Two-sided markets with multihoming and exclusive dealing

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Abstract

We analyze some models of price competition in two-sided markets, allowing for the possibility of agents to purchase from multiple platforms (multihome) and for prices to be restricted to be non-negative. The models are used to illustrate a number of different effects which influence the structure of prices in two-sided markets, as well as to explore the role of exclusive dealing in these markets.

1 Introduction

Two-sided markets involve two distinct types (or groups) of agents, each of whom obtains value from interacting with agents of the opposite type. In these markets, platforms deal with both types of agents in a way that allows them to influence the extent to which cross-group externalities are enjoyed. Commonly cited examples include directory services such as classifieds and Yellow Pages which cater to potential buyers and sellers; entertainment platforms such as cable TV and video game platforms which cater to users and content providers; matching markets such as (heterosexual) dating agencies which cater to men and women; payment schemes such as debit and credit card schemes which cater to cardholders and retailers; and trading posts such as auctions, business-to-business markets, car fairs, flea markets, and shopping malls which cater to buyers and sellers.1

This paper follows Armstrong (2004), analyzing two-sided markets in the context of a standard Hotelling model of price competition between two platforms. There are two groups of agents, these agents make a subscription decision, and network effects run between the two groups. Unlike Armstrong, in this paper agents choose whether to subscribe to a single platform (singlehome) or to both platforms (multihome).2 Additionally, we explore the implications of what happens if subscription prices must be

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1Some slightly less obvious examples include academic journals with readers and authors; computer operating systems with users and software developers; conferences with audiences and speakers; employment agencies with job seekers and employers; expos and trade fairs with potential buyers and firms promoting their goods; magazines, newspapers, public TV operators and web portals with information/entertainment seekers and advertisers; quality assurance providers such as ETS which offer GRE and TOEFL exams to students and universities; real estate agencies with home buyers and sellers (as well as tenants and landlords); search engines with searchers and websites; stock markets with companies wishing to list and to investors/traders (through brokers); and text processors such as Adobe Acrobat with readers and writers.

2Some other papers that allow for endogenous multihoming in two-sided markets, but in different contexts, include Caillaud and Jullien (2003), Gabszewicz et al. (2001), Gabszewicz and Wauthy (2004), Guthrie and Wright (2003), Hagiu
non-negative (which we argue is a reasonable restriction in many settings), and what happens if platforms can offer exclusive contracts? Relative to the existing literature, our framework offers a number of new insights.

Where there is strong product differentiation on each side of the market, the model predicts all agents buy from a single platform (singlehome). With all agents singlehoming, as Armstrong shows, the price-cost margin for each group is equal to the product differentiation parameter for that group minus the externality that joining the platform has on the other group. If attracting one group of agents (say buyers) makes the platform particularly attractive to the other group of agents (say sellers), then buyers will be “subsidized”. Given subscription prices must be non-negative, this implies that when the cost of serving buyers is not too large, buyers will be charged nothing to subscribe, while sellers will be charged a price that decreases in the extent to which the platform would like to set a negative price for buyers.

In the case where product differentiation arises only with respect to one side of the market (say, the buyer side), an equilibrium exists whereby all agents on the other side of the market (the sellers’ side) will subscribe to both platforms (multihome). This case represents that of a competitive bottleneck — platforms compete aggressively to sign up buyers, charging them less than cost (perhaps nothing), and then make their profits from sellers who want to reach these buyers and do not have a choice of which platform to join in order to reach them. We argue this case, especially that in which the buyers side pays nothing, fits the observed pricing and choice of multihoming in a number of different two-sided markets. We also show how a similar outcome arises in the case of pure network effects (no product differentiation on either side).

We show this competitive bottleneck equilibrium can be undermined when platforms have the ability to offer exclusive contracts to the seller side. Exclusive contracts allow a platform to bribe sellers not to subscribe to the rival platform. This allows the platform to attract more buyers, which reinforces the sellers’ decision to sign up exclusively. Competition between platforms results in sellers capturing most, if not all, of the surplus, while buyers and the rival platform are unambiguously worse off. The result is an equilibrium which reverses the properties of the competitive bottleneck outcome.

Our model has differing predictions for the social desirability of exclusive contracts depending on the extent of product differentiation on the buyer side. In our model, with strong product differentiation on the buyer side, exclusive contracts result in all sellers joining a single platform, but some buyers sticking to their preferred platform. Not only does this result in lower network benefits (for those buyers loyal to the excluded platform), but it also results in higher transportation costs for those buyers that do not stick to their preferred platform. We show that under the assumptions of our model, the added transportation costs and reduced network benefits exceed the cost savings to sellers, who no longer subscribe to both platforms. Exclusive contracts are inefficient. In contrast, with pure network effects exclusive contracts are efficient, since they eliminate the duplication of costs that arises under seller multihoming, and result in maximal network benefits given buyers and sellers all subscribe to a single platform.

The rest of the paper proceeds as follows. Section 2 introduces our general framework. Section 3 analyzes a version of the framework with a high degree of product differentiation on both sides of the (2004) and Rochet and Tirole (2003). Several other papers model pricing in two-sided markets without allowing for multihoming. Early examples include Parker and Van Alstyne (2000) and Schiff (2003).
market, Section 4 assumes only one side has high product differentiation, while Section 5 assumes neither side has any product differentiation. Section 6 concludes.

2 A simple model of two-sided markets

Suppose there is a measure 1 of agents of group $A$ and likewise for group $B$. We will often refer to the group $A$ types as sellers and group $B$ types as buyers. Consistent with the standard assumption of two-sided markets, we assume each type of agent values the number of agents of the other type but not the number of agents of its own type. In particular, agents get benefits $b_kn$ of subscribing to a service which allows them to access $n$ agents of the other type, where $k = A, B$.

There are two symmetric platforms, $i = 1, 2$. They both offer a service to the two types of agents. Agents of type $A$ and $B$ can subscribe to a service from either platform 1, platform 2, or both platforms if they multihome. Agents of type $k$ obtain an intrinsic benefit of $v_k^0$ from subscription regardless of whether they subscribe to a single platform or both platforms. Allowing duplicated intrinsic benefits provides one rationale for multihoming. We prefer to consider multihoming that arises for network reasons, and so allow agents to obtain intrinsic benefits of the platform only once. In our model, agents will only multihome if doing so allows them to connect with more agents of the other type.

Platforms also differ in the normal Hotelling way, so they are located at either end of a unit interval and agents are located uniformly along the unit interval. Agents of type $k$ face transportation costs $t_kx$ of travelling a distance $x$ to the platform(s) they purchase from. Transportation costs of an agent located at $x$ for a purchase from platform 1 are $t_kx$, while they are $t_k(1-x)$ for a purchase from platform 2. When multihoming, it is natural to add transportation costs together given transportation costs are treated as a real cost to agents of buying a more “distant” product. We adopt this approach.

The measure of agents of type $k$ that buy from platform $i$ exclusively is denoted $n_k^i$, while the number that multihome is denoted $N_k$. Suppose platform $i$ sets a subscription price $p_k^i$ to agents of type $k$. Unless stated otherwise, we assume prices must be non-negative. This is a reasonable restriction for many examples of two-sided markets. (Gabriel et al., 2001 make a similar restriction in analyzing newspaper subscription in a two-sided market context.) Finally, assume it costs each platform $f_k$ to provide the service to each agent of type $k$.

In summary, the net utility of an agent of type $k$ with network benefits parameter $b_k$ located at $x \in [0, 1]$ when she subscribes only to platform 1 is given by

$$v_k^1 = v_k^0 - p_k^1 - t_kx + b_k(n_k^1 + N_l),$$

for $k = A, B$, $l = A, B \neq k$. When the same agent subscribes only to platform 2 she gets net utility of

$$v_k^2 = v_k^0 - p_k^2 - t_k(1-x) + b_k(n_k^2 + N_l).$$

\[3\] If people were paid to accept Yellow Pages directories, or to enter shopping malls, or to own a games console, there would be obvious adverse selection problems. In some cases, consumers can obtain a small rebate or gift for joining a platform, but typically this requires them to make some other commitment or purchase. Since ours is a pure subscription model, there is no opportunity to exploit such pricing. In any case, our analysis can be extended to the case of small “gifts” for signing up, provided the size of such gifts is bounded.
When an agent multihomes, subscribing to both platforms, the net utility she gets is

\[ v_k^{12} = v_k^0 - p_k^1 - p_k^2 - t_k + b_k (n_k^l + n_k^2 + N_k) . \]

Platform \( i \)'s profit is

\[ \pi_i = (p_i^A - f_A) (n_i^A + N_A) + (p_i^B - f_B) (n_i^B + N_B) . \]

Platforms simultaneously choose prices, and then after observing these prices, agents simultaneously decide on which platform(s) they join. Equilibria considered are Subgame Perfect Nash Equilibria.

In the following sections we analyze various special cases of this framework.

3 Strong product differentiation on both sides

We are interested in whether there are some sufficient conditions to ensure a unique equilibrium exists in which all agents choose to buy from a single platform (singlehome). We consider both the case in which both sides face a positive price, and the case in which one group is charged nothing due to a strong asymmetry in the network effects between the two groups. We make the following assumptions.

A1 \( v_A^0 \) and \( v_B^0 \) are sufficiently high such that all agents will always wish to subscribe to at least one of the platforms in equilibrium

A2 \( t_A > b_A \) and \( t_B > b_B \)

A3 \( 4t_A t_B > (b_A + b_B)^2 \)

The first assumption ensures we do not have to consider the case where some agents prefer not to subscribe (we relax it when considering pure network effects, in Section 5). The second assumption ensures that transportation costs are sufficiently high so that agents will never want to multihome at non-negative prices. This result is demonstrated in Lemma 1. It also ensures that there is a unique consistent demand configuration for any given set of prices.\(^4\) Thus, in contrast to subsequent sections, here we do not need to define how agents choose among multiple consistent demand configurations for a given set of prices. The third assumption is needed so that the platforms’ profit functions, which are quadratic functions of prices, are concave. The assumptions (A1)-(A3) are sufficient conditions for our results below. Necessary conditions are more complicated (see Appendix A for some discussion).

Lemma 1 Assume (A2). No agent will ever multihome at any non-negative prices set by the two platforms.

Proof. For any set of prices \( p_k^1 \) and \( p_k^2 \), \( k = A, B \), the agent of type \( k \) most likely to want to multihome must be located at \( x \) where \( v_k^1 (x) = v_k^2 (x) \), provided \( x \in (0, 1) \). This represents the lowest utility agents can get from singlehoming (the utility from multihoming does not depend on \( x \)). Evaluated at this location, the additional benefit from multihoming relative to singlehoming is \((b_k (n_k^l + n_k^2) - t_k - p_k^1 - p_k^2)/2.\)

\(^4\)A consistent demand configuration is just one in which for given prices in stage 1, the agents’ choice of platform(s) represent a Nash equilibrium in the second stage subgame.
Given the assumption of non-negative prices and \( t_k > b_k \), this additional benefit is always negative. Consequently, no agent of type \( k \) with \( x \in (0, 1) \) will ever want to multihome.

When \( v_k^1(1) > v_k^2(1) \), the agent most likely to multihome is the agent located at \( x = 1 \), who would otherwise buy from platform 1 exclusively. Evaluated at \( x = 1 \), the additional benefit of multihoming relative to singlehoming is \( b_k n_k^2 - p_k^2 \). However, for \( v_k^1(1) > v_k^2(1) \), it must be that \( b_k n_k^1 - t_k - p_k^1 > b_k n_k^2 - p_k^2 \), so that the additional benefit of multihoming relative to singlehoming is less than \( b_k n_k^1 - t_k - p_k^1 \). Since \( t_k > b_k \), \( n_k^1 \leq 1 \), and \( p_k^1 \geq 0 \), this expression is negative, and no agent of type \( k \) will ever want to multihome. By symmetry, the same is true if \( v_k^2(0) > v_k^1(0) \). ■

Lemma 1 allows us to focus only on the case in which all agents singlehome. Proposition 1 characterizes the possible equilibria for this case.\(^5\)

**Proposition 1.** Assume (A1)-(A3). A unique equilibrium exists in which all users singlehome. Half the agents join each platform. If \( f_A + t_A \geq b_B \) and \( f_B + t_B \geq b_A \), equilibrium prices are

\[
p_A^* = p_A^1 = p_A^2 = f_A + t_A - b_B
\]

\[
p_B^* = p_B^1 = p_B^2 = f_B + t_B - b_A
\]

and equilibrium profits are

\[
\pi^* = \pi^1 = \pi^2 = \frac{t_A + t_B - b_A - b_B}{2} > 0.
\]

If \( f_A + t_A < b_B \), equilibrium prices are

\[
p_A^* = p_A^1 = p_A^2 = 0
\]

\[
p_B^* = p_B^1 = p_B^2 = f_B + t_B - \frac{b_A (b_B - f_A)}{t_A} > 0
\]

and equilibrium profits are

\[
\pi^* = \pi^1 = \pi^2 = \frac{(t_A t_B - b_A b_B) - f_A (t_A - b_A)}{2t_A}.
\]

If \( f_B + t_B < b_A \), equilibrium prices are

\[
p_A^* = p_A^1 = p_A^2 = f_A + t_A - \frac{b_B (b_A - f_B)}{t_B} > 0
\]

\[
p_B^* = p_B^1 = p_B^2 = 0
\]

and equilibrium profits are

\[
\pi^* = \pi^1 = \pi^2 = \frac{(t_A t_B - b_A b_B) - f_B (t_B - b_B)}{2t_B}.
\]

**Proof.** Given all agents singlehome from Lemma 1, we have \( N_A = N_B = 0 \) and \( n_k^1 = 1 - n_k^2 \). Equating the utilities of \( A \) types from joining each of the platforms (and likewise for \( B \) types) to find the location of the agent of each type indifferent between the two platforms, we get that

\[
v_A^0 - p_A^1 - t_A x_A + b_A n_k^1 = v_A^0 - p_A^2 - t_A (1 - x_A) + b_A (1 - n_k^1)
\]

\[
v_B^0 - p_B^1 - t_B x_B + b_B n_k^1 = v_B^0 - p_B^2 - t_B (1 - x_B) + b_B (1 - n_k^1).
\]

\(^5\)The three cases in the proposition are mutually exclusive and exhaustive. The other case \( f_B + t_B < b_A \) and \( f_A + t_A < b_B \) cannot arise since from (A2) this implies \( t_A > b_A > t_B \) and \( t_B > b_B > t_A \), which is a contradiction.
The demand configuration is consistent provided \( x_A = n_A^1 \) and \( x_B = n_B^1 \). Solving these two equations implies the equilibrium number of agents of each type on platform 1 are

\[
\begin{align*}
    n_A^1 &= \frac{1}{2} + \frac{b_A (p_B^2 - p_A^1) + t_B (p_A^2 - p_A^1)}{2(t_A t_B - b_A b_B)} \\
    n_B^1 &= \frac{1}{2} + \frac{b_B (p_A^2 - p_B^1) + t_A (p_B^2 - p_B^1)}{2(t_A t_B - b_A b_B)}.
\end{align*}
\]

Assumption (A2) ensures that \( t_A t_B > b_A b_B \), so that rational expectations demands are well defined.

Substituting these share functions into the profit functions, and solving \( d\pi_i / dp_i = 0 \) for \( i = 1, 2 \) and \( k = A, B \) implies a set of four first-order conditions that are linear in prices. Given (A3), firm \( \hat{\pi} \)'s profit is concave in prices. The unconstrained solution to the first-order conditions corresponds to the Nash equilibrium prices, which are characterized in (1) and (2) provided these prices are non-negative. This is true if \( f_A + t_A \geq b_B \) and \( f_B + t_B \geq b_A \). At these prices, the platforms’ share the market equally, and their equilibrium profits are given in (3).

When \( f_A + t_A < b_B \), the prices charged to group \( A \) at this equilibrium are negative. Given that prices must be non-negative, we instead solve the four first-order conditions resulting from the constrained profit maximization problem, with the relevant constraints being \( p_A^1 \geq 0 \) and \( p_B^1 \geq 0 \). Following the same logic as above, this leads to the equilibrium prices (4) and (5). Note the price set to \( B \) types can be written as \((t_A t_B - b_A b_B + b_A f_A + t_A f_B )/ t_B \), which is positive from (A2). At these prices the platforms’ share the market equally, and their equilibrium profits are given in (6). Since \( f_A < b_B - t_A \), equilibrium profits are also positive given (A2). By symmetry, when \( b_A > f_B + t_B \), the equilibrium prices are given in (7) and (8), with corresponding (positive) equilibrium profits given in (9).

Like the case of a one-sided market with network effects, the competition to capture additional users so as to realize the network benefits of doing so drives prices and profits below those without network effects. In all cases, equilibrium prices and profits are decreasing in the network benefit parameters. Apart from affecting the overall level of prices, the structure of prices is also affected by these network effects. In particular, with both prices positive, the price-cost margin for each group is equal to the product differentiation parameter for that group minus the externality that joining the platform has on the other group. If attracting one group of user (say buyers) makes the platform particularly attractive to the other group (sellers), then buyers will be “subsidized”. Subsidizing buyers will increase the number of buyers subscribing to the platform, and this will lead to an increase in utility (and so demand) from sellers. For example, if magazine readers value the number of advertisements less than the advertisers value the number of readers, then competing magazine publishers will subsidize readers relative to advertisers to exploit the greater demand from advertisers for readers than vice-versa.

When the asymmetry in the strength of network effects between the two groups above is sufficiently large, and when the cost of serving buyers is not too great, platforms will want to pay buyers to join given this increases the demand from sellers a lot. With our assumption of non-negative prices, this results in platforms offering buyers free subscription (a common pricing strategy in two-sided markets). The

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6There is another solution to the set of first order conditions involving one platform having its price (to one side) constrained to zero. Using (A2) and (A3) it can be shown this implies the rival platform must set a negative price to this side of the market, contradicting the fact the solution has all prices non-negative.
more the platforms would like to set negative prices to buyers, the lower the prices that sellers will enjoy. If platforms are prevented from further subsidizing buyers due to a non-negativity constraint on prices, they will instead compete more aggressively for sellers (so as to attract more buyers, and therefore more sellers, due to the positive externalities running in both directions).  

For instance, if the extent of price competition becomes stronger on side B of the market ($t_B$ decreases), then the platforms will want to set lower prices to $B$ types. However, if subscription prices are already set to zero, this will instead result in each platform reducing their price to $A$ types. Figure 1 illustrates. (A similar effect arises if the cost of serving group B goes down, or the network benefits parameter increases for group A.)

Moving right to left in this figure, as competition increases for $B$ types ($t_B$ decreases), initially just the price to $B$ types falls (the price to $A$ types remains constant). However, once the price to $B$ types falls to zero, then any further increase in competition for $B$ types, decreases the price charged to $A$ types.

These results have implications for analyzing markets where one side is not charged. They suggest that such markets may be able to be treated as though they are one-sided markets in the sense that greater competition, lower costs, or stronger externalities on either side of the market will have the same qualitative effects. This can be seen from (7), in which all the parameters for $B$ types enter with the same sign as those for $A$ types.

The results also imply that not taking into account how the parameters on the buyers’ side influence prices on the sellers’ side could lead to erroneous conclusions, in the case buyers are charged nothing. For  

\[ f_A + t_A > b_B \]  
\[ f_B + t_B > b_A \]
instance, a merger analysis which concludes that a merger between platforms has no detrimental effect on competition for buyers simply because buyers continue to be charged nothing would be incorrect. With buyers paying nothing, reduced competition for buyers may not increase prices for buyers, but it may increase prices for sellers instead. Similarly, an empirical analysis that tries to explain prices to sellers should include cost and product characteristics from the buyers’ side even if buyers are charged nothing. For example, differences in the costs of distributing Yellow Pages to readers, or in the extent of differentiation between directories from the readers’ perspective could help explain variations in how much directories charge advertisers across different markets. (See Rysman, 2004 for an empirical analysis of the market for Yellow Pages).

4 Product differentiation on one side

The assumption employed in the previous section, that transportation costs are stronger than the corresponding network benefits, ensured no agents wanted to multihome, even in the case in which platforms charge nothing. When this assumption is relaxed, it introduces the possibility of multihoming, which is the focus of this section. With lower levels of product differentiation (so network effects dominate) and provided costs are not too high, there may be some agents who will want to buy from both platforms if agents on the other side of the market do not. Since not everyone will multihome, it seems natural to look for an equilibrium in which agents multihome on one side and singlehome on the other.\footnote{If every other agent multihomes, there is no incentive for any individual agent to multihome unless they face negative prices, which we have ruled out.}

In this section, we consider a market where on one side the two platforms are treated as homogenous, while the other side involves product differentiation. This captures the fact in many two-sided markets involving buyers and sellers it is natural to assume that, controlling for the size of the network benefits, sellers (firms) view the competing platforms as more or less homogenous, while buyers (consumers) may have strong preferences for using a particular platform. Possible examples include certain types of directories, listing services, entertainment platforms, payment schemes, search engines, softwares and trading posts. Where the product differentiation on the buyers’ side is high relative to network benefits, buyers will tend to split between the two platforms. This gives an incentive for sellers, who anyway view both platforms as homogenous, to multihome in order to capture maximal network benefits.

Formally, we replace assumption (A2) with

\[ A_2' \quad t_A = 0 \text{ and } t_B > b_B \]

To characterize the equilibria of this game, the consistent demand configurations for given prices need to be characterized. Given we are only interested in subgame perfect equilibria, we must ensure the agents’ choices of platform(s) are Nash equilibria in the second-stage subgame. Given \((A_2')\), lemma 1 still applies with respect to side \(B\), so that type \(B\) agents will never want to multihome regardless of the (non-negative) prices. Thus, restricting attention to the case agents choose pure strategies, there are three possible consistent demand configurations to consider.
1. All $A$ types multihome. The proportion of $B$ types that purchase from platform 1 is then determined by the normal Hotelling formula

$$n^1_B = \frac{1}{2} + \frac{p^2_B - p_B^1}{2t_B},$$

where $1 - n^1_B$ purchase from platform 2. Type $A$ agents will want to multihome if $v_A^0 - p_A^1 - p_A^2 + b_A \geq v_A^0 - p_A^1 + b_A n^1_B$ and $v_A^0 - p_A^1 - p_A^2 + b_A \geq v_A^0 - p_A^1 + b_A (1 - n^1_B)$. This requires

$$p^1_A \leq \left( \frac{1}{2} + \frac{p^2_B - p_B^1}{2t_B} \right) b_A$$

(10)

$$p^2_A \leq \left( \frac{1}{2} + \frac{p^1_B - p_B^2}{2t_B} \right) b_A.$$  

(11)

The platforms’ profits are

$$\pi^1 = p_A^1 - f_A + (p_B^1 - f_B) \left( \frac{1}{2} + \frac{p_B^2 - p_B^1}{2t_B} \right)$$

(12)

$$\pi^2 = p_A^2 - f_A + (p_B^2 - f_B) \left( \frac{1}{2} + \frac{p_B^1 - p_B^2}{2t_B} \right).$$

(13)

2. All $A$ types singlehome on platform 1. The proportion of $B$ types that will buy from platform 1 is then determined by

$$n^1_B = \frac{1}{2} + \frac{p^2_B - p_B^1 + b_B}{2t_B},$$

where $1 - n^1_B$ purchase from platform 2. Type $A$ agents will want to singlehome on platform 1 if $v_A^0 - p_A^1 + b_A n^1_B \geq v_A^0 - p_A^1 - p_A^2 + b_A$ and $v_A^0 - p_A^1 + b_A n^1_B \geq v_A^0 - p_A^1 + b_A (1 - n^1_B)$. This requires

$$p^2_A \geq \left( \frac{1}{2} + \frac{p_B^1 - p_B^2 - b_B}{2t_B} \right) b_A$$

(14)

$$p^1_A \geq p_A^1 + \left( \frac{p_B^2 - p_B^1 + b_B}{t_B} \right) b_A.$$  

(15)

The platforms’ profits are

$$\pi^1 = p_A^1 - f_A + (p_B^1 - f_B) \left( \frac{1}{2} + \frac{p_B^2 - p_B^1 + b_B}{2t_B} \right)$$

(16)

$$\pi^2 = (p_B^2 - f_B) \left( \frac{1}{2} + \frac{p_B^1 - p_B^2 - b_B}{2t_B} \right).$$

(17)

3. All $A$ types singlehome on platform 2. The proportion of $B$ types that will buy from platform 1 is then determined by

$$n^1_B = \frac{1}{2} + \frac{p^2_B - p_B^1 - b_B}{2t_B},$$

where $1 - n^1_B$ purchase from platform 2. Type $A$ agents will want to singlehome on platform 2 if $v_A^0 - p_A^2 + b_A (1 - n^1_B) \geq v_A^0 - p_A^1 - p_A^2 + b_A$ and $v_A^0 - p_A^2 + b_A (1 - n^1_B) \geq v_A^0 - p_A^1 + b_A n^1_B$. This requires

$$p^1_A \geq \left( \frac{1}{2} + \frac{p_B^2 - p_B^1 - b_B}{2t_B} \right) b_A$$

(18)

$$p^2_A \leq p_A^1 + \left( \frac{p_B^1 - p_B^2 + b_B}{t_B} \right) b_A.$$  

(19)

The platforms’ profits are

$$\pi^1 = (p_B^1 - f_B) \left( \frac{1}{2} + \frac{p_B^2 - p_B^1 - b_B}{2t_B} \right)$$

(20)

$$\pi^2 = p_A^2 - f_A + (p_B^2 - f_B) \left( \frac{1}{2} + \frac{p_B^1 - p_B^2 + b_B}{2t_B} \right).$$

(21)
Notice that, unlike the situation in section 3, for a range of prices there is more than one consistent demand configuration. For instance, consider the case where prices are symmetric on the two platforms, with $p_A$ and $p_B$ being the platforms’ prices to the two groups. Then configuration 1 is consistent if $p_A < b_A/2$ while the other two configurations are consistent if, in addition, $p_A > (1 - b_B/t_B)b_A/2$. Essentially, these different consistent configurations correspond to different assumptions about how agents coordinate on different configurations for various prices.

We start by imposing only a weak restriction on how agents coordinate, by doing so only with respect to deviations from equilibrium prices. Specifically, assume any equilibrium must have the property that if one platform changes its prices and the original demand configuration is still an equilibrium in the subgame, then this configuration should still be selected. Anything else requires the simultaneous coordination of agents to move to a new demand configuration, since it is not in the interests of either group of agent to change demands if the other does not. We refer to this as an inertia condition. In simple terms, the inertia condition ensures any equilibrium must at least be robust to a deviating platform changing its prices when agents find it difficult to coordinate.

With just this condition, it turns out any subgame perfect equilibrium must involve $A$ types multihoming, and in which their network benefits are fully extracted.

**Proposition 2** Assume (A1) and (A2′) hold, and the inertia condition applies. The only equilibria involve $A$ types multihoming. In any such equilibrium, platforms will extract the network benefits enjoyed by $A$ types.

**Proof.** Assumptions (A1) and (A2′) rule out $B$ types multihoming, or some agents not subscribing to either platform. Consider instead a possible equilibrium with all $A$ types singlehoming on platform 1 (the case with platform 2 follows by symmetry). Such an equilibrium implies (15) holds with equality, otherwise platform 1 could increase its price to $A$ types, and by the inertia condition still face the same demand, thereby obtaining higher profits. Combining $p_A^1 = p_B^2 + (p_B^2 - p_B^1 + b_B) b_A/t_B$ with the inequality (14) gives

$$p_A^1 \geq \left( \frac{1}{2} + \frac{p_B^2 - p_B^1 + b_B}{2t_B} \right) b_A. \quad (22)$$

In any such equilibrium, platform 2 will set $p_A^2 \geq f_A$ (given it faces no $A$ types in equilibrium, it seems unreasonable to consider an equilibrium in which platform 1 dominates and which only exists because platform 2 willingly sets its price $p_A^2$ below cost), while it will always set $p_B^2 > f_B$ to obtain a positive profit on $B$ types.

Suppose platform 2 deviates from the proposed equilibrium by lowering its price to $A$ types by some arbitrarily small amount $\varepsilon_A$. This causes (15) to no longer hold since it was previously just binding, so configuration 2 is no longer feasible. Configuration 1 is also not feasible, since (22) contradicts (10). On the other hand, configuration 3 is feasible, and so will apply. The change in profits for platform 2 as a
result is
\[ p_A^2 - f_A - \varepsilon_A + \left(p_B^2 - f_B\right) \frac{b_B}{t_B}. \]

Given \( p_A^2 \geq f_A \) and \( p_B^2 > f_B \), this expression can always be made positive by setting \( \varepsilon_A \) sufficiently small. Thus, platform 2 can always do better by slightly lowering its price to \( A \) types so that \( A \) types instead singlehome with it. As a result, there is no singlehoming equilibrium.

Alternatively, consider any equilibrium involving multihoming (configuration 1). Then the inertia condition implies (10) and (11) must both hold with equality, otherwise one or other platform can increase its price to \( A \) types without losing any demand, thereby increasing its profits. This shows any equilibrium must involve multihoming and must involve \( A \) types having all their network benefits extracted by the platforms.

Since any such equilibrium involves (10) and (11) holding with equality, it is natural to focus on such equilibria in which each platform then sets its price to the \( B \) types to maximize its respective profit given these two constraints. Specifically, given all \( A \) types multihome, the share of \( B \) types on each platform is just the normal Hotelling market share. Then given no transportation costs for \( A \) types, the amount an \( A \) type is willing to pay to each platform is just \( b_A \) multiplied by this market share. Thus, platform 1 sets \( p_B^1 \) to maximize
\[ \left(\frac{1}{2} + \frac{p_B^1 - p_B^2}{2t_B}\right) (b_A + p_B^1 - f_B) - f_A \]
and platform 2 sets \( p_B^2 \) to maximize
\[ \left(\frac{1}{2} + \frac{p_B^2 - p_B^1}{2t_B}\right) (b_A + p_B^2 - f_B) - f_A. \]

When \( b_A \leq f_B + t_B \), the result is
\[ p_A^* = p_B^1 = p_B^2 = \frac{b_A}{2} \quad (23) \]
\[ p_B^* = p_B^1 = p_B^2 = f_B + t_B - b_A. \quad (24) \]

This characterizes a candidate equilibrium in which \( A \) types multihome and \( B \) types singlehome. Profits are
\[ \pi^* = \frac{t_B}{2} - f_A. \quad (25) \]

Alternatively, if \( b_A > f_B + t_B \), given our assumption of non-negative prices, the candidate equilibrium has prices
\[ p_A^* = p_B^1 = p_B^2 = \frac{b_A}{2} \quad (26) \]
\[ p_B^* = p_B^1 = p_B^2 = 0, \quad (27) \]
in which case profits are
\[ \pi^* = \frac{b_A}{2} - f_A - \frac{f_B}{2}. \quad (28) \]

With the additional assumption that
\[ A3' \quad b_B \geq 2f_A \text{ and } b_B \geq f_B \]
Proposition 3 shows such equilibria indeed exist. The first part of the assumption when combined with (A2') ensures profits in (25) and (28) are non-negative. Both parts of the assumption also help rule out particular deviations (detailed in the proof) that can otherwise be profitable for certain parameter values. We refer to the resulting equilibria as a competitive bottleneck equilibrium, following the use of this terminology by Armstrong (2004).\footnote{Earlier models of competitive bottlenecks are given in Armstrong (2002, Section 3.1) and Wright (2002), in the context of call termination in telecommunication networks.} Platforms behave as though they do not compete directly for the multihoming $A$ types, instead choosing to compete indirectly by subsidizing $B$ types to join. Having attracted $B$ types, they then exploit $A$ types surplus from connecting with these $B$ types.

**Proposition 3** Assume (A1), (A2') and (A3') hold. (i) With $t_B \leq b_A \leq f_B + t_B$ there is a competitive bottleneck equilibrium characterized by (23) and (24). All $A$ types multihome, while $B$ types split evenly between the two platforms. Equilibrium profits are given in (25). (ii) With $b_A > f_B + t_B$ there is a competitive bottleneck equilibrium characterized by (26) and (27). Equilibrium profits are given in (28).

**Proof.** Consider the case $t_B \leq b_A \leq f_B + t_B$ first. To see whether the proposed equilibrium is indeed an equilibrium, consider whether platform 1 would want to change its prices given that the prices of platform 2 are set at (23) and (24).

If platform 1 changes its prices so that (10) and (11) still hold, then we assume agents continue to select configuration 1 (consistent with the inertia condition), in which case profits are necessarily lower (given the prices $p_1^B = p_2^B = f_B + t_B - b_A$ were found by maximizing profits subject to these constraints). If platform 1 changes its prices so that configuration 1 no longer holds, then this requires $p_1^A < f_B + t_B - b_A$ and/or $p_1^B > b_A/2 + b_A (f_B + t_B - b_A - p_2^B) / (2t_B)$. There are two cases to consider.

If platform decreases at least one price, and does not increase either price, then configuration 2 will always be a consistent demand configuration, and we assume it applies. This is consistent with the implications of the monotonicity condition used by Caillaud and Jullien, 2003, which loosely speaking rules out the case demand is increasing in price wherever non-increasing demand functions also exist for price changes from the proposed equilibrium. This requires $p_1^A = b_A/2 - \varepsilon_A$ for some $\varepsilon_A \geq 0$ and $p_1^B = f_B + t_B - b_A - \varepsilon_B$ for some $\varepsilon_B \geq 0$, with one of the inequalities being strict. The resulting profits are

$$\pi^1 = \frac{b_A}{2} - f_A - \varepsilon_A + (t_B - b_A - \varepsilon_B) \left( \frac{1}{2} + \frac{b_B + \varepsilon_B}{2t_B} \right)$$

$$= \frac{t_B}{2} - f_A + (t_B - b_A - \varepsilon_B) \left( \frac{b_B + \varepsilon_B}{2t_B} \right) - \varepsilon_A - \frac{\varepsilon_B}{2}.$$ 

Thus, provided $b_A \geq t_B$, it follows that $\pi^1 < \pi^*$ and platform 1 would not want to undercut on both prices in this way.\footnote{If $0 < b_A < t_B$ and the monotonicity condition applies, there will always be some sufficiently small $\varepsilon_B$, so that platform 1 does better lowering its prices to $B$ types while keeping the price to $A$ types unchanged (thereby attracting many more $B$ types, as $A$ types now subscribe exclusively to platform 1). This is profitable since the condition $0 < b_A < t_B$ ensures $B$ types attract a positive margin at the original prices. Thus, the candidate competitive bottleneck equilibrium does not exist in this case (at least, when monotonicity applies).}
consistent demand configuration. (Without the condition $b_B > f_B$ there will be some such prices where only configuration 2 holds, in which case there are parameter values for which platform 1 will want to deviate). In this case configuration 3 is assumed to be selected, regardless of whether configuration 2 is also a consistent demand configuration. For the case in which neither price decreases, this is consistent with monotonicity. For other cases, where configuration 2 and 3 both hold, there is no obvious criteria to use to select one or other configuration, so to prove existence of the proposed equilibria we just choose the configuration which most easily supports it.\footnote{The particular selection of configurations used in this proof is consistent with a simple sequential tâtonnement process that defines how agents choose which platform(s) to join when, starting from a proposed equilibrium, one platform changes its prices. See Appendix B for the details.}

Given configuration 3 is selected, platform 1 does not sell to any $A$ types, and only sells to those $B$ types who have a strong preference for its network. Its resulting profits are

$$\pi^1 = (p_B^1 - f_B) \left( \frac{1}{2} + \frac{f_B + t_B - b_A - p_B^1 - b_B}{2t_B} \right),$$

which is maximized by setting $p_B^1 = f_B + t_B - (b_A + b_B)/2$. At this price, profits are

$$\pi^1 = \frac{t_B}{2} \left( 1 - \frac{b_A + b_B}{2t_B} \right)^2.$$

Since platform 1 can only make profits from attracting some consumers, we only need to consider the case

$$t_B > \frac{b_A + b_B}{2}. \quad (29)$$

Platform 1’s change in profits from the deviation is

$$\pi^1 - \pi^* = \frac{t_B}{2} \left( 1 - \frac{b_A + b_B}{2t_B} \right)^2 - \left( \frac{t_B}{2} - f_A \right)$$

$$= f_A + \frac{(b_A + b_B)^2}{8t_B} - \frac{b_A + b_B}{2}$$

$$< f_A - \frac{b_A + b_B}{4}$$

$$< f_A - \frac{b_B}{2}$$

$$< 0$$

where the first inequality follows from (29), the second follows from the assumption $b_A \geq t_B$ together with (A2′), and the last follows from the first part of (A3′). Thus, platform 1 is worse off if it changes its price in this way. This proves result (i) in the proposition.

Next we consider the case $b_A > f_B + t_B$. To see whether the proposed equilibrium is indeed an equilibrium, consider whether platform 1 would want to change its prices given that the prices of platform 2 are set at (26) and (27).

If platform 1 changes its prices so that the constraints (10) and (11) still hold, then we assume agents continue to select configuration 1 (consistent with the inertia condition), in which case profits are necessarily lower (given the prices $p_B^1 = p_B^2 = 0$ were found by maximizing profits subject to these constraints). If platform 1 changes its prices so that configuration 1 no longer holds, then it is straightforward to check
that configuration 3 is always a consistent demand configuration. As before, to support the equilibrium we assume that configuration 3 is selected (see Appendix B for a particular justification).

Under configuration 3, platform 1’s profit is

$$\pi_1 = (p_1^* - f_B)^2 \left( \frac{1}{2} - \frac{p_1^* + b_B}{2t_B} \right),$$

which is maximized by setting $p_1^* = (f_B + t_B - b_B)/2$. At this price, profit becomes

$$\pi_1 = \frac{(t_B - b_B - f_B)^2}{8t_B}.$$ 

Since platform 1 can only make profits from attracting some consumers, we only need to consider the case $t_B > f_B + b_B$. (30)

Platform 1’s change in profits from the deviation is

$$\pi_1 - \pi^* = \frac{(t_B - b_B - f_B)^2}{8t_B} - \left( \frac{b_A}{2} - \frac{f_A}{2} \right)^2$$

$$= f_A + \frac{(f_B + b_B)^2}{8t_B} - \left( \frac{b_A}{2} - \frac{f_B}{4} - \frac{t_B}{8} + \frac{b_B}{4} \right)^2$$

$$< f_A + \frac{t_B}{8} - \left( \frac{b_A}{2} - \frac{f_B}{4} - \frac{t_B}{8} + \frac{b_B}{4} \right)^2$$

$$= f_A - \left( \frac{b_A}{2} - \frac{f_B}{4} - \frac{t_B}{8} + \frac{b_B}{4} \right)^2$$

$$< f_A - \frac{b_A + b_B}{4}$$

$$< f_A - \frac{b_B}{2}$$

$$< 0$$

where the first inequality follows from (30), the second follows from the condition $b_A > f_B + t_B$, the third follows from the condition $b_A \geq f_B + t_B > t_B$ together with (A2'), and the last follows from the first part of (A3'). Thus, platform 1 is worse off if it changes its price in this way. This proves result (ii) in the proposition.

The above proposition shows that provided agents of type A (sellers) value agents of type B (buyers) sufficiently strongly, there is an equilibrium in which all sellers multihome. In this equilibrium, sellers have their network benefits fully exploited (they still get their intrinsic utility), while buyers face a price that is set below cost. Despite prices favouring buyers, it is sellers that choose to multihome. Where sellers value buyers sufficiently strongly, the equilibrium implies the platforms give away their services to buyers.13

13Independently, Gabszewicz and Wauthy (2004) have obtained a similar result in quite a different framework. They assume agents on each side are heterogenous in their network benefits (but are otherwise homogenous), and characterize the fulfilled expectations equilibrium (so platforms take expectations as given when setting their prices). They find an equilibrium in which all agents on one side multihome and are charged the monopoly price, while those on the other side are charged cost (which in their model is set to zero). The equilibrium we characterize shows that when platforms take into account how expectations change with their prices, they will subsidize the singlehoming side in order to attract the more lucrative multihoming side.
The equilibrium captures the idea of a competitive bottleneck. There is strong competition to sign up buyers, but conditional on the demand from buyers, platforms do not compete to attract sellers. Platforms make a loss on buyers, which they recover from sellers who want to reach buyers and do not have a choice of which platform to join in order to do so. Although sellers have all their surplus extracted, it is not the case that platforms necessarily have any market power. If we define the services provided to sellers as a separate market, then this follows since the prices here were derived assuming the seller side was perfectly competitive (platforms competed in price assuming no product differentiation). If instead we define the market more generally so as to include services provided to both types of agents, the same conclusion applies, since removing product differentiation from the buyer side can in fact lead to higher prices.\textsuperscript{14} This is not to say, in general, that the “high” prices charged to sellers in a competitive bottleneck are reasonable or efficient. Rather, what these results imply is that the bottleneck is not necessarily resolved by increasing the extent of competition. This motivates the description “competitive bottleneck”.

Proposition 3 provides an explanation for why two-sided markets may involve one side of the market being charged nothing. The explanation is based on the case in which agents on the other side of the market choose to multihome. If the homogenous agents represent sellers, then the equilibrium above corresponds to the case in which sellers choose to multihome, and the buyers pay nothing to subscribe (but do not multihome). This would seem to fit the case of directories like Yellow Pages in which most sellers pay to list in both directories while buyers generally use just one even though it is free (and similarly for search engines, shopping malls, and other trading posts).\textsuperscript{15}

The analysis up to this point has assumed that platforms offer fixed subscription charges that do not depend upon how well the platforms do in attracting the other group. In practice, charges are sometimes levied as a function of the platform’s performance on the other side of the market. In the current context, suppose platforms charge group A a charge $P_A$ per member of group B that joins the same platform. Assuming that $v_A^0 = 0$ then it is clear that every group A agent will join any platform as long as $P_A < b_A$. (This is the case regardless of group A’s beliefs about what group B does.) Therefore, each platform will set the price $P_A = b_A$ per member of group B that joins, and group B will behave in the normal Hotelling way. The resulting prices to group B (and in fact group A) are then exactly as stated in Proposition 3.\textsuperscript{16}

An additional reason to lower prices to A types would arise if this allowed platform 1 to attract A types exclusively, and thereby charge more to B types. This is not possible in the competitive bottleneck equilibria, as platform 2 always leaves A types (just) willing to subscribe to it, regardless of what platform 1 does. However, if platform 1 can offer exclusive deals, setting a different price for A types that agree

\textsuperscript{14}Section 5 does exactly this, and comparing prices across the two cases we find prices are not necessarily lower (to either side), and can in fact be higher in the more competitive setting.

\textsuperscript{15}Other possible applications include some employment websites, classified newspapers, web portals and magazines, where searchers/shoppers may just choose one platform to use even though it is offered for free, while businesses “list” on both competing platforms despite being charged for the privilege. A somewhat different application is to competing software which are offered free to private users, who stick to one software, but sold to commercial users, who buy both softwares so they can “communicate” with a larger number of private users. Some text editors which are offered free to readers roughly fit this characterization.

\textsuperscript{16}See section 5 in Armstrong (2004) for a further discussion of the effects of different charging regimes.
not to multihome, then singlehoming equilibria naturally arise, as we now demonstrate.

4.1 Exclusive dealing

Cross-market effects are often used to explain the anti-competitive effects of exclusive dealing. Such cross-market effects naturally arise in two-sided markets. When agents on one side of the market multihome, platforms might offer exclusive contracts to them to prevent them multihomeing, thereby profiting from the increased demand from users on the other side of the market. Such exclusive contracts can be “cheap” to offer, since by tying up one side of the market, the platform attracts the other side, which reinforces the decision of the former side to sign up exclusively. In this section we use the model above to show how such contracts work, and furthermore to examine the efficiency implications of these contracts.

We focus on exclusive contracts being offered only to one side — the side that would otherwise multihome. Recall the multihoming agents in the previous section are those for whom the platforms are not differentiated (side A). Such agents (sellers) could be advertisers in directories, retailers in shopping malls, content providers for cable TV platforms, or video-game developers for game platforms. Platforms should be able to write contracts for such firms so as to prevent multihoming (that is, to enforce exclusivity). On the other hand, the other side of the market (side B) is represented by buyers (consumers) who view the platforms as differentiated and for whom it may be relatively difficult (or costly) to enforce exclusivity. These “buyers” could be readers of directories, shoppers in shopping malls, viewers of cable TV, and players of video games.

We revisit the case analyzed in Proposition 3, which gave rise to a competitive bottleneck equilibrium with multihoming by sellers. To allow for the possibility of platforms to offer exclusive deals, we add two prior stages to the game. In the first stage of the game, platforms offer sellers a subscription price $p_A^E$ if they sign up with the platform exclusively. We allow for the possibility this price is negative, reflecting compensation for signing up exclusively with the platform, but also consider the case in which it is restricted to be non-negative. In the second stage, sellers decide whether to take up one or other of these exclusive deals. The remaining two stages of the game proceed as before — in stage three, platforms set prices for the remaining agents, and in stage four, the remaining agents make their subscription decisions.

Some tie-breaking rules are employed. We assume if sellers are indifferent between an exclusive contract or not, they take the exclusive contract, while if they are indifferent between an exclusive contract with platform 1 and one with platform 2, they will take the one with platform 1 (thus, we refer to platform 1 as the dominant platform). With this assumption, we find:

**Proposition 4** Assume (A1), (A2′) and (A3′) hold, and that $b_A \geq t_B$. Assume without exclusive contracts, the equilibrium in Proposition 3 applies. (i) Platforms will offer exclusive contracts, with platform 1 attracting all A types. (ii) As a result of these contracts being offered, A types will be better off and B types will be worse off. (iii) Such contracts are inefficient.

**Proof.** Since all A types are identical, and given our tie-breaking assumptions, any equilibrium in the stage 2 subgame will either involve all A types signing an exclusive contract with one platform, or none signing such a contract.
If \(A\) types do not sign up to the exclusive contract in stage 2, then Proposition 3 applies to the stage 3 subgame. In this case platform 1’s profits \(\pi^*\) are defined in (25) if \(t_B \leq b_A \leq f_B + t_B\) and in (28) if \(b_A > f_B + t_B\). The consumer surplus of \(A\) types is \(v^0_A\) in both cases (they have all their network benefits exploited under multihoming).

If a platform (say platform 1) signs type \(A\) agents exclusively in stage 2, it will enjoy demand configuration 2 regardless of the prices it sets, with corresponding profits in (16). The rival platform will only obtain profits in (17). In stage 3, the platforms will set prices to \(B\) types of \(p^1_B = f_B + t_B + b_B/3\) and \(p^2_B = f_B + t_B - b_B/3\) respectively. Platform 1’s corresponding profit is

\[
\pi^E = \frac{t_B}{2} \left( 1 + \frac{b_B}{3t_B} \right)^2 - f_A
\]

plus the exclusive price \(p^E_A\) it charges to \(A\) types, while platform 2’s profit is

\[
\pi^0 = \frac{t_B}{2} \left( 1 + \frac{b_B}{6t_B} \right)^2.
\]

Consumer surplus for \(A\) types is

\[
CS_A = v^0_A + b_A \left( \frac{1}{2} + \frac{b_B}{6t_B} \right) - p^E_A.
\]

For \(A\) types to want to sign the exclusive contract we require

\[
p^E_A \leq \frac{b_A}{2} + \frac{b_A b_B}{6t_B}.
\]

For platform 1 to want to offer the exclusive contract (when the other platform does not) we require

\[
p^E_A \geq \pi^* - \pi^E,
\]

where using (25) we get

\[
p^E_A \geq \frac{t_B}{2} \left( 1 - \left( 1 + \frac{b_B}{3t_B} \right)^2 \right)
\]

if \(t_B \leq b_A \leq f_B + t_B\), and using (28) we get

\[
p^E_A \geq \frac{b_A}{2} - \frac{f_B}{2} - \frac{t_B}{2} \left( 1 + \frac{b_B}{3t_B} \right)^2
\]

if \(b_A > f_B + t_B\). For both ranges of \(b_A\), there are prices \(p^E_A\) such that both the platform and \(A\) types will want to sign an exclusive contract. This shows that any subgame perfect equilibrium in the full game must involve \(A\) types signing exclusive contracts.

Competition between the platforms to attract the \(A\) types exclusively will lead platforms to offer more and more attractive contracts to these users. Due to this competition, any equilibrium involving exclusive contracts must leave platforms indifferent between offering the winning contract, or not, and just serving the remaining \(B\) types. Such an equilibrium must satisfy \(\pi^E + p^E_A = \pi^0\), which implies

\[
p^E_A = f_A - \frac{2}{3} b_B.
\]

Given the tie-breaking rule, it is platform 1 that will attract all \(A\) types through such an exclusive contract in equilibrium. This proves result (i) in the proposition.
Given \((A3')\) the resulting price in \((31)\) is negative, so that sellers will be paid to subscribe exclusively to platform 1’s service. If for some reason such payments are not possible, the price \(p_{E}^{A}\) will be set to zero. In the former case, both platforms obtain profits equal to \(\pi^{0}\), which are lower than those obtained under the competitive bottleneck equilibrium. In the latter case, platform 1 obtains profits equal to \(\pi^{E}\), which are higher than those obtained under the competitive bottleneck equilibrium (and those obtained by platform 2).

Regardless of whether \(p_{E}^{A}\) is negative, or constrained to zero, type \(A\) consumers are better off in the equilibrium with exclusive contracts since \(CS_{A} > v_{0}^{A}\). In contrast, \(B\) types now pay more (in fact, more than costs \(f_{B}\)), get lower network benefits (due to a lack of multihoming on the \(A\) side), and face higher average transportation costs. They are unambiguously made worse off by the introduction of exclusive contracts. This confirms result (ii) in the proposition.

When all \(A\) types multihome and \(B\) types split, welfare is

\[
W_{M} = v_{A}^{0} + v_{B}^{0} + b_{A} + b_{B} - 2f_{A} - f_{B} - \frac{t_{B}}{4}.
\]

When all \(A\) types exclusively belong to platform 1, and \(B\) types face the prices \(p_{1}^{B} = f_{B} + t_{B} + b_{B}/3\) and \(p_{2}^{B} = f_{B} + t_{B} - b_{B}/3\), welfare is

\[
W_{E} = v_{A}^{0} + v_{B}^{0} + (b_{A} + b_{B}) \left( \frac{1}{2} + \frac{b_{B}}{6t_{B}} \right) - f_{A} - f_{B} - \frac{t_{B}}{4} - \frac{b_{B}^{2}}{36t_{B}}.
\]

Then

\[
W_{M} - W_{E} = (b_{A} + b_{B}) \left( \frac{1}{2} - \frac{b_{B}}{6t_{B}} \right) - f_{A} + \frac{b_{B}^{2}}{36t_{B}}
\]

\[
> \frac{b_{A} + b_{B}}{3} - f_{A} + \frac{b_{B}^{2}}{36t_{B}}
\]

\[
> \frac{2b_{B}}{3} - f_{A} + \frac{b_{B}^{2}}{36t_{B}}
\]

\[
> \frac{b_{B}^{2}}{36t_{B}}.
\]

where the first inequality follows from \((A2')\), the second inequality follows from \(b_{A} \geq t_{B}\) together with \((A2')\), and the final inequality follows from \((A3')\). This proves result (iii) in the proposition. \(\blacksquare\)

The proposition suggests that exclusive contracts will be used by platforms to prevent multihoming, thereby allowing them to attract one side exclusively (here, the sellers). The reason the (dominant) platform and sellers are willing to write such a contract, is they can extract some surplus from outsiders from doing so (the rival platform, and the buyers). In contrast, without exclusive contracts, sellers have all their surplus extracted by platforms. Competition to offer the exclusive contract which sellers sign up to, leads to a more competitive outcome than the competitive bottleneck equilibrium, since the winning platform ends up paying \(A\) types to get them to join exclusively. However, such exclusive contracts are inefficient compared to multihoming, since many buyers are forced to access sellers through a platform they do not like, and the remaining (loyal) buyers no longer enjoy any network benefits. The result is that, compared to the competitive bottleneck equilibrium, the sellers are now better off at the expense of end-users (buyers) and the platforms.
It is worthwhile understanding why platforms cannot achieve the same result simply by offering better terms to sellers but not signing them up exclusively. Starting from the equilibrium in proposition 3, consider the case platform 1 tries to lower its price to sellers to attract them exclusively. Given buyers are split between the two sides, sellers will still want to subscribe to platform 2 (in addition to platform 1), regardless of the price charged by platform 1. The alternative facing platform 1 is to lower its price to the buyer side and not increase its price to sellers. While this can attract more buyers, and hence cause sellers to want to sign up exclusively, since buyers are already being subsidised in our competitive bottleneck equilibrium this strategy is not profitable. In contrast, exclusive contracts provide platforms with a different strategy. They can set their price to sellers contingent on them not subscribing to the rival platform, and this allows them to exploit the resulting network effects through the buyer side.

Such exclusive contracts involve partial foreclosure. Although the rival platform is foreclosed from one side of the market, it is still able to service some $B$ type users. This reflects the assumption of strong product differentiation on the $B$ side. This could capture that even without any “sellers” on its platform, the excluded platform still offers some services that buyers value. For example, if sellers represent content providers, perhaps the platform offers some of its own content (or content that cannot legally be made exclusive). When we turn to pure network effects in the next section, such services will be absent, in which case exclusive contacts will allow a platform to foreclose its rival from both sides of the market.

We close this section by noting that the identical result to Proposition 4 holds for the alternative case in which sellers get no network benefits. With $b_A = 0$, exclusive deals necessarily involve a payment being paid to $A$ types to get them to subscribe since without exclusive deals they will already pay nothing to join. The absence of network benefits seems reasonable for certain types of sellers such as content providers who provide content for cable TV. Content providers may not value the number of subscribers to the platform directly, but rather just the revenue they can get from selling their service to platforms. Without exclusive deals, content providers would just cover the cost of their programs (leaving them with zero surplus). With exclusive deals, the TV platforms would pay them (above their costs of providing these programs) to obtain their content exclusively. Cable subscribers would end up paying for this, both in terms of higher subscription prices, but also in terms of a reduced choice over which platform they can view content over. Those viewers loyal to the excluded platform will face reduced access to content. This assumes cable subscribers care more about their choice of cable platform than they do the extent of content on the platform. In other cases, it may be that subscribers do not care about the choice of platform so much as how much content they can access (one way or another), which is the scenario we turn to next.

5 Pure network effects

A model without product differentiation (in fact with pure network effects) has already been examined in Caillaud and Jullien’s (2003) model of competing matching agencies. We revisit this case, in particular to explore the role of exclusive contracts. Our model coincides with a special case of theirs (assuming their matching technology is perfect and that there are no transaction fees), when we make the following
assumptions:

\[ A_1' \quad v_0^A = v_0^B = 0 \]

\[ A_2' \quad t_A = t_B = 0 \]

Given the assumption of no intrinsic benefits and no transportation costs, we will focus attention to prices for which \( 0 \leq p_1^A \leq b_A, 0 \leq p_1^B \leq b_B, 0 \leq p_2^A \leq b_A, 0 \leq p_2^B \leq b_B \) since regardless of network benefits, consumers will not purchase a service if its price exceeds the maximal network benefit, and we have assumed prices cannot be negative.

With pure network effects, there are twenty-five different demand configurations which can arise. On each side of the market, agents can subscribe to platform 1, platform 2, multihome, randomize between the two platforms, or not subscribe at all. For each of the five different possibilities for \( A \) types, there are five possibilities for \( B \) types. To work out consistent demand configurations we conjecture a given configuration (say configuration 4, in which all \( A \) types subscribe to platform 1 and \( B \) types subscribe to platform 2) and then check whether there is some range of the prices \( p_1^A, p_1^B, p_2^A, p_2^B \) for which if each agent believes others will behave in this way, they will choose to act consistently with the others’ beliefs. This gives rise to the twenty-five consistent demand configurations listed in Table 1.

[Table 1 about here]

The multiplicity in possible demands for given prices gives rise to the possibility of many different equilibria. To be consistent with Section 4, here we retain \((A3')\) and focus on multihoming equilibria in which all type \( A \) agents (sellers) multihome and type \( B \) agents (buyers) split between the two platforms (configuration 7 in Table 1).

**Proposition 5** Assume \((A1''), (A2'')\) and \((A3')\) hold. A set of equilibria exist in which \( A \) types multihome and \( B \) types randomize between the two platforms. The equilibria are characterized by the prices

\[
\begin{align*}
    p_1^A & \leq b_A/2 \text{ and } p_2^A \leq b_A/2 \\
    p_1^B & = p_2^B \leq f_B.
\end{align*}
\]  \hspace{1cm} (32) \hspace{1cm} (33)

**Proof.** The proposed equilibrium implies maximal profits of

\[
\pi^* = b_A/2 - f_A,
\]  \hspace{1cm} (34)

which can only be non-negative if the condition \( b_A \geq 2f_A \) is met. Given \((A3')\), both types of agents get non-negative utility at these prices. To see why this is an equilibrium note that at any candidate equilibrium involving prices for \( A \) types above \( b_A/2 \), table 1 implies \( A \) types will prefer to singlehome since the price of joining the second platform exceeds the associated network benefits. At any candidate equilibrium involving prices for \( B \) types above \( f_B \), either platform can undercut the prices on both sides in such a way that (by monotonicity) it will attract all agents and will increase profits. At prices which satisfy (32) and (33), neither platform wants to undercut prices so as to attract all \( B \) types, since this

\[17\] For brevity we restrict attention to randomization that involves agents choosing each platform with equal probability. The results naturally extend to the more general case.
will involve a greater subsidy to $B$ types and no gain on $A$ types. Similarly, neither platform wants to undercut prices to $A$ types, since each platform already attracts all such agents (they multihome). On the other hand, a platform which raises the price to either (or both) type of user loses all its users (pessimistic beliefs against such a deviating platform), which makes such a deviation unprofitable.

Caillaud and Jullien obtain essentially the same result (see their Proposition 11). One can view this result as the natural extension of the competitive bottleneck equilibrium in Proposition 3 to the case of pure network effects. Under the same assumption ($A3'$), the prices and equilibrium demand configuration in Proposition 3 also characterize an equilibrium here. The result shows that the possibility of a competitive bottleneck, and the associated positive profits for platforms, does not depend on any limitation in the extent of competition — here we have homogenous goods and price competition. This reinforces the idea discussed in Section 4, that in such two-sided markets there is a disconnect between market power (here there is none, by construction) and the ability of platforms to profitably sustain prices, at least to one type of user, above cost. Wright (2004) discusses other cases in which standard market analysis needs to be revisited to take into account the special features of two-sided markets.

5.1 Exclusive dealing

Using the framework above of pure network effects, we again examine whether a platform would want to offer an exclusive deal to $A$ types, to upset such a competitive bottleneck equilibrium. To do so we focus on the equilibrium above which implies maximal platform profits when no exclusive deals are possible. (It is straightforward to adjust the analysis to consider other cases). As was the case in the earlier setting, exclusive deals will be offered. However, here the new equilibrium with exclusive dealing is more efficient than the competitive bottleneck equilibrium.

**Proposition 6** Assume ($A1''$), ($A2''$) and ($A3'$) hold. Assume without exclusive contracts, the equilibrium in Proposition 5 applies with maximal profits. (i) Platforms will offer exclusive contracts, with platform 1 attracting all $A$ types. (ii) As a result of these contracts being offered, $A$ types will be better off and $B$ types will be worse off. (iii) Such contracts are efficient.

**Proof.** Since all $A$ types are identical, and given our tie-breaking assumptions, any equilibrium in the stage 2 subgame will either involve all $A$ types signing an exclusive contract with one platform, or none signing such a contract.

If $A$ types do not sign up to the exclusive contract in stage 2, then Proposition 5 applies to the stage 3 subgame. In this case platform 1’s profits $\pi^*$ are defined in (34). $A$ types receive no consumer surplus.

If a platform (say platform 1) signs type $A$ agents exclusively in stage 2, then the stage 3 equilibrium involves all $B$ types joining platform 1, even at the maximal price $p^*_B = b_B$, since the rival platform does not offer any network benefits. Platform 1’s corresponding profit is

$$\pi^E = b_B - f_A - f_B$$

plus the exclusive price $p^E_A$ it charges to $A$ types. Consumer surplus for $A$ types is

$$CS_A = b_A - p^E_A.$$
For $A$ types to want to sign the exclusive contract we require

$$p^E_A \leq b_A.$$ 

For platform 1 to want to offer the exclusive contract (when the other platform does not) we require

$$p^E_A \geq \pi^* - \pi^E,$$

where using (34) we get

$$p^E_A \geq b_A / 2 - (b_B - f_B).$$

Given (A3) there is a $p^E_A$ such that $b_A / 2 - (b_B - f_B) \leq p^E_A \leq b_A$, so that the platform and $A$ types will want to sign an exclusive contract. This shows that any subgame perfect equilibrium in the full game must involve $A$ types signing exclusive contracts.

Competition between the platforms to attract the $A$ types exclusively will leave platforms indifferent between offering the winning contract, or not, and earning nothing. Such an equilibrium must satisfy

$$\pi^E + p^E_A = 0,$$

which implies

$$p^E_A = f_A + f_B - b_B. \quad (35)$$

Given the tie-breaking rule, it is platform 1 that will attract all $A$ types through this exclusive contract in equilibrium. This proves result (i) in the proposition.

The resulting price in (35) may or may not be negative. In the case it is negative, and such a payment to sellers is not feasible, the price $p^E_A$ will be set to zero, thus allowing platform 1 a positive profit in equilibrium. Otherwise, competition forces both profits to be zero. As a result of the exclusive contracts, $A$ types enjoy a transfer of $b_A + b_B - f_A - f_B$ from the platforms and the $B$ types. The $B$ types have all their surplus extracted, and so given (A3) are worse off, confirming result (ii) in the proposition.

When all $A$ types multihome and $B$ types split, welfare is

$$W_M = b_A + b_B - 2f_A - f_B.$$ 

Under the exclusive deal, all agents join platform 1, in which case welfare is

$$W_E = b_A + b_B - f_A - f_B.$$ 

Welfare is increased by the saving of duplicated costs that arise under multihoming of $A$ types without exclusive contracts. This proves result (iii) in the proposition.

The proposition shows that with pure network effects, exclusive contracts to sign up $A$ types (to stop them multihoming) will again be used by platform 1 to foreclose the market to platform 2. In this case foreclosure is complete in the sense both sides of the market join platform 1 exclusively. In contrast to the previous setting, such exclusive contracts are efficient here. Given there is no product differentiation, the only efficient outcome is for all agents to subscribe to a single platform, which is what the use of exclusive contracts ensures.
5.2 Some other equilibria

In closing, it is worth pointing out some other equilibria of interest in this setting. First, note that the efficient outcome in which platform 1 takes the whole market is also an equilibrium when agents have pessimistic beliefs against platform 2, even without the use of exclusive contracts. There are two cases to consider, depending on whether negative prices are allowed or not. Without the possibility of negative prices, the equilibrium with maximal profits for the dominant platform involves platform 1 setting $p_A^1 = b_A$ and $p_B^1 = b_B$, and platform 1 obtaining a profit of $\pi^1 = b_A + b_B - f_A - f_B$. Platform 1 obtains the full surplus in this case.

Alternatively, when platforms can set negative prices this outcome is not an equilibrium, as Proposition 10 in Caillaud and Jullien demonstrates. Then the dominated platform will set a negative price to one side and undercut slightly on the other side (a divide-and-conquer strategy), so as to take the whole market and still make a positive profit. This constrains each of the prices the dominant platform can set to $f_A + f_B$. The maximum profits that can be sustained in a dominant-firm equilibrium where negative prices are allowed is thus $f_A + f_B$. Where negative prices are not feasible, an entrant will not be able to use a divide-and-conquer strategy, which thus allows the dominant firm to potentially obtain higher profits.

Finally, there is another equilibrium in the pure network effects setting which is of interest given it resembles that of a competitive bottleneck. Suppose that all agents believe that $A$ types will all go to the platform with the lowest group $A$ price (and with a tie, all will go to platform 1, say). These beliefs are consistent with optimal behavior, since if $A$ types behave in this way, $B$ types will also want to choose the same platform, which reinforces $A$’s choice. Then the following prices characterize an equilibrium $p_A^1 = p_B^1 = f_A + f_B - b_B$ and $p_A^2 = p_B^2 = b_B$, with both types singlehoming on platform 1. (This assumes $f_A + f_B \geq b_B$. Otherwise, a similar equilibrium arises in which $p_A^1 = p_A^2 = 0$ and $p_B^1 = p_B^2 = f_A + f_B$). At this equilibrium there are no profits, nor any multihoming, and the outcome is efficient. Yet this situation could also be called a “competitive bottleneck” equilibrium: in this case group $A$ is targeted in order to exploit group $B$. This reinforces an earlier point — that a competitive bottleneck should not necessarily be thought of as some kind of market failure.

6 Conclusion

This paper was motivated by two observations about two-sided markets. As Evans (2003) notes, a feature of many two-sided markets is that agents on one or both sides of the market can (and often do) multihome. Most merchants accept MasterCard and Visa, most businesses advertise in competing Yellow Pages directories, and most game developers write games for multiple systems (XBox, PlayStation etc). Moreover, in many such markets, platforms charge little or nothing to one side of the market (auctions, employment agencies, real-estate agents, search engines, shopping malls, Yellow Pages etc).

Our model can explain both observations by showing how a competitive bottleneck arises endogenously as an equilibrium. In this equilibrium, sellers have their network benefits fully exploited, while buyers face a price that is set below cost. Despite prices favouring buyers, it is sellers that choose to multihome.
Where sellers value buyers sufficiently strongly, the equilibrium implies the platforms give away their services to buyers. Essentially, platforms make a loss on buyers, which they recover from sellers who want to reach buyers and do not have a choice of which platform to join in order to do so given buyers have strong preferences for one or other platform.

Our model can also be used to explain why some two-sided markets involve exclusive contracts even though such contracts may be inefficient. The analysis shows that exclusive contracts will be used by platforms to prevent multihoming, thereby allowing them to attract one side exclusively (generically the sellers), and so capture the network benefits generated to the other side (the buyers). Although buyers are always made worse off as a result of these contracts, whether such contacts are inefficient depends on the extent of product differentiation on the buyer side. With no product differentiation on the buyers’ side, exclusive contracts allow the dominant platform to foreclose both sides of the market, which results in all agents buying from a single platform, the most efficient outcome in a world of pure network effects. In contrast, where buyers have heterogenous tastes for the competing platforms, exclusive contracts can be inefficient since compared to a multihoming solution they result in reduced network benefits and reduced variety.

7 References


Appendix A

Consider the equilibrium in Proposition 1 in which both prices are positive. This corresponds to the case considered by Armstrong (2004). He assumes (A3) and that agents singlehome, in which case assumption (A2) is not necessary to obtain the same result. Here we explain why we are unable to relax (A2) given that we allow for endogenous multihoming.

If (A2) is relaxed, then we need the conditions $3t_A \geq b_A + 2b_B - 2f_A$ and $3t_B \geq b_B + 2b_A - 2f_B$, in addition to (A3), so that agents will not want to multihome at the proposed equilibrium. For instance, consider the parameter values $t_A = 1$, $b_A = 2$, $f_A = 0$, $t_B = 2.5$, $b_B = 0.75$ and $f_B = 1$. Then even though the necessary condition (A3) holds and prices implied by the singlehoming equilibrium are positive, some type A agents are better off multihoming at the proposed equilibrium prices. Assumption (A2) ensures agents would not want to multihome.

The second possibility arises even if agents do not want to multihome at the proposed equilibrium prices. We still need to ensure that a platform cannot increase profits by lowering its price to one side so as to induce agents on that side to multihome, and raising its price on the other (singlehoming) side of the market, given the “equilibrium” prices set by the rival platform. Such a price change could be profitable since it allows the platform to increase market share more than it could for an equivalent change in prices when agents are assumed to continue to singlehome. For instance if $t_A = 1$, $b_A = 3$, $f_A = 0$, $t_B > 3$ and $b_B = 0$ then (A3) holds, but platform 1 can increase its profits by unilaterally changing its prices from those in (1) and (2) to

$$p^1_A = \frac{6t_B - 9}{8t_B - 9}$$
$$p^1_B = \frac{27 - 30t_B - 9f_B + 8f_B t_B + 8t_B^2}{8t_B - 9} > 0.$$
The increase in platform 1’s profits from such a deviation is
\[
\frac{t_B}{2(s_B - 9)} > 0.
\]
The deviation involves platform 1 lowering its price to side A and increasing its price to side B. Instead of sharing half of each side of the market, platform 1 will obtain more than half of A types (since some agents multihome) and more than half of B types (since it sets a lower price to side B than platform 2). Assumption (A2) rules out this possibility by ensuring agents will not want to multihome, even at these new prices.

9 Appendix B

For the competitive bottleneck equilibrium derived in Proposition 3, in order to support the equilibrium, we assumed a particular choice of configurations in the case a platform deviated by changing its prices. Here we show the choice of configurations is that implied by the following sequential tatönemment process (we also show some other ways the equilibrium is supported). First A types (who recall are all identical) decide which platform(s) to join, taking the B types’ choices as given. Then B types decide which platform to join, taking the A types’ choices as given, and so on, until a fixed point in the agents’ choices is reached. In the process, agents are assumed to only change their choice of platform(s) if doing so makes them strictly better off.

Proposition 7 The equilibrium defined in Proposition 3 is an equilibrium when agents’ choose which platform to join according to the sequential tatönemment process defined above.

Proof. Starting from the proposed equilibrium characterized by (23) and (24), consider whether platform 1 would want to deviate by changing one or both prices. There are six distinct cases that have to be considered.

(i) Platform 1 increases \(p_A^1\) and increases \(p_B^1\) (or leaves it unchanged). In this case A types would want to respond by singlehoming on platform 2, which would cause more B types to want to join platform 2, thereby confirming the choice of A types. The fixed point reached by this process is configuration 3. This is consistent with Proposition 3, which assumed agents would choose configuration 3 by monotonicity in this case.

(ii) Platform 1 leaves \(p_A^1\) unchanged and increases \(p_B^1\). In this case, A types would want to continue to multihome. The higher price to B types would cause more B types to want to join platform 2. This would cause A types to want to instead singlehome on platform 2, which would cause even more B types to join platform 2, thereby confirming the choice of A types. The fixed point reached by this process is configuration 3. This is consistent with Proposition 3, which assumed agents would choose configuration 3 by monotonicity in this case.

(iii) Platform 1 decreases \(p_A^1\) (or leaves it unchanged) and decreases \(p_B^1\). In this case A types would want to continue to multihome. The lower price to B types would cause more B types to want to join platform 1. This would cause A types to want to instead singlehome on platform 1, which would cause even more B types to want to join platform 1, thereby confirming the choice of A types. The fixed point
reached by this process is configuration 2. This is consistent with Proposition 3, which assumed agents would choose configuration 2 by monotonicity in this case.

(iv) Platform 1 increases $p_A^1$ and decreases $p_B^1$. In this case $A$ types would want to singlehome on platform 2, which would cause more $B$ types to want to join platform 2, thereby confirming the choice of the $A$ types. The fixed point reached by this process is configuration 3. This is consistent with Proposition 3, which assumed agents would always coordinate on configuration 3 in this case.

(v) Platform 1 decreases $p_A^1$ and increases $p_B^1$ (or leaves it unchanged) such that the condition for configuration 1,  

\[ p_A^1 \leq b_A/2 + b_A (f_B + t_B - b_A - p_B^1) / (2t_B) \]  

(36) still holds. Facing lower prices, $A$ types would want to continue to multihome. The higher price to $B$ types would cause fewer $B$ types to want to join platform 1, but despite this, given (36), $A$ types will continue to want to multihome. (The same is true when the price to $B$ types is left unchanged). The fixed point reached by this process is configuration 1. This is consistent with Proposition 3, which assumed agents would always coordinate on configuration 1 wherever possible (the inertia condition).

(vi) Platform 1 decreases $p_A^1$ and increases $p_B^1$ such that the condition for configuration 1 does not hold. Facing lower prices, $A$ types would want to continue to multihome. The higher price to $B$ types would cause fewer $B$ types to want to join platform 1. Given (36) does not hold, this would cause $A$ types to want to instead singlehome on platform 2, which would cause even more $B$ types to want to join platform 2, thereby confirming the choice of the $A$ types. The fixed point reached by this process is configuration 3. This is consistent with Proposition 3, which assumed agents would always coordinate on configuration 3 in this case.

For the equilibrium characterized by (26) and (27), the logic is identical, except now cases (iii) and (iv) do not need to be considered, as these are no longer feasible.

The above result shows that the “beliefs” that underlie the competitive bottleneck equilibrium we found in Proposition 3 can be given some plausible interpretation. The particular competitive bottleneck equilibrium can also be derived under some alternative conditions, which we briefly spell out here.

One can show, that for the case with $b_A \geq f_B + t_B$, and with the stronger condition that $b_B \geq 4f_A$, the same equilibrium in Proposition 3 holds as a subgame perfect equilibrium when agents have beliefs that uniquely define demands for any given prices. The particular beliefs which give this result are such that whenever there are multiple consistent configurations for given prices, agents coordinate on the configuration which maximizes total surplus (if total surplus is the same, they coordinate on the configuration which maximizes the total surplus of the agents, ignoring the profits of the platforms).

The equilibrium corresponding to $t_B \leq b_A < f_B + t_B$ also holds in the special case in which $A$ types obtain no network benefits ($b_A = 0$), although the analysis here is somewhat different as agents no longer need to coordinate their choice of configuration ($A$ types will make their choice independent of what $B$ types will do, and $B$ types know this). Thus, the implied equilibrium, in which $A$ types multihome at the prices $p_A^1 = p_A^2 = 0$ and $B$ types split between the two platforms at the prices $p_B^1 = p_B^2 = f_B + t_B$ does not depend on any particular specification of beliefs. However, the equilibrium can still be thought

\[ \text{(This requires $p_B^1$ strictly increases, since otherwise the condition for configuration 1 to hold is met given $p_A^1$ decreases.} \]
of as in the same way as the competitive bottleneck in Proposition 3 — that given they multihome, A
types have their network benefits fully exploited (it is just their surplus is zero here).
Table 1: Consistent demand configurations

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<td>(0 \leq p_A \leq b_A, 0 \leq p_A \leq b_A, 0 \leq p_B \leq b_B, 0 \leq p_B \leq b_B)</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(0 \leq p_A \leq b_A, p_A = 0, 0 \leq p_B \leq b_B, p_B = b_B)</td>
</tr>
<tr>
<td>21</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>(p_A = b_A, 0 \leq p_A \leq b_A, p_B = 0, 0 \leq p_B \leq b_B)</td>
</tr>
<tr>
<td>22</td>
<td>0</td>
<td>0</td>
<td>(\frac{1}{2})</td>
<td>(\frac{1}{2})</td>
<td>0</td>
<td>(p_A = p_A = \frac{b_B}{2}, p_B = p_B = 0)</td>
</tr>
<tr>
<td>23</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>(p_A = p_A = b_A, p_B = 0, p_B = 0)</td>
</tr>
<tr>
<td>24</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>(0 \leq p_A \leq b_A, p_A = b_A, 0 \leq p_B \leq b_B, p_B = 0)</td>
</tr>
<tr>
<td>25</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>(0 \leq p_A \leq b_A, 0 \leq p_A \leq b_A, 0 \leq p_B \leq b_B, 0 \leq p_B \leq b_B)</td>
</tr>
</tbody>
</table>