

Equilibrium Size in Network with Indirect Network Externalities

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May 2004

Abstract

I present a simple model of determination of the equilibrium size of a network with indirect network externalities. Indirect network externalities can generate complementarity between goods, and then the demand functions for the network good are those of the complementary goods: the result in the determination of the equilibrium size in a market with indirect network externalities is the "classical" result with complementary goods. I calculate the number of consumers of each group that should be optimal for firms in different market structures, as perfect competition, monopoly and duopoly. The result is that the equilibrium size of the network with indirect network externalities depends on the market structure; it is wider in perfect competition than in monopoly and duopoly; because of the externalities, perfect competition is inefficient, that is, the equilibrium size in this market structure is smaller than the equilibrium size chosen by a social planner; prices charged in a duopolistic market with indirect network externalities are greater than prices charged in a monopoly market, and then, the equilibrium size in duopoly is smaller than in monopoly; this is the classical result that we obtain with complementary goods, generated by the indirect network externalities, and it is different from the result in markets with direct network externalities in which the equilibrium size in duopoly is greater than monopoly because the competition with substitute goods.

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1 Introduction

In this paper I present a simple model of determination of the equilibrium size of a network with indirect network externalities, that is the size of the network which maximises the profits of firms (providing network good) in different market structures.

Networks exhibit positive network externalities: "the value of a unit of the good increases with the expected number of units to be sold"¹. This is the reason why the demand for network goods slopes downward but shifts upward with increases in the number of units expected to be sold.

The sources of the network externalities is the complementarity between the components of a network. The crucial assumption is the "compatibility", i.e., that various links and nodes on the network are costlessly combinable to produce demanded goods. Compatibility makes complementarity actual, because for many complex products, actual complementarity can be achieved only through the adherence to specific technical compatibility standards.

Network externalities are *direct* and *indirect*. *Direct network externalities* are generated through a direct effect of the number of agents, consuming the same good, on the utility function of agents themselves (through a creation of new goods that directly and positively affects the utility function of every participant to the network). This is the case of telecommunication and fax network².

Network externalities can also be *indirect*: the value of a good increases as the number, or varieties, of complementary goods increases: the addition of new varieties of one type of components affects positively but indirectly the utility of all participants through the reduction of prices. More generally, most markets with indirect network externalities are characterized by the presence of two distinct sides which benefit from the interaction among them. Typical examples are the PC market and the credit cards network³.

Now, I want to describe in details the example of credit card market. It provides a useful example of market with indirect network externalities, and an illustration of how growth dynamics, in the market for a network good, can change over time when the good's value to consumers depends

¹Economides N. (1996)

²For example, purchasing a fax machine directly benefits existing fax machine owners, who now can have an additional person with whom they may communicate. If there are n fax machines in the network, each owner has $n(n - 1)$ potential interlocutors; an additional fax machine adds $2n$ potential communications within the system, and thus it enhances the value of memberships, assuming that each owner may, at some point, wish to communicate with every other owner. See Economides 1996.

³In the PC market, if many consumers buy the same kind of hardware, software producers will produce a lot of varieties of them, all compatible with the same hardware; on the contrary, an increase of compatible software varieties, will promote the hardware demand and the hardware price will decrease. Another examples of network with indirect network externalities is the e-commerce B2B.

on the size of the network. Credit card are said to exhibit indirect network externalities because, the average consumer's benefit from using them in transactions, depends on how many other consumers and businesses are using the same vehicle. The total benefits associated with the use of credit cards exceed the benefits accruing directly to an individual consumer. By extending the network, one more person's participation increase the benefits to the other participants.

In this market, the nature of the demand interdependency is determined by the total number of market participants and by the relative numbers of payors and payees. The explanation is: the benefit to the consumer (payor) to have a credit card is a function of how many establishments will accept it in payment for goods and services. Merchants (payees), on the other hand, are like to invest in the systems necessary to accept credit cards only when there is a sufficiently large demand for using this payments instrument. Therefore, the benefit that a consumer expects to derive depends *directly* on the number of establishments accepting credit cards, and *indirectly*, on the number of consumers using them. A merchant's benefit, conversely, depends *directly* on the number of consumers using credit cards, and *indirectly* on the number of establishments accepting them. I make this description to clarify the concept of indirect network externalities, and to explain how indirect network externalities are a way to introduce the complementarity between goods, different from the standard idea of complementary goods.

Technologies which present strong network externalities have a swift growth, at least up to some *critical mass*. This is the consequence of a "positive feedback effect" of increased network size, that makes the larger network much more attractive to new purchasers, and the network good much more valuable. The value of the network good also depends on the purchaser's expectations about the future size of the network since each consumer's demand depends on the number of others purchasing the same good. The demand for a fax machine, for example, is a function of its price and of the expected size of the network to which the fax machine will be connected.

As the demand for a network good is a function of both its price and the expected size of the network, the problem of startup arises when consumers expect that no one would buy the good or that no complementary goods would be available in the market. The existence of a critical mass in markets with network externalities suggests that a sustainable growth of the network requires a minimal nonzero equilibrium size (the critical mass)⁴. Economides

⁴The concept of critical mass is linked to the "chicken-and-egg" paradox: many consumers are not interested in purchasing the good because the installed base is too small; otherwise the installed base is too small because an insufficiently small number of consumers have purchased the good. For example, in the videogame market a platform cannot sell the consoles without games to play on, and it cannot attract game developers without the prospect of an installed base of consumers.

and Himmelberg (1995) defined the critical mass as "the smallest network size that can be sustained in equilibrium, given cost and market structure"; they determined this critical mass and the equilibrium size of the network with network externalities under different market structures. I will describe the Economides and Himmelberg (1995) model in section 2 of this paper.

Existence of positive network externalities in a network, implies that, the private marginal benefit of purchasing a network good is lower than the social benefit; in deciding whether to buy a network good, the agent compares the price only with his private benefit, not with the benefit that his purchase confers on other users. Consequently, the critical mass under perfect competition, with network externalities, may be smaller than the social optimum.

I want to present a model of determination of the equilibrium size in networks, with indirect network externalities in different market structures. This is a simple model to explain how indirect network externalities can generate complementarity between goods. With indirect network externalities, the demand functions for network goods are the complementary ones; the result of the equilibrium size determination in a market with indirect network externalities is the "classical" result with complementary goods. Indirect network externalities markets must be treated in a different way with respect to direct network externalities markets in determination of the equilibrium size. The sources of the externalities is different. With direct network externalities, the value of the network good depends on the total number of consumers purchasing the same good while, with indirect network externalities there are two sides, or two groups, and the value of the network good depends on the agents of each group. Therefore, the demand's interdependence is different. In a fax market, the fax utility for a user depends only on the number of other purchasers, and the value of the fax machine does not depend on its functions (send and/or receive documents). In the credit card network, instead, there are two groups, consumers and merchants, and then two kinds of utility depending on which side they are (while a fax user can either send or receive documents). Economides and Himmelberg (1995) in their model of critical mass and the equilibrium size of network determination, do not distinguish between direct and indirect network externalities, they speak about network externalities. Their model only refers to direct network externalities, because they use an external-ity function which depends on the total expected number of agents in the network, and the network goods are homogeneous, and then substitute.

In my model, the indirect network externalities source is the interdependence of demands of two consumers groups. I assume an utility function of consumers of each group which depends positively on the expected size of the other one. In this way I introduce indirect network externalities in the model, that makes as in the credit card market, the network goods complementary. I calculate the optimal number of consumers of each group in

different market structures, as perfect competition, monopoly and duopoly, and I make a comparison with an efficient situation represented by the number of consumers of each group chosen by a social planner who maximizes social welfare. The result is that, the equilibrium size of the network, depends on the market structure and is wider in perfect competition than in monopoly and duopoly. The equilibrium size in monopoly is greater than the equilibrium size in duopoly because of the complementarity between goods generated by network externalities. For the externalities, perfect competition is inefficient, that is the equilibrium size in this market structure is smaller than the equilibrium size chosen by a social planner.

2 The literature

In this section I analyse the literature about network externalities and, especially, the paper of Economides and Himmelberg (1995) to make some comparisons.

First, the paper of Economides (1996) analyses the salient features of networks and point out the similarities between the economic structure of networks and the structure of vertically related industries. The analysis focuses on network externalities. He discusses their sources and their effects on pricing and market structure. There are two approaches in the analysis of network externalities. The "macro" approach assumes that network externalities exist, and attempts to model their consequences; it was the predominant approach during the 80s. The "micro" approach attempts to find the root cause of the network externalities; it started with the analysis of mix-and-match models and has evolved to the analysis of various structures of vertically related markets. He distinguishes between results that do not depend on the underlying industry microstructure and those that do. He analyses the issues of compatibility, coordination to technical standards, interconnection and interoperability, and their effects on pricing and quality of services and on the value of network links in various ownership structures.

A paper that analyses the effect of indirect network externalities is that of Caillaud and Jullien (2001) who present a model of imperfect competition among providers of informational intermediation services with a particular relevance for intermediation on the Internet. They propose a model of competition by price-discriminating intermediaries that are subject to indirect network externalities.

The combination of network externalities across the two categories of users and different prices for each category open the possibility of rich business strategies. For instance, an intermediary may subsidise the participation of some users in order to attract other participants.

They investigate a Bertrand duopoly competition game between two matchmakers who propose to match two sides of a market. The demand

addressed by users of one side of the market to a given matchmaker depends on matchmakers' prices and on the number of users of the other side who demand intermediation from this matchmaker. In this model matchmakers can rely on two pricing instruments, registration fees paid ex-ante, and a transaction fee paid ex-post when a transaction takes place between two matched parties.

They characterize the different equilibrium outcomes of imperfect competition situations, depending on the pricing instruments that intermediaries use and on the possibility of users using several intermediaries' services simultaneously. They start by assuming that users can only demand intermediation from one matchmaker. It is shown that one best strategy to protect market share, as well as to conquer the market, requires the setting of the maximal feasible transaction fee along with low registration fee. With the possibility of multiple registration by users, they show that inefficient equilibria can exist where users on only one side of the market ask for intermediation from both matchmakers. Moreover these equilibria may generate the highest level of profits for both intermediaries.

Another paper which treats price allocation between the two sides of the market is that of Rochet and Tirole (2003). They study how this price allocation is affected by platform governance (for profit vs not-for-profit) and they show that a symmetric equilibrium (both competing platform set the same price for each buyers and for each sellers), of the competition between associations, even if the downstream markets (the markets of buyers and sellers members) are perfectly competitive, and so the price level is socially optimal, is not characterized by an efficient outcome.

I want to describe more in detail the model of Economides and Himmelberg (1995) because the aim of this paper is similar to our. They assume that network externalities exist and attempt to model their consequences. As said in the introduction, Economides and Himmelberg (1995) define the critical mass as "the smallest network size that can be sustained in equilibrium given cost and market structure", and they derive the critical mass and the equilibrium size of the network with direct network externalities under different market structures. They do not explicitly refer their model to a market with direct network externalities, but they use an externality function (which depends on the total expected number of agents in the network) that shows it. Indeed, the network externality function that they specify is $h(n^e) = k + \delta f(n^e)$ where k is the value of the good in the absence of network externalities, δ is an indicator function taking value 1 if there are network externalities and $f(n^e)$ measures the network effects. As we can notice, the network externality function depends on the expected size n^e of the network as a whole; it implies that this analysis is addressed to network with direct network externalities. Their most important result is that the critical mass is independent of market structure, that is, it is the same in perfect competition, monopoly and duopoly and, when marginal cost decreases, the optimal

size of the network is different in these three market structures. More in detail, they assume that the willingness to pay for the n th unit of the good, when n^e units are expected to be sold, be $p(n; n^e)$. This is a decreasing function in its first argument because the demand slopes downward, and an increasing function in n^e because of the network externalities. At a market equilibrium, expectations are fulfilled, $n = n^e$, thus defining the *fulfilled expectation demand curve* $p(n; n)$. They impose $\lim_{n \rightarrow \infty} p(n; n) = 0$ so $p(n; n)$ is decreasing for large n . Economides and Himmelberg (1995) show that the fulfilled expectation demand curve is increasing for small n if either one of three conditions hold: (1) the utility of every consumer in a network of zero size is zero (this condition holds in network with direct network externalities); (2) there are immediate and large external benefits to network expansion for very small network; (3) there is a significant density of high willingness to pay consumers who are just indifferent on joining a network of approximately zero size.

When the fulfilled expectation demand curve increases for small n , they say that *the network exhibits a positive critical mass under perfect competition*. That is, with a constant marginal cost c , the network will start at a positive and significant n^0 (corresponding to marginal cost c^0); n^0 is the critical mass, that corresponds to the maximum of $p(n; n)$; for each smaller marginal cost there are three network size consistent with marginal cost pricing: a zero size network, an unstable network size at a first intersection of the horizontal through c with $p(n; n)$, and the Pareto optimal stable network size at the largest intersection of the horizontal with $p(n; n)$.

They show that in the presence of network externalities, perfect competition is inefficient: marginal social benefit of network expansion is larger than the benefit that accrues to a particular firm under perfect competition; thus perfect competition will provide a smaller network than is socially optimal.

They analyse the case of monopoly and duopoly. They show that a monopolist who is unable to price discriminate will support a smaller network and charge higher prices than perfectly competitive firms, but the critical mass in monopoly coincides with that of perfect competition; at the critical mass only, monopolist prices at marginal cost.

In the analysis of Cournot duopolists producing homogeneous compatible components, they assume that every duopolist takes the output of the other as given and sets the expectation of consumers of his own output. The result is that Cournot duopolists support a network of size between monopoly and perfect competition; the critical mass is the same as in the others market structures.

I demonstrate that the equilibrium size of the network with indirect network externalities depends on the market structure, that is, it is different in perfect competition, monopoly and duopoly. To show this, I use a network externality function that do not depends on the expected size of the network

as a whole (which characterises network with direct network externalities), but I introduce two groups of consumers to create the interdependence of demands for the network good, and I specify an externality function of each group that depends on the expected size of the other group adhering to the network (typical of network with indirect network externalities); the derived demand functions are those of the complementary goods. Moreover, from a social welfare point of view, the equilibrium of monopolist is more efficient than the duopolists outcome, differently from the prediction of Economides and Himmelberg (1995) model, due to the complementarity of the network good.

3 The Model

I analyze a two-stage game: in the first stage the producers of the network good choose the prices p_i , and in the second stage, after having observed prices, consumers will choose whether to buy or not the good.

Consumers are heterogeneous; I define a network externality function that captures the influence of the network size expectations of the one side of the market on the willingness to pay of the one side, for the good provided through the network. Let the network externality function for group i be kn_j : since the externalities are positive, larger expected size of network j gives higher individual utility to consumers of group i (and vice versa). In this way I introduce indirect network externalities: the utility of each agent indirectly depends on the number of agents of his own group that operated the same choice.

I indicate with y_i the income of consumers in group i ($i = 1, 2$); y_i is uniformly distributed between 0 and 1, with $f(y) = 1$. The willingness to pay of each agent in a group is constituted by two elements: his *income*, and an *externality function*; therefore, be income equals, the greater the number of consumers of the other group in the network, the greater the willingness to pay of the consumer for the good.

There are two goods in the market and each consumer of each group can buy just one good.

I write the indirect utility functions of consumers of the two groups, if they purchase the good as

$$\begin{cases} u_1 = y_1 - p_1 + kn_2^e \\ u_2 = y_2 - p_2 + kn_1^e \end{cases}$$

where n_i^e ($i = 1, 2$) is the expected number of consumers of group i which purchases the good; I normalise n_i^e to be between 0 and 1; $k > 0$ represents the degree of network externalities in the market. At equilibrium expectations are fulfilled, that is $n_i^e = n_i$, so the utility functions of the two groups become

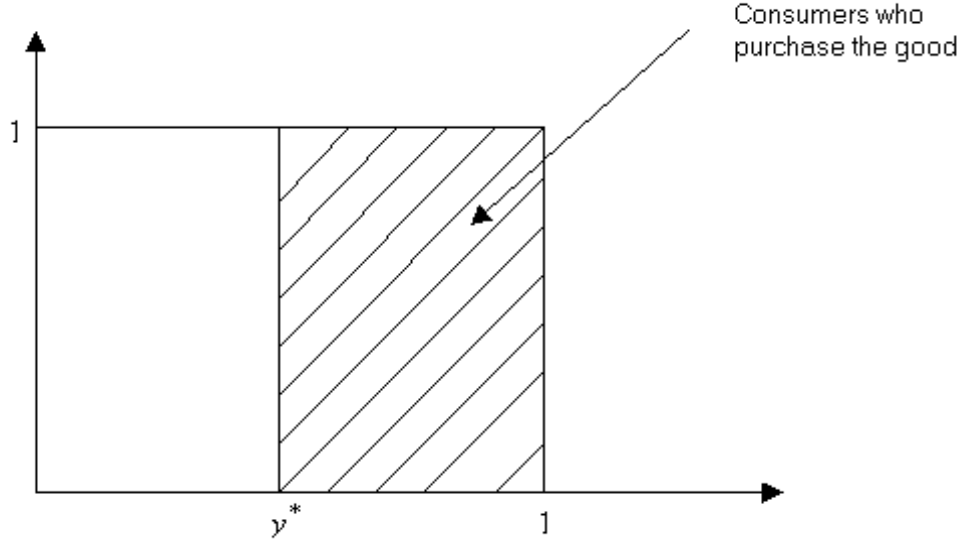


Figure 1:

$$\begin{cases} u_1 = y_1 - p_1 + kn_2 \\ u_2 = y_2 - p_2 + kn_1 \end{cases}$$

The equilibrium size of the network is the number n_i^* of consumers of both groups which buys the good.

The demand for the network good by consumers of each group is given by the condition $u_i \geq 0$: consumers with willingness to pay greater than the price of the good will purchase it. The indifference condition is

$$u_i = 0 \implies y_i - p_i + kn_j = 0 \implies y_i^* = p_i - kn_j \quad (1)$$

Given n_j and p , all consumers with willingness to pay, $y \geq y^*$, will purchase the good; given the uniform distribution of y_i between 0 and 1, the number of consumers in each group that will buy the good at price p will be:

$$n_i = 1 - y^* \quad (2)$$

this expression defines the demand for the good at price p .
We have

$$y_i^* = p_i - kn_j \implies 1 - n_i = p_i - kn_j \implies n_i = 1 - p_i + kn_j$$

which represents the size of the network at price p_i . We can invert this expression to obtain the inverse demand curve of group i ⁵

$$p_i = 1 - n_i + kn_j \quad (3)$$

The size of the network for the two groups is

$$\begin{cases} n_1 = 1 + kn_2 - p_1 \\ n_2 = 1 + kn_1 - p_2 \end{cases} \quad (4)$$

We derive the expressions of the size of each group only in function of prices, that is, we solve the previous system of equations. The result is

$$\begin{cases} n_1 = \frac{1+k-kp_2-p_1}{1-k^2} \\ n_2 = \frac{1+k-kp_1-p_2}{1-k^2} \end{cases} \quad (5)$$

The demand for the network good by one side of the market depends not only on prices but also on the demand of the other group; therefore, the producer of network good recognizes that the variation of price in one market does not influence only that market but also the other market, because of the externalities.

We calculate the equilibrium size of the network in three possible market structures for the provision of network good: perfect competition, monopoly, duopoly, and we compare these with the size of the network chosen by a social planner.

3.1 Perfect Competition

Consider that the market for the network good is perfectly competitive. Consumers of both groups pay to firms a price p_i to buy the good; Let the constant marginal cost of production be $c \geq 0$. To explain, we recall the example of credit card market. The payors are consumers of group 1 and the payees are merchants of group 2. Payors of group 1 buy from firms the credit card at price p_1 and payees buy from firms the systems necessary to accept a credit card at price p_2 . In a perfectly competitive environment, firms set prices equal to marginal cost and offer an infinitely elastic supply. Therefore, at equilibrium,

$$p(n) = c$$

⁵ $\frac{\delta p_i}{\delta n_i} < 0$; $\frac{\delta p_i}{\delta n_j} > 0$ for the externalities, and then, for the complementary of the two goods.

In this model we assume that the marginal cost of each firm c is equal for the two groups, so the previous condition becomes:

$$p_1 = p_2 = c$$

At the first stage of the game firms choose prices. We take the expressions for p_1 and p_2 from (3) and we obtain

$$\begin{cases} p_1 = 1 + kn_2 - n_1 = c \\ p_2 = 1 + kn_1 - n_2 = c \end{cases} \implies \begin{cases} n_1 = 1 + kn_2 - c \\ n_2 = 1 + kn_1 - c \end{cases} \quad (6)$$

The solution of this system gives the equilibrium size of the network in perfect competition⁶:

$$n_1^C = n_2^C = \frac{1 - c}{1 - k}$$

We notice, as expected, that the equilibrium size positively depends on the degree of externalities k : the greater the degree of externalities, the greater the incentive for consumers to adhere to the network.

3.2 Monopoly

A monopolist produces the network good. Consumers of both groups pay to the monopolist a price p_i to buy the good; the monopolist produce at a constant marginal cost $c \geq 0$. In the credit card market example payors of group 1 buy from the monopolist the credit card at price p_1 and payees buy from the monopolist the systems necessary to accept a credit card at price p_2 .

In the first stage of the game the monopolist fixes prices for the two groups to maximize its own profit:

$$\Pi = p_1 n_1 + p_2 n_2 - c(n_1 + n_2)$$

By substituting in the profit the expressions of n_1 and n_2 in [5] and deriving with respect to p_1 and p_2 , we obtain the two first order conditions⁷

$$\begin{cases} \frac{\delta \Pi}{\delta p_1} = \frac{1+k-kp_2-p_1}{1-k^2} + p_1(-\frac{1}{1-k^2}) - \frac{kp_2}{1-k^2} + \frac{c}{1-k^2} + \frac{ck}{1-k^2} = 0 \\ \frac{\delta \Pi}{\delta p_2} = \frac{1+k-kp_1-p_2}{1-k^2} + p_2(-\frac{1}{1-k^2}) - \frac{kp_1}{1-k^2} + \frac{c}{1-k^2} + \frac{ck}{1-k^2} = 0 \end{cases}$$

⁶From the equilibrium values of n_i^C we derive a restrictions on the parameters: $c > k$ because $0 < n_i < 1$. **($c > k$ non è un'assunzione, ma una conseguenza del fatto che n deve essere compreso tra 0 e 1. Punto 4) refer 1)**

⁷The second order condition are verified for $k < 1$. See the appendix.

and the solution gives the equilibrium prices⁸ :

$$\begin{cases} p_1^M = \frac{1+c}{2} \\ p_2^M = \frac{1+c}{2} \end{cases} \quad (7)$$

that are greater than prices in perfect competition (under the restriction made on c).

Now we can calculate the equilibrium size of the network for the monopolist by substituting the value of prices inside expression (5), and we have

$$n_1^M = n_2^M = \frac{1-c}{2(1-k)} \quad (8)$$

n_1^M and n_2^M represent the equilibrium number of consumers of each group which buys the good. Also in this case the equilibrium size positively depends on the degree of externalities k .

Now it is possible to compare the equilibrium size in the two analysed market structures.

Proposition 1 $n_i^C > n_i^M$

the equilibrium size of the network with indirect network externalities in perfect competition is always greater than the equilibrium size in monopoly, for each value of c and k ⁹.

We can think that, for the presence of network externalities, could be optimal for firms (in monopoly as in perfect competition) to expand the production until satisfy total demand, including in the network every consumer of each group. But if network grows, it will include consumers of very low willingness to pay for the good; therefore, after a given n , the lower willingness to pay more than offsets the effects of network externalities.

3.3 Duopoly with complementary goods

Consider a kind of duopoly that produces network good. There are two firms, 1 and 2, which share the market. Firms produce two different but

⁸The monopoly prices do not depend on the degree of the externalities k . This result is linked to the linearity of the utility functions and to the uniform distribution of y . n_i^M must be inside 0 and 1. This condition is verified (given $k < 1$ by the second order condition) for $2k - 1 < c < 1$ if $\frac{1}{2} < k < 1$, and for $0 < c < 1$ if $0 < k < \frac{1}{2}$. We consider only the values of k in 0 and $\frac{1}{2}$ to compare the results, as we clarify in the follow. With this restrictions on k and c , the condition on y^* in 0 and 1 is verified. Indeed

$0 < y^* = \frac{(1+c)-2k}{2-2k} < 1$.

⁹We notice that the equilibrium size in perfect competition is twice the equilibrium size in monopoly, that is, even with indirect network externalities (but under the hypothesis about the form of the utility function and the distribution of revenue), the standard result that "the quantity in perfect competition is twice the quantity in monopoly" holds.

complementary goods which form together the network good. Recalling the example of the credit card market, we can think that firm 1 produces credit cards to sell to payors of group 1 and firm 2 produces systems to accept credit cards to sell to payees of group 2 (in the previous case of monopoly, this two goods are produced by the same firm).¹⁰

Therefore, p_1 is the price that firm 1 charges for consumers of group 1 if they purchase the good, and the same for p_2 .

In the first stage of the game firms set prices, respectively p_1 and p_2 , maximizing their own profit. The profit of firm 1 is

$$\Pi_1 = p_1 n_1 - c n_1$$

and the profit of firm 2 is

$$\Pi_2 = p_2 n_2 - c n_2$$

Recalling the expressions of n_1 and n_2 as function of prices in (5), we substitute them in the profit of firm 1 and firm 2.

Firm 1 maximizes its own profit for p_1 given p_2 . The first order condition is

$$\frac{\delta \Pi_1}{\delta p_1} = \frac{1 + k - k p_2 - p_1}{1 - k^2} + p_1 \left(-\frac{1}{1 - k^2} \right) + \frac{c}{1 - k^2} = 0$$

this is the *reaction function* of firm 1. Firm 2 does the same and its reaction function is

$$\frac{\delta \Pi_2}{\delta p_2} = \frac{1 + k - k p_1 - p_2}{1 - k^2} + p_2 \left(-\frac{1}{1 - k^2} \right) + \frac{c}{1 - k^2} = 0$$

Equilibrium prices come from the solution of the system of the two reaction functions¹¹

$$p_1^D = p_2^D = \frac{k^2 - (k + 2) + c(k - 2)}{k^2 - 4}$$

Prices positively depend on the value of the externalities, indeed $\delta p / \delta k > 0$ for each value of k and c in the admissible interval: if the value of the externality is high, consumers have a greater utility if they adhere to the network and they are willing to pay an higher price.

Now we can calculate the equilibrium size in duopoly network by substituting equilibrium prices in the expressions of n_1 and n_2 in (5)¹²

¹⁰If we consider a scheme of duopoly in which agents can choose between the two firms, we reproduce a Bertrand competition and the only equilibrium is the competitive one.

¹¹For $0 < k < 1$ these prices are always positive. Indeed, because of the denominator of p_i^D is always negative, p_i^D is positive if $k^2 - (k + 2) + c(k - 2) < 0$; this inequality holds if k is in the interval $(-c - 2; 2)$ and then for k inside $(0; 1)$.

¹²For $0 < k < \frac{1}{2}$, n_i^D is inside 0 and 1 for $0 < c < 1$.

$$n_1^D = n_2^D = \frac{1 - c}{(1 - k)(k + 2)}$$

In this case too, the equilibrium size of the network positively depends on the value of the network externalities k .

We can see that

$$n_i^D < n_i^M$$

Proposition 2 *the equilibrium size of the network in duopoly is always smaller than the equilibrium size in monopoly and in perfect competition, for every admissible values of c and k .*

We can verify that

$$p^M < p^D$$

for $0 < k < 1$ and $0 < c < 1$ ¹³; so we expect that equilibrium size in monopoly is greater than in duopoly. This is the "classical" result for complementary goods. Prices charged by the monopolist are lower than prices charged by duopolists because the monopolist, by a price reduction in a market, increases its base of users not only in that market, but also in the other market because of the externalities (the complementarity), and he entirely benefits of a larger base; in the duopoly market the other duopolist too will benefit of a price reduction charged by the duopolist, because of the externalities, and this does not incentive to set lower prices.

In this case of duopoly, we have shown that with indirect network externalities the equilibrium size of the network in duopoly is smaller than in monopoly. Comparing this result with that of Economides and Himmelberg (1995) paper, in which the authors showed that, with direct network externalities, duopoly producing homogeneous compatible goods has an equilibrium size greater than the equilibrium size in monopoly (standard result with substitute goods), I want to underlying how indirect network externalities must be treated in a different way with respect to direct network externalities.

This result is in line with the nature of the network externalities, that is, the complementarity of the network good: if goods are complements, we expect that the monopolist expand the production more than a duopolist.

¹³See the appendix.

3.4 Social Planner

Now we suppose that a social planner manages the network. His objective is to maximize a social welfare function given by the sum of consumer surplus and profit of firms which produce the good. The utility function of group i is

$$u_i = y_i - p_i + kn_j$$

and surplus of consumer inside group i is

$$S = \begin{cases} 0 & \text{if } y_i \leq y_i^* \\ y_i - p_i + kn_j & \text{if } y_i \geq y_i^* \end{cases}$$

(with y^* as in (1)): only consumers with willingness to pay greater than price, will adhere to the network and then will have a positive surplus.

Remembering that y_i is a random variable uniformly distributed between 0 and 1, the surplus of group 1 is

$$ST_i = \int_{y_i^*}^1 (y_i - p_i + kn_j) dy_i = (1 - y_i^*) \left(\frac{1 + y_i^*}{2} - p_i + kn_j \right) \quad (9)$$

Recalling the expressions of the size of each group at price p given by (4)

$$n_i = 1 - p_i + kn_j$$

we can obtain the aggregate demand of group i

$$p_i = 1 - n_i + kn_j$$

Now total surplus of each group can be expressed only as a function of n_i and n_j by substituting in (8) the expressions of y_i^* and p_i . We have

$$ST_i = n_i \left(\frac{n_i}{2} \right)$$

For the two groups, total surplus is

$$\begin{cases} ST_1 = n_1 \left(\frac{n_1}{2} \right) \\ ST_2 = n_2 \left(\frac{n_2}{2} \right) \end{cases}$$

We suppose that there are two firms which produce the network good respectively for group 1 and group 2 (considering only one firm producing the good for both groups will be the same). Profits of the two firms are

$$\begin{cases} \Pi_1 = p_1 n_1 - cn_1 \\ \Pi_2 = p_2 n_2 - cn_2 \end{cases}$$

The welfare function is¹⁴

$$W = ST_1 + ST_1 + \Pi_1 + \Pi_2$$

We substitute the expressions of surplus and profits derived above and we obtain

$$W = n_1\left(\frac{n_1}{2}\right) + n_2\left(\frac{n_2}{2}\right) + n_1(1 + kn_2 - n_1) - cn_1 + n_2(1 + kn_1 - n_2) - cn_2$$

that social planner will maximize for n_1 and n_2 . The first order conditions are

$$\begin{cases} \frac{\delta W}{\delta n_1} = 2kn_2 - c + 1 - n_1 \\ \frac{\delta W}{\delta n_2} = 2kn_1 - c + 1 - n_2 \end{cases}$$

and, by solving, the equilibrium size is¹⁵

$$n_1^P = n_2^P = \frac{1 - c}{1 - 2k}$$

Even in this case equilibrium size positively depends on the degree of externalities in the market.

Now we can compare this equilibrium size with that in perfect competition. We can see that

$$n_i^P > n_i^C$$

for any admissible value of k and c .

Proposition 3 : *the equilibrium size in perfect competition is always smaller than the equilibrium size chosen by a social planner who wants to maximize social welfare, therefore perfect competition is inefficient*¹⁶

Therefore, also with indirect network externalities (as with direct network externalities) perfect competition is inefficient.

¹⁴Social planner separately maximizes surplus of the two groups of consumers because we have taken into account of the reciprocal influence, due to externalities, in the computation of the surplus of each group.

¹⁵Second order conditions are verified for $k < 1/2$. n_i^P is between 0 and 1 for $2k < c < 1$. **(non sono assunzioni ma restrizioni sui paramentri. Punto 4) refer 1)**

¹⁶ n_i^P verifies the condition on y^* ($0 \leq y^* \leq 1$). Prices are positive if $\frac{c}{1+c} > k$ and this inequality always holds because $k < \frac{c}{2} < \frac{c}{1+c}$.

4 Conclusions

Many two sides markets present indirect network externalities: the utility for a consumer of each group depends, directly, on the number of consumers in the other group that he meets and, indirectly, on the number of consumers of his own group which decide to buy the same good or service.

I presented a simple model of determination of the equilibrium size of a network with indirect network externalities in three market structures, perfect competition, monopoly and duopoly with complementary goods.

The leading idea is that markets with indirect network externalities must be treated differently from markets with direct network externalities, because of the sources of the externalities. That is, indirect network externalities can be viewed as a source of complementarity between goods, different from the standard examples of them.

I showed that perfect competition is inefficient: the equilibrium size in this market structure is always smaller than that chosen by a social planner for any admissible value of the marginal cost and the network externalities.

Moreover, the prices charged in a duopolistic market with indirect network externalities are greater than the prices charged in a monopoly market, and then, the equilibrium size in duopoly is smaller than in monopoly; this is the classical result that we obtain with complementary goods, generated by the indirect network externalities, and it is different from the result in markets with direct network externalities in which the equilibrium size in duopoly is greater than monopoly because the competition is with substitute goods.

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5 Appendix

We verify the second order conditions for the monopoly case. The Hessian matrix must be negative semi-definite. The Hessian matrix is the following

$$\begin{bmatrix} \frac{\delta \Pi}{\delta p_1 \delta p_1} & \frac{\delta \Pi}{\delta p_1 \delta p_2} \\ \frac{\delta \Pi}{\delta p_2 \delta p_1} & \frac{\delta \Pi}{\delta p_2 \delta p_2} \end{bmatrix}$$

by substituting the values, it is

$$\begin{bmatrix} -\frac{2}{1-k^2} & -\frac{2k}{1-k^2} \\ -\frac{2k}{1-k^2} & -\frac{2}{1-k^2} \end{bmatrix}$$

It must be

$$\frac{4}{(1-k^2)^2} - \frac{4k^2}{(1-k^2)^2} \geq 0$$

that is verified for $k < 1$.

We show that $p^M < p^D$.

$$\begin{aligned} \frac{1+c}{2} &< \frac{k^2-(k+2)+c(k-2)}{k^2-4} \\ \implies \frac{1+c}{2} - \frac{k^2-(k+2)+c(k-2)}{k^2-4} &< 0 \\ \implies \frac{-k^2+ck^2+2k-2ck}{2(k^2-4)} &< 0 \end{aligned}$$

because of the denominator is always negative, the inequality holds if the numerator is positive, that is

$$-k^2 + ck^2 + 2k - 2ck > 0 \implies (c-1)(k^2 - 2k) > 0 \implies k^2 - 2k < 0$$

(given that $c < 1$) which is verified for $0 < k < 2$, and then for $0 < k < 1$ too.