Selling Reputation
When Going out of Business†

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Abstract: Is the reputation of a firm tradeable when the previous owner has to retire? We consider a competitive market in which a share of owners must retire in each period. New owners, observing only recent profits, bid for the firms on sale. Customers are concerned with the owners’ type, which reflects the quality of the good or service provided. If a customer observes an ownership change, he may have an incentive to switch to a different firm even if his past experience was good. However, we show that, in equilibrium, customers believe that also the new owner is of the good type. Hence reputation is tradeable although ownership change is observable. In our model, reputation is an intangible asset embodied in an attractive customer base. Firms owned by a good type sell at a premium.

Keywords: reputation, ownership change, intangible asset, theory of the firm.

JEL-Classification: D40, D82, L14, L15.

This Version: March 15, 2004.

†We would like to thank Heski Bar-Isaac and Nicola Persico for helpful comments.
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1 Introduction

In many markets the customer side is only partially informed about the product’s characteristics. This typically applies to experience goods. A firm’s address is then valuable because customers, who have previously consumed a product or service provided by that firm, can easily recognize the product or service. For instance, if they learned that the product is of good quality, they can repurchase the good without risk in the future if the quality of the good cannot be downgraded over time. Suppose the owner, who is also the provider of the good or service, changes while keeping the firm’s address or its customer records, and this change of provider is observed by the customer. Then new owners are keen to inherit the reputation for e.g. good quality, reliable service, or good treatment. However, it is a priori not clear why customers should trust the new owner who may as well be of bad quality. If good addresses sell at a premium, the question is whether new good-quality providers are willing to pay more for the firm’s address than bad quality providers. In other words, the question is whether a firm’s reputation can be preserved and used by new owners. If this is the case, consumers can trust the new provider.

In this paper we consider a market in which reputation manifests itself in an attractive customer base. In other words, reputation is an intangible asset. Our main result is that there exists a positive price different from zero for the intangible asset. Our argument applies to the sale of a business when the ownership change but not the price paid by the new owner is publicly observable. Observable ownership change implies here that customers can base their decision whether to stay with the firm or to switch on the observation whether an ownership change has or has not happened. In equilibrium, customers ignore this information in effect and reputation lives on and supports a positive price for a firm with an attractive customer base. We show that new owners self-select because an attractive customer base is more valuable to new owners who do not exploit the intangible asset.

We make our argument in a competitive, infinitely lived market with an inflow and outflow of market participants on both sides of the market. To focus on the price for reputation we treat the price for market transactions as fixed.\textsuperscript{1} This is indeed often the case for services provided by doctors, dentists, pharmacists and professions such as lawyers, tax advisors and notaries. Let us now present our exemplary story: the sale of a doctor’s practice.\textsuperscript{2} Because institutional arrangements differ across countries, we do so by first collecting a number of facts for a specific country, namely Germany.\textsuperscript{3} We then spell out the setup of the model.

\textsuperscript{1}Note that this is only for the sake of the argument. Alternatively, one may introduce bargaining between customers and owners. Crucial for the argument is that one type of customers is more attractive for owners than the other (see the discussion section).

\textsuperscript{2}Mailath and Samuelson (2001) informally discuss that a doctor’s practice may sell at a higher price if its reputation is good. They also observe that their model, where ownership is unobservable, does not apply to this example.

\textsuperscript{3}A good source on the German health care system for non-German speakers is “Health Care
Patients and Practices in Germany: Some Facts. In the ambulatory health care sector, office-based for-profit physicians – we call them doctors – play a dominant role. They typically run their own practice. Therefore, a change in ownership is an abrupt change for patients. This ownership change is clearly observable to patients.

Fact 1. Ownership change is observable.

Fact 1 is the starting point of our modeling effort and the main distinguishing feature from related literature. A practice is typically sold when the old doctor retires, which can be seen as an exogenous event. A retiring doctor puts the practice for sale which is acquired by a new doctor. Among other factors, the price of the practice depends on the number and type of treatments in a particular period and on the composition of the patient base.

In Germany, health insurance is compulsory. Essentially, there exist two types of patients: those insured by a statutory sickness fund (around 51 million members plus dependents in 1999) and those insured by a private health insurance company (around 7 million fully insured in 1999). Patients with a private insurance typically have a wider coverage for treatments; in addition, for any treatment a doctor charges more if the patient has private insurance. This means that a privately insured patient is of high value to a doctor.

Fact 2. There exist two types of patients: high-value and low-value patients.

Guide books for doctors caution new doctors to pay a too high price for a large share of privately insured patients. The reason behind this warning is that although both types of patients can choose their practice, it is the group of privately insured patients which is the more mobile. In theory, the German ambulatory system is characterized by the patient’s freedom to choose and the doctor’s obligation to treat. In practice, however, some doctors reject low-value patients whereas they accept high-value patients.⁴

Fact 3. High-value patients are more likely to change practice.

A patient-doctor relationship however tends to be long-term. There does not seem to exist much market transparency, for example, no ranking or outside advice is available which would provide recommendation about the quality of a doctor. Patients therefore typically have a much better information about the quality of “their” doctor than of other doctors.

⁴This is often done in a subtle way. For instance, the next available date for a treatment of a new patient depends on whether he is privately insured or not. Also, a new patient calling up a practice is often told that unfortunately, a practice cannot take more patients. Matters change when the patient tells that he is privately insured.
**Fact 4.** Patients tend to have local information about the quality of doctors.

This fact implies that patients stay with a good doctor. The open question is whether new doctors can at least partly benefit from a large share of high-value patients of the retiring doctor. In practitioners guides for German doctors it is recognized that a practice for sale with a large share of high-value patients is potentially more profitable. Checking announcements for the sale of practices, we found that a large share of high-value patients is advertised as a selling point for practices. Hence, a retiring doctor with a large share of high-value patients can expect to receive a premium.

**Fact 5.** Practices with a large share of high-value patients are sold at a premium.

This fact suggests that new doctors must be confident to retain at least a part of them. Together with fact 4 this implies that a good doctor is more likely to enjoy a long-term profit from these patients than a bad doctor. This makes a good doctor to bid higher than a bad doctor, ceteris paribus. However, a good doctor may buy a practice with a small share of high-value patients and gradually improve his base. Since new patients arrive in each period and among experienced patients high-value patients are more likely to switch (fact 3) this makes a practice with a small share of high-value patients more profitable for a good than a bad doctor. This in effect reduces a new doctor’s willingness-to-pay for a practice with a large vis-a-vis a small share of high-value patients. Our analysis captures these two countervailing effects. Furthermore, our model accommodates the five facts stated above. Facts 3 and 5 are equilibrium outcomes. Facts 1, 2, and 4 are reflected in our model assumptions.

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**The Sale of a Doctor’s Practice: Our Model.** A doctor is interested in utilizing the full capacity of his practice and in attracting high-value patients. In our model total capacity equals demand and there are no frictions which leave some patients without treatment.\(^5\) Hence, all practices operate under full capacity and only differ in their composition of patients.

Patients can be of high or of low value to doctors. They prefer good doctors, but can tell the quality of a doctor only from experience. Patients only obtain “local” information: they learn the quality of “their” doctor but do not know the quality of the others. Clearly, if patients experience that their doctor is good and this doctor remains active in the next period, they have no reason to switch practice. If, however, patients observe an ownership change, they initially lack knowledge about the new doctor’s quality. Still, they can use their past experience with the old doctor to form beliefs about the quality of the new doctor’s quality.

Suppose that patients believe that the new doctor’s quality coincides with the old doctor’s quality. In this case a practice with a larger share of high-value patients is

\(^5\)We also shortly analyze the case that there is a single patient type and excess capacity. Our main findings are robust to this modification.
more valuable. This holds true for good and bad new doctors alike. However, the difference in profits for the two types of practices is greater for good than for bad new doctors. The reason is essentially that a bad new doctor is more likely to lose many high-value patients after one period whereas a good new doctor retains them and that building up an attractive patient base takes a lot of time for a good doctor. Consequently, good new doctors are willing to pay a higher price for a practice with a large share of high-value patients than bad new doctors.

The key prediction of the model is the self-selection of doctors. This implies that the share of high-value patients at a given practice is stable over time and little affected by ownership change. This is in line with casual observations in the German ambulatory health care sector.6

Related Literature. Our paper contributes to the literature on the firm as a bearer of reputation – this literature is reviewed in Bar-Isaac (2003). A first attempt to model reputation as a tradeable intangible asset, represented by the firm’s name, is the repeated game model by Kreps (1990). The basic idea of Kreps is that reputation can survive ownership change because owners have an interest to preserve the reputation and pass it on to new owners. A firm maintains reputation if customers believe so. However, a shortcoming of his model is that there exist multiple equilibria including equilibria in which the firm’s name has no value. In our model, such an ambiguity does not arise.

Our paper analyzes a competitive market for reputation with adverse selection. Closest in this spirit is the work by Tadelis (1999, 2003) who studies the market for the name of a firm. He also analyzes a large population of firms and customers under adverse selection. However, in his model ownership change is, at least partly, unobservable. In his model the non-observability of ownership change is a necessary condition for the market for names to be active in all periods. In addition to the observability or non-observability of ownership change, there is another important difference between Tadelis’ and our framework. In Tadelis’ work the firm’s name summarizes the reputation of a firm; this name is known to all potential customers. In our framework the name of the firm only has a meaning for the (previous) customers of that firm; it is meaningless for all other customers. Tadelis shows that since a good new owner can build up reputation on his own, whereas a bad new owner cannot, bad new owners have an interest in buying good names. In his model, this effect is strong enough to destroy the sorting of types: some bad new owners buy good names in equilibrium. In contrast, in our model types are completely sorted.7

Also related to our paper is the work on umbrella branding and brand stretching. In a contribution by Wernerfelt (1988) brand stretching works because the owner wants

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6We are, however, not aware of empirical studies addressing this issue.
7Mailath and Samuelson (2001) and Tadelis (2002) present related analyses in a moral hazard environment. They show that reputation transfer alleviates the moral hazard problem.
to avoid a reputation loss of the original product. Closer in spirit to our work is the model by Choi (1998), in which the quality of the established products is beyond doubt. Brand extension works here because an owner who uses the established products’ brand name does not want to forego the earning possibilities from that brand for future products and therefore only uses the brand name for the current new product if the quality of this product is high. Here, the argument is reminiscent of the ones by Klein and Leffler (1981) and Shapiro (1983) where the firm does not deceive customers because otherwise it would suffer a long-term loss of reputation.\footnote{See also Andersson (2002) for an analysis in a moral hazard environment.}

Finally, Cabral (2000) considers an adverse selection environment which, in some aspects, is similar to Tadelis (1999). In his model product quality is positively correlated across different products. He shows that brand stretching occurs if the quality is sufficiently high. The main difference to the work on brand stretching is that in our work, there exists an information problem between old and new owner. In particular, old owners do not know the new owner’s type. The market price for reputation then separates types. In the literature on brand stretching, there is no need to introduce a market for reputation.

With respect to consumer behavior, our setup is related to models with switching costs – for an overview of the literature see Klemperer (1995). In our model we have two kinds of switching costs. First, a customer who experiences that the firm is good is inclined to stay because he is taking a gamble when switching firm. This means that such a customer behaves as if he faces switching costs.\footnote{Fishman and Rob (2002) consider a firm’s decision to invest in quality over time, where only past realized qualities are observable. With respect to the customer behavior our models are similar. Fishman and Rob postulate that a share of customers are experienced: these are customers who learn about the past quality of one of the firms. If a reputation customer leaves that firm, he goes to a different firm at random. The remaining share of customers do not have previous knowledge, in our model these are new customers.} Note that these switching costs are endogenous and may depend on the type of customer. Second, in addition to these switching costs arising from asymmetric information, there exist exogenous switching costs which do not depend on the type of customer. This makes it costly for customers to leave the status quo independent of his and his firm’s type.

The plan of the paper is as follows. In section 2 we present the model. In section 3 we characterize an equilibrium in which there exists a positive price for the sale of an intangible asset. In this equilibrium, new owners are sorted, that is, good new owners buy firms with a high reputation and bad new owners buy firms with a low reputation. Although a good new owner can build up reputation over time on his own, this is less profitable than buying reputation. We furthermore show that there do not exist non-informative equilibria. In section 4 we discuss our results and possible model extensions. The proofs of the lemmas and the analysis under an alternative parameter constellation are relegated to the appendix.
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2 The Model

We consider a stationary environment where time is discrete, \( t \in \mathbb{Z} \). There are two groups of agents, doctors and patients. Here, doctors are the providers of a service, and patients are the customers. Doctors are of good or bad type, denoted by \( d \in \{G, B\} \). Patients are of high or of low value for a doctor, denoted by \( p \in \{H, L\} \).\(^{10}\)

**Doctors.** A fraction \( \lambda_G \) of doctors is of type \( G \), the remaining fraction \( \lambda_B = 1 - \lambda_G \) is of type \( B \).\(^{11}\) The type \( d \) of a doctor is observable to the patient after a treatment. Hence the treatment is an experience good in the extreme form that goodwill has the maximum duration of one period. Doctors provide treatments, for which they charge fixed fees \( f_p > 0 \) that depend on the patient’s type. Each doctor has a fixed capacity which, without loss of generality, is set equal to 1. The opportunity cost of providing a treatment is set equal to zero. Suppose that a doctor has a share \( \chi_d \) of high-value patients. Then his per-period profit is \( \chi_d f_H + (1 - \chi_d) f_L \).

In each period, a doctor realizes that he becomes too old to run his practice with probability \( \delta_d > 0 \), in which case he must sell and retire.\(^{12}\) Assuming stochastic independence, this results in a share of \( \delta_d \) doctors retiring in each period. In each period, a continuum of mass \( \delta_d \) of young doctors arrives, hence at each point in time there is a continuum of doctors of mass 1. Young doctors do not observe the type of the retiring doctor but learn the composition of the patients at the practice for sale. The practices then are sold at some price \( T \).

Doctors maximize the sum of the net present value of fees and the net present value of the sale of practice. There is no discounting.

**Patients.** The continuum of patients has mass ‘1 \times 1’. There is thus a continuum of mass 1 per practice. Patients frequent a doctor once in a period. Hence, the total capacity of all practices just covers the total demand for treatments. This means that even if a share of patients leaves a particular practice, that practice does not suffer from under- or overcapacity. As a result, we may focus on the composition of patients at a practice.

For both types of patients, the utility derived from a treatment of a good doctor is higher than that from that of a bad doctor, \( u_G > u_B \). A fraction \( \lambda_H \) of patients is of type \( H \), the remaining fraction \( \lambda_L = 1 - \lambda_H \) is of type \( L \). High- and low-value

\(^{10}\)Note that we use the letter \( d \) as index for doctors as well as for the doctors type. Similarly, for letter \( p \) and patients.

\(^{11}\)Greek letters are used for probabilities and rates.

\(^{12}\)Note that it can be shown that the doctor does not have an incentive to sell the practice before he reaches retirement.
patients differ in the fees $f_p$ that they pay for a treatment, $f_H > f_L$. As a special case we can consider the model with a single type of patients and excess capacity (see remark 3 below).

In each period, a patient exits with probability $\delta_p > 0$. Assuming stochastic independence, this results in a share of $\delta_p$ patients exiting in each period. In each period, a continuum of mass $\delta_p$ of new patients arrives, hence at each point in time there is a continuum of doctors of mass 1 of which $1 - \delta_p$ are experienced and $\delta_p$ are new.

Patients who do not exit may decide to switch practice. If a patient switches practice he incurs an exogenous switching cost $s > 0$. This switching cost is independent of the current practice of a patient and independent of any loss of information. For instance, when accepted at a new practice, the newly arrived patient must fill out forms, and medical records have to be transferred from the practice where the patient used to be before. Patients maximize expected utility, there is no discounting.

A share of patients $\chi_d$ at a particular practice of type $d$ are are of high value, and the remaining share $1 - \chi_d$ is of low value. In a stationary world in which the patient base does not change over time and only depends on the doctor’s type, we can describe it by $\chi_G$ and $\chi_B$, respectively. In table 1, we have summarized the notation of the model. Note that $\chi_G$, $\chi_B$ and $T(\chi_{t-1})$ are endogenous. Figure 1 displays the weight of good and bad practices and the composition of their clienteles. White areas add up to $\lambda_L$, gray areas add up to $\lambda_H$. The figure can be used to understand the streams of patients between practices.

**Time Structure.** The sequence of events is the same in each period. First, all doctors observe whether they must retire. They then put their practice for sale and reveal the composition of their patient base $\chi$ to entering young doctors. The latter make take-it-or-leave-it offers to retiring doctors, which can depend on their own

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\textsuperscript{13}Note that we model a good treatment as life-improving but not as life-prolonging.
Figure 1: Fractions of Doctors and Patients

Type \((d \in \{G, B\})\) and on the composition of the patients at the practice, \(\chi\). The bid is denoted by \(T = T(d, \chi_{t-1})\), where \(\chi_{t-1}\) contains the information a bidder can gather about the sold practice.\(^{14}\) Retiring doctors then accept the highest bid. In case of a tie between a good and a bad doctor, we use the tie-breaking rule that the doctor who would be willing to pay infinitesimally more wins and pays his bid price.

The ownership change, but not the price paid by the new doctor, is observed by the patients of that practice. Conditional on that information and the doctor’s type of the previous period and provided they do not leave the market, they choose whether to stay or switch practice. We assume that if patients change practice or newly arrive in the market, they apply at new practices with uniform distribution, knowing neither the reputation of the practice nor their chances of being accepted. Hence reputation is only local.

Remark 1 If reputation were publicly observable, then patients could coordinate on this information. This would give rise to the emergence of multiple equilibria. For example, the patients’ equilibrium beliefs may be that practices previously run by good doctors are always bought by bad doctors. With local reputation such counterintuitive equilibria do not exist.

Doctors observe the patient’s type and can accept or reject the patient. If patients switch practice, they incur a switching cost \(s > 0\), which we have introduced above. This switching cost is, for simplicity, assumed to be independent of the number of attempts made to be accepted at another practice. If the switching cost was increasing in the number of attempts this would make switching less attractive for low-value patients because, as we will show, they are rejected with positive probability, whereas high-value patients are always accepted.

Finally, once all patients are allocated to practices, each patient receives a treatment

\(^{14}\)Note that \(\chi_{t-1}\) need not necessarily be equal to \(\chi_t\), because patients may want to switch because of the ownership change.
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old doctor observes whether he must retire, announces retirement, reveals private clientele $\chi$ to young doctors
young doctor makes bid $T(d, \chi_{t-1})$ for a practice of a retiring doctor, potentially depending on his own type $d$
highest bid is accepted, ownership changes. In the case of a tie between a good and a bad doctor, one who would be willing to bid infinitesimally more wins.
patients observe whether doctor continues or ownership changes
patients of measure $\delta_p$ leave the system, patients of measure $\delta_p$ enter the system, remaining patients of measure $1 - \delta_p$ stay or switch the practice, doctors observe the type of applying patients and accept or reject application
treatment
patients observe the type of their doctor

Figure 2: Time Structure of the Game

after which he observes the doctor’s type. Figure 2 summarizes the sequence of events in one period $t$.

Strategies. Patients observe $d_{t-1}$ and whether an ownership change has happened in period $t$, where $Y$ stands for “yes” and $N$ for “no”. Hence, the patients’ information is an element of $\{G, B\} \times \{Y, N\}$. Given the information of a patient of type $p$, his strategy is to “stay”, to “switch” or to randomize between those. $\sigma_t^p(\cdot, \cdot) \in [0; 1]$ denotes the switching probability. Doctors always accept high-value patients and randomly fill up remaining capacity with low-value patients.

The strategy of retiring doctors does not need to be formalized, because they simply accept the highest offer from young doctors. The strategy of young doctors is to “make offer $T$”. Formally, $\sigma_t^H(\chi) \in \mathbb{R}_+$ and $\sigma_t^L(\chi) \in \mathbb{R}_+$.

Beliefs. Each patient must hold beliefs for three different situations that can occur: he stays and the practice does not change hands, he stays and the practice does change hands, he does not stay. First, his beliefs about the probability to be accepted by a good doctor if he stays at the practice which is not for sale is denoted by $\beta^t_H(d_{t-1}, N) \in [0; 1]$ and $\beta^t_L(d_{t-1}, N) \in [0; 1]$. Because the type is perfectly observable after a treatment, we must have

\[
\beta^t_H(d_{t-1}, N) = \beta^t_L(d_{t-1}, N) = \begin{cases} 
1 & \text{if } d_{t-1} = G \\
0 & \text{if } d_{t-1} = B 
\end{cases}.
\]

Second, each patient holds beliefs about the probability with which the next doctor will be good if the practice is sold, $\beta^t_H(d_{t-1}, Y) \in [0, 1]$ and $\beta^t_L(d_{t-1}, Y) \in [0, 1]$. If reputation transfer is successful beliefs depend on the quality of the retiring doctor.
Finally, patients hold beliefs about the probability with which they end up at a good doctor if they switch, denoted by $\beta^t_p(\emptyset, \emptyset)$ where $(\emptyset, \emptyset)$ stands for the lack of information regarding the doctor. Note that beliefs possibly depend on the patient’s type, i.e. $\beta^t_H(\emptyset, \emptyset) \neq \beta^t_L(\emptyset, \emptyset)$ because the probability to be accepted may differ between patients of different type. We will see that, in equilibrium, high-value patients are always accepted, while low-value patients are sometimes rejected by good doctors.

Retiring doctors do not need to hold any beliefs, they possess all payoff relevant information. Young doctors observe the fraction of high-value patients of the practices they bid for, $\chi_{t-1}$. However, they must form beliefs about whether high-value patients are going to stay at the practice. This in turn depends on the old doctor’s quality.\footnote{The information problem for the new doctor would not arise if either new doctors could directly observe the old doctor’s type or draw unambiguous inferences from the clientele base $\chi_{t-1}$ on the old doctor’s quality (see the discussion).}

**Equilibrium.** We refer to an equilibrium if agents maximize their utility given their beliefs while anticipating future behavior of other agents, if beliefs are confirmed by the behavior of agents, and if the market for practices clears in each period. Agents update their beliefs using Bayes rule whenever applicable. Hence in particular, equilibria are perfect Bayesian.

**The (Lack of) Information of Patients and New Doctors – Summary.** When a practice is sold, new doctors have to infer the value of a practice. To do so, they have to predict the behavior of the patients previously under treatment at the practice. Similarly, patients have to infer the value of a treatment. To do so, they have to predict the behavior of the new doctors. In the model formulated above, we made a number of assumptions on the information the different agents possess. They are summarized as follows:

- **retiring doctors**
  - do not know quality of young (bidding) doctors $d$
  - do not know next period’s composition of clientele $\chi_t$

- **young doctors**
  - do not know quality of retiring doctor
  - know composition $\chi_{t-1}$ of clientele of retiring doctor
  - do not know next period’s composition of clientele $\chi_t$

- **patients**
  - do not know quality of new doctor $d$
  - know quality of retiring doctor
3 Symmetric Stationary Equilibria

In this section we characterize stationary equilibria. In doing so, we must specify how large the fractions of good doctors and of high-value patients are. The reason is that the availability of good doctors determines the behavior of consumers. For example, if good doctors were so scarce that their was insufficient to cater to all high-value patients, low-value patients would always be rejected under perfect information – this case is analyzed in the appendix. In the main text, we will focus on the reverse situation, where high-value patients are always accepted when asking for a treatment at a particular practice. In this case, also low-value patients have a chance to be treated by a good doctor under perfect information. Because of market clearing \( \lambda_G + \lambda_B = \lambda_H + \lambda_L \) this must also hold under asymmetric information. In particular, a low-value patient who switches practice has a positive probability to be accepted by a good doctor. This implies that, in the absence of switching costs, all patients who learn that their doctor is bad and continues will change practice. Such a massive turnover of patients is avoided if moderate switching costs exist. We will describe exactly such a situation of switching costs, in which only high-value patients switch after learning that a doctor was bad. To be precise, the main point of this section is to construct an equilibrium in which there exists a strictly positive price for a practice with high reputation. In this equilibrium, good new doctors buy practices with high reputation, and bad new doctors buy practices with low reputation. We know that it is not worthwhile for a good new doctor to buy a cheap practice with low reputation and build up the reputation himself.

3.1 Informative Equilibrium

In this section we consider a particular class of equilibria which we call informative. An equilibrium is said to be informative if several properties are met, as spelled out in the definition below. We focus on parameter constellations so that patients do not switch practice unless a high-value patient learns that his doctor is bad.

Definition 1 (Informative Equilibrium) An equilibrium is called informative if it is characterized by the following observable strategies (along the equilibrium path).

1. Old low-value patients stay at their practice. Old high-value patients stay if their doctor was good, otherwise they switch.

2. New low- and new high-value patients choose their practice at random.

3. New good doctors buy from old good doctors at a high transfer price, new bad doctors buy from old bad doctors at a low transfer price.

Here, item 1 implies especially that the switching behavior of patients is independent from whether an ownership change takes place. Item 2 is obvious because new
patients do not have any information that they can base any decision on. Item 3 really characterizes the informativeness of the equilibrium: The type of the buying doctor is identical to that of the selling doctor, even though buying doctors cannot observe the type of the selling doctor. The market mechanism alone provides for perfect sorting.

**Strategies and Beliefs in the Informative Equilibrium.** We define strategies and beliefs of doctors and patients. We will show that these strategies and beliefs form an informative equilibrium for certain parameter constellations.

Possible information sets of patients are: \{ \{G, B\} \times \{Y, N\} \cup \{\emptyset, \emptyset\}\}: Either patient has experienced a good \(G\) or a bad \(B\) doctor who either keeps his practice \(N\) or retires \(Y\), or he comes to an unknown practice and knows nothing \(\{\emptyset, \emptyset\}\).

The strategies of high-value and low-value patients are given by

\[
\sigma^t_H(G, Y) = 0, \quad \sigma^t_H(G, N) = 0, \\
\sigma^t_H(B, Y) = 1, \quad \sigma^t_H(B, N) = 1, \\
\sigma^t_L(\cdot, \cdot) = 0.
\]

(1) Here, (1) means that high-value patients stay if they detect good quality, independently from an ownership change. The probability of a change is 0. (2) denotes that high-value patients switch if they detect bad quality, independently from an ownership change. The probability of a change is 1. (3) signifies that low-value patients never switch. Retiring doctors have no strategy, they simply accept the highest offer.

\[
\sigma^t_G(\chi_G) = T_{\text{min}}, \quad \sigma^t_G(\chi_B) = 0, \\
\sigma^t_B(\chi_G) = T_{\text{min}}, \quad \sigma^t_B(\chi_B) = 0,
\]

where \(T_{\text{min}} > 0\) will be defined in the following. As a tie breaking rule, if good and bad doctors bid the same price for a practice, the good doctor gets the bid.

The beliefs of patients are given by probability distributions,

\[
\beta^t_H(d_{t-1}, N) = \beta^t_L(d_{t-1}, N) = \begin{cases} 
1 & \text{if } d_{t-1} = G \\
0 & \text{if } d_{t-1} = B
\end{cases}, \\
\beta^t_H(d_{t-1}, Y) = \beta^t_L(d_{t-1}, Y) = \begin{cases} 
1 & \text{if } d_{t-1} = G \\
0 & \text{if } d_{t-1} = B
\end{cases}, \\
\beta^t_H(\emptyset, \emptyset) = \lambda_G, \\
\beta^t_L(\emptyset, \emptyset) = \lambda_G \Pr\{\text{accepted at } G|L\}.
\]

(4) (5) (6) (7)
Here, (4) means that high-value and low-value patients, if they stay at their practice and the doctor does not retire, then his type remains unchanged almost surely. (5) means that even if the doctor retires, patients believe that the new doctor will have the same type almost surely. (6) means that a switching high-value patient believes he will find a good doctor with probability $\lambda_G$, and a bad doctor otherwise. Finally, (7) means that a switching low-value patient believes he will find a good doctor only with probability $\lambda_G \Pr\{\text{accepted at } G|L\}$ (calculated in the following), and a bad doctor otherwise. Note that the patients’ beliefs depend only on the latest performance of their doctor, not on his former quality. In equilibrium, this is irrelevant. The out-of-equilibrium analysis may well depend on these beliefs.\footnote{However, one may endogenize the short memory by assuming that memorizing earlier information on quality bears an (infinitesimal) cost $\epsilon$. In this case, the patient forgets everything that dates back more than one period, because such information is irrelevant in equilibrium.}

Finally, if $\chi_{t-1} \in \{\chi_B, \chi_G\}$, beliefs of young doctors are

$$\beta^G_t(\chi_{t-1}) = \beta^B_t(\chi_{t-1}) = \chi_{t-1} \text{ with prob. } 1.$$ 

Otherwise, if $\chi_B < \chi_{t-1} < \chi_G$, beliefs are

$$\beta^G_t(\chi_{t-1}) = \beta^B_t(\chi_{t-1}) = \begin{cases} 
\chi_B & \text{with prob. } \delta_d \\
\chi_B + (1 - \delta_p) \chi_{t-1} & \text{with prob. } 1 - \delta_d 
\end{cases}.$$

Hence young doctors believe that if they buy a practice with either clientele $\chi_B$ or $\chi_G$, the clientele will stay unchanged for the next period. If the clientele is in between, they infer that they are dealing with a practice that used to be bad but has been bought by a good doctor at some point. They also infer that the practice is still possessed by a good doctor with probability $1 - \delta_d$, but is run by a bad doctor for the recent period with probability $\delta_d$. In this case, the practice is worthless, because all high-value patients will switch. We will now calculate the clientele of practices, $\chi_G$ and $\chi_B$, and the equilibrium transfer price $T_G$.

**The Clientele of Practices.** The dominant strategy for a high-value patient is to leave his doctor in case he is bad, if the probability that he will be accepted by a good doctor is positive (and if switching costs are reasonably low). In each period, the number of high-value patients that apply for places at good practices is

$$\delta_p \lambda_G \lambda_H + \chi_B (1 - \delta_p) (1 - \lambda_G) \lambda_G,$$

because of the newly born patients $\delta_p$, a fraction $\lambda_H$ is high-value, and of these a fraction $\lambda_G$ applies at a good practice. Be $\chi_B$ the equilibrium fraction of high-value patients at bad practices. A fraction $(1 - \delta_p)$ of them does not exit, a fraction $\lambda_G$ of them arrives at a good practice, and $1 - \lambda_G = \lambda_B$ is the weight for the mass of bad practices.
Because of the stationarity of equilibrium, this number must be equal to that of exiting high-value patients in good practices, \( \delta_p \chi_G \lambda_G \), where \( \chi_G \) is the fraction of high-value patients at good practices. Hence

\[
\begin{align*}
\delta_p \chi_G \lambda_G &= \delta_p \lambda_G \lambda_H + \chi_B \left(1 - \delta_p\right) \left(1 - \lambda_G\right) \lambda_G, \\
\delta_p \chi_G &= \delta_p \lambda_H + \chi_B \left(1 - \delta_p\right) \left(1 - \lambda_G\right), \\
\chi_G &= \lambda_H + \chi_B \left(1 - \lambda_G\right) \frac{1 - \delta_p}{\delta_p}.
\end{align*}
\]

The number of high-value patients at good and bad practices must sum up to the number of high-value patients in the economy, thus

\[
\chi_G \lambda_G + \chi_B \left(1 - \lambda_G\right) = \lambda_H. \tag{9}
\]

Substitution yields

\[
\begin{align*}
\lambda_H &= \left(\lambda_H + \chi_B \left(1 - \lambda_G\right) \frac{1 - \delta_p}{\delta_p}\right) \lambda_G + \chi_B \left(1 - \lambda_G\right), \\
\lambda_H &= \lambda_G \lambda_H + \chi_B \left(\lambda_G \left(1 - \lambda_G\right) \frac{1 - \delta_p}{\delta_p} + \left(1 - \lambda_G\right)\right), \\
\chi_B &= \frac{\lambda_H \left(1 - \lambda_G\right)}{(1 - \lambda_G) (1 + \lambda_G \frac{1 - \delta_p}{\delta_p})} = \frac{\delta_p \lambda_H}{\delta_p + \lambda_G \left(1 - \delta_p\right)} \tag{10}
\end{align*}
\]

Here, \( \delta_p \to 0 \) implies \( \chi_B \to 0 \): If patients never exit, all high-value patients land at a good doctor after some time. Furthermore, \( \delta_p \to 1 \) implies \( \chi_B \to \lambda_H \): If patients live for one period only, the fraction of high-value patients at a bad doctors equals the population mix. Substituting (10) into (9) yields

\[
\begin{align*}
\chi_G &= \frac{\lambda_H}{\lambda_G} - \chi_B \frac{1 - \lambda_G}{\lambda_G} = \frac{\lambda_H}{\lambda_G} - \frac{\delta_p \lambda_H}{\delta_p + \lambda_G \left(1 - \delta_p\right)} \frac{1 - \lambda_G}{\lambda_G} \\
&= \frac{\lambda_H}{\lambda_G} \left(1 - \frac{\delta_p \left(1 - \lambda_G\right)}{\delta_p + \lambda_G \left(1 - \delta_p\right)}\right) = \frac{\lambda_H}{\delta_p + \lambda_G \left(1 - \delta_p\right)} = \frac{\chi_B}{\delta_p}. \tag{11}
\end{align*}
\]

Here, \( \delta_p \to 0 \) implies \( \chi_G \to \lambda_H/\lambda_G \): If patients never exit, high-value patients are eventually shared between good doctors. Furthermore, \( \delta_p \to 1 \) implies \( \chi_G \to \lambda_H \): Analogously to bad doctors, also good doctors get only the population mix if patients live for one period only, and no information/reputation can be passed on. One can say that \( \delta_p \) is a proxy for how much information is handed down from one generation of patients to the next.

From (11), we can derive a condition that guarantees that high-value patients are always accepted from good doctors,

\[
\chi_G < 1 \iff \lambda_H < \delta_p + \lambda_G \left(1 - \delta_p\right). \tag{12}
\]

For the rest of the section, assume that (12) holds.
A Lower Bound on Switching Costs $s$. If switching costs are very low, then not only high-value patients try to switch to good practices, but also low-value patients. The only difference between the types is that high-value patients are accepted with probability 1 (their only problem is to find a good practice). Low-value patients run the risk to be rejected if they find a good practice. We now look for the minimum switching cost that guarantees no switching for low-value patients, making use of the single deviation principle.\(^{17}\) The critical switching cost is given by

$$s_L = \frac{\lambda_G}{\delta_p} \left( 1 - \frac{\lambda_H}{1 - \lambda_H} \left( 1 - \lambda_G \right) \frac{1 - \delta_p}{\delta_p + \lambda_G \left( 1 - \delta_p \right)} \right) (u_G - u_B).$$  \hspace{1cm} (13)

For details see the proof of lemma 1 in the appendix.

**Lemma 1** If $s \geq s_L$ all low-value patients stay at their old practice.

For lower switching costs a positive share of low-value patients switches after encountering a bad doctor. Below a critical value $\tilde{s}_L$ all low-value patients switch practice (see Figure 3 and Remark 2).

An Upper Bound on Switching Costs $s$. Clearly, if $s$ is sufficiently high, even high-value patients do not want to switch, despite their relatively high probability to be accepted from a good doctor. In this case reputation transfer is not possible. Denote the critical switching cost by

$$s_H = \frac{\lambda_G}{\delta_p} (u_G - u_B) > s_L.$$  \hspace{1cm} (14)

**Lemma 2** If $s \leq s_H$ all high-value patients switch practice after encountering a bad doctor.

Because we want to focus on equilibria in which low-value patients do not switch, we only consider the region $s \in [s_L, s_H]$ as defined by (13) and (13). For lower switching costs, we distinguish between two intervals. In the interval $(\tilde{s}_L, s_L)$, a positive share of low-value patients would switch practice in each period. If $s = \tilde{s}_L$, low-value patients are indifferent between switching and not even if all other low-value patients switch. In the range $[0, \tilde{s}_L]$, even all low-value patients would switch. Note that our qualitative results remain unchanged for $s < s_L$. In particular, a doctor’s profits and the decisions of high-value patients are unaffected by the switching costs $s$, as long $s < s_H$. The probabilities to switch from a bad practice are illustrated in Figure 3.

\(^{17}\)The single deviation principle implies that only one-time deviations need to be considered. Furthermore, the notion of equilibrium implies that only unilateral deviations must be taken into account.
The Separating Transfer Price $T$. Bad young doctors buy practices at a price $T_B = 0$. The reason is that they can make the retiring doctors take-it-or-leave-it offers, hence they choose the lowest possible transfer price, $T_B = 0$. If a bad practice is bought from a bad young doctor, expected profits amount to

$$
\Pi_B(\chi) = \sum_{t=0}^{\infty} (1 - \delta d)^t \left( \chi f_H + (1 - \chi) f_L \right)
$$

$$
= \chi f_H + (1 - \chi) f_L.
$$

(15)

How much does a bad young doctor gain if he bids for an old good practice? In this case, his expected profits consist of four components: First, the profits of a good practice, $\chi f_H + (1 - \chi) f_L$, are earned for one period. Second, with probability $\delta d$, the bad doctor retires after one period and is able to regain the transfer price $T_G$ before his high-value patients switch. Third, if he retires after more than one period, he gets the profits of a bad practice and a low transfer price $T_B = 0$ in the end, and fourth, he must pay the high transfer price $T_G$ right away:

$$
\Pi_B(\chi_G) = (\chi_G f_H + (1 - \chi_G) f_L) + \delta d T_G + (1 - \delta d) \frac{\chi_B f_H + (1 - \chi_B) f_L}{\delta d} - T_G.
$$

In a separating equilibrium, $\Pi_B(\chi_G) \leq \Pi_B(\chi_B)$ must hold, which can be rewritten as

$$
(1 - \delta d) T_G \geq (\chi_G - \chi_B) (f_H - f_L).
$$

(16)

One may also assume that, as an outside option, young doctors can found their own practices. This has exactly the same value as purchasing an old bad practice: If an old bad practice is bought, the high-value patients are anticipated to leave before the next treatment. Therefore in the next period, the only high-value patients are the ones just arriving. If a new practice is founded, the only high-value patients are again the ones just arriving. The proportion is identical.
Therefore,

\[
T_G \geq \frac{1}{1 - \delta_d} (\chi_G - \chi_B) (f_H - f_L)
\]

\[
= \frac{1}{1 - \delta_d} \left( \frac{\lambda_H}{\delta_p + \lambda_G (1 - \delta_p)} - \frac{\delta_p \lambda_H}{\delta_p + \lambda_G (1 - \delta_p)} \right) (f_H - f_L)
\]

\[
= \frac{1}{1 - \delta_d} \left( \frac{\lambda_H}{\delta_p + \lambda_G (1 - \delta_p)} \right) (f_H - f_L) =: T_{\text{min}},
\]

where \(T_{\text{min}}\) is the minimum transfer price that guarantees separating. Note that if \(\delta_d \to 1\), nearly all doctors retire after one period. Therefore, the probability that bad doctors can resell good practices after one period and regain the high transfer price \(T_G\) is close to 1. Only if \(T_G \to \infty\), they can be deterred from mimicking. On the other hand, if \(\delta_d \to 0\), the probability that bad doctors can regain \(T_G\) is close to 0. The transfer price that suffices to deter mimicking is only slightly higher than the one-time gains for the bad doctor, \(T_{\text{min}} \to (\chi_G - \chi_B) (f_H - f_L)\). If \(\delta_p \to 1\), nearly all patients exit after one period, therefore hardly any information is passed to the next generation, and \(T_{\text{min}} \to 0\), because good and bad practices earn the same money. If \(\delta_p \to 0\), patients never exit, and \(T_{\text{min}}\) becomes maximal, \(T_{\text{min}} = \frac{\lambda_H}{1 - \delta_d \lambda_G} (f_H - f_L)\). This analysis confirms that \(\delta_p\) contains the information permeability over time.

**The Decision Problem of a Young Good Doctor.** So far, we have examined how high the transfer price of a good practice \(T_G\) must be to deter bad doctors from buying it (i.e., \(T_{\text{min}}\)). But at this transfer price, is the purchase profitable for a good doctor? If a good doctor buys a good practice, his expected profits are:  

\[
\Pi_G(\chi_G) = \sum_{t=0}^{\infty} (1 - \delta_d)^t (\chi_G f_H + (1 - \chi_G) f_L)
\]

\[
= \frac{\chi_G f_H + (1 - \chi_G) f_L}{\delta_d}. \tag{18}
\]

**Building up Goodwill.** Instead of buying a good practice, a good doctor may buy a bad practice with clientele \(\chi_B\) and high-value patients accumulate at the practice by and by. That is, he builds up reputation over time. As we will show later, the good doctor prefers to buy a good practice at price \(T_{\text{min}}\) rather than a bad practice at price 0 if the following two properties hold: \(\chi_t \in [\chi_B, \chi_G]\) and \(T_t \in [0, T_{\text{min}}]\). We now will show that there exists a perfect Bayesian equilibrium off the equilibrium path with these properties; we show this by fully characterizing it. In period \(t = 0\), the fraction of high-value patients is \(\chi_B\). In \(t = 1\), all surviving high-value patients stay, and additionally a fraction \(\chi_B\) of high-value patients arrive.

\[\text{Note that } T_{\text{min}} \text{ does not appear in the equation, because it is spent in at the date of purchase and regained at the date of retirement with probability 1.}\]
Because every high-value patient leaves a bad practice immediately, the fraction of high-value patients at a bad practice is identical to those arriving at a practice in one period. The fraction goes up to $\chi_B \left(1 - \delta_p\right) + \chi_B$. In $t = 2$, the fraction is $\chi_B \left(1 - \delta_p\right)^2 + \chi_B \left(1 - \delta_p\right) + \chi_B$. In period $\tau$, it amounts to

$$\chi_\tau = \sum_{t=0}^{\tau} \chi_B \left(1 - \delta_p\right)^t = \chi_B \frac{1 - \left(1 - \delta_p\right)^{\tau+1}}{\delta_p}. \quad (19)$$

As $\tau \to \infty$, this fraction approaches $\chi_\infty = \chi_G = \chi_B / \delta_p$, and the good doctor has fully built up his reputation.

We now construct a perfect Bayesian equilibrium, regarding the off-equilibrium action by a good doctor buying a bad practice. The main difficulty consists in constructing a sequence $T_\tau$ which supports a perfect Bayesian equilibrium, i.e., the price a doctor gets if he sells this practice (with an accrued clientele $\chi_t < \chi_G$).

The buying doctor must consider the probability with which he buys from a good doctor. If he buys from a bad doctor, patients will leave the practice before the new doctor can convince them of his quality. The practice is hence worthless. If $\chi_t$ is observed by the new doctor, the relative probability that the old doctor is good and has built up this clientele on his own is

$$\delta_d \left(1 - \delta_d\right)^{t-1},$$

because the doctor must have “survived” for $t - 1$ periods, and retired in period $t$. The relative probability that the doctor is already bad and that the practice is hence worthless is

$$\delta_d^2 \left(1 - \delta_d\right)^{t-2}.$$

Hence no matter which $\chi_t$ a doctor observes, the probability that the current doctor is still good (thus that he can reap some of the gains from this clientele) is

$$\frac{\delta_d \left(1 - \delta_d\right)^{t-1}}{\delta_d \left(1 - \delta_d\right)^{t-1} + \delta_d^2 \left(1 - \delta_d\right)^{t-2}} = \frac{1 - \delta_d}{(1 - \delta_d) + \delta_d} = 1 - \delta_d.$$

Now the crucial question is at which transfer price $T(\chi_t)$ a practice with a fraction $\chi_t$ of high-value patients is traded. Be $T_\tau$ the transfer price that is paid for a practice with a clientele that has been built up for $\tau \geq 1$ periods, hence $T_\tau = T(\chi_\tau)$. For a bad doctor considering the purchase of a practice with some clientele $\chi_t$, we have (at the indifference point)

$$T_\tau = (1 - \delta_d) \left((\chi_\tau - \chi_B) \left(f_H - f_L\right) + \delta_d T_{\tau+1}\right). \quad (20)$$

On the one hand, the bad doctor pays the transfer price $T_\tau$, on the other hand he has a chance $1 - \delta_d$ to get a practice from a good doctor, in which case he receives higher fees for one period and can resale the practice for a high transfer price with probability $\delta_d$. 
Lemma 3 (Transfer Price for a Practice with Medium Clientele) The transfer price that a bad doctor is willing to pay for a practice with medium clientele \((\chi_\tau \in (\chi_G, \chi_B))\) is given by

\[
T_\tau = \chi_B (f_H - f_L) (1 - \delta_d) \frac{1 - \delta_p}{\delta_p} \left( \frac{1}{1 - \delta_d (1 - \delta_d)} - \frac{(1 - \delta_d)^\tau}{1 - \delta_d (1 - \delta_d) (1 - \delta_p)} \right)
\]

if \(\tau \geq 1\). However, \(T_0 = 0\).

Note that \(T_\tau\) increases with \(\tau\), and \(\lim_{\tau \to \infty} T_\tau \to T_\infty\) as derived in (4a). The reason that \(T_1 = 0\) is that if a good doctor sells directly after the first period, his clientele is still \(\chi_0 = \chi_B\). Therefore, buying doctors infer that the selling doctor is bad with probability 1. In later periods, \(\chi_t > \chi_B\), and buying doctors believe that they buy from a good doctor with positive probability. Because \(\chi_\tau\) is just a monotone function of \(\tau\), we can write \(\tau\) as a function of the clientele \(\chi\) and hence derive \(T(\chi)\) from \(T_\tau\).

From (19), we get

\[
\tau(\chi) = \frac{\log(1 - \delta_p \chi/\chi_B)}{\log(1 - \delta_p)} - 1.
\]

Substitution into (21) yields

\[
T(\chi) = \chi_B (f_H - f_L) (1 - \delta_d) \frac{1 - \delta_p}{\delta_p} \left( \frac{1}{1 - \delta_d (1 - \delta_d)} - \frac{(1 - \delta_p \chi/\chi_B)/(1 - \delta_p)}{1 - \delta_d (1 - \delta_d) (1 - \delta_p)} \right)
= (f_H - f_L) \frac{1 - \delta_d}{\delta_p} \left( \frac{\chi_B (1 - \delta_p) + \delta_p \chi - \chi_B}{1 - \delta_d (1 - \delta_d) (1 - \delta_p)} \right).
\]

Hence the transfer price \(T(\chi)\) is an affine linear function of the clientele \(\chi\) of a practice (given that \(\chi \not\in \{\chi_B, \chi_G\}\)).

Two properties of the sequence \(T_\tau\) remain to be shown. First, in order to prove that \(T_\tau\) really renders the out-of-equilibrium transfer prices for a practice that has accumulated a clientele \(\chi_\tau\), we must prove that it really is the bad doctors that buy a practice with this clientele, i.e., that good doctors prefer to buy good practices. Second, we must show that good doctors never buy bad practices, given the sequence \(T_\tau\) of expected future transfer prices. Both question can be answered at once. We prove that a good doctor would not buy a practice with clientele \(\chi_\tau\) for any \(\tau \geq 0\). Because \(T_0 = T_B = 0\), we know in particular that he would not buy a bad practice for \(T_0 = 0\). We have

\[
T_\tau - T_{\min} \leq \sum_{t=0}^{\infty} (1 + \delta_d)^t (\chi_{t+\tau} - \chi_G) (f_H - f_L) + (1 - \delta_d)^t \delta_d (T_{t+\tau} - T_{\min})
\]

which is implied by

\[
0 \leq 1 - (1 - \delta_d) (\delta_d + \delta_p - 2 \delta_d \delta_p) \quad \text{and} \quad 0 \leq 1 - \delta_d (1 - \delta_d) (1 - \delta_p).
\]

Both is true for \(\delta_d, \delta_p \in [0, 1]\).
**Sorting of Doctors’ Types.** To complete the analysis, we have to show that at the price $T_{\min}$, good doctors buy good practices, given that if they would buy bad practices, they could sell at a transfer price $T_{\tau}$. In other words, we have to check $\Pi_G(\chi_G) > \Pi_G(\chi_B)$. Because at the transfer price $T_{\min}$ the bad doctor is indifferent between buying a good and a bad practice. Keeping in mind that $T_1 = T_B = 0$, this is equivalent to showing

$$\Pi_G(\chi_G) - \Pi_B(\chi_G) > \Pi_B(\chi_B) - \Pi_B(\chi_B),$$

(23)

$$\left((f_H - f_L) \sum_{t=0}^{\infty} (1 - \delta_d)^t (\chi_G - \chi_{t+1}) + \sum_{t=0}^{\infty} (1 - \delta_d)^t \delta_d (T_{\min} - T_t)\right)$$

$$> (f_H - f_L) (\chi_G - \chi_B) + \delta_d T_{\min};$$

$$\left((f_H - f_L) \sum_{t=1}^{\infty} (1 - \delta_d)^t (\chi_G - \chi_t) + \sum_{t=1}^{\infty} (1 - \delta_d)^t \delta_d (T_{\min} - T_{t+1})\right) > 0.$$  

This is true because $T_{\min} > T_t$ and $\chi_G > \chi_t$ for all $t \geq 1$. Both $\chi_t$ and $T_t$ rise monotonously, $\chi_t$ converges against $\chi_G$, and $T_t$ is even bounded away from $T_{\min}$, cf. (5a). The intuition is that in the first period, both good and bad doctor benefit equally from buying a good practice. Henceforth, the bad doctor does not benefit any longer, whereas benefits keep accruing for the good doctor.

**The Informative Equilibrium – Main Results.** Summing up our previous analysis, we have the following results.

**Proposition 1 (Informative Equilibrium)** Suppose $\lambda_H < \lambda_G + \delta_P (1 - \lambda_G)$. If $s \in [s_L, s_H]$ then there exists an informative equilibrium.

Good new doctors pay a transfer price $T(\chi_G) = T_{\min}$, bad new doctors pay $T(\chi_B) = 0$.

**Remark 2 (Low Switching Costs)** If switching costs $s$ are low ($s < s_L$), a different sort of informative equilibrium exists (see appendix). The only difference to the equilibrium of Proposition 1 is that also low-value patients switch with a certain positive probability $\sigma_L(B, \cdot) > 0$. Their strategies have to be modified accordingly. If $s \leq \tilde{s}_L$, this probability is 1, with

$$\tilde{s}_L = \left(\frac{u_G - u_B}{\delta_p + \delta_d (1 - \delta_p)}\right).$$

A proof is given in the appendix.

**Remark 3 (Single Patient Type)** Suppose that only a single type of patients exists but that there is excess capacity in the market. This means that doctors are interested in a large patient base. A new doctor then makes his bid for a practice
dependent upon the number of treatments in the previous period. Such a situation is a special case of our model, where low-value patient are effectively ignored (by setting $f_L = 0$). In equilibrium, good and bad doctors have excess capacity but good doctors provide more treatments. Bad doctors pick up patients who after the treatment switch immediately. If there are more patients such that the parameter condition of Appendix A.2 holds good doctors operate under full capacity and only bad doctors have excess capacity.

What happens if $\lambda_H < \lambda_G + \delta_p(1 - \lambda_G)$? Then, in equilibrium low-value patients are always rejected by a good doctor and a high-value patient runs the risk of being rejected by a good doctor. This means that the calculations have to be substantially modified. The new feature under this parameter constellation is that a good doctor can build up reputation in finite time. Still, there exists an informative equilibrium so that building up reputation is not an equilibrium phenomenon. The formal analysis is relegated to section A.2 of the appendix.

### 3.2 Non-Existence of Uninformative Equilibrium

In this part of the paper we show that equilibria in which types do not sort do not exist.

**Definition 2 (Uninformative Equilibrium)** An equilibrium is called uninformative if it is characterized by the following observable strategies (along the equilibrium path).

1. Old low-value patients stay at their practice. Old high-value patients stay if their doctor was good or retires, otherwise they switch.
2. New low- and high-value patients choose their practice at random.
3. New doctors buy from old doctors at random. The type of the old doctor contains no information on the new doctor.

In comparison to the first item of definition 1, item 1 here says that high-value patients switch away from bad doctors unless ownership changes. Item 2 is the same as in definition 1. Item 3 justifies the name “uninformative equilibrium”. There is no relation between the type of the selling and that of the buying doctor.

**Proposition 2 (Uninformative Equilibria)** Assume that

$$s < (u_G - u_B) \frac{\lambda_H}{\delta_p + \delta_d (1 - \delta_p)}.$$  

Then there is no uninformative stationary equilibrium.
Intuitively, the proof can be made by contradiction: Assume there is an uninformative equilibrium. Then patients consistently have to believe that the expected quality of a new doctor is average, independently of whether the practice used to be good or bad before. The type of the retiring doctor contains no information about the type of the new doctor. Now patients always stay at good doctors, whereas high-value patients switch away from bad doctors. This implies that during their professional life, good doctors accumulate a better clientele than bad doctors. Because patients expect that their probability to get a good doctor is independent from whether they stay or switch after the retirement of their doctor, they stay because of the (possibly infinitesimal) switching costs $s$. This implies that doctors must believe that they can inherit a good clientele and turn it into profit. As in section 3.1, a high-value clientele is relatively more valuable for good doctors. As a result, good doctors bid higher transfer prices, hence patients of good practices tend to remain in the hands of good doctors. This contradicts the uninformativeness of stationary equilibrium. A formal proof follows.

Proof of Proposition 2: Assume that at a point in time, there are practices with differing clienteles $\chi_B$ and $\chi_\tau$ for sale. We show that a good doctor benefits more from buying the $\chi_\tau$-practice. Expected profits of a good doctor buying the $\chi_B$-practice are

$$\Pi_G(\chi_B) = \sum_{t=0}^{\infty} (1 - \delta_d)^t (\chi_t f_H + (1 - \chi_t) f_L) + \sum_{t=0}^{\infty} (1 - \delta_d)^t \delta_d T_{t+1},$$

where $T_1 = 0$, and $T_\tau$ increases strictly with $\tau$.

For a bad doctor, profits buying the $\chi_B$-practice are

$$\Pi_B(\chi_B) = \sum_{t=0}^{\infty} (1 - \delta_d)^t (\chi_B f_H + (1 - \chi_B) f_L),$$

whereas profits when buying the $\chi_\tau$-practice are

$$\Pi_B(\chi_\tau) = -T_\tau + \delta_d T_{\tau+1} + (\chi_t f_H + (1 - \chi_t) f_L) + \sum_{t=1}^{\infty} (1 - \delta_d)^t (\chi_B f_H + (1 - \chi_B) f_L).$$

Comparing profits, both good and bad doctors pay the same transfer price $T_\tau$ for the $\chi_\tau$-practice, their additional revenue in the first period is the same, and if they sell immediately after one period, the transfer $T_{\tau+1}$ they get is the same. Yet for the remaining time, the good doctor benefits more. Comparably to (23),

$$\left( \Pi_G(\chi_\tau) - \Pi_G(\chi_B) \right) - \left( \Pi_B(\chi_\tau) - \Pi_B(\chi_B) \right)$$

$$= \left( \sum_{t=0}^{\infty} (1 - \delta_d)^t (\chi_{t+\tau} - \chi_t) (f_H - f_L) + \sum_{t=0}^{\infty} (1 - \delta_d)^t \delta_d (T_{t+\tau+1} - T_{t+1}) \right)$$

$$- \left( (\chi_\tau - \chi_B) (f_H - f_L) + \delta_d T_{\tau+1} \right)$$

$$= \sum_{t=1}^{\infty} (1 - \delta_d)^t (\chi_{t+\tau} - \chi_t) (f_H - f_L) + \sum_{t=1}^{\infty} (1 - \delta_d)^t \delta_d (T_{t+\tau+1} - T_{t+1}), \quad (24)$$
which is strictly positive if $\chi_t$ is strictly increasing in $t$, hence if high-value patients switch from bad doctors.\footnote{The maximal switching costs $s_H$ under which high-value patients switch from bad doctors are lower in the uninformative equilibrium. The reason is that if they find a good doctor, they know that the practice stays good for sure only until the old doctor retires. Hence}

Note that the proof is even more general than Proposition 2. It shows that not only perfectly uninformative equilibria are impossible. It also demonstrates that the informative equilibrium from Proposition 1 is unique in the following sense. In a stationary equilibrium, good doctors always buy good practices, bad doctors always buy bad practices. Otherwise, both good and bad doctors would have to adopt a mixed strategy. This is impeded by (24).

In addition, the proof indicates that the informative equilibrium from Proposition 1 is even unique in a non-stationary framework. Starting from any distribution of clienteles, good doctors always buy the practices with high-value clienteles. At retirement, they leave behind a practice with even better clientele. Bad doctors buy practices with low-value clientele, and with a large probability, this clientele even breaks down to $\chi_B$. Hence from any initial distribution, we have convergence towards the informative equilibrium.

In particular, there is no equilibrium in which $T_G = T_B = 0$, in which the price of an old practice is independent from its clientele. However, if switching costs $s$ are considerably large (\((u_G - u_B) \frac{\lambda_H}{\delta_p + \delta_d (1-\delta_p)} < s < (u_G - u_B) \frac{\lambda_G}{\delta_p} \)), informative and uninformative equilibrium may coexist.

\chapter{Discussion and Conclusion}

In this paper, we presented a model of reputation as a tradeable intangible asset. We placed the model in the context of the sale of a doctor’s practice. We have shown that there exist stationary equilibria with a price difference between practices sold by good doctors and practices sold by bad doctors. In equilibrium, the two types of doctors are separated. Two properties are needed:

\[ s_H = \lambda_G (u_G - u_B) \sum_{t=0}^{\infty} (1-\delta_p)^t (1-\delta_d)^t \]

\[ = (u_G - u_B) \frac{\lambda_G}{\delta_p + \delta_d (1-\delta_p)} = (u_G - u_B) \frac{\lambda_G}{\delta_p}. \]

In any case, profits of a good doctor buying the $\chi_T$-practice are

\[ \Pi_G(\chi_T) = -T_T + \sum_{t=0}^{\infty} (1-\delta_d)^t \left( \chi_{T+t} f_H + (1 - \chi_{T+t}) f_L \right) + \sum_{t=0}^{\infty} (1-\delta_d)^t \delta_d T_{t+T+1}. \]
1. bad new doctors do not buy practices from good old doctors,

2. good new doctors do not buy practices from bad new doctors.

With respect to the first property to hold note that there exists a free-riding problem because the quality of a doctor is an experience good so that patients can leave only after experiencing that the current doctor is bad. However, if the price for a practice of a good old doctor is sufficiently high, bad new doctors do not have an incentive to free-ride on the reputation of a good old doctor. With respect to the second property, note that new doctors can save on buying a practice with a good reputation and build up their reputation on their own because over time, more and more high-value patients visit their practice and stay. Here the share of high value patients can be seen as the intangible asset: since high-value patients stay unless the treatment is bad, reputation can be traded. Indeed, if the price is sufficiently low, buying reputation is more attractive than building up reputation over time. The equilibrium price satisfies both properties.

To transfer reputation it is crucial that new doctors can distinguish between practices with a good and with a bad reputation. Although new doctors do not observe the old doctors’ types, they can make informed decisions because they observe the composition of patients within a practice – a large share of high value patients is an indicator that the old doctor is good. Otherwise, new doctors would not be able to distinguish between the different practices put for sale.

Patients have only local information about a practicing doctor so that reputation can only be local: Only patients frequenting a practice of a doctor in a particular period learn the quality of that doctor in that period. Otherwise, patients do not have any idea about the quality of other doctors. This endogenously creates switching costs (in addition to exogenous switching costs) because of quality uncertainty. In equilibrium, these switching costs are present even when patients observe that the old doctor is replaced by some new doctor. In other words, patients believe that good doctors are followed by good doctors and bad doctors are followed by bad doctors.

From a normative point of view, an uninformative equilibrium has the attractive feature that switching costs are avoided. However, switching may be socially desirable if there are efficiency gains from matching high-value patients to good doctors. This can be exemplified by the one-type model (see remark 3): The informative equilibrium corresponds to a situation in which patients are likely to visit a good doctor, \( \chi_G > \chi_B \). In contrast, fewer patients visit good doctors in an uninformative equilibrium.

Our model can be modified in several ways without affecting our main insights:

\[21\text{Other model versions could be constructed, in which the old doctor’s type is directly observable to new doctors or in which some indicators for the attractiveness of a practice are available before the sale of the practice is completed (see below).}\]
• A simpler setting would be to postulate that the type of old doctors can be observed by young doctors. In this case, new doctors perfectly learn the type of the old doctor of a particular practice also off-equilibrium. Young doctors then must infer the retiring doctor’s type. Our main insights also hold in this simpler setting. This minimizes information requirements.

• Alternatively to our model, new doctors may also observe how long the retiring doctor was in charge of the practice. This means that although young doctors do not directly observe the type of the old doctor, they know that if a doctor has attracted a high share of high-value patients and if that doctor was active for more than one period, he must be a good doctor. Again, the off-equilibrium analysis would need to be modified. Our result on reputation transfer is robust.

• Worsening the precision of information new doctors receive about a practice for sale, we may introduce a probability that the books of old doctors are cooked, so that they are uninformative with a certain probability. On the whole, our mechanism of reputation transfer is robust to such an extension.

• In our model, patients have a memory of one period only. Note that a bounded memory could be made endogenous because more memory would lead to no additional utility along the equilibrium path.

• In reality, word-of-mouth communication is likely to be relevant as a means to acquire information on doctors who are not personally known. This information typically will be noisy. Suppose that patients can get noisy signals about doctors at other practices, then the informative equilibrium survives. However, for low switching costs, also an uninformative equilibrium may coexist. In this equilibrium, high-value patients switch after ownership change to a recommended doctor, whose conditional expected quality is above average.

• Word-of-mouth communication may be predominant as a means for new patients to obtain information from experienced patients. If a share of new patients received information about the doctor’s type from old patients and if this information was reliable, we can simply relabel the groups of old and new patients by including new patients who receive information, in the group of experienced patients.

• In our analysis, we focused on the composition of patients at a practice. As we have pointed out, a special case is to consider a model in which there is an overcapacity for treatments and a single type of patients. Our result of reputation transfer can also be shown in a model with two types of patients and an overcapacity for treatments.

• Patients learn the doctor’s type after one period. We could extend the model and allow for noisy signals for patients about their doctor’s quality. For example, only certain types of treatments reveal the quality of the doctor. If the
quality is only observed with probability less than 1, one obtains a smooth deterioration of the clientele in case a bad doctor acquires a good practice. Our results remain valid.

Population shares and the price of a treatment are parameters in our model. In an extended model, these can be endogenized:

- We postulated that there exists a fixed share of good and bad doctors. The model could be extended to allow for investments in education, so that those who invested are of good quality, whereas those who do not are of bad. This investment occurs before a doctor starts a practice and must be unobservable to patients. With ex ante homogeneous doctors the cost of investment must equal the expected difference in lifetime profits for good compared to bad doctors. In a model with heterogeneous doctors this must hold for the marginal doctor.

- The analysis could also be extended to combine adverse selection and moral hazard (as e.g. in Mailath and Samuelson (2001)). A share $\lambda_G$ of doctors, the potentially good doctors, decides whether to invest in education leading to good quality, whereas a share $\lambda_B = 1 - \lambda_G$ is not able to improve quality by investing in education. Then an informative equilibrium corresponds to a situation in which potentially good doctors do invest in education. A reputation transfer alleviates the moral hazard problem. In an uninformative equilibrium, doctors have a weaker incentive to invest in education, which is an additional source for a welfare loss.

- We also postulated that there exists a fixed share of high value and low value patients. The model could be extended to allow for patients selecting the insurance contract (e.g. full insurance/partial insurance). With ex ante homogeneous patients the extra cost of full insurance must equal the expected lifetime gain from the higher probability to be treated by a good doctor. In a model with heterogeneous patients this must hold for the marginal patient.

- In our model the price of a treatment was exogenously given and larger for high value than for low value patients. This price can be endogenized by assuming that low value patients have a lower willingness to pay than high value patients. Then Nash bargaining between doctors and patients leads to a higher price

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22 This partly accommodates the view that treatments have credence goods characteristics.
23 As is well understood in the literature on network effects, there may exist situations with multiple equilibria: If all other doctors are good, a bad doctor has a very unattractive composition of patients, whereas if all others are bad, he will get the population average. This makes a new doctor more inclined to invest in education if others do.
24 The analogous remark about network effects as in the case of the endogenous number of doctors applies here.
for high value patient. Clearly, there are other ways to endogenize the price of a treatment that are consistent with our assumption that high value patients pay more than low value patients. Also recall that for our argument to work we do not need two types of patients. With one type of patients, all patients pay the same price but there exists an overcapacity so that at least bad doctors cannot operate their practice under full capacity; this is formally equivalent to low value patients paying only the cost per treatment.

While we framed the possibility of reputation transfer in the context of the sale of a doctor’s practice, our argument can be applied to a wide variety of industries. In many industries in which businesses are run by the owner, ownership changes frequently occur (and are exogenous events). Furthermore, the change in ownership of a firm is often observable to customers. Nevertheless, because the customer base is an intangible asset, reputation can be transferred at a positive price from one generation of owners to the next.

A Technical Appendix

A.1 Proofs

Proof of lemma 1: Given that low-value patients at bad doctors switch with probability $\pi_L = 0$, the number of low-value patients applying at good practices is equal to the rate of newborn patients, weighted with the fractions of good practices and low-value patients,

$$\delta_p \lambda_G \lambda_L = \delta_p \lambda_G (1 - \lambda_H).$$

The number of applying high-value patients can be taken from (8), it is

$$\delta_p \lambda_G \lambda_H + \chi_B (1 - \delta_p) (1 - \lambda_G) \lambda_G = \delta_p \lambda_G \lambda_H + \frac{\delta_p \lambda_H}{\delta_p + \lambda_G (1 - \delta_p)} (1 - \delta_p) (1 - \lambda_G) \lambda_G$$

$$= \frac{\delta_p \lambda_G \lambda_H}{\delta_p + \lambda_G (1 - \delta_p)}.$$

After all these high-value patients have been accepted, the spare capacity $\kappa_G$ of a good doctor is

$$\kappa_G = \delta_p \lambda_G - (\delta_p \lambda_G \lambda_H + \chi_B (1 - \delta_p) (1 - \lambda_G) \lambda_G)$$

$$= \delta_p \lambda_G (1 - \lambda_H) - \frac{\delta_p \lambda_H}{\delta_p + \lambda_G (1 - \delta_p)} (1 - \delta_p) (1 - \lambda_G) \lambda_G$$

(1a)
The probability for a low-value patient to be accepted after having applied at a good practice is

$$\Pr\{\text{accepted at G} | L\} = \frac{\kappa_G}{\delta_p \lambda_G (1 - \lambda_H)} = 1 - \frac{\lambda_H}{1 - \lambda_H} \frac{1 - \delta_p}{\delta_p + \lambda_G (1 - \delta_p)}.$$  \hspace{1cm} \text{(2a)}

For low-value patients, there is no incentive to try to change a bad practice if the costs of a trial $s$ are sufficiently large, $s \geq s_L := \lambda_G \Pr\{\text{accepted at G} | L\} (u_G - u_B) \sum_{t=0}^{\infty} (1 - \delta_p)^t$

$$= \frac{\lambda_G}{\delta_p} \Pr\{\text{accepted at G} | L\} (u_G - u_B).$$  \hspace{1cm} \text{(3a)}

where $\Pr\{\text{accepted at G} | L\}$ is taken from (2a).

Proof of lemma 2: (14) is structurally similar to (13). The only difference is that for a high-value patient, the probability to be accepted is 1, hence we have to set $\Pr\{\text{accepted at G} | L\} = 1$ in (13).

Proof of lemma 3: We are looking for a solution of (20), which gives us a recursive formula: We can derive $T_{\tau+1}$ from $T_\tau$. However, we would need some beginning for the recursion. Although $T(\chi_0) = T(\chi_B) = 0$, this does not serve, because (20) does not hold for the step from $T_0$ to $T_1$. The reason is that in equilibrium, a bad doctor buying a practice at $T_0 = 0$ cannot expect to sell the practice at a higher price after one period with some probability. We can analyze the property of $T_\tau$ as $\tau \to \infty$, though. Because $\chi_\tau \to \chi_G$, we get

$$T_\infty = (1 - \delta_d) ((\chi_G - \chi_B) (f_H - f_L) + \delta_d T_\infty)$$

$$T_\infty = \frac{1 - \delta_d}{1 - \delta_d (1 - \delta_d)} (\chi_G - \chi_B) (f_H - f_L).$$  \hspace{1cm} \text{(4a)}

Interestingly, $T_\infty < T_{\min}$, although the clientele after many periods is the same. This is because a doctor that buys a practice that has only infinitesimally lower than $\chi_G$ runs the risk to get a valueless practice with probability $\delta_d$. We have

$$T_\infty < T_{\min} \iff \frac{1 - \delta_d}{1 - \delta_d (1 - \delta_d)} (\chi_G - \chi_B) (f_H - f_L) < \frac{1}{1 - \delta_d} \frac{(1 - \delta_p) \lambda_H}{\delta_p + \lambda_G (1 - \delta_p)} (f_H - f_L)$$

$$1 - \delta_d (1 - \delta_d) < (1 - \delta_d)^2$$

$$1 - \delta_d + \delta_d^2 < 1 - 2 \delta_d + \delta_d^2.$$  \hspace{1cm} \text{(5a)}

We now find a solution for $T_\tau$ by an “educated” guess, derived from the following intuition. Imagine the doctors determine their $T_\tau$ “dynastywise”, i.e., in a manner...
that a doctor, instead of regarding a transfer price that he can get after one period (which is unknown) takes into account the gains a buying doctor expects after one period (which is known). In (6a), \(1 - \delta_d\) is the probability with which a practice is valuable at all, and \(\delta_d\) the probability that it is sold after one period.

\[
T_\tau = \sum_{t=0}^{\infty} (1 - \delta_d)^{t+1} \delta_d^t (\chi_{t+\tau} - \chi_B) (f_H - f_L)
\]

\[
= (f_H - f_L) \sum_{t=0}^{\infty} (1 - \delta_d)^{t+1} \delta_d^t \chi_B \left( \frac{1 - (1 - \delta_p)^{t+\tau+1}}{\delta_p} - 1 \right)
\]

\[
= \chi_B (f_H - f_L) \sum_{t=0}^{\infty} (1 - \delta_d)^{t+1} \delta_d^t \left( (1 - \delta_p) - (1 - \delta_p)^{t+\tau+1} \right)
\]

\[
= \chi_B (f_H - f_L) (1 - \delta_d) \frac{1 - \delta_p}{\delta_p} \sum_{t=0}^{\infty} (1 - \delta_d)^{t+1} \delta_d^t \left( 1 - (1 - \delta_p)^{t+\tau} \right)
\]

From (6a), one can see directly that the recursion equation (20) is satisfied. Dividing (20) by \((f_H - f_L)\) yields

\[
\sum_{t=0}^{\infty} (1 - \delta_d)^{t+1} \delta_d^t (\chi_{t+\tau} - \chi_B)
\]

\[
= (\chi_{t+\tau} - \chi_B) (1 - \delta_d) + (1 - \delta_d) \delta_d \sum_{t=0}^{\infty} (1 - \delta_d)^{t+1} \delta_d^t (\chi_{t+\tau+1} - \chi_B)
\]

\[
= (\chi_{t+\tau} - \chi_B) (1 - \delta_d) + (1 - \delta_d) \delta_d \sum_{t=1}^{\infty} (1 - \delta_d)^{t} \delta_d^{t-1} (\chi_{t+\tau} - \chi_B)
\]

\[
= (\chi_{t+\tau} - \chi_B) (1 - \delta_d) + \sum_{t=1}^{\infty} (1 - \delta_d)^{t+1} \delta_d^t (\chi_{t+\tau} - \chi_B) \iff (1 - \delta_d) (\chi_{\tau} - \chi_B) = (\chi_{t+\tau} - \chi_B) (1 - \delta_d).
\]

This completes the proof. \(\Box\)

**Proof** of remark 2: We first calculate the probability \(\pi_L\) with which a low-value patients switches away from a bad practice in equilibrium.

In analogy to (8), the number of low-value patients arriving at good practices is

\[
\delta_p \lambda_G (1 - \lambda_H) + (1 - \chi_B) (1 - \delta_p) (1 - \lambda_G) \lambda_G \pi_L.
\]

The spare capacity of good doctors after all arriving high-value patients have been accepted is given in (1a), it is independent from how many low-value patients arrive.
Hence for a low-value patient, the probability to be accepted after having applied at a good practice is

$$\Pr\{\text{accepted at } G|L\} = \frac{\kappa_G}{\delta_p \lambda_G (1 - \lambda_H) + (1 - \chi_B) (1 - \delta_p) (1 - \lambda_G) \lambda_G \pi_L}. $$

In equilibrium, the low-value patient must be indifferent between switching or not, hence necessarily

$$s = \frac{\lambda_G}{\delta_p} \Pr\{\text{accepted at } G|L\} (u_G - u_B).$$

When setting $\pi_L = 0$, we receive $s_L$, which obviously is equal to that calculated in (3a). When setting $\pi_L = 1$, we get $\tilde{s}_L$. Because $\Pr\{\text{accepted at } G|L\}$ rises with $\pi_L$, it is clear that $s_L > \tilde{s}_L$. Furthermore, one can easily prove that $\tilde{s}_L > 0$ iff $\lambda_H < \delta_p + \lambda_G (1 - \delta_p)$, which is true because of (12).

We now argue why we obtain an informative equilibrium. For doctors and high-value patients, expected profits are independent from how many low-value patients try to switch practices. Therefore, also the clientele of practices remain the same. Therefore, all three properties of an informative equilibrium (definition 1) remain unchanged. □

### A.2 More High-Value Patients than Good Doctors

Because there are more high-value patients than good doctors, good practices may possibly be filled with high-value patients. In this case, it is no longer optimal for high-value patients at bad doctors always to switch practice – if their probability to be accepted at a good practice is close to zero, they should rather stay. In the following, we analyze a mixed equilibrium in which high-value patients at bad practices switch with probability $\pi_H$.

**The Clientele of Practices.** First, note that in a mixed stationary equilibrium, good doctors treat only high-value patients. If they treated also low-value patients, this would mean that they had spare capacity for more high-value patients, thus a high-value patient would be accepted at a good practice with certainty. Consequently, $\chi_G = 1$, and from the market clearing condition (analogously to (9)), we receive

$$\lambda_H = \lambda_G + \chi_B (1 - \lambda_G) \iff \chi_B = \frac{\lambda_H - \lambda_G}{1 - \lambda_G}. \tag{7a}$$
Boundaries on Switching Costs $s$. Like in section 3.1, we have qualitatively different switching behavior for different values of $s$. For very large $s$ (if $s \geq \tilde{s}_H$, where $\tilde{s}_H$ is to be specified), neither type of patients switches. For smaller $s$ (if $s \in (s_H, \tilde{s}_H)$), low-value patients never switch, and high-value patients adopt a mixed strategy, switching away from bad doctors with probability $\pi_H$. For $s < s_H$, high-value patients always switch away from bad doctors. Even for infinitesimal $s$, low-value patients do not attempt to switch from bad doctors, because their probability to be accepted from a good doctor is zero.

The number of good patients that switch practice is $(1 - \delta_p) \lambda_B \chi_B \pi_H$, as $1 - \delta_p$ is the fraction of surviving patients, $\lambda_B$ is the fraction of bad practices, $\chi_B$ the fraction of high-value patients. Additionally, there are $\delta_p \lambda_H$ newborn patients looking for practices. The total number of high-value patients on search is thus

$$\delta_p \lambda_H + (1 - \delta_p) \lambda_B \chi_B \pi_H$$

$$= \delta_p \lambda_H + (1 - \delta_p) (1 - \lambda_G) \frac{\lambda_H - \lambda_G}{1 - \lambda_G} \pi_H$$

$$= \delta_p \lambda_H + (1 - \delta_p) (\lambda_H - \lambda_G) \pi_H.$$

(8a)

Each patient catches a good practice only with probability $\lambda_G$, and this good practice has free capacity $\delta_p$, the number of patients that have exited recently. Hence the probability to be accepted after having applied at a good practice is

$$\Pr\{\text{accepted at G} | H\} = \frac{\delta_p}{\delta_p \lambda_H + (1 - \delta_p) (\lambda_H - \lambda_G) \pi_H}.$$  

(9a)

In equilibrium, expected utilities must coincide, hence

$$s = (u_G - u_B) \Pr\{\text{accepted at G} | H\} \sum_{t=0}^{\infty} (1 - \delta_p)^t$$

$$= (u_G - u_B) \frac{\delta_p}{\delta_p \lambda_H + (1 - \delta_p) (\lambda_H - \lambda_G) \pi_H} \frac{1}{\delta_p}$$

$$= \frac{u_G - u_B}{\delta_p \lambda_H + (1 - \delta_p) (\lambda_H - \lambda_G) \pi_H} \Rightarrow$$

$$\pi_H^* = \frac{(u_G - u_B) / s - \delta_p \lambda_H}{(1 - \delta_p) (\lambda_H - \lambda_G)}.$$ 

(10a)

We can now derive the critical $s_H$ (for which $\pi_H^* = 1$) and $\tilde{s}_H$ (for which $\pi_H^* = 0$),

$$\pi_H^* = 1 \Rightarrow s_H = \frac{u_G - u_B}{\lambda_G},$$

$$\pi_H^* = 0 \Rightarrow \tilde{s}_H = \frac{u_G - u_B}{\delta_p \lambda_H}.$$ 

Again, we concentrate on pure equilibria, hence we assume that $s < s_H$, hence high-value patients always switch away from bad practices.
The Separating Transfer Price $T$. In complete analogy to (16), a transfer price $T_G$ that guarantees a separating equilibrium must satisfy

$$T_G \geq \frac{1}{1-\delta_d} (\chi_G - \chi_B) (f_H - f_L)$$

$$= \frac{1}{1-\delta_d} (1 - \frac{\lambda_H - \lambda_G}{1 - \lambda_G}) (f_H - f_L)$$

$$= \frac{1}{1-\delta_d} \frac{1-\lambda_H}{1-\lambda_G} (f_H - f_L) =: T_{\text{min}}. \quad (11a)$$

Building up Goodwill. We now consider the case that a good doctor buys a bad practice. The issue now becomes even more involved than in section 3, because a good doctor will reach a clientele of 100% high-value patients already after finite time. Integer problems arise.

In $t = 0$, the fraction of high-value patients is $\chi_B$. In $t = 1$, it goes up to $\chi_B (1 - \delta_p) + \chi_B$. In $t = 2$, it is then $\chi_B (1 - \delta_p)^2 + \chi_B (1 - \delta_p) + \chi_B$. In period $\tau$, it amounts to

$$\chi_{\tau} = \min \left\{ 1, \chi_B \sum_{t=0}^{\tau} (1 - \delta_p)^t \right\}$$

$$= \min \left\{ 1, \frac{\lambda_H - \lambda_G}{1 - \lambda_G} \frac{1 - (1 - \delta_p)^{\tau+1}}{\delta_p} \right\}.$$ 

One can see immediately that $\chi_{\tau}$ reaches 1 in finite time iff

$$\frac{\lambda_H - \lambda_G}{1 - \lambda_G} \frac{1}{\delta_p} > 1,$$

$$\lambda_H > \delta_p + \lambda_G (1 - \delta_p).$$
We again found (12), using the opposite approach. In this case, a good doctor has regained 100% high-value patients after 

\[
\tau_1 = \left\lceil \frac{\log(1 - \delta_p/\chi_B)}{\log(1 - \delta_p)} - 1 \right\rceil
\]

\[
= \left\lceil \frac{\log \left(1 - \delta_p \frac{1 - \lambda_G}{\lambda_H - \lambda_G} \right)}{\log(1 - \delta_p)} \right\rceil - 1,
\]

where \([\cdot]\) stands for rounded up numbers.

Because of the integer problem, further calculations with \(\chi\) become rather tedious. All we have to show is that for a good doctor, buying a bad practice and building up reputation is inferior to buying a good practice right away. We can follow the same approach as for Proposition 2. If \(\chi_t\) and \(T_t\) increase with time, the benefit of good young doctors from buying a good practice is higher than that of bad doctors (analogously to (24), bearing in mind that sums end at time \(\tau_1\)). However, there is one additional prerequisite: If there are abundantly many high-value patients, practices of good doctors reach the maximal clientele \(\chi = 1\) already after one period \((\tau_1 = 1)\). In this case, benefits from buying a good practice are the same for both types of young doctors. We have

\[
\tau_1 > 1 \iff \left\lceil \frac{\log \left(1 - \delta_p \frac{1 - \lambda_G}{\lambda_H - \lambda_G} \right)}{\log(1 - \delta_p)} \right\rceil > 2 \\
\log \left(1 - \delta_p \frac{1 - \lambda_G}{\lambda_H - \lambda_G} \right) > 2 \log(1 - \delta_p) \\
1 - \delta_p \frac{1 - \lambda_G}{\lambda_H - \lambda_G} > (1 - \delta_p)^2 = 1 - 2\delta_p + \delta_p^2 \\
\lambda_H < \frac{1 + (1 - \delta_p) \lambda_G}{2 - \delta_p}.
\]

Resuming, we have the following remark.

**Remark 4 (Informative Equilibrium)** Assume that

\[
\lambda_H < \frac{1 + (1 - \delta_p) \lambda_G}{2 - \delta_p}.
\]

Then an informative equilibrium as described in Proposition 1 exists.

Again, for relatively low switching costs, there is no uninformative equilibrium.
References


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