Tariff Protection, Intellectual Property Rights and North-South Trade with Perfect Price Flexibility

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Abstract

We study a North-South trade situation in which prices are perfectly flexible and knowledge spills over from Northern to Southern firms. In a partial equilibrium setup we analyse the role of tariffs in preventing technological leakages and their impact on domestic (Northern) consumers and producers. We show that, unlike in conventional results, tariff protection preserves or raises consumer’s welfare relative to free trade. Moreover, tariff protection keeps foreign (Southern) firms out of the market, and therefore, R&D becomes appropriable. Even though the innovation level remains the same as under free trade, total industry efficiency increases.

Keywords: tariff protection, intellectual property rights, consumer’s welfare

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1. INTRODUCTION

Markets in which firms actively innovate are in general imperfectly competitive. Since R&D investments are in fact fixed costs, only in concentrated, imperfectly competitive markets firms can recoup these costs. In addition, R&D intensive markets are prone to public policy intervention as, from the social point of view, two major problems arise in such markets. First, because firms undervalue the social surplus created by R&D investments, they tend to undertake lower investment in innovation than it is socially optimal. Second, the newly created knowledge and technology can easily spill over into the public domain. When R&D is not appropriable, the innovation is reduced further down from its socially optimal value (see Arrow, 1962, for a discussion of these two problems).

Governments can restore, at least partially, the appropriability of innovation by introducing a strong intellectual property rights (IPR) regulation. This is usually the case in developed (henceforth called North) countries. However, if foreign firms export their products into the domestic market, domestic IPR protection might be ineffective. Firms originating from countries with a loose IPR regulation, as is often the case with developing (henceforth called South) countries, might decipher the technology incorporated in their rivals’ product and adopt it without facing any punishment. Then governments have to implement additional policies in order to restore the R&D appropriability in a North-South trade situation. As Žigić (2000) has shown, one such policy is tariff protection. Even though the introduction of tariff barriers is highly discouraged by WTO, when domestic IPR regulation is infringed by foreign firms, governments are allowed to set “punitive” tariffs.

Most studies that assess the impact of government policies on market structure and social welfare in imperfectly competitive markets assume that firms interact either in Bertrand or Cournot manner. We know very little about the impact of tariff protection under different market conducts. However, in some markets, the pricing or the production strategies might be different than those arising from Cournot or Bertrand interaction. For example in some markets sellers can change a quoted price directly after a price cut by some competitor (Farm and Weibull, 1987). In these cases a dynamic price adjustment occurs before products are sold in the market. Such situations were present in the corn market and nowadays, to some extent, occur in conventional or internet retail markets.

This paper attempts to fill this gap in the literature by studying a North-South trade situation in which prices in the North market are flexible and knowledge spill over from North to South firms. We concentrate our attention on only one industry from the whole North
economy and therefore we do only a partial equilibrium analysis. In this context, we analyse the role of tariff in preventing the technological leakages and we assess its impact on domestic firms and consumers, and on the overall domestic social welfare.

Even though sellers react only with some delay to certain prices and have to incur some adjustment costs, we study a perfect price flexibility situation in which price changes are costless and instantaneous. By studying such a limiting case, we can isolate the effect of tariff policy on social welfare, consumer and producer surpluses from the impact of adjustment costs and price stickiness. We model perfect price flexibility by relying on the auction-based decision process over prices developed by Boone (2002). In Boone’s set-up firms in the market can instantaneously react to an existing price by undercutting it or leaving the market. Only after no remaining firm has an incentive to decrease the price, production and consumption occur. In this set-up the toughness of the competition is driven by the cost distribution of firms in the market. More precisely, when firms are similar undercutting the market price so as to drive rivals out of the market is very costly. Therefore it is more profitable for firms to behave “softly” towards their rivals and bid high prices. However, if some firms are much more efficient than their competitors, it might pay off to price aggressively in order to keep the less efficient firms out of the market.¹

We show that unlike in perfectly competitive environments or in imperfectly competitive ones in which firms compete à la Cournot or Bertrand, when prices are perfectly flexible, the optimal tariff protection does not lead to any consumption distortion. On the contrary, the optimal tariff preserves or raises consumer’s welfare relative to the free trade situation. Due to tariffs, the cost differential between domestic and foreign firms increases, triggering a more aggressive pricing strategy.² Consequently, consumers benefit from tariff protection. A similar result was previously obtained by Kabiraj and Marjit (2003). However, Kabiraj and Marjit have studied a North-South trade situation in which IPR was not an issue and moreover, the target market was located in the South country. In their model, the augment in consumer surplus occurs when tariff protection induces foreign firms to transfer its superior technology to its domestic rival. On the contrary, in our model tariff protection is a policy aiming to prevent technological transfer.

Under a tariff protection policy, domestic producers invest in innovation as much as under free trade and get the same level of profit as in the case of free trade. This is because

¹ Farm and Weibull (1987) have also modelled a perfect price flexibility situation. However, in their model all firms are symmetric. This assumption is not suitable for a context in which innovation and spillovers occur.
² In different contexts, Jones and Takemori (1989), Helpman and Krugman (1989), and Rosenthal (1980) have also showed that tariffs may have a favourable price effect.
their aggressive pricing strategy drives the foreign firms out of the market. The gain in market share associated with entry deterrence exactly offsets the negative effect that a low price has on domestic firms’ price mark-up, and through it, on profits. Since only the low cost firms end up producing in the market, total efficiency in the industry increases.

The paper is organized in the following way. First we introduce the core model and discuss the role of government intervention. Next, we describe the pricing game and its equilibrium. Then in Section 4 we solve the model for the case when there is no government intervention in the market. We use this free trade case as a benchmark for assessing the performance of the trade protection policy. The optimal tariff protection and the characteristics of the market equilibrium under government intervention are derived in Section 5. In Section 6 we compare the social welfare under tariff protection and free trade. Main findings and conclusions are presented in Section 7.

2. The Set-up

2.1. Supply, demand and the pricing game

Consider an industry in which $N$ “domestic” firms located in the North and $n$ “foreign” firms located in the South produce a homogenous good. Initially, all firms hold the same technology and can produce one unit of good at the same constant cost, $\alpha \geq 0$. However, domestic firms possess enough knowledge and resources to conduct process R&D activities. Innovation has an impact on the production efficiency rather than on the type or on the quality of the provided goods.\(^3\) To get an $x$ decrease in its marginal cost, a domestic firm must invest $i(x)$ in R&D, where $i(0) = i'(0) = 0$ and $i''(x) > 0$, $i'''(x) \geq 0$ on $(0, \alpha)$. Any attempt to decrease the marginal cost to less than 0 brings the R&D cost to $\infty$. We assume that there is no “lick” of information between domestic firms or, alternatively, the IPR are perfectly enforced in the domestic country. Also, we assume that all domestic firms have the same knowledge and capabilities to innovate, so they are symmetric even in this respect. Foreign firms have no capabilities to innovate on their own. Yet, they can imperfectly imitate the innovation of domestic firms and adapt it to their production process. The extent to which this imitation can occur is capture by the spillover parameter $\beta \in [0,1]$.

We focus on the interaction between domestic and foreign firms that take place in the

\(^3\)Spence (1984) has noticed that, “if one thinks of the product as the services it delivers to the customer (in the way that Lancaster pioneered), then product development often is just cost reduction” (p.101).
domestic (North) market.\textsuperscript{4} We follow Boone’s (2002) approach in modelling this interaction and domestic market conditions.

Domestic demand in this industry is $D(p)$, where $D()$ is a continuous and decreasing function of the market price, $p$. We assume that for any $x \in [0, \alpha]$, $D(p) \ (p - \alpha + x)$ is concave in $p \in \mathbb{R}^+$, and that

$$\alpha \leq \text{argmax}_p \ \{D(p) \ (p - \alpha)\}, \ \forall \ x \in [0, \alpha]. \quad \text{(a1)}$$

Consequently, in this market, a monopolist that has a marginal cost of production $\alpha - x$ gets positive profits and has a concave profit function. Further, we assume that prices, $p$, the marginal cost, $\alpha$, the cost reductions due to R&D activities, $x$, and the spillover parameter, $\beta$, belong to the denumerable set of rational numbers, $\mathbb{Q}$. This assumption ensures that the pricing game considered below is finite.

The domestic market price is established after repeating rounds of bidding. More precisely, the bidding process starts at $p^0 = \text{argmax}_p \ \{D(p) \ (p - \alpha)\}$ with all $N$ domestic firms and all $n$ foreign firms participating in the first bid. Each round $s$ is defined by the numbers of players still in the game, $N^s$ and $n^s$ respectively, and the current price, $p^s$. A round has two stages. First, the firms still in the game bid new prices that either undercut or preserve the current price. The resulting new price, $p^{s+1}$, equals the lowest bid. Second, the firms that do not want to follow the price $p^{s+1}$ can leave the game. The bidding rounds continue until a round $s^*$ in which there is no more undercutting is reached. Then, the market price equals $p^* = p^{s^*}$ and the number of firms is $N^* = N^{s^*}$ in the domestic country and $n^* = n^{s^*}$ in the foreign country. Once the market price is set, each firm that remained in the market has to produce $\frac{D(p^*)}{N^* + n^*}$ units of homogenous good and gets a $\frac{D(p^*)}{N^* + n^*} \ (p^* - c_f)$ payoff, where $c_f$ denotes firm’s unit cost.\textsuperscript{5} Firms that have already left the market do not produce and get zero profits. These rules regarding the distribution of production and profits ensure that the bidding process does not become a “cheap talk”.

\textbf{2.2. The role of the domestic government (the timing of the game)}

A value of the spillover parameter, $\beta$, bigger than 0 indicates a violation of IPR. In order to restrain the leakage of information from domestic to foreign firms and to enhance the

\textsuperscript{4} We have in mind the case when the domestic economy is a developed, North economy, while the foreign country is a South economy. We further consider that the size of the market in the developing country is negligible compared with the market size of the North country. Consequently, the decisions of North firms are mostly based on the interactions that take place in their domestic markets.

\textsuperscript{5} One could imagine that, since all firms deliver the same product at the same price, consumers pick randomly their provider.
social welfare, the government from the North considers setting a tariff, $t$. Domestic government’s decision is based on a social welfare function that assigns equal weights to consumer surplus, domestic profits, and tariff revenue. We assume that this decision follows domestic firms’ R&D investment stage, but precedes the pricing game. Carmichael (1987) has notice that in some circumstances governments set the level of their policies only after domestic firms have chosen the level of some strategic variable. If to accommodate this observation in our model we would also have to consider the case when the tariff protection level is set before domestic firms invest in R&D. However, tariffs that are introduced with the aim of reducing knowledge leakages are usually set only after the innovation is already in place.

Foreign firms might avoid paying tariff duties by establishing a plant in the domestic country (Motta, 1992). However, since in the North country IPRs are perfectly enforced, besides paying set-up costs associated with direct investment, foreign firms that establish subsidiaries in the domestic country have to forgo the benefits associated with spillover. In what follows, we exclude foreign direct investment from our analysis by assuming that entry costs together with the negative effect of a strong IPR protection are high enough to prevent foreign firms to undertake direct investment. This assumption is supported by stylised facts: less than 5% of the developed countries inward foreign direct investment stocks originate in developing countries (own calculations based on reports from OECD, UNCTAD, the U.S. Bureau of Economic Analysis, and South African Reserve Bank). Given these considerations, the sequence of actions in our model is the following. First to move are domestic firms by investing in R&D. Next, domestic government announces the tariff level. At the end the market price is set and production ensues.

The above specifications regarding tariff protection are consistent with “punitive” tariffs. According to the WTO rules when IPRs are violated, a government might introduce a “punitive” tariff in order to alleviate the harm that IPR infringement have on the domestic country (for more on punitive tariffs see Žigić, 2000). As such tariffs are enacted only after a leakage in technology or knowledge has occurred, firms invest in R&D before the domestic government announces the level of tariff protection. In addition, given that the reasoning and the behaviour of the government is monitored by its trading partners and by WTO, a government that sets a “punitive” tariff is likely to behave as a benevolent one.

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6 When taking their decision on the optimal innovation level, Northern firms might already foresee that Southern firms will imitate their technology and that this will trigger a “punitive” action from their government. Therefore, even in such case one might consider that Northern firms’ decision on the R&D takes into account that government will interfere in the market by setting a tariff protection policy.
To assess the efficiency of tariff policy in protecting IPRs in the domestic country, we compare the government intervention case with the outcome that free trade policy yields, therefore using this latter case as a benchmark. For each of these two games we look for its symmetric subgame perfect equilibrium (SPE) – the equilibrium in which all domestic firms choose the same investment level. Consequently, we solve these games backwards, starting with their last stage. Since regardless of the set-up, the last stage is always the pricing game, first we derive the market price and the corresponding profits for a given level of tariff and R&D, and then we close each case by identifying their overall equilibrium; first we discuss the benchmark case – free trade – then we continue with the government intervention case.

In order to simplify the analytical analysis of the game, we consider that the market demand is linear, namely $D(p) = A - p$. The assumption (a1) states that regardless the level of innovation, a monopolist makes always positive profits in this market. Accordingly, the initial marginal cost $\alpha$ should be smaller than the market size $A$.

3. Pricing Game – Its Equilibrium

When the pricing game is reached, the domestic government has already announced the tariff protection level $t = 0$ for free trade, domestic firms have already invested in R&D $i(x)$ and therefore their marginal cost equals $C = \alpha - x$, and foreign firms have imitated the foreign technology, so they have reached a marginal cost of $\alpha - \beta x$. However, for each unit of good exported to the domestic country foreign firms have to pay a tariff $t$, so each of these firms has a total unit cost $c = (\alpha - \beta x) + t$. Given that the spillover parameter, $\beta$, is never higher than 1, domestic producers can deliver the homogenous good at a lower, at the most equal cost compared with the foreign firms. Then domestic firms’ price level that maximizes their profits is smaller than the corresponding price for the foreign firms (all proofs are consigned to the Appendix):

**Lemma 1.** Let $P^* = \arg \max_p \frac{D(p)}{N(p) + n(p)}(p - C)$ and $P^* = \arg \max_p \frac{D(p)}{N(p) + n(p)}(p - c)$ where $N(p) = \begin{cases} N, & C \leq p \\ 0, & C > p \end{cases}$ and $n(p) = \begin{cases} n, & c \leq p \\ 0, & c > p \end{cases}$ Then $P^* \leq P^*$.

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7 We assume that a unilaterally deviation in the innovation level from the equilibrium value, is not profitable. As the only reasoning for deviating from the equilibrium R&D level would be to deter the entry of all firms, foreign as well as domestic, when the number domestic firms, $N$, is low enough and the innovation is expensive, such deviation is not profitable.
Firms that are more efficient have a higher price mark-up and therefore benefit more than less efficient firms from the increase in output that follows a decline in price.

We can now characterize the SPE for this pricing game:

**Theorem 1.** Assume that firms do not play weakly dominated strategies. Then the pricing game has a unique SPE in pure strategies,

\((P^*, N(P^*), n(P^*))\).

In any round \(s\) of the pricing game, the domestic firms choose their prices according to the following rules:

- if \(p^s > P^*\) => bid \(P^*\)
- if \(p^s \leq P^*\) => do not undercut \(p^*\)
- if \(p^s > C\) => stay in the game
- if \(p^s \leq C\) => leave the game

while the foreign firms act as follows:

- if \(p^s > p^*\) => bid \(p^*\)
- if \(p^s \leq p^*\) => do not undercut \(p^*\)
- if \(p^s > c\) => stay in the game
- if \(p^s \leq c\) => leave the game.

Thus, no firm has any incentive to undercut the optimum price of the firms that have the lowest unit cost and therefore these most efficient firms impose their optimum price as the market price. From now on we refer to the most efficient firms, in our case to the domestic firms, as price leaders. The notion of price leadership that we use in this pricing game is different than that used in Stakelberg games, where the price leader is the first to commit to a price. On the contrary, here, the possible reaction of the price leaders to undercutting from \(P^*\) ensures their position.

By replacing the actual demand function in the formula for \(P^*\) (see Lemma 1), we can further describe the SPE defined in the above theorem. In the case of linear demand, the monopolist price is \(P^m = \arg\max_p \{D(p) (p - C)\} = \frac{4ac}{\pi}\). Depending on the relationship between \(P^*, c\) and \(P^m\) three different situations can occur: domestic firms might be unconstrained or constrained “monopolists” in the market thus preventing the participation of foreign firms into the market, or they might accommodate the entry of their foreign rivals. First, if
domestic firms are unconstrained “monopolists”, the SPE is given by $\left(\frac{d\alpha c}{2}, N, 0\right)$, and each domestic firm gets a profit

$$\pi^m(t, x) = \frac{(A - C)^2}{4N} - i(x).$$  \hspace{1cm} (2)

Due to the assumption (a1), for high levels of spillovers ($\beta > \frac{1}{2}$) the condition (1) cannot hold unless the government sets a high enough level of tariff protection $(\frac{1}{2}(A - \alpha - (1 - 2\beta)x) > 0)$.

Yet, for low level of spillovers ($\beta < \frac{1}{2}$) domestic firms can on their own drive foreign firms out of the market by investing enough resources in R&D. In this case, when the innovation effort increases the cost efficiency gap between domestic and foreign firms builds up while the monopoly price, $P^m$, decreases. If the soar in innovation is high enough, the monopoly price declines to a level that is lower than foreign firms’ marginal cost. However, such a case can occur only when through their R&D effort domestic firms get a high cost advantage with respect to their foreign competitors. This can happen only when all firms are initially highly inefficient so that $2(1-\beta)\alpha > A$. If this condition is fulfilled, in the absence of a tariff protection, domestic firms can impede on their own IPR violation by deterring foreign entry.

We rule out this situation by refining the assumption (a1) to

$$A > \max\{\alpha, 2(1-\beta)\alpha\}. \hspace{1cm} (a2)$$

Given the assumption (a2), if the tariff level is not high enough condition (1) does not hold and domestic firms, as price leaders, have two alternatives. First, is to price at $c$ and drive foreign firms out of the market. In this case each domestic firm gets a profit of

$$\pi^{cm}(t, x) = \frac{(A - c)(c - C)}{N} - i(x).$$  \hspace{1cm} (3)

Second, domestic firms can price at $P^m$, and thus allow for foreign competition. Then each domestic and each foreign firm gets a

$$\pi^{c,d}(t, x) = \frac{(A - C)^2}{4(N + n)} - i(x), \hspace{1cm} (4)$$

$$\pi^{c,f}(t, x) = \frac{(A - C)(A + C - 2c)}{4(N + n)} \hspace{1cm} (4')$$

profit, respectively. When

$$4(N + n)(A - c)(c - C) \geq N(A - C)^2$$  \hspace{1cm} (5)
domestic firms get a higher profit when they price at \( c \). In this case the SPE is \((c, N, 0)\) and domestic firms are constrained “monopolists”. If condition (5) does not hold, domestic firms’ optimum price is \( P^* = P^m \), the SPE is \( \left( \frac{4c}{1 + \beta}, N, n \right) \) and foreign entry is accommodated.

The social welfare is the sum of domestic profit, of consumer surplus, and of tariff rents:

\[
W(t, x) = CS(t, x) + N \pi^d(t, x) + R(t, x) = \frac{1}{2} (A - p)^2 + N \pi^d(t, x) + R(t, x)
\]

where \( CS(t, x) \) is the consumer surplus, \( \pi^d(t, x) \) is the profit of one domestic firm, and \( R(t, x) \) is the tariff revenue. Given our specifications, \( CS(t, x) = \frac{1}{2} (A - p)^2 \). Depending on domestic firms’ relative cost efficiency and pricing strategy in the market, \( \pi^d(t, x) \) is given by one of the formulas (2), (3) or (4). In the case \( t > 0 \), \( R(t, x) = tQ(t, x) \), where \( Q(t, x) \) is the total quantity produced by foreign firms; otherwise \( R(t, x) = 0 \).

4. FREEDOM TRADE

As we discussed above, when the tariff level is set to 0 and the assumption (a2) holds, domestic firms cannot behave as unconstrained “monopolists”. Depending if condition (5) is satisfied or not, they either behave as constrained “monopolists” or accommodate foreign entry.

Since \( c > C \), from (5) follows that \( 4(N + n)(c - C) > N(A - c) \). By replacing the formulas for \( C \) and \( c \) in this inequality, we can rewrite it as

\[
x[4(1 - \beta)(N + n) - N] > N(A - \alpha).
\]

Unless the expression \( 4(1 - \beta)(N + n) - N \) is positive, condition (5') does not hold. Therefore, for high levels of spillover,

\[
\beta \geq \frac{3N + 4n}{4(N + n)},
\]

domestic firms always accommodate foreign entry. When R&D investment gives only a mild marginal cost advantage to domestic firms with respect to foreign firms, fighting foreign entry becomes too expensive, as domestic firms have to price very near their own production costs. The alternative option, splitting the monopoly profit with their foreign rivals yields higher profits per domestic firm. Using Boone’s (2002) terminology, when local firms can only

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8 In order for (1) to hold for \( t = 0 \), the level of increase in efficiency \( x \) should be high enough: \( x > \frac{1}{\frac{1}{1 - \beta}} (A - \alpha) \). However, \( x \) cannot be higher than \( \alpha \). When \( A > 2(1 - \beta)\alpha \), \( \frac{1}{1 - 2\beta} (A - \alpha) > \alpha \).
reach a small cost advantage over foreign firms it doesn’t pay off to fight. On the contrary, it
pays off to be nice, to charge a high price and share the market with the foreign firms.

Before the pricing game takes place, domestic firms invest in R&D. If domestic firms
were to behave as constrained “monopolists” in the pricing stage, they would count on a profit
\( \pi^\text{cm}(0,x) \) given by (3) and therefore, they would opt for a decrease in marginal cost \( x_{fi}^\text{cm} \) that
maximizes this level of profit:
\[
\frac{1}{N}(1 - \beta)(A - \alpha + 2x_{fi}^\text{cm} \beta) = i'(x_{fi}^\text{cm}) .
\]  
(8)
The second order necessary condition requires \( \frac{1}{N} \beta(1 - \beta) - i''(x_{fi}^\text{cm}) < 0 \). The corresponding
domestic welfare and consumer surplus are given by
\[
W_{fi}^\text{cm} = (A - \alpha + x_{fi}^\text{cm} \beta)\left[x_{fi}^\text{cm} (1 - \beta) + \frac{1}{N} (A - \alpha + x_{fi}^\text{cm} \beta)\right] - Ni(x_{fi}^\text{cm}) ,
\]  
(9) and
\[
CS_{fi}^\text{cm} = \frac{1}{2} (A - \alpha + \beta x_{fi}^\text{cm})^2 ,
\]  
(10) respectively.

If domestic firms were to split production and profits with foreign firms, the best that
they could do is to choose an innovation level, \( i(x_{fi}^e) \), that maximizes the profit \( \pi^\text{e,d}(0,x) \) given by (4):
\[
\frac{1}{2(N+n)} (A - \alpha + x_{fi}^e) = i'(x_{fi}^e) .
\]  
(11) The corresponding second order necessary condition is \( \frac{1}{2(N+n)} - i''(x_{fi}^e) < 0 \). The social welfare
in this case is given by
\[
W_{fi}^e = \frac{3N+n}{8(N+n)} (A - \alpha + x_{fi}^e)^2 - Ni(x_{fi}^e) ,
\]  
(12) and the consumer surplus equals
\[
CS_{fi}^e = \frac{1}{8} (A - \alpha + x_{fi}^e)^2 .
\]  
(13)

For levels of spillover that are higher than \( \frac{N+2n}{2(N+n)} \), for any level of R&D \( x \in [0,\alpha] \), the
right hand side of the first order condition (8) is always smaller that the one of condition (11).
Therefore, when \( \beta > \frac{N+2n}{2(N+n)} \), \( x_{fi}^e > x_{fi}^\text{cm} \). An increase in the level of spillover lowers the price
that constraint “monopolists” charge, pushes down their profits and consequently, lowers their

\[9\] In all the following sections we assume that parameters’ values are such that the second order necessary
conditions for all welfare maximization problems hold.
innovative effort. For high levels of spillover this negative impact on R&D is high enough to drive its level below \( x^c_{Rt} \). In addition, when \( \beta \geq \frac{3N+4n}{4(N+n)} \) condition (5') holds for any given level of \( x \), and thus domestic firms accommodate entry. In particular this is true for \( x = x^c_{Rt} \), so \( \pi^e_d(0,x^c_{Rt}) > \pi^c_{Rt} \) where \( \pi^c_{Rt} = \pi^c(0,x^c_{Rt}) \). Since \( \pi^e_d \) increases as \( x \) increases towards \( x^c_{Rt} \), for high enough level of spillover domestic profits as well as the R&D level are higher if the domestic firms are “nice” towards foreign firms and accommodate their entry in the market. Thus, at least for levels of spillovers that are higher than \( \frac{3N+4n}{4(N+n)} \) domestic firms bid the monopoly price during the pricing game. At such high price foreign firms participate as well in the market.

5. Government Intervention

If the government interferes in the market by setting an optimal tariff protection, the interaction between different agents in the market takes place in the following order: first, domestic firms invested in R&D, next, the government announces the level of tariff protection, and at the end the market price is establish in the bidding process described in Section 3. The outcome of the pricing game was already discussed in the same section. We showed that, for a given level of tariff, \( t \), and R&D investment, \( i(x) \), the market price is either equal with foreign firms’ marginal cost or with domestic firms’ monopoly price. The former outcome is the consequence of domestic firms’ aggressive pricing strategy which deters foreign entry and ensures them a “constraint” monopolist position in the market. In the latter situation, depending on the relative cost advantage that domestic firms have on their rivals, either foreign entry is deterred and domestic firms are unconstrained “monopolists” or foreign entry is accommodated. In our search for the SPE, we continue to solve backwards the entire game by analysing the government intervention stage.

Through the level of its policy instrument, the government can influence the outcome of the pricing game. More precisely, given the existent R&D investment the government can tilt the balance towards the most socially desirable market structure (unconstrained “monopolists” vs. constrained “monopolists” vs. entry accommodation). When taking its decision, for each of the three possible situations that can arise in the pricing game the

\[ \frac{dx^c_{Rt}}{d\beta} = \frac{4-N-2x^c_{Rt}(1-2\beta)}{2\beta(1-\beta)-N(1-x^c_{Rt})} \]

Due to the second order conditions, the denominator is always smaller than 0. Since the assumption (a2) holds, for \( \beta > \frac{1}{2} \), the innovation

\[ 10 \]
domestic government first maximizes the social welfare given by formula \( (6) \), and then compares the outcomes in order to induce the market structure that yields the highest social welfare.

5.1. Unconstrained monopoly

If in the pricing game domestic firms were to behave as unconstrained “monopolists”, the tariff level should be so that foreign firms’ price mark-up, \( p^m - c = \frac{1}{2} \left( A - \alpha - 2t - x(1 - 2\beta) \right) \), is lower or equal to zero. Consequently, the tariff protection should be at least as high as

\[
 t^m = \frac{1}{2} [A - \alpha - x(1 - 2\beta)].
\]  
(14)

For any level of tariff that equals or exceeds \( t^m \), domestic firms charge consumers the monopoly price \( (P^* = P^m) \). Meanwhile, foreign entry is deterred.

The impact of an increase in the cost efficiency level on \( t^m \) is

\[
 \frac{\partial t^m}{\partial x} = -\frac{1}{2}(1 - 2\beta).
\]

For low levels of spillover \( (\beta < \frac{1}{2}) \) a rise in domestic firms’ innovation level drives a drop in \( t^m \). On the contrary, when the level of spillovers is above \( \frac{1}{2} \), an increase in the R&D level brings an increase in \( t^m \). This is because when spillovers are lower than \( \frac{1}{2} \), both tariff protection and R&D investment have a negative impact on the foreign firms’ price mark-up, \( P^m - c \), and therefore act as substitutes in driving this mark-up down to zero. When spillovers are higher than \( \frac{1}{2} \), the negative effect that an increase in R&D have on the monopoly price, \( P^m \), is offset by the positive effect that innovation has on foreign firms’ cost efficiency, \( c \), and therefore, an increase in the domestic R&D investment has a positive impact on \( P^m - c \). Since the impact of an increase in tariff protection on \( P^m - c \) remains negative, when the target is to bring \( P^m - c \) down to zero, an increase in the innovation level should be complemented by an increase in tariff protection.

When tariff protection is at least as high as \( t^m \), domestic welfare equals

\[
 W_{t^m}^d(t^m, x) = \frac{1}{2} (A - \alpha + x)^2 - Ni(x),
\]  
(15)

while the consumer surplus is given by

\[
 CS_{t^m}^d(t^m, x) = \frac{1}{2} (A - \alpha + x)^2.
\]  
(16)

Being protected by a tariff level at least as high as \( t^m \), a domestic firm expects a profit \( \pi_{t^m}^d(t^m, x) \) given by \( (2) \). Therefore it will invest in R&D so that the decrease in marginal cost, \( x_{t^m}^m \), verifies

\[
 \text{chosen by constrained “monopolists” decreases with the level of spillover}
\]
\[
\frac{1}{x_i^m}(A - \alpha + x_i^m) = \ell'(x_i^m).
\]

The second order necessary condition requires \(\frac{1}{x_i^m} - \ell''(x_i^m) < 0\).

### 5.2. Constrained monopoly

Domestic firms act as constrained “monopolists” only if tariff protection is high enough so that the inequality (5) is satisfied, but still not higher than \(\ell''\). The smallest level of tariff for which (5) holds with equality is

\[
t^p = \frac{1}{2}[A - \alpha - x(1 - 2\beta)] - \frac{\sqrt{n}}{2N + n}(A - \alpha + x) = \ell'' - \frac{\sqrt{n}}{2N + n}(A - \alpha + x) .
\]

This level of tariff protection is smaller than \(\ell''\). In addition, for high levels of spillover, \(\beta\), \(\ell^p\) is positive: in the case of free trade, when \(\beta \geq \frac{3N + 4n}{4(N + n)}\), regardless of the innovation level domestic profits under entry accommodation are higher than under constraint monopoly (see Section 4). Therefore, \(\ell^p\) has to be positive in order to induce domestic firm to price as constraint “monopolists”. Yet, if \(\beta\) is small, domestic firms might prefer to act as constraint monopolist rather than to accommodate foreign entry even under free trade. Then \(\ell^p \leq 0\).

To see when \(\ell^p\) could become negative, we rewrite \(\ell^p\) as

\[
t^p = \frac{\sqrt{n}}{2N + n}(A - \alpha) - \frac{(1 - 2\beta)\sqrt{N + n} + \sqrt{n}}{2N + n}x .
\]

Unless \(\frac{(1 - 2\beta)\sqrt{N + n} + \sqrt{n}}{2N + n}\) is positive, \(\ell^p\) is always positive. Consequently, for low levels of spillovers (\(\beta < \frac{1}{2} + \frac{\sqrt{n}}{2N + n}\)), \(\ell^p\) might become negative. Only for such spillovers, without government protection, domestic firms might be able insure a higher profit under a constraint monopoly strategy than under entry accommodation.

The effect of an increase in cost efficiency, \(x\), on \(\ell^p\) is

\[
\frac{\partial \ell^p}{\partial x} = -\frac{1}{2N + n}[(1 - 2\beta)\sqrt{N + n} + \sqrt{n}].
\]

This impact is always negative for small levels of spillover (\(\beta < \frac{1}{2} + \frac{\sqrt{n}}{2N + n}\)) but becomes positive once \(\beta\) is high (\(\beta > \frac{1}{2} + \frac{\sqrt{n}}{2N + n}\)). The underling intuition is the following. In order for the domestic firms to be optimal to price aggressively and become constraint “monopolist” rather than to accommodate foreign entry they need to have enough unit cost advantage with respect to their foreign rivals. Such cost advantage could be gained through an increase in tariff protection. In addition, for low level of spillovers a high enough unit cost advantage could be obtained out of a rise in the R&D investment as in this case a rise in the R&D cost is completely offset by the gains in profits stemming from market share expansion. However, when spillovers are high, an additional increase in innovation brings an increase in cost that cannot be offset by the positive effects related with the gain in market share. Thus, much like in the case of unconstraint monopoly, for low levels
of spillover \( (\beta < \frac{1}{2} + \frac{\sqrt{6}}{2J_{N+n}}) \) R&D investment and tariff protection can be used as substitutes in imposing a more monopolistic outcome in the market while for high level of spillovers \( (\beta > \frac{1}{2} + \frac{\sqrt{6}}{2J_{N+n}}) \) innovation and tariff protection act as complements.

The impact of an increase in the tariff level on the social welfare given in formula (6) is 
\[
\frac{\partial W^C}{\partial t} = -t - x(1 - \beta),
\]
which is always negative. An increase in the level of tariff, \( t \), has a negative effect on the consumer surplus, \( CS^C_{it} = \frac{1}{2}(A - \alpha + \beta x - t)^2 \). In the case of linear demand, this effect offsets the positive impact that a rise in tariff has on domestic profits. Since an increase in tariff has an overall negative impact on the social welfare, the optimal level of tariff protection under constraint monopoly should equal the lower tariff under which domestic firms still price as constraint “monopolists”. Therefore, the government should impose a tariff of \( t^p \) (zero if this level is negative). This level of tariff protection is higher than zero only when under a free trade regime it is not optimal for domestic firms to price as constraint “monopolists”.

When \( t^p > 0 \), domestic welfare at \( t^p \) equals
\[
W^C_{it}(t^p, x) = \frac{3N + 2N^2 + 2N(N+n)}{8(N+n)}(A - \alpha + x)^2 - Ni(x),
\]
while the consumer surplus is given by
\[
CS^C_{it}(t^p, x) = \frac{N + 2N^2 + 2N(N+n)}{8(N+n)}(A - \alpha + x)^2.
\]
In the case \( t^p = 0 \), domestic welfare and consumer surplus equal their counterparts under free trade and are given by formula (9) and (10), respectively.

Protected by a tariff of \( t^p > 0 \), each domestic firm earns a profit of
\[
\pi^C_{it}(t^p, x) = \frac{1}{4N(N+n)}(A - \alpha + x)^2 - i(x),
\]
Anticipating this level of profit, domestic firms invest in a cost reduction, \( x^C_{it} \), that maximizes (21)
\[
\frac{1}{2N(N+n)}(A - \alpha + x^C_{it})^2 = l'(x^C_{it}),
\]
The second order necessary condition in this case requires \( \frac{1}{2N(N+n)} - l''(x^C_{it}) < 0 \). The optimal level of R&D investment does not depend on the spillover level. Thus tariff protection completely eliminates the effect that spillovers have on innovation. It does so by enforcing domestic firms to undertake the same investment and to benefit from similar profits as under entry accommodation. However, this outcome is reached at a lower market price than the monopoly price. In addition, for different levels of technological leakages, \( \beta \), this outcome is
achieved with different levels of tariff protection: \( \frac{\partial \pi_c}{\partial \tau} \bigg|_{\tau_0} = x_{cm}^t \) (see Figure 3.1). When \( \beta \) increases, it becomes more costly for domestic firms to deter the entry of the foreign rivals through their level of R&D investment. Therefore, a higher tariff is needed to supplement \( (\beta < \frac{1}{2} + \frac{\Delta}{2N + \alpha}) \) or to encourage \( (\beta > \frac{1}{2} + \frac{\Delta}{2N + \alpha}) \) firms’ innovative effort.

Figure 3.1. Predatory and monopoly tariffs as functions of the spillover level, \( \beta \).

When \( \tau^p = 0 \), each domestic firms gets a profit of \( \pi^c_{ji} (0, x_{cm}^{ji}) \) where \( x_{cm}^{ji} \) is implicitly given by (8).

5.3. Foreign entry

If domestic firms were to accommodate foreign entry, any tariff between 0 and \( \tau^p \) induces such behaviour (however, when the level of \( \tau^p \) is negative, domestic firms prefer not to accommodate foreign and act as constraint “monopolists”). In this case domestic firms bid the monopoly price. Since neither this price nor domestic profits \( \pi^{c,d} (t, x) \) given in formula (4) do not depend on the tariff protection level, domestic innovation and consumer surplus equal their corresponding counterparts under free trade – formulas (11), and (13) respectively. However, the corresponding social welfare is different than the free trade one since under protection domestic government extracts tariff rents. Therefore, the level of social welfare equals \( W^e \) (given in formula (12)) plus the tariff revenue:

\[
W^e_t (t, x) = \frac{3N + \alpha}{N(N + \alpha)} (A - \alpha + x)^2 - N i(x) + \frac{\alpha}{N(N + \alpha)} (A - \alpha + x)t .
\] (23)

Since the tariff level does not impinge on domestic profits and consumer welfare, its impact on the social welfare is due to its effect on tariff rents. As these rents increase with an increase in tariff protection, the government should charge the highest tariff that still induces foreign
entry. Therefore, it sets a tariff protection level right below \( \ell^p \) given in formula (18). As a result, when \( \ell^p > 0 \) the social welfare equals

\[
W_e^e(\ell^p, x) = \left( \frac{1}{2} - \frac{\sqrt{n(N + \alpha)}}{4(N + \alpha)^2} \right)(A - \alpha + x)^2 - \frac{n}{2(N + \alpha)}(1 - \beta)(A - \alpha + x)x - Ni(x).
\]

(24)

Otherwise the social welfare equals \( W_e^e \) given in formula (12).

In Section 5.2, we pointed out that a constraint monopoly outcome under government intervention and an entry deterrence one under free trade are characterized by the same level of R&D investment and the same level of domestic profits. Since irrespective of the level of tariff, the entry deterrence yields a constant level of innovation and a constant level of firms’ profit, profits and R&D investments under entry accommodation and constraint “monopolists” are the same. However, a lower than monopoly price is necessary to reach a constraint monopoly situation. Therefore, the consumer surplus under entry accommodation is lower than under entry deterrence.

5.4. The optimal tariff protection

For any given \( x \), among the left hand sides of the formulas (17), (22) and (11) that implicitly define the investments levels under the three different situations – unconstraint and constraint “monopolists”, and entry accommodation – the highest is the one in formula (17). Since the right hand sides are all the same, this imply that the highest R&D investment occurs under the highest government protection – domestic firms act as unconstrained “monopolists”. Meanwhile when \( \ell^p > 0 \), accommodating entry or acting as constrained “monopolists” bring the same level of investment in innovation (\( x_p^e = x_{m} \)). Also, unless the tariff protection is zero, the levels of R&D investment and domestic profits are not influenced by the spillover parameter, \( \beta \). By introducing an optimal tariff protection, domestic government completely eliminates the effect that spillovers have on domestic firms.

Domestic firms choose to behave as constraint “monopolists” only if the tariff protection is between \( \ell^p \) and \( \ell^m \). When the tariff equals the highest bound, \( \ell^m \), constraint monopoly outcome is equivalent with the unconstraint monopoly outcome. Since under a constraint monopoly the government sets a tariff level that equals \( \ell^p \) (or 0), domestic social welfare under unconstraint monopoly is lower than under a constraint one. Even though the innovation level and domestic profits are higher under unconstraint “monopolists”, the loss in consumer surplus due to high, monopoly price, more than offsets these benefits of having a higher tariff protection.

When tariff protection is lower than \( \ell^p \), in the pricing stage domestic firms bid the
monopoly price. Despite this high price, domestic firms put the same effort in innovation and obtain the same profits as when they act as constraint “monopolists”. Nonetheless, due to the high price, consumer surplus is lower than when domestic firms price aggressively and deter foreign entry. The gain in tariff revenue cannot offset this decrease is consumers’ welfare. Hence, the overall social welfare under constraint monopoly surpasses the one under entry accommodation (see in the Appendix the proof for Theorem 2).

The above discussions are summarized in Theorem 2.

**Theorem 2.** The optimal protection level, $t^*$, is just above $t^p$, if $t^p > 0$, and zero, otherwise. Correspondingly, domestic firms behave as constraint monopolist.

Thus the government chooses a tariff protection level that eliminates leakages in technology and meanwhile induces the highest consumer and overall social welfare. This result still holds for social welfare functions in which, in order to avoid lobbing, the government assigns a higher weight to consumer surplus than to tariff revenue and producer surplus.

### 6. Tariff Protection versus Free Trade

Due to the nature of tariffs that are set in order to prevent leakages in knowledge and technology, in our model we considered that the level of tariff protection is announced only after domestic firms have already invested in innovation. Grossman and Maggi (1998) and Ionașcu and Žigić (2001) have shown that when a domestic government sets the level of its policy only after domestic firms have invested in R&D, through the level of their R&D effort local firms can manipulate government’s choice. This manipulative action might drive the innovation far away from its socially optimal value, and therefore, avoiding such a situation by committing in advance to free trade might become optimal.

Theorem 2 indicates that the best strategy under tariff protection is to enact a tariff level that is just high enough to drive foreign firms out of the market. In some cases domestic firms can themselves achieve this outcome. Then the optimal protection level is 0 and thus firms act in a free trade environment. However the interesting question is what happens with the social welfare when intervention prescribes a protection level higher than 0. Therefore, in the remaining part of this section we concentrate our attention on the case $t^* > 0$.

By setting a tariff of $t^*$, the government gives enough incentives to domestic firms to
fight foreign entry. Therefore under intervention the only outcome that arises in the market is the constraint monopoly one. Under free trade, however, the optimal strategy for domestic firms is to accommodate foreign entry. We have already showed in Section 5.4. that this strategy yields the same level of innovation, and profits as tariff protection. Yet, the level of consumer surplus and therefore of the overall welfare is higher under government intervention than under entry accommodation.

Consequently, in comparison with free trade policy, protection increases or preserves domestic social welfare at the same level.

**Theorem 3.** When \( t^* > 0 \), a tariff protection policy yields a higher social welfare than free trade. If the innovation levels and profits are the same under tariff protection and under free trade, the former case is associated with a higher consumer surplus.

By introducing a tariff protection regime, the government tilts the balance towards the most competitive outcome in terms of market price and toughness of competition without hurting domestic firms’ profits. Therefore, unlike in the case of perfect competition or Bertrand or Cournot oligopolistic competition, the introduction of a tariff protection does not induce any consumption distortion.

**7. Conclusions**

In this paper we consider a North-South trade set-up in which firms from a developed, North country compete with firms from a less developed, South, country. Competition takes place in the North market. In this market prices are perfectly flexible and are set during a bidding process. Firms from North engage in innovative activities, in order to reduce their production costs. There is no technological leakage between firms from North as the IPR rights are perfectly enforced in this country. Firms from South do not have the capacity to conduct R&D. However, due to lax IPR protection in their home country, they can imperfectly imitate the new technology developed by firms from North. Given this problem of R&D appropriability that emerges in this situation, the government from North might want to introduce a tariff protection in order to prevent technological spillovers. In this context, we analysed the role of tariff in preventing the technological leakages and its impact on domestic profits, innovation, consumer surplus, and the social welfare.

In contrast to previous results that were obtained under different pricing strategies, we
show that tariff protection preserves or enhances domestic consumers’ welfare with respect to the free trade situation. The rational for this result stays in the relation between cost distribution and prices. In the case of perfect flexible prices that we considered in this model, when domestic and foreign firms have similar cost efficiency levels, entry deterrence is costly. As a result the competition in the market is soft and the market price is high. Due to tariffs, the cost differential between domestic and foreign firms increases, triggering a more aggressive domestic pricing strategy that drives foreign firms out of the market. Thus, the R&D becomes appropriable. What is more, tariff protection reduces the monopoly power of participants by inducing higher competition and lower prices (interestingly, this is achieved with lower number of firms). Consequently, consumers benefit from tariff protection. In addition, the gain in market share associated with entry deterrence exactly offsets the negative effect that a low price has on domestic firms’ price mark-up, and through it, on profits. Therefore, tariff protection preserves the same level of innovation and domestic profits as in the case of free trade. Since only the low cost firms end up producing in the market, the total efficiency in the industry increases.
REFERENCES


APPENDIX

PROOF OF LEMMA 1

We prove it by contradiction. Assume that \( p^* < P^* \). Then \( D(p^*) > D(P^*) \). Also, given the definitions of \( N(p) \) and \( n(p) \) this relation between prices implies that \( n(p^*) + N(p^*) \leq n(P^*) + N(P^*) \). By using the definition of \( P^* \) and the fact that \( C \leq c \) we get:

\[
0 \leq \frac{D(P^*)}{N(P^*) + n(P^*)} (P^* - C) - \frac{D(p^*)}{N(p^*) + n(p^*)} (p^* - C) =
\]

\[
= \frac{D(P^*)}{N(P^*) + n(P^*)} P^* - \frac{D(p^*)}{N(p^*) + n(p^*)} p^* + c \left( \frac{D(P^*)}{N(P^*) + n(P^*)} - \frac{D(p^*)}{N(p^*) + n(p^*)} \right) \leq
\]

\[
\leq \frac{D(P^*)}{N(P^*) + n(P^*)} P^* - \frac{D(p^*)}{N(p^*) + n(p^*)} p^* + c \left( \frac{D(P^*)}{N(P^*) + n(P^*)} - \frac{D(p^*)}{N(p^*) + n(p^*)} \right) =
\]

\[
= \frac{D(P^*)}{N(P^*) + n(P^*)} (P^* - c) - \frac{D(p^*)}{N(p^*) + n(p^*)} (p^* - c).
\]

This implies that the foreign firms can get higher profits from choosing \( P^* \) rather than \( p^* \). Therefore \( p^* \) cannot be the \( \arg \max_p \frac{D(p)}{N(p) + n(p)} (p - c) \).

PROOF OF THEOREM 1

1. First we will prove that \((P^*, N(P^*), n(P^*))\) together with the strategies described in Theorem 1 for each round \( s \) for the domestic and foreign firms form is a SPE of the pricing game.

   By definition of \( P^* (p^*) \), a domestic (foreign) firm cannot do better by quoting another price. Therefore, if the price at stage \( s \) is higher than \( P^* (p^*) \), a domestic (foreign) firm will undercut it to \( P^* (p^*) \). Moreover, since \( D(.) (\cdot - \alpha + x) \) is a concave function on \( \mathbb{R}^+ \) for any \( x \in [0, \alpha] \) (\( D(.) \) is in fact a linear function), given that the other firms will keep the strategies specified in the theorem, a domestic (foreign) firm’s profit is increasing in \( p \) as long as \( p < P^* (p < p^*) \). Therefore, a domestic (foreign) firm will not undercut a price \( p \leq P^* (p \leq p^*) \). Also, a firm cannot gain by changing unilaterally its exit strategy.

   Given the equilibrium strategies and the result from Lemma 1, the SPE of the pricing game will be \((P^*, N(P^*), n(P^*))\).

2. We will show that the above SPE is unique.

   Since \( D(.) \) is a linear function, \( D(.) (\cdot - \alpha + x) \) is a concave function on \( \mathbb{R}^+ \) for any \( x \in [0, \alpha] \). This implies that \( P^* \) is a singleton and the profit function \( \frac{D(p)}{N(p) + n(p)} (p - C) \) is
single peaked on \([C, P^*]\). We will again prove by contradiction that \((P^*, N(P^*), n(P^*))\).

Suppose that there is a price \(p' \neq P^*\) which is also a SPE price. Since firms do not play weakly dominated strategies, this implies that firms with cost below this price are active in the market while firms with cost higher than this price, have left the market. Therefore, at this price, there will be \(n(p') + N(p')\) firms in the market, where \(N(p') = \begin{cases} N, & C \leq p' \\ 0, & C > p' \end{cases}\)

and \(n(p) = \begin{cases} n, & c \leq p' \\ 0, & c > p' \end{cases}\).

First, let’s assume that \(p' < P^*\). If only one firm has reduced its price to \(p'\), since the profit function is concave and \(p' < P^* \leq p^*\), that firm could have been better off by bidding \(P^*\). If at least 2 firms have quoted the price \(p'\), each of these firm can never loose by bidding \(P^*\) (on the contrary, they might even gain from quoting this higher price). Thus \(p'\) can form a SPE only if firms choose weakly dominated strategies, case which is rule out by our assumptions.

Now, let’s suppose that \(p' > P^*\). Unless there is a credible threat that a price reduction to \(P^*\) will trigger a further reduction in the price, the domestic firm can do strictly better by lowering the price to \(P^*\). However, in our game, such a credible threat can exist only if firms are allowed to play weakly dominated strategies (see the argument above).

**Proof of Theorem 2**

We have already shown that \(W_{i,m}^c < W_{i,m}^w\). What we still need to prove is that \(W_{i}^w < W_{i}^c\).

Since a constraint monopoly situation and entry accommodation are characterized by the same R&D investment level, it is enough to show that for any \(x \in [0, \alpha]\), \(W_{i}^w(t^p, x) < W_{i}^c(t^p, x)\). When we subtract \(W_{i}^w(t^p, x)\) given in formula (24) from \(W_{i}^c(t^p, x)\) given in formula (19) we get:

\[
W_{i}^c(t^p, x) - W_{i}^w(t^p, x) = \sqrt{n(N+n)(2N+2n)-n(N+n)}(A-\alpha+x)^2 + \frac{n}{2(2N+n)}(1-\beta)(A-\alpha+x)x.
\]

Since \(2(N+2n) > \sqrt{n(N+n)}\), both terms of the right hand side of the above equality are positive. Therefore \(W_{i}^w(t^p, x) < W_{i}^c(t^p, x)\) for any \(x \in [0, \alpha]\).