Real and Virtual Competition

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Abstract

While the Internet reduces market frictions by making it easier for consumers to obtain information about prices and product offerings, goods sold by electronic firms are not perfect substitutes to otherwise identical goods sold by conventional stores. Online purchases, due to non-zero shipping time, are associated with waiting costs, and they do not allow consumers to inspect the product prior to purchase. Visiting a conventional store, on the other hand, involves positive travelling costs. A model extending the circular-city paradigm with two types of firms, conventional and electronic, is studied. Under the benchmark setting with only conventional firms in the market, each consumer visits the nearest store and purchases the product there. When electronic firms enter the market, an intriguing type of market segmentation may arise. First, each consumer travels to the nearest conventional store to "try on" the product. Second, conventional retailers increase their prices and sell the good only to consumers who discover that they have high valuations; consumers with low valuations return "home" and order the good online. In spite of the increased competition from Internet retailers, welfare decreases.

JEL classifications: D43 (Oligopoly and other Forms of Market Imperfections), D81 (Criteria for Decision-Making under Risk and Uncertainty), and L11 (Production, Pricing and Market Structure).

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1 Introduction

Most economic analyses of the Internet have focused on its role as an information retrieval system that reduces consumer costs of obtaining information about prices and product offerings. Bakos’s (1997) seminal article on electronic marketplaces, for instance, shows that reducing search costs typically will improve market efficiency as a result of the increased competition among electronic retailers.¹

While the Internet undoubtedly reduces market frictions by making it easier for consumers to compare prices and product availability, goods sold over the Internet are clearly not perfect substitutes for otherwise identical goods sold by brick-and-mortar retailers. Hence, the focus of this paper is on the characteristics of goods that are associated with their modes of marketing and distribution.

First, due to non-zero shipping time, there are waiting costs associated with online purchases. When these costs are substantial, electronic retailers “lose points” to conventional stores, where the buyer immediately has access to the product.

Second, goods purchased online cannot be inspected beforehand. This becomes a problem when (i) there is uncertainty about how well the product “fits”, which can only be resolved by physical inspection (search goods in Nelson 1970’s terms) and (ii) it is costly, if not impossible, for consumers to return poorly fitting products to electronic firms for refunds.

The economics literature on Internet price competition typically takes the prices charged by conventional retailers as given; this paper, however, highlights the equilibrium interaction between electronic firms and their off-line counterparts. How do consumers make their purchasing decisions: do they buy a good from an ordinary “brick-and-mortar” firm or order the product online? What is the impact of the Internet on the prices charged by ordinary firms? How is economic welfare affected by the Internet?

To address these questions, a model extending the circular-city paradigm introduced by Salop (1979) with two types of firms, ordinary stores and electronic retailers, is studied. The firms sell physically identical products and possess constant marginal cost technology; the

fixed cost of entry is positive for brick-and-mortar stores and zero for electronic retailers. Consumers want to buy one unit of the product in question. A consumer can only learn his valuation for the good prior to purchase by travelling to a brick-and-mortar firm and physically inspecting the product, which entails transportation costs. Consumers cannot return poorly fitting products to electronic retailers, at least not at reasonable cost. Also, it is assumed that each consumer has access to the Internet and can visit an electronic firm without incurring any transportation costs, although there are waiting costs associated with ordering the product online. The waiting costs take the form of a discount factor and are, therefore, proportional to a consumer’s valuation.

Standard Bertrand-style arguments establish that the electronic segment of the market is competitive: at least two firms set the price equal to marginal cost. In a symmetric Nash Equilibrium, conventional stores are located equidistantly around the circle and charge the same price, and the equilibrium number of conventional stores is determined from the zero-profit condition.

Two settings are investigated: the No-Internet setting, in which only brick-and-mortar firms operate; and the Internet setting, in which both types of firms, conventional and electronic, exist in the market. With certain parameter restrictions in place, under the No-Internet setting each consumer visits the nearest brick-and-mortar firm and purchases the product there. When electronic firms enter the market, an intriguing type of market segmentation arises. First, each consumer travels to the nearest conventional store to “try on” the product. Second, conventional retailers actually raise their prices and sell the good only to consumers who discover that they have high valuations. Consumers who learn that they have low valuations return ”home” and order the good online.

The result that brick-and-mortar firms raise prices when electronic firms enter the market contrasts with the common view that increased competition leads to lower prices\(^2\). This result can be explained by the effect electronic firms have on the elasticity of demand faced by conventional retailers. Since the demand becomes less elastic, brick-and-mortar firms raise their prices in equilibrium. Moreover, economic welfare actually goes down when electronic

\(^2\)For other examples in which increasing competition leads to higher equilibrium prices see Satterthwaite (1979) and Rosenthal (1980).
firms enter the market, as consumers with low valuations incur positive waiting costs when ordering the good online.

In the next section, the formal model is presented. The No-Internet setting is analyzed in Section 3. In Section 4, the Internet setting is studied, and the effect of electronic firms on off-line prices and economic welfare is investigated. Concluding remarks appear in Section 5. All proofs are in an Appendix.

2 The Basic Model

In this section the technology, the preferences of the agents, and the equilibrium concept are presented.

2.1 Supply Side

Two types of firms operate in the market: ordinary brick-and-mortar firms (b-firms), and electronic retailers (e-firms) that market via the Internet. The two types produce the same physical good using different technologies. Electronic firms possess constant marginal cost technology,

$$C_e(q) = cq.$$  

Brick-and-mortar firms possess constant marginal and fixed cost technology,

$$C_b(q) = \begin{cases} \phi + cq, & \text{if } q > 0 \\ 0, & \text{if } q = 0. \end{cases}$$

Fixed cost $\phi$ refers to the building costs incurred if a b-firm enters the market.

2.2 Demand Side

Consumers with total mass $L$ are distributed uniformly on a circle with a perimeter equal to 1. Each consumer wants to buy one unit of the product. Consumer $i$ derives utility of $v_i$ if he buys the good from a b-firm, and $\delta v_i$ if he purchases from an e-firm; $\delta \in (0, 1)$ is the discount factor. That is, the consumer incurs waiting costs of $(1 - \delta)v_i$ if he purchases the good online$^3$.

$^3$Even though shipping time may involve only a few days, the "discount factor", $\delta$, may be significantly less than one if – by waiting – the consumer will miss an important opportunity to use the good.
Consumer $i$'s valuation $v_i$ is random and can take one of two values: it is high, $v_H$, with probability $\lambda$, and low, $v_L$, with probability $1 - \lambda$. Consumer $i$ does not know his valuation of the good at the outset. He can, however, learn $v_i$ prior to purchase by travelling to one of the brick-and-mortar firms located around the circle, which entails transportation cost $t$ per unit of length.

Each consumer has an Internet connection and can visit an e-firm without incurring any additional costs. For simplicity, it is assumed that the cost to a consumer of returning a poorly fitting product to an e-firm for a refund is prohibitive.

In short, visiting a b-firm involves positive travelling costs but allows consumers to inspect the product prior to purchase. Ordering the good from an e-firm entails no travelling costs, but positive waiting costs, and does not allow consumers to resolve uncertainty before buying the product.

### 2.3 Equilibrium Concept

Electronic retailers are perfectly competitive. Standard Bertrand-style arguments establish that in equilibrium at least two e-firms set price equal to marginal cost. Brick-and-mortar firms compete with each other, taking into account that consumers can always order the product on the Internet.

The equilibrium concept employed is a symmetric Nash equilibrium (referred to as just the equilibrium below), in which b-firms are located equidistantly around the circle and charge the same price. Under the free entry assumption, the equilibrium number of b-firms in the market is determined from the zero-profit condition.

### 3 Benchmark: No Internet

In this section, the setting in which only brick-and-mortar firms operate in the market is analyzed. For ease of exposition, this setting is called the No-Internet setting.

Suppose consumer $i$ has travelled to a b-firm and learned his valuation $v_i$. The consumer will purchase the good if the price charged by the firm, $p_b$, does not exceed $v_i$. Thus, the
expected value of visiting the b-firm to the consumer is
\[
\lambda \max\{v_H - p_b, 0\} + (1 - \lambda) \max\{v_L - p_b, 0\} - tx,
\]
where \(x\) is the distance to the firm. Since in equilibrium the brick-and-mortar firms charge the same price, the optimizing consumer will visit the closest b-firm if his expected value of doing so is non-negative. Otherwise, the consumer will not visit any of the firms, which yields zero payoff.

**Assumption 1.**
\[
v_H - c > \frac{3}{2\lambda} \sqrt{\frac{t\phi}{L}}.
\]

Assumption 1 is likely to hold for low values of the entry costs \(\phi\). It guarantees that there will be enough b-firms operating in the market, so that the firms will "indeed" compete with each other. That is, even the consumers who live in the middle between two neighboring firms (i.e., the consumers who travel the most) get strictly positive expected payoffs from visiting a b-firm. Specifically, let \(p_b^*\) be the equilibrium price and \(n_b^*\) be the equilibrium number of b-firms (these are determined below). Then, Assumption 1 ensures that
\[
\lambda \max\{v_H - p_b^*, 0\} + (1 - \lambda) \max\{v_L - p_b^*, 0\} - \frac{t}{2n_b^*} > 0.
\]

The main result of this section is Proposition 1 (below), which fully characterizes the equilibrium under Assumption 1. If \(v_L\) is low, the **Exclusive equilibrium** obtains in which the firms charge price \(p_b^* \in (v_L, v_H)\); each consumer visits the closest b-firm and "tries on" the good; only consumers with high valuations end up buying the product. If \(v_L\) is moderate, then the **Non-Exclusive Corner equilibrium** obtains in which the firms set the price to \(v_L\); both consumer types buy the good. Finally, if \(v_L\) is high, the **Non-Exclusive equilibrium** obtains in which the firms charge price \(p_b^* < v_L\); both types purchase the product.

It is notationally convenient to define the constants
\[
v_L \equiv c + \frac{1 - \sqrt{1 - \lambda}}{\lambda} \sqrt{\frac{t\phi}{L}}
\]
and
\[
v_L \equiv c + \sqrt{\frac{t\phi}{L}}.
\]
Observe that \(v_L < v_L\) for any \(\lambda \in (0, 1)\).
Proposition 1 (Equilibrium under the No-Internet Setting). Suppose Assumption 1 holds. Under the No-Internet setting, there are three types of equilibrium, depending on the value of parameter \( v_L \).

(i) The Exclusive Equilibrium. If \( v_L < v_{L}^* \), then the equilibrium price and the equilibrium number of b-firms are given by

\[
\begin{align*}
    p^*_b &= c + \frac{1}{\lambda} \sqrt{\frac{t\phi}{L}}, \\
    n^*_b &= \sqrt{\frac{L\phi}{v_L}}.
\end{align*}
\]

Each consumer visits the closest b-firm; only \( v_H \)-type consumers purchase the good.

(ii) The Non-Exclusive Corner Equilibrium. If \( v_L \in (v_{L}^*, \overline{v}_L) \), then the equilibrium price and the equilibrium number of b-firms are given by

\[
\begin{align*}
    p^*_b &= v_L, \\
    n^*_b &= \frac{L}{\phi}(v_L - c).
\end{align*}
\]

Each consumer visits the closest b-firm; both consumer types purchase the good.

(iii) The Non-Exclusive Equilibrium. If \( v_L > \overline{v}_L \), then the equilibrium price and the equilibrium number of b-firms are given by

\[
\begin{align*}
    p^*_b &= c + \frac{1}{\lambda} \sqrt{\frac{t\phi}{L}}, \\
    n^*_b &= \sqrt{\frac{L\phi}{v_L}}.
\end{align*}
\]

Each consumer visits the closest b-firm; both consumer types purchase the good.

Proof. See the Appendix. \( \square \)

It is interesting to compare the No-Internet setting with the standard circular city model in which consumers have the same valuation known from the outset\(^4\). When the Exclusive

\[ v - c > \frac{3}{2} \sqrt{\frac{t\phi}{L}}. \]
equilibrium obtains, the equilibrium number of b-firms is the same as in the standard circular
city model, while the price margin, \( p_b^* - c \), is higher by the factor of \( 1/\lambda \). This factor is the
inverse of the probability that a consumer purchases the product in the Exclusive equilibrium.

When the Non-Exclusive equilibrium obtains, both the equilibrium price and number of
b-firms are the same as in the standard circular city model. This finding is sensible, given that
the No-Internet setting converges to the standard circular city model as \( v_L \) goes to \( v_H \).

4 The Internet

In this setting both types of firms, brick-and-mortar stores and electronic retailers, operate in
the market. For ease of exposition, this setting is called the Internet setting.

First, suppose consumer \( i \) has ordered the good from an online retailer. Since electronic
firms are perfectly competitive and set their prices equal to marginal cost, the consumer’s
expected payoff is simply

\[
\delta (\lambda v_H + (1 - \lambda) v_L) - c.
\]

Second, suppose consumer \( i \) has travelled to a b-firm and learned his valuation \( v_i \). At this
point, he has three alternatives: buy the good at the store, return home and purchase from
an online retailer, or not buy the good. Thus, the expected value of visiting the b-firm to the
consumer is

\[
\lambda \max\{v_H - p_b, \delta v_H - c, 0\} + (1 - \lambda) \max\{v_L - p_b, \delta v_L - c, 0\} - tx,
\]

where \( x \) is the distance to the firm.

Observe that if the consumer finds it optimal to purchase the good from the b-firm when
\( v_i = v_L \), then he will buy the product from the firm when \( v_i = v_H \). However, if the consumer
purchases the good online when \( v_i = v_L \), then he still might find it optimal to purchase the
good from the b-firm when \( v_i = v_H \) (the waiting costs, \( (1 - \delta)v_H \), might be too high).

Proposition 2 (below) shows that, under certain parameter restrictions, the following type
of market segmentation arises in equilibrium. Each consumer visits a b-firm and learns his
valuation; consumers with low valuations return home and purchase the good online, while
consumers with high valuations buy the product from b-firms.
Assumption 2.

\[ c < \frac{\delta(1 - \sqrt{1 - \lambda})}{(1 - \delta)\lambda} \sqrt{\frac{t\phi}{L}}. \]

Assumption 2 is analogous to Assumption 1 of the No-Internet setting. It guarantees that b-firms are indeed competing with each other. Consumers who live in the middle between two neighboring b-firms get strictly higher expected payoffs from visiting a b-firm then from purchasing the good online at the first place:

\[ \lambda(v_H - p_b^{**}) + (1 - \lambda)(\delta v_L - c) - \frac{t}{2n_b^{**}} > \delta(\lambda v_H + (1 - \lambda)v_L) - c, \]

where \( p_b^{**} \) and \( n_b^{**} \) are the equilibrium price and number of b-firms (given below).

The next assumption ensures that consumers with high valuations purchase the good from the b-firms,

\[ (1 - \delta)v_H > p_b^{**} - c. \]

Assumption 3.

\[ v_H > \frac{3}{2(1 - \delta)\lambda} \sqrt{\frac{t\phi}{L}}. \]

It is notationally convenient to define the constant

\[ \hat{v}_L \equiv \frac{1 - \sqrt{1 - \lambda}}{(1 - \delta)\lambda} \sqrt{\frac{t\phi}{L}}. \]

Assumption 2 implies \( \hat{v}_L > v_L > c/\delta \) for any \( \lambda \in (0, 1) \).

Proposition 2 (Market Segmentation). Suppose assumptions 2 and 3 hold, and \( v_L \in (c/\delta, \hat{v}_L) \). Under the Internet setting, the equilibrium price and the equilibrium number of b-firms are given by

\[
\begin{aligned}
p_b^{**} &= c + \frac{1}{2}\sqrt{\frac{t\phi}{T}}, \\
n_b^{**} &= \sqrt{\frac{tL}{\hat{v}_L}}.
\end{aligned}
\]

Each consumer visits the closest b-firm and learns his valuation; \( v_H \)-type consumers buy the good from b-firms, while \( v_L \)-type consumers return home and order the product online.

Proof. See the Appendix.
Note that the results of Proposition 1 are valid under the assumptions of Proposition 2, since assumptions 2 and 3 imply

\[
v_H - c > \left( \frac{3}{2(1-\delta)\lambda} - \frac{\delta(1 - \sqrt{1 - \lambda})}{(1-\delta)\lambda} \right) \sqrt{\frac{t\phi}{L}} = \frac{3}{2\lambda} \sqrt{\frac{t\phi}{L}} + \frac{\delta(1 + 2\sqrt{1 - \lambda})}{2(1-\delta)\lambda} \sqrt{\frac{t\phi}{L}} > \frac{3}{2\lambda} \sqrt{\frac{t\phi}{L}},
\]

i.e., Assumption 1 holds.

What is the impact of the Internet on the prices charged by brick-and-mortar firms? How is economic welfare affected by the Internet? (Note that economic welfare coincides with consumer welfare, as both types of firms make zero profits in equilibrium.) Proposition 3 follows more or less directly from propositions 1 and 2. It shows that, in certain cases, welfare will actually fall when electronic firms enter the market.

**Proposition 3 (Welfare).** Suppose assumptions 2 and 3 hold, and \(v_L \in (c/\delta, \tilde{v}_L)\). The following welfare comparison holds between the two settings.

(i) **Increasing Welfare.** If \(v_L \in (c/\delta, \tilde{v}_L)\), then the Exclusive equilibrium obtains under the No-Internet setting. Welfare goes up when e-firms enter the market.

(ii) **Declining Welfare.** If \(v_L \in (\tilde{v}_L, v_L)\), then either the Non-Exclusive or the Non-Exclusive Corner equilibrium obtains under the No-Internet setting. In both cases, welfare goes down when e-firms enter the market.

**Proof.** See the Appendix.

First, consider \(v_L \in (c/\delta, \tilde{v}_L)\). Both the equilibrium price and number of b-firms remain unchanged when e-firms enter the market. Under the No-Internet setting, only \(v_H\)-type consumers purchase the good from b-firms. Under the Internet setting, \(v_H\)-type consumers purchase the good from b-firms, \(v_L\)-type consumers order the product online. Thus, welfare goes up by

\[\Delta W = L(1 - \lambda)(\delta v_L - c) > 0.\]

Next, consider \(v_L \in (\tilde{v}_L, \min\{\tilde{v}_L, \bar{v}_L\})\). Under the No-Internet setting, b-firms set prices equal to \(v_L\) and sell the good to both consumer types. When e-firms enter the market, b-firms would have to lower the price to \(c + (1 - \delta)v_L\) to attract both types; i.e. individual expected
demand for the product becomes less elastic. Brick-and-mortar firms respond by price increase. Under the Internet setting, \( v_H \)-type consumers still purchase the good from b-firms, while \( v_L \)-type consumers switch to electronic retailers. The equilibrium number of b-firms increases. The change in welfare is equal to

\[
\Delta W = -L(1 - \lambda)(1 - \delta)v_L - \left( \frac{tL}{4n^*_b} + \phi n^*_b - \frac{tL}{4n^*_b} - \phi n^*_b \right) < 0.
\]

(See the Appendix for the detailed explanation.)

Finally, consider \( v_L \in (\min \{ \tilde{v}_L, v_L \}, \tilde{v}_L) \). Under the No-Internet setting, b-firms set the prices below \( v_L \) and sell the good to both consumer types. Individual expected demand for the product becomes less elastic when electronic firms enter the market. Brick-and-mortar firms respond by increasing their price. Under the Internet setting, \( v_H \)-type consumers still purchase the good from b-firms, while \( v_L \)-type consumers switch to online firms. The equilibrium number of b-firms remains unchanged. Thus, welfare goes down by

\[
\Delta W = -L(1 - \lambda)(1 - \delta)v_L < 0.
\]

## 5 Conclusion

This paper examines the impact of the Internet on prices charged by conventional retailers and economic welfare. Two settings were investigated in the context of a circular city model, the No-Internet setting with brick-and-mortar stores, and the Internet setting with two types of firms, conventional and electronic, in the market.

Under the No-Internet setting, when individual expected demand is inelastic, brick-and-mortar firms charge the price accepted by consumers who discover they have high valuations. When individual expected demand is elastic, the firms charge the price accepted by all consumers.

Under the Internet setting, parameter restrictions were derived that give rise to the following type of market segmentation. Each consumer visits a conventional store and inspects

\[5\]This set is non-empty if and only if

\[
e < \frac{\delta(1 - \sqrt{1 - \lambda})}{(1 - \delta)\lambda} \sqrt{\frac{t\phi}{L}} - \frac{\sqrt{1 - \lambda} - (1 - \lambda)}{\lambda} \sqrt{\frac{t\phi}{L}},
\]

which is stronger than Assumption 2.
the product; consumers with high valuations buy the good there, while consumers with low valuations return home and order the product online.

With these parameter restrictions in place, the impact of the Internet on economic welfare was explored. In the case of inelastic demand, welfare rises when electronic firms enter the market, as consumers with low valuations now order the good on the Internet. In the case of elastic demand, welfare falls, as consumers with low valuations switch their purchases from conventional retailers to Internet firms and, therefore, experience waiting costs.

The main message of this paper is that goods that are physically identical may often, nevertheless, be differentiated by the modes through which they are marketed and sold. Electronic retailing, while reducing consumer search and transportation costs, is not frictionless. Consumers do not always know what they are getting when they purchase a product online and they must often experience significant delays between the time they order a good and the time they receive it. Moreover, when these frictions interact with the market imperfections coming in conventional retailing, the resulting increase in competition need not lead to a superior social outcome.
Appendix

Proof of Proposition 1

Each part is proven in turn.

(i) Suppose $v_L < \underline{v}_L$. Consider the representative firm that charges price $p_b$ which is accepted only by $v_H$-type consumers. Its rivals, located at distance $1/n_b^*$, charge price $p_b^*$ which is also accepted only by $v_H$-type consumers. The firm captures consumers living within distance $x$ defined by

$$\lambda(v_H - p_b) - tx = \lambda(v_H - p_b^*) - t \left( \frac{1}{n_b} - x \right),$$

or

$$x(p_b, p_b^*) = \frac{1}{2t} \left( \frac{t}{n_b} + \lambda p_b^* - \lambda p_b \right).$$

The firm makes expected profit

$$\Pi(p_b, p_b^*) = L\lambda 2x(p_b, p_b^*)(p_b - c) - \phi = \frac{\lambda L}{t} \left( \frac{t}{n_b} + \lambda p_b^* - \lambda p_b \right) (p_b - c) - \phi.$$

The equilibrium price satisfies

$$p_b^* \in \arg \max_{p_b} \Pi(p_b, p_b^*).$$

The first-order condition is

$$\frac{t}{n_b^*} - \lambda (p_b^* - c) = 0,$$

or

$$p_b^* = c + \frac{t}{\lambda n_b^*}.$$

The equilibrium number of firms is defined by the zero-profit condition

$$\frac{\lambda L}{n_b^*} \frac{t}{\lambda n_b^*} = \phi,$$

or

$$n_b^* = \sqrt{\frac{tL}{\phi}}.$$

Substituting $n_b^*$ into the equilibrium price yields

$$p_b^* = c + \frac{1}{\lambda} \sqrt{\frac{t\phi}{L}}.$$
Assumption 1 guarantees that b-firms are indeed competing with each other. Consumers living in the middle between two neighboring firms get strictly positive expected payoff:
\[ \lambda(v_H - p_b^*) - \frac{t}{2n_b^*} = \lambda \left( v_H - c - \frac{1}{\lambda} \sqrt{\frac{t \phi}{L}} \right) - \frac{1}{2} \sqrt{\frac{t \phi}{L}} = \lambda \left( v_H - c - \frac{3}{2\lambda} \sqrt{\frac{t \phi}{L}} \right) > 0. \]

Finally, the representative firm will not benefit from charging a price equal or below \( v_L \) (so that both consumer types buy the good). To see this, suppose \( v_L = v_L \). By charging \( p_b \leq v_L \), the firm captures consumers living within distance \( y \) defined by
\[ \lambda v_H + (1 - \lambda)v_L - p_b - ty = \lambda(v_H - p_b^*) - t \left( \frac{1}{n_b^*} - y \right), \]
or
\[ y(p_b, p_b^*) = \frac{1}{2t} \left( \frac{t}{n_b^*} + (1 - \lambda)v_L + \lambda p_b^* - p_b \right). \]

The firm maximizes
\[ \Pi(p_b, p_b^*) = L2y(p_b, p_b^*) (p_b - c) - \phi = \frac{L}{t} \left( \frac{t}{n_b^*} + (1 - \lambda)v_L + \lambda p_b^* - p_b \right) (p_b - c) - \phi, \]
subject to
\[ p_b \leq v_L. \]

The solution is
\[ \hat{p}_b = \min \left\{ v_L, \frac{1}{2} \left( \frac{t}{n_b^*} + (1 - \lambda)v_L + \lambda p_b^* + c \right) \right\} \]
\[ = \min \left\{ v_L, v_L + \frac{1}{2} \left( \frac{t}{n_b^*} - (1 + \lambda)(v_L - c) + \lambda(p_b^* - c) \right) \right\} \]
\[ = \min \left\{ v_L, v_L + \frac{1}{2} \left( \frac{t \phi}{L} - (1 + \lambda) \frac{1 - \sqrt{1 - \lambda}}{\lambda} \sqrt{\frac{t \phi}{L}} + \lambda \frac{1}{\lambda} \sqrt{\frac{t \phi}{L}} \right) \right\} \]
\[ = \min \left\{ v_L, v_L + \frac{1}{2} \frac{\sqrt{1 - \lambda}(1 + \lambda) - \sqrt{1 - \lambda}}{2\lambda} \sqrt{\frac{t \phi}{L}} \right\} = v_L. \]

Substituting \( \hat{p}_b \) into the profit function gives
\[ \Pi(\hat{p}_b, p_b^*) = \frac{L}{t} \left( \frac{t}{n_b^*} + (1 - \lambda)v_L + \lambda p_b^* - v_L \right) (v_L - c) - \phi \]
\[ = \frac{L}{t} \left( \frac{t}{n_b^*} + \lambda(p_b^* - v_L) \right) (v_L - c) - \phi \]
\[ = \frac{L}{t} \left( \sqrt{\frac{t \phi}{L}} + \lambda \left( \frac{1 - \sqrt{1 - \lambda}}{\lambda} \right) \sqrt{\frac{t \phi}{L}} \right) 1 - \sqrt{1 - \lambda} \sqrt{\frac{t \phi}{L}} - \phi = 0. \]

The firm will not benefit from charging a price equal or below \( v_L \) when \( v_L = v_L \). Obviously, it will not benefit from charging a price equal or below \( v_L \) when \( v_L < v_L \).
(ii) Suppose $v_L \in (v_L, \overline{v}_L)$. Consider the representative firm that charges price $p_b$ which is accepted by both consumer types. Its rivals, located at distance $1/n_b^*$, charge price $p_b^*$ which is also accepted by both consumer types. The firm captures consumers living within distance $x$ defined by

$$\lambda v_H + (1 - \lambda)v_L - p_b - tx = \lambda v_H + (1 - \lambda)v_L - p_b^* - t\left(\frac{1}{n_b^*} - x\right),$$

or

$$x(p_b, p_b^*) = \frac{1}{2t}\left(\frac{t}{n_b^*} + p_b^* - p_b\right).$$

The firm maximizes

$$\Pi(p_b, p_b^*) = L\frac{2}{tx}(p_b, p_b^*)(p_b - c) - \phi = \frac{L}{t}\left(\frac{t}{n_b^*} + p_b^* - p_b\right)(p_b - c) - \phi,$$

subject to

$$p_b \leq v_L.$$

The solution is

$$\hat{p}_b = \min \left\{ v_L, \frac{1}{2}\left(\frac{t}{n_b^*} + p_b^* + c\right) \right\}.$$

Substituting $p_b^* = v_L$ and $n_b^* = L(v_L - c)/\phi$ yields

$$\hat{p}_b = \min \left\{ v_L, \frac{1}{2}\left(\frac{t\phi}{L(v_L - c)} + v_L + c\right) \right\} = \min \left\{ v_L, \frac{1}{2(v_L - c)}\left(\frac{t\phi}{L} - (v_L - c)^2\right) \right\} = v_L.$$

Assumption 1 guarantees that b-firms are indeed competing with each other. Consumers living in the middle between two neighboring firms get strictly positive expected payoff:

$$\lambda(v_H - v_L) - \frac{t}{2n_b^*} = \lambda(v_H - v_L) - \frac{t\phi}{2L(v_L - c)}$$

$$= \frac{t\phi}{2L(v_L - c)}\left(2\lambda(v_H - v_L)(v_L - c)\frac{L}{t\phi} - 1\right)$$

$$\geq \frac{t\phi}{2L(v_L - c)}\min\left\{2\lambda(v_H - \overline{v}_L)(\overline{v}_L - c)\frac{L}{t\phi} - 1, 2\lambda(v_H - \overline{v}_L)(\overline{v}_L - c)\frac{L}{t\phi} - 1\right\}$$

$$\geq \frac{t\phi}{2L(v_L - c)}\min\left\{2\lambda\left(\frac{3}{2\lambda} - 1 - \sqrt{1 - \lambda}\right)\frac{1 - \sqrt{1 - \lambda}}{\lambda} - 1, 2\lambda\left(\frac{3}{2\lambda} - 1\right) - 1\right\}$$

$$= \frac{t\phi}{2L(v_L - c)}\min\left\{\frac{1}{\lambda}(\sqrt{1 - \lambda} - (1 - \lambda)), 2(1 - \lambda)\right\} > 0.$$
Finally, the representative firm will not benefit from charging a price above \( v_L \) (so that only \( v_H \)-type consumers buy the good). To see this, suppose \( v_L = \underline{v}_L \). By charging \( p_b > \underline{v}_L \), the firm captures consumers living within distance \( y \) defined by

\[
\lambda(v_H - p_b) - ty = \lambda v_H + (1 - \lambda) \underline{v}_L - p_b - t \left( \frac{1}{n_b} - y \right),
\]

or

\[
y(p_b, p_b^*) = \frac{1}{2t} \left( \frac{t}{n_b} - (1 - \lambda) \underline{v}_L + p_b^* - \lambda p_b \right).
\]

The firm maximizes

\[
\Pi(p_b, p_b^*) = \lambda L y(p_b, p_b^*)(p_b - c) - \phi = \frac{\lambda L}{t} \left( \frac{t}{n_b} - (1 - \lambda) \underline{v}_L + p_b^* - \lambda p_b \right)(p_b - c) - \phi,
\]

subject to

\[ p_b > \underline{v}_L. \]

The solution is

\[
\hat{p}_b = \frac{1}{2\lambda} \left( \frac{t}{n_b} - (1 - \lambda) \underline{v}_L + p_b^* + \lambda c \right)
\]

\[
= \frac{1}{2\lambda} \left( \frac{t\phi}{L(\underline{v}_L - c)} - (1 - \lambda) \underline{v}_L + \underline{v}_L + \lambda c \right)
\]

\[
= c + \frac{1}{2\lambda} \left( \frac{t\phi}{L(\underline{v}_L - c)} + \lambda(\underline{v}_L - c) \right)
\]

\[
= c + \frac{1}{2\lambda} \left( \frac{\lambda}{1 - \sqrt{1 - \lambda}} \sqrt{\frac{t\phi}{L}} + \frac{1 - \sqrt{1 - \lambda}}{\lambda} \sqrt{\frac{t\phi}{L}} \right)
\]

\[
= c + \frac{1}{\lambda} \sqrt{\frac{t\phi}{L}}.
\]

Substituting \( \hat{p}_b \) into the profit function gives

\[
\Pi(\hat{p}_b, p_b^*) = \frac{\lambda L}{t} \left( \frac{t}{n_b} - (1 - \lambda) \underline{v}_L + \underline{v}_L - \lambda \hat{p}_b \right)(\hat{p}_b - c) - \phi
\]

\[
= \frac{\lambda L}{t} \left( \frac{t}{n_b} - \lambda(\hat{p}_b - \underline{v}_L) \right)(\hat{p}_b - c) - \phi
\]

\[
= \frac{\lambda L}{t} \left( \frac{\lambda}{1 - \sqrt{1 - \lambda}} \sqrt{\frac{t\phi}{L}} - \lambda \left( \frac{1}{\lambda} - \frac{1 - \sqrt{1 - \lambda}}{\lambda} \right) \sqrt{\frac{t\phi}{L}} \right) \frac{1}{\lambda} \sqrt{\frac{t\phi}{L}} - \phi = 0.
\]

The firm will not benefit from charging a price above \( v_L \) when \( v_L = \underline{v}_L \). Obviously, it will not benefit from charging a price above \( v_L \) when \( v_L \in (\underline{v}_L, \overline{v}_L) \).
(iii) Suppose \( v_L > v_L \). Consider the representative firm that charges price \( p_b \) which is accepted by both consumer types. Its rivals, located at distance \( 1/n_b^* \), charge price \( p_b^* \) which is also accepted by both consumer types. The firm captures consumers living within distance \( x \) defined by

\[
\lambda v_H + (1 - \lambda) v_L - p_b - tx = \lambda v_H + (1 - \lambda) v_L - p_b^* - t \left( \frac{1}{n_b^*} - x \right),
\]

or

\[
x(p_b, p_b^*) = \frac{1}{2t} \left( \frac{t}{n_b^*} + p_b^* - p_b \right).
\]

The firm makes profit

\[
\Pi(p_b, p_b^*) = Lx(p_b, p_b^*)(p_b - c) - \phi = \frac{L}{t} \left( \frac{t}{n_b^*} + p_b^* - p_b \right) (p_b - c) - \phi.
\]

The equilibrium price satisfies

\[
p_b^* \in \arg \max_{p_b} \Pi(p_b, p_b^*).
\]

The first-order condition is

\[
\frac{t}{n_b^*} - (p_b^* - c) = 0,
\]

or

\[
p_b^* = c + \frac{t}{n_b^*}.
\]

The equilibrium number of firms is defined by the zero-profit condition

\[
\frac{L}{n_b^*} = \phi,
\]

or

\[
n_b^* = \sqrt{\frac{tL}{\phi}}.
\]

Substituting \( n_b^* \) into the equilibrium price yields

\[
p_b^* = c + \sqrt{\frac{t\phi}{L}}.
\]

Assumption 1 guarantees that b-firms are indeed competing with each other. Consumers living in the middle between two neighboring firms get strictly positive expected payoff:

\[
\lambda v_H + (1 - \lambda) v_L - p_b^* - \frac{t}{2n_b^*} > \lambda v_H + (1 - \lambda) \left( c + \sqrt{\frac{t\phi}{L}} \right) - c - \sqrt{\frac{t\phi}{L}} - \frac{1}{2} \sqrt{\frac{t\phi}{L}}
\]

\[
= \lambda \left( v_H - c - \frac{3}{2\lambda} \sqrt{\frac{t\phi}{L}} \right) + (1 - \lambda) \sqrt{\frac{t\phi}{L}} > 0.
\]
Finally, the representative firm will not benefit from charging a price strictly above \( v_L \) (so that only \( v_H \)-type consumers buy the good). To see this, suppose \( v_L = \bar{v}_L \). By charging \( p_b > \bar{v}_L \), the firm captures consumers living within distance \( y \) defined by

\[
\lambda(v_H - p_b) - ty = \lambda v_H + (1 - \lambda)\bar{v}_L - p_b^* - t \left( \frac{1}{n_b^*} - y \right),
\]

or

\[
y(p_b, p_b^*) = \frac{1}{2t} \left( \frac{t}{n_b^*} - (1 - \lambda)\bar{v}_L + p_b^* - \lambda p_b \right).
\]

The firm maximizes

\[
\Pi(p_b, p_b^*) = \lambda L 2y(p_b, p_b^*)(p_b - c) - \phi = \frac{\lambda L}{t} \left( \frac{t}{n_b^*} - (1 - \lambda)\bar{v}_L + p_b^* - \lambda p_b \right)(p_b - c) - \phi,
\]

subject to

\( p_b > \bar{v}_L \).

The solution is

\[
\hat{p}_b = \frac{1}{2\lambda} \left( \frac{t}{n_b^*} - (1 - \lambda)\bar{v}_L + p_b^* + \lambda c \right)
\]

\[
= c + \frac{1}{2\lambda} \left( \frac{t}{n_b^*} + (p_b^* - c) - (1 - \lambda)(\bar{v}_L - c) \right)
\]

\[
= c + \frac{1}{2\lambda} \left( \sqrt{\frac{t\phi}{L}} + \sqrt{\frac{t\phi}{L}} - (1 - \lambda)\sqrt{\frac{t\phi}{L}} \right)
\]

\[
= c + \frac{1 + \lambda}{2\lambda} \sqrt{\frac{t\phi}{L}}.
\]

Substituting \( \hat{p}_b \) into the profit function gives

\[
\Pi(\hat{p}_b, p_b^*) = \frac{L\lambda}{t} \left( \frac{t}{n_b^*} - (1 - \lambda)\bar{v}_L + p_b^* - \lambda \hat{p}_b \right)(\hat{p}_b - c) - \phi
\]

\[
= \frac{L\lambda}{t} \left( \sqrt{\frac{t\phi}{L}} - (1 - \lambda)\sqrt{\frac{t\phi}{L}} + \sqrt{\frac{t\phi}{L}} - \lambda \frac{1 + \lambda}{2\lambda} \sqrt{\frac{t\phi}{L}} \right) \frac{1 + \lambda}{2\lambda} \sqrt{\frac{t\phi}{L}} - \phi
\]

\[
= \left( \frac{(1 + \lambda)^2}{4} - 1 \right) \phi < 0.
\]

The firm will not benefit from charging a price strictly above \( v_L \) when \( v_L = \bar{v}_L \). Obviously, it will not benefit from charging a price strictly above \( v_L \) when \( v_L > \bar{v}_L \).
Proof of Proposition 2

Facing price $p_b^{**}$ charged by b-firms, consumers with low valuations order the good on the Internet if

$$
\delta v_L - c > \max\{0, v_L - p_b^{**}\} = \max\left\{0, v_L - c - \frac{1}{\lambda}\sqrt{\frac{t\phi}{L}} \right\}.
$$

This holds for any $v_L \in (c/\delta, \tilde{v}_L)$.

Consumers with high valuations buy the good from b-firms if

$$
v_H - p_b^{**} > \delta v_H - c,
$$
or

$$
v_H > \frac{p_b^{**} - c}{1 - \delta} = \frac{1}{(1 - \delta)\lambda}\sqrt{\frac{t\phi}{L}}.
$$

This holds by Assumption 3.

Consider the representative b-firm which charges the price accepted only by $v_H$-type consumers. The firm captures consumers living within distance $x$ defined by

$$
\lambda(v_H - p_b) + (1 - \lambda)(\delta v_L - c) - tx = \lambda(v_H - p_b^{**}) + (1 - \lambda)(\delta v_L - c) - t\left(\frac{1}{n_b^{**}} - x\right),
$$
or

$$
x(p_b, p_b^{**}) = \frac{1}{2t}\left(\frac{t}{n_b^{**}} + \lambda p_b^{**} - \lambda p_b\right).
$$

The firm makes expected profit

$$
\Pi(p_b, p_b^{**}) = \lambda L 2x(p_b, p_b^{**})(p_b - c) - \phi = \frac{\lambda L}{t}\left(\frac{t}{n_b^{**}} + \lambda p_b^{**} - \lambda p_b\right)(p_b - c) - \phi.
$$

The solution is

$$
\hat{p}_b = \frac{1}{2}\left(\frac{t}{\lambda n_b^{**}} + p_b^{**} + c\right)
= \frac{1}{2}\left(\frac{1}{\lambda}\sqrt{\frac{t\phi}{L}} + \frac{1}{\lambda}\sqrt{\frac{t\phi}{L}} + c\right) = c + \frac{1}{\alpha}\sqrt{\frac{t\phi}{L}} = p_b^{**}.
$$

Consumers living in the middle between two neighboring b-firms get higher expected payoff from visiting a b-firm than ordering the good on the Internet in the first place if

$$
\lambda(v_H - p_b^{**}) + (1 - \lambda)(\delta v_L - c) - \frac{t}{2n_b^{**}} > \lambda(\delta v_H - c) + (1 - \lambda)(\delta v_L - c),
$$

20
or
\[
\lambda \left( v_H - c - \frac{1}{\lambda} \sqrt{\frac{t\phi}{L}} \right) - \frac{1}{2} \sqrt{\frac{t\phi}{L}} > \lambda (\delta v_H - c),
\]
\[
v_H > \frac{3}{2(1 - \delta)\lambda} \sqrt{\frac{t\phi}{L}}.
\]
This holds by Assumption 3.

Finally, the representative b-firm will not benefit from charging \( p_b \leq c + (1 - \delta)v_L \) (so that both consumer types buy the good from it). The firm captures consumers living within distance \( y \) defined by
\[
\lambda v_H + (1 - \lambda)v_L - p_b - ty = \lambda (v_H - p_b^*) + (1 - \lambda)(\delta v_L - c) - t \left( \frac{1}{n_b^*} - y \right),
\]
or
\[
y(p_b, p_b^*) = \frac{1}{2t} \left( \frac{t}{n_b^*} + (1 - \lambda)(c + (1 - \delta)v_L) + \lambda p_b^* - p_b \right).
\]
The firm maximizes
\[
\Pi(p_b, p_b^*) = L2y(p_b, p_b^*)(p_b - c) - \phi
\]
\[
= \frac{L}{t} \left( \frac{t}{n_b^*} + (1 - \lambda)(c + (1 - \delta)v_L) + \lambda p_b^* - p_b \right) (p_b - c) - \phi,
\]
subject to
\[
p_b \leq c + (1 - \delta)v_L.
\]
Let
\[
v'_L \equiv c + (1 - \delta)v_L.
\]
The firm’s problem can be rewritten as
\[
\Pi(p_b, p_b^*) = \frac{L}{t} \left( \frac{t}{n_b^*} + (1 - \lambda)v'_L + \lambda p_b^* - p_b \right) (p_b - c) - \phi,
\]
subject to
\[
p_b \leq v'_L.
\]
Observe that \( v'_L < v_L \):
\[
c + (1 - \delta)v_L < c + \frac{1 - \sqrt{1 - \lambda}}{\lambda} \sqrt{\frac{t\phi}{L}},
\]
or
\[
v_L < \frac{1 - \sqrt{1 - \lambda}}{(1 - \delta)\lambda} \sqrt{\frac{t\phi}{L}} = \tilde{v}_L.
\]
The proof (deviation to \( p_b \leq v'_L \) is unprofitable) involves exactly the same algebra as the final part for the Exclusive equilibrium.

**Proof of Proposition 3**

First, consider the Internet setting. By Proposition 2, \( v_L \)-type consumers order the good on the Internet, while \( v_H \)-type consumers purchase the good from the b-firms. The surplus generated by producing and selling the good is

\[
S^{**} = L(\lambda v_H + (1 - \lambda)\delta v_L - c).
\]

Summing up the entry and travelling costs (consumers travel \( 1/(4n_b^{**}) \) on average) yields

\[
C^{**} = \frac{tL}{4n_b^{**}} + \phi n_b^{**} = \frac{tL}{4} \left( \sqrt{\frac{tL}{\phi}} \right)^{-1} + \phi \sqrt{\frac{tL}{\phi}} = \frac{5L}{4} \sqrt{\frac{t\phi}{L}}.
\]

Subtracting the costs, \( C^{**} \), from the surplus, \( S^{**} \), gives welfare under the Internet setting:

\[
W^{**} = S^{**} - C^{**} = L(\lambda v_H + (1 - \lambda)\delta v_L - c) - \frac{5L}{4} \sqrt{\frac{t\phi}{L}}.
\]

Next, consider the No-Internet setting.

(i) If \( v_L \in (c/\delta, v_L) \), the Exclusive equilibrium obtains, in which only \( v_H \)-type consumers purchase the good. Thus, the surplus generated by producing and selling the good is

\[
S^* = L\lambda(v_H - c).
\]

Summing up the entry and travelling costs yields

\[
C^* = \frac{tL}{4n_b} + \phi n_b^* = \frac{tL}{4} \left( \sqrt{\frac{tL}{\phi}} \right)^{-1} + \phi \sqrt{\frac{tL}{\phi}} = \frac{5L}{4} \sqrt{\frac{t\phi}{L}}.
\]

Subtracting the costs, \( C^* \), from the surplus, \( S^* \), gives welfare under the No-Internet setting:

\[
W^* = S^* - C^* = L\lambda(v_H - c) - \frac{5L}{4} \sqrt{\frac{t\phi}{L}}.
\]

The change in welfare is strictly positive:

\[
\Delta W = W^{**} - W^* = L(1 - \lambda)(\delta v_L - c) > 0.
\]
Suppose \( v_L \in \langle \bar{v}_L, \tilde{v}_L \rangle \). Note that Assumption 2 alone does not imply \( v_L > \tilde{v}_L \), the sign can be reverse. If \( v_L \in \langle \bar{v}_L, \min\{\tilde{v}_L, \bar{v}_L\} \rangle \), the Non-Exclusive Corner equilibrium obtains. In this case, the surplus generated by producing and selling the good is

\[
S^* = L(\lambda v_H + (1 - \lambda)v_L - c).
\]

Summing up the entry and travelling costs yields

\[
C^* = \frac{tL}{4n^*_b} + \phi n^*_b = \frac{tL}{4} \left( \frac{L}{\phi} (v_L - c) \right)^{-1} + \phi \frac{L}{\phi} (v_L - c) = \frac{t\phi}{4(v_L - c)} + L(v_L - c).
\]

Therefore,

\[
W^* = S^* - C^* = L(\lambda v_H + (1 - \lambda)v_L - c) - \left( \frac{t\phi}{4(v_L - c)} + L(v_L - c) \right),
\]

and

\[
\Delta W = W^{**} - W^* = -L(1 - \lambda)(1 - \delta)v_L - \left[ \left( \frac{t\phi}{4(v_L - c)} + L(v_L - c) \right) - \frac{5L}{4} \sqrt{\frac{t\phi}{L}} \right].
\]

The term in square brackets is strictly positive for any \( v_L \in \langle \tilde{v}_L, \min\{\bar{v}_L, \bar{v}_L\} \rangle \) (the algebra is straightforward but tedious). Thus, the change in welfare is strictly negative, \( \Delta W < 0 \).

If \( v_L \in \langle \min\{\tilde{v}_L, \bar{v}_L\}, \tilde{v}_L \rangle \), the Non-Exclusive equilibrium obtains. (The set is non-empty if and only if

\[
c < \frac{\delta(1 - \sqrt{1 - \lambda})}{(1 - \delta)\lambda} \sqrt{\frac{t\phi}{L}} - \frac{\sqrt{1 - \lambda} - (1 - \lambda)}{\lambda} \sqrt{\frac{t\phi}{L}},
\]

which is stronger than Assumption 2.) In this case, the surplus generated by producing and selling the good is

\[
S^* = L(\lambda v_H + (1 - \lambda)v_L - c).
\]

Summing up the entry and travelling costs yields

\[
C^* = \frac{tL}{4n^*_b} + \phi n^*_b = \frac{tL}{4} \left( \sqrt{\frac{tL}{\phi}} \right)^{-1} + \phi \sqrt{\frac{tL}{\phi}} = \frac{5L}{4} \sqrt{\frac{t\phi}{L}}.
\]

Therefore,

\[
W^* = S^* - C^* = L(\lambda v_H + (1 - \lambda)v_L - c) - \frac{5L}{4} \sqrt{\frac{t\phi}{L}}.
\]

The change in welfare is strictly negative:

\[
\Delta W \equiv W^{**} - W^* = -L(1 - \lambda)(1 - \delta)v_L < 0.
\]
References


