Abstract. The present paper analyses different formulations of individual rights within a traditional social choice-theoretic framework and a game form framework. It proposes a theory of social situations as a basis for some new formulations of individual rights that better incorporates some positive aspects of liberty.

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1. Introduction

In his seminal paper Sen (1970) has introduced a subject of individual rights into social choice theory. Sen’s paradox of a Paretian liberal has provoked some controversies among economists, political scientists and philosophers (for a comprehensive survey, see Sen (1976), (1983), Suzumura (1983), or Wriglesworth (1985)). Immediately some authors (see Nozick (1974) or Bernholz (1974)) have challenged Sen’s original formulation of rights within the social choice-theoretic framework. As a result of this criticism, Gardenfors (1981) and Sugden (1985) have suggested an alternative formulation of rights within the game-theoretic framework. Both have claimed that their formulations could better capture our intuition about rights. Indeed, since the publication of a joint paper by Gaertner, Pattanaik and Suzumura (1992) the focus of most researchers has shifted from producing impossibility (possibility) results to discussing an adequacy (or inadequacy) of different formulations of individual rights (see Deb (1994), Dowding and van Hees (2003), Hammond (1996), Pattanaik (1996), Pattanaik and Suzumura (1996), Riley (1992), Sen (1992), van Hees (2000) among others).

In this paper we propose to formulate individual rights within the framework of the theory of social situations developed by Greenberg (1990). Our proposal is motivated by Gardenfors’ (1981) suggestion that an individual right can be described as a possibility for an individual i to restrict the set of social states X to a subset Y of X. However, instead of representing individual rights as effectivity functions (see Deb (1994), Peleg (1998), van Hees (1999) for such representation), we formalize them using rights-exercising protocols within the theory of social situations. Such representation allows us to avoid some technical problems associated with the formulation of individual rights.
within cooperative or non-cooperative game-theoretic frameworks. Moreover, it forces us to specify precisely the nature of different rights-exercising protocols that reflect the differences in social environment or institutions. The theory of social situations also provides us with a rather attractive stability or consistency criterion that can be applied uniformly to all rights-exercising protocols.

The plan of this paper is as follows. Section 2 introduces Sen’s and a game form formulations of individual rights and highlights some tensions between them. Section 3 presents an analysis of individual rights within the framework of the theory of social situations. Section 4 concludes with some brief remarks.

**Sen’s and game form formulations of rights**

Sen (1983) explicitly adopts the following formulation of individual rights:

(2.1) If two social states, x and y, differ only with respect to some aspect which belongs to individual i’s recognized personal sphere (RPS), and if i strictly prefers x to y, then in any choice situation where x is available, the society should not choose y.

It should be pointed out that (2.1) is only a necessary condition for individual i to have a right. It restricts social choice, based on individual i’s preferences over two social states which differ only with respect to i’s RPS. Gaertner et al (1992), Pattanaik (1996) highlight, using Gibbard’s (1974) example, some intuitive problems with Sen’s formulation of individual rights. They claim that each individual should be able to determine a certain feature or aspect of any social state provided that an aspect is within that individual’s RPS. For example, each person has a right to choose the colour of his own shirt. Then it seems that the game form formulation of individual rights adequately captures this intuition. Recall that a game form specifies: (i) a finite set N of players; (ii) a
finite set $S_i$ of strategies for each player $i$ in $N$; (iii) a finite set $X$ of all feasible outcomes; (iv) an outcome function which specifies precisely one outcome for each $n$-tuple of strategies (for more on game form, see Moulin (1983) or Peleg (1984)). If, in addition, we specify preferences of the players, then we have a game in normal (strategic) form.

Continuing with the example of the right to choose the colour of his own shirt, suppose that players 1 and 2 each have two permissible strategies: wear white shirt (w) or wear blue shirt (b). Then we have four possible social states: $x=(w,w)$, $y=(b,w)$, $z=(w,b)$ and $w=(b,b)$. Notice that by choosing $w$, player 1 can ensure that only social states $x$ and $z$ will prevail where he wears white shirt. Similarly, by choosing $b$, he can ensure that social states $y$ and $w$ prevail where he wears blue shirt. Of course, what strategies players 1 and 2 will choose will depend on their preferences. Following Gibbard, assume that 1 wants to conform while 2 insists on being different. Then the following preference orderings may result:

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Suppose also that each person is completely ignorant about the other person’s preferences. Given this uncertainty, it is reasonable to assume that both 1 and 2 may follow the maximin rule. In this case 1 will choose $b$, while 2 will choose $w$. Then the social state $y$ will emerge as an outcome of their choices. Notice, however, that the pair of social states $(x,y)$ belongs to 1’s RPS and he strictly prefers $x$ over $y$. Hence according to (2.1), $y$ should not be chosen. The emergence of $y$, therefore, could be construed as a violation of 1’s right to choose the colour of his shirt.
Sen (1992), however, does not believe that this example brings out the tension between the game form formulation and his own formulation of individual rights. In fact, he believes that the game form formulation entails his own formulation of rights under some proviso (such as absence of uncertainty, or in the presence of uncertainty, the desire rankings were such that each individual would have a dominant strategy). According to Sen, “the violations of liberty with which political philosophy has been traditionally concerned have not been particularly geared to decision problems under uncertainty, or to the gap between the individual’s own choices and desires.” Hence the contribution of Gaertner et al is in enriching the classical account of liberty by bringing uncertainty into the story.

We believe that Gaertner et al are right in insisting that the game form formulation of individual rights is not in general compatible with Sen’s formulation of rights. We will try to bring out the tension between these two formulations of individual rights by using another Gibbard’s (1974) example which involves no uncertainty. Recall that Angelina wants to marry Edwin, but she will settle for the judge, who wants whatever Angelina wants. Edwin wants to remain single, but he would rather wed Angelina than see her wed the judge. Denote “both Edwin and Angelina remain single” as x, “Edwin weds Angelina” as y, and “the judge weds Angelina and Edwin remains single” as z. Then we have the following pattern of their preferences over $X=\{x,y,z\}$:

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<th>Angelina</th>
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Notice that the pair $(x,z)$ is in Angelina’s RPS and she strictly prefers $z$ over $x$. Hence according to (2.1), $x$ should not be chosen. Next, the pair $(x,y)$ is in Edwin’s RPS and he
strictly prefers x over y. Hence according to (2.1), y should not be chosen. The outcome that will emerge after Angelina and Edwin exercise their individual rights is z. Yet z is the outcome that Edwin likes the least. Gibbard (1974) suggests that in this case Edwin might be better off by bargaining his rights away or by waiving them. However, the game form formulation of individual rights provides a natural solution. Both Angelina and Edwin have two permissible strategies: remain single (s) or marry (m) (notice that the judge is a fictitious player who simply grants Angelina’s wishes). Then we have the following 2 x 2 game in normal form:

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<tr>
<td>s</td>
<td>x</td>
<td>x</td>
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<tr>
<td>m</td>
<td>z</td>
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Given the preference orderings above, the strategy m for Angelina strictly dominates her strategy s. Hence given common knowledge of rationality, Edwin knows that Angelina will not play her strategy s, which can be eliminated. Since Edwin strictly prefers y over z, his strategy m also strictly dominates his strategy s in the reduced game and hence s can be eliminated. Therefore, Angelina and Edwin will choose their strategies m leading to the outcome y. Of course, y=(m,m) is also a unique Nash equilibrium of the game. The emergence of y does not constitute any violation of Edwin’s rights under the game form formulation. On the contrary, Edwin rationally chooses his permissible strategy to marry Angelina rather than to remain single and see her wedding the judge.

If we slightly change the original Gibbard’s story by strengthening Edwin’s desire to remain single as follows: Edwin wants to remain single and feels so strongly about it that he would rather see Angelina wed the judge than to marry her. In this case we have the
Given this preference orderings, the strategy $m$ for Angelina still strictly dominates her strategy $s$ which can be eliminated. However, now Edwin strictly prefers $z$ over $y$, and hence his strategy $s$ strictly dominates his strategy $m$ in the reduced game. Hence $m$ can be eliminated. Therefore, the outcome $z=(m,s)$ will emerge which is also a unique Nash equilibrium of the game. In this case the exercise of rights under the game form formulation will coincide with the social choice or Sen’s formulation of individual rights.

3. **Individual rights and the theory of social situations**

Gardenfors (1981) has tried to formalize Nozick’s account of rights that refers to legal and philosophical analysis of the concept of a right. “The type of individual right that Sen and Nozick seem to have in mind can be presented in the form ‘i may see to it that F’ where $F$ is not a particular social state, but a condition on (or a property of) a social state (see, for example, Kanger and Kanger (1966) and Lindahl (1977)). Rights do not establish an ordering of social states but divide them into classes; if a right is exercised, some class of possible social states is excluded from further consideration and the remaining class of social states may be subject of further restrictions by the exercising of other rights (p.343).”

The game form formulation of individual rights might be considered as one possible formalization of such an intuition. For example, Edwin by exercising his right to remain single may force the choice of social states where he remains single, that is $\{x,z\}$. Notice, however, that the normal game form formulation of individual rights abstracts from the
dynamic or sequential aspects of rights-exercising. We shall see below that Gardenfors also abstracts from the sequential aspects of rights-exercising by imposing a rather artificial condition on the rights combination.

Gardenfors’ formalization takes a right as a possibility for an individual $i$ to restrict the set of social states $X$ to a subset $Y$ of $X$. A right system is defined then as a set of pairs $(i, Y)$. First, Gardenfors requires that rights of different individuals be mutually consistent, that is, different individuals should not have conflicting rights. However, in many cases we are interested in the analysis of individual rights in the situations where these rights are conflicting. For example, the right to smoke for one individual may conflict with the right to a clean air of another individual. In order to simplify his formalization of rights, Gardenfors also requires that the consistent rights of an individual could be combined without restrictions. This requirement is too strong and fails in many situations. As Gardenfors himself notices “I may at a certain point of time have the right to drink a bottle of whisky and also have the right to drive my car, but I may not exercise both of these rights at the given point of time (p. 345).”

According to Gardenfors, what rights an individual would choose to exercise will depend on his or her preferences over social states. However, since the rights-exercising may induce the sets of social states, Gardenfors extends preferences over single states to preferences over the sets of social states. He views the exercising (or waiving) of a right assigned to an individual as a move in a game. The condition on combinations of rights allows Gardenfors to ignore the sequential aspects of a set of moves and to identify the set of strategies available to an individual with the set of rights assigned to him.

In this paper, following the lead of Gardenfors, we propose to formulate and analyze
individual rights within the framework of the theory of social situations. Similarly to Gardenfors’ and the game form formulations of individual rights we distinguish between having a right and exercising that right. However, unlike Gardenfors (1981), we do not impose any restrictions on individual rights. In particular, we will allow different individuals to possess conflicting rights, and we also allow individuals to exercise their rights sequentially. Specifically, we assume that a society with rights consists of an ordered quadruple $\Omega \equiv (N,X,\{R_i\}_{i \in N},\rho)$, where $N$ is a finite set of individuals, $X$ is a finite set of social states, $R_i$ is a weak preference relation of individual $i$ over $X$ ($P_i$ is the asymmetric part of $R_i$), and $\rho$ is a rights-assignment in a society. Like in Gardenfors (1981) (see also Deb (1994), Peleg (1998), or van Hees (1999)), we assume that if individual $i$ exercises his or her rights, then the original set of alternatives $X$ is restricted to a subset $Y$ of $X$. Hence for every $Y$ we define a sub-society $\Omega^Y \equiv (N,Y,\{R'_i\}_{i \in N},\rho')$, where $R'_i$ and $\rho'$ are the restrictions of $R_i$ and $\rho$ to $Y$. With each sub-society $\Omega^Y$ we will associate a position $G^Y \equiv (N,Y,\{R'_i\}_{i \in N})$. The set of all possible positions is denoted by $\Gamma$, that is, $\Gamma \equiv \{G^Y : Y \subseteq X\}$.

The most important concept of the framework is a definition of the right-exercising protocol. By the *rights-exercising protocol* we understand a mapping $\delta$ that for every coalition $S$, $S \subseteq N$, and every sub-society $G^Y$, and each alternative $x \in Y$ specifies the set $\delta(S \mid G^Y, x) \subseteq \Gamma$. Next, we define a *standard of behaviour* as a mapping $\sigma$ that specifies the subset of $Y$ for every sub-society $G^Y$. The only requirement that we impose on the standard of behaviour is stability (or consistency). Formally, given $(N,X,\{R_i\}_{i \in N},\rho)$ and $\delta$, the standard of behaviour $\sigma$ is conservatively stable (CSSB for short) if for every $G^Y \in \Gamma$ the following condition holds:
\( x \in \sigma(G^Y) \) if and only if there exist no coalition \( S \subseteq N \) and \( G^Z \in \delta(S \mid G^Y, x) \) such that \( \sigma(G^Z) \neq \emptyset \) and moreover for all \( i \in S, yP^i \preceq x \) for all \( y \in \sigma(G^Z) \).

Alternatively, we can define a CSSB \( \sigma \) as follows. First, we define the dominion of \( G^Y \) (CDOM for short) as

\[
\text{CDOM}(\sigma, G^Y) = \{ x \in Y : \text{there exist } S \subseteq N \text{ and } G^Z \in \delta(S \mid G^Y, x) \text{ such that } \sigma(G^Z) \neq \emptyset, \text{ and for all } i \in S, yP^i \preceq x \text{ for all } y \in \sigma(G^Z) \}.
\]

Then the standard of behaviour \( \sigma \) is a CSSB if and only if for \( G^Y \in \Gamma \), \( \sigma(G^Y) = Y \setminus \text{CDOM}(\sigma, G^Y) \).

The theory of social situations framework allows different formulations of the rights-exercising protocols that we might want to employ in our analysis of individual rights. For example, individuals might want to know if sequential rights-exercising is permissible or not, or whether they can form coalitions and exercise their rights collectively by signing some binding agreements (in this case the existence of institutions enforcing such agreements is naturally presupposed). It is this flexibility that underscores certain advantages of this framework over the traditional social choice-theoretic framework.

In this paper we will limit ourselves to only three rights-exercising protocols (many more could be easily defined). Specifically, the first rights-exercising protocol prohibits both sequential rights-exercising and the formation of coalitions, that is, an individual \( i \) can exercise her right to restrict the set \( X \) to \( Y \subseteq X \setminus \rho(i) \) only if some element from \( \rho(i) \) is currently under consideration, and no prior rights-exercising was performed. Formally, this protocol is described by the correspondence \( \delta^1 \) as follows:

For every \( G^Y \in \Gamma \) for every coalition \( S \subseteq N \) and each alternative \( x \in Y \),
\[ \delta^1({i} \mid G^Y, x) = \emptyset \text{ if } Y \neq X, \]
\[ \delta^1({i} \mid G^X, x) = \{G^Z \in \Gamma: Z = X\setminus \rho(i) \text{ and } x \notin Z\}, \]
\[ \delta^1(S \mid G^Y, x) = \emptyset \text{ otherwise.} \]

The next rights-exercising protocol allows individuals to exercise their rights even if other members of society did so before. However, it still prohibits the formation of coalitions. Formally, this protocol can be defined as follows:

For every \( G^Y \in \Gamma \), for every coalition \( S \subseteq N \) and each alternative \( x \in Y \),
\[ \delta^2({i} \mid G^Y, x) = \{G^Z \in \Gamma: Z = Y\setminus \rho(i) \text{ and } x \notin Z\}, \]
\[ \delta^2(S \mid G^Y, x) = \emptyset \text{ otherwise.} \]

The third rights-exercising protocol allows both the formation of coalitions and sequential rights-exercising. Formally, this protocol might be described as follows:

For every \( G^Y \in \Gamma \), for every coalition \( S \subseteq N \) and each alternative \( x \in Y \),
\[ \delta^3(S \mid G^Y, x) = \{G^Z \in \Gamma: Z = Y \setminus \bigcup \{\rho(i) : i \in S\} \text{ and } x \notin Z\}. \]

Using Theorem 5.4.1 from Greenberg (1990), the following result can be easily established:

**Proposition 1.** Every situation \((\delta^k, \Gamma), k = 1,2,3\) admits a unique CSSB.

As an illustration of the flexibility of the theory of social situations framework, we apply it to the Gibbard’s example from Section 2. We have the following society with rights: \( N = \{A(\text{Angelina}), E(\text{Edwin})\}; X = \{x,y,z\}; \) the preferences are exactly like in the Gibbard’s example; we will first examine the situation where Angelina and Edwin both have the right to marry, that is \( \rho(A) = \text{marry}, \rho(E) = \text{marry} \). In this case, by exercising her right, Angelina may restrict the original set \( X \) to \( Y = \{y,z\} \) and similarly Edwin by exercising his right may restrict the set \( X \) to \( Y = \{x,y\} \). We want to invest
tigate how different rights-exercising protocols may affect Angelina’s and Edwin’s exercising of their rights. The following proposition provides an answer.

**Proposition 2.** The unique CSSB, $\sigma^k$, for the associated situation $(\delta^k, \Gamma)$, $k = 1,2,3$ satisfies:

1. $\sigma^1(G^X) = \{y\}$; 
2. $\sigma^2(G^X) = \{y,z\}$; 
3. $\sigma^3(G^X) = \emptyset$.

**Proof.** We shall prove only (i) ((ii) and (iii) can be handled similarly). After Angelina exercises her rights, $x \in \text{CDOM}(\sigma^1, G^X)$. Similarly, after Edwin exercises his rights, $z \in \text{CDOM}(\sigma^1, G^X)$. Therefore, $\sigma^1(G^X) = X \setminus \text{CDOM}(\sigma^1, G^X) = \{y\}$. Notice that regardless of the differences in the rights-exercising protocols the outcome of the rights-exercising through the theory of social situations coincides with the outcome of the rights-exercising through the normal game form. It is not compatible with the outcome emerging through the exercise of rights within the social choice-theoretic framework. What will happen, however, if we will endow Edwin with the right to remain single along with his right to marry? Now Edwin by exercising his right to remain single can restrict the original set $X$ to $Y = \{x,z\}$. He can also restrict the set $X$ to $Y = \{x,y\}$ by exercising his right to marry. How does the difference in the rights-exercising protocols affect Angelina’s and Edwin’s exercising of their rights in this case? The following proposition answers this question.

**Proposition 3.** The unique CSSB, $\sigma^k$, for the associated situation $(\delta^k, \Gamma)$, $k = 1,2,3$ satisfies:

1. $\sigma^1(G^X) = \{y\}$; 
2. $\sigma^2(G^X) = \{y,z\}$; 
3. $\sigma^3(G^X) = \emptyset$.

**Proof.** We will prove (i) and (ii) ((iii) is left to the reader). In case of (i), Angelina will exercise her right to marry, hence we have $x \in \text{CDOM}(\sigma^1, G^X)$. Similarly, Edwin will
exercise his right to marry, therefore \( z \in \text{CDOM}(\sigma^1, G^X) \). However, Edwin will not exercise his right to remain single under the rights-exercise protocol \( \delta^1 \). The reason is simple. If he chooses to exercise this right he will induce the sub-society containing the outcome \( z \) which might make him worse off. Hence \( y \notin \text{CDOM}(\sigma^1, G^X) \). Therefore we can conclude that \( \sigma^1(G^X) = X \setminus \text{CDOM}(\sigma^1, G^X) = \{y\} \).

In case of (ii), we have to explain why Edwin will refrain from exercising his right to marry as well as his right to remain single. Indeed, Edwin has the right to remain single. However, if he chooses to exercise this right he will induce a new agenda \( Y = \{x, z\} \).

Notice that Edwin and Angelina have opposing preferences over \( \{x, z\} \), and each will exercise their rights to eliminate both \( x \) and \( z \). Hence \( \sigma^2(G^Y) = \emptyset \) and by exercising his right to remain single, Edwin will create “chaos”. Therefore, \( y \notin \text{CDOM}(\sigma^2, G^X) \). Similarly, Edwin will refrain from exercising his right to marry. Indeed, he must realize that in case he will choose to exercise his right to marry, he will induce a new agenda \( Y = \{x, y\} \). However, Edwin and Angelina again have opposing preferences over \( Y \), and each will exercise their rights to eliminate both \( x \) and \( y \). As a result, \( \sigma^2(G^Y) = \emptyset \). Hence by exercising his right to marry, Edwin creates the “chaotic situation”. Therefore, \( z \notin \text{CDOM}(\sigma^2, G^X) \). Since Angelina will exercise her right to marry we have \( x \in \text{CDOM}(\sigma^2, G^X) \). Hence \( \sigma^2(G^X) = X \setminus \text{CDOM}(\sigma^2, G^X) = \{y, z\} \). □

Notice that the endowment of Edwin with an additional right to remain single does not matter in the case of the rights-exercising protocol \( \delta^1 \). Remember that under this protocol he can exercise his rights only once (sequential or successive rights-exercising is prohibited). Naturally then Edwin will continue to exercise his right to marry waiving his right to remain single. Hence the result is the same as when he had only one right to
marry. However, the situation is entirely different in the case when sequential rights-exercising is allowed. In this case, not only Edwin will waive his right to remain single, but perhaps paradoxically, he will also waive his right to marry. It is indeed puzzling that giving more rights as well as providing more power to exercise them might lead to rights alienation.

Suppose now that Edwin’s desire to remain single is so strong that he would rather see Angelina wed the judge than to marry her. We will first examine the situation where Edwin will be endowed only with the right to remain single while Angelina will be endowed with her right to marry, that is, \( \rho(A) = \text{marry} \), \( \rho(E) = \text{remain single} \). How does this change in the rights endowment and in Edwin’s preference ordering affect Angelina’s and Edwin’s exercising of their rights?

**Proposition 4.** The unique CSSB, \( \sigma^k \), for the associated situation \((\delta^k, \Gamma)\), \( k = 1,2,3 \) satisfies:

(i) \( \sigma^1(G^X) = \{z\} \); (ii) \( \sigma^2(G^X) = \{z\} \); (iii) \( \sigma^3(G^X) = \{z\} \).

**Proof.** It is similar to the proof of Proposition 2 and is therefore omitted. \( \square \)

Again notice that regardless of the differences in the right-exercising protocols, the outcome of the rights-exercising through the theory of social situations coincides with the outcome of the rights-exercising through the normal game form. However, this time it also coincides with the outcome that has emerged within the social choice-theoretic framework. We want again to investigate what will happen if we endow Edwin with an additional right to marry. How does the difference in the rights-exercising protocols affect Edwin’s and Angelina’s exercising of their rights in this case?

**Proposition 5.** The unique CSSB, \( \sigma^k \), for the associated situation \((\delta^k, \Gamma)\), \( k = 1,2,3 \)
satisfies:

(i) $\sigma^1(G^X) = \{z\}$; (ii) $\sigma^2(G^X) = \{y,z\}$; (iii) $\sigma^3(G^X) = \{\emptyset\}$.

Proof. It is similar to that of Proposition 3 and is therefore omitted.

Again granting more rights as well as providing more power to exercise them leads to an alienation of Edwin’s individual rights. This time, despite his strong desire to remain single, he would refrain from exercising his right to do so.

Notice that so far all our rights-assignments were in the form of a certain feature of the state that was in the private sphere of an individual i. For example, Edwin by exercising his rights to remain single could exclude y from the set of all social states. Similarly, by exercising his right to marry, he could exclude z from the set of all social states. However, Edwin did not have a power to exclude x from the set of all social states. On the other hand, Angelina did not have a power to exclude z from the set of all social states. And yet it is precisely this power that both Edwin and Angelina may enjoy under the social choice formulation of individual rights. Remember that the pair $\{x,y\}$ is in Edwin’s RPS and depending on his preferences he can eliminate both x or y. Similarly, the pair $\{x,z\}$ is in Angelina’s RPS and depending on her preferences she can knock out both x or z. What will happen if we will mimic the social choice-theoretic rights-assignments within the framework of the theory of social situations but will exercise those rights using different rights-exercising protocols? Suppose first that Angelina, by exercising her right, can eliminate x from the set of all social states and Edwin, by exercising his right can knock out y from the set X. In effect, we assume one-sided decisiveness and since Angelina strictly prefers z over x, we will allow her to eliminate x only. Similarly, since Edwin strictly prefers x over y, we will allow him to knock out y
only. Given the modified Gibbard’s example where Edwin strongly desires to remain single, we have already established that $\sigma^k(G^X) = \{z\}$, $k = 1,2,3$ in the case of this rights-assignment (see Proposition 4). However, what will happen if in addition to our previous assignment of rights we will allow Angelina to eliminate $z$ and Edwin to eliminate $x$ from the set $X$ by exercising their rights? The following proposition provides an answer.

**Proposition 6.** The unique CSSB, $\sigma^k$, for the associated situation $(\delta^k,\Gamma)$, $k = 1,2,3$ satisfies:

(i) $\sigma^1(G^X) = \{z\}$; (ii) $\sigma^2(G^X) = \{x,z\}$; (iii) $\sigma^3(G^X) = \{\emptyset\}$.

**Proof.** We will prove (ii) and (iii) ((i) is left to the reader). In case of (ii), we will show that both Angelina and Edwin will refrain from exercising their rights to eliminate $x$.

First, we will show that Angelina will not choose to eliminate $x$. Because if she will, then she will induce a new agenda $Y = \{y,z\}$, and Edwin and Angelina have opposing preferences over $Y$. By exercising their rights, they can eliminate both $y$ and $z$ leading to $\sigma^2(G^Y) = \emptyset$. Hence by exercising her right to eliminate $x$, Angelina will create “chaos”.

For similar reason, Edwin will not choose to eliminate $x$. Therefore, $x \notin \text{CDOM}(\sigma^2, G^X)$. Angelina also will not choose to eliminate $z$. Because if she will, then she will induce a new agenda $Y = \{x,y\}$, and Edwin and Angelina have opposing preferences over $\{x,y\}$. Hence by exercising their rights, they can eliminate both $x$ and $y$ leading to $\sigma^2(G^Y) = \emptyset$. Therefore, $z \notin \text{CDOM}(\sigma^2, G^Y)$. Finally, Edwin will exercise his right to eliminate $y$. By eliminating $y$, he will induce a new agenda $Y = \{x,z\}$. Angelina strictly prefers $z$ over $x$ and she has a right to eliminate $x$ that she will exercise. Edwin, on the other hand, will not use his right to eliminate $x$ since he strictly prefers $x$ over $z$. As a result, $z$ would emerge as the final outcome of their rights exercising. Because Edwin strictly prefers $z$
over y he will choose to exercise his right to eliminate y. Hence \( y \in \text{CDOM}(\sigma^2,G^X) \) and 
\[
\sigma^2(G^X) = X \setminus \text{CDOM}(\sigma^2,G^X) = \{x,z\}.
\]

In case of (iii), Angelina can combine her rights to exclude both x and z, since she prefers y to both of them. Hence both x and z belong to \( \text{CDOM}(\sigma^3,G^X) \). Edwin will choose to exercise his right to eliminate y. Because by eliminating y, he will induce a new agenda \( Y = \{x,z\} \). Angelina strictly prefers z over x and she has a right to knock out x which she will use. Edwin, on the other hand, strictly prefers x over z and therefore will waive his right to eliminate x. Therefore, z will emerge as an outcome of rights-exercising. Since Edwin strictly prefers z over y, he will use his right to exclude y from X. Hence \( y \in \text{CDOM}(\sigma^3,G^X) \) and 
\[
\sigma^3(G^X) = X \setminus \text{CDOM}(\sigma^3,G^X) = \emptyset.
\]

Notice that the endowment of both Angelina and Edwin with the additional rights would not make any difference in the case of the rights-exercising protocol \( \delta^1 \). However, the situation is somewhat different in the case of \( \delta^2 \). In this case, perhaps paradoxically, Angelina will waive all her rights. Again a familiar pattern has emerged, namely, granting more rights as well as providing more power to exercise them leads to an alienation of Angelina’s individual rights.

After our examination of the Gibbard’s example and its modification within the framework of the theory of social situations, the reader might think that a somewhat paradoxical phenomenon of rights alienation could be even established formally, that is, it should be possible to prove, for example, that \( \sigma^1 \subseteq \sigma^2 \). This result would indeed be paradoxical because intuitively we believe that giving more rights to an individual as well as providing more power to exercise them should lead in general to people exercising more of their individual rights rather than less, that is, intuitively it should be true that
\(\sigma^2 \subseteq \sigma^1\). However, the following example establishes that neither inclusion holds.

Suppose that Angelina and Edwin are the only people in the train compartment. We also assume that smoking on the train is not prohibited unless someone would object to it. Angelina is addicted to smoking and will smoke provided no one would object. On the other hand, Edwin is a non-smoker who values a clean air. At stake here is the right to smoke (or the right to continue smoking) for Angelina and the right to clean air (or the right to object to smoking) for Edwin. Angelina, being addicted to smoking, will definitely start smoking. Edwin then faces two choices: either to object to smoking \((o)\) or refrain from an objection and let her continue to enjoy her habit \((o')\). In case of objection, Angelina either will comply and stop smoking \((s')\) or she will ignore it and will continue to smoke \((s)\). Denote “Edwin objects and Angelina ignores it” as \(x\), “Edwin objects and Angelina complies” as \(y\), and “Edwin refrains from an objection and Angelina enjoys her habit” as \(z\). The reader should easily come up with a story justifying the following preferences:

<table>
<thead>
<tr>
<th>Edwin</th>
<th>Angelina</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>z</td>
</tr>
<tr>
<td>z</td>
<td>y</td>
</tr>
<tr>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

Notice that the pair \((x,z)\) is in Edwin’s RPS, and he strictly prefers \(z\) over \(x\). Therefore, according to (2.1), \(x\) should not be chosen. The pair \((x,y)\) is in Angelina’s RPS, and she strictly prefers \(y\) over \(x\). Hence, again according to (2.1), \(x\) should not be chosen. The outcomes that will emerge after Edwin and Angelina will exercise their rights are \(\{y,z\}\).

If we want to formulate their rights using a game form formulation, it is perhaps more natural for this example to utilize an extensive game form. However, we will continue to use a normal game form. Edwin has two permissible strategies: object to smoking \((o)\) or
refrain from an objection (o’). Angelina also has two permissible strategies: continue
smoking (s) (more precisely start smoking and continue) and stop smoking (s’) (again
more precisely start smoking and then stop). Hence we have the following 2 x 2 game in
normal form:

<table>
<thead>
<tr>
<th></th>
<th>Angelina</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edwin</td>
<td>o</td>
</tr>
<tr>
<td></td>
<td>s</td>
</tr>
<tr>
<td>o’</td>
<td>x</td>
</tr>
<tr>
<td></td>
<td>z</td>
</tr>
</tbody>
</table>

Given the preference orderings above, we have two Nash equilibria in this game,
y = (o,s’) and z = (o’,s). However, only y is a subgame perfect Nash equilibrium.

Within the theory of social situations we first examine the case when Angelina will
have her right to continue smoking while Edwin will have his right to object to it, that is
ρ(E) = object to smoking, ρ(A) = continue smoking. Hence by exercising her right
Angelina may restrict the original set X = \{x,y,z\} to Y = \{x,z\}, and similarly Edwin by
exercising his right might restrict the set X to Y = \{x,y\}. What would be the result of
their rights exercising in this case? The following proposition provides an answer.

*Proposition 7.* The unique CSSB, \( \sigma^k \), for the associated situation \( (\delta^k, \Gamma) \), \( k = 1,2,3 \)
satisfies:

(i) \( \sigma^1(G^X) = \{x,y,z\} \); (ii) \( \sigma^2(G^X) = \{x,y\} \); (iii) \( \sigma^3(G^X) = \{x,y\} \).

*Proof.* We will prove (i) and (ii) ((iii) can be handled similarly). In case of (i), Edwin
will refrain from exercising his right to object to smoking. Because if he will exercise his
right he will induce a sub-society containing social state x, and Edwin strictly prefers z
over x. Hence z \( \not\in \text{CDOM}(\sigma^1,G^X) \). Similarly, and for the same reason, Angelina will
refrain from exercising her right to continue smoking. Therefore, y \( \not\in \text{CDOM}(\sigma^1,G^X) \) and
\[ \sigma^1(G^X) = X \setminus \text{CDOM}(\sigma^1, G^X) = \{x,y,z\} \].

In case of (ii), Edwin will choose to exercise his right to object to smoking. By exercising his right, he will induce a new agenda \( Y = \{x,y\} \). Angelina strictly prefers \( y \) over \( x \) and by exercising her right will eliminate \( x \). Since Edwin strictly prefers \( y \) over \( z \), he will exercise his right, that is, \( z \in \text{CDOM}(\sigma^2, G^X) \). On the other hand, Angelina will not exercise her right to continue smoking. Because by exercising her right, she will induce a new agenda \( Y = \{x,z\} \). Edwin strictly prefers \( z \) over \( x \), and will not exercise his right to eliminate \( z \). Hence by exercising her right, Angelina will induce a sub-society containing social state \( x \), and she strictly prefers \( y \) over \( x \). Therefore, \( y \notin \text{CDOM}(\sigma^2, G^X) \) and \( \sigma^2(G^X) = X \setminus \text{CDOM}(\sigma^2, G^X) = \{x,y\} \).

At last we have a situation where providing more power to exercise individual rights will actually lead people to exercising more of them. Also notice that in this case, \( \sigma^2(G^X) \subset \sigma^1(G^X) \) which corresponds to our intuition about rights-exercising. However, will this inclusion hold if we will grant some additional rights to the participants? Specifically, suppose that Angelina will not only have her right to continue smoking, but she will also have the right to stop smoking, that is \( \rho(A) = \text{continue smoking}, \rho(A) = \text{stop smoking}, \rho(E) = \text{object to smoking} \). The preference orderings are exactly as before. The following proposition provides a negative answer.

**Proposition 8.** The unique CSSB, \( \sigma^k \), for the associated situation \((\delta^k, \Gamma)\), \( k = 1,2,3 \) satisfies:

(i) \( \sigma^1(G^X) = \{y,z\} \); (ii) \( \sigma^2(G^X) = \{x\} \); (iii) \( \sigma^3(G^X) = \emptyset \).

**Proof.** We will prove (i) and (ii) ((iii) is left to the reader). In case of (i), Angelina will exercise her right to stop smoking and will eliminate \( x \). Hence \( x \in \text{CDOM}(\sigma^1, G^X) \).
However, she will refrain from exercising her right to continue smoking. Because if she will, she will induce a sub-society containing social state x, and she strictly prefers y over x. Therefore, \( y \not\in \text{CDOM}(\sigma^1, G^X) \). Similarly, and for the same reason, Edwin will refrain from exercising his right to object to smoking. Hence \( z \not\in \text{CDOM}(\sigma^1, G^X) \) and \( \sigma^1(G^X) = X \setminus \text{CDOM}(\sigma^1, G^X) = \{y, z\} \).

In case of (ii), we will show first that Edwin will choose to exercise his right to object to smoking. Indeed, if he will, he will induce a new agenda \( Y = \{x, y\} \). Angelina strictly prefers y over x and by exercising her right not to smoke, she will eliminate x. Since Edwin strictly prefers y over z, he will choose to eliminate z by exercising his right. Therefore, \( z \in \text{CDOM}(\sigma^2, G^X) \). Similarly, Angelina will choose to exercise her right to continue smoking. By exercising her right, she will induce a new agenda \( Y = \{x, z\} \).

Edwin and Angelina have similar preferences over \( \{x, z\} \), and since Edwin strictly prefers z over x, he will not exercise his right to eliminate z. On the other hand, by exercising her right to stop smoking, Angelina will eliminate x because she also strictly prefers z over x. Hence z will emerge as an outcome and Angelina strictly prefers z over y. Therefore, she will exercise her right to continue smoking and eliminate y, that is, \( y \in \text{CDOM}(\sigma^2, G^X) \).

However, Angelina will refrain from exercising her right to stop smoking. Because if she will exercise this right, she will induce a new agenda \( Y = \{y, z\} \), and Edwin and Angelina have opposing preferences over Y. By exercising their individual rights, they will eliminate both y and z leading to \( \sigma^2(G^Y) = \emptyset \). Hence by exercising her right to stop smoking Angelina will create “chaos”. Therefore \( x \not\in \text{CDOM}(\sigma^2, G^X) \), and \( \sigma^2(G^X) = X \setminus \text{CDOM}(\sigma^2, G^X) = \{x\} \).

Notice that the original rights-assignment in this example does not allow Angelina and
Edwin to knock out $x$ by exercising their individual rights. However, granting Angelina an additional right to stop smoking, does provide her with an opportunity to eliminate $x$ from the set $X$. It turns out that whether she will use this opportunity or not will depend on the nature of the rights-exercising protocols. For example, Angelina will exercise her right to stop smoking under the rights-exercising protocol $\delta^1$, and she will waive this right under the rights-exercising protocol $\delta^2$. As a result, we have established that $\sigma^1 \cap \sigma^2 = \emptyset$, that is, neither $\sigma^1 \subseteq \sigma^2$ nor $\sigma^2 \subseteq \sigma^1$ holds. Also notice that the outcome $x$ that has emerged under the rights-exercising protocol $\delta^2$ clashes with the outcomes that have emerged under the game form formulation of individual rights. Perhaps, even more fundamentally, the emergence of $x$ could be construed as a violation of Edwin’s and Angelina’s rights under the social choice-theoretic formulation of individual rights. However, the close examination of the proof of Proposition 8 shows that both Angelina and Edwin will exercise their individual rights (to smoke and to object to smoking respectively). The emergence of $x$, therefore, reflects the fact that when individual rights are in conflict someone’s rights must give in (for more on the co-possibility as well as compossibility of individual rights, see Dowding and van Hees (2003)).

4. Concluding remarks

In this paper we have suggested an alternative formulation of rights within the framework of the theory of social situations. We have focused primarily on the analysis of individual rights within this framework leaving the detailed examination of group rights and their relationship to individual rights until next occasion. However, Pattanaik (1988) has already shown that the clash may arise between individual and group rights, and that, perhaps, group rights deserve more scrutiny than they have received so far.
In his paper, Riley (1989) has suggested that any cooperation in the rights-exercising games should be based on the reasonable non-cooperative foundation. Furthermore, he insists that “if the possibility of coalition formation is taken seriously in a non-cooperative setting, then any solution is appropriately required to be a strong Nash equilibrium” (p. 145). However, Deb, Pattanaik and Razzolini (1997) argue that a strong Nash equilibrium is perhaps too “strong” in many rights-exercising situations. Instead they suggest a k-strong equilibrium or a coalition-proof Nash equilibrium to model rights-exercising games (Greenberg’s (1990) notion of an equilibrium with coalitional commitments or an equilibrium with coalitional contingent threat might also be relevant). Perhaps, an additional advantage of the theory of social situations is that it allows a rather uniform treatment of both individual and group rights within the same framework using the same stability or consistency criterion.
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