Stochastic Equilibria in Spatial Multi-Party Competition

Thomas Plümper† and Christian W. Martin‡

†Department of Politics and Management
University of Konstanz
Box D 86
D-78457 Konstanz, Germany
Tel: +49-7531-88 2608/3081
Fax: +49-7531-88 2774
thomas.pluemper@uni-konstanz.de

‡Max Planck Institute for Research into Economic Systems
Kahlaische Straße 10
D-07745 Jena, Germany
Tel: +49-3641-686 736
Fax: +49-3641-686 710
martin@mpiew-jena.mpg.de

Abstract:

The analysis of multi-dimensional partisan competition in multi-party systems has been hampered by the notorious problem of disequilibria. This paper offers a solution for the ‘problem’ of disequilibria in spatial voting models which is based on ‘condensed’ Markov equilibria: By eliminating the time-dimension from a Markov chain of partisan platforms we are able to identify probability zones dubbed ‘stochastic equilibria’. We then impose controlled parameter variations in computational models to allow for the use of statistical tools with which we derive hypotheses from a spatial model of multi-party competition. The usefulness of this method is illustrated by three examples dealing with the impact of voters’ memory, abstention and the number of parties on endogenous partisan preferences.

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1. Introduction

Political scientists have since long fully recognized the median-voter equilibrium to be an "artifact" (Hinich 1977) of the restrictive assumptions built into Hotel ling-Downsian type models of spatial political competition. The main result of Down's (1957) model – two parties competing for votes in a unidimensional policy space converge to the median of voters' preferences – is conditional upon strict adherence to the initial model formulation. Once changes to assumptions are introduced, such as permitting two dimensions instead of one, three parties instead of two or assuming partly ideological parties, platforms do no longer converge (Palfrey 1984).

This sobering insight became the starting point of a quest for more general models of endogenous partisan positioning. But instead of finding solutions to the problem at hand, scholars noticed that multi-party equilibria are “just as rare in one dimension as are two-candidate equilibria in many dimensions” (Cox 1990, 183). The situation becomes even more complicated if a combination of both problems is studied, i.e. multi-party competition in a policy space with more than one dimension. Such a setup matches real-world electoral contests much more closely than the Downsian model – especially when we study political systems outside the USA.

Failure to identify unique equilibria has in most instances not given way to solutions which depart from trodden paths. Instead, existing research merely turned to refining standard concepts. For instance, using probabilistic voting models instead of models that assume deterministic voting became increasingly popular because such models "more often result in stable equilibria in multi-dimensional space" (Burden 1997: 1151).

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1 This fact has already been formally recognized by B. Curtis Eaton and Richard Lipsey (1975) for the case of spatial competitions among three profit maximizing firms.
Unfortunately, the derivation of equilibria in probabilistic voting models does not come without costs. For one thing, the stochastic element rendering these models "probabilistic" has to grow rather large in order to produce stable outcomes in multi-party competition. Second, and more importantly, probabilistic voting models succeed in generating multiple equilibria instead of disequilibria, without producing a unique Nash equilibrium in pure strategies. Therefore, researchers are still left with the question of how to select point predictions.

While this situation is certainly unsatisfactory from a theoretical point of view, the empirical deficiencies loom large as well: Real-world political competitions rarely match the predictions of the few theoretical models that bring forth equilibria. This has led some scholars in the field to abandon multi-dimensional or multi-party spatial modeling altogether.

This paper offers a solution for the problem of disequilibria in spatial voting models based on ‘condensed’ Markov equilibria: By eliminating the time-dimension from a Markov chain equilibrium of partisan platforms we are able to identify probability zones which we dub ‘stochastic equilibria’. We then impose controlled parameter variations in computational models to permit the use of simple statistical tools with which we derive hypotheses from a spatial model of multi-party competition.

The advantages of our procedure are therefore manifold:

1. This procedure takes assumptions more seriously. It allows for both a richer set of assumptions and the specification of more complex interactions among agents. We define the situation faced by parties in a multi-party system according to the literature on party competition, transform these statements into a set of mathematically formulated assumptions and then analyze the emerging disequilibria by a technique based on multiple simulations. As a result, we do not necessarily get a unique and stable equilibrium. Instead, we are able to submit the variance of the stochastic equilibria to standard statistical procedures, thereby gaining insight into the systematic properties of the model.
2. The technique enables us to explore the relationship between all parameters in the model. It thus serves as a tool to refine existing theories, and possibly build new ones.

3. The technique allows to narrow the search for empirically relevant parameters, thus permitting to interknit theory and empirics more closely.

4. Although not contained in this paper, the method proposed allows in principle to identify the conditions under which a) a system in which all possible outcomes have the same a priori probability of being realized, b) stochastic equilibria, c) a unique Nash equilibria in mixed strategies and d) a unique Nash equilibrium in pure strategies emerge.

The method developed in this paper gives researchers the freedom to analyze the model they have in mind without restricting modeling assumptions to the ones necessary for equilibrium generation. Furthermore, the method we suggest is not restricted to the analysis of the specific model that we formulate in this paper. One may disagree with the model we use here and still employ the technique of using stochastic equilibria for the derivation of hypotheses.

The remainder of this paper is organized as follows: In the next section we argue that existing models of multi-dimensional multi-party competition remain unsatisfactory. Section 3 briefly discusses the computational model we use. Section 4 discusses Markov perfect and stochastic equilibria and outlines how these can be used to derive hypotheses about multi-party competition. Section 5 gives a few examples illustrating the method we specified before and section 6 concludes.

2. The Search for Equilibria and the Rise of Probabilistic Voting Models

Elections are probably the single most important event in democratic politics. Political parties competing for votes and office seeking politicians announce policy platforms in order to attract voter's support. On election day, voters decide for which party to cast their ballot. It seems only natural that political
science should try to capture party positioning and voting behavior in a unified framework. Both factors are inseparably linked.

At the very same time, building models urges researchers to focus on a limited number of carefully selected causal mechanisms. In this respect, a good formal model satisfies four conditions: First, it has to parsimoniously isolate its explanandum from seemingly or even actually connected phenomena. In developing an explanans, a good model, secondly, ought to formulize stylized accounts of the ontological units constituting the artificial world that serves as a functional analogy to reality. Third, the model has to make simplifying and sometimes wrong assumptions about the interaction between its ontological units. And finally, researchers have to choose, and sometimes invent, a reliable mechanism of making predictions re-connecting the model to reality.

How do existing models of multi-party electoral competition deal with the simultaneous trade-off between the complexity of real political systems and the academic goal to simplify reality? How do these models measure up against the benchmark?

As we examine the literature, we are left with the impression that the search for equilibrium predictions dominates modelers’ choices of phenomena, ontology, and assumptions. From our four criteria outlined above, the fourth – connecting the model to reality by deriving unique equilibria – causes three deliberate choices. First, researchers typically analyze two-party single-dimension competitions instead of multi-party and/or multi-dimensional contests, simply because the former allow to identify a unique equilibrium while the latter do not. Second, choice of actors and actions is biased towards the desired results. For instance, it makes a potentially huge difference if parties are modeled as unitary actors or if we assume inner party fractionalization. Third, the preference

\[\text{To quote Morton once again: } \text{"[...] almost perversely [formal modelers] actually often desire to derive unique equilibrium point predictions and discard and criticize models that do not provide them." (Morton 1999: 165)}\]
for probabilistic over deterministic voting models can well be explained by scholars' desire to produce single equilibria.

As Rebecca Morton notes: "Probabilistic voting was originally devised to explain how equilibrium may occur in majority voting models of candidate competition in multi-dimensional issue space. Early theoretical research showed that, when the issue space moved beyond the single dimension of the Hotelling-Downsian model of candidate or party competition, disequilibrium results were predicted. Researcher began to add randomness to voting in order to solve the disequilibrium problem." (Morton 1999: 170). However, when assumptions of these simple two-parties-in-one-dimension-models are justified it is rarely with reference to the possibility of deriving equilibrium results. Robert Erikson and David Romero, for instance, note: "Probabilistic voting theory offers a potential escape from the inevitability of disequilibrium." (Erikson/ Romero 1990). James Adams, too, argues for the choice of probabilistic over deterministic models: "[...] Probabilistic voting models enhance the possibility of equilibrium, particularly in cases where the random component of voter decision making is sufficiently

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3 However, scholars more often than not employed behavioral arguments to reconcile the choice of probabilistic models and claimed that probabilistic models are superior because assumptions are more realistic. Dennis Mueller, for instance, summarizes the concerns about deterministic models: "Deterministic models assume that vote choices gyrate schizophrenically as candidates move about competing for votes." (Mueller 1989: 199). And Peter Coughlin argues: "First, candidates tend to rely on polls for information about how voters will vote, but information from public opinion surveys is not error-free. (...) Second, even when economists and political scientists have developed sophisticated statistical models of voters’ choices (...) there has consistently been a residual amount of unexplained variance." (Coughlin 1990: 145)

4 If voting is supposed to be deterministic and more than one policy dimension exists the emergence of an equilibrium crucially depends on the existence of a point with the property that "every hyperplane through it divides the distribution of ideal points in two equal parts." (Osborne 1995: 267) This condition is satisfied if the same individual is always the median in all policy-dimensions. This point is known as the Condorcet equilibrium (Plott 1967). Two parties competing in a multi-dimensional policy space will choose this point if it exists (Davis, deGroot, Hinich 1972). If a Condorcet winner (CW) does not exist, equilibria may emerge in mixed strategies (Kramer 1978). Unfortunately, the probability that this point exists is

\[ p(CW) = \frac{1}{N}\binom{N}{D} \] 

where \( N \) is the number of voters and \( D \) is the number of independent policy dimensions. Thus, in a political system with 50 million voters and three relevant policy-dimensions is 1 to \( 2.5^{-15} \). Needless to say that is not very promising to construct a theory of partisan politics based on this possibility.
large.” (Adams 1999: 260). More frequently, the justification put forward for including a stochastic element is couched in terms of some kind of uncertainty: Either parties do not know the true preferences of voters (Ordeshook 1986, 179-180), or voters do not know the policy parties implement once empowered (Morton 1999: 172), or both.  

Deterministic models assume that voters choose the party whose position is nearest to their own ideal point in the policy space with certainty; in probabilistic voting models only the likelihood of voting for one party increases with decreasing distance between voter and partisan preferences. As a consequence, a vote against the nearest party becomes possible in probabilistic models, because of the inclusion of a stochastic element in voters’ choice. Probabilistic models are flexible: If the probabilistic term approaches zero, the model more and more resembles its deterministic counterpart. On the other hand, with ‘randomness’ in voters’ decisions becoming more important, the distance based on policy positions is rendered increasingly irrelevant.

Technically, distance between a voter $i$ and a party $j$ in an Euclidian policy space is given by:

$$d_{ij} = \left( \sum_{k=1}^{K} (x_{i,k} - x_{j,k})^2 \right)^{1/2},$$

(1)

where $k \in K$ is a policy dimension, $x_{i,k}$ is the ideal point of voter $i$ in dimension $k$, and $x_{j,k}$ is the position of party $j$ in dimension $k$. While in deterministic models voters vote for the party minimizing this distance, probabilistic models assume that a voter’s utility of voting for party $k$ is

$$u_i(x_{j,k} | x_{i,k}) = d_{ij} + \mu_{ij},$$

(2)

where $\mu_{i,k}$ is drawn from a mathematically convenient distribution with mean zero and a variance $\sigma^2$.

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5 An alternative assumption is that voters do not vote for the party closest to their own position, because they are “biased” (Plümper/ Martin 2003a) or because they calculate past positions of the parties (Martin/ Plümper 2003b).
These assumptions are variants of those used by Lin, Enelow, and Dorussen (1999) as well as by Adams (1999), who analyze if (Lin et al) and under what conditions (Adams) convergence towards the voter mean is an equilibrium in a multi-dimensional policy space with more than two parties. Both articles demonstrate that with an increasing relative weight of the stochastic term in the voters’ utility function, the likelihood of a convergent equilibrium becomes larger.

To see why (and under what conditions) these models bring about a convergent equilibrium, consider the relation between the expected difference between \( \mu_{i,j} \) and \( \mu_{i,-j} \) on the one hand and \( d_y \) and \( d_{i,j} \) on the other. If the expected difference between the stochastic term is large (which is a function of \( \sigma^2 \)) compared to the average difference between the voters’ positions and the parties’ positions (which is a function of the distribution of voters and the maximum distance between parties), almost all combinations of partisan strategies (including of course the strategy combination that converges to the mean voter’s position) are equilibria. In other words, if the distance between two parties is smaller than the average random factor, parties must necessarily assume that voter vote randomly. It is only under conditions resembling an electoral lottery that a convergent equilibrium emerges. As an unavoidable consequence of this type of models, all combinations of partisan strategies – including of course the convergent mean equilibrium – become equilibria if the stochastic term is relatively large.

Thus, equilibria in a multi-dimensional policy space and multi-party systems are possible, but this possibility carries a hefty price tag: These models typically (and perhaps inevitably) satisfy only the first and the second of our criteria of a ‘good’ model outlined above. Despite their success in isolating an important explanandum and in singling out the essential ontological units, they miss to formulize the strategic interaction between parties. Most importantly, albeit such models bring about equilibria, they still fail to identify a unique equilibrium that

\[ \text{id} \]

Ironically, researchers have also employed probabilistic voting models to demonstrate that divergent equilibria in an one-dimensional policy space are possible.
would enable researchers to cast point predictions. Rather, they prove the existence of only one among many equilibria.

Upon closer investigation of these deficiencies, we find parties’ maximization strategies to be dependent on the other parties being fixed at some point in the policy space (e.g. Lin et al 1999; Adams 1999). This assumption decides over whether the equilibrium points found by these models are stable. When we allow parties to take turns in choosing policy positions, we see that the equilibrium obtained is not unique, but only one among many. The absolute number of equilibria is a function of the size of the stochastic term relative to the distance between parties’ positions in the policy space. It is far from obvious why the mean of voters’ preferences should be chosen as the point of equilibrium. When the stochastic term in voters’ utility calculation grows sufficiently large, any point in the policy space turns into an equilibrium. Thus, the mean is merely a special case and – as evidence from partisan competition in continental European countries suggests – it is obviously not the equilibrium ‘real’ parties choose. For real world elections, it is easy to find contradicting empirical evidence.

Therefore, alternatives to the search for Nash equilibria are needed. In the remainder of the paper, we discuss the promising alternative of Markov perfect equilibria and stochastic equilibria. Before we do so, we briefly illustrate the computational model we use throughout this paper.

3. Setup of the Simulation

Before we turn to discuss how Markov perfect equilibria and stochastic equilibria can be used to analyze games that do not result in unique pure or mixed strategy Nash equilibria, we briefly describe the basic setup of our version of the usual multi-party, n-dimensional partisan competition. This model is similar to but not identical with the models employed by Lin et al. (1999) and Adams (1999).
As commonly used in the literature we employ a computational model to explore the properties of multi-party competition in a two-dimensional policy space. The political arena of our simulation is made up of two types of actors, voters and parties. We assume 1000 virtual voters with preferences drawn from a bivariate normal distribution. Accordingly, there are two uncorrelated policy dimensions, \( x \) and \( y \). The policy space is restricted to a range from 0 to 1 on both dimensions, while the density function of voter preferences is bell shaped over both dimensions, with means of around 0.5, respectively.

Altogether, we ran 696 experiments. Each experiment consists of 100 iterations. The basic rules are simple: Parties maximize their shares of votes by searching the whole policy space and by comparing the electoral outcomes of each position, given voters' preferences and the positions of the other parties. In other words, parties choose the global maximum rather than a local maximum. In our program parties move in steps of 0.01. This amounts to \( 100 \times 100 = 10'000 \) positions per iteration being assessed by each party. For every position a party takes, it assesses the vote share it would receive if it were to stick to this position. In

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7 The simulation was written in Object Pascal, using the Borland Delphi 7 development environment. A version of the program with reduced options can be obtained from the authors upon request.

8 Parties' initial positions are assigned randomly for each experiment.

9 The latter search criterion is not uncommon in the literature and it is known as 'up the hill' maximization behavior. In these models parties change platforms in one dimension as long as their votes or share of votes increases. They stop if further moves in that direction reduce expected votes. This assumption does not guarantee that parties maximize votes, because parties choose local rather than global maxima (see, for instance, Kollman/Miller/Page 1997; Kollman/Miller/Page 1992)

10 The position of a party is augmented by a small error drawn from a normal distribution with mean 0 and standard deviation 1, divided by 10'000. This serves to prevent parties from moving to exactly the same position, whereby a voting decision based on minimum distance considerations would be made impossible.
doing so, the party takes as given the positions of the other parties. At the end of the search, the party moves to the position where its share of votes is largest. Parties take turns in optimizing their positions; their move order is randomly assigned.

Given this setup, parties are prone to make decisions which are not strategic. While they are fully informed with respect to voters’ ideal points, they consider only current positions of their competitors, rendering their maximization behavior non-strategic. Thus, our parties do not use forward induction strategies. We have opted for this setup, because allowing for strategic behavior of parties would exhaust the computational capacities of most modern computers.

Votes are awarded to parties based on minimum distance considerations. Each voter votes for the party coming closest to the voter’s policy position. Distance between voters and parties is assessed based on

\[ d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} , \]

where \( d_{ij} \) denotes distance between voter \( i \) and party \( j \), \( x \) and \( y \) stand for the positions on the two policy dimensions. Voters maximize utility by minimizing policy distance. Thus, we assume a standard deterministic voting approach.

In order to grasp the properties of our computational model, we allowed a number of parameters to vary across experiments, namely the number of parties, the degree to which voters take into account partisan positions from the previous iteration, and voters’ inclination to abstain from voting due to alienation motives. The values for each parameter were assigned by random, drawn from a uniform distribution. Table 1 gives an overview of the value ranges for these parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Parties</td>
<td>2</td>
<td>6</td>
<td>4.49</td>
<td>1.30</td>
</tr>
<tr>
<td>Importance of past position</td>
<td>0</td>
<td>1</td>
<td>0.52</td>
<td>0.28</td>
</tr>
<tr>
<td>Inclination to abstain from voting</td>
<td>0.1</td>
<td>1.99</td>
<td>1.04</td>
<td>.54</td>
</tr>
</tbody>
</table>

*Table 1: Varying parameters and their value range*
Additionally, the order in which parties move was assigned by random for each single iteration, as already mentioned above.

Let’s take a closer look at what these varying parameters actually mean for the behavior of our virtual voters. While, from a voter’s perspective, the number of parties does not seem to matter much (the voter can simply choose among a larger number of competing political alternatives), the impact of the importance of past positions ("memory") and the inclination to abstain from voting due to alienation motives ("alienation") is potentially huge.

The "memory" parameter determines the degree to which voters "remember" the positions of a party from the previous iteration. That is, they assess the distance between their ideal point and the position a party takes not only on the basis of where the party currently locates, but also take into account a party’s past position. The meaning of this is straightforward: Parties pay a (varying) price for changing positions too swiftly. The minimum distance assessment from eq. 3, therefore, becomes:

\[ d_{i,j} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + \delta \sqrt{(x_i - x_{i-1,j})^2 + (y_i - y_{i-1,j})^2}} \],

where \( x_{i-1,j} \) and \( y_{i-1,j} \) denote position of party \( j \) in the previous iteration on policy dimensions \( x \) and \( y \), respectively, and \( 0 \leq \delta \leq 1 \) captures the size of the "memory" parameter.

Turning to vote abstention due to "alienation", observe, first, that this parameter captures only one out of two possible motives for abstention. Voters may choose not to vote either because they feel that none of the parties is close enough to their own ideal point, or else that the parties are too close to each other, and that this closeness renders them indistinguishable. These two motives, "alienation" and "indifference", are frequently discussed in the literature (Hinich/ Ordeshook 1970). In our computational model, we consider only "alienation" motives for abstention. Note, second, that with larger values of the "alienation" parameter, voters' inclination not to vote actually declines. This is brought about by the model specification, where a voter votes for a party if and only if a) the distance between this party and the voter's ideal point is smaller than the distance
between the voter’s ideal point and all other parties, and, b) this minimum distance is smaller than the "alienation" parameter. In other words: When the "alienation" parameter assumes large values, parties are allowed to move far away from a voter's ideal point while that voter still casts its ballot for the party, as long as all other parties are even farther away. Conversely, if the parameter that captures "alienation" is small, parties have to move closely to a voter's ideal point in order to get his or her vote. In case a party moves away too far, the voter chooses not to vote at all, even though no other party is closer to its most preferred policy position.

With these varying parameters, we are able to generate data that are open to analysis by statistical means. Before we actually turn to taking a closer look at the properties of the model, the following section lays the ground for such analysis by exploring two analytical tools that have so far been largely neglected.

4. Deriving Hypotheses in the Absence of Unique Nash Equilibria

As we have contended in the literature review, scholar’s obsession with unique Nash equilibria results from the widespread belief that models inducing multiple equilibria or even disequilibria do not make clear point predictions. As Rebecca Morton has put it: “[...] disequilibria is the case where no equilibrium exists in the model. Therefore, we can make no predictions about the likely outcome of the model” (Morton 1999: 182). This belief stems from the late 1970s, when William Riker (1980) dubbed Political Science the “dismal science”, simply because disequilibria were so pervasive.

It is certainly correct that the absence of Nash equilibria precludes the formulation of point predictions. However, contrary to common wisdom the absence of Nash equilibria does not preclude the derivation of hypotheses from a model.

Specifically, even if there are no Nash equilibria, researchers can identify Markov perfect equilibria (MPE) on which probabilistic predictions of an outcome can be based (see, for instance Fudenberg/ Tirole 1995: 503ff.). In this section, we
introduce an analytical mechanism deduced from MPEs. We suppress the time dependency of MPEs, thereby transforming them into stochastic equilibria. In stochastic equilibria, parties’ strategies oscillate around a mean position. Hence, stochastic equilibria permit researchers to obtain probabilistic point predictions.\textsuperscript{11} The next subsection discusses this point more thoroughly. This procedure allows to derive multiple hypotheses, because we may vary parameters of the model under inspection and analyze possibly resulting differences in the stochastic equilibria by standard statistical techniques.

4.1. Deriving Probabilistic Predictions From Markov Perfect Equilibria

Markov prefect equilibria are strategy combinations chosen by actors $i \in N$ (for analytical convenience the number of actors should be small, but this is not a necessity), where each single strategy combination is the set of strategies chosen by the actors. In comparison to Nash equilibria, MPEs do not solely depend on the outcomes of various strategy combinations. Rather, each single MPE is determined by the state variables, which in turn are influenced by the history of interactions among actors.

More specifically, in Markov perfect equilibria the utility of actor $i$ depends on the set of available strategies and the state variables including the strategies chosen by the other actors. The game is defined by state variables $k \in K$, action spaces $A_i(k)$ and possibly a transition function which gives the probability that the next state will be $k_{t+1}$ if the current state is $k_t$ and actor $i$ plays $a_i$ (Fudenberg/Tirole 1993: 503). Both the state and the other actors’ strategies are a function of the history of the game.\textsuperscript{12} Thus, a MPE is defined by $\{a_{i,t}, a_{i+1,t}, a_{N,t}\}$.

\textsuperscript{11} Admittedly, we cannot longer explain why a particular strategy set is a MPE if we ignore the time dimension. Given the advantages of our method we are, however, confident that it is worth to pay this price.

\textsuperscript{12} As in Fudenberg and Tirole (1993: 503) we are simplifying actors’ choices to depend only on the state of the game instead of being based upon the entire history of the game.
To illustrate the logic of MPEs, table 2 presents a limited number of iterations cut out of one of the simulations we discuss in greater depth in section 5.

<table>
<thead>
<tr>
<th>Iteration</th>
<th>Party 1</th>
<th>Party 2</th>
<th>Party 3</th>
<th>Party 4</th>
<th>Move Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>0.51, 0.46</td>
<td>0.51, 0.47</td>
<td>0.51, 0.47</td>
<td>0.51, 0.49</td>
<td>3, 2, 1, 4</td>
</tr>
<tr>
<td>81</td>
<td>0.5, 0.53</td>
<td>0.51, 0.51</td>
<td>0.51, 0.5</td>
<td>0.5, 0.53</td>
<td>3, 2, 1, 4</td>
</tr>
<tr>
<td>82</td>
<td>0.49, 0.55</td>
<td>0.52, 0.48</td>
<td>0.52, 0.49</td>
<td>0.48, 0.56</td>
<td>3, 2, 1, 4</td>
</tr>
<tr>
<td>83</td>
<td>0.47, 0.57</td>
<td>0.52, 0.49</td>
<td>0.52, 0.48</td>
<td>0.47, 0.57</td>
<td>1, 2, 3, 4</td>
</tr>
<tr>
<td>84</td>
<td>0.46, 0.58</td>
<td>0.53, 0.47</td>
<td>0.54, 0.46</td>
<td>0.44, 0.59</td>
<td>1, 2, 3, 4</td>
</tr>
</tbody>
</table>

First number in each cell denotes party position on x; the second number in each cell stands for party positions on y.

Table 2: Markov perfect Equilibria in Spatial Partisan Competition: An Example

Table 2 exhibits a typical Markov chain of equilibria in spatial competition. Observe the absence of stability and convergence. However, the lack thereof does not necessarily lead to a situation in which parties choose their platforms by random. Rather, the position each party takes shows regularities and relative stability. For instance, party 1 varies its position between 0.46 and 0.51 on x and between 0.46 and 0.58 on y.\(^\text{13}\)

Even from this relatively straightforward and unsophisticated example we derive two hypotheses: First, in multi-party competition pervasive centrifugal forces prevent parties from approaching the mean voter position. Our model therefore predicts observable deviations from the mean voter outcome and thus from the ‘convergent equilibrium’ identified by Lin et al. (1999) and Adams (1999). And second, in four-party systems party positions tend to converge in pairs but diverge between pairs.

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\(^{13}\) This degree of stability is a consequence of the assumption that voters ‘remember’ the party position of the previous election and consider this ‘historical’ fact when casting their vote. We analyze the impact of voters remembering past partisan positions more closely in section 5.2. For a description of the implementation of this feature see section 3.
In comparison to the simulation model suggested by Lin et al. and Adams, our model does not include a random element in voters’ utility functions. While Lin et al. and Adams end up with results where almost all combinations of partisan positions are Nash equilibria, we obtain no Nash equilibria, but can identify systematic patterns in partisan behavior. Whereas Lin et al. and Adams, are unsuccessful in eliminating unlikely outcomes, our method largely narrows down the possibility space. We therefore claim that our approach lends itself more readily to empirical investigations about how opportunistic parties choose electoral platforms.

4.2. Deriving Hypotheses by the Analyses of Stochastic Equilibria

The analysis of Markov perfect equilibria is not limited to the identification of outcomes that are more probable than others. MPEs also allow to win hypotheses from comparison between different Markov dynamics. Variants of Markov dynamics can be generated by running a connected set of simulations where one or more parameters vary. To the extent parameters have real world equivalents, the hypotheses can easily be tested against real world evidence.

Markov equilibria are combinations of strategy choices of all actors at each time interval \( t \). When we ignore the time information, these MPEs condense into ‘stochastic equilibria’. In doing so, we perceive the strategy choice of each single actor as if it included a stochastic element, even though the actors’ choices are determined by the current state. For instance, the strategy choices of actor 1 in table 1 can be described as oscillating around the voters’ mean positions \( x=0.497 \), \( y=0.515 \) with average distance from the mean position of 0.049.

To ignore the time dimension of MPEs renders computation of each single equilibrium impossible. What do we gain by doing so? Instead of highlighting the peculiarities of each single outcome and the time dimension of MPEs, stochastic equilibria stress the average behavior of the system under analysis. The move from MPEs to stochastic equilibria offers insights into the logic governing macro systems such as the party system.
Condensing MPEs to stochastic equilibria would still be pointless if all available strategy combinations had the same ex ante likelihood regardless of the history of the game $h_i \equiv \sum_{t=1}^{T} a_i$. In this case, the stochastic equilibria of each single actor would spread over the entire action space and we would not be able to formulate hypotheses. But, as our examples demonstrate, this must not necessarily be the case. With no exception, we found stochastic equilibria in simulations of spatial partisan competition that had significantly smaller average distances from the mean position of voters’ than half of maximum distance of the policy space. In other words, we rarely observe random distributions of party positions. Consequently, we are able to identify ‘subsets’ of each party’s action space $S_i(k) \subset A_i(k)$.

To illustrate the logic of stochastic equilibria, consider the following example, again drawn from one of the computational experiments we analyze more closely in section 5:14

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14 In this example, we consider only the final 40 out of 100 iterations. Distortions brought about by parties' initial moves when they relocate from their randomly assigned starting positions can thus be neglected.
Figure 1: Endogenous Partisan Preferences in Spatial Politics

Figure 1 is based on a simulation of electoral competition between four parties that maximize vote shares in a two-dimensional policy space. This example is typical insofar as we choose an experiment where both the "alienation" and the "memory" parameter are close to their sample mean.\(^{15}\)

We observe that the four parties neither converge to the origin and that parties do not vary their platforms over the entire action space. Quite to the contrary, party positions concentrate in two well defined clusters to the right and to the left of the mean voter’s position. Each cluster encompasses two parties with convergent positions.

\(^{15}\) The values are 0.505 for the degree to which voters take into account parties’ positions from the previous iteration (sample mean = 0.522), and 1.06 for the "alienation" parameter that determines voters’ inclination to abstain from voting (sample mean = 1.04)
Apparently, though we cannot obtain Nash equilibria in pure or repeated strategies, this does not mean that partisan behavior is chaotic or purely random. Figure 1 identifies two relatively small subsets in the entire action space where all Markov perfect equilibria are located. The boundaries of the areas of Markov perfect equilibria form clearly identifiable stochastic equilibria. They share the undesired characteristics of games with multiple equilibria in the sense that they do not allow to make unique point predictions. Nevertheless, stochastic equilibria enable researchers to derive predictions based on probabilistic assessment of partisan behavior. For instance, in figure 1, party 1 chooses a position which oscillates around $x=0.553$ and $y=0.428$ with standard deviations of 0.012 and 0.017, respectively. Table 3 presents summary statistics for all parties in our example.

<table>
<thead>
<tr>
<th>Party</th>
<th>$x_i$ Min-Max</th>
<th>$y_i$ Min-Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Party 1</td>
<td>0.53 - 0.57</td>
<td>0.40 - 0.46</td>
</tr>
<tr>
<td>Party 2</td>
<td>0.51 - 0.58</td>
<td>0.39 - 0.47</td>
</tr>
<tr>
<td>Party 3</td>
<td>0.41 - 0.46</td>
<td>0.57 - 0.63</td>
</tr>
<tr>
<td>Party 4</td>
<td>0.41 - 0.45</td>
<td>0.57 - 0.63</td>
</tr>
</tbody>
</table>

Table 3: Example of average partisan positions

When analyzed over all iterations in this particular experiment, parties chose a mere 108 out of 10'000 possible strategy combinations. 98.92 percent of possible positions are thus left uncovered. Considering only the last 40 iterations – the iterations on which figure 1 and table 3 are based – the number of positions parties actually adopt drops to 55. Of these, the 18 strategy pairs most frequently assumed account for almost 63 percent of the positions where parties located during iterations 60 to 99 in the experiment under investigation.

Apparently, there are clear cut boundaries to the positions parties adopt. Just by visual inspection we are already able to identify regular patterns far from
random. However, as the following subsection demonstrates, this research strategy can also be explored more systematically.

4.3. Exploring Stochastic Equilibria by Standard Statistical Techniques

The analysis of stochastic equilibria is not restricted to comparative statics. If we analyze the different outcomes of the simulation over time, we are able to derive ‘comparative dynamics’. We get a picture of the strategy choices across iterations, treating the various action sets chosen by the parties as stochastic equilibria. These can in turn be analyzed by standard statistical techniques.

To start with, consider the analogy between real world data and computationally generated data. If researchers are faced with real world data, statistical analysis serves as the standard tool with which to summarize many observations by a small amount of mathematical concepts. This becomes especially important when hypotheses about the influence of some variable on another quantity of interest are to be assessed. The task we are facing when we deal with data stemming from computational experiments is not unequal: Quantitative methods can reduce complexity in the data and help to identify patterns.

Most importantly, statistical analyses of computed data can resolve the aggregation problem common to social science models. Even if the computational model does not include stochastic elements so that agents’ behavior on the micro level is fully determined by the assumptions, the interactions between single agents can easily result in the emergence of behavior at the macro level that cannot be worked out analytically. Yet, there are limits to the analogy between real and computed data, because computed data cannot be used to ‘reject’ a hypothesis; it only allows to assess the consistency of assumptions and hypotheses.

Still, analyzing data produced from computer experiments by statistical means serves an important purpose: Statistical methods can identify the connection between assumptions about behavior on the micro level and macro phenomena. To give an example: The behavior of parties in the simulation we already briefly
discussed is determined by the rationality assumption, the distribution of voters, and the ‘state’ of the game, i.e. the position of the other parties, denoted \(-i\), when party \(i\) moves. Thus, if we know the current state, we can compute the Markov equilibrium. After turning to the analysis of stochastic equilibria, neither the micro behavior nor the outcome on the macro level are fully determined; but we can now use the location and character of these stochastic equilibria across different computational experiments as the dependent variable in research questions that hypothesize about the influence of different aspects of multi-party competition.

The easiest way to introduce variance across experiments is to take different parameter values. We could, for example, change the size of the political system by defining alternative numbers of parties in the arena. Other parameters potentially subject to change include the distribution of voters’ preferences, the number of policy dimensions, and the order in which parties optimize their positions (fixed move order vs. random move order vs. simultaneous optimization). Another approach consists of adding features to the computational model, i.e. to make it more complex, thereby possibly enhancing its resemblance to real world phenomena. If, for instance, we dropped the assumption of perfectly informed and rational voters that cast their vote inevitable for the party minimizing the distance to their respective ideal points, we would get a different picture of the artificially produced electoral outcomes (Plümper/ Martin 2003).

To illustrate the use of combining stochastic equilibria with varying parameters and parameter values we will briefly elaborate on the example of vote abstention. As mentioned above, our computational model includes a parameter that captures abstention out of alienation motives, i.e., a voter may not cast his or her vote if the distance between their ideal policy position and the positions of every party is larger than some randomly assigned value – which, in our model, is called the "alienation" parameter. Translated into real world phenomena, voters feel alienated in the sense that none of the parties in the political arena matches their interests, thus rendering voting a meaningless effort.
We capture this phenomenon by including abstention into the model and by varying the threshold for this feature across experiments. Figures 2 and 3 show the locations of stochastic equilibria for two alienation parameter values. Note that higher alienation parameter values result in higher turnout rates, i.e. vote abstention drops with increasing values for "alienation".

![Stochastic Equilibria](image)

*Figure 2a: Stochastic Equilibria when alienation = 0.64 (high electoral turnout)*
Visual inspection reveals that dispersion of stochastic equilibria changes with varying parameter values. In the first case (low vote abstention; turnout rates at about 95 percent) the equilibrium is clearly concentrated in two areas of the policy space, while in the second case (high vote abstention; turnout rates at about 63 percent), parties' locations are much more dispersed. We therefore expect the alienation parameter to influence the mean distance between partisan platforms. Note that this result is brought about solely by the variation of the "alienation" parameter, because other parameters were kept constant: The number of parties is four in both experiments on which figures 2a and 2b are based, while the "memory" parameter that determines importance of past partisan position varies only slightly from 0.83 in experiment 147 (low turnout) to 0.88 in experiment 630 (high turnout).

5. **Analysis of the Model’s Properties**
Computational models can be viewed as equivalents to controlled experiments in the laboratory, because we can vary parameters of interest and use the output of the simulations as data that lend themselves readily to the analysis by conventional statistical means. In this section we analyze data generated by the computer simulation we ran with relatively standard statistical means. This serves to identify the properties of multi-party competition in a two-dimensional policy space. Using the concepts of stochastic equilibria outlined above, we show that, despite the absence of unique equilibria in pure strategies, these computationally generated data allow to derive hypotheses which can be tested against data from real world phenomena. Specifically, we derive three types of hypotheses – on radicalism of partisan platforms, on volatility of partisan platforms, and on the average distance between parties – from our model.

We start by briefly describing the type of data generated by the model and discuss the estimation techniques we employ.

Data drawn from computational models are in one way not different from data that are assembled by observing and recording real world phenomena. Data from both sources can be transformed into datasets with columns and rows where each column is assigned a variable and each intersection between columns and rows contains a single observation. Such datasets can be analyzed with conventional statistical software packages. Viewed from another angle, however, the differences between data generated by computer experiments and real world data are huge. Where real world data are prone to error and missing observations due to incorrect measurement, lack of information and a host of other factors, artificially produced data contain only what has been coded into the computer simulation. Each variation in outcome can be traced to variation of parameters over which control is fully guaranteed. Thus, researchers can be confident that inferences drawn from the data under analysis are not questionable artifacts of measurement or coding errors, but are solely based on perfectly controlled variation of parameters that have been plugged into the model. Because the costs of computing time have declined so dramatically, it is now even possible to run experiments that contain more than just a few
parameters with a wide value range and still make sure that the results of (almost) every conceivable parameter combination show up in the data.

In our computational model of multi-party competition in a two-dimensional policy space, the most important parameters that were varied over experiments are the number of parties competing for vote shares, the inclination to abstain from voting out of alienation motives, and voters’ memory of past partisan positions. Also subject to variation was the initial position of parties in each experiment (which was assigned by random), and the order in which parties took turns in maximizing their vote shares, taking as given the position of the other parties in the political arena.

Each of these experiments consists of 100 iterations. Admittedly, this is an arbitrary number and we could have decided to run 65 or 253 iterations per experiment. However, initial inspection of the data reveals that once the first 10 or so iterations are over and parties have adjusted from their randomly assigned initial positions to vote share maximizing locations in the policy space, outcomes are no longer very volatile.

An iteration is made up of one move of each party. That is, in every iteration, all parties choose their position exactly once. After the last party has moved to the position that it has assessed as maximizing its electoral result, the positions of all parties on both policy dimensions are recorded and written to file. One observation, therefore, consists of the positions of one party in a given experiment in a given iteration.

Altogether, we (or rather: the computer) ran 696 experiments with 100 iterations each. This leaves us with 69'600 iterations which sum up to 278'700 observations. Note that this number is only slightly off the mark when compared to the

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16 This does not rule out that, say, in iteration 812'453 a dramatic change of results could occur. However, this is the same problem that potentially plagues all experiments, and in fact, all research that draws on observations, be they taken from the real world or from artificially crafted environments. All we can say is that we assess the probability of such changes as rather low in the context of our computational model.
theoretically expected 278,400 observations with an average of four parties per experiment.

For every observation, we generated a number of variables that serve as dependent variables in the regressions below. Specifically, from the positions each party took on the two policy dimensions, three variables were calculated that capture a) the distance between each partisan position and the mean positions of voters ("radicalism"); b) the difference between a party’s position in the current iteration and its position in the iteration before that ("volatility"), and; c) the mean distance between a party's position and the positions of all other parties ("mean distance").

Of the 100 iterations in each experiment, we use only the last 25 for analysis by regression techniques. Still, we end up with a number of observations that is large, perhaps so large that estimations are overconfident. We therefore demonstrate in appendix B that a drastic reduction in the number of observations has an impact on the estimated coefficients and standard errors of some variables. A conservative interpretation of our model would therefore allow to derive less hypotheses than we actually formulate.

The simulation allows the identification of a variety of parameters. To derive hypotheses on the maximizing behavior of parties in multi-party systems we use the discount parameter \( \delta \), the alienation parameter, the number of parties, a dummy taking the value 1 if the number of parties is even (0 else), and radicalism, that is: the deviation of a party’s position from the voters’ mean. As dependent variables we use radicalism (for obvious reasons, we do not consider radicalism as a covariate in this model), the change in the parties’ positions from iteration \( t \) to iteration \( t+1 \), and the mean distance between a party and all other parties in one iteration.

An important concern arises from the distribution of the dependent variable: Neither our measure for political radicalism, nor our measures for political volatility and proximity between parties is normally distributed. To avoid potential bias, we employ a robust Huber-White-sandwich estimator in the regressing political radicalism and proximity on a battery of right-hand side
variables and we use a truncated model (which fits a regression model from a sample drawn from a restricted part of the population). Note that we do not have theoretical priors about the distribution of the dependent variables. All choices of estimators are the results of visual inspection of the dependent variable rather than of theoretical information. However, a simple OLS regression would not yield substantially different results. All procedures we employ are consistent and efficient.

Table 4 displays the results of the analysis of our computational model.

<table>
<thead>
<tr>
<th>Estimation Technique</th>
<th>Radicalism of Parties</th>
<th>Platform Volatility</th>
<th>Mean Distance Between Parties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>-0.0493 (0.0005) ***</td>
<td>0.0244 (0.0007) ***</td>
<td>-0.0438 (0.0004) ***</td>
</tr>
<tr>
<td>Discount Parameter</td>
<td>0.0374 (0.0004) ***</td>
<td>-0.0021 (0.0005) ***</td>
<td>0.0265 (0.0003) ***</td>
</tr>
<tr>
<td>Number of Parties</td>
<td>0.0273 (0.0001) ***</td>
<td>0.0075 (0.0001) ***</td>
<td>0.0185 (0.0001) ***</td>
</tr>
<tr>
<td>Even Number of Parties</td>
<td>0.0000 (0.0002)</td>
<td>-0.0248 (0.0003) ***</td>
<td>0.0022 (0.0002) ***</td>
</tr>
<tr>
<td>Move Order</td>
<td>-0.0005 (0.0003) *</td>
<td>0.0050 (0.0004) ***</td>
<td>0.0025 (0.0002) ***</td>
</tr>
<tr>
<td>Radicalism</td>
<td>-0.0378 (0.0037) ***</td>
<td>-0.0378 (0.0037) ***</td>
<td>0.7867 (0.0022) ***</td>
</tr>
<tr>
<td>Alienation</td>
<td>-0.0002 (0.0002)</td>
<td>-0.0011 (0.0005) ***</td>
<td>-0.0011 (0.0003) ***</td>
</tr>
<tr>
<td>Nobs</td>
<td>69675</td>
<td>69675</td>
<td>69675</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>131546.48</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>22311.52 ***</td>
<td>10272.18 ***</td>
<td>87786.53 *** ***</td>
</tr>
</tbody>
</table>

Table 4: Inductive Analysis of Model Properties (variables that turn out to be not robust in the permutation test reported in appendix B are in italics)

We discuss all three regression models in turn.

In respect to political radicalism our model allows to derive three hypotheses. First of all, the radicalism of parties increases with the extent to which voters consider past partisan ideologies. To our knowledge, no formal model of endogenous partisan preferences has as yet assumed voters remembering past
positions of parties. Thus, the hypothesis seems to be a novelty of our model. It can easily be tested against real world data if we analyze partisan behavior in new democracies. Our model predicts that when new democracies mature, new parties become more radical and increase the distance between each other. Secondly, we derive the hypothesis that the number of parties tends to raise the average radicalism of political platforms. This result appears to be self-evident and can already be indirectly derived from the median-voter theorem: If platforms converge to the median voter in the 2-parties case but not in the 3-parties case, radicalism has to increase if the number of parties exceeds two. As our model suggests, this hypothesis is generalizable beyond the median voter artifact. Thirdly and finally, parties that later can change their program are slightly less radical than early movers. Clearly, our model reveals a small last mover advantage. However, the effect appears to be negligibly small. We would thus be surprised if this effect was detectable in real world data but this would nevertheless constitute an interesting piece of research.

Our second ‘dependent’ variable is political volatility. We find four substantive positive effects on political volatility: Political volatility increases with voters’ propensity to abstain and with the number of parties. In addition, the last moving party makes bigger changes in its program than earlier movers. Moreover, we find that a larger discount parameter (as expected) stabilizes partisan positions. The higher voters’ judge past positions of parties, the less attractive are changes in partisan platforms. Hence, in political systems where credibility of partisan positions is not backed by some observable record, we expect some parties to ‘jump’ through the policy space, while on average. It is therefore promising to test this hypothesis by analyzing political volatility in new democracies such as the transition countries. Finally, we observe lower changes in electoral platforms if there is an even number of parties. This result – astonishing as it may seem – mirrors known outcomes of game-theoretical models, where actors’ behavior depends on whether the number of players is even or odd. For the time being, we have to leave to empirical research to figure out if this result has real world equivalences.
Eventually, we consider the impact of the battery of ‘exogenous’ variables on the proximity of parties. We find that parties move further apart from each other the more parties do exist. At a first glance, this result seems to be counter-intuitive, but it stems from the fact that radicality also increases with the number of parties. When (some) parties move further away from the center of the policy space, they also increase the distance between them. Voter’s memory and alienation seem to have a similar effect: Both factors increase radicalism and thereby tend to raise the distance between parties. Obviously, if voters abstain from voting when parties are too far away from their ideal point, parties feel more strongly compelled to move to the ends of the distribution because there they can gain some votes which would otherwise be lost. The same is true with indifference motives for abstention: If voters are assumed to abstain when partisan platforms are very close, party leaders have an incentive to push platforms apart if (but only if) more than two parties compete in the elections. Note that in our simulation, we solely consider alienation motives for vote abstention.

Table 5 summarizes the central results of the analysis of our model:

<table>
<thead>
<tr>
<th></th>
<th>radicalism</th>
<th>platform volatility</th>
<th>partisan distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>discount parameter</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>number of parties</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>even number of parties</td>
<td>0</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>move order</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>radicalism</td>
<td>n.a.</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>alienation parameter</td>
<td>0</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

*Table 5: Summary of Hypotheses derived from the Model*

Observe that even the analysis of our relatively simple model allows the formulation of 15 hypotheses – most of which appear to be testable. If we had restricted our analysis on Nash equilibria, we would not have been able to derive
a single hypothesis. In our view, this amply demonstrates the power of our method.

6. Conclusion

The quest for equilibria, and, more generally, for analytically accessible formal models should not be taken as a 'natural' starting point for doing analytical work in political science. When unique Nash equilibria in pure strategies do not emerge 'naturally', scholars are not restricted to refinements or the choice of assumptions that produce equilibria but at the very same time lack connection to real world phenomena.

This paper has suggested a method of analysis that takes advantage of Markov perfect equilibria. As we have shown, MPEs can be condensed into stochastic equilibria, which can be analyzed with all standard statistical tools including regression techniques. Our method is not confined to analytically tractable models. In cases where the model becomes too complex to be solved analytically, computer simulations serve as a valuable tool. The combination of computationally generated data, the notion of stochastic equilibria and conventional statistical means potentially paves political science a way out of its 'dismal' state. Therefore, Riker's assertion can no longer claim to be authoritative. To the contrary, our method establishes the prospect of a political science no longer confined to the treatment of disequilibria as problems. Disequilibria are pervasive in the world we are living in. Refining away these disequilibria is therefore not necessarily a good idea.

On a more substantive account, we have identified a number of factors that can be expected to influence rational partisan strategies in multi-party, multi-dimensional competition. These insights are of potential value to the explanation of, for instance, observed differences in parties' radicalism across different countries. Such empirical research that draws on the hypotheses stated in this paper could not only lead to interesting results when assessing the relative importance of different factors influencing partisan behavior. It would also subject the method proposed herein to a test each model must undergo: the confrontation with reality.
Appendix A: Party Positioning and Number of Parties

Figures A1-A5: Party Positions when Number of Parties increases from 2 to 6

Appendix B: Test of Robustness by downsizing the Sample

We compute the bandwidth of the estimated coefficients from 25 subsample regression analyses, where each single iteration between iteration 75 and 99 serve as a subsample. Had we drawn subsamples randomly, results would not differ.

Radicalism
<table>
<thead>
<tr>
<th>coefficients</th>
<th>standard errors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>min</td>
</tr>
<tr>
<td>discount parameter</td>
<td>0.0335</td>
</tr>
<tr>
<td>number of parties</td>
<td>0.0267</td>
</tr>
<tr>
<td>even number of parties</td>
<td>-0.0016</td>
</tr>
<tr>
<td>move order</td>
<td>-0.0023</td>
</tr>
<tr>
<td>radicalism</td>
<td></td>
</tr>
<tr>
<td>alienation parameter</td>
<td>-0.0018</td>
</tr>
</tbody>
</table>

Table A2.1: Subsample Permutation Robustness Test (depvar: radicalism)

<table>
<thead>
<tr>
<th>coefficients</th>
<th>standard errors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tr>
<tr>
<td>discount parameter</td>
<td>-0.0084</td>
</tr>
<tr>
<td>number of parties</td>
<td>0.0056</td>
</tr>
<tr>
<td>even number of parties</td>
<td>-0.0274</td>
</tr>
<tr>
<td>move order</td>
<td>0.0022</td>
</tr>
<tr>
<td>radicalism</td>
<td>-0.0832</td>
</tr>
<tr>
<td>alienation parameter</td>
<td>-0.0014</td>
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</table>

Table A2.2: Subsample Permutation Robustness Test (depvar: dlastpos)
<table>
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<th>standard errors</th>
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<tbody>
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<td>mean</td>
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<td>discount parameter</td>
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<td>number of parties</td>
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</tr>
<tr>
<td>even number of parties</td>
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<td>0.0022</td>
</tr>
<tr>
<td>move order</td>
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<td>0.0025</td>
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<tr>
<td>radicalism</td>
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<tr>
<td>alienation parameter</td>
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<td>-0.0011</td>
</tr>
</tbody>
</table>

Table A2.3: Subsample Permutation Robustness Test (depvar: meandist)

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