Abuse of Authority and Collusion in Organizations

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Abstract

We investigate an agency relationship with moral hazard where a principal relies on a supervisor to obtain verifiable information about an agent’s output. The supervisor’s discretionary power allows him to engage in both collusion and abuse of authority. Collusion occurs when the supervisor accepts a bribe from the agent to conceal information whereas abuse of authority occurs when instead the supervisor asks the agent a tribute to reveal information. We characterize the optimal incentive contracts in this environment and study how collusion and abuse of authority interact. We show that, compare to the case where either collusion or abuse of authority may occur, the presence of both of these unofficial activities increases the scope as well as the intensity of the efficiency loss sustained by the organization. We also identify the condition under which the presence of two forms of unofficial activity entails cost super-additivity.

Keywords: Moral hazard; Hierarchy; Abuse of authority; Collusion; Bureaucracy.

JEL Classification: D82; L20; M12.

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1 Introduction

Empirical studies and news reports reveal that organizations are exposed to multiple forms of unofficial activity. Yet, the principal-supervisor-agent literature to date has only investigated a single form of unofficial activity, namely collusion between the supervisor and the agent. In the standard model of this literature, the supervisor and the agent may form a coalition in order to manipulate information to the detriment of the principal. We extend this model by allowing the supervisor to engage in both collusion and abuse of authority.

Including abuse of authority in the analysis of multi-level hierarchies is an important step towards a better understanding of the functioning of these organizations.

Indeed, abuse of authority is one of the most widespread and harmful forms of unofficial activity in hierarchies. Anecdotal evidence on minor forms of abuse of authority abounds. There is also a great deal of evidence on severe forms of abuse of authority such as sexual harassment. In a comprehensive study conducted by the United States Merit Systems Protection Board in 1989, 42 percent of women and 15 percent of men reported that they had been sexually harassed on the job (Flynn 1991). Similar results have been obtained by Timmerman and Bajema (1999) in their investigation of sexual harassment in Northwest European countries.

For our analysis, we consider a three-level hierarchy with hard (i.e., verifiable) information in the tradition of Tirole (1986, 1992). In such an environment information can only be concealed but not forged. More formally, we investigate a principal-supervisor-agent hierarchy with moral hazard where hard information about the output produced by the agent is obtained through supervision. The role of the supervisor is therefore to produce a verifiable report on the agent’s output. Since the supervision technology is imperfect, the supervisor obtains information only with a certain probability. The informed supervisor then has the discretion to conceal information and pretend that he has not observed anything. This allows him to engage either in collusion or abuse of authority. When supervision reveals the evidence that the agent has produced a low output, the supervisor may collude with the agent and, in exchange for a bribe, conceal his information from

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2 Tirole (1992) and Laffont and Rochet (1997) provide extensive surveys of the literature on collusion.
3 See also Edwards (1979) and Johnson and Libecap (1989) for evidence on other forms of abuse of authority in organizations.
4 Sexual harassment falls into two categories: quid pro quo harassment and hostile work environment harassment. Quid pro quo harassment, as the name implies, occurs when an employee must submit to unwelcome sexual advances as part of his or her employment (for example, sexual favors in return for increase in pay). “Hostile work environment” harassment occurs when an employee is subject to sexual misconduct (be it physical actions by another or others, or verbal sexual comments or innuendos) which creates a work environment so intolerable that a “reasonable person” would not be able to work under those conditions. The present paper is concerned with quid pro quo sexual harassment.
5 A different strand of literature investigates organizations where information is soft - that is, forgeable - and where collusion takes place under asymmetric information (e.g., Laffont and Martimort, 1997; Faure-Grimaud, Laffont, and Martimort, 2003).
the principal. When instead supervision reveals the evidence that a high output has been produced, the supervisor may abuse his authority by asking the agent a tribute to reveal the information he has obtained.\footnote{The paper’s terminology is borrowed from the sociological literature (see, for example, Breton, 1995). In this literature, “authority” is used to describe a superior control over his subordinate.} In this setting, it is possible and optimal to simultaneously deter both forms of unofficial activity.\footnote{This may not be systematically the case in alternative environments. Indeed, in the very different context of crime and punishment, Marjit and Shi (1998) show that it may even be impossible to deter a single form of unofficial activity. They present a model that synthesizes the classical and the strategic approaches to crime and punishment. Unlike us, they consider external collusion, that is, collusion between a member of an organization (the law enforcing agent) and an outsider (the criminal). In our model, the supervisor and the agent are both members of the same organization, and hence both receive their wages from the same principal. Our model is thus one of internal collusion in the tradition of Tirole (1986, 1992).}

We characterize the optimal contracts in this environment. To investigate the additional modifications that the presence of two types of unofficial activity introduces in the organization, we characterize the optimal contracts in the cases where only a single type of unofficial activity may take place and in the case where both types are feasible.

Our results suggest that considering a single type of unofficial activity in organizations may be deceptive. Indeed, we first show that, in our moral hazard environment, when only collusion may occur in the organization, it can be costlessly deterred, and is hence harmless. We next find that when abuse of authority is the only possible type of unofficial activity, it is not systematically costly to deter it. Abuse of authority becomes harmful only when the supervision technology is sufficiently inefficient. Therefore, though abuse of authority is not systematically harmful, it is more harmful than collusion.

We then consider both types of unofficial activity and show that, compare to the case where only a single type of unofficial activity is possible, the presence of multiple types of unofficial activity increases both the scope and the intensity of the efficiency loss sustained by the organization. Indeed, preventing unofficial activities becomes systematically costly in this case and reduces more severely the efficiency of the organization. This is because, in the presence of both collusion and abuse of authority, preventing abuse of authority makes collusion deterrence costly. Collusion is not harmless anymore.

Finally, we identify the condition under which the cost of preventing both forms of unofficial activity is larger than the sum of the costs of preventing a single form of unofficial activity. That is, we identify the condition under which the presence of multiple forms of unofficial activity entails cost super-additivity.

Although unofficial activities can take many forms, principal-supervisor-agent models have rarely considered other types of unofficial activity beside collusion.\footnote{Other types of unofficial activity than collusion have been studied by economists. For example, Milgrom (1988) and Shleifer and Vishny (1989) consider respectively influence activities and managerial entrenchment in firms. Unlike these authors, we are concerned with unofficial activities taking place in a} There is however a burgeoning...
literature that investigates optimal contracting in the presence of two types of unofficial activity. Lambert-Mogiliansky (1997) studies extortion and collusion in the context, very different from the present, of public procurement. She considers a three-level government (principal)-agency (supervisor)-firm (agent) procurement relationship in the presence of both collusion between the agency and the firm and extortion of the firm by the agency. Unlike us, she shows that extortion is not generally an issue in such relationships whereas collusion is. Indeed, in her model extortion is harmless in the sole presence of information asymmetries. Extortion becomes an issue only when there are both information asymmetries and a community of interest between the principal and the agent. By contrast, in our moral hazard environment the sole presence of information asymmetries is enough to make abuse of authority noxious.

Vafaï (1994, 2002) investigates collusion and abuse of authority in a similar environment to the one considered here. However, since in his models there is no uncertainty in production, collusion and abuse of authority do not interact. Collusion is costless to deter even in the presence of abuse of authority.\(^9\)

Hindricks et al. (1999) and Marjit et al. (2000) analyze extortion and collusion in the context, very different from ours, of tax evasion. They consider a hierarchy composed of a government, a tax inspector, and a taxpayer in which the tax inspector may either collude with the taxpayer or extort him. Since in their models the taxpayer is not a member of the organization, and is hence not paid by the government, these authors are concerned with external unofficial activities while we focus on internal unofficial activities. Moreover, these authors consider an environment where the supervisor’s information is soft while we are concerned with a hard information environment in the tradition of Tirole (1986, 1992).

The plan of the paper is as follows. The model is presented in Section 2. Section 3 characterizes the optimal incentive contracts in the absence of unofficial activities. Sections 4 and 5 characterize the optimal incentive contracts respectively when a single form of unofficial activity is possible and when two types of unofficial activity are possible. Section 6 concludes the paper. All proofs are in the Appendix.

2 The model

We consider a hierarchical organization under moral hazard consisting of three risk neutral players: a principal (it), a supervisor (he) and an agent (she). The organization has the following characteristics.

2.1 Production technology

The agent, who is in charge of the production task, has the choice between two effort levels, \( e \in \{0, 1\} \); i.e., she may either shirk \((e = 0)\) or work \((e = 1)\). Neither the principal nor the supervisor can observe the agent’s effort level. The production technology is such that if the agent decides to work, she produces a high output \(x_H > 0\) with probability \(\pi \in (0, 1)\) and a low output \(x_L \equiv 0\) with probability \(1 - \pi\). If instead the agent decides to shirk, she produces \(x_L\).10

2.2 Supervision technology

The principal can obtain hard information (i.e., verifiable by a third party) about the output produced by the agent only through supervision. The agent and the principal are unable - due, for example, to a lack of time or expertise - to perform the supervisory task. The principal therefore relies on the supervisor to obtain a verifiable report on the agent’s output.11 Since the supervision technology is imperfect, the supervisor obtains information about the output only with probability \(p \in (0, 1)\). The supervisor’s report, \(r\), thus belongs to \(I = \{x_L, \emptyset, x_H\}\), where \(r = \emptyset\) indicates that supervision has been inconclusive. Given that the information obtained by the supervisor is hard evidence, it can be concealed but not forged. That is, it is not possible for the informed supervisor to misreport the low output as high output or the high output as low output.

2.3 Payoffs

The agent’s and the supervisor’s utility functions are respectively, \(U^A(w, e) = w - \gamma e\) and \(U^S(s) = s\), where \(w\) and \(s\) are the transfers received from the principal and \(\gamma > 0\) is the agent’s disutility of effort. The agent’s reservation utility is \(u > 0\). For simplicity and without loss of generality, the supervisor’s reservation utility is normalized to zero. We assume that \(x_H\) is large enough for it to be in the principal’s interest to engage in production. The principal must therefore elicit the production effort level \(e = 1\) for its organization to be valuable. The principal’s utility function is then \(U^P(w, s) = \pi x_H - C(w, s)\), where \(C(w, s) \equiv p [\pi (w_H + s_H) + (1 - \pi)(w_L + s_L)] + (1 - p)(w_\emptyset + s_\emptyset)\) is the expected cost of production and supervision.

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10 Relaxing the assumption that there are two levels of output (failure, success) and the probability of success is zero when the agent shirks will make the model much messier without affecting qualitative results and without gaining more understanding.

11 The agent observes the output but given that hard information about the output can only be obtained through supervision, she is unable to provide hard information about it.

12 Introducing costly supervision would not affect qualitative results.
2.4 Contracts

Since hard information about the output is exclusively obtained through supervision, contracts are contingent on the supervisor’s report. The principal offers a contract \((w_L, w_∅, w_H)\) to the agent, where \(w_L\) and \(w_H\) are the wages he receives when \(r = x_L\) and \(r = x_H\), respectively, and \(w_∅\) is the wage he receives when \(r = ∅\). Similarly, the principal offers a contract \((s_L, s_∅, s_H)\) to the supervisor. The agent and the supervisor are protected by limited liability. We simply assume that the principal cannot impose negative wages on them.

2.5 Unofficial Activities

Given that the supervision technology is imperfect, the supervisor has discretion to conceal information from the principal. The supervisor’s discretion allows him to engage in two types of unofficial activity. When supervision reveals the evidence that the agent has produced a low output, the supervisor may collude with the agent and, in exchange for a bribe, report \(r = ∅\) to the principal. When instead supervision reveals the evidence that a high output has been produced, the supervisor may abuse his authority and threaten the agent with an uninformative report, \(r = ∅\), if she refuses to pay him a tribute.\(^{13}\)

Concerning unofficial activities, we make the following assumptions.

2.5.1 Observability

Unofficial activities are only observable to the involved parties.

2.5.2 Side transfer technology

Following the literature on collusion, we assume that the technology used to transfer bribes and tributes, which we refer to as side transfer technology, is less efficient than the official transfer technology (i.e., the transfer technology used by the principal to pay its employees). Formally, unofficial income can be transferred to the supervisor at a rate \(k \in (0, 1)\). That is, the side transfer creates a deadweight loss. A side transfer of size \(t\) is then only worth \(kt\) to the supervisor. There are therefore transaction costs connected to side contracting. The transaction costs of side contracting reflect the fact that unofficial activities are costly to organize. These transaction costs can also reflect the non-monetary nature of bribes and tributes.\(^{14}\) Observe that if \(k = 0\), the side transfer technology is totally inefficient, and thus unofficial activities do not occur.

Finally, note that the technology of the organization is fully characterized by the vector \((π, p, k)\).

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\(^{13}\) This paper focuses on unofficial activities involving the supervisor and the agent. We therefore assume that it is common knowledge that the principal is honest.

\(^{14}\) See Tirole (1992) for a discussion of different kinds of transaction costs.
2.5.3 Unofficial bargaining

In concert with Tirole (1992) and others, we assume that the supervisor has all the bargaining power when engaging in unofficial activities.

2.5.4 Unofficial commitments

One very important feature of unofficial activities is that they involve unofficial commitments (i.e., threats and promises). When colluding with the supervisor, the agent has to be able to credibly abide by her promise to pay a bribe to the supervisor after the supervisor has reported \( r = \emptyset \). Similarly, when the supervisor abuses his authority, his threat of reporting \( r = \emptyset \) instead of \( r = x_H \) if the agent refuses to pay him a tribute has to be credible. The agent must also be able to credibly abide by her promise to pay the demanded tribute to the supervisor in the case where the supervisor abuses his authority. Since these unofficial commitments cannot be enforced by a third party, they have to be enforced by various mechanisms.

Following the existing literature on unofficial activities in organizations, and more particularly Vafaï (2002), we do not explicitly model the mechanism that ensures the credibility of unofficial commitments (i.e., the supervisor’s threat of concealing information and the agent’s promises to pay a bribe or a tribute to the supervisor). Recent findings in experimental economics and evidence derived from case studies of abuse of authority in organizations show that reputational as well as various non-monetary mechanisms sustain the credibility of commitments even in one-shot interactions (e.g., Loewenstein, 1996; Fehr and Gächter, 1999, 2000; Bosman et al., 2001; Bosman and van Winden, 2002).15

We therefore take these findings and evidence as the starting point of our modelling. That is, we do not explicitly model the mechanisms that ensure the credibility of unofficial threats and promises. Following the existing literature, we assume that three possible mechanisms may guarantee the credibility of the supervisor’s and the agent’s unofficial commitments, namely reputation, reciprocity, and emotions.16 For example, if the supervisor has developed a reputation of being vengeful, the agent complies when the supervisor threatens her with an uninformative report and keeps her promise to pay a bribe or a tribute to the supervisor in the unofficial subgames. Similarly, if it is known that the supervisor is subject to fits of destructive anger, unofficial threats and promises are credible. Indeed, as Hirshleifer (2000, p. 18) puts it: ““Hot” anger serves to deter undesired behavior by making the threat of a punishing response credible, even if executing that punishment is not actually materially profitable.”17

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15See also Hirshleifer (1987) and Frank (1987, 1988).

16For a detailed discussion of these mechanisms, see Vafaï (2002).

17As discussed in Vafaï (2002), further evidence in support of the assumption that long term as well as non-monetary devices sustain the credibility of the supervisor’s threat can be derived from case studies of sexual harassment.
2.6 Timing

The timing is as follows: (1) The principal offers a contract \((w_L, w_∅, w_H)\) to the agent and a contract \((s_L, s_∅, s_H)\) to the supervisor. (2) The agent and the supervisor decide whether to accept or refuse the contract. If either refuses, the game ends and they both get their reservation utility. If instead contracts are accepted, the game continues as follows. (3) The agent decides whether to work or to shirk and supervision takes place simultaneously. (4) If (i) supervision reveals the evidence that the agent has produced a low output, the agent and the supervisor decide whether or not to collude; (ii) supervision reveals the evidence that a high output has been produced, the supervisor decides whether or not to abuse his authority. (5) The supervisor produces a report. (6) Transfers and side transfers take place.

We look for a Subgame Perfect Equilibrium of this game.

3 Optimal contracts in the absence of unofficial activities

As a benchmark, let us consider the case where unofficial activities are not possible (i.e., \(k = 0\)). The agent’s incentive compatibility constraint is then, 
\[
p(w_H + (1 - \pi)w_L) + (1 - p)w_∅ - \gamma \geq pw_L + (1 - p)w_∅, \quad \text{or equivalently,} 
\]
\[
  w_H - w_L \geq \frac{\gamma}{p\pi}. 
\]  
(1)

This equation makes the agent prefer to exert effort in equilibrium.\(^{18}\)

The agent’s contract must also satisfy her participation constraint,
\[
p(w_H + (1 - \pi)w_L) + (1 - p)w_∅ - \gamma \geq w. 
\]  
(2)

Given that the agent and the supervisor are protected by limited liability, we have
\[
w_L \geq 0, w_∅ \geq 0, w_H \geq 0, s_L \geq 0, s_∅ \geq 0, s_H \geq 0. 
\]  
(3)

Since supervision is costless, the supervisor accepts any contract \((s_L, s_∅, s_H) \in \mathbb{R}^3_+\). That is, any contract \((s_L, s_∅, s_H) \in \mathbb{R}^3_+\) is individually rational.

In order to provide incentives to the supervisor to reveal the information he has obtained, the principal must also set \(s_L \geq s_∅\) and \(s_H \geq s_∅.\(^{19}\)) Given that the optimal contract offered to the supervisor in this case and in the subsequent cases automatically satisfies these constraints we will systematically disregard them.

When unofficial activities are not possible, the program of the organization can thus be written as

\(^{18}\)We assume that the agent chooses to work when she is indifferent. \(^{19}\)We assume that the supervisor reports truthfully when he is indifferent.
\[ [P_0] \min p[\pi(w_H + s_H) + (1 - \pi)(w_L + s_L)] + (1 - p)(w_\emptyset + s_\emptyset) \]

\( (w_L, w_\emptyset, w_H), (s_L, s_\emptyset, s_H) \)

s.t. (1), (2) and (3).

Proposition 1 summarizes the set of solutions to this program.

Proposition 1. If unofficial activities are not possible, the optimal contracts are such that

\( w_\emptyset \in [0, \frac{\gamma}{1 - p}], \quad w_L \in [0, \frac{\gamma}{p} - \frac{1 - p}{p} w_\emptyset], \quad w_H = \frac{\gamma + w_\emptyset}{\pi} - \frac{1 - p}{p} w_\emptyset - \frac{(1 - \pi)}{\pi} w_L, \quad \text{and} \quad (s_L, s_\emptyset, s_H) = (0, 0, 0). \)

The expected cost of production and supervision is \( C^0 = \gamma + \mu. \)

The proposition states that, in the absence of unofficial activities, the principal does not need to provide rents to obtain information. We now study if this result still holds when the supervisor may engage in unofficial activities.

4 Optimal contracts in the presence of a single type of unofficial activity

In this section, we study the cases where the supervisor may only engage in a single form of unofficial activity. We respectively consider the case where only collusion is possible and the case where the supervisor may only engage in abuse of authority.

4.1 Collusion

In an organization where only collusion is possible, the supervisor and the agent may collude when supervision reveals the evidence that a low output has been produced. This takes place either when the agent shirks or when she works hard but is unlucky. In both of these cases, the agent promises to pay a bribe to the supervisor if he accepts to conceal information and report \( r = \emptyset \) instead of \( r = x_L. \)

We now turn to the determination of the additional constraint that the deterrence of collusion imposes on the organization.\(^{20}\)

If collusion occurs, the agent’s utility is \( w_\emptyset - b \) if she has shirked, and \( w_\emptyset - b - \gamma \) if she has worked but has been unlucky, where \( b \) is the bribe offered to the supervisor. If instead collusion does not occur, the agent’s utility is \( w_L \) if she has shirked, and \( w_L - \gamma \) if she has worked but has been unlucky. The agent is thus ready to form a coalition with the supervisor if \( w_\emptyset - b \geq w_L, \)

\(^{20}\)Note that in the environment considered here, there is no loss of generality in restricting attention to contracts that deter collusion.
that is, if $b \leq w_\emptyset - w_L$. The maximum bribe, $b^M$, the agent is willing to offer for an uninformative report is therefore $b^M \equiv w_\emptyset - w_L$. Given that the supervisor has all the bargaining power, he can extract $b^M$ from the agent. Observe that we make the standard assumption that the supervisor does not engage in unofficial activities when he is indifferent. Collusion between the supervisor and the agent will hence not occur if the supervisor’s utility from making a truthful report, $s_L$, exceeds his utility from concealing information, $s_\emptyset + kb^M$, that is, if

$$s_L \geq s_\emptyset + k(w_\emptyset - w_L),$$

where $w_\emptyset - w_L$ is the stake of supervisor/agent collusion. This constraint is the no-collusion constraint. It expresses that collusion can be prevented either by creating incentive payments for the supervisor or by destroying its stake, that is, by setting $w_\emptyset \leq w_L$. The no-collusion constraint must be added into program $[P_0]$ when looking for the optimal contracts in the presence of collusion.

Since it is optimal for the principal to set $s_\emptyset = 0$ and one must have $s_L \geq 0$, collusion can be costlessly prevented if the principal sets $w_\emptyset \leq w_L$, and hence $s_L = 0$. In other words, all collusion-free contracts of Proposition 1 verifying $w_\emptyset \leq w_L$ are collusion-proof. We therefore have:

Proposition 2. When collusion is the only possible type of unofficial activity, it does not affect the efficiency of the organization.

In the next section we show that collusion is not harmless anymore when the supervisor may also engage in abuse of authority.

Observe that a similar result as that of Proposition 2 is proven in Vafaï (1994, 2002). However, the reason behind Vafaï’s (1994, 2002) result is different from the one here. Indeed, collusion is harmless in Vafaï (1994, 2002) because $\pi = 1$, and hence the supervisor’s wage $s_L$ does not enter the objective function. This wage can then be set as large as to deter collusion without affecting the efficiency of the organization.

We are not of course claiming that collusion is systematically costless to prevent. Our result suggests that, unlike in certain adverse selection environments, collusion, when occurring alone, may be completely harmless in a moral hazard setting while, as will be shown below, abuse of authority is not always costless to prevent in such a setting. As discussed in Vafaï (2002), collusion can also be harmless in adverse selection organizations. Indeed, recent papers show that the harmfulness of collusion is very dependent on the environment in which collusion may occur (e.g., Olsen and Torsvik, 1998; Kessler, 2000). These papers prove that if the standard adverse selection model of collusion is slightly modified collusion becomes harmless or even beneficial.\footnote{For instance, Kessler (2000) proves that collusion becomes costless to deter if the timing of the observation of the supervisor in the standard collusion model with adverse selection is modified.}
4.2 Abuse of authority

In the case where abuse of authority is the only possible type of unofficial activity, the supervisor may abuse his authority when supervision reveals the evidence that the agent has produced a high output. The supervisor then threatens the agent with an uninformative report if she refuses to pay him a tribute. The agent’s utility is $w_H - t - \gamma$ if she accepts to pay the demanded tribute, denoted $t$, and $w_\emptyset - \gamma$ if she refuses. The agent therefore accepts to pay a tribute for information revelation if $w_H - t - \gamma \geq w_\emptyset - \gamma$, that is, if $t \leq w_H - w_\emptyset$. Since the supervisor can extract the maximum amount of $t_M \equiv w_H - w_\emptyset$ from the agent, his utilities corresponding respectively to abusing and not his authority are $s_H + ktM$ and $s_H$. The supervisor thus engages in abuse of authority whenever he has a stake in it, that is, whenever $t_M > 0$.22

Accordingly, when the supervisor may engage in abuse of authority, the agent’s incentive compatibility constraint is, $p \left[ \pi(w_H - tM) + (1 - \pi)w_L \right] + (1 - p)w_\emptyset - \gamma \geq pw_L + (1 - p)w_\emptyset$, or equivalently,

$$w_\emptyset - w_L \geq \frac{\gamma}{p\pi}. \quad (5)$$

Similarly, the agent’s participation constraint is, $p \left[ \pi(w_H - tM) + (1 - \pi)w_L \right] + (1 - p)w_\emptyset - \gamma \geq w_\emptyset$, or equivalently,

$$(1 - p(1 - \pi))w_\emptyset + p(1 - \pi)w_L \geq \gamma + w_\emptyset. \quad (6)$$

Since abuse of authority takes place as long as $t_M > 0$, the principal also faces the following constraint,

$$w_H - w_\emptyset \geq 0. \quad (7)$$

When only abuse of authority is possible, the program of the organization is therefore

$$\begin{align*}
\min_{[PA]} & \quad p \left[ \pi(w_H + s_H) + (1 - \pi)(w_L + s_L) \right] + (1 - p)(w_\emptyset + s_\emptyset) \\
\text{s.t.} & \quad (3), (5), (6) \text{ and } (7).
\end{align*}$$

22Note that we do not account for the possibility of the agent appealing to the principal or lodging a complaint against her superior. Indeed, the agent cannot prove her allegations if she decides to lodge a complaint against the supervisor since abuse of authority cannot be observed and the information provided by the supervisor is the only available information. This accords with empirical evidence that lodging a complaint is often ineffective. For example, in the case of sexual harassment, Husbands (1992, p. 556) writes: “A complaint may encounter a number of practical obstacles in litigating a sexual harassment case. In pursuing any type of civil case in the countries surveyed, the burden of proofs falls on the complainant alleging the harassment... the proposition asserted by the complainant may be difficult to prove in a sexual harassment case. Most propositions for tangible job benefits in exchange for sexual favors are not made with witnesses present, so it may often be the complainant’s word against the alleged harasser’s.” He then notes: “There is evidence that a great deal of sexual harassment goes unreported.” (p. 538). As observed by Roberts and Mann (1996, p. 270): “...most cases of sexual harassment still go unreported: as many as ninety-five percent of all such incidents may not be brought to light.” On this issue, see also Peirce et al. (1998).
Define $\tilde{p} \equiv \frac{\gamma}{\gamma+u}$. The solution to this program is summarized in Proposition 3.

**Proposition 3.** The organization where only abuse of authority is possible has the following properties:

1. For $p \leq \tilde{p}$, the optimal contracts are $(w_L, w_\emptyset, w_H) = (0, \frac{\gamma}{p\pi}, \frac{\gamma}{p\pi})$ and $(s_L, s_\emptyset, s_H) = (0, 0, 0)$. Abuse of authority reduces the efficiency of the organization. The expected cost of production and supervision is $C^A = \left[1 - p(1-\pi)\right] \frac{\gamma}{p\pi}$.

2. For $p > \tilde{p}$, the optimal contracts are such that $w_L \in \left[0, \frac{\gamma u - (1-p)\gamma}{p\pi}\right]$, $w_\emptyset = w_H = \frac{\gamma + u}{1 - p(1-\pi)} - \frac{p(1-\pi)}{1 - p(1-\pi)} w_L$, and $(s_L, s_\emptyset, s_H) = (0, 0, 0)$. Abuse of authority does not affect the efficiency of the organization. The expected cost of production and supervision is $C^0$.

The principal has the choice between tolerating and deterring abuse of authority. Recall that the supervisor engages in abuse of authority whenever $w_H > w_\emptyset$. Since the expected cost of production and supervision is increasing in $w_H$, it is clearly optimal for the principal to deter abuse of authority by setting $w_H = w_\emptyset$, that is, by destroying its stake, rather than tolerating it by instead setting $w_H > w_\emptyset$.

There are less instruments available to deter abuse of authority than to deter collusion. Indeed, unlike collusion, abuse of authority cannot be deterred by an incentive policy, that is, a policy which creates incentive payments for the supervisor to reveal his information. An incentive policy is ineffective against abuse of authority because when the supervisor engages in this type of unofficial activity he does not have to trade-off a low wage (i.e., a wage associated with an uninformative report) plus a tribute, and a high wage (i.e., a wage associated with an informative report) since he can get both.

As expressed above, the only available policy against abuse of authority is a policy that destroys its stake, that is, that reduces the supervisor’s discretion. When adopting this policy, the principal makes the incentive payments to the agent less sensitive to the supervisor’s report. The principal then offers a flat wage to the agent whenever $r \neq x_L$. That is, the principal sets $w_H = w_\emptyset$.

Proposition 3 also states that, unlike collusion, abuse of authority cannot always be costlessly deterred. This type of unofficial activity affects the efficiency of the organization whenever the supervision technology is sufficiently inefficient (i.e., $p \leq \tilde{p}$). In this case, the agent receives a rent. However, when $p > \tilde{p}$, abuse of authority is harmless. Abuse of authority is thus sometimes harmless in the case where the supervisor may only engage in this type of unofficial activity. In the next section, we show that when the organization is exposed to both collusion and abuse of authority, unofficial activities are never harmless.

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23 As shown by Vafaï (2002), preventing abuse of authority is also optimal in the case where supervision is costly.
Before proceeding, it is important to observe that the assumption of limited liability plays an essential role in the results of Proposition 3.24 Indeed, as shown in the proof of this proposition, relaxing the assumption of limited liability results in abuse of authority becoming harmless. This is because the principal can then systematically set \( w_L \) as low as to keep the agent at her reservation utility level. Unlike collusion, abuse of authority is harmless only in the absence of limited liability.

5 Optimal contracts in the presence of multiple types of unofficial activity

We now turn to the case where both collusion and abuse of authority are possible.

Constraint (4) must be added into program \([P^A]\) when looking for the optimal contracts in an organization where both collusion and abuse of authority are possible. Let \([P^{CA}]\) denote this new program. Proposition 4 then summarizes the solution to \([P^{CA}]\).

Proposition 4. The organization where both collusion and abuse of authority are possible has the following properties:

1. For \( p \leq \bar{p} \), the optimal contracts are \( (w_L, w_0, w_H) = (0, \frac{\gamma p}{p \pi}, \frac{u}{p \pi}) \) and \( (s_L, s_0, s_H) = (\frac{k \gamma p}{p \pi}, 0, 0) \). The expected cost of production and supervision is \( C_1^{CA} = \frac{(1 - p(1 - \pi)(1 - k))\gamma}{p \pi} \).

2. For \( p > \bar{p} \), the optimal contracts are \( (w_L, w_0, w_H) = (\frac{\pi \gamma k - (1 - p)\gamma}{p \pi}, \frac{\pi \gamma k + u}{p \pi}, \frac{\pi \gamma k - \frac{u}{p \pi}}{p \pi}) \) and \( (s_L, s_0, s_H) = (\frac{k \gamma p}{p \pi}, 0, 0) \). The expected cost of production and supervision is \( C_2^{CA} = \frac{(1 - \pi)(\frac{k \gamma p}{p \pi} + u)}{p \pi} \).

3. Compared to the case where only abuse of authority may occur, the presence of multiple types of unofficial activity increases both the scope and the intensity of the efficiency loss sustained by the organization.

As in the previous section, it is optimal to prevent abuse of authority by setting \( w_H = w_0 \). However, unlike in the absence of abuse of authority, collusion is now harmful. The intuition for this is as follows.

When collusion is the only unofficial activity that may occur, there are two policies available to deter it. Indeed, as explained in Section 4, in the absence of abuse of authority collusion can be deterred either by creating incentive payments for the supervisor or by destroying its stake, that is, by setting \( w_0 \leq w_L \) - or equivalently, by eliminating the gap between \( w_0 \) and \( w_L \) or by specifying a negative gap between these two wages. When both forms of unofficial activity may take place, this second policy is not available anymore. As established above, the possibility of abuse of authority modifies the agent’s incentive compatibility constraint which becomes \( w_0 - w_L \geq \frac{k \gamma p}{p \pi} \).

The possibility of abuse of authority thus makes it impossible for the principal to deter collusion.

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24This assumption systematically plays an essential role in models of unofficial activities in organizations.
by setting $w_0 \leq w_L$. The only available policy for preventing collusion is then to create incentive payments for the supervisor, that is, to set $s_L = k(w_0 - w_L) > 0$. The presence of abuse of authority therefore makes collusion costly to deter. Both the supervisor and the agent now receive a rent.

Compared to the case where only abuse of authority is possible, the presence of both collusion and abuse of authority has two important consequences.

First, it increases the scope of the efficiency loss sustained by the organization. In other words, while the sole presence of abuse of authority only reduces the efficiency of the organization when $p \leq \tilde{p}$, the presence of both abuse of authority and collusion also reduces the efficiency of the organization when $p > \tilde{p}$. Unofficial activities are now always harmful.

Second, as it is easy to verify, the expected cost of production and supervision increases when the supervisor may engage in both collusion and abuse of authority. That is, $C^{CA} > C^A$ and $C^{CA} > C^0$. The presence of multiple types of unofficial activity thus also increases the intensity of the efficiency loss sustained by the organization.

Another interesting point is to study if the cost of preventing both forms of unofficial activity, $C^{CA}$, is larger than the sum of the costs of preventing a single form of unofficial activity, $C^A + C^0$. Formally, one must verify if $C^{CA} \geq C^0 + C^A$ when $p \leq \tilde{p}$, and if $C^{CA} \geq 2C^0$ when $p > \tilde{p}$. In words, it is interesting to investigate if the presence of multiple forms of unofficial activity entails cost super-additivity. The following corollary identifies the condition under which this is the case.

Define $\tilde{\pi} \equiv \frac{k\gamma}{(1+k)\gamma+2}$. One then has:

**Corollary.** For $\pi \leq \tilde{\pi}$, the presence of multiple forms of unofficial activity entails cost super-additivity.

The intuition for this result is based on the observation that the above cost comparisons boil down to $p(1-\pi)\frac{k\gamma}{p\sigma} \geq C^0$, or equivalently, $\frac{(1-\pi)k\gamma}{\pi} \geq C^0$. The left-hand side of this inequality is the extra expected cost of collusion deterrence in an organization with both collusion and abuse of authority whereas the right-hand side is the expected cost of an organization where only collusion may occur. Given that the additional expected cost of collusion deterrence increases as $\pi$ decreases while the expected cost $C^0$ is a constant, one has that when $\pi$ is relatively small (i.e. $\pi \leq \tilde{\pi}$), the presence of multiple forms of unofficial activity entails cost super-additivity. The opposite holds for $\pi$ relatively large.

As discussed in the previous section, recent studies of collusion in hierarchies have shown that this type of unofficial activity does not systematically reduce the efficiency of organizations. We have found that this is also the case with abuse of authority. This form of unofficial activity is

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25 More formally, one has $C_1^{CA} > C^A$ for $p \leq \tilde{p}$, and $C_2^{CA} > C^0$ for $p > \tilde{p}$.
not systematically harmful when employees are protected by limited liability and is harmless in the absence of limited liability. Things are however different when the organization is exposed to multiple forms of unofficial activity. Indeed, as shown in the proof of Proposition 4, when employees are not protected by limited liability the optimal contracts are identical to those of case 2 of Proposition 4. Therefore, when both collusion and abuse of authority may occur, unofficial activities are always harmful even in the absence of limited liability. This is another illustration of the fact that, unlike the presence of a single form of unofficial activity, the presence of multiple forms of unofficial activity may systematically reduce the efficiency of organizations.

6 Conclusion

Recently, investigations of optimal incentive contracts in principal-supervisor-agent hierarchies have accounted for the possibility of collusion between the supervisor and the agent. In this research, we have extended the analysis of these hierarchies by allowing the supervisor to engage in both collusion and abuse of authority. We have characterized the optimal incentive contracts in such an environment, and have analyzed how collusion and abuse of authority interact.

Our results suggest that investigating a single type of unofficial activity in organizations may be misleading. Indeed, we have found that, while in the absence of abuse of authority collusion is harmless, this is not true anymore when both types of unofficial activity are possible. We have shown that, compare to the cases where a single type of unofficial activity may occur, the presence of collusion and abuse of authority increases both the scope and the intensity of the efficiency loss sustained by the organization. We have also proved that the presence of multiple forms of unofficial activity may entail cost super-additivity.

The approach taken in this paper should be considered a first step in the study of the interaction between abuse of authority and collusion in organizations. Since in many real world situations evidence is not verifiable, and thus can be forged, an important extension would be to investigate collusion and abuse of authority in a moral hazard hierarchy with soft information. Another interesting extension would be to relax the simplifying assumptions of a binary effort decision and of the existence of an effort target to account for more complex environments. Finally, in this paper we have dealt exclusively with moral hazard. The study of unofficial activities in an adverse selection environment is also an important topic for future research.
Appendix

Proof of Proposition 1. Before turning to the derivation of the optimal incentive contracts in the different scenarios, it is straightforward to see that, in each of these scenarios, the principal sets \( s_0 \) as low as allowed by the limited liability constraints, that is, \( s_0 = 0 \). This is because an uninformative report is undesirable from the principal’s point of view.

As explained in the text, any contract \((s_L, s_0, s_H) \in \mathbb{R}^3_+\) is individually rational. Given that the expected cost of production and supervision is increasing in wages paid, the principal sets the supervisor’s wages \( s_L \) and \( s_H \) as low as allowed by the limited liability constraints, that is, \( s_L = s_H = 0 \).

Rewriting the agent’s incentive and participation constraints respectively as \( w_H \geq \frac{L}{p} + w_L \) and \( w_H \geq \frac{L}{p} - \frac{1-p}{p} \), one has two constraints on \( w_H \) (plus the limited liability constraints).

To find which of the incentive compatibility constraint and the participation constraint is more restrictive, one must compare \( \frac{L}{p} + w_L \) to \( \frac{L}{p} - \frac{1-p}{p} w_0 - \frac{(1-\gamma)}{\pi} w_L \). One has that the incentive compatibility constraint is more restrictive than the participation constraint if \( \frac{L}{p} + w_L \geq \frac{L}{p} - \frac{1-p}{p} w_0 - \frac{(1-\gamma)}{\pi} w_L \), that is, if \( w_L \geq \frac{L}{p} - \frac{1-p}{p} w_0 \). The converse is true if \( w_L \leq \frac{L}{p} - \frac{1-p}{p} w_0 \). Therefore,

1. If \( w_L \geq \frac{L}{p} - \frac{1-p}{p} w_0 \), the relevant constraint is the agent’s incentive compatibility constraint.

Since limited liability requires \( w_L \geq 0 \), one must then have \( w_0 \leq \frac{L}{p} \). Given that the objective function is increasing with \( w_H \) and \( w_L \), the principal sets \( w_H \) as low as allowed by the agent’s incentive compatibility constraint, that is, \( w_H = \frac{L}{p} + w_L \). Similarly, the constraint \( w_L \geq \frac{L}{p} - \frac{1-p}{p} w_0 \) binds, otherwise the principal can reduce the expected cost of production and supervision by lowering \( w_L \) only slightly, while still satisfying the constraint in question. Substituting \( w_L = \frac{L}{p} - \frac{1-p}{p} w_0 \) into the agent’s incentive compatibility constraint then yields \( w_H = \frac{L}{p} + \frac{1-p}{p} w_0 \).

Finally, substituting \( w_L = \frac{L}{p} - \frac{1-p}{p} w_0 \) and \( w_H = \frac{L}{p} + \frac{1-p}{p} w_0 \) into the objective function of program \([P_0]\), one obtains the new objective function \( \gamma + w_L \). One must then minimize this function under the constraint \( w_0 \in \left[ 0, \frac{L}{1-p} \right] \). Since \( w_0 \) does not enter the new objective function, the set of solutions to this program is \( w_0 \in \left[ 0, \frac{L}{1-p} \right], w_H = \frac{L}{p} + \frac{1-p}{p} w_0, \) and \( w_L = \frac{L}{p} - \frac{1-p}{p} w_0 \).

2. If \( w_L \leq \frac{L}{p} - \frac{1-p}{p} w_0 \), the relevant constraint is the agent’s participation constraint. As above, one must have \( w_0 \leq \frac{L}{1-p} \). The agent’s participation constraint binds, otherwise the principal can reduce the expected cost of production and supervision by lowering \( w_H \) only slightly, while still motivating the agent to participate. One then has \( w_H = \frac{L}{p} + \frac{1-p}{p} w_0 - \frac{(1-\gamma)}{\pi} w_L \). Since limited liability requires \( w_H \geq 0 \), one must have \( w_L \leq \frac{L}{p} + \frac{1-p}{p} w_0 - \frac{(1-\gamma)}{\pi} w_L \). Similarly, one must have \( w_L \geq 0 \), and hence \( w_0 \leq \frac{L}{1-p} \). One now has two constraints on \( w_L \): \( w_L \leq \frac{L}{p} - \frac{1-p}{p} w_0 \) and \( w_L \leq \frac{L}{p} + \frac{1-p}{p} w_0 \). One has \( \frac{L}{p} - \frac{1-p}{p} w_0 \) \( \frac{L}{p} + \frac{1-p}{p} w_0 \) \( \gamma + w_H \). Therefore, given that limited liability requires that \( w_0 \) be smaller than \( \frac{L}{1-p} \), the relevant constraint on \( w_L \) is \( w_L \leq \frac{L}{p} - \frac{1-p}{p} w_0 \). Substituting \( w_H = \frac{L}{p} + \frac{1-p}{p} w_0 - \frac{(1-\gamma)}{\pi} w_L \) into
the objective function of program \([P_0]\), one obtains the new objective function \(\gamma + \frac{u}{p}\). One must then minimize this function under the constraints \(w_L \in [0, \frac{\mu}{p} - \frac{1-p}{p} w_\emptyset]\) and \(w_\emptyset \in [0, \frac{\mu}{1-p}]\). Given that \(w_L\) and \(w_\emptyset\) do not enter the new objective function, the set of solutions to the principal’s program is \(w_\emptyset \in [0, \frac{\mu}{1-p}]\), \(w_L \in [0, \frac{\mu}{p} - \frac{1-p}{p} w_\emptyset]\), and \(w_H = \frac{\gamma + u}{p\pi} - \frac{1-p}{p\pi} w_\emptyset - \frac{(1-\pi)}{\pi} w_L\). Since it includes the above set of solutions, this set of solutions is the optimal set of solutions to program \([P_0]\). The expected cost of production and supervision is therefore \(C^0 = \gamma + \frac{u}{p}\).

**Proof of Proposition 3.** Given that the expected cost of production and supervision is increasing in \(w_H\) and one must have \(w_H \geq w_\emptyset\), it is optimal for the principal to set \(w_H = w_\emptyset\).

The agent’s incentive compatibility constraint and participation constraints can be respectively rewritten as \(w_\emptyset \geq \frac{\gamma + u}{p\pi} + w_L\) and \(w_\emptyset \geq \frac{\gamma + u}{1-p(1-\pi)} - \frac{p(1-\pi)}{1-p(1-\pi)} w_L\). To find which of these two constraints is more restrictive, one must compare \(\frac{\gamma + u}{p\pi} + w_L\) to \(\frac{\gamma + u}{1-p(1-\pi)} - \frac{p(1-\pi)}{1-p(1-\pi)} w_L\). One has that the agent’s incentive compatibility constraint is more restrictive if \(w_L \geq \frac{p\pi - (1-p)\gamma}{p\pi}\). The converse is true if \(w_L \leq \frac{p\pi - (1-p)\gamma}{p\pi}\). Since limited liability requires \(w_L \geq 0\) and one has \(\frac{p\pi - (1-p)\gamma}{p\pi} \leq 0\) if \(p \leq \bar{p} \equiv \frac{\gamma + u}{\gamma + \frac{u}{p\pi}}\), the agent’s incentive compatibility constraint is always the relevant constraint when \(p \leq \bar{p}\). In this case the agent’s incentive compatibility constraint binds, otherwise the principal can reduce the expected cost of production and supervision by lowering \(w_\emptyset\) only slightly, while still motivating the agent to work hard. One thus has \(w_\emptyset = \frac{\gamma + u}{p\pi} + w_L\). Clearly the wage \(w_L\) must then be set as low as allowed by the limited liability constraint, that is, \(w_L = 0\), and hence \(w_\emptyset = \frac{\gamma + u}{p\pi}\) (\(= w_H\)).

When instead \(p > \bar{p}\), two cases must be distinguished.

(a) If the principal sets \(w_L \geq \frac{p\pi - (1-p)\gamma}{p\pi}\), the agent’s incentive compatibility constraint is the relevant constraint. From the same reasoning as above, one then has \(w_\emptyset = \frac{\gamma + u}{p\pi} + w_L\). Substituting \(w_\emptyset = \frac{\gamma + u}{p\pi} + w_L\) into the objective function of program \([P^A]\), one obtains the new objective function \(\frac{(1-p(1-\pi))\gamma}{p\pi} + w_L\). One must then minimize this function under the constraint \(w_L \geq \frac{p\pi - (1-p)\gamma}{p\pi}\). Since the objective function is increasing in \(w_L\), it is optimal to set \(w_L\) as low as allowed by the constraint, that is, \(w_L = \frac{p\pi - (1-p)\gamma}{p\pi}\). One thus has \(w_\emptyset = \frac{\gamma + u}{p\pi}\) (\(= w_H\)).

(b) If the principal sets \(w_L \leq \frac{p\pi - (1-p)\gamma}{p\pi}\), the agent’s participation constraint is the relevant constraint. Since limited liability requires \(w_\emptyset \geq 0\), one must then have \(w_L \leq \frac{\gamma + u}{1-p(1-\pi)} - \frac{p(1-\pi)}{1-p(1-\pi)} w_L \geq 0\), that is, \(w_L \leq \frac{\gamma + u}{p(1-\pi)}\). Given that one must also have \(w_L \leq \frac{p\pi - (1-p)\gamma}{p\pi}\), one has \(w_L \leq \min\left\{\frac{p\pi u - (1-p)\gamma}{p\pi}, \frac{\gamma + u}{p(1-\pi)}\right\}\). From the above reasoning, one has that the agent’s participation constraint binds. Substituting \(w_\emptyset = \frac{\gamma + u}{1-p(1-\pi)} - \frac{p(1-\pi)}{1-p(1-\pi)} w_L\) into the objective function of program \([P^A]\), one obtains the new objective function \(\gamma + \frac{u}{p}\). This new objective function must then be minimized under the constraint \(w_L \in \left[0, \frac{p\pi - (1-p)\gamma}{p\pi}\right]\). Given that \(w_L\) does not enter this function, any \(w_L \in \left[0, \frac{p\pi - (1-p)\gamma}{p\pi}\right]\) is solution to this program. To sum up, when \(p > \bar{p}\), one has \(w_L \in \left[0, \frac{p\pi - (1-p)\gamma}{p\pi}\right]\) and \(w_\emptyset = \frac{\gamma + u}{1-p(1-\pi)} - \frac{p(1-\pi)}{1-p(1-\pi)} w_L\).
As in the previous proof, the principal sets the supervisor’s wages \( s_L \) and \( s_H \) as low as allowed by the limited liability constraints, that is, \( s_L = s_H = 0 \). The expected cost of production and supervision is then \( C^A = \frac{(1 - p(1 - \pi))}{p \pi} \) if \( p \leq \bar{p} \) and \( C^0 \) otherwise.

Before turning to the proof of Proposition 4, observe that abuse of authority is harmless in the case where employees are not protected by limited liability. The reason behind this is that the principal can then always set \( w_L \) as low as to keep the agent at her reservation utility level. Clearly, in the absence of limited liability the principal can systematically set \( w_L \leq \frac{p \pi u - (1 - p) \gamma}{p \pi} \), where \( \frac{p \pi u - (1 - p) \gamma}{p \pi} \in \mathbb{R} \). As explained above, the relevant constraint is then the agent’s participation constraint and this constraint binds. One thus has \( w_H = w_H = \frac{(\gamma + u)}{1 - p(1 - \pi)} = \frac{p(1 - \pi)}{1 - p(1 - \pi)} w_L, s_L = s_H = 0 \), and the expected cost of production and supervision is always \( C^0 \).

**Proof of Proposition 4.** As in the preceding proofs, one has \( s_H = 0 \). Since the expected cost of production and supervision is increasing in \( s_l \), and one has \( w_H - w_L > 0 \) from the agent’s incentive compatibility constraint, the no-collusion constraint binds. That is, \( s_L = k(w_H - w_L) \). Given that one must have \( s_L \geq 0 \), one then has \( w_H \geq w_L \). This constraint is less restrictive than the agent’s incentive compatibility constraint, and hence can be ignored. As in the previous proof, one must compare the agent’s participation constraint to her incentive compatibility constraint. Following the lines of the proof of Proposition 3, one has that when \( p \leq \bar{p} \), the agent’s incentive compatibility constraint is the relevant constraint and this constraint binds. Substituting \( w_H = \frac{\gamma + u}{p \pi} + w_L \) into the no-collusion constraint then yields \( s_L = \frac{k \gamma}{p \pi} \). Finally, from the same reasoning as in the previous proof, one has \( w_L = 0 \) and \( w_H = w_H = \frac{\gamma}{p \pi} \) when \( p \leq \bar{p} \).

Again, as in the proof of Proposition 3, two cases must be considered when \( p > \bar{p} \).

(a) Following the lines of the preceding proof, if the principal sets \( w_L \geq \frac{p \pi u - (1 - p) \gamma}{p \pi} \), the agent’s incentive compatibility constraint is the relevant constraint. As explained above, this constraint binds, and hence one has \( s_L = \frac{k \gamma}{p \pi} \). Then, from the same reasoning as in the proof of Proposition 3, one has \( w_L = \frac{p \pi u - (1 - p) \gamma}{p \pi} \), and \( w_H = \frac{\gamma}{p \pi} (= w_H) \).

(b) If instead the principal sets \( w_L \leq \frac{p \pi u - (1 - p) \gamma}{p \pi} \), the agent’s participation constraint is the relevant constraint. As in the previous proof, one has that this constraint binds. One must then substitute \( w_H = \frac{\gamma + u}{1 - p(1 - \pi)} - \frac{p(1 - \pi)}{1 - p(1 - \pi)} w_L \) into \( s_L = k(w_H - w_L) \). This yields \( s_L = \frac{k(\gamma + u)}{1 - p(1 - \pi)} - \frac{k(1 - p)}{1 - p(1 - \pi)} w_L \). Given that limited liability requires \( w_H \geq 0 \) and \( s_L \geq 0 \), one must then have \( \frac{k(\gamma + u)}{1 - p(1 - \pi)} - \frac{k(1 - p)}{1 - p(1 - \pi)} w_L \geq 0 \). That is, \( w_L \leq \min \left\{ \frac{\gamma + u}{1 - p(1 - \pi)}, \gamma \right\} = \frac{\gamma + u}{p(1 - \pi)} \).

Since one must also have \( w_L \leq \frac{p \pi u - (1 - p) \gamma}{p \pi} \), one has \( w_L \leq \min \left\{ \frac{p \pi u - (1 - p) \gamma}{p \pi}, \frac{\gamma + u}{1 - p(1 - \pi)} \right\} = \frac{p \pi u - (1 - p) \gamma}{p \pi} \).

Substituting \( w_H = \frac{\gamma + u}{1 - p(1 - \pi)} - \frac{p(1 - \pi)}{1 - p(1 - \pi)} w_L \) and \( s_L = \frac{k(\gamma + u)}{1 - p(1 - \pi)} - \frac{k(1 - p)}{1 - p(1 - \pi)} w_L \) into the objective function of program \([PC\pi A]\), one obtains the new objective function \( \frac{(1 - p(1 - \pi))(\gamma + u)}{1 - p(1 - \pi)} - \frac{pk(1 - \pi)}{1 - p(1 - \pi)} w_L \).

This new objective function must be minimized under the constraint \( w_L \in \left[ 0, \frac{p \pi u - (1 - p) \gamma}{p \pi} \right] \). Given
that this objective function is decreasing in $w_L$, it is optimal to set $w_L = \frac{p\pi u - (1-p)\gamma}{p\pi}$. One therefore has $w_\emptyset = \frac{\pi u + \gamma}{\pi} (= w_H)$ and $s_L = \frac{k\gamma}{p\pi}$.

Since cases (a) and (b) yield the same result, the expected cost of production and supervision is $C^{CA}_1 = \frac{[1-p(1-\pi)(1-k)]\gamma}{p\pi}$ if $p \leq \bar{p}$ and $C^{CA}_2 = \frac{(1-(1-\pi)k+\pi)\gamma}{\pi} + \Pi$ otherwise.

As it straightforward to verify, one has $C^{CA}_1 > C^A$ when $p \leq \bar{p}$, and $C^{CA}_2 > C^0$ when $p > \bar{p}$. This proves the third result of Proposition 4.

Before turning to the proof of the Corollary, observe that even in the absence of limited liability unofficial activities are always harmful. Indeed, as it is straightforward to verify, dropping the assumption of limited liability and following the above reasoning yield $w_L = \frac{p\pi u - (1-p)\gamma}{p\pi}$, $w_\emptyset = \frac{\pi u + \gamma}{\pi}$ (= $w_H$), and $s_L = \frac{k\gamma}{p\pi}$. In the absence of limited liability the expected cost of production and supervision is then always $C^{CA}_2$. Since one clearly has $C^{CA}_2 > C^0$, the presence of multiple forms of unofficial activity thus systematically reduces the efficiency of the organization.

**Proof of the Corollary.** To see if the presence of multiple forms of unofficial activity entails cost super-additivity, one must check if $C^{CA}_1 \geq C^0 + C^A$ when $p \leq \bar{p}$; and if $C^{CA}_2 \geq 2C^0$ when $p > \bar{p}$. One has both $C^{CA}_1 \geq C^0 + C^A$ and $C^{CA}_2 \geq 2C^0$ when $\pi \leq \bar{\pi} \equiv \frac{k\gamma}{(1+k)\gamma + \Pi}$. 

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References


