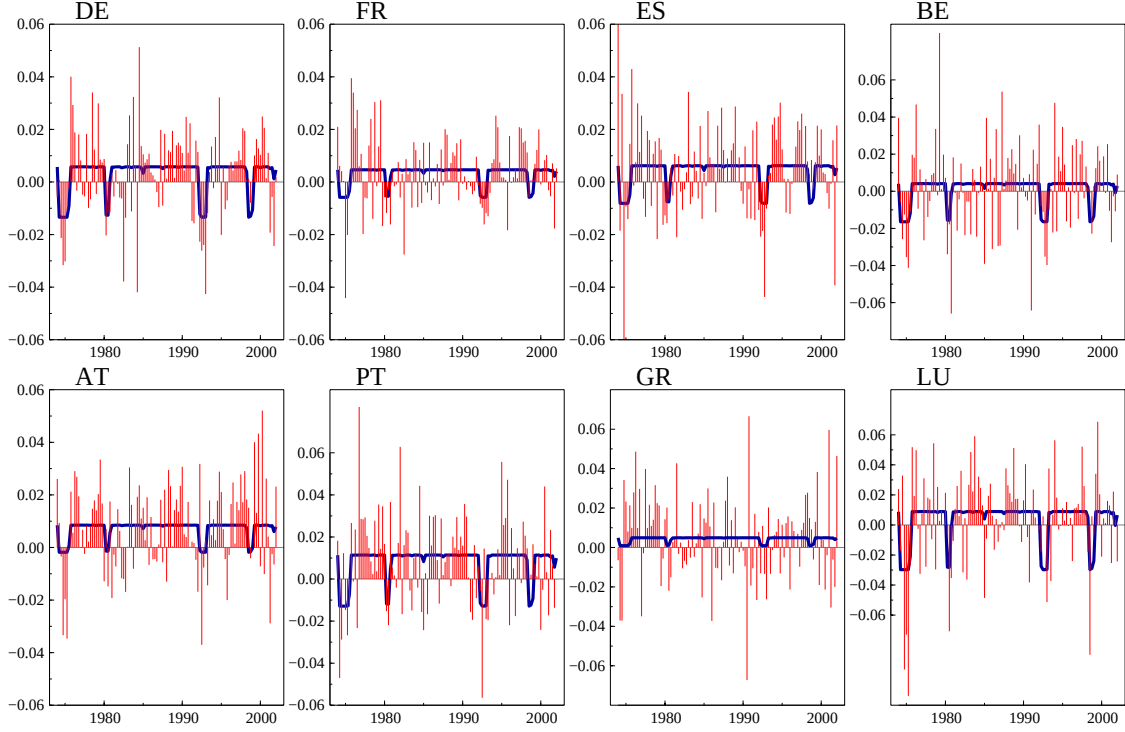


Figure 13 IIP growth data (OECD) and their common cycle.

The time series exhibit a more erratic behavior than the GDP growth rates, which might indicate the presence of outliers. It could also be due to moving-average errors, which have been dealt with in Artis *et al.* (2003) by pre-smoothing.

In contrast to Artis *et al.* (2003), we failed in finding a meaningful three-regime model of the euro-zone business cycle. Interestingly, the problems in modelling euro-zone IIP growth are also reflected in Anas and Ferrara (2002), who analyze aggregated IIP data for the euro-zone. We therefore invoke the classical two-regime model. An MSM(2)-VAR(1) model with shifts in the mean growth rate of industrial production is considered:

$$\Delta \mathbf{y}_t - \boldsymbol{\mu}(s_t) = A_1 (\Delta \mathbf{y}_{t-1} - \boldsymbol{\mu}(s_{t-1})) + \mathbf{u}_t, \quad \mathbf{u}_t | s_t \sim \text{NID}(\mathbf{0}, \Sigma), \quad (23)$$

where $\Delta \mathbf{y}_t$ is an (8×1) vector of IIP growth rates and $s_t \in \{1, 2\}$ is again generated by a hidden Markov chain. The regimes are distinguished by the vectors $\boldsymbol{\mu}_1$ and $\boldsymbol{\mu}_2$ defining the mean growth rates of $\Delta \mathbf{y}_t$ in ‘contractions’ and ‘expansions’.

The estimated parameters are displayed in Table 4. Regime 1 is associated with contractions in industrial production in all considered countries but Greece, where output stagnates. The transition probabilities are estimated as:

$$[\tilde{p}_{ij}] = \begin{bmatrix} 0.709 & 0.291 \\ 0.047 & 0.953 \end{bmatrix},$$

resulting in the regime properties reported in Table 5. The persistence of recessions is in line with the results for MS-VAR model of GDP growth.

Visual inspection of the smoothed probabilities of recessions plotted in Figure 15 reveals a much shorter recession in the 1980s, no recessionary tendencies in 1986/87, and strong indication of a recession in 1998/99. Due to the pronounced difference to the regime classification produced by the previous

Table 4 ML estimate of the MSM(2)-VAR(2) in industrial production (1974Q1 – 2002Q1).

	DE	FR	ES	BE	AT	PT	GR	LU
<i>Mean growth rate</i>								
Recession	-1.341	-0.583	-0.812	-1.645	-0.184	-1.287	0.092	-2.979
Expansion	0.576	0.464	0.618	0.414	0.849	1.140	0.492	0.891
<i>Short-run dynamics</i>								
DE	-0.149	0.192	-0.101	-0.020	0.374	-0.306	0.347	0.002
FR	-0.015	-0.282	-0.254	0.361	0.178	0.189	0.432	0.271
ES	0.238	0.385	0.322	0.258	0.225	0.223	-0.198	0.137
BE	0.005	0.052	-0.041	-0.356	-0.061	-0.091	-0.189	0.116
AT	0.362	0.036	0.251	0.444	-0.105	0.289	-0.254	0.258
PT	-0.064	-0.086	-0.114	-0.071	0.029	-0.037	0.047	-0.024
GR	0.189	0.190	0.368	0.043	0.144	0.161	-0.079	-0.029
LU	-0.084	0.080	-0.081	-0.024	0.107	-0.027	0.041	-0.147
<i>Standard errors</i>								
σ	1.321	0.956	1.337	1.915	1.338	1.881	1.888	2.503

The coefficients of the short-run dynamics report $\hat{A}_1 + \hat{A}_2$.

Table 5 Regime properties of the MSM(2)-VAR(2) in industrial production (1974Q1 – 2002Q1).

	observations	ergodic probability	expected duration
Regime 1	15.5	0.138	3.44
Regime 2	97.5	0.862	21.39

model and the limited data quality, we have investigated the robustness of the results by the removing Luxembourg (LU) from the system. The revised model fails in detecting the recession in the early eighties. Recessions are called for the periods 1974Q1 – 1975Q2, 1992Q2 – 1993Q2 and finally for 2001Q2 – 2001Q4. The lack of robustness is a reason for concern. The results derived from this model, therefore, should be considered with some caution.

5.3.3 A Two-Regime Model of GDP Growth (1980-2002)

As the third model we analyze GDP growth data provided by Eurostat over the period 1980Q3 – 2002Q1 plotted in Figure 14. The data are seasonally adjusted by X-12-ARIMA and outlier corrected using PcGets. We are again interested in detecting the shifts in the mean growth rate of real DGP representing turning-points of the euro-zone business cycle. The cycle is characterized by smoother transitions than in the IIP case, so we use a model with switching intercepts. Given the evidence in the literature, it can be assumed that for sample period considered here, two-regime models are an adequate description of the business cycle. A first-order model with shifts in the intercept is used:

$$\Delta \mathbf{y}_t = \boldsymbol{\nu}(s_t) + \mathbf{A}_1 \Delta \mathbf{y}_{t-1} + \mathbf{u}_t, \quad \mathbf{u}_t | s_t \sim \text{NID}(\mathbf{0}, \boldsymbol{\Sigma}). \quad (24)$$

The ML estimates of this MSI(2)-VAR(1) model are reported in Table 6.

The estimated transition probabilities are given by:

$$[\tilde{p}_{ij}] = \begin{bmatrix} 0.898 & 0.102 \\ 0.053 & 0.947 \end{bmatrix},$$

implying the regime properties reported in Table 7.

Only in the case of Finland and the Netherlands, real GDP contracts in the first regime. Figure 14 reveals that the very severe recession in Finland during the early nineties is indeed the mostly remarkable

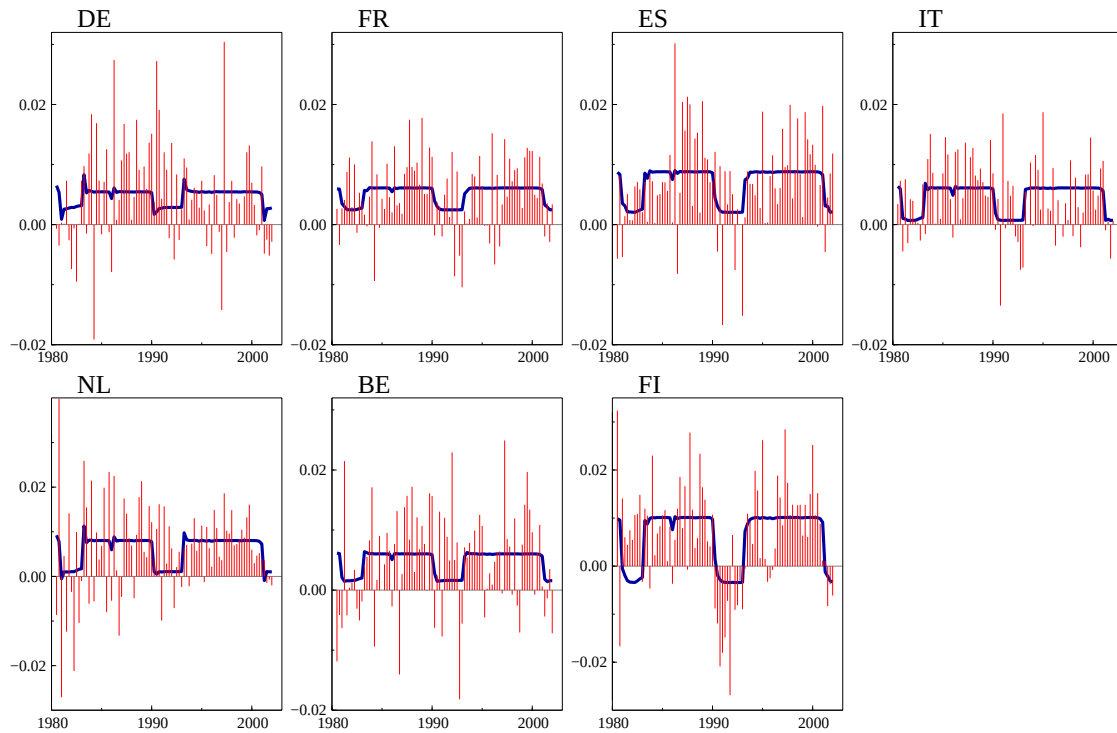


Figure 14 GDP growth data (Eurostat) and their common cycle.

feature of the data set. In contrast, the Markov component of GDP growth is positive for all other euro-zone countries over the entire sample period. It is therefore more appropriate to think of this model as a representation of the euro-zone *growth* cycle. This finding is confirmed by the surprisingly high estimate of the expected duration of the ‘low-growth’ regime. The average duration of a regime 1 is with 10 quarters twice as long as the ‘euro-zone recessions’ found by the first two models. However, the estimated duration is very similar to the result in Anas and Ferrara (2002), when analyzing aggregated euro-zone GDP data. Also, the probabilities of the ‘low-growth’ regime follow closely their results (see Figure 15). Apart from the outlier in 1986Q1, there are three distinguished recessions within the sample period: 1981Q1 – 1983Q1, 1990Q2 – 1993Q1, and 2001Q2 – 2002Q2.

In § 5.5, we will compare the regime classifications derived here to the business cycle chronologies proposed in the literature. Next, we investigate the degree of business cycle synchronization in the three estimated models of the euro-zone business cycle.

5.4 Business Cycle Synchronization

The synchronization of the business cycles in the EMU member states is an important indicator for the optimality of a single monetary policy. A lack of business cycle synchronization within the euro area could complicate the operation of monetary policy in the union. For non-member states, it constitutes a negative indicator for joining EMU. Therefore, business cycle synchronism deserves careful screening.

Here we analyze whether the cyclical shifts in the mean growth rate of the euro-zone countries are traceable to the exposed common euro-zone cycle. The hypothesis of a common euro-zone cycle is investigated by testing for the significance of the regime shifts in the mean growth rate of the economies in the euro-zone. Under the null hypothesis, the mean growth rate of country k in recessions (regime 1) is identical to its growth rate in expansions (regime 2): $\mu_{k1} = \mu_{k2}$ for $k = 1, \dots, K$. Since the regime-

Table 6 ML estimate of the MSI(2)-VAR(1) of real GDP growth (1980Q3 – 2002Q1).

	DE	FR	IT	ES	NL	BE	FI
<i>Mean growth rate (intercept)</i>							
Recession	0.162	0.245	0.020	0.220	-0.051	0.049	-0.400
Expansion	0.734	0.568	0.647	0.874	1.008	0.532	0.895
<i>Short-run dynamics</i>							
DE	-0.056	-0.155	-0.011	-0.245	0.316	0.124	-0.326
FR	-0.079	-0.232	0.036	0.183	0.435	0.064	0.251
IT	0.244	0.124	-0.188	0.065	-0.106	0.112	-0.111
ES	0.220	0.206	0.100	-0.100	-0.018	0.097	0.282
NL	-0.117	-0.090	-0.053	-0.153	-0.344	-0.081	-0.079
BE	-0.031	0.241	0.137	0.300	-0.243	-0.168	0.156
FI	-0.335	-0.061	-0.063	0.018	-0.135	-0.121	-0.068
<i>Standard errors</i>							
σ	0.750	0.522	0.537	0.652	0.852	0.777	0.897

Table 7 Regime properties of the MSI(2)-VAR(1) of real GDP growth (1980Q3 – 2002Q1).

	observations	ergodic probability	expected duration
Regime 1	24.8	0.3405	9.78
Regime 2	62.2	0.6595	18.93

dependent means in the remaining equations of the system are unrestricted, the model is identified under the null hypothesis. Hence, the tests are nuisance parameter free and classical likelihood theory can be invoked. The asymptotic null distribution of the Wald test is $\chi^2(r)$, where $r = 1$ is the number of linearly independent restrictions.

Since testing for the number of regimes is a difficult enterprise in Markov-switching models, it is important to note that we are here only interested in showing that individual euro-zone countries share the inferred cycle as episodes of economic expansion and contraction (or stagnation). In contrast, formal tests of the Markov-switching model against linear alternatives are confronted with unidentified nuisance parameters under the null, hampering the applicability of conventional testing procedures. Suppose a two-regime model with Markov-switching means is subjected to the test hypothesis $\mu_1 = \mu_2$. Then, the transition probabilities p_{12} and p_{21} are not identified. Standardized likelihood ratio (LR) tests designed to deliver (asymptotically) valid inference have been proposed by Hansen (1992, 1996) and Garcia (1998). Hansen's approach delivers a bound on the asymptotic distribution of the standardized LR test, but the test is conservative, tending to be under-sized in practice and of low power, and computationally demanding. In comparison, the tests presented here can be easily implemented (see Krolzig, 1997b, ch.7, for details) and were found useful in empirical research.

Table 8 reports the results of the Wald specification tests for the regime invariance of the mean growth rates for the three models discussed in this paper:

- i. MSIH(3)-VAR(4) of GDP growth (OECD data): the test hypothesis $\nu_{k1} = \nu_{k2}$ is strongly rejected for all countries and the system as a whole. However, the support for a third regime is not very strong. Only in the case of Spain (ES), the differences in the regime-dependent intercept turn out to be statistically significant.⁵ Due to the regime dependence of the variance matrices, also system tests of the form $\nu_1 = \nu_2 = \nu_3$ are free of nuisance parameters. The test results provide strong support for the presence of regime shifts in the mean growth rate of the analyzed economies.

⁵Note, however, that regime 3 is also characterized by a sharply increased volatility for Italy (IT), the Netherlands (NL) and Austria (AT).

Table 8 Wald tests of the regime-invariance hypothesis.

Model	MSIH(3)-VAR(4)				MSM(2)-VAR(2)				MSI(2)-VAR(1)	
Data	GDP growth				IIP growth				GDP growth	
Source	OECD				OECD				Eurostat	
Sample	1973Q3 – 2002Q1				1974Q1 – 2002Q1				1980Q3 – 2002Q1	
Test	$\mu_1=\mu_2$	$\chi^2(1)$	$\mu_2=\mu_3$	$\chi^2(1)$	$\mu_1=\mu_2=\mu_3$	$\chi^2(2)$	$\mu_1=\mu_2$	$\chi^2(1)$	$\mu_1=\mu_2$	$\chi^2(1)$
DE	13.62	[0.0002]**	0.82	[0.3646]	16.27	[0.0003]**	11.79	[0.0006]**	5.70	[0.0169]*
FR	13.16	[0.0003]**	0.93	[0.3332]	16.07	[0.0003]**	7.51	[0.0061]**	4.23	[0.0395]*
IT	14.04	[0.0002]**	2.36	[0.1239]	17.71	[0.0001]**			14.77	[0.0001]**
ES	9.32	[0.0023]**	13.14	[0.0003]**	25.94	[0.0000]**	9.60	[0.0019]**	11.03	[0.0009]**
NL	24.43	[0.0000]**	0.39	[0.5300]	25.05	[0.0000]**			15.75	[0.0001]**
BE							2.30	[0.1291]	22.04	[0.0000]**
AT	28.07	[0.0000]**	0.69	[0.4034]	30.39	[0.0000]**	3.68	[0.0550]		
FI									4.02	[0.0448]*
PT							4.80	[0.0283]*		
GR							0.38	[0.5334]		
LU							8.93	[0.0028]**		
System	180.21	[0.0000]**	57.32	[0.0000]**	659.50	[0.0000]**				

The reported system tests are free of nuisance parameters since the regimes are identified under the null by switching covariance matrices $\Sigma_i \neq \Sigma_j$ for $i \neq j$. The degree of freedoms are 6 and 12, respectively.

- ii. MSM(2)-VAR(2) of IIP growth (OECD data): the test hypothesis of regime-invariant means, $\mu_{k1} = \mu_{k2}$, is strongly rejected for Germany (DE), France (FR), Spain (ES) and Luxembourg (LU). It can also be rejected for Portugal (PT), with Austria (AT) being a marginal case. Belgium (BE) and Greece (GR), however, are not significantly affected by the euro-zone *growth* cycle.
- iii. MSI(2)-VAR(1) of GDP growth (Eurostat data): the regime-invariance hypothesis $\mu_{k1} = \mu_{k2}$ can be rejected for all countries, though the common cycle is less pronounced in Germany (DE), France (FR) and Austria (AT).

Overall, the notion of a common euro-zone business cycle is justified. There is overwhelming evidence for the relevance of the identified euro-zone *growth* and *business* cycles. With the exception of Greece, the common cycle contributes to the mean growth rate in all countries of the euro-zone.

5.5 Turning Point Chronologies of the Euro-Zone Business Cycle

As discussed in § 3, the construction of a turning point chronology with Markov-switching models is based on the time paths of the smoothed regime probabilities, which are a by-product of the EM algorithm. For the dating of the euro-zone business cycle, we start by reconsidering the smoothed and filtered regime probabilities derived with the three models considered above. The regime probabilities are plotted in Figure 15; the filtered ones are shown with bars, and the smooth ones are represented by a bold line.

It is worth recalling that the filtered regime probability can be understood as an optimal inference on the state variable (whether the system is in a boom or recession) at time t using only the information available up to this point in time, i.e. $\Pr(s_t = m \mid \mathbf{Y}_t)$, where m stands for a given regime; the smoothed regime probability is the optimal inference on the regime at time t using the full sample information $\mathbf{Y}_T$: $\Pr(s_t = m \mid \mathbf{Y}_T)$. The difference between the two measures indicates the amount of revisions required when new observations become available. The classification of the regimes and the dating of the business cycle amounts to assigning $\mathbf{y}_t$ to the regime m with the highest smoothed probability. The resulting chronology of the euro-zone business cycle is reported in Table 9 for the three measures of macroeconomic activity analyzed in this paper. The peak date denotes the period t just before the beginning of a recession, i.e. $\Pr(\text{'recession' at time } t \mid \mathbf{Y}_T) < 0.5$ and $\Pr(\text{'recession' at time } t + 1 \mid \mathbf{Y}_T) > 0.5$; the trough is the last period of the recession.

Table 9 Datings of the euro-zone business cycle.

Model	MSIH(3)-VAR(4)			MSM(2)-VAR(2)			MSI(2)-VAR(1)		
Data	GDP growth			IIP growth			GDP growth		
Source	OECD			OECD			Eurostat		
Sample	1973Q3 – 2002Q1			1974Q1 – 2002Q1			1980Q3 – 2002Q1		
Cycle	Peak	Trough	h	Peak	Trough	h	Peak	Trough	h
A	1974Q2	1975Q1	0.75	[1974Q1]	1975Q3	1.50			
B									
C	1979Q4	1981Q3	1.75	1980Q1	1980Q3	0.50	1980Q4	—	
D	1981Q4	1982Q4	1.00				—	1983Q1	2.25
E	1986Q2	1987Q1	0.75						
F	1992Q1	1993Q3	1.50	1992Q1	1993Q1	1.00	1990Q1	1993Q1	3.00
G				1998Q2	1999Q1	0.75			
H							2001Q1	[2002Q2]	1.25

Notes: [·] Period before the beginning or at the end of the sample;
 h Duration of recessions in years.

Table 10 Datings of the euro-zone business cycle in the literature.

	K1			K2 [†]			A1			A2		
Cycle	Peak	Trough	h	Peak	Trough	h	Peak	Trough	h	Peak	Trough	h
A										1974M8	1975M8	1.00
B										1977M2	1977M7	0.41
C	[1980Q1]	1981Q1	1.00	[1980Q1]	1981Q1	1.00	[1980Q1]	1982Q4	3.00	1980M4	1980M10	0.50
D										1982M5	1982M8	0.25
E												
F	1992Q1	1993Q1	1.00	1992Q2	1993Q3	1.25	1992Q1	1993Q2	1.00	1992M4	1993M2	0.83
G												
H							2001Q1	2002Q2	1.00	2001M10	2001M12	0.17

	AKT			KT*			PS		
Cycle	Peak	Trough	h	Peak	Trough	h	Peak	Trough	h
A	1974M7	1975M7	1.00	1974Q1	1975Q2	1.25			
B									
C	1979M10			1980Q1	—		1979Q4	—	
D		1982M8	2.83	—	1982Q4	2.75	—	1983Q1	4.25
E							1985Q4	1987Q1	1.25
F	1990M9	1992M9	2.00	1992Q2	1993Q2	1.00	1990Q1	1992Q3	2.50
G							1995Q2	1996Q1	0.75
H									

Notes: [·] Period before the beginning or at the end of the sample;
 h Duration of recessions in years;
* Dummy variables being 1 in 1984Q2 and 1987Q1 and –1 in the subsequent period included;
[†] Dummy variable being 1 1987Q1 and –1 in the subsequent period included.

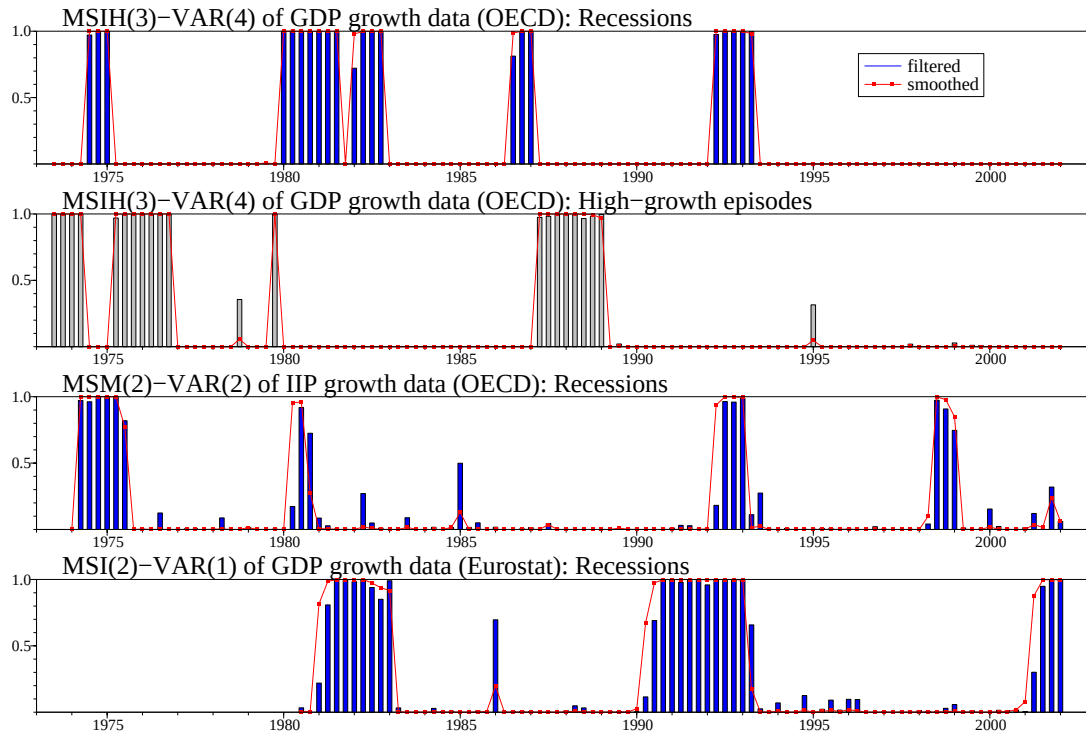


Figure 15 Regime probabilities.

The derived business cycle chronologies should be compared with the datings in Table 10, which reports the datings of the European business cycle derived from Markov-switching models proposed in the literature (see the overview in Table 1 of § 5.1). Given the lack of an NBER-type reference cycle for the euro zone, it is worth to elaborate the robust features of the euro-cycle datings in the literature and to compare them to the turning points of the euro-zone business cycle identified in this paper. The most striking result is that the GDP data based business cycle classifications of Krolzig (2001b) and Anas and Ferrara (2002) coincide when the different sample length is taken into consideration.

In contrast, the MS-AR mode considered by Anas and Ferrara (2002) for euro-zone IIP generates recessions which are too short and too frequent. The main difference with regard to the results of Artis *et al.* (2003), Krolzig and Toro (2000) and Krolzig (2001a) is the relative short expected duration of recessions. As these papers find some indication for incomplete business cycle synchronization, the prolonged duration of recessions may reflect the need to subsume the UK and continental cycle. An outlier is marked by the study of Peersman and Smets (2001) as the common cycle identified in their study has more the characteristics of a *growth* cycle. As can be seen in Table 10, their (growth) ‘recessions’ have much a longer duration and are found much more frequently.

Altogether, we can conclude that a robust pattern in the chronologies of the euro-zone business cycle produced by Markov-switching models has emerged. The turning point chronology of the euro-zone business cycle is dominated by three recessions:

- i. the recession following the 1973/74 oil price shock (from 1974Q2/Q3 to 1975Q1/Q3);
- ii. possibly a double dip recession in the early eighties (1980Q1/1981Q1 to 1982Q4/1983Q1), coinciding with Volcker disinflation in the US;
- iii. a recession in the early nineties (1990Q2/1992Q2 to 1993Q1/Q3) associated with greater uncertainty regarding its dating, presumably caused by monetary tightening and fiscal expansion in the

aftermath of the German reunification;

- iv. there is some indication of a euro-zone recession commencing in 2001Q2.

Some differences in the precise dating of the turning points remain. Among other causes, they are likely due to the choice of the measure of macroeconomic activity and the set of countries, for which the data have been available. In line with the literature⁶, we have identified various characteristics of the data that matter for the construction of the business cycle chronology:

- i. Provision of data for the largest possible set of countries;
- ii. Availability of long historic time series;
- iii. Data frequency;
- iv. Pre-filtering:
 - seasonal adjustment;
 - outlier correction;
 - smoothing (for monthly data sets).

The importance of the quality of the data for the dating of the euro-zone business cycle underlines the pivotal role of Eurostat in the creation and distribution of high-quality statistical data for the EMU member states. The case for further research is clearly granted.

6 Conclusions

In this paper, we have addressed the issue whether Markov-switching vector autoregressive models can be used for the extraction and dating of the euro-zone business cycle. We have shown that this generalization of the approach innovated by Hamilton in his analysis of the US business cycle allows the identification of a common business cycle in the euro zone. The regime identification scheme distinguished between states of recession and expansion in the euro area, where different expansionary regimes may be considered to embed structural change.

The main features of the proposed MS-VAR modelling procedure are:

- i. *Approach*
 - Congruent statistical model of the time series of interest;
 - Meaningful definition of a common (euro-zone) business cycle.
- ii. *Business chronologies*
 - Dating of turning point based on the smoothed probability of being in a recession;
 - Multi-regime models feasible.
- iii. *Advantages of the regime-switching approach*
 - Quantification of the uncertainty associated with the regime classification;
 - Detection of new turning points in real time;
 - Systematic revision when new data arrive.
- iv. *Advantages of modelling systems*
 - Euro-zone business cycle as a common feature of the macroeconomic dynamics in the EMU member states;

⁶For example, it is well known that seasonal adjustment procedures can severely affect the determination of business cycle turning points (see, *inter alia* Astolfi, Ladiray and Mazzi, 2001, and Lommatzsch and Stephan, 2001).

- Missing invariance of regime classification under aggregation.

The empirical analysis consisted in fitting an MS-VAR model to a system of real GDP or IIP growth rates of EMU member states. The models have been found statistically congruent and economically meaningful, though room for improvements and refinements remains. The results shed light on the issue of which countries drive the euro-zone cycle. We have been able to demonstrate the relevance of the common cycle for all EMU member states except Greece. So the evidence for the presence of a common, Markovian euro-zone business cycle shared by the EMU member states is very robust. The estimated Markov-switching VAR models were then used to construct turning point chronologies of the euro-zone business cycle, differing regarding the measurement of macroeconomic activity, the set of countries available, and the time period considered. While the pattern of recessions and expansions is robust, differences in the precise dating of events have been acknowledged. In this context, we have also emphasized the importance of data quality. Altogether, we believe that the findings of this study justify a recommendation of the Markov-switching vector autoregressive model for the construction of turning point chronologies of the euro-zone business cycle.

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