Power, Outcomes and Preferences

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Abstract: There is an ongoing discussion about the relationship of power and preferences: Is power reflected in what the agents can do and what they want to do, or, alternatively, are preferences and power two separate dimensions of determining the outcome of decision making? In the latter case decision making is troubled with all kinds of paradoxes which do not allow to derive well-defined outcomes which can relate preferences to resources (votes), decision rules, and power - if we do not subscribe to the rather rigorous assumption of single-peaked preferences on a one-dimensional preference space. This paper raises the question whether these paradoxes do not undermine a power concept which combines preferences and collective decision rules, described by games, with resulting outcomes. A discussion of the Public Good Index with respect to decision rules concludes the paper.
1. Introduction

There is an ongoing discussion about the relationship of power and preferences. One party maintains that power is reflected in what the agents can do and what they want to do, the other party sees preferences and power as two separate dimensions of determining the outcome of decision making. In this paper, we will subscribe to the latter view. This is not to deny that it can be helpful to consider, factual or hypothetical, preferences of a decision maker when we study his or her power. Often the full range of possible preference orders which agents can have on the set of alternatives are taken into consideration in order to analyse the potential of decisions which is available – this is like following the headlight of a car to see where it may be heading to. However, it could be disastrous to confuse the headlight with the car.

In a series of recent articles,¹ Tsebelis and Garrett claim that power indices, like the Banzhaf index or the Shapley-Shubik index, cannot explain decisionmaking in the European Union. Their main argument is that power indices do not take into consideration the preferences of the member countries. In a critical review of this claim, Holler and Widgrén (1999a) argue that power indices are not meant to forecast decision making in the EU but to be of use to identify structural properties of the decision body: for instance, what impact has the modification of the decision rule to the distribution of voting power in the Council of Ministers or, will the power of all incumbent members of the EU decrease by the entry of new members? As preferences of the national governments may change over time and are hardly foreseeable for the distant future, such questions should be answered without reference to specific preferences. In fact, in order to guarantee some degree of fairness, one could well argue that the structure of EU decision bodies should be selected behind a veil of ignorance and preferences of its members with respect to specific political issues or coalition partners should be ignored.

However, if one chooses a non-cooperative approach in order to analyze specific decision situations and to forecast for them, then the incorporation of preferences are necessary to formulate the game. To some extent, these games can also be used for structural analysis of decision bodies if preferences can be assumed to be constant for some time and allow to derive decision outcomes. The latter constraint is often circumvented by the rather rigorous assumption of single-peaked preferences on a one-dimensional preference space. However, there is ample evidence from social choice literature that if we drop this assumption, decision making is troubled with all kinds of paradoxes which do not allow to derive well-defined outcomes which can relate preferences to resources (votes), decision rules, and power.

In Section 4, we will discuss a selection of voting problems to illustrate the issues of voting paradoxes and path dependence of outcomes which have been widely neglected by the literature which proposes to consider preferences in the measurement of power. Section 2 illustrates the power-preference relationship with reference to the Shapley-Shubik index and its relationship to preferences and the value of the characteristic function: What does i get if it wins \[v(S)-v(S\{i\})\]? To clarify the properties and interpretation of \(v(S)\), Section 3 theorizes on the concept of characteristic function, referring to Max Weber's quite popular definition of power. It is argued that the value of a coalition is power and describes the coalition members' potential to enjoy an outcome which concurs with their preferences - technically speaking, to gain utility from cooperation. In Section 5, in order to contrast the private good view implicit to the Shapley-Shubik index, we assume that the coalition value is a collective good and apply the Public Good Index to the power discussion.

2. The Shapley-Shubik Index and Preferences

The Shapley-Shubik index (SSI) is not only the pioneer in the history of power indices,\(^2\) but also serves as a point of reference for the great variety of measures which have been

\(^2\) Felsenthal and Machover (1998, 6ff.) point out that a version of the Banzhaf index has already been proposed by Lionel Penrose in an article of 1946.
since developed. It derived from the Shapley value which had been proposed in Shapley (1953) as solution concept for cooperative games. Shapley and Shubik (1954) applied this concept to voting games which forms a subclass of simple games, i.e., games which are defined by the two alternatives of winning and losing coalitions.

The SSI is based on the concept of a pivot player. We are given a simple game \((N, v)\), where \(N\) is the set of players and \(v\) is the characteristic function which assigns the values \(v(S) = 1\), if \(S\) is a winning coalition, and \(v(S) = 0\), if \(S\) is a losing coalition. Player \(i\) is pivotal if \(i\) turns a losing coalition \(S\setminus\{i\}\), which consists of the first \(s-1\) elements of an ordering, into a winning coalition \(S\) by joining coalition \(S\setminus\{i\}\) such that \(i\) is the \(s\)th element of coalition \(S\).

The expression \#pivots(i) counts the number of orderings for which \(i\) is a pivotal member (i.e. permutations of the elements of the set of payers \(N\)). The contribution of a pivotal player to the winning of a coalition \(S\) is measured by \(v(S) - v(S\setminus\{i\}) = 1\). Then SSI is a vector \(\Phi(v) = (\Phi_1(v), \ldots, \Phi_n(v))\) with components defined as the ratio \#pivots(i)/\(\sum\#pivots(i)\) where the summing is over all players \(i\) in \(N\). Obviously, the SSI values sum up to 1 and a probability interpretation is straightforward.

In order to calculate \(\Phi_i(v)\) we look at all subsets \(S\) of \(N\) which have \(i\) as an element and ask, for each such \(S\), (a) whether \(S\) is a winning coalition and, if yes, (b) whether \(i\) can turn a winning coalition into a losing coalition by defecting from \(S\). If the answer is yes again then we get \(v(S) - v(S\setminus\{i\}) = 1\) for the specific \(S\). Next, we take into account that the SSI is based on orderings and ask how likely it is that \(i\) is in the pivot position if coalition \(S\) forms: it is \((s-1)!(n-s)!/n!\), where \(s\) and \(n\) are the numbers of players in \(S\) and \(N\), respectively, and \(n!\) is the number of total orderings which can be formed by \(n\) players. Finally, we sum the resulting product (of the probability that \(i\) is pivotal and the value of \(i\) being pivotal) over all subsets \(S\) of \(N\) which have \(i\) as an element. For instance, consider a weighted voting game \(v^* = (d; w) = (55; 50,30,20)\) where \(d = 55\) is the decision (i.e., majority) rule and \(w = (50,30,20)\) represents the vote distribution. The set of permutations has \(n! = 6\) elements. Player 1, characterized by voting weight \(w_1 = 50\), is the pivot element of 4 permutations while players 2 and 3 are
pivot elements of only one permutation each. The SSI of \((N, v^*)\) therefore is: \(\Phi(v^*) = (2/3, 1/6, 1/6)\).

It is not easy to justify the application of orderings intuitively when only coalitions, i.e. unordered sets seem to matter. This is all the more so because the fact that each permutation is taken into account with equal probability gives unequal weights to coalitions if they have different numbers of members. There are \(2^n\) coalitions which correspond to a set of players of \(n\) members, including the null coalition and the grand coalition. The same set allows for \(n!\) different permutations.

The classical story to justify the focus on permutations, given in Shapley and Shubik (1954), assumes that those players with stronger preferences to form a winning coalition \(S\) enter the coalition formation process first, followed by players with weaker preferences. Therefore, the pivotal player \(i\), the last player in the sequence who turns a losing coalition \(S\setminus\{i\}\) into a winning coalition \(S\) is the member of \(S\) with the weakest preferences concerning this coalition: it is assumed that \(i\) will get the undivided coalition value \([v(S)-v(S\setminus\{i\})] = 1\) because the members of \(S\setminus\{i\}\) will get "nothing" if \(i\) does not join them. However, since no specific information on preferences is given with respect to the various winning coalitions \(S\), all \(n!\) permutations of \(n\) players are considered. This amounts to saying that \(i\) is only with a certain probability in the pivotal position with respect to coalition \(S\) in which \(i\) is a member.

In the case of SSI, the assumption of ordered preference profiles serves as headlights. Alternative and perhaps more convincing stories could be told to justify the SSI, e.g., the probabilistic approach in Straffin (1977). Shapley and Shubik (1954, p.790) themselves considered the arranging of the voters in all possible orderings to be "just a convenient conceptual device". Another problem is to interpret the value \(\Phi_i(v)\). Is it the (expected) worth of a player to participate in a (voting) game? If \(v(S)\) were the value of the coalition \(S\), to be shared among the members of \(S\), and utilities were perfectly transferable then \(\Phi_i(v)\) could be identified with the payoff (i.e. von Neumann Morgenstern utilities) of player \(i\) in the voting game. This however seems to be at odds with the variation in preferences implied in the story of ordered preference profiles.
Moreover, the implication that the members of coalition S with high preferences for the
forming of S get nothing out of S, while the member i with the weakest preference for S
gets "all" raises the question whether \( v(S) \) expresses the collective value of the coalition
S or just some private spoils which result from the formation of coalition S.

The latter interpretation seems to concur with the pivot player concept which
characterizes SSI. As we see from the above formula, \( \Phi_i(v) \) is the result of adding up
\[ v(S) - v(S\{i\}) \] for all coalitions S of N which have i as a member. However, an
interpretation of the function \( v(.) \) and what it means with respect to power and
preferences is in itself not straightforward. It is not at all obvious what i gets if he or she
wins \[ v(S) - v(S\{i\}) \].

3. Characteristic Function and Power

Generally, a characteristic function is a map from the set of coalitions of a specific
social situation into the space of real numbers. Widgrén and Holler (1999b) suggest that
value of a coalition S, \( v(S) \), is power. This coincides with the view that coalitions do not
have preferences, but merely represent the preferences of their members. It is therefore
inappropriate to assign utility values (or payoffs) to coalitions. Individual agents have
preferences. A coalition can represent the preferences of its members, i.e. of the agents
which form the coalition. If, however, the members' preferences are not identical then
the aggregation of the individual preferences has to be analyzed in order to understand
this representation and to apply it adequately. The aggregation defines the coalition's
objective, i.e. its "will".

A coalition is an agent which tries to carry out its will despite resistance.
Resistance results either from nature, when the cooperation of several people is needed
to produce something, or, within a social relationship, from other agents which have
conflicting desires. The coalition's potential to enforce its will despite resistance is the
power of the coalition: It is not the will which defines the value of a coalition but the power, i.e. the potential to carry it out.

Of course, this view derives from Max Weber well-known definition of power "...as the probability that one actor within a social relationship will be in a position to carry out his own will despite resistance" (Weber 1947 [1919], p.152). We do not have to accept this definition only because Weber is one of the founding fathers of Sociology and a less rewarded prophet of Public Choice. However, as it is widely quoted, it seems to be a relevant starting point to talk about power. But, obviously, it needs further interpretation. What does it mean to have one's will? Is it that the result concurs with ones preferences? Why "against resistance"? At the first glance, these qualifications seems rather obscure, but there are strong arguments in favor. If we take resistance into consideration, we get an operational concept which allows to derive power measures from the characteristic function of coalitional games. The coalition game reduces to a constant-sum game played over the outcomes of "winning" and "losing" between S and its complement N\S. Even if this game can have multiple equilibria in terms of strategies (which are not modelled in the characteristic function form of the game) we know ever since von Neumann and Morgenstern (1944) that all equilibria of such a game are related to the same payoff values. Note, however, that the assumption of a game is just a construct to derive values for coalitions; it is a hypothetical game as coalitions do not enjoy payoffs.

The construction of a constant-sum game presupposes that the coalitional game is decisive, i.e., if S is a winning coalition then N\S is a losing coalition, and vice versa, i.e., the simple game (N,v) is a decisive game if v(N\S) = 1 - v(S) for all subsets S of N. There might be other ways to guarantee an unequivocal representation, but the reference to resistance seems straightforward and the relation to power, as defined by Max Weber,
seems obvious, i.e., if we apply the power notion of coalition value then the
characteristic function of the von Neumann Morgenstern type defines a power relation.

4. Power and Aggregation of Preferences

The view that connects power with preferences is problematic on the count that it leads
to some paradoxical situations. We are here referring to monotonicity paradoxes in the
theory of voting. The basic feature of these paradoxes is the counterintuitive change in
voting outcomes resulting from specific types of changes in preference profiles, i.e. in
the voter opinions. Consider the classic additional support paradox. This occurs when
additional support, ceteris paribus, diminishes rather than increases an alternative’s
chance of being elected. Consider the preference profile of three alternatives A, B and
C in Table 1.

<table>
<thead>
<tr>
<th>34% of voters</th>
<th>35% of voters</th>
<th>31% of voters</th>
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</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
</tr>
<tr>
<td>C</td>
<td>C</td>
<td>B</td>
</tr>
<tr>
<td>B</td>
<td>A</td>
<td>A</td>
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Table 1. Additional Support Paradox (Nurmi 1999, 57).

Suppose that the plurality runoff system is being used and that all voters vote according
to their preferences. Since no alternative gets more than 50% of the votes on the first
round, there will be a second one between A and B, the two largest vote-getters. This
round will result in B being chosen (as those 31% whose most preferred alternative did
not make it to the second round prefer B to A).

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6An illuminating and entertaining account of these types of paradoxes is given by Fishburn and Brams 1983. See
also Brams 1976; Fishburn 1982; Nurmi 1999; Richelson 1979; Straffin 1980. Explanations to these and many other
voting paradoxes are provided in Saari 2001a and Saari 2001b.
Suppose now that 4% of the electorate with preference ranking ACB had increased their support of B by placing it first in their ranking thereby having the ranking BAC. Under this modified profile, B would be facing C rather than A on the second round. On this round, C would defeat B. Thus, the increase of the winner’s (B’s) support, *ceteris paribus*, would turn it non-winner. If we identify the voter groups by their first preferences, then, in the above example, the supporters of B would increase their voting resources from the first profile to the second. Yet, their first ranked alternative would be achieved under the first profile, but not in the second. In terms of preference-satisfaction the power of the supporters of B is smaller in the latter profile than in the former one. Yet, B’s position is the same in both profiles and so is the position of the other groups excepting those voters who join the B-supporters.

To this one could conceivably counter that BAC voters should not be seen to belong to the same group as BCA voters because of the difference between second and third preferences. Thus, the configuration in the second profile is fundamentally different from the first one. Consequently, it is conceivable that the power of the BCA group is indeed smaller in the latter than in the former profile despite the increase in the number of voters who rank B first. Another paradox to be discussed next will however show that the preference-satisfaction view of power is not compatible with *any* resource-based view that assumes that some resources yield better outcomes than none at all. In other words, if one wants to embrace the former view, one has to abandon any concept that connects power with resources.

The paradox that will demonstrate this is called the no-show paradox. It takes on two versions: the weak and strong one. The former occurs whenever a voter group prefers the outcome resulting from its abstaining to the outcome that would result if it would vote according to its preferences. The strong paradox is a situation yielding the group’s first ranked alternative when it abstains, but something else when it participates in voting. Consider the following profile of three alternatives and 19 voters (Table 2). The assumed preferences of the voters are listed from the most (uppermost) to the least preferred one. The procedure being used is Coombs’ one. It operates on the preference
rankings of voters. If an alternative is ranked first by more than 50% of the voters, it is elected. Otherwise, the alternative ranked last by more voters than any other alternative is eliminated. One then looks at the reduced preference profile again to see if any of the remaining alternatives is ranked first by more than half of the electorate. The elimination continues until such an alternative is found.⁷

<table>
<thead>
<tr>
<th>5 voters</th>
<th>5 voters</th>
<th>6 voters</th>
<th>3 voters</th>
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<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>B</td>
<td>C</td>
<td>A</td>
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</tr>
<tr>
<td>C</td>
<td>A</td>
<td>B</td>
<td>A</td>
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Table 2. Coomb’s Procedure and the Strong No-Show Paradox (Nurmi 2002, 97).

Since no alternative is ranked first by more than 50% of the voters, A is eliminated since it has the largest number (8) of last ranks. Thereupon B becomes the winner. Suppose now that the 3 voters with ranking CBA had not voted at all. Then B with 6 last place ranks had been eliminated whereupon C – the abstainers’ first ranked alternative – would have won.

Both versions of the no-show paradox show that a group of voters may be better off by not having any votes at all than by participating in the vote. In the power index jargon, in terms of preference-satisfaction, a dummy player may be more powerful than a player with a considerable portion of the total votes. For instance, in the example of Table 2, the dummy player (the group of 3 voters abstaining) is more powerful than a player with more than 25% of the total votes (the voter group with BCA ranking). So, the conflict between preference- and resource-based power concepts is apparent.

It could be argued, though, that the examples of Table 1 and 2 are based on specific procedures and are, consequently, not general. This is undoubtedly the case, but

⁷ An up-to-date discussion of a wide variety of voting procedures is given in Brams and Fishburn (2002).
also the preference-based power concepts have to be based on voting procedures. Otherwise, it would not be possible to measure the distance between voting outcomes and voter ideal points. But what is the voting procedure underlying preference-based power indices? The most common procedure discussed is the amendment procedure used e.g. in the Congress of the United States and several parliaments in Europe. It is an agenda-based procedure in which the alternatives are voted upon in pairs. The agenda first singles out two alternatives and each voter can vote for one and only one of them. Whichever receives more votes faces the next alternative in the agenda in the second binary vote. With k alternatives, the procedure consists of k-1 pairwise majority comparisons so that the winner of the last one is declared the overall winner.

Now, one may ask what is the relationship between this voting procedure and the above paradoxes. The answer is that the amendment procedure is vulnerable not only to the no-show paradox, but to the strong version thereof as well. Thus, the preference-based indices are based on precisely such procedures that are vulnerable to no-show paradoxes. This means that dummy players may – in terms of preference satisfaction – be more powerful than players whose votes account for more than ¼ of the total voting weight.

<table>
<thead>
<tr>
<th>22 voters</th>
<th>33 voters</th>
<th>23 voters</th>
<th>22 voters</th>
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<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>C</td>
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<td>B</td>
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<tr>
<td>C</td>
<td>A</td>
<td>B</td>
<td>A</td>
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Table 3. The Strong No-Show Paradox in Amendment Procedure (adapted from Nurmi and Hosli 2003).

The vulnerability of the amendment procedure to the strong no-show paradox is illustrated by the following 100-voter example (Table 3). Suppose that the agenda singles out A and B for the first pairwise vote, whereupon C takes on the winner. It turns out that
B first defeats A and then, in the second pairwise vote, C defeats B so that C becomes the overall winner. Assume now that the 22 right-most voters had abstained. This would have resulted in A’s victory in the first vote to be followed by C’s victory in the second. Thus, by abstaining the 22 voters would have ended up with their favorite C winning, while by participating they would have seen B winning.

It could be objected that the above analysis pertains to non-equilibrium behavior and is, thus, not likely to persist. Indeed, while the choice of B in the 100 voter profile is the equilibrium outcome, the choice of C in the reduced one isn’t. This can be seen by resorting to backwards induction which necessarily ends up with the Condorcet winner when one exists (see McKelvey and Niemi 1978). It does exist in the 100 voter profile, but the reduced profile exhibits a majority cycle through alternatives and thus there is no Condorcet winner. In the reduced profile; the induction results again in B. Thus, if the voters are sophisticated instead of sincere C is not chosen.\(^8\)

It can, however, be shown that the amendment procedure is vulnerable to the strong no-show paradox in equilibrium as well. This can be seen by considering Table 3 again and assuming that the agenda is: (1) B versus C, and (2) the winner of (1) versus A. The backwards induction would again give outcome B as the outcome. In the reduced profile, however, B is no longer the equilibrium outcome, but C is. Thus, the vulnerability holds in equilibrium. In other words, both sincere and sophisticated voting behavior may lead to the strong no-show paradox when the amendment procedure is used.

5. Power, Preferences and the Production of Public Goods

Above we have raised the question whether, in the case of SSI, \(v(S)\) expresses the value of the coalition S or just some spoils which result from the formation of coalition S.\(^9\) We proposed that a private good perspective is inherent to the Shapley-Shubik index. Given this perspective, the vector \(\Phi(v) = (\Phi_1(v),..., \Phi_n(v))\) describes the expected allocation of spoils. However, the core of exerting social power is given by a collective

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\(^8\) In the cyclic majority cases the backward induction outcome depends on the agenda of binary comparisons.
\(^9\) This is related to the distinction of I- and P-power in Felsenthal and Machover (1998, 35f and 172f).
property: those who are its target are subject to it irrespective of their preferences. Criminals are target by police and tax payers enjoy highways whether they like it or not. Of course, they can try to resist the power wielder $i$ and constrain $i$'s possibilities defined by the set of public goods $i$ can produce and their radius of impact. (The radius of impact describes the relevant society.)

The set of possible public goods which can be produced with $i$'s support, despite resistance, define $i$'s power. To measure this power, the Public Good Index will look at all coalitions which have $i$ as a member such that they form a decisive set: that is, coalitions which have the power to produce a public good (i.e., are winning coalitions) and are small enough to avoid free-riding in production.\footnote{Free-riding is an essential impediment for the voluntary production of public goods. Unfortunately, it is widely ignored in the discussion of collective action as Dixit and Olson (2000) pointed out in a recent publication on the Coase Theorem.}

Technically speaking, a decisive sets is identical with a minimum coalition (sometimes also classified as strict minimum winning coalition) which contains crucial players, only, i.e. players who can turn a winning coalition $S$ into a losing by dropping their support for $S$. That is, if $S$ is a decisive set then any true subset of $S$ is a losing coalition.

A decisive set, however, is not about winning, only: it is assumed that different decisive sets produce different collective goods; coalitions which give support identical goods either are identical or they contain surplus players and thus invite free-riding. Consequently, if $\{A, B\}$ and $\{A, C\}$ are two decisive sets then the public good determined by $\{A, B\}$ is different from the public good determined by $\{A, C\}$ while it is identical with the public good determined by coalition $\{A, B, D\}$ where $D$ is a surplus player - however, is likely to form only by "luck"\footnote{See Barry (1982) for a discussion of the relationship between power, decisiveness and luck.} and should not considered when measuring power. The latter also applies to coalition $\{A, B, C\}$ which obviously has at least two surplus player: it therefore does not matter whether the public good related to $\{A, B, C\}$ corresponds with the possible outcome of decisive set $\{A, B\}$ or $\{A, C\}$.

If we consider non-rivalry in consumption and non-exclusiveness for pure public goods then it is straightforward that all members of a decisive set $S$ "enjoy" the same
public good, irrespective of their preferences, which is identical with the value of the coalition, \( v(S) \). If we give all coalitions an identical value, which is not inappropriate as preferences coalition members are not taken into consideration, then the power of a player \( i \) is determined by the number of decisive sets of which he is a member. Let us call this number \( m_i \). Dividing \( m_i \) by the sum of all crucial positions in decisive sets of the game \( (N, v) \) gives us the Public Good Index (PGI)\(^\text{12} \) \( h_i = m_i / \sum m_i \) where summation is over all players \( i \) in \( N \).

It has been discussed whether the normalization which results from the division procedures makes sense - as it has effects on, e.g., the monotonicity properties of the measure. (See Holler and Li (1995) for such effects.) However, in this we are concerned with the relationship of preferences and the paradoxical results offered by social choice theory. Let us first illustrate the decisive sets of the game in Table 3 under the assumption that there are two bloc of voters, 1 and 4, with 22 votes and a bloc 2 with 33 votes and a bloc 3 with 23 votes. Given simple majority voting, the decisive sets are \{1, 3, 4\}, \{1, 2\}, \{2, 3\} and \{2, 4\}. Given a 56 per cent decision rule, \{1, 3, 4\} and \{2, 3\} are the decisive sets while for a 2/3 rule every coalition of three members is a decisive set. For 4/5 and 5/6 rules which are applied in some contemporary parliaments the only decisive set consists of all voters.

Firstly, it should be noted that the set of decisive sets is never empty if the set of winning coalitions is non-empty. Secondly, many decision rules select identical sets of decisive sets. Thirdly, decision rules which select a single weighted voter, like the pairwise comparison of candidates, imply decisive sets which are singletons.

In principle one can consider decisive sets also in contexts where voter preferences are taken into account, but these are necessarily profile-dependent. Given a preference profile, voting strategy n-tuple and voting procedure, one can determine which voters’ preferences are satisfied (i.e. who get their first ranked alternatives elected). These voters form a decisive set in the profile under consideration. On the basis of this information, one can proceed to compute the relative frequency of each

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\(^\text{12}\)The Public Good Index has been proposed by Holler (1982) and axiomatized in Holler and Packel (1983). It is also called Holler index or Holler-Packel index.
voter having his/her preferences satisfied, provided that we know the relative frequency of each profile. Assuming that each profile is equally likely leads to a profile-based version of the Shapley-Shubik index. It is, however, necessary to know not only the preference profiles, but also the voting strategy n-tuple as well as the voting system being used. This approach differs, however, considerably from the current spatial power indices.

6. Conclusion

We have argued that power indices, such as the Shapley-Shubik index and the Public Good Index are not designed to explain or to forecast the decisionmaking in a specific collective decision situation but to analyze the a priori voting power which results from relationship of the number of votes (i.e. voting weights) and the majority rule (i.e., decision rule) in case that decisionmaking is based on voting. Power indices are designed to explain how the evaluation of the players change if the vote distribution changes or a new decision rule is applied. Therefore they seem to be valuable instruments to analyse institutional changes and effects of alternative institutional design behind a veil of ignorance, when the preferences of the decisionmakers are unknown. Moreover, we can derive results and insights from the application of power indices which ignore preferences when other measures are indecisive because of voting paradoxes which render the relationship of votes to outcome inconclusive or path dependent.
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