Abstract: The aim of this paper is to develop a model that considers the existence of two tiers of government providing public goods with the same tax base to finance them. Their rent is related to the level of competition. Citizens need to maximize their own utility starting from these different levels of competition. Therefore, they can decide to turn down the governments to induce them to behave efficiently. Moreover, governments can choose whether to accept the behaviour proposed by citizens or to maximize their rent for a single period of office of the legislature and consequently lose the next elections.

Jel classification: H11, H21, H71, H77.
1. Introduction

The choice among private goods and public goods is a topic developed by many handbooks\(^1\) in economics and in public economics. The aim of this paper is to investigate about this choice analysing two particular hypotheses. First of all, we study the presence of a multi – tiered governmental system with governments that provide different public goods. Secondly, we deal with the presence of competition among governments. We consider vertical competition among governments of different tiers and horizontal competition among governments located at the same level.

In particular, we want to show the best choice for a representative voter/citizen according to the following hypotheses. We will consider the consumption of three goods: a private good, produced in a perfectly competitive market, a local public good, produced by governments located at a lower tier and a central public good, produced by a single high – tiered government. The local public good is health, the central public good is security and the private good is food. The utility of citizens is positive if the quantity of health, security and food are positive. As assumed before, it exists a given level of competition among governments situated at the lower level and another given level of competition between the low – tiered governments and the high – tiered government. Then, the model will be developed starting from a situation in which we have two governments located at different tiers and a median voter that chooses the optimal allocation of a private good and the two public goods. In fact, we choose a simple model with the presence of two political parties as in Downs (1957), because we know that an optimal solution for a community is very difficult to achieve when we speak about public goods. Therefore, we need to develop a second best – solution, as in the rest of the paper.

The paper will be developed as follows. Section two will be dedicated to present some related literature. In the third section I will outline some hypotheses of the model and in the fourth one the model will be illustrated. Section five will show the behaviour of governments after the choice of the optimal tax rates and in the sixth one some conclusions will be outlined.

\(^1\) For example: Stiglitz and Varian (1990), chapter 31.
2. The related literature

The literature on decentralization is wider and wider. Some preliminary references to develop my paper can be found in Hettich and Winer (1999) and in Breton (1996) that analyse democratic choice. In particular, this paper is inspired by some contributions about the literature on decentralization and incomplete contracts (Seabright, 1996), vertical competition and Cournot solution (Flowers, 1988), and vertical and horizontal yardstick competition among governments (Bodenstein and Ursprung, 2001). Vertical and horizontal competition will be used to generate an efficient allocation of private and public goods. This allocation will be chosen by a median voter. Incomplete contracts will be used to understand the behaviour of governments that can choose between the efficient solution proposed by voters and an aggressive behaviour maximising their rent for a single period of office of the legislature with the consequence of being voted out the office. We consider that no one (a judge, for example) can oblige governments in office to behave as citizens need. The Cournot solution will be used to analyse the allocation of the tax revenue when all governments opt for the aggressive solution proposed before. In fact, we consider governments as duopolists maximising simultaneously their own rent and, then, their tax revenues.
3. Hypotheses of the model

Following Bodenstein and Ursprung (2001), we will consider two parameters: \( \phi \) and \( \theta \). These two parameters represent the possibility to compare the performances of different governments. In our model, when we assume that local governments produce a good and central government provides another good, we have a direct way to compare the performance of local governments and an indirect way to compare the performances of local governments versus central government. In particular, we assume the possibility to compare the performance of central government versus local government in producing two different goods that are consumed by the same citizen. Furthermore, we assume the possibility to compare the performance of local governments that produce the same good consumed by different citizens in different localities. \( \phi \) is used to measure the first comparison and is associated, by hypothesis, to the rent that central government can extract by citizens. \( \theta \) represents the second comparison and is associated to the rent of local governments.

We define \( \phi \) as the level of vertical competition among different tiers of government and \( \theta \) as the level of horizontal competition among government located at the lower tier. The value of the two parameters, as in Bodenstein and Ursprung, can be equal to 1, with perfect competition (perfect comparability), or higher.

Obviously, and also very probably, the two parameters can assume different values. Different values of competition produce different values of politicians’ rents. With a higher value of \( \theta \) local governments can earn a higher rent. Similarly, a higher value of \( \phi \) generates a higher rent for the central government. The two following formulas can explain how competition can affect the provision of public goods starting from the tax revenue. In fact, we assume that:

\[
G_L = \left( \frac{W}{wT_L} \right)^{1/\phi} \quad \text{with } T_L > 1
\]

and

\[
G_C = \left( \frac{W}{wT_C} \right)^{1/\phi} \quad \text{with } T_L > 1
\]

where \( G_i \) is the quantity of public goods provided by each government, \( T_i \) is the tax revenue that government \( i \) receives from the median voter, \( W \) is the total income and \( w \) is the median voter’s income. \( T_i \) is obtained by a proportional income tax and is equal to \( \tau_i w \), where \( \tau_i \) is the tax rate of
government $i$ multiplied by the median voter’s income (represented by his wage) living in the locality considered by the analysis. As we said before, we consider a median voter in our model. He knows the levels of competition among governments and the rent of the governments. The number of citizens of a locality does not influence its rent. The median voter also chooses the level of public goods and private good starting from his utility function, that is similar to the utility function used by Hettich and Winer (1999)\textsuperscript{2}: we consider two public goods instead of one but we do not consider leisure. We deal with an exogenous level of leisure because our model differs from a standard model where taxes do not influence the utility of a consumer. In particular, when our model is explicit, it would be difficult to assign an explicit preference to the variable leisure. As argued by Hettich and Winer, we consider the income (represented by $w$). In particular, we assume a given value of $w$ to maximize the utility of the median voter and we also assume that this value can be lower if governments do not choose the optimal allocations proposed by our median voter.

$$U_m = (x, G_L, G_C)$$ (3)

where $x$ is the quantity of private good. Starting from our hypotheses, we can re-write this function as a generic Cobb–Douglas function:

$$U_m = x^{\alpha} \cdot G_L^\beta \cdot G_C^\gamma$$ (4)

where $\alpha, \beta$ and $\gamma$ represents the preferences associated to the private good, the local public good and the central public good respectively.

4. The model

From equations (1), (2) and (4), we obtain:

\[ U_m = x^\alpha \cdot \left( \frac{W}{w}T_L \right)^{\frac{\beta}{\phi}} \cdot \left( \frac{W}{w}T_C \right)^{\frac{\gamma}{\phi}}. \]  

(5)

The constraint, as in Hettich and Winer (1999), of our median voter is:

\[ x = w(1 - \tau_L - \tau_C)^3 \]  

(6)

that can be re-written as

\[ x + T_L + T_C = w. \]  

(7)

where \( w \) is the wage of the median voter. As we can easily note in the constraint, \( w \) represents the total income of the median voter.

Thus, our maximization problem becomes:

\[
\max U_m = x^\alpha \cdot \left( \frac{W}{w}T_L \right)^{\frac{\beta}{\phi}} \cdot \left( \frac{W}{w}T_C \right)^{\frac{\gamma}{\phi}}
\quad \text{s. t.} \quad x + T_L + T_C = w
\]  

(8)

that can be solved using the Lagrangian function:

\[
L = x^\alpha \cdot \left( \frac{W}{w}T_L \right)^{\frac{\beta}{\phi}} \cdot \left( \frac{W}{w}T_C \right)^{\frac{\gamma}{\phi}} - \lambda (x + T_L + T_C - w).
\]  

(9)

Our First Order Conditions are:

\[
\frac{\partial L}{\partial x} = 0 : \alpha \frac{W^{\frac{\beta}{\phi}}}{w} x^{\alpha - 1} \cdot \frac{T_L^{\frac{\beta}{\phi}}}{T_L^{\frac{\gamma}{\phi}}} \cdot \frac{T_C^{\frac{\gamma}{\phi}}}{T_C^{\frac{\gamma}{\phi}}} = \lambda
\]  

(10)

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3 See note 2.
\[
\frac{\partial L}{\partial T_L} = 0 : \beta W^{\frac{\beta + \gamma}{\alpha \theta}} T_L^{\theta -1} \cdot x^\alpha \cdot T_L^\theta = \lambda 
\]  
(11)

\[
\frac{\partial L}{\partial T_C} = 0 : \frac{\gamma W^{\frac{\beta + \gamma}{\alpha \theta}}}{\phi} T_C^{\theta -1} \cdot x^\alpha \cdot T_C^\theta = \lambda 
\]  
(12)

\[
\frac{\partial L}{\partial \lambda} = 0 : x + T_L + T_C = w. 
\]  
(13)

From equations (10), (11) and (12) we can write the values of \( T_L \) e \( T_C \) as a function of the level of \( x \):

\[ T_L = \frac{\beta}{\alpha \theta} x \]  
(14)

and

\[ T_C = \frac{\gamma}{\alpha \phi} x. \]  
(15)

We can substitute these values in equation (13) to obtain the optimal value of \( x \) and, consequently, \( T_L \) and \( T_C \):

\[ x^* = \frac{w \alpha \theta \phi}{\alpha \theta \phi + \beta \phi + \gamma \theta} \]  
(16)

\[ T_L^* = \frac{w \beta \phi}{\alpha \theta \phi + \beta \phi + \gamma \theta} \]  
(17 a)

\[ T_C^* = \frac{w \gamma \theta}{\alpha \theta \phi + \beta \phi + \gamma \theta}. \]  
(18 a)

Starting from \( T_L \) and \( T_C \) we can obtain the values of \( G_L \) and \( G_C \):
\[ G_L^* = \left( \frac{W \beta \varphi}{\alpha \theta \varphi + \beta \varphi + \gamma \theta} \right)^\frac{1}{\delta} \]  

(17 b)

\[ G_C^* = \left( \frac{W \gamma \vartheta}{\alpha \theta \varphi + \beta \varphi + \gamma \theta} \right)^\frac{1}{\delta} \]  

(18 b)

We know that \( T_L = \tau_L w \) and \( T_C = \tau_C w \), so the optimal values of the two tax rates are:

\[ \tau_L^* = \frac{\beta \varphi}{\alpha \theta \varphi + \beta \varphi + \gamma \theta} \]  

(19)

and

\[ \tau_C^* = \frac{\gamma \vartheta}{\alpha \theta \varphi + \beta \varphi + \gamma \theta} \]  

(20)

We can easily note that parameters \( \theta \) and \( \varphi \) affect positively the quantity of the private good demanded by the median voter because a lower level of intergovernmental competition reduces the quantity of public goods provided by governments with the same tax revenue. Moreover, we can see that the preference for public good \( G_L \) depends on the value of the associated preference \( \beta \) and the level of vertical competition \( \varphi \). It is due to the assumption that competition is perfect when \( \varphi = 1 \) and its level decreases when \( \varphi \) increases. The same way of reasoning can be applied to \( G_L \), using \( \gamma \) and \( \theta \). In this case, we can note that \( G_L \) and \( G_C \) are substitutes. We can also note that lower levels of intergovernmental competition reduce the level of the utility of the median voter.

We can note that a point in figure 1 represents the optimal allocation of the two tax rates. This result is different if we do not consider given values of \( \varphi \) and \( \theta \). In fact, if we consider the presence of a continuous range of values with a lower and an upper limit, we will have a space as an optimal result. Figure 2 represents an example of this case.
Figure 1

Figure 2
5. The behaviour of governments

If we consider given values of $\theta$ and $\varphi$, as in the model before, we can note that we have two optimal solutions for the tax rates of the two governments: $\tau^*_L$ (the optimal tax rate for the local government) and $\tau^*_C$ (the optimal tax rate for the central government). If each government wants to be re-elected, it needs to choose the optimal tax rate chosen by the median voter.

The rent of a re-elected local government is

$$R_L^* = \left( \frac{W}{w} T_L^* - G_L^* \right) \cdot \frac{1}{r} = \left( \frac{W}{w} T_L^* - \left( \frac{W}{w} T_L^* \right)^{1/\varphi} \right) \cdot \frac{1}{r}$$ \hspace{1cm} (21)

and the rent of the re-elected central government is

$$R_C^* = \left( \frac{W}{w} T_C^* - G_C^* \right) \cdot \frac{1}{r} = \left( \frac{W}{w} T_C^* - \left( \frac{W}{w} T_C^* \right)^{1/\varphi} \right) \cdot \frac{1}{r}$$ \hspace{1cm} (22)

where $r$ represents the discount rate of the period of office of the legislature.

So, each government can choose $\tau^*_i$ (with $i = L, C$) and being re-elected or choose a higher tax rate and being voted out of office at the end of the period of office of the legislature. Then, in this first case there is no direct competition between the two levels of government to have the lion’s share of the tax revenue. The median voter decides the allocation of the tax revenue among governments and they accept this allocation (“good” governments).

If a government chooses to be turned down (“bad” government), its tax rate will depend on the choice of the other government. In particular, if a single government chooses to be turned down and the other chooses to stay in office, its optimal tax rate would be:

$$\tau_i = 1 - \tau_j^*$$ \hspace{1cm} (23)

where $\tau_i$ represents the “bad” government tax rate and $\tau_j^*$ the “good” government tax rate. At the same way, if all the two governments choose to be voted out at the next elections and maximize their rents in the present period we have:

$$\tau_L + \tau_C = 1.$$ \hspace{1cm} (24)
In both the cases presented before the two governments absorb all the income of the citizens. This means that the level of the private good is equal to 0 and the level of the utility of the median voter is 0 consequently.

This result, in my opinion, is not very interesting. We need a new hypothesis to better develop this problem. We must consider the fact that citizens prefer a higher level of leisure when taxes increase, in particular when all the taxes are not converted in public goods, as in our case. Therefore, governments have to manage with distortionary taxes when they maximize their rent.

Applying this hypothesis to our model, we can note that the level of income of a locality is not given, but it decreases when the total amount of the taxes \((T_L + T_C)\) increases. Therefore, we cannot consider a given value of the total income. In particular, we know that our income has a given value if the level of taxes is \((T_L^* + T_C^*)\). If the level is \((T_L + T_C) > (T_L^* + T_C^*)\) we know that our aggregate income is lower than the aggregate income that represents the optimal solution. In terms of equations (21) and (22), a higher value of \((T_L + T_C)\) under the new assumption produces a lower value of \(W\) and a lower value of \(w\). We do not know the sign of the effect on \(W/w\), because we do not know the preferences of each citizen. Furthermore, we do not know if the level of \(T_i = \tau_i w\) is higher or lower, because we know that an increase in \(\tau_i\) causes a decrease of \(w\).

Let us assume that an increase in \(\tau_i\) causes an increase in \(T_i\) if the sum of the tax rates is:

\[
\tau_L + \tau_C < \bar{\tau}.
\]

(25)

It the last equation \(\bar{\tau}\) represents the upper point of a Laffer Curve (Laffer, 1981). If the two governments can collude, this level of the tax rate is the alternative point that they can choose instead of \(\tau_L + \tau_C\). We have this solution if there is at least an allocation \(\tau_L + \tau_C = \bar{\tau}\) that allows to have rents for the two governments higher than \(R_L^* + R_C^*\).

If collusion is not available governments may behave as duopolists that maximize their own rent. In this case the sum of their tax rates is higher than \(\bar{\tau}\) and we have this solution if the rent of the local government is higher than \(R_L^*\) and the rent of the central government is higher than \(R_C^*\).

In the two cases showed before all the governments choose to be turned down.

There is a third case where only one government acts to maximize its rent in one period and is turned down and the other chooses the tax rate proposed by the median voter. In this case, the “good” government has a rent of \(R_i^*\) and the other maximizes its revenue starting from the tax rate of the “good” government. The aggregate tax rate is lower than the “duopoly case” but higher than
the “monopoly case”. The rent for the “bad” government would be higher than its rent if it were “good”, $R_i^*$. 

6. Conclusions

In this paper we analyse a situation where citizens need to maximize their utilities in an imperfect intergovernmental competition model. As argued in the introduction, we need a second – best solution. Consequently, this model describes an imperfect competition situation. In fact, with values of $\theta$ and $\varphi$ equal to 1 we have rents equal to 0, so no governments would exist.

If we consider the same citizens’ preferences for local and central public goods and private good, the different levels of competition can be used to determine the efficient amount of public goods provided by central and local governments. In particular, a higher degree of vertical competition can justify a wide assignment of powers and functions to the central level. On the other hand, a higher degree of horizontal competition can be used to justify an assignment of powers to the lower tier of government, when interjurisdictional spillovers are relatively low.

At the same time, governments can choose if they want to accept the median voter's allocation or choose an aggressive behaviour to maximize its rent in only one period. This choice cannot be influenced by voters and it can lead to four results. In the first case, governments behave as citizens need and the utility of the median voter is maximized. In the second cases only one government acts as median voter wants but the other one imposes a very high tax rate. In the last two cases governments act aggressively. In the first one (monopoly case), they act aggressively with respect to citizens but they act yieldingly to each other. In the second case (duopoly case), they always behave aggressively: this is the worst case for citizens.
References


