Profit Tax Competition
and Formula Apportionment

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Abstract

We analyse tax competition with corporate income taxes in a common market where tax revenues are allocated to governments according to an apportionment formula. We compare Nash equilibria of the tax competition game for different apportionment methods. If labour input is fixed, equilibrium tax rates turn out to be lowest when apportionment is based on property-shares, followed by the sales- and payroll-shares. In the first case tax rates are suboptimally low, in the latter two cases they are inefficiently high. If both capital and labour are variable inputs and technologies are Cobb-Douglas, equilibrium tax rates under a property- and a payroll-share rule are lower than with a sales-share formula. Factor elasticities determine whether payroll- or property-share apportionment generates sharper tax competition.

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1 Introduction

It is widely acknowledged that the current system of corporate taxation in the European Union (EU) hampers cross-border economic activities and impedes the creation and operations of multinational business structures in Europe (Cnossen, 2002). Differences in national tax rates and capital income tax systems imply that investment and location decisions are, to a measurable degree, driven by tax considerations rather than by gross (social) returns on investment (Devereux and Griffith, 2001). Moreover, since due to the EU member states’ uncoordinated tax schemes double taxation cannot be excluded, investment projects extending over several countries are often taxed more heavily than purely national projects. Multinationals within the EU have to set up separate accounts for each country where they operate (separate accounting, SA). This creates strong incentives for firms to shift profits to low-tax countries.\(^1\) Moreover, SA incurs high administrative costs for EU companies since they have to deal with 15 tax systems in the EU, and high monitoring costs for the tax authorities of the EU member states.

In response to these problems the European Commission recently launched another report on company taxation in the EU (European Commission, 2001, Bolkestein Report). This report examines a number of remedies to remove obstacles to EU cross-border investment and suggests four comprehensive alternative taxation methods, deemed workable and politically feasible, to replace the current disarray of 15 (or, in the future, 25) corporate tax systems. Under each of these methods, firms face a single tax base for all their EU-wide activities such that corporate profits can be determined on the basis of the company’s consolidated accounts rather than by separate accounting in each member state.\(^2\) Obviously, any such unitary method must determine

\(^1\) Usually, profit shifting is achieved via transfer pricing (arm’s length pricing) of intra-firm activities — which has, in turn, become a key instrument in firms’ strive to evade taxes. See, e.g., Schjelderup and Sørgard (1997) for a theoretical analysis and Bartelsman and Beetsma (2000) for empirical evidence.

\(^2\) The Commission’s proposals comprise Common Base Taxation (CBT), a European Corporate Income Tax (EUCIT), Home State Taxation (HST), and the compulsory harmonization of all national corporate tax codes. Under CBT, the member states would harmonize their rules for computing the taxable profits of firms with cross-border operations, possibly maintaining national rules for purely domestic businesses. This would be similar with a EUCIT where, however, tax revenues would (at least partly) accrue to the EU budget. Under HST, member states keep their own rules for profit determination, but companies with cross-border operations would be taxed according to the rules of the member state where their headquarters are located. For a survey on these concepts (and other proposals in the EU report) see Weiner (2002a).
how tax revenues emerging from this uniform tax base are allocated to EU member states (and possibly to the EU budget). Following the example of the U.S. and Canada for the profit taxation of multi-state or multi-province firms, the Bolkestein report suggests the sharing of taxes levied on consolidated profits on the basis of what is called *formula apportionment (FA)*.

Under a FA method, a corporation’s tax liability to each country where it operates is determined as the product of its consolidated (total) international profits times that country’s tax rate times the fraction of the corporation’s activities in that country. This fraction is calculated by an *apportionment formula* made up of one or more indicators for business activities. Most common in the U.S. is a three-part apportionment formula that calculates, for each state, a weighted average of the shares of a firm’s total sales (gross receipts), payroll, and property that can be attributed to that state (see Wildasin, 2000, for a survey). Canada uses a two-factor, payroll- and sales-shares formula. Under FA, no internal transfer prices are needed any more and, thus, the scope for tax evasion and tax exportation may be expected to be limited, as compared to the existing SA method. It is hoped that narrowing possibilities of tax shifting for firms translates, on the side of governments, into a weakening of the incentives for strategic tinkering with the tax code. A tax regime with smaller spillover effects might, thus, alleviate the problems of allocative inefficiencies and of harmful tax competition in the EU, both problems being induced by the current methods of taxation in the EU member states.

From a theoretical perspective, such hopes may not be well-grounded. Several authors have analysed and discussed spillover effects and distortions generated by FA taxation (i.e., the combination of unitary taxation and revenue apportionment) and showed that FA taxation does not necessarily go along with smaller cross-border externalities and distortions, compared to the SA method (see, e.g., Gordon and Wilson, 1986; Goolsbee and Maydew, 2000; Anand and Sansing, 2000; Mintz, 2002; Nielsen et al., 2003; Wellisch, 2003). Hence, the goads to strategic taxation for governments may be sharpened rather than blunted. In recent papers, Nielsen et al. (2001)

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3These studies and the present one deal with externalities and distortions that arise among the jurisdictions that participate in the FA method. Spillovers to non-participating jurisdictions are not discussed. It is a controversial issue associated with FA methods whether they should be applied only to corporate income earned within the group of participating countries in the FA method (“water’s edge” taxation) or to worldwide income; see Weiner (2002b) for a discussion in the European context. A theoretical analysis of this question is, to our knowledge, still missing.
and Sørensen (2003) compare tax spillovers and their implications for tax competition under SA and under the FA method that uses property (capital) shares to apportion profits. They conclude that no clear-cut ranking of SA and FA taxation is possible with respect to externalities and tax competition. Indeed, the problem of harmful tax competition might acerbate due to a switch from SA to FA. Even more strikingly, Sørensen (2003) conjectures that a switch to FA might change the nature of the tax competition game entirely. While under the current SA system, tax competition is of the “race to the bottom”-nature, tax rates in tax competition under FA might be set at inefficiently high levels (i.e., on the downward-sloping part of the Laffer curve).

While Nielsen et al. (2001) and Sørensen (2003) focus on a single and one-factor apportionment formula, namely the property-shares formula, and compare its performance relative to SA, the present paper compares different methods of formula apportionment with respect to their allocative features and strategic incentives. In an economy with two taxing jurisdictions (called countries) and a common market, a representative multinational corporation locates and operates in both countries. Profit taxation is unitary and revenues are distributed according to a FA method. Leviathan-type governments set their tax rates strategically, taking the FA method as given. The outcome of the associated tax competition game depends on the apportionment method. Our general result is that tax competition is sharper (i.e., equilibrium tax rates are lower) the more elastic the formula share is with respect to tax changes. To substantiate this observation, we identify the factors determining the tax elasticity of the formula shares and show that they are determined by curvature properties of the underlying production technologies. This enables us to show that in a scenario where capital is the only variable input tax competition is sharpest under the property-share rule, followed by the sales-share rule and the payroll-share rule. With two variable inputs tax competition under property- or payroll-apportionment is sharper than under the output-share formula. Whether the payroll- or the property-share formula generates tougher tax competition, depends on partial factor elasticities of production. While these findings relate to equilibrium tax rates, we also show that the nature of the tax competition game may change for different formulas: Tax competition under property-share apportionment leads to

\[\text{Eggert and Schjelderup (2003) compare tax competition under FA with tax competition when governments can use both residence- and source-based capital taxes and find that, while there are no inefficiencies associated with the latter one, FA tax competition leads to an inefficient outcome.}\]
inefficiently low tax rates, but equilibria under sales- and payroll-apportionment typically yield inefficiently high tax rates. Put differently: Tax competition typically leads to inefficient results due to a fiscal externality generated by national tax changes. Under traditional SA taxation the externality is generally positive. With FA taxation, whether the externality is a positive or a negative one depends on the apportionment method applied.

The remainder of this paper is organized in six sections. Section 2 sets out a model of Leviathan-type tax competition under FA with two countries and one representative multinational firm. Section 3 analyzes the tax competition game when capital is the only variable factor of production. Section 4 then generalizes the analysis to the case of two variable factors. Section 5 concludes.

2 The Model

2.1 The Multinational Firm

Following Nielsen et al. (2001) and Sørensen (2003), we consider a common market with two identical countries A and B. There is a single multinational firm that operates in both countries. In each of its two entities, the firm produces a single numéraire output, using the same technology $F$ that requires capital $K$ and labour $L$ as inputs.\(^5\)

Capital invested in country $i$ will be denoted by $K_i$ ($i = A, B$). Capital is in perfectly elastic supply and can be rented on world capital markets at an exogenous rate $\rho > 0$ (for a leveraged firm) or must yield that rate of return $\rho$ if it is equity capital.\(^6\) The input of labour in country $i$ is $L_i$. Denoting the common wage rate for both (identical) countries by $\omega$, the firm’s labour costs (payroll) in country $i$ is $\omega \cdot L_i$. Total output and, thus, sales revenues of the firm amount to $F(K_A, L_A) + F(K_B, L_B)$ while total expenditures for inputs sum up to $\rho(K_A + K_B) + \omega(L_A + L_B)$.

For brevity, we will write $F^i := F(K_i, L_i)$ for $i = A, B$. The production technology is assumed

\(^5\)In Nielsen et al. (2001) and Sørensen (2003) production requires a further factor, called “services”. This factor is a public input for the corporation as a whole. For its usage in the foreign affiliate an intra-firm transfer price can be charged which opens up opportunities for profit shifting, an important tax feature under SA. Under the types of FA discussed in this paper, profit shifting does not play a role; hence we can omit the public input without loss of generality.

\(^6\)We assume that the multinational is too small to impact on prices on (world) capital markets and on labour markets.
to be well-behaved. In particular, marginal productivities are positive and strictly decreasing: $F_x > 0$ and $F_{xx} < 0$ for $x = K, L$. Moreover, we assume that $F$ is strictly concave, implying $F_{KK}F_{LL} - F_{KL}^2 > 0$ for all $(K, L)$. Strict concavity ensures that corporate profits are strictly positive, which is an obvious requirement for any meaningful analysis of profit tax competition. We also assume that capital and labour are essential for production: $F(0, L) = F(K, 0) = 0$ for all $K, L$.

### 2.2 FA Taxation and Tax Competition

A multinational firm pays profit taxes to the government of each country where it operates. We assume that both countries share common rules of how to calculate consolidated profits of multinational enterprises\(^7\) and adopt a FA method to calculate the shares of the firms’ activities in their jurisdiction. A country’s tax revenues will then be calculated by multiplying the country’s profit tax rate with the fraction of total profits that is assigned to the country through the formula share. Denote country $A$’s [B’s] share of the firm’s activities by $\alpha$ [$\beta$]. In the countries’ profit tax laws, the values of $\alpha$ and $\beta$ are made dependent, in some way, on how the firm allocates its activities to the two countries. More specifically, there exist functions $\alpha$ and $\beta$ such that

$$\alpha = \alpha(K_A, K_B, L_A, L_B) \quad \text{and} \quad \beta = \beta(K_B, K_A, L_B, L_A)$$

are the formula shares of profits assigned, respectively, to country $A$ and country $B$. For the time being, no constraints are imposed on these functional forms of $\alpha$ and $\beta$. As special cases these functions include the most widely used parametric formulas (exemplified for $\alpha$ below):

- **FA according to property (or capital) shares**: $\alpha = \frac{K_A}{K_A + K_B}$;

- **FA according to sales (or output) shares**: $\alpha = \frac{F^A(K_A, L_A)}{F^A(K_A, L_A) + F^B(K_B, L_B)}$;

- **FA according to payroll shares**: $\alpha = \frac{\omega L_A}{\omega(L_A + L_B)} = \frac{L_A}{L_A + L_B}$;

\(^7\)To date, EU member states apply different rules for calculating profits as a base for (national) taxation. To incorporate such differences into the formal model would render the analysis much more complex.

\(^8\)Throughout we assume that plants only produce for the national markets where they are located. Thus, sales and outputs coincide. Given that we will mainly work in a setting with identical countries below, the assumption of zero ex- and imports (at least in equilibrium) is not overly restrictive.
and convex combinations thereof (as in the US or Canada). Given their widespread application we will refer to the three formulas above as *standard* formulas; however, other and more complex formulas are also well conceivable.

In general, it is not clear at all why the two countries should adopt formulas of the same type. In fact, this is a matter of strategic consideration or bilateral agreement (see Anand and Sansing, 2000; Wellisch, 2003). Therefore $\alpha$ and $\beta$ need generally not add up to unity. We will say that a tax regime under FA uses *uniform formulas* if the tax shares of the participating countries add up to one. In our two-country world this will be the case whenever

$$\alpha(K_A, K_B, L_A, L_B) + \beta(K_A, K_B, L_A, L_B) = 1$$

(1)

for all $(K_A, K_B, L_A, L_B)$. Obviously, if both countries adopt one and the same standard formula or the same convex combination thereof, then their formulas are uniform. With uniform formulas, the symbol $\beta$ can be dispensed with.

As the unitary base $\Phi$ for the corporate tax we use earnings before income and taxes (EBIT):

$$\Phi(K_A, K_B, L_A, L_B) := F^A + F^B - \omega(L_A + L_B) > 0.$$  

(2)

Hence, capital costs are assumed to be non-deductible in the corporate tax base.\(^9\) Denoting by $t^i$ country $i$’s statutory tax rate for corporate profits, the firm’s total tax bill under FA is

$$(\alpha \cdot t^A + \beta \cdot t^B) \cdot \Phi.$$  

The bracketed expression denotes the effective average tax rate on the firm’s operations in the common market; we will refer to it as $\tau$. Observe that the effective tax rate is, via $\alpha$ and $\beta$, a function of the firm’s activities:

$$\tau = \tau(K_A, K_B, L_A, L_B) = \alpha(K_A, K_B, L_A, L_B) \cdot t^A + \beta(K_A, K_B, L_A, L_B) \cdot t^B$$

The firm maximizes its net profits

$$\Pi = (1 - \tau(K_A, K_B, L_A, L_B)) \cdot \Phi(K_A, K_B, L_A, L_B) - \rho(K_A + K_B).$$

(3)

Tax revenues for governments $A$ and $B$ are given by

$$T^A = t^A \cdot \alpha \cdot \Phi \quad \text{and} \quad T^B = t^B \cdot \beta \cdot \Phi,$$

\(^9\)Our analysis would not have to undergo substantial changes if capital costs were fully or partly tax-deductible (also see Section 4.2 below).
respectively. We assume that governments aim at maximizing tax revenues, using the statutory tax rates \( t^A \) and \( t^B \) as policy instruments. The Leviathan assumption of revenue maximization is less restrictive than it may appear to be. As long as the ownership of the multinational firm is balanced between the two identical countries, our findings go through for more general government social welfare functions, provided that higher tax revenues \textit{ceteris paribus} increase social welfare (on this point also see Nielsen et al. (2001)).

Governments are strategically linked since both the common tax base \( \Phi \) and the formula shares \( \alpha \) and \( \beta \) are affected by both countries’ tax rates. Hence, a tax competition game will emerge, and we aim at comparing the Nash equilibria of this game under various FA specifications.

We will analyse the tax competition game in two scenarios. In the first scenario we assume that labour is a fixed input factor while in the second and more general scenario both capital and labour are variable factors of production. We adopt this procedure mainly for expositional purposes; one could, however, also make the case for the fixed-labour scenario by arguing that labour input cannot (or can less easily) be adjusted in the short run due to labour protection or other labour market inflexibilities.

3 Tax Competition when Labour Input is Fixed

3.1 Profit Maximization

Assuming that labour input is fixed at \( L_A = L_B = \bar{L} \), the multinational firm chooses \( K_A \) and \( K_B \) as to maximize (3). The FOCs for an interior solution to this program are given by (subscripts to functions denote partial derivatives):

\[
- \frac{\partial \tau}{\partial K_i} \cdot \Phi + (1 - \tau) \cdot F_{K_i}^i - \rho = 0 \quad \text{for } i = A, B. \quad (4)
\]

Here, the changes in the effective tax rates

\[
\frac{\partial \tau}{\partial x} = t^A \cdot \alpha_x + t^B \cdot \beta_x
\]

\((x = K_A, K_B)\) can be quite complicated expressions, depending on the formula applied. Observe that for uniform formulas \((\alpha + \beta = 1)\), we have

\[
\frac{\partial \tau}{\partial x} = (t^A - t^B) \cdot \alpha_x
\]
for $x = K_A, K_B$. As shown in (4), FA taxation does not only affect the factor allocation through its direct effects on net marginal factor returns (the second term on the LHS in (4)), but also indirectly: The firm influences its apportionment-determined, effective tax rates $\tau^i$ through input choices (the first terms on the LHS of (4)). If $\partial \tau / \partial x \neq 0$ for $x = K_A, K_B$ a profit-maximizing firm needs to account for the dependence of its effective tax rate on its choice variables. In fact, the firm would unnecessarily increase its overall tax load by disregarding this relationship. It is not \textit{a priori} clear whether this indirect effect is positive or negative.\textsuperscript{10}

3.2 Comparative Statics

The multinational firm’s responses to tax changes under a FA method are more difficult to determine than under separate accounting since in the former case firms do not only influence the tax base, but also their effective tax rate.\textsuperscript{11} To facilitate the analysis we will impose further restrictions:

- Full symmetry: Not only are both countries identical, but their governments have also set equal tax rates in the initial situation: $t^A = t^B = t$.
- Uniform formulas: Condition (1) holds.

As a consequence of these assumptions, the initial situation will be characterized, first, by equal marginal productivities of capital ($F^A_K = F^B_K = \rho$), second, by identical amounts of investment ($K_A = K_B$), third, by $\tau_x = 0$ for $x = K_A, K_B$, and fourth, by $\tau = t$.

A further implication of full symmetry will turn out to be extremely helpful for comparative statics: We can confine our analysis to changes of $t^A$. Variations in $t^B$ follow one-by-one according to the pattern:

$$\frac{\partial K_B}{\partial t^B} = \frac{\partial K_A}{\partial t^A} \quad \text{and} \quad \frac{\partial K_A}{\partial t^B} = \frac{\partial K_B}{\partial t^A}. \quad (5)$$

\textsuperscript{10}Weiner (2002a, b) refers to positive and negative effects as an additional “tax” or a “subsidy” in the FA method. This is, strictly speaking, only appropriate when firms would not change their production plans upon a switch to a FA tax rule. See Weiner (2002b) for more details on corporate tax planning under FA tax regimes.

\textsuperscript{11}Basically, it is the firm’s impact on its own effective tax rate that might render FA taxation more distortionary than SA taxation.
Under full symmetry, the comparative statics of (4) with respect to a tax change in \( A \) are described by the following equations:\(^{12}\)

\[
\frac{\partial K_A}{\partial t^A} = \frac{1}{(1-t)F_{KK}} \cdot (\alpha F_K + \alpha K_A \Phi) \\
\frac{\partial K_B}{\partial t^A} = \frac{1}{(1-t)F_{KK}} \cdot (\alpha F_K + \alpha K_B \Phi).
\]

Hence, we obtain

**Proposition 1**

\( a) \) A tax increase in a country drives capital out of that country whenever the country’s formula share does not decline upon an increase in its capital stock:

\[ \alpha K_A \geq 0 \implies \frac{\partial K_A}{\partial t^A} < 0. \]

\( b) \) For all formulas satisfying \( \alpha K_A = -\alpha K_B \) in a symmetric allocation, a tax increase reduces total investments by the same amount:

\[ \frac{\partial (K_A + K_B)}{\partial t^A} = \frac{2\alpha F_K}{(1-t)F_{KK}} < 0. \]

The assumption that higher investments in \( A \) do not reduce that country’s share according to the apportionment formula (i.e., \( \alpha K_A \geq 0 \)) seems plausible; in particular, it is satisfied by the three standard formulas and their convex combinations. With this feature, higher taxes drive investment out of the country (first item of Proposition 1). The standard formulas also exhibit the property that starting from a symmetric allocation an inflow of capital to country \( A \) increases that country’s formula share by the same amount by which it reduces the other country’s share (\( \alpha K_A = -\alpha K_B \)). Under all formulas with this feature a tax increase leads to an identical drop in worldwide investment.

A tax increase in one country increases the effective tax rate \( \tau \) and thereby reduces the marginal return to investment everywhere. This leads to a fall in total investment. Moreover, the country where the tax increase occurs becomes less attractive for hosting investments, relative to the foreign country. Hence, domestic investment always reduces upon a domestic tax increase. However, two opposing effects are at work in the foreign country. That leaves it generally unclear

\(^{12}\)It is worth noting that without the assumptions of full symmetry and uniform formulas these expressions would be considerably more complex and hardly tractable.
whether investment increases or falls there (unless one unplausibly assumes that \( \alpha_{KB} > 0 \)). It is easy to see that a tax raise in \( A \) reduces capital investments in \( B \) if and only if, in absolute terms, the elasticity of country \( A \)'s tax share \( \alpha \) with respect to \( K_B \) is larger than the elasticity of EBIT with respect to \( K_B \):

\[
\frac{\partial K_B}{\partial t^A} > 0 \iff \frac{\alpha_{KB} K_B}{\alpha} > \frac{F_K K_B}{\Phi}.
\]

(7)

Going through the list of the standard formulas in a symmetric situation, we see that foreign investments may indeed rise or fall upon a domestic tax increase. We will index variables relating to \( FA \) via property shares, sales shares and payroll shares by, respectively, \( K, F \) and \( W \).

**FA with property shares.** Assume that \( \alpha = K_A/(K_A + K_B) \). For the symmetric case, we then get \( \alpha = 1/2 \) and \( \alpha_{KA} = -\alpha_{KB} = 1/(4K) \). Inserting this specification into (6a) and (6b), we obtain:

\[
\left. \frac{\partial K_A}{\partial t^A} \right|_K = \frac{1}{2(1-t)F_{KK}} \left( F_K + \frac{\Phi}{2K} \right) < 0 \quad (8a)
\]

\[
\left. \frac{\partial K_B}{\partial t^A} \right|_K = \frac{1}{2(1-t)F_{KK}} \left( F_K - \frac{\Phi}{2K} \right) > 0. \quad (8b)
\]

Hence, an increase in \( t^A \) leads to an inflow of capital in country \( B \). The positive sign of (8b) follows from the observation that the tax base is smaller than total receipts. Tax increases in \( A \) reduce capital investment in both \( A \) and in \( B \).

**FA with sales shares.** Let \( \alpha = F_A/(F_A + F_B) \). Again, \( \alpha = 1/2 \) for the symmetric case. Furthermore, \( \alpha_{KA} = -\alpha_{KB} = F_K/(4F) \). Hence, eqs. (6a) and (6b) are turned into

\[
\left. \frac{\partial K_A}{\partial t^A} \right|_F = \frac{F_K}{2(1-t)F_{KK}} \left( 1 + \frac{\Phi}{2F} \right) < 0 \quad (9a)
\]

\[
\left. \frac{\partial K_B}{\partial t^A} \right|_F = \frac{F_K}{2(1-t)F_{KK}} \left( 1 - \frac{\Phi}{2F} \right) < 0. \quad (9b)
\]

The negative sign of (9b) follows from the observation that the tax base is smaller than total receipts. Tax increases in \( A \) reduce capital investment in both \( A \) and in \( B \).

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13 Eqs. (8a) and (8b) correspond to eq. (15) in Nielsen et al. (2001) where, however, an additional factor of production is present. This additional factor prevents Nielsen et al. (2001) from unambiguously signing their analogue to (8b).
FA with payroll shares. In this case, $\alpha = 1/2$ is constant. Trivially, $\alpha_{KA} = \alpha_{KB} \equiv 0$. Obviously, then, investments drop by the same amount in both countries when tax rates rise:

$$\frac{\partial K_A}{\partial t^A} \bigg|_W = \frac{\partial K_B}{\partial t^A} \bigg|_W = \frac{F_K}{2(1-t)F_{KK}} < 0. \quad (10)$$

We can now compare the comparative static effects of tax changes across the three standard apportionment formulas. Starting from a symmetric situation with $t^A = t^B$, the comparison across formulas is indeed meaningful since in the initial situation the standard formulas lead to the same factor allocation.

**Proposition 2** Starting from a symmetric situation with uniform formulas,

a) the negative impact of a tax increase on domestic investment is, in absolute terms, largest under the property-share formula, followed by the sales-share and the payroll-share formula:

$$\frac{\partial K_A}{\partial t^A} \bigg|_K < \frac{\partial K_A}{\partial t^A} \bigg|_F < \frac{\partial K_A}{\partial t^A} \bigg|_W < 0; \quad (11)$$

b) the impact of a tax increase in one country on investment abroad is, in absolute terms, largest under a property-share formula, followed by the output-share and the payroll-share formula:

$$\frac{\partial K_B}{\partial t^A} \bigg|_W < \frac{\partial K_B}{\partial t^A} \bigg|_F < \frac{\partial K_B}{\partial t^A} \bigg|_K; \quad (12)$$

c) the negative impact of a tax increase on the firm’s total investment is the same for all three formulas:

$$\frac{\partial (K_A + K_B)}{\partial t^A} \bigg|_W = \frac{\partial (K_A + K_B)}{\partial t^A} \bigg|_F = \frac{\partial (K_A + K_B)}{\partial t^A} \bigg|_K < 0. \quad (13)$$

**Proof:** The result follows from comparing equations (8a) through (10). Observe that, due to $F(0, L) = 0$ for all $L$, the marginal productivity of capital always falls short of its average productivity: $F_K(K, L) < F(K, L)/K$ for all $(K, L)$.

Proposition 2 states for the FA rules under consideration that the negative impact of an increase in $t^A$ on domestic investment is (among the three standard rules) strongest for property-share apportionment and weakest for the payroll-share formula. This ranking reverses in case of foreign
investment. The aggregate effect, i.e., the effect on the sum of domestic and foreign investment, is the same for all rules. Observe that foreign investment reacts quite differently under the different formulas. A tax increase leads to an increase in foreign investment when a property-share formula is applied while the same tax increase decreases foreign investment under sales- and payroll-based apportionment.\footnote{In their comparison of the effects of tax increases under separate accounting and FA with a property-share formula, Nielsen et al. (2001) find that the effects on domestic [foreign] investment are stronger [weaker] under FA than under SA. Our results suggest that the formula scrutinized by Nielsen et al. (2001) is the most favorable for such a comparison and that it is not clear, therefore, whether their finding also carries over to other apportionment methods.}

3.3 Equilibria in Tax Competition Games under FA

Government $A$ chooses its tax rate $t^A$ as to maximize tax revenues $T^A = \alpha \cdot t^A \cdot \Phi$. After some re-arrangements we calculate the marginal impact of a change in $t^A$ on government $A$’s tax revenues as

$$\frac{\partial T^A}{\partial t^A} = \left\{ \alpha \Phi + t^A \alpha \left( F^K_A \frac{\partial K_A}{\partial t^A} + F^K_B \frac{\partial K_B}{\partial t^A} \right) \right\} + t^A \Phi \left[ \alpha_K^A \frac{\partial K_A}{\partial t^A} + \alpha_K^B \frac{\partial K_B}{\partial t^A} \right]. \quad (14)$$

Obviously, a zero tax rate can never be optimal as $\frac{\partial T^A}{\partial t^A} |_{t^A=0} > 0$. Since the setting is fully symmetric, so will be Nash equilibria. Verify that, by Proposition 2, items a) and c), the terms in curly brackets in (14) are identical for property, payroll, and sales apportionment. Further recall that in a symmetric situation $\alpha_K^A = -\alpha_K^B > 0$ holds for all rules. Hence, the only source of difference in the marginal revenue effects of $t^A$ under the various rules is the difference $(\partial K_A/\partial t^A) - (\partial K_B/\partial t^A)$. Now use Proposition 2, items a) and b), to obtain

$$\frac{\partial T^A}{\partial t^A} \bigg|_K < \frac{\partial T^A}{\partial t^A} \bigg|_F < \frac{\partial T^A}{\partial t^A} \bigg|_W \quad (15)$$

at all symmetric pairs of tax rates $(t^A, t^B) = (t, t)$. The same pattern holds for country $B$. Hence, tax rates in a symmetric Nash equilibrium (i.e., the tax level $t$ for which $\partial T^A(t, t)/\partial t^A = \partial T^B(t, t)/\partial t^B = 0$) are higher for payroll apportionment than for sales apportionment which in turn lies above that for property apportionment. Therefore, we obtain

**Proposition 3** In a symmetric Nash equilibrium of the Leviathan tax competition game with FA, the level of taxation will be highest with a payroll formula, followed by sales and property...
According to Proposition 3, tax competition is sharpest (in the sense that tax rates are lowest) with a FA method that uses capital shares, followed by sales shares, and finally payroll shares. The economic explanation for this ranking can be based on the intuitive principle that tax competition is stiffer the more elastic is the tax base with respect to tax changes. This principle is at work in many tax-competition models. E.g., in standard models of capital tax competition (surveyed in Wilson, 1999), the tax-base elasticity is determined by the elasticity of capital demand with respect to its gross return, and it turns out that the level of public good provision is lower the more elastic is capital demand. To see that the elasticity principle also drives Proposition 3, recall from (14) that, starting from a symmetric allocation, differences in the effects of tax changes on the tax base are fully determined by the reaction of the formula share $\alpha$ on changes in $t^A$; the terms outside the curly brackets in (14) can be written as

$$t^A \Phi \left[ \alpha_{KA} \frac{\partial K_A}{\partial t^A} + \alpha_{KB} \frac{\partial K_B}{\partial t^A} \right] = \alpha \cdot \Phi \cdot \left( \frac{\partial \alpha}{\partial t^A} \cdot \frac{t^A}{\alpha} \right),$$

where the RHS term is the elasticity of the formula share with respect to the tax rate. This elasticity is, in absolute terms, lowest (precisely and trivially: zero) for the payroll formula, since labor inputs are constant by assumption. Furthermore, as the proof shows the elasticity is, in absolute terms, higher for property-share apportionment than for the sales-share formula whenever $F > F_K K$, which holds for all concave technologies. As the same pattern also holds for country $B$, *mutatis mutandis*, tax competition between $A$ and $B$ turns out to be sharpest with property-share apportionment.

### 3.4 Assessing and Comparing Nash Equilibria

We now briefly compare the Nash equilibrium to the cooperative solution of joint revenue maximization. First observe that the (symmetric) cooperative solution $(t^*, t^*)$ (i.e., the maximizer of the problem $\max \{ T^A + T^B \}$) is common to all three formulas we are discussing here. Whether tax rates in the cooperative solution lie below or above those in the non-cooperative solution depends on whether the revenue externality $\partial T^A / \partial t^B$ is, respectively, negative or positive, if
evaluated at the Nash equilibrium. We calculate:

\[
\frac{\partial T^A}{\partial t^B} = t^A \cdot \alpha \cdot \left( F_K \frac{\partial K_A}{\partial t^B} + F_K \frac{\partial K_B}{\partial t^B} \right) + t^A \Phi \left[ \alpha_K \frac{\partial K_A}{\partial t^B} + \alpha_K \frac{\partial K_B}{\partial t^B} \right]
\]

\[
= \frac{t}{2(1-t)F_K} \cdot \left( F_K^2 - 2 \alpha_K (\alpha_K - \alpha_K) \Phi^2 \right),
\]

where the second line follows from (5), (6a), and (6b), presupposing symmetry \((t^A = t^B = t)\).

**FA with property shares.** For \(\alpha = K_A / (K_A + K_B)\) we get \(\alpha_K (\alpha_K - \alpha_K) = -1/(8K^2)\) in a symmetric allocation. Hence, from (17),

\[
\frac{\partial T^A}{\partial t^B} \bigg|_{K} = \frac{t}{2(1-t)F_K} \cdot \left( F_K^2 - \Phi^2/(4K^2) \right) > 0,
\]

by the same argument as in (8b). Therefore, under FA with a property-share rule, the Nash equilibrium tax rate is too low, relative to the cooperative one.

**FA with sales shares.** If \(\alpha = F^A / (F^A + F^B)\) we obtain \(\alpha_K (\alpha_K - \alpha_K) = -F_K^2/(8F^2)\) in a symmetric allocation. Hence, from (17),

\[
\frac{\partial T^A}{\partial t^B} \bigg|_{F} = \frac{tF_K^2}{2(1-t)F_K} \cdot \left( 1 - \Phi^2/(4F^2)\right) < 0,
\]

since \(\Phi < 2F\). Consequently, the Nash equilibrium tax rate is too high under FA with a sales-share formula, relative to the cooperative one.

**FA with payroll shares.** In this case, \(\alpha\) is constant. It is therefore immediate from (17) that the fiscal externality is negative,

\[
\frac{\partial T^A}{\partial t^B} \bigg|_{W} = \frac{tF_K^2}{2(1-t)F_K} < 0,
\]

driving tax rates down to a suboptimally high level in the Nash equilibrium.

Observe that for all tax pairs \((t, t)\) we have \(\partial T^A/\partial t^B|_W < \partial T^A/\partial t^B|_F < 0\). Combining this with Proposition 2, we notice that in the Nash equilibrium under payroll-share apportionment joint and, due to symmetry, also national tax revenues are lower than under sales-share apportionment. However, it is not possible to compare the tax revenues under property-share apportionment with those under the two other formulas since equilibrium tax rates are located on different sides of the peak of the (joint-revenue) Laffer curve. We sum up our findings in
**Proposition 4** In the Nash equilibrium of tax competition with property-share apportionment tax rates are inefficiently low, while equilibrium tax rates under sales- and payroll-share apportionment are inefficiently high, relative to the cooperative level. Tax revenues in the Nash equilibrium are lower under payroll-share than under sales-share apportionment.

### 3.5 Optimally Weighted Formulas

The observation in Proposition 4 that a positive fiscal externality prevails under property-share apportionment while under the two other standard formulas the fiscal externality is negative readily suggests to look for suitable convex mixtures of standard formulas which balance the incentives for over- and undertaxation such as to non-cooperatively implement the cooperative outcome. Do there exist non-negative weights $m_F$, $m_W$, and $m_K$ with $m_F + m_W + m_K = 1$ such that tax competition under the apportionment formula

$$
\alpha = m_F \cdot \alpha_F + m_W \cdot \alpha_W + m_K \cdot \alpha_K
$$

is efficient? (Superscripts to $\alpha$ indicate the “pure” formula shares.) Since both pure payroll- and pure sales-share apportionment lead to suboptimally high tax rates (Proposition 4), we can set either $m_W = 0$ or $m_F = 0$ without loss of generality. We choose to dispense with sales-share apportionment ($m_F = 0$) and therefore proceed by considering formulas of type

$$
\alpha(m_K) = m_K \cdot \alpha_K + (1 - m_K) \cdot \alpha_W
$$

(21)

with $m_K \in [0,1]$. For simplicity, we assume that both countries adopt the same weights in their formulas. All marginal effects and comparative statics for $\text{FA}$ according to $\alpha(m_K)$ can now be obtained from weighing and summing up the pure effects derived in the previous section for property- and payroll-share apportionment with $m_K$ and $(1-m_K)$. Fiscal externalities will consequently be internalised if and only if $m_K$ is set to satisfy

$$
m_K \cdot \frac{\partial T_A}{\partial t_B} \bigg|_K + (1 - m_K) \cdot \frac{\partial T_A}{\partial t_B} \bigg|_W = 0.
$$

Invoking (18) and (20) we determine the “efficient” weight $m_K$ as

$$
m_K^* = \frac{4F^2_K K^2}{\Phi^2 - 4F^2_K K^2} \in (0, 1).
$$

Hence, we established
Proposition 5 There exist convex combinations of property-share apportionment with payroll-share and/or sales-share apportionment such that FA tax competition under the resultant mixed formula share implements the cooperative level of taxation.

To assess the policy relevance of this result we need to point out, however, that the term on the RHS of (22) has to be evaluated at the level of capital prevailing in the cooperative solution \((t^*, t^*)\). It is apparent, therefore, that in practice the search for \(m^*_K\) will be fraught with unsurmountable informational problems, requiring precise knowledge not only of the efficient level of taxation but also of production technologies.

4 Tax Competition when Capital and Labour Choices are Endogenous

4.1 Profit Maximization and Comparative Statics

So far we assumed labour to be a fixed factor. Owing to this assumption the formula apportionment with payroll shares degenerated to a constant rule. Although the assumption of constant labour inputs might be warranted in a short-run perspective by inflexibilities in the labour market, it ought to be dropped if we wish to assess the full and long-run effects of alternative FA methods. Therefore we now assume that not only capital but also labour is in perfectly elastic supply and the multinational chooses both labour and capital inputs for its plants in countries A and B. As before, our analysis will start from an initial equilibrium with full symmetry. With a uniform formula, the set of FOCs for profit maximization on behalf of the firm reads:

\[
\begin{align*}
- (t^A - t^B) \alpha_{KA} \Phi + (1 - \tau) F_K^A - \rho &= 0; \\
- (t^A - t^B) \alpha_{KB} \Phi + (1 - \tau) F_K^B - \rho &= 0; \\
- (t^A - t^B) \alpha_{LA} \Phi + (1 - \tau) [F_L^A - \omega] &= 0; \\
- (t^A - t^B) \alpha_{LB} \Phi + (1 - \tau) [F_L^B - \omega] &= 0.
\end{align*}
\]

In a symmetric situation with \(t^A = t^B\) and under a uniform formula (23a) to (23d) yield \(F_K^A = F_K^B\) and \(F_L^A = F_L^B = \omega\). Consequently, \(K_A = K_B = K\) and \(L_A = L_B = L\).

With \(t^A = t^B\) in the initial situation, the comparative statics of (23a) through (23d) for a tax
The change in a are described by the following system of equations:

\[
(1 - \tau) \cdot \begin{pmatrix}
F_{KK}^A & 0 & F_{KL}^A & 0 \\
0 & F_{KK}^B & 0 & F_{KL}^B \\
F_{KL}^A & 0 & F_{LL}^A & 0 \\
0 & F_{KL}^B & 0 & F_{LL}^B \\
\end{pmatrix}
\cdot \begin{pmatrix}
\partial K_A / \partial t^A \\
\partial K_B / \partial t^A \\
\partial L_A / \partial t^A \\
\partial L_B / \partial t^A \\
\end{pmatrix} = \begin{pmatrix}
\gamma_1 \\
\gamma_2 \\
\gamma_3 \\
\gamma_4 \\
\end{pmatrix}
\] (24)

where

\[
\gamma_1 := \alpha_{K_A} \Phi + F_{K}^A \alpha \\
\gamma_2 := \alpha_{K_B} \Phi + F_{K}^B \alpha \\
\gamma_3 := \alpha_{L_A} \Phi + [F_{L}^A - \omega] \alpha = \alpha_{L_A} \Phi \\
\gamma_4 := \alpha_{L_B} \Phi + [F_{L}^B - \omega] \alpha = \alpha_{L_B} \Phi,
\]

where \(\gamma_3\) and \(\gamma_4\) account for a symmetric setting. Due to symmetry we can dispense with country indexes again. It is then straightforward to show

\[
\det M = (F_{KK}^A F_{LL}^A - F_{KL}^A)^2 > 0,
\]

where \(M\) is the matrix on the LHS of (24). Denote by \(M_i\) the matrix that emerges from replacing the \(i\)-th column in \(M\) by the \(\gamma\)-vector. From Cramer’s Rule,

\[
\frac{\partial K_A}{\partial t^A} = \frac{\det M_1}{(1 - t) \det M} = \frac{(F_{KK}^A F_{LL}^A - F_{KL}^A) \cdot [\gamma_1 F_{LL}^A - \gamma_3 F_{KL}^A]}{(1 - t) (F_{KK}^A F_{LL}^A - F_{KL}^A)^2} = N^{-1} \cdot (\gamma_1 F_{LL}^A - \gamma_3 F_{KL}^A),
\]

(25a)

\[
\frac{\partial K_B}{\partial t^A} = N^{-1} \cdot (\gamma_2 F_{LL}^A - \gamma_4 F_{KL}^A),
\]

(25b)

\[
\frac{\partial L_A}{\partial t^A} = N^{-1} \cdot (\gamma_3 F_{KK}^A - \gamma_1 F_{KL}^A),
\]

(25c)

\[
\frac{\partial L_B}{\partial t^A} = N^{-1} \cdot (\gamma_4 F_{KK}^A - \gamma_2 F_{KL}^A),
\]

(25d)

where

\[
N := (1 - t) (F_{KK}^A F_{LL}^A - F_{KL}^A)^2 > 0.
\]

(26)

It is reasonable to assume that higher investment or labour input in country \(A [B]\) does not reduce [does not increase] country \(A\)'s tax share:

\[
\alpha_{K_A} \geq 0 \geq \alpha_{K_B} \quad \text{and} \quad \alpha_{L_A} \geq 0 \geq \alpha_{L_B}.
\]

(27)

This provided, we obtain

17
Proposition 6 Suppose that (27) holds.

- If $FKL \geq 0$, then a tax increase in country $A$ reduces investment $KA$, employment $LA$, and output $FA$ in country $A$.

If furthermore $\alpha_A = -\alpha_B$ (with $x = K, L$) in a symmetric allocation, then a tax hike decreases total investments $KA + KB$, total employment $LA + LB$ and, thus, total output $FA + FB$.

- For $FKL < 0$ the effects of an increase in $t_A$ on $KA, KB, LA, and LB$ are ambiguous.

\textbf{Proof:} If (27) holds, $\gamma_1 > 0, \gamma_3 \geq 0$, and $\gamma_4 \leq 0$ while the sign of $\gamma_2$ is unclear.\textsuperscript{15} Combined with $FKL \geq 0$, this leads to unambiguously negative comparative static effects for $KA$ and $LA$. Otherwise the impact is unclear.

To determine the effects on $KA + KB$ and $LA + LB$ verify that, if $\alpha_K = -\alpha_B, \gamma_1 + \gamma_2 = 2\alpha F_K = F_K$ and $\gamma_3 + \gamma_4 = 0$. Use this when summing up the partial effects. ■

Proposition 6 covers the three standard formulas: $FA$ according to capital shares, payroll shares, and sales shares. The assumption that the marginal productivity of either factor increases with an increase in the input of the other ($FKL > 0$) is satisfied by most commonly used production functions. In particular, it holds for Cobb-Douglas technologies.

Proposition 6 is the analogue of Proposition 1 and both propositions yield about the same message: If a government raises its profit tax rate, it will drive economic activities out of its jurisdiction. The cross-border effects of tax increases remain ambiguous, however, and may vary with the formula that is applied.

\subsection{4.2 Digression: Tax-Deductible Cost of Capital}

So far, we assumed that the costs of capital are not deductible from the profit tax base, while labour expenditures are. We now briefly discuss the implications of allowing interest payments to be tax-deductible. As a first consequence, the FOCs (23a) and (23b) for profit maximization with respect to investments are modified to

\[-(t^A - t^B)\alpha_K A \Phi + (1 - \tau)(FA^A - \rho) = 0;\]

\textsuperscript{15}Recall from eq. (6b) in the previous section that $\gamma_2$ might indeed take different signs for different formulas.
\[-(t^A - t^B)\alpha_{KB}\Phi + (1 - \tau)[F^B_K - \rho] = 0.\]

In a symmetric initial situation we have \(F^A_K = F^B_K = \rho\). The first two components in the vector \(\gamma\) in (24) will then read \(\gamma_1 = \alpha_{K_A}\Phi > 0\) and \(\gamma_2 = \alpha_{K_B}\Phi < 0\). As a consequence, a tax increase in \(A\) will unambiguously trigger additional investments of capital in \(B\) when \(F_{KL} > 0\). Hence, we can tighten the corresponding result of Proposition 6. Moreover, if \(\alpha_{KB} = -\alpha_{KA}\) in a symmetric allocation, a tax increase in \(A\) neither affects total investment nor total employment (since \(\gamma_1 + \gamma_2 = \gamma_3 + \gamma_4 = 0\) in this case). Yet, total output \(F^A + F^B\) will decrease: The initial situation is characterized by international production efficiency. Shifting factors from \(A\) to \(B\) will then cause a decline in worldwide production. We sum this up in

**Proposition 7** Assume that interest costs are deductible from the profit tax base. Suppose that (27) holds and that \(F_{KL} \geq 0\). Then a tax increase in country \(A\) leads to smaller investment \(K_A\), lower employment \(L_A\), and lower output \(F^A\). It increases investment \(K_B\), employment \(L_B\) and output \(F^B\).

If furthermore \(\alpha_{xA} = -\alpha_{xB}\) for \(x = K, L\) in a symmetric allocation, then a tax increase in \(A\) leaves total investment and total employment unchanged. Total output \(F^A + F^B\), however, decreases:

\[
\frac{\partial K_A}{\partial t^A} = -\frac{\partial K_B}{\partial t^A}, \quad \frac{\partial L_A}{\partial t^A} = -\frac{\partial L_B}{\partial t^A}, \quad \text{and} \quad \frac{\partial (F^A + F^B)}{\partial t^A} < 0.
\]

The results on comparative statics and on equilibria in tax competition games to be presented below do not depend on the asymmetry in the tax treatment of capital and labour costs (see footnotes 16 and 18 below). We therefore continue to take interest costs as non-deductible in the tax base definition.

### 4.3 Comparing Formulas

We now compare the comparative statics of tax changes for the standard formulas. Table 1 (see page 20) summarizes the numerators of (25a) to (25d) for the three formulas.\(^{16}\) Comparing the numerators suffices for a comparison since in a symmetric setting the denominators are identical for all three formulas. The following result is immediate from Table 1:

\(^{16}\) In case we allowed for interest cost deductibility we would get \(Q_1 = Q_2 = 0\) in Table 1. It is easy to see that this would not affect any of the comparisons across formulas presented in the following propositions.
\[ \begin{align*} 
\partial K_A/\partial t^A & \quad (25a) \\
\partial K_B/\partial t^A & \quad (25b) \\
\partial L_A/\partial t^A & \quad (25c) \\
\partial L_B/\partial t^A & \quad (25d) 
\end{align*} \]

<table>
<thead>
<tr>
<th>Formula</th>
<th>( \frac{F_{LL}}{4K} \Phi + Q_1 )</th>
<th>( \frac{F_{LL}}{4K} \Phi + Q_1 )</th>
<th>( \frac{F_{KL}}{4K} \Phi + Q_2 )</th>
<th>( \frac{F_{KL}}{4K} \Phi + Q_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K )</td>
<td>( \frac{F_{KL}F_{LL} - F_{L}F_{KL}}{4F} \Phi + Q_1 )</td>
<td>( \frac{F_{KL}F_{LL} - F_{L}F_{KL}}{4F} \Phi + Q_1 )</td>
<td>( \frac{F_{L}F_{KK} - F_{KL}F_{KL}}{4F} \Phi + Q_2 )</td>
<td>( \frac{F_{L}F_{KK} - F_{KL}F_{KL}}{4F} \Phi + Q_2 )</td>
</tr>
<tr>
<td>( F )</td>
<td>( \frac{F_{KL}}{4L} \Phi + Q_1 )</td>
<td>( \frac{F_{KL}}{4L} \Phi + Q_1 )</td>
<td>( \frac{F_{KK}}{4L} \Phi + Q_2 )</td>
<td>( \frac{F_{KK}}{4L} \Phi + Q_2 )</td>
</tr>
<tr>
<td>( W )</td>
<td>( \frac{F_{KL}}{4L} \Phi + Q_1 )</td>
<td>( \frac{F_{KL}}{4L} \Phi + Q_1 )</td>
<td>( \frac{F_{KL}}{4L} \Phi + Q_2 )</td>
<td>( \frac{F_{KL}}{4L} \Phi + Q_2 )</td>
</tr>
</tbody>
</table>

with \( Q_1 := \frac{1}{2} \cdot (F_{KL}F_{LL} - (F_{L} - w)F_{KL}) = F_{KL}F_{LL}/2 \) and \( Q_2 := \frac{1}{2} \cdot ((F_{L} - w)F_{KK} - F_{KL}F_{KL}) = -F_{KL}F_{KL}/2 \).

All effects share the common positive denominator \( N \) (defined in (26)) which is suppressed in this table.

Table 1: Comparing comparative statics across formulas
Proposition 8  In a symmetric situation with $t^A = t^B$ the following holds:

a) Comparison between property-share and payroll-share formulas:

\[
\frac{\partial K_A}{\partial t^A} \bigg|_{K < \frac{\partial K_A}{\partial t^A}} \mid W \iff \frac{\partial K_B}{\partial t^A} \bigg|_{K > \frac{\partial K_B}{\partial t^A}} \mid W \iff \frac{F_{LL}}{K} > \frac{F_{KL}}{L}
\]

\[
\frac{\partial L_A}{\partial t^A} \bigg|_{K < \frac{\partial L_A}{\partial t^A}} \mid W \iff \frac{\partial L_B}{\partial t^A} \bigg|_{K > \frac{\partial L_B}{\partial t^A}} \mid W \iff -\frac{F_{KL}}{K} > \frac{F_{KK}}{L}
\]

b) Comparison between property-share and sales-share formulas:

\[
\frac{\partial K_A}{\partial t^A} \bigg|_{K < \frac{\partial K_A}{\partial t^A}} \mid W \iff \frac{\partial K_B}{\partial t^A} \bigg|_{K > \frac{\partial K_B}{\partial t^A}} \mid W \iff \frac{F_{LL}}{K} > \frac{F_{KL}F_{LL} - F_{LL}F_{KL}}{F}
\]

\[
\frac{\partial L_A}{\partial t^A} \bigg|_{K < \frac{\partial L_A}{\partial t^A}} \mid W \iff \frac{\partial L_B}{\partial t^A} \bigg|_{K > \frac{\partial L_B}{\partial t^A}} \mid W \iff -\frac{F_{KL}}{K} > \frac{F_{KK}F_{KL} - F_{KL}F_{KL}}{F}
\]

c) Comparison between payroll-share and sales-share formulas:

\[
\frac{\partial K_A}{\partial t^A} \bigg|_{W < \frac{\partial K_A}{\partial t^A}} \mid F \iff \frac{\partial K_B}{\partial t^A} \bigg|_{W > \frac{\partial K_B}{\partial t^A}} \mid F \iff -\frac{F_{KL}}{L} < \frac{F_{KL}F_{LL} - F_{LL}F_{KL}}{F}
\]

\[
\frac{\partial L_A}{\partial t^A} \bigg|_{W < \frac{\partial L_A}{\partial t^A}} \mid F \iff \frac{\partial L_B}{\partial t^A} \bigg|_{W > \frac{\partial L_B}{\partial t^A}} \mid F \iff \frac{F_{KK}}{L} < -\frac{F_{LL}F_{KK} - F_{KK}F_{KL}}{F}
\]

d) Comparison of total effects:17

The effects of a tax increase on total investment $K_A + K_B$ and on total employment $L_A + L_B$ are identical across all three formulas.

Although the results of Proposition 8 appear to be quite involved, they are, in essence, very similar to those reported in Proposition 2. First, Proposition 8 shows that the comparative statics under different formulas can be neatly ranked by simple terms involving production and the properties of technology. We will further comment on this below. Second, Proposition 8 demonstrates that the ranking of the formulas with respect to the strength of domestic tax effects is exactly inverse to the ranking with respect to cross-border effects.

More specific results can be obtained from Proposition 8 by assuming that technologies are homogeneous of degree $r < 1$:

17 From (25a) to (25d) it is easy to verify that total effects are equal for all uniform formulas satisfying $\alpha_{K_A} = -\alpha_{K_B}$ and $\alpha_{L_A} = -\alpha_{L_B}$ under symmetry.
Corollary 8.1 If technologies are homogeneous of degree \( r < 1 \), the following inequalities will hold in a symmetric situation:

\[
\left. \frac{\partial K_A}{\partial t^A} \right|_K < \left. \frac{\partial K_A}{\partial t^A} \right|_W < 0 \quad \text{and} \quad \left. \frac{\partial K_B}{\partial t^A} \right|_K > \left. \frac{\partial K_B}{\partial t^A} \right|_W \nabla 0 \left. \frac{\partial L_A}{\partial t^A} \right|_K > \left. \frac{\partial L_A}{\partial t^A} \right|_W \quad \text{and} \quad \left. \frac{\partial L_B}{\partial t^A} \right|_K < \left. \frac{\partial L_B}{\partial t^A} \right|_W .
\]

Proof: Homogeneity of degree less than one is formally characterized by \( KF_K + LF_L = r \cdot F \) for some constant \( r < 1 \). Differentiating this equation with respect to \( L \) yields \( KF_K L + LF_LL = (r-1)F_L < 0 \), and differentiation with respect to \( K \) yields \( LF_K L + KF_K K = (r-1)F_K < 0 \). The corollary follows from invoking these inequalities in the first item of Proposition 8. ■

The ranking established in Corollary 8.1 makes intuitive sense: If a formula targets directly at one factor (as with payroll and property shares), then profit tax changes have a greater impact on that factor than under a formula that does not target this factor. Corollary 8.1 does not allow to insert the sales-share formula into the ranking. Yet, further restricting technologies to Cobb-Douglas functions enables us to explicitly calculate the expressions on the far RHS in Proposition 8. Straightforwardly, this leads to

Corollary 8.2 Assume that technologies are Cobb-Douglas: \( F(K, L) = K^\beta L^\delta \) with \( \beta, \delta \in (0, 1) \) and \( \beta + \delta < 1 \). In a symmetric situation the following is true:

\[
\left. \frac{\partial K_A}{\partial t^A} \right|_K < \left. \frac{\partial K_A}{\partial t^A} \right|_F < \left. \frac{\partial K_A}{\partial t^A} \right|_W < 0 \quad \text{and} \quad \left. \frac{\partial L_A}{\partial t^A} \right|_K > \left. \frac{\partial L_A}{\partial t^A} \right|_F > \left. \frac{\partial L_A}{\partial t^A} \right|_W .
\]

The reverse ranking holds regarding the effects of \( t^A \) on \( K_B \) and \( L_B \).

This corollary confirms the intuition we already provided for Corollary 8.1: Sales-share apportionment targets both factors; the tax impacts under that regime are therefore more moderate, lying in between those of the payroll- and property-share formulas.

It is worth emphasizing that Corollaries 8.1 and 8.2 were established for special technologies only. As Proposition 8 reveals, in general the ranking of effects across formulas depends, in a
non-trivial way, on the concavity properties of the production function. It is well conceivable that under some technologies the tax effect on capital is larger with a formula that is only indirectly targeted at capital than with an explicit property-shares rule.

4.4 Leviathan Tax Competition

Let us now turn to a comparison of governments’ incentives in a Leviathan-type tax competition game. As before, the tax revenues government $A$ aims to maximize are given by $T^A = t^A \cdot \alpha \cdot \Phi$; similar for government $B$. In a symmetric setting, Nash equilibria are symmetric too.

**Proposition 9**

- Under the property-share formula tax competition leads to lower tax rates in the Nash equilibrium than under the sales-share rule, $(t^A, t^B)|_K < (t^A, t^B)|_F$, if and only if
  \[ \frac{F_{LL}}{K^2} < \frac{F^2_K F_{LL} - 2F_L F_K F_{KL} + F^2_F F_{KK}}{F^2} \tag{28a} \]

- Under the payroll-share formula tax competition leads to lower tax rates in the Nash equilibrium than under the sales-share rule, $(t^A, t^B)|_L < (t^A, t^B)|_F$, if and only if
  \[ \frac{F_{LL}}{K^2} < \frac{F_{KK}}{L^2} \tag{28b} \]

- Under the property-share formula tax competition leads to lower tax rates in the Nash equilibrium than under the sales-share rule, $(t^A, t^B)|_K < (t^A, t^B)|_F$, if and only if
  \[ \frac{F_{KK}}{L^2} < \frac{F^2_K F_{LL} - 2F_L F_K F_{KL} + F^2_F F_{KK}}{F^2} \tag{28c} \]

**Proof:** Taking the partial derivative of $T^A$ with respect to $t^A$ yields:

\[ \frac{\partial T^A}{\partial t^A} = \left\{ \alpha \Phi + t^A \alpha \left( \Phi_{KA} \frac{\partial K_A}{\partial t^A} + \Phi_{KB} \frac{\partial K_B}{\partial t^A} + \Phi_{LA} \frac{\partial L_A}{\partial t^A} + \Phi_{LB} \frac{\partial L_B}{\partial t^A} \right) \right\} + \Phi t^A \Omega, \tag{29} \]

where

\[ \Omega := \frac{\partial \alpha}{\partial t^A} = \alpha_{KA} \frac{\partial K_A}{\partial t^A} + \alpha_{KB} \frac{\partial K_B}{\partial t^A} + \alpha_{LA} \frac{\partial L_A}{\partial t^A} + \alpha_{LB} \frac{\partial L_B}{\partial t^A} = \alpha_{KA} \left( \frac{\partial K_A}{\partial t^A} - \frac{\partial K_B}{\partial t^A} \right) + \alpha_{LA} \left( \frac{\partial L_A}{\partial t^A} - \frac{\partial L_B}{\partial t^A} \right) \tag{30} \]
Verify that, by the last item of Proposition 8, the curly-bracketed terms in (29) are identical for property, payroll, and sales apportionment methods when tax rates are identical (recall that \( \Phi_{LA} = \Phi_{LB} = 0, \Phi_{KA} = \Phi_{KB} = F_{K}, \)\(^{18} \alpha_{KA} = -\alpha_{KB}, \) and \( \alpha_{LA} = -\alpha_{LB} \)). Hence, the partial effects only differ by the term labelled \( \Omega \). From (30) (where we also employed the equalities \( \alpha_{KA} = -\alpha_{KB} \) and \( \alpha_{LA} = -\alpha_{LB} \)) and Table 1 we calculate:

\[
\begin{align*}
\Omega_K &= \Phi \frac{F_{LL}}{8N} K^2 \\
\Omega_F &= \Phi \frac{F_{K}^2 F_{LL} - 2F_L F_K F_{KL} + F_L^2 F_{KK}}{F^2} \\
\Omega_W &= \Phi \frac{F_{KK}}{8N} L^2
\end{align*}
\]

where subscripts to \( \Omega \) indicate the formula under consideration and \( N > 0 \) was defined in (26).

Thus, in every symmetric situation,

\[
\begin{align*}
\frac{\partial T^A}{\partial t^A} \bigg|_{K} &< \frac{\partial T^A}{\partial t^A} \bigg|_{F} & \iff & & F_{LL} < \frac{F_{K}^2 F_{LL} - 2F_L F_K F_{KL} + F_L^2 F_{KK}}{F^2} \\
\frac{\partial T^A}{\partial t^A} \bigg|_{K} &> \frac{\partial T^A}{\partial t^A} \bigg|_{W} & \iff & & F_{KK} < \frac{F_{LL}^2 - 2F_L F_K F_{KL} + F_L^2 F_{KK}}{F^2} \\
\frac{\partial T^A}{\partial t^A} \bigg|_{W} &> \frac{\partial T^A}{\partial t^A} \bigg|_{F} & \iff & & \frac{F_{KK}}{L^2} < \frac{F_{LL}^2 - 2F_L F_K F_{KL} + F_L^2 F_{KK}}{F^2}
\end{align*}
\]

Due to symmetry, country B’s revenue functions possess corresponding properties. The symmetric Nash equilibrium under FA method \( x = K, F, W \) is defined by \( \frac{\partial T^A(t, t)}{\partial t^A}|_x = \frac{\partial T^B(t, t)}{\partial t^B}|_x = 0 \). Hence, conditions (32a) to (32c) allow us to compare Nash equilibria. E.g., from (32a) it follows that reaction functions under property apportionment intersect at a higher point along the 45-degrees-line than under sales apportionment. Going through this argument for all cases proves the claim.

Proposition 9 relates the tax rates emerging in the equilibrium of a Leviathan-type tax competition game to concavity features of the production technology. In principle, conditions (28a) to (28c) are easy to check. It should be noted, however, that these conditions will not be globally satisfied in general.

\(^{18}\)In case that interest costs were deductible from the profit tax base we would obtain \( \Phi_{KA} = \Phi_{KB} = F_{K} - \rho = 0, \) which obviously would not affect the argument.
The proof of Proposition 9 reveals that for alternative formulas differences in the strategic incentives for taxation only arise from the term

\[ \Phi t^A \Omega = \alpha \cdot \Phi \left( \frac{\partial \alpha}{\partial t^A} \cdot t^A \right) \]

in (29). This observation suggests an elasticity interpretation of Proposition 9 analogous to Proposition 3: \textit{Tax competition is sharper the greater the elasticity of the tax formula with respect to tax changes.} Further insights into this result can be obtained by restricting technologies to Cobb-Douglas functions:

**Proposition 10** Suppose that technologies are Cobb-Douglas: \( F(K, L) = K^\beta \cdot L^\delta \) with \( \delta + \beta < 1 \).

- Tax competition under the property-share rule leads to lower tax rates than tax competition under the payroll-share rule if and only if \( \beta < \delta \).
- Tax competition under the sales-share always leads to higher tax rates than tax competition under either the payroll- or the property-share rule.

**Proof:** For Cobb-Douglas functions calculate from (31a) to (31c)

\[
\begin{align*}
\Omega_K &= \delta(\delta - 1) \cdot K^{\beta - 2} L^{\delta - 2} \cdot \frac{\Phi}{8N}; \\
\Omega_W &= \beta(\beta - 1) \cdot K^{\beta - 2} L^{\delta - 2} \cdot \frac{\Phi}{8N}; \\
\Omega_F &= -\delta \beta(\delta + \beta) \cdot K^{\beta - 2} L^{\delta - 2} \cdot \frac{\Phi}{8N}.
\end{align*}
\]

The claim follows from observing that \( \delta + \beta < 1 \).\textsuperscript{19}

Proposition 10 conveys two important messages:

- When formulas directly targeted at one factor are used (such as payroll- and property-share rules), then the technological factor elasticities \( \beta \) and \( \delta \) determine the sharpness of international tax competition. Given that empirically the labour share exceeds the capital share, \( \delta > \beta \), we would conclude that the property-share rule leads to lower tax rates than the payroll-share rule.

\textsuperscript{19}The function \( h(x) = x(x - 1) \) is increasing in \( x \) if and only if \( x > 0.5 \). Hence, for \( \beta > 0.5 \), the inequality \( \delta(\delta - 1) > \beta(\beta - 1) \) only holds if \( \delta > \beta > 0.5 \). But then \( \beta + \delta > 1 \) which violates the presupposition \( \beta + \delta < 1 \).
• When the sales-share formula is implemented that targets both factors of production by
design, tax competition is mitigated (regardless of the magnitudes of the factor elasticities).

4.5 Fiscal Externalities

Let us briefly consider the fiscal externalities (i.e., the cross-border effects) involved in the tax
competition game with two variable factors. For a small change in $t^B$ the fiscal externality can
be written as

$$\frac{\partial T_A}{\partial t^B} = 2t^A \cdot [F_{LL} \gamma_1 \gamma_2 + F_{KK} \gamma_3 \gamma_4 - F_{KL} (\gamma_1 \gamma_4 - \gamma_2 \gamma_3)].$$

Recalling that $\gamma_1 > 0$, $\gamma_3 \geq 0$, $\gamma_4 \leq 0$, this expression is unambiguously positive whenever
$\gamma_2 = \alpha_K \Phi + F^B_K \alpha$ is negative. As shown above, this is the case for property-share apportionment.
However, $\gamma_2$ is positive for both payroll- and sales-share apportionment, leaving the sign of the
fiscal externality unclear in these cases. Hence,

**Proposition 11** Under property-share apportionment, tax rates in the Nash equilibrium of the
tax competition game are inefficiently low. For payroll- and sales-share apportionment, tax com-
petition may but need not result in excessively high tax rates.

5 Concluding Remarks

We analysed the effects of different FA methods on the strategic incentives for Leviathan govern-
ments to set their profit tax rates. Focussing on the level of tax rates, our model yields the general
insight that tax competition is stiffer (i.e., equilibrium tax rates are lower) the more elastic the
formula share in the apportionment method is with respect to tax changes. While the thrust
of this finding sounds very intuitive, the challenge in our setting is to identify the conditions
determining the order of magnitude of the formula-share elasticities. We trace back these con-
ditions to properties of production technologies and provide some general, yet simple-to-verify,
qualifiers.

A second major insight that emerges from our analysis is that switching from the existing system
of SA taxation to FA taxation may alter the character of the implied tax competition game

\[30\] We owe this formulation to Silke Gottschalk and Wolfgang Peters.
considerably, from a race-to-the bottom to excessively high tax rates where the Laffer curve is downward sloping. This confirms a conjecture put forward by Sørensen (2003). However, our analysis also shows that excessive taxation can be prevented when property-share apportionment is applied (or has a sufficiently great weight in a mixed system).

Our simple model can be extended in various directions:

- **Endogenous choice of formulas**: We did not address the important issue as to how strategic governments would choose the apportionment formula or the weights therein. Investigating this question could relate our approach to Anand and Sansing (2000) who show that governments have an incentive to employ different formulas while a uniform formula would be the preferred choice from a co-operative perspective.

- **Asymmetric formulas and countries**: Since governments are unlikely to agree on a uniform formula it is important to know the impact on tax competition among governments that use different formulas. Likewise, tax competition among countries with different characteristics is a further step towards realism. Both extensions introduce asymmetries into the model. In view of the complexities we already encountered in the symmetric settings studied here, these extensions can be expected to pose massive analytical challenges.

- **Government objectives other than revenue maximization**: Nielsen et al. (2001) show that a comparison of tax competition under SA and under property-share FA basically yields the same results for both revenue- and for welfare-maximizing governments. One may want to check whether this is also true for comparing tax competition equilibria under various methods of FA.

- **Comparison between FA and separate accounting**: Nielsen et al. (2001) find that, under property-share, FA tax increases may have a stronger [weaker] impact on domestic [foreign] investment than under separate accounting. From our analysis, it is questionable whether this observation carries over to other FA methods as well. This issue calls for clarification.

Nielsen et al. (2001) have a central policy message concerning the EU’s plan to replace its current, messy system of corporate taxation by a unitary method: If that reform involves formula apportionment to allocate tax revenues to EU member states, harmful tax competition cannot
be expected to be mitigated within Europe. The principal message from our analysis is that the strategic incentives in tax competition games under FA crucially depend on the attributes of the apportionment formula to be implemented. Hence, designing the appropriate apportionment formula is a crucial policy issue should the EU move from separate to unitary corporate taxation.

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