Access Charges and Quality Choice in Competing Networks*

Carlo Cambini (Politecnico di Torino)

and

Tommaso M. Valletti (Imperial College London and CEPR)***

Abstract

We study the impact of reciprocal access charges on the incentives to invest in networks of higher quality. We show how private and social preferences always diverge once investments are endogenized. Private negotiations never lead to charges being set at their marginal cost. Whether or not marginal cost charges have good dynamic properties depends on the way investments in quality impact on traffic generated on the networks.

Keywords: Telecommunications, Interconnection, Investments, Quality.

JEL Classification: L41, L51, L96.

* We thank Julian Wright for very useful comments. We acknowledge support from the Italian Ministry for Scientific Research MIUR Grant No. 2002 131535_005.

*** Corresponding author. The Business School, Imperial College London, 53 Prince's Gate, Exhibition Road, London SW7 2PG, UK. Tel. +44-20-75949215, Fax +44-20-78237685, e-mail t.valletti@ic.ac.uk, http://www.ms.ic.ac.uk/tommaso.
1. Introduction

A recent but expanding literature on two-way access pricing has addressed the question of interconnection in a deregulated environment. When competing operators have sufficient network coverage to reach all customers, they still need access to rival networks in order to terminate calls originated by own customers but destined to rival's subscribers. The policy concern is that firms might use interconnection terms in order to weaken the intensity of price competition. The answer to this problem was first provided by the seminal works of Armstrong (1998) and Laffont et al. (1998) (hereafter ALRT) and depends on the form of pricing strategies that firms follow. In the realistic case of two-part tariffs ALRT find a result of "profit neutrality": access terms cannot be used to sustain higher profits. This can be understood by noting that, with two-part tariffs, operators can build market shares without having to inflate their outflow charges. A high access would give profits to a network when it terminates a call, however all the profits thus generated would be used to lower the fixed component. These effects cancel out, producing the result of profit neutrality with respect to access charges.

Profit neutrality depends on a number of assumptions. In the ALRT benchmark firms are symmetric, they charge only for outgoing calls and consumers do not get utility from receiving calls, and the market is mature, i.e. there is 100% participation rate. In particular, by relaxing the first of such assumption, Carter and Wright (2003, hereafter CW) show that asymmetries do matter. In particular, they show that the larger network always prefers the reciprocal interconnection charge be set at cost, which is efficient in their model under many reasonable situations. Hence they conclude that it is possible to achieve desirable outcomes with minimal regulatory intervention by letting the parties

---

1 See Armstrong (2002) for a survey of variants of the basic ALRT framework.
negotiate and, in case of disagreement, by allowing the larger network to choose the reciprocal interconnection rate.

As the degree of asymmetry between networks is generated by investments in brand image, quality and so forth, a natural question arises as to what impact interconnection charges have on the incentives to invest. While most of the existing literature is static, in Valletti and Cambini (2002) and Cambini and Valletti (2003) we provide a first analysis of the dynamics of access pricing. We extend the framework of ALRT by introducing an investment stage - prior to price competition - where the level of quality chosen is the source of asymmetries in later stages of competition. When the regulator can commit to access rules prior to the investment stage and consumers’ traffic is affected by network quality, we show that, in order to induce firms to invest in an efficient manner, socially optimal access charges should be set below costs. On the contrary, if firms are left alone to negotiate reciprocal termination charges, they would set reciprocal termination charges above cost, in order to underinvest and avoid a costly investment battle. Above-cost access charges are then "tacitly collusive" as they make a firm more reluctant to become big and overtake the rival: the reason is that an increase of the own investment in quality, relative to the rival, generates more outgoing calls and therefore creates a costly access deficit.

The purpose of this paper is to study the similarities and differences between our approach, that leads to a negative view of unregulated charges, and that of CW whose results imply a rather non-intrusive role for regulation. We first study the impact of access charges on investments when quality gives a fixed benefit to consumers, but does not alter their calling patterns. We show that, to the extent to which firms or the regulator can commit to access charges prior to investments, then CW's results are reversed: there is never an alignment of private and social interests over the optimal level of access charges.
We then extend our prior work by analyzing an alternative specification of utility where quality impacts on traffic. Also in this case we confirm the divergence between social and private preferences and we show how socially optimal charges depend on the way traffic is generated.

The rest of the paper is organized as follows. Section 2 introduces a model of network competition based on ALRT with a quality feature that enters additively in consumers' utility. Section 3 recalls CW's results for a given level of investments, while Section 4 endogenizes quality choices and analyzes the impact of access charges on investments. Section 5 extends the analysis to the case where quality affects the number of calls made at a certain price. Section 6 concludes and discusses the main findings.

2. The model

The model follows ALRT. There are two networks (differentiated à la Hotelling) competing in a telecommunications market. A unit mass of consumers is uniformly located on the segment [0,1] while the network operators are located at the two extremities. We denote by 1 (respectively 2) the firm located at the origin (respectively at the end) of the line. Network operators compete in two-part tariff, i.e. \( T_i(q) = F_i + p_i q \), \( i = 1, 2 \), where the fixed fee \( F_i \) can be interpreted as a subscriber line charge and \( p_i \) as the marginal price for a call.

When a consumer located at \( x \) buys from firm \( i \) located at \( x_i \), he enjoys a utility given by:

\[
y + v_0 - |x - x_i|/(2\sigma) + v(p_i) + k_i - F_i
\]  

(1)
where $y$ is the income, $v_0$ is a fixed surplus component from subscribing (sufficiently large such that all customers always choose to subscribe to a network) and $v(p_i)$ is the indirect utility derived from making calls at a price $p_i$. The parameter $\sigma$ represents an index of substitutability between the networks.

In eq. (1) there is also a quality parameter $k_i$ that is increasing in operator $i$’s investment. We assume that the quality parameter enters the utility function additively. This makes the framework of this section similar to CW’s model where a parameter ($\beta$ in their notation) is introduced to measure an (exogenous) degree of asymmetry between the networks (brand loyalty). We will endogenize this asymmetry in this paper, as we will explicitly look for the level of quality provided by network operators.

Notice that, under an additive specification, quality is a fixed effect that enters the utility but does not influence directly the traffic level, i.e. the quantities of minutes of calls originated and terminated on the networks. This can describe cases where quality represents a benefit deriving from having additional services on a particular network (e.g. wake-up services, voice mail, simultaneous conversations), or a just a specific brand preference, as in CW. If we refer to competition between ISPs, this assumption implies that consumers get a benefit from services such as web assistance, simple set-up software, availability of mail boxes, special on-line information services and so on. These services enhance the value of the network perceived by customers but do not alter their volume of originated traffic.

Both networks can potentially serve every customer. Serving a customer involves a fixed cost $f$ of connection and billing. Each call has to be originated and terminated. The marginal cost is $c$ per call at the originating end and $t$ at the terminating end. The total marginal cost for a call is thus $c + t$. Networks pay each other a reciprocal two-way access
charge, denoted by $a$, for terminating each other’s calls. Finally, each network incurs a fixed cost $I(k)$ to provide a service of quality $k$, with $I'(\cdot) > 0$ and $I''(\cdot) > 0$.

The timing of the game is as follows. Interconnection terms are set initially. Operators then first invest in quality, followed by price competition.

3. Price Competition

The consumer indifferent between the two networks determines the market share of the two firms. In particular firm $i$’s share is $\alpha_i$ where:

$$
\alpha_i = \alpha(w_i, w_2) = 1/2 + \sigma(w_i - w_j)
$$

(2)

where $w_i = \nu(p_i) + k_i - F_i$ is the net surplus for customers connected to network $i$.

In the last stage investments are fixed, hence network $i$ has to solve:

$$
\max_{F_i, p_i} \pi_i = \max \left[ \pi_i - I(k_i) \right]
$$

$$
\pi_i = \alpha_i \left( (p_i - c - t)q(p_i) + F_i - f' \right) + \alpha_i(1 - \alpha_i)(a - t) \left[ q(p_j) - q(p_i) \right]
$$

(3)

Since our model in stage II corresponds to CW’s model by replacing their brand parameter with $\beta = 2\sigma(k_i - k_j)$, we borrow from their analysis the equilibrium prices:

$$
p_i^* = c + t + (a - t)\alpha_j^*,
$$

(3)

$$
F_i^* = \alpha_i^* / \sigma + f - \left( p_i^* - c - t \right)q(p_i^*) - (\alpha_i^* - \alpha_j^*)(a - t) \left( q(p_j^*) - q(p_i^*) \right)
$$

(4)
i.e. the usage fee is equal to the ‘perceived’ marginal cost. This is a typical result when
firms compete in two-part prices. From eq. (3) it is immediate to observe that, when
access charges are set above cost, then both firms would charge above marginal cost. In
addition, the firm with a market share above 50% would charge less than the rival. This is
because, being the larger firm, it would terminate more calls on-net than the rival, hence
the perceived marginal cost for the larger firm would be smaller.

In equilibrium, the profit (gross of investment) of network operator $i$ is given by:

$$\pi_i^* = \frac{\alpha_i^2}{\sigma} - \alpha_i^2 (a - t)\left(q(p_i^*) - q(p_j^*)\right).$$

(5)

where market share of firm $i$ at equilibrium is:

$$\alpha_i^* = \frac{1}{2} + \frac{\sigma}{3} \left[k_i - k_j + v(p_i^*) - v(p_j^*) + (a - t)\left(\alpha_j^* q(p_i^*) - \alpha_i^* q(p_j^*)\right)\right]^2.$$  

(6)

Since cost-based (LRIC) regulation is the typical regulatory benchmark, we are
interested to analyze the impact of stage-I parameters on stage-II equilibrium when access
charges are slightly increased above (or decreased below) termination costs. This is
developed in CW, where it is shown:

**Proposition 1.** A small variation of the access charge above the marginal cost of
termination has:

- A positive impact on prices: \( \frac{\partial p_i^*}{\partial a} \bigg|_{a=a_j} = \alpha_j^* \);  

(7)

\(^2\) Equations (5) and (6) can be obtained respectively from eq. (6) and (A.1) in CW.
• No local effect on market shares: \( \frac{\partial \alpha_i^*}{\partial a} \bigg|_{a=t} = 0 \); (8)

• No impact on gross profits if firms have the same quality. If firms have different levels of quality, then the higher quality firms always prefers \( a = t \), while the low quality firms prefers \( a = t \) only if its market share is less than 1/3;

• Finally, prices are not affected by quality, while market shares are:

\[
\frac{\partial p_{j}^*}{\partial k_j} \bigg|_{a=t} = \frac{\partial p_{j}^*}{\partial k_j} \bigg|_{a=t} = 0 ; \tag{9}
\]

\[
\frac{\partial \alpha_i}{\partial k_i} \bigg|_{a=t} = \frac{\partial \alpha_i}{\partial k_j} \bigg|_{a=t} = \frac{\sigma}{3} = \frac{\partial \alpha_i^*}{\partial k_j} \bigg|_{a=t} . \tag{10}
\]

Proof. Omitted as it would follow the same lines as CW (all expressions can be retrieved from their Appendix A).

A small increase in the termination charge implies that the perceived marginal cost increases and this effect is magnified for the smaller firm, since it has a higher proportion of off-net calls than the rival (eq. (7)). As far as the market share is concerned, the effect of a variation of \( a \) on prices is always off-set by a variation in the fixed charge, giving the result summarized by eq. (8). When access is set at cost, the perceived marginal cost is equal to the true marginal cost, independently from the firm’s share of the market and the relative differences in quality, which explains eq. (9). On the other hand, quality has an impact on utility of customers and a higher quality allows a firm to obtain a bigger share of the market (eq. (10)).
The most interesting result is the effect on gross profits. Asymmetries imply that the neutrality result of ALRT no longer holds. As shown by CW, starting from \( a = t \), a small increase in access charge implies that the high quality network has a net outflow of calls which is costly; similarly, if \( a < t \), the high quality network has a net inflow of calls which is unprofitable. Then, the high quality firm always wants to set \( a = t \).

A direct implication of these results stressed by CW is that the setting of access charges can be left to the parties when they are not too different. If they are very asymmetric, and absent any fear of foreclosure, then the larger operator should be entitled to select the reciprocal access price. In both cases \( a = t \) would be chosen, which is optimal for social welfare in the pricing stage as long as asymmetries between the two firms are not too big (see Proposition 2 in CW for details).

This strong policy conclusion however may depend on the degree of asymmetry that has been taken as exogenous so far. Do the networks have an incentive to invest in quality in order to reduce (or increase) this asymmetry? How does the access charge impact on investments? This is addressed in the next section.

4. Investment decision

In stage I, the FOC w.r.t. quality of firm \( i \) can be obtained subtracting from eq. (5) the cost of investing, leading to this expression:

\[
\frac{\partial \Pi}{\partial k_i} = 2 \alpha_i^* \frac{\partial \alpha_i^*}{\partial k_i} - (a - t) \frac{\partial \Omega}{\partial k_i} - I'(k_i) = 0 \quad \text{where} \quad \Omega = (\alpha_i^*)^2 \left( p_i^* - q(p_i^*) - q(p_i^*) \right).
\]

As before, we take LRIC as the starting benchmark. We are now interested to see if, starting at \( a = t \), a firm would increase or decrease quality when the access charge is slightly increased above cost. In the Appendix we are able to obtain this important result:
**Proposition 2.** In a neighborhood of \( a = t \), investments of both firms reach a maximum. In a symmetric equilibrium, firms’ profits reach a minimum at \( a = t \).

*Proof.* See the Appendix.

Intuitively, imagine firms start from a symmetric choice of quality. The firm that decides to invest a bit more thus becomes the high quality firm. From Proposition 1 we know that the high quality firm maximizes its gross profit when \( a = t \). Then, in stage I, the incentive to compete, via higher investments, is stronger when \( a = t \). On the contrary, whenever the access charge diverges from its marginal cost, the (gross) profit and investments both decrease. In a symmetric equilibrium, the networks perfectly share the market and the gross profit is fixed at \( 1/(4\sigma) \); then, for \( a = t \), the net profit is minimized since the level of investment reaches its maximum.

The implication of Proposition 2 is that networks do not have any incentive to negotiate a termination charge at cost since it reduces their net profits. This finding makes CW’s conclusion sensitive to the timing of the game (in our framework firms - or the regulator - can commit to access terms prior to making investments) when asymmetries that induce fixed effects on utility are endogenized. In fact, the conclusion on the coincidence of wants between private and social interests is reversed.

To see this, we still have to show what is the socially optimal level of access charges. When \( a = t \), in a symmetric equilibrium, it can be seen from eq. (11) that the optimal private level of investment is given by \( k |_{a=t} = I^{-1}(1/3) \). From a social point of
view, the optimal level of investment is derived from the maximization of total welfare, given by the following expression:

$$\max_{k_i,k_j} W = \alpha_i (v(p_i) + k_i) + \alpha_j (v(p_j) + k_j) - (c + t)(\alpha_i q(p_i) + \alpha_j q(p_j)) - I(k_i) - I(k_j)$$

$$- (a_i^2 + a_j^2)/(4\sigma) + v_0 - f$$

The socially optimal level of investment, denoted with $k^W$, that network $i$ should undertake is given by the following condition:

$$\frac{\partial W}{\partial k_i} = \alpha_i \left(1 + \frac{\partial v(p_i)}{\partial p_i} \frac{\partial p_i}{\partial k_i}\right) + \alpha_j \left(\frac{\partial v(p_j)}{\partial p_j} \frac{\partial p_j}{\partial k_i}\right) + \frac{\partial \alpha_i}{\partial k_i} (v(p_i) + k_i) + \frac{\partial \alpha_j}{\partial k_i} (v(p_j) + k_j)$$

$$- (c + t) \frac{\partial (\alpha_i q(p_i) + \alpha_j q(p_j))}{\partial k_i} - I'(k_i) = \left\{ \alpha_i \frac{\partial \alpha_i}{\partial k_i} + \alpha_j \frac{\partial \alpha_j}{\partial k_i} \right\} / (2\sigma) = 0.$$

Evaluating the above equation in $a = t$, given eq. (9) and eq. (10), when $k_i = k_j$, we obtain $k^W\bigg|_{a=t} = I^{-1}(1/2) > k^*\bigg|_{a=t}$. Thus, privately chosen investments are lower than the second best. In order to give incentives to invest, a social planner would set access charge at cost since – as shown in Proposition 2 – private investments are maximized in $a = t$: any other access charge would make investments depart even further from their socially optimal level.

Notice that in our analysis the common regulatory benchmark LRIC is optimal both from a static and from a dynamic point of view. LRIC is optimal in a symmetric equilibrium in stage II as market shares are optimal and any departure from costs would induce a sub-optimal allocation of calls. This is reinforced in stage I, as any departure from cost would worsen the level of investment, which is already too low. In other words,
LRIC is optimal independently from whether or not the regulatory authority can ex ante credibly commit to set access rules before investment takes place. On the contrary, unregulated network operators would always agree not to set access charge at cost in the first stage, since in $a = t$ their net profits are minimized.

In conclusion, endogenizing the asymmetry between networks, turns CW’s results quite dramatically: in equilibrium, the interest of a social planner and the private ones are never aligned. The former would like to set $a = t$, precisely the level of access charges that would never be chosen by the firms. However, contrary to Valletti and Cambini (2002), the additive model is elusive on the way firms would prefer to negotiate the access charge above or below termination cost. While both the additive and multiplicative specification of endogenous quality lead to the conclusion that unregulated agreements on reciprocal access charge are detrimental for social welfare, the multiplicative form is crucial in defining the below-cost social optimum charge and the above-cost collusive result. In the next section we develop a model of quality interaction between network, in order to test the robustness of our earlier findings.

5. Quality choice and interaction between networks: a multiplicative quality setting

In this section we consider the case where the quantity of calls is influenced by the investment in quality. Imagine situations that involve a high-speed network like ISDN, DSL or another broadband connection. These technologies require expensive network investments and permit the users to surf the Internet, receive calls, download documents, web pages, MP3 files or video clips. In all these examples, for a given price, a higher quality feeds into higher traffic generated by users. It is then clear that an additive

---

3 With an additive specification of utility, the net profit function is U-shaped with a minimum in $a = t$. 

specification of quality such as eq. (1) cannot capture this link between quality and quantities.

In Valletti and Cambini (2002) we adopt a multiplicative model and study two different quality settings: in the first one the quality of on-net and off-net calls are both exclusively controlled by the network users subscribe to. In the second one quality depends on the interaction between the networks, in particular quality of off-net calls has a strong bottleneck feature and it depends on the minimum quality provided by the two networks. In the following we develop an alternative model of interacting qualities. We now denote with $k_i$ the quality provided by network $i$ for its on-net calls, while the quality for off-net calls depends on a weighted average of the qualities offered by both networks. The quality for calls from network $i$ to network $j$ is thus $\Lambda_j = \delta k_i + (1 - \delta) k_j$, where $\delta \in [0, 1]$ is an exogenous parameter.\footnote{$\delta = 1$ corresponds to the basic model in Valletti and Cambini (2002).} This assumption seems to be particularly relevant for the interconnection between ISPs in order to ensure global connectivity: since providers are interconnected, every packet that travels has to pass across several networks. A natural assumption to describe this is to assume that the quality level (QoS) of the off-net traffic depends on a weighted average of the quality of the networks that carry traffic.

Firm $i$’s market share is still given by eq. (2), where now $w_i = \nu_i(p_i) - F_i = \alpha_i k_i \nu(p_i) + \alpha_j \Lambda_i \nu(p_i) - F_i$ is the net surplus for customers connected to network $i$. Notice the multiplicative specification of quality. Rearranging, we can write:

$$\alpha_i = \frac{M_j + \sigma(F_j - F_i)}{M_i + M_j}$$

(12)
where \( M_i = 1/2 + \sigma(\Lambda_i \nu(p_i) - k_j \nu(p_j)) \). Network \( i \) gross profit is given by:

\[
\pi_i = \alpha_i \left[ \alpha_i (p_i - c - t)k_i q(p_i) + \alpha_i (p_i - c - a)\Lambda_i q(p_i) + F_i - f \right] + \alpha_i \alpha_j (a - t)\Lambda_j q(p_j)
\]

After maximization, we obtain at the equilibrium the following prices:\(^5\)

\[
p_i^* = c + t + (a - t)\alpha_i^* \Lambda_i / (\alpha_i^* k_i + \alpha_j^* \Lambda_i)
\]

\[
F_i^* = f + \frac{\alpha_i^*}{\sigma} (M_i + M_j) - (p_i^* - c - t)\left[ \Lambda_i + 2\alpha_i^* (k_i - \Lambda_i) \right] q(p_i^*)
\]

\[
- (\alpha_i^* - \alpha_j^*) (a - t) \left( \Lambda_i q(p_i^*) - \Lambda_j q(p_j^*) \right)
\]

where \( \alpha_i^* \) is the equilibrium market share of network \( i \). Note that, as before, call prices are set to maximize joint surplus between network \( i \) and its customers. As off-net quality is weighted down by the lower quality derived by interconnection, call prices are scaled down by a “quality adjusted factor” \( \alpha_i^* \Lambda_i / (\alpha_i^* k_i + \alpha_j^* \Lambda_i) \). Substituting these prices into (12), one gets market share in equilibrium. This results in a complicated expression that simplifies when access is set at cost into the following:

\[
\alpha_i^* \bigg|_{\text{as} \rightarrow} = 1/2 + \sigma \nu(c + t)(k_i - k_j) \delta / 3
\]

Notice that in the extreme case when \( \delta = 0 \), market share is set at 50% independently from any quality difference. To confirm this, when \( a = t \), the perceived marginal cost is equal to the true marginal cost everywhere, both for on- and off-net calls,
hence networks will offer the same call price. Imagine now the market is indeed shared equally: net surplus from subscribing to firm $i$ simplifies to $\alpha_i k_i v(p_i) + \alpha_j k_j v(p_j) - F_i = (k_i + k_j) v(c + t) / 2 - F_i$. Hence, even if firms are asymmetric, the call surplus is the same for the customers. Competition is then driven only by transportation costs and firms will in fact share the market equally. On the contrary, when $\delta \neq 0$, the high quality network still gets more than 50% of the market even if it is penalized by the reduced quality of interconnection.

We have the following comparative statics on prices, market shares and profits in $a = t$, summarized in the following Proposition:

**Proposition 3.** A small variation of the access charge above its marginal cost has:

- A positive impact on prices:
  $$\left. \frac{\partial p_i^*}{\partial a} \right|_{a=t} = \frac{\alpha_i^* \Lambda_i}{\alpha_i k_i + \alpha_j^* \Lambda_i}$$  (16)

- A positive (negative) impact on the market share of the higher (lower) quality firm:
  $$\left. \frac{\partial \alpha_j^*}{\partial a} \right|_{a=t} = \frac{\alpha q(c + t)}{3} (1 - \delta)(k_i - k_j) \left[ \alpha_j^* \left( \left. \frac{\partial p_j}{\partial a} \right|_{a=t} \right) + \alpha_i^* \left( \left. \frac{\partial p_j}{\partial a} \right|_{a=t} \right) \right]$$  (17)

- The following impact on gross profits:
  $$\left. \frac{\partial \pi_i}{\partial a} \right|_{a=t} = \frac{q(c + t)}{3} (1 - \delta)(k_i - k_j) \alpha_i^* \left[ 2 \alpha_j^* \left( \left. \frac{\partial p_j}{\partial a} \right|_{a=t} \right) - \alpha_j^* \left( \left. \frac{\partial p_j}{\partial a} \right|_{a=t} \right) \right]$$  (18)

- Finally, prices are not affected by quality, while market shares are:

5 See the Appendix for calculations.
\[
\frac{\partial p_i^*}{\partial k_i} = \frac{\partial p_j^*}{\partial k_j} = 0 \tag{19}
\]

\[
\frac{\partial \alpha_i^*}{\partial k_i} = \frac{\alpha}{3} \nu(c + t) \delta \tag{20}
\]

**Proof.** See the Appendix

As before, the impact of a higher termination charge is to increase both call prices (eq. (16)) and to reduce fixed fees since part of the interconnection mark up is passed on to customers. Contrary to the previous case, however, an increase in access charge has an impact on market shares (eq. (17)). In fact, while the reduction in fixed fees is almost the same for both networks, the increase in call prices is diluted for the higher quality firms due to the presence of the “quality adjusted factor” which limits the raise in price. Then, the higher quality network is able to increase its share of customers. In addition, an access charge raised slightly above cost has an impact also on gross profit. Once again, the result of ALRT of “profit neutrality” of the reciprocal access charge under competition in two-part tariffs does not hold any longer if networks are asymmetric.

5.1. Investment decision

We look at a symmetric equilibrium and ask what firms and a social planner would likely to do around \(a = t\). In the first stage, the symmetric equilibrium when \(a = t\) is given by the FOC:

\[
\frac{\partial \Pi_j}{\partial k_j} = 0 = 0
\]

\[
\frac{\partial \Pi_j}{\partial k_j} = 0 = 0
\]
The social optimal is \( k^W \bigg|_{a=t} = I^{-1}(v/2) \). Hence compared to the previous additive case the under-investment problem is exacerbated for low values of \( \delta \) (a firm would invest even less since it confers benefits to the rival users calling off-net when it sets a high quality). The impact on investments when \( a \) is increased slightly above \( t \) is summarized in the following result:

**Proposition 4.** In a symmetric equilibrium, when access charges are set slightly above cost, firms decrease their investment when \( \delta \) is high enough:

\[
\frac{dk_i}{da} \bigg|_{a=t}^{a=t+k_j} < 0 \quad \text{iff} \quad \delta > 7/13.
\]

Thus, when \( \delta \) is high enough, firms would want to collude on above-cost access charges while a social planner would want to set below-cost access charges.

**Proof.** See the Appendix.

This result confirms what has been shown both in our earlier work and in the previous section: firms can use access charges to increase their profits. Thus, even in a symmetric equilibrium with two-part pricing we can restore the “tacit collusion” statement shown by ALRT. In our framework, however, tacit collusion is due not to the classical “raise each other’s cost” effect as in ALRT (with linear pricing), but to “diminish each other’s incentives to invest”. In conclusion, unregulated negotiations over reciprocal access charges are detrimental to social welfare.
It is worth noting that in a multiplicative setting the preference of network operators is clear: they would negotiate access charges above cost if $\delta$ is high enough. From a social point of view, contrary to the additive specification, LRIC is not optimal anymore since investments are now monotonic around $a = t$. When $\delta$ is high enough, the optimal policy is to set access charges below cost as this induces firms to invest more. This has a first-order gain on welfare, despite introducing a second-order loss since calls are now misallocated.

This results hold if the parameter $\delta$, i.e. the way traffic is passed from on network to the other, is high enough. For $\delta = 1$ the model described corresponds to our previous basic model. On the contrary, let $\delta = 0$; in this case, the quality of off-net calls depends only on the other network’s quality choice. Then, if a network unilaterally invest in quality, the quantity of its on-net calls increases, while the quantity of off-net calls remain the same. However, the quantity of calls to be terminated (i.e. off-net calls originated by the rival network) raises as the increase in own quality prompts users on the rival network to call more often own users. In the end, the network receives a net inflow of calls, which is remunerative if the access charge is set above the termination cost. This gives to the network an incentive to invest in quality, which has a negative overall impact in a symmetric equilibrium since it induces a costly investment battle without making extra gross profits in stage II. When off-net quality is mostly dependent on the quality chosen by the rival, if networks are unregulated, they prefer a bill-and-keep mechanism since it leads the firms to collude not to increase their investment. In this case, a social planner would prefer to allow for a termination mark up in order to induce operators to undertake more investments.
6. Conclusions

This paper has derived one robust result: once investments are endogenized, there is never an alignment between private and social interests over the setting of reciprocal interconnection charges. In particular, in all the specifications that we have studied, LRIC would never be chosen by firms, while in most cases they would agree on above cost termination charges. Whether or not LRIC has good dynamic properties for total welfare depends on how investments affect traffic. If investments do not affect traffic, then LRIC has both good static and dynamic properties. On the other hand, if higher investments lead consumers to generate more traffic, then below-cost charges are second best as long as own investments have a major impact on both on- and off-net quality. Our results are in contrast with CW's finding that LRIC is a candidate access price that is likely to emerge in private negotiations.

There are two critical factors in our analysis that explain the stark difference in the policy conclusions:

a) the commitment role of access charges, and

b) whether investment in quality has an impact on the quantity of calls.

The first point is the more relevant one. Our results apply when access charges are set prior to investments, i.e. firms (or the regulator) can commit to an access pricing rule. In practice, how realistic is this assumption? The answer must distinguish between the type of commitment a regulator can achieve as opposed to the one that unregulated firms might have. It can be argued that the access price set by regulation provides some commitment. To give an example, a regulator can announce that it will set access charges at a certain level (e.g. LRIC) for a certain period - for instance until the next review. In this case our timing makes sense: if firms undertake investments during the same period, they will take as given the access rule set by the regulator. Once such commitment is
present our results then say that LRIC has good dynamic properties only if quality has no impact on quantities (additive specification of the utility function), while below-cost charges are good if quality affects quantity of calls (multiplicative specification with a strong off-net impact of own investment). In this latter case, while optimal charges require a lot of demand and cost information on the regulator's side, we note that commitment value could be achieved by introducing a simple system such as a "bill-and-keep" rule that effectively sets to zero the termination charge.6

When access charges involve commitment, we have also derived a "collusive" result for symmetric unregulated firms. This commitment value could be obtained by allowing operators to write long-term contracts over access terms. However, such commitment is much more difficult to achieve for firms as they can change, or reverse, a decision on access prices relatively costlessly. On the contrary, investment which builds up brand loyalty, or delivers a better service, is a sunk asset in later stages of the game and should be viewed as a commitment device. In this context it makes sense to consider the alternative timing where investments are set first, then access charges, followed by price competition. If investments can be taken as exogenous (for instance because large investments have already occurred in the past), then CW's analysis holds for the additive case. Lean regulation can be introduced leaving to the parties (or to the bigger network in case negotiations fail) the setting of reciprocal access charges, as this leads to LRIC being chosen. In fact, this result can also be extended to the case where endogenous investments occur first, and then reciprocal access charges are decided by the larger firm and set prior to competition.7

---

6 See also DeGraba (2002).

7 There is a technical problem to reach this conclusion: to solve the first stage (when investments are made), a solution is needed for the second stage (when the larger firm sets access terms) for any level of
However, this conclusion does not hold for the case of multiplicative quality.\(^8\)

This brings us to the second crucial factor, whether an additive or a multiplicative specification of quality in the utility function makes more sense. Both have their own merits. In a standard local telephone network, unless the number and/or length of an individual calling causes a capacity constraint to bind, investment in more capacity is not likely to influence how many calls a person makes, but could result in more callers joining the network. Investments in brand image also do not impact on the quantity of calls. However, if investments allow a network to have greater and/or better coverage (e.g. in mobile telephony), then a subscriber is going to call more per unit of time at a given price. Similarly, if investments in fixed telephony allow a subscriber to surf the net and call at the same time (e.g. the subscriber gets multiple lines), again quantities will increase with the level of investment. Which specification is more realistic is largely an empirical matter.

\(^8\) This can be seen from Proposition 3. The preferences of the two networks over access terms always diverge. When \(\delta\) is high the high quality/bigger network prefers a charge below cost while the low quality/smaller network prefers a charge above cost (see eq. (18)). As the market share of the high quality network is sub-optimally low, welfare is increased by setting the access term above cost in order to try to expand the incumbent's market share (see eq. (17)). Hence, in this case, it should be the smaller network to set interconnection terms in order to produce such positive mark-up over termination, although there is nothing to ensure that the private choice of the smaller network would be socially optimal.
References


Appendix

Proof of Proposition 2.

The effect we want to study is given by the following expression:

\[
\frac{dk_i}{da} \bigg|_{a=0} = -\frac{\partial^2 \Pi_i}{\partial k_i \partial a} \bigg|_{a=0} \frac{\partial^2 \Pi_i}{\partial k_i^2} \bigg|_{a=0} \tag{a1}
\]

Differentiating eq. (11) w.r.t. \( k_i \), we have:

\[
\frac{\partial \Pi_i}{\partial k_i} = \frac{2\alpha_i^*}{\sigma} \frac{\partial \alpha_i^*}{\partial k_i} - (a-t) \left[ 2\alpha_i^* \frac{\partial \alpha_i^*}{\partial k_i} (q(p_i) - q(p_j)) + \alpha_i^* \left( q'(p_i) \frac{\partial p_j}{\partial k_i} - q'(p_j) \frac{\partial p_j}{\partial k_i} \right) \right] - I'(k_i) = 0
\]

Then the denominator of (a1) is \( \frac{\partial^2 \Pi_i}{\partial k_i^2} \bigg|_{a=0} = 2\sigma / 9 - I''(k_i) < 0 \) which we assume to be satisfied at a symmetric equilibrium for the SOC to hold. Now we must calculate:

\[
\frac{\partial^2 \Pi_i}{\partial k_i \partial a} = \frac{2}{\sigma} \frac{\partial \alpha_i^*}{\partial a} \frac{\partial \alpha_i^*}{\partial k_i} + \frac{2 \alpha_i^*}{\sigma} \frac{\partial^2 \alpha_i^*}{\partial k_i \partial a} - 2(a-t) \left[ \frac{\partial \alpha_i^*}{\partial a} \frac{\partial \alpha_i^*}{\partial k_i} + \alpha_i^* \frac{\partial^2 \alpha_i^*}{\partial k_i \partial a} \right] (q(p_i) - q(p_j)) \\
+ \alpha_i^* \frac{\partial \alpha_i^*}{\partial k_i} \left( q'(p_i) \frac{\partial p_i}{\partial a} - q'(p_j) \frac{\partial p_i}{\partial a} \right) - 2\alpha_i^* \frac{\partial \alpha_i^*}{\partial k_i} (q(p_i) - q(p_j)) \tag{a2}
\]

\[
-(a-t) \left[ 2\alpha_i^* \frac{\partial \alpha_i^*}{\partial a} \Gamma + \alpha_i^* \frac{\partial^2 \Gamma}{\partial a} \right] - \alpha_i^* \left( q'(p_i) \frac{\partial p_i}{\partial k_i} - q'(p_j) \frac{\partial p_i}{\partial k_i} \right)
\]

where \( \Gamma = q'(p_i) \frac{\partial p_i}{\partial k_i} - q'(p_j) \frac{\partial p_j}{\partial k_i} \).
Since \( \frac{\partial^2 \alpha_i^*}{\partial k_i \partial a} \bigg|_{a=t} = 0 \), and using the results of Proposition 1, we get:

\[
\frac{\partial^2 \Pi_i}{\partial k_i \partial a} \bigg|_{a=t} = 0 \Rightarrow \frac{dk_i}{da} \bigg|_{a=t} = 0.
\]  

(a3)

Given the above, we now analyze if the optimum just defined is a minimum or a maximum. To this regard, we must calculate in a neighborhood of \( a = t \):

\[
\frac{d^2 k_i}{da^2} \bigg|_{a=t} = \left[ \left( \frac{\partial^2 \Pi_i}{\partial k_i \partial a^2} \bigg|_{a=t} \right) - \frac{\partial^2 \Pi_i}{\partial k_i \partial a} \bigg|_{a=t} \right] \left( \frac{\partial^2 \Pi_i}{\partial k_i^2} \bigg|_{a=t} \right)^2
\]

that, given eq. (a3), reduces to:

\[
\frac{d^2 k_i}{da^2} \bigg|_{a=t} = -\frac{\partial^3 \Pi_i}{\partial k_i \partial a^2} \bigg|_{a=t} \left( \frac{\partial^2 \Pi_i}{\partial k_i^2} \bigg|_{a=t} \right)^2
\]  

(a4)

As the denominator is negative, the sign of (a4) strictly depends on the sign of the numerator. Thus, we have to calculate:

\[
\frac{\partial^3 \Pi_i}{\partial k_i \partial a^2} = \frac{2}{\sigma} \frac{\partial^2 \alpha_i^*}{\partial a^2} \frac{\partial \alpha_i^*}{\partial k_i} + \left( \frac{4}{\sigma} \frac{\partial \alpha_i^*}{\partial a} \frac{\partial^2 \alpha_i^*}{\partial k_i \partial a} + \frac{2}{\sigma} \frac{\partial \alpha_i^*}{\partial k_i} \frac{\partial^2 \alpha_i^*}{\partial a^2} \right) \left( a - t \right) \left( \frac{\partial \Lambda}{\partial a} + \frac{\partial \Theta}{\partial a} \right)
\]  

(a5)

where:
\[ \Lambda = \left( \frac{\partial \alpha_i^*}{\partial a} + \frac{\partial \alpha_i^*}{\partial k_i} \right) (q(p_i) - q(p_j)) + \alpha_i^* \frac{\partial \alpha_i^*}{\partial k_i} \left( q'(p_i) \frac{\partial p_i}{\partial a} - q'(p_j) \frac{\partial p_j}{\partial a} \right) \]

\[ \Theta = 2 \alpha_i^* \frac{\partial \alpha_i^*}{\partial a} + \alpha_i^* \frac{\partial \Gamma}{\partial a} . \]

In \( a = t \), it results \( \frac{\partial^2 \alpha_i^*}{\partial a^2} \bigg|_{a=t} = \frac{\sigma}{3} q'(1 - 2 \alpha_i^*) \), \( \frac{\partial^3 \alpha_i^*}{\partial k_i \partial a^2} \bigg|_{a=t} = 0 \), \( \Lambda \bigg|_{a=t} = \frac{\sigma}{3} q' \alpha_i^* \bigg|_{a=t} (1 - 2 \alpha_i^* \bigg|_{a=t} \). In addition:

\[ \frac{\partial \Gamma}{\partial a} = q''(p_i) \frac{\partial p_i}{\partial k_i} \frac{\partial p_i}{\partial a} + q'(p_i) \frac{\partial^2 p_i}{\partial k_i \partial a} - q''(p_j) \frac{\partial p_j}{\partial k_i} \frac{\partial p_j}{\partial a} - q'(p_j) \frac{\partial^2 p_j}{\partial k_i \partial a} \]

where:

\[ \frac{\partial^2 p_i}{\partial k_i \partial a} \bigg|_{a=t} = -\frac{\partial^2 p_i}{\partial k_i \partial a} \bigg|_{a=t} = \frac{\partial \alpha_i^*}{\partial k_i} \bigg|_{a=t} = -\frac{\sigma}{3} . \]

Hence \( \frac{\partial \Gamma}{\partial a} \bigg|_{a=t} = -\frac{2}{3} \sigma q' \) and \( \Theta \bigg|_{a=t} = -\frac{2}{3} \sigma q' \alpha_i^* \bigg|_{a=t}^2 \). Finally, substituting the above expressions into (a5) and simplifying, we obtain:

\[ \frac{\partial^3 \Pi}{\partial k_i \partial a^2} \bigg|_{a=t} = \frac{2}{9} \sigma q' \left( 18 \alpha_i^* \bigg|_{a=t}^2 - 8 \alpha_i^* \bigg|_{a=t} \right) + 1 \]
where the bracket is always positive as \(0 \leq \alpha_j^* \bigg|_{a=t} \leq 1\). Then, eq. (a6) is always negative.

This implies that \(\frac{d^2 k}{da^2} \bigg|_{a=t} < 0\), i.e. the investment function is concave w.r.t. \(a\) in \(a = t\) the firm’s investment reaches a local maximum in \(a = t\). This proves the first part of Proposition 2. To prove the second part, it suffices to notice that, in \(a = t\), the gross profit is constant at the Hotelling level while investments are maximized. A small departure away from cost (either below or above) does not change the gross profits in a symmetric equilibrium while it induces firms to invest less. Hence the net profit of each network is minimized at \(a = t\).

**Proof of Proposition 3.**

Substituting eq. (13) and (14) into (12), market share in equilibrium is given by:

\[
\alpha_i^* = \frac{1}{3} + \frac{1}{3} \frac{M_i + \sigma \Xi}{M_i + M_j}
\]

where \(M_i\) and \(M_j\) are now evaluated in \(p_i^*\), and

\[
\Xi = (p_i^* - c - t)q(p_i^*)\left[2\alpha_i^*k_i + (1 - 2\alpha_i^*)\Lambda_i\right] - (p_j^* - c - t)\left[2(1 - \alpha_j^*)k_j - (1 - 2\alpha_j^*)\Lambda_j\right]g(p_j^*).
\]

Evaluating the equilibrium market share in \(a = t\), since \(\Xi \bigg|_{a=t} = 0\), \(M_i \bigg|_{a=t} = 1/2 + \sigma v(c + t)\delta(k_i - k_j)\), and \((M_i + M_j) \bigg|_{a=t} = 1\), we obtain eq. (15).
The equilibrium gross profit is given by the following expression:

\[ \pi_i = \frac{\alpha_i^2}{\sigma} (M_i + M_j) + \alpha_i^2 (a - t)(\Lambda_i q(p_i^*) - \Lambda_j q(p_j^*)) - \alpha_i^2 (1 - \delta)(p_i^* - c - t)q(p_i^*)(k_i - k_j) \]  

(a8)

From eq. (13) we have the comparative statics on prices summarized by eq. (16) and (19).

We now analyze the effect of \( a \) on the equilibrium market share given by (a7):

\[ \frac{\partial \alpha_i^*}{\partial a} = \left[ \frac{\partial M_i}{\partial a} + \sigma \frac{\partial \Xi}{\partial a} (M_j + M_j) - \frac{\partial (M_i + M_j)}{\partial a} \frac{\partial (M_i + M_j)}{\partial \sigma} \right] \left[ \frac{3(M_j + M_j)^2}{(M_j + M_j)^2} \right] \]  

(a9)

Evaluating all the above terms in \( a = t \), it results:

\[ \left. \frac{\partial M_i}{\partial a} \right|_{a=t} + \sigma \left. \frac{\partial \Xi}{\partial a} \right|_{a=t} = \sigma q(c + t)(1 - \delta)(k_i - k_j) \left[ 2\alpha_i^* \left. \left( \frac{\partial p_i}{\partial a} \right|_{a=t} \right) \right] + (1 - 2\alpha_i^* \left. \left( \frac{\partial p_i}{\partial a} \right|_{a=t} \right) \right] \]

\[ \left. \frac{\partial (M_j + M_j)}{\partial a} \right|_{a=t} = \sigma q(c + t)(1 - \delta)(k_i - k_j) \left[ \left. \frac{\partial p_i}{\partial a} \right|_{a=t} - \left. \frac{\partial p_j}{\partial a} \right|_{a=t} \right] \]

Substituting in (a9) and rearranging we obtain eq. (17) which is always positive if \( k_1 > k_2 \). The effect of \( k_i \) on the equilibrium market share is given by:

\[ \frac{\partial \alpha_i^*}{\partial k_i} = \left[ \frac{\partial M_i}{\partial k_i} + \sigma \frac{\partial \Xi}{\partial k_i} (M_i + M_j) - \frac{\partial (M_i + M_j)}{\partial k_i} \frac{\partial (M_j + M_j)}{\partial \sigma} \right] \left[ \frac{3(M_j + M_j)^2}{(M_j + M_j)^2} \right] \]
Since \( \frac{\partial M_i}{\partial k_i} \bigg|_{a=t} = \frac{\partial M_j}{\partial k_i} \bigg|_{a=t} = \sigma \delta \nu (c + t) \), \( \frac{\partial \Xi}{\partial k_i} \bigg|_{a=t} = 0 \), we have eq. (20). We can finally analyze the effects of access charges on profits. From eq. (a8):

\[
\frac{\partial \pi_i}{\partial a} = 2\alpha^*_i \frac{\partial \alpha^*_i}{\partial a} (M_i + M_j) + \frac{\alpha^*_i}{\sigma} \left( \frac{\partial M_i}{\partial a} + \frac{\partial M_j}{\partial a} \right) + (a - t) \frac{\partial \left( \alpha^*_i (\Lambda_j q(p_j) - \Lambda_i q(p_i)) \right)}{\partial a} + \\
\alpha^*_i (\Lambda_j q(p_j) - \Lambda_i q(p_i)) - \alpha^*_i (1 - \delta) q(p_i) (k_i - k_j) \frac{\partial p_i}{\partial a} - (p_i^* - c - t) \frac{\partial \left( \alpha^*_i q(p_i) (1 - \delta) (k_i - k_j) \right)}{\partial a}
\]

Then, in \( a = t \), given the above expressions and rearranging, we obtain eq. (18).

**Proof of Proposition 4.**

In order to analyze what happens to investments when \( a \) is increased slightly above \( t \), we must determine the sign of:

\[
\frac{dk_i}{da} \bigg|_{a=t, k_i=k_j} = -\frac{\partial^3 \Pi_i}{\partial k_i \partial a \partial a} \bigg|_{a=t, k_i=k_j} / \frac{\partial^2 \Pi_i}{\partial k_i^2} \bigg|_{a=t, k_i=k_j}
\]

Differentiating (a8) w.r.t. \( k_i \), we have:

\[
\frac{\partial \Pi_i}{\partial k_i} = \frac{1}{\sigma} \left[ 2\alpha^*_i \frac{\partial \alpha^*_i}{\partial k_i} (M_i + M_j) + \frac{\partial (M_i + M_j)}{\partial k_i} \alpha^*_i \right] + \\
(a - t) \frac{\partial \left( \alpha^*_i (\Lambda_j q(p_j^*) - \Lambda_i q(p_i^*)) \right)}{\partial k_i} - (p_i^* - c - t)(1 - \delta) \frac{\partial \left( \alpha^*_i (k_i - k_j) q(p_i^*) \right)}{\partial k_i}
\]

\[
-\alpha^*_i (1 - \delta) (k_i - k_j) q(p_i^*) \frac{\partial p_i^*}{\partial k_i} - I'(k_i)
\]
which, in \( a = t \), simplifies to:

\[
\frac{\partial \Pi}{\partial k_i} \bigg|_{a=t} = \frac{2}{3} \nu(c + t) \alpha_i^* \bigg|_{a=t} - I'(k_i). 
\]

In a symmetric equilibrium, \( \frac{\partial^2 (M_i + M_j)}{\partial k_i^2} \bigg|_{a=t, k_i = k_j} = 0 \), and \( \frac{\partial^2 \Xi}{\partial k_i^2} \bigg|_{a=t, k_i = k_j} = 0 \) implying

\[
\frac{\partial^2 \alpha_i^*}{\partial k_i^2} = 0, \quad \text{then the denominator of (a10) becomes}
\]

\[
\frac{\partial^2 \Pi}{\partial k_i^2} \bigg|_{a=t, k_i = k_j} = \frac{2\nu(c + t)^2 \delta^2}{9} - I^*(k_i), \quad \text{which is assumed to be negative for the SOC to hold.}
\]

Now we must calculate:

\[
\frac{\partial^2 \Pi}{\partial k_i \partial a} = \frac{2}{\sigma} \left\{ \frac{\partial \alpha_i^*}{\partial k_i} \frac{\partial \alpha_i^*}{\partial a} (M_i + M_j) + \alpha_i^* \frac{\partial^2 \alpha_i^*}{\partial k_i \partial a} (M_i + M_j) + \alpha_i^* \frac{\partial \alpha_i^*}{\partial k_i} \frac{\partial (M_i + M_j)}{\partial a} \right\}
\]

\[
+ \frac{\partial^2 (M_i + M_j)}{\partial k_i \partial a} \alpha_i^2 + \frac{2\alpha_i^*}{\sigma} \frac{\partial \alpha_i^*}{\partial a} \frac{\partial (M_i + M_j)}{\partial k_j} - (a - t) \frac{\partial \Gamma}{\partial a} - \Gamma - \frac{\partial \alpha_i^2}{\partial a} q(p_i^*) \frac{\partial p_i^*}{\partial k_i} 
\]

(a12)

where:

\[
(p_i^* - c - t)(1 - \delta) \frac{\partial \Phi}{\partial a} - (1 - \delta) \Phi \frac{\partial p_i^*}{\partial a} - (1 - \delta)(k_i - k_j) \frac{\partial \alpha_i^2}{\partial a} q(p_i^*) \frac{\partial p_i^*}{\partial k_i}
\]
\[ \Gamma = \frac{\partial (\alpha_i^2 (\Lambda_i q(p_i^*) - \Lambda_i q(p_j^*))}{\partial k_i} = 2\alpha_i^* \frac{\partial \alpha_i^*}{\partial k_i} \left( \Lambda_i q(p_i^*) - \Lambda_i q(p_j^*) \right) + \alpha_i^2 \left( \delta(q(p_i^*) + q(p_j^*) - q(p_j^*)) \right) \]

\[ \Phi = \frac{\partial (\alpha_i^2 (k_i - k_j) q(p_i^*))}{\partial k_i} = 2\alpha_i^* \frac{\partial \alpha_i^*}{\partial k_i} \left( k_i - k_j \right) q(p_i^*) + \alpha_i^2 q(p_i^*) + \alpha_i^2 (k_i - k_j) q'(p_i^*) \frac{\partial p_i^*}{\partial k_i} \]

We have \( \Gamma \bigg|_{a=t \atop k_i = k_j} = \frac{q(c + t)(2\delta - 1)}{4}, \Phi \bigg|_{a=t \atop k_i = k_j} = \frac{q(c + t)}{4}, \frac{\partial \alpha_i^*}{\partial a} \bigg|_{a=t \atop k_i = k_j} = 0, \frac{\partial p_i^*}{\partial a} \bigg|_{a=t \atop k_i = k_j} = \frac{1}{2} \).

\[ \frac{\partial^2 (M_i + M_j)}{\partial k_i \partial a} \bigg|_{a=t \atop k_i = k_j} = \sigma(1 - \delta)q(c + t) \left( \frac{\partial p_i^*}{\partial a} \bigg|_{a=t \atop k_i = k_j} - \frac{\partial p_j^*}{\partial a} \bigg|_{a=t \atop k_i = k_j} \right) = 0, \frac{\partial (M_i + M_j)}{\partial a} \bigg|_{a=t \atop k_i = k_j} = 0. \]

Hence (a12) simplifies to:

\[ \frac{\partial^2 \Pi_i}{\partial k_i \partial a} \bigg|_{a=t \atop k_i = k_j} = \frac{1}{\sigma} \frac{\partial^2 \alpha_i^*}{\partial k_i \partial a} \bigg|_{a=t \atop k_i = k_j} - \frac{q(c + t)(3\delta - 1)}{8} \quad \text{(a13)} \]

The last term we have to analyze is:

\[
\begin{align*}
\frac{\partial^2 \alpha_i^*}{\partial k_i \partial a} &= \frac{1}{3(M_i + M_j)^2} \left[ \left( \frac{\partial^2 M_i}{\partial k_i \partial a} + \sigma \frac{\partial^2 \Xi}{\partial k_i \partial a} \left( M_i + M_j \right) \right) + \left( \frac{\partial M_i}{\partial k_i} + \sigma \frac{\partial \Xi}{\partial a} \right) \left( \frac{\partial (M_i + M_j)}{\partial a} \right) \right. \\
&\quad - \frac{\partial^2 \Xi(M_i + M_j)}{\partial k_i \partial a} \left( M_i + \sigma \Xi \right) - \frac{\partial (M_i + M_j)}{\partial k_i} \left( \frac{\partial M_i}{\partial a} + \sigma \frac{\partial \Xi}{\partial a} \right) \right] \\
&\quad - 2 \frac{\partial (M_i + M_j)}{\partial a} \left[ \left( \frac{\partial M_i}{\partial k_i} + \sigma \frac{\partial \Xi}{\partial a} \right) \left( M_i + M_j \right) - \frac{\partial (M_i + M_j)}{\partial k_i} \left( M_i + \sigma \Xi \right) \right] \\
&\quad \text{ (a14)}
\end{align*}
\]
Given the above results, eq. (a14) reduces to
\[ \frac{\partial^2 \alpha_i}{\partial k_i \partial \alpha_j^{\text{phot}}} = \frac{\sigma q(c + t)(1 - \delta)}{6} \]
and eq. (a13) becomes
\[ \frac{\partial^2 \Pi_j}{\partial k_i \partial \alpha_j^{\text{phot}}} = \frac{q(c + t)(7 - 13\delta)}{24} \]. In conclusion, we have:

\[ \frac{dk_i}{da^{\text{phot}}} = -\frac{g(c + t)(7 - 13\delta) / 24}{\frac{1}{2} \sigma \nu (c + t)^2 \delta^2 - I^*(k_i)} \]

which is always negative if \( \delta > 7/13 \equiv 0.54 \).