Yet Another Model for the Federal Funds Rate Target
An Alternative Methodology and Forecast Evaluation

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Abstract

Forecasting interest rate rules of central banks is one of the challenging topics in empirical monetary economics. It has recently been shown that the forecast of the federal funds rate target can be considerably improved if its evolution is modeled as a marked point process (MPP) in which the irregularly spacing between target change events is accounted for. This paper proposes a new MPP model for the target that combines the autoregressive conditional hazard (ACH) and the autoregressive conditional multinomial (ACM) model. The ACM, a dynamic model for discrete data, is appealing from a methodological point of view, since the time series properties of target change sizes are explicitly modeled. We find that the ACH-ACM approach improves the accuracy of target forecasts in the short and medium term. Furthermore, we show by adapting methods for the evaluation of density forecasts that MPP models are appropriate to predict target changes on a forecast horizon of three months. However, the forecast quality deteriorates beyond this forecast horizon. These results suggest that MPP models are useful for predicting the target in the short and medium term.

Keywords: federal funds rate target, marked point process, ACH, ACM, density forecast evaluation

JEL classification: C41, C52, C53, E47, E52

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1 Introduction

By setting a target for the effective federal funds rate, the executive body of the Federal Reserve (Fed) influences a widespread range of economic variables and the financial markets. Therefore, if and how much the Fed changes the target is of great interest for policy makers and investors. Hamilton and Jordà (2002) have recently shown that the accuracy of target forecasts can be considerably improved if the sequence of target change events and the associated target change sizes are modeled as a marked point process (MPP). The MPP methodology accounts for both the irregularly spacing between target change events and the discreteness of the target change size.

This paper draws on Hamilton and Jordà’s (2002) seminal work. We present a new MPP model for the federal funds rate target and compare its forecast performance with existing approaches. Our methodology combines Hamilton and Jordà’s (2002) autoregressive conditional hazard (ACH) model and the autoregressive conditional multinomial (ACM) model introduced by Russell and Engle (2005). We also adapt methods for evaluating density forecasts in order to provide an assessment of the quality of target forecasts delivered by MPP models.

The paper contributes to the literature which focuses on the estimation of empirical reaction functions, i.e. the response of the Fed to economic developments (see Judd and Rudebusch, 1998; Khoury, 1990). For that purpose other papers have employed vector-autoregressive models (VARs) (e.g. Bernanke and Blinder, 1992; Evans and Marshall, 1998; Sack, 1998). However, since target changes occur in discrete steps, and because the time interval between change events is irregular, using a VAR can be criticized on methodological grounds (Rudebusch, 1998; Evans and Kuttner, 1998). A popular econometric approach that takes into account the discreteness of the target change sizes is the ordered probit (OP) model. Analyses of the Fed’s, the Bank of England’s and ECB’s monetary policy using OP models include Eichengreen, Watson, and Grossman (1985), Davutyan and Parke (1995), Dueker (1999), Carstensen (2006), and Gerlach (2005).

A major contribution to that literature has been delivered by Hamilton and Jordà (2002) (henceforth H&J). They introduce a model that accounts for both the irregularly spacing of the target change events and the discreteness of the target change size. Drawing on recent contributions to the econometrics of ultra high frequency data, H&J propose to conceive the
time series of target changes as a realization of a marked point process.\(^1\)

Engle (2000) proposes to separate the modeling of a marked point process into a model for the duration between events (here: observing a target change) and another model for the marks (the variables observed when the event occurs, here: the size of the target change). To model the duration between target changes Hamilton and Jordà (2002) put forth a dynamic discrete time duration model. They refer to it as the autoregressive conditional hazard (ACH) model which enjoys, due to its versatility, increasing popularity (e.g. Demiralp and Jordà, 1999; Bergin and Jordà, 2004; Davis and Hamilton, 2004; Dolado and María-Dolores, 2002). Applied for modeling the Fed response function, the ACH model delivers an estimate of the probability that a target change will be observed within the next week. H&J follow the previous literature and choose an ordered probit to model the target change size. The ACH-OP methodology delivers one- and multi-period target forecasts. One-step ahead forecasts are simple, but for multi-period target forecasts computer-intensive simulation techniques have to be employed. However, the additional effort pays off. As a matter of fact, the ACH-OP methodology was a major leap forward in terms of improving the accuracy of the target forecast. Compared to a VAR model, the mean squared error (MSE) of the ACH-OP target forecast is reduced by an order of magnitude at all forecast horizons.

This paper offers two contributions to this literature. We first motivate an alternative MPP model for the target that combines the ACH with the autoregressive conditional multinomial (ACM) model introduced by Russell and Engle (2005) (henceforth R&E), and compare its empirical performance with the ACH-OP approach. R&E’s main objective was to provide a model for discrete transaction price changes (“ticks”).\(^2\) Those tick changes occur at a much higher frequency (with only seconds between events), but also with irregular intervals between trade events. Hence, similarities with the present data generating process are obvious. To model the duration between trade events R&E use the autoregressive conditional duration (ACD) model introduced by Engle and Russell (1998). As a matter of fact, the ACD model has been the starting point for H&J’s ACH in the first place. It thus seems natural to combine ACH and ACM to form a new MPP model for the target. The ACH-ACM combination is

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\(^1\) The idea of using market point processes to model ultra-high frequency data in economics can be attributed to Engle (2000). MPP models have become a popular tool especially in modeling financial transactions data (e.g. Grammig and Wellner, 2002; Russell and Engle, 2005).

\(^2\) Other applications of the ACM model can be found in Liesenfeld, Nolte, and Pohlmeier (2006) and Prigent, Renault, and Scaillet (2004).
appealing from a methodological point of view. Specially, the ACM is a discrete-state time series model which is precisely what is needed in the present context. Russell and Engle (2005) argue that the ACM model is able to deal with a more complex intertemporal dependencies of the transaction price changes, and that it is more flexible than the ordered probit model concerning the impact of new information on the probability of a certain size of a price change.\footnote{Similar to the present application, the ordered probit also served as the standard model for modeling discrete price tick changes, see Campbell, Lo, and MacKinlay (1997) Chapter 3.} We conjecture that this flexibility is also rewarded when modeling the dynamics of the target.

The second contribution of the paper is an assessment of the forecast quality of MPP models for the target. Despite their superiority over VAR models documented by Hamilton and Jordà (2002), an analysis of the overall quality of the forecasts of MPP models for the target has not yet been delivered. For that purpose we draw on techniques for the evaluation of density forecasts revived and extended by Diebold, Gunther, and Tay (1998). This methodology is particularly useful since it provides diagnostic tools to detect forecast failures and specification problems. However, a direct application of the density forecast evaluation techniques is not possible. The reason is that both ACH-ACM and ACH-OP models for the target deliver probability function forecasts and not density forecasts and the results for continuous forecast variables do not readily apply to discrete variables. We therefore adapt the density forecasting evaluation methods and assess the probability function forecasts of MPP models for the target at various horizons. The objective is to reach a conclusion at which forecast horizons MPP models deliver sensible results and offer recommendations for their practical use.

The main findings of this paper can be summarized as follows. The ACH-ACM methodology delivers encouraging results and qualifies as an interesting alternative to model and forecast the evolution of the target. Given the relative small number of events, parsimony of the ACM specification is called for. We thus motivate an ACM specification, referred to as response symmetric ACM, that imposes restrictions on the responses to previous target change shocks. This parsimonious specification improves on the ACH-OP both in terms of information criteria and in terms of MSE of the target forecast. The estimates are economically plausible. Specifically, the probability of observing a large positive target change increases with the spread of the six month Treasury Bill rate and the federal funds rate target. A large
positive target change shock increases the probability of observing another positive target change and vice versa. Furthermore, there is persistence in target change sizes. For a forecast horizon up to three months the MSE of the ACH-ACM reduces the MSE delivered by the ACH-OP by on average about 13%. The evaluation of the probability forecasts of MPP models (ACH-ACM and ACH-OP) does not hint at any specification problems for a forecast horizon up to three months. However, beyond this horizon the forecast quality of both ACH-OP and ACH-ACM models deteriorates. The long run dynamics of the target are not well captured and MPP models tend to underpredict large negative and large positive target values. Our recommendation is thus to use MPP models for forecast horizons up to three months, but not for longer term target forecasts. Our results suggest that for this forecast horizon a parsimoniously specified ACH-ACM model is a good choice.

The remainder of this paper is structured as follows. Section 2 describes the data and institutional details. The econometric background for the ACH-ACM methodology is presented in Section 3. In Section 4 we explain how density forecast evaluation techniques can be adapted to the present application. In Section 5 we discuss the estimation results, document a specification search and model selection criteria and we compare the empirical performance of ACH-OP and ACH-ACM models. Section 6 evaluates the forecast performance of MPP models for the target at various horizons. Section 7 concludes with a summary of the main results.

2 Data and Institutional Details

The Federal Reserve (Fed) uses three principal tools to implement monetary policy: the reserve requirement ratio, the discount rate and open market operations. The latter, the sales and purchases of government securities, is the most flexible and most frequently used. The executive organ of the Fed, the Federal Open Market Committee (FOMC), is responsible for the implementation of open market operations. Specifically, the FOMC sets a target for the effective federal funds rate, which is the rate at which depository institutions lend reserves at the Fed to other depository institutions overnight. At the Trading Desk of the Federal Reserve Bank of New York the target decisions of the FOMC are carried out by open

\footnote{In the case of a purchase (sale) of securities by the Fed, the reserves increase (decrease) and money supply extends (contracts). See Hamilton and Jordà (2002) and Meulendyke (1998) for details on the Fed’s monetary policy implementation and history.}
market operations that influence the demand for, and supply of, balances that depository institutions hold at the Federal Reserve banks.

Figure 1 depicts the data used in this paper. They are the same as in H&J’s (2002) application. A few comments regarding these data are in order. The top panel shows the target for the federal funds rate and the effective federal funds rate from 1984 to 2001. The third time series depicted in the top panel is the six-month Treasury Bill rate on the secondary market. The lower panel shows the corresponding target change sizes. Before 1994 the target level was not announced in public, though, by observing the activity at the Domestic Trading Desk at the Federal Reserve Bank of New York the objective of the Fed was inferred by public and, thereafter, the speculations about the Fed’s intended target level were publicized in press. These official trading desk data were compiled by Rudebusch (1995) and updated by Volker Wieland. Hamilton and Jordà (2002) transformed these daily into weekly data by defining a seven-day period from Thursday until Wednesday.

During the sample period the implementation of the Fed’s monetary policy have altered. In the earlier sample period the Fed influenced the effective federal funds rate indirectly by targeting the borrowed reserves. This period, as it can be seen from the lower panel in Figure 1, was characterized by small and frequent target changes. In 1989 the Fed changed its objective and directly set a specific level for the federal funds rate. In this sample period the target changes occurred less frequently and were of larger magnitude. The distinguishing feature of the target is recognizable in the lower panel in Figure 1. The target changes are discrete and irregularly spaced in time. This motivated H&J to model the target evolution as a marked point process. In their model the FOMC meeting dates affect the duration between the target changes and the size of a change. Furthermore, the spread variable (SP) turned out to be useful in explaining the evolution of the target series. The spread is defined as the difference between the six-month Treasury Bill rate (TB6) and the effective federal funds rate (FFR), i.e. SP = TB6-FFR. Both time series are depicted in the top panel of Figure 1.

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5 We thank J. D. Hamilton and Ò. Jordà for provide these data on the homepage http://www.econ.ucdavis.edu/faculty/jorda/pubs.html.
6 For eight weeks, there were two target changes within one week, which is treated as a single large change. The affected weeks are: 5/16/85, 9/3/87, 10/22/87, 8/8/88, 11/17/88, 2/9/89, 2/23/89, and 10/31/91.
7 Before 1990 the Fed changed the target in increments of 6.25 basis points and later in increments of 25 basis points.
Figure 2 depicts the frequency distribution of target changes in the sample period 1984-2001. As H&J we consolidate the observed target changes \( \{y_t^\#\} \) into five categories. Specifically, we construct a series of target changes \( \{y_t\} \) in the following way:

\[
y_t = \begin{cases} 
-0.50 & \text{if } -\infty < y_t^\# \leq -0.4375 \\
-0.25 & \text{if } -0.4375 < y_t^\# \leq -0.125 \\
0.00 & \text{if } -0.125 < y_t^\# < 0.125 \\
0.25 & \text{if } 0.125 \leq y_t^\# < 0.4375 \\
0.50 & \text{if } 0.4375 \leq y_t^\# < \infty 
\end{cases}
\]  

(1)

The lower panel of Figure 2 depicts the frequency distribution of target changes \( \{y_t\} \) after the consolidation.

### 3 Econometric Methodology

We follow Hamilton and Jordà (2002) and conceive the evolution of the target as a marked point process and specify a model that accounts for the time between successive target changes (the points) and a model for the size of the target change (the mark). We will retain H&J’s autoregressive conditional hazard (ACH) specification for the point process and use the autoregressive conditional multinomial (ACM) approach introduced by Russell and Engle (2005) to model the discrete target change sizes. This section describes how the ACH and the ACM model can be combined to provide a new model for the federal funds rate target.

We begin with a brief review of the ACH model. As in Engle and Russell’s (1998) ACD framework, the ACH assumes an autoregressive specification for the expected time between two events (here: target changes) conditional on previous durations. In contrast to the ACD, the ACH model is formulated as a discrete time model. In the context of modeling target change durations, the smallest time interval between events is one week.

To sketch the model framework, let us denote by \( \tau_n \) the duration in number of weeks between the \( n^{th} \) and \( (n + 1)^{th} \) target change. To provide a link between event and calendar time it is convenient to introduce a step function, denoted \( N(t) \), which counts the number of target changes that occurred as of week \( t \). \( N(t) \) jumps by one if a target change occurs during
week \( t \) and remains the same as in week \((t - 1)\) if no target change occurs. The sequence of conditional expected durations \( \psi_{N(t)} \equiv \mathbb{E}(\tau_{N(t)}|\tau_{N(t)-1}, \tau_{N(t)-2},...) \) is assumed to evolve as an autoregressive process which can be written as

\[
\psi_{N(t)} = \alpha \tau_{N(t)-1} + \beta \psi_{N(t)-1} ,
\]

(2)

where \( \alpha \) and \( \beta \) are parameters. Equation (2) implies that the expected duration is updated only if a target change occurs.

The conditional probability of a target change during week \( t \) given the information available in \((t - 1)\) is referred to as the hazard rate,

\[
h_t = \text{Pr}[N(t) \neq N(t - 1)|\mathbf{Y}_{t-1}] .
\]

(3)

If the information set \( \mathbf{Y}_{t-1} \) only consists of past durations, the hazard rate will remain the same until the next target change occurs. Hamilton and Jordà (2002) show that in this case hazard rate and conditional expected durations are inversely related,

\[
h_t = \frac{1}{\psi_{N(t-1)}} .
\]

(4)

In the ACH model lagged predetermined variables \( z \) observed in \((t - 1)\) may have an impact on the probability of a target change which implies a hazard rate that varies in calendar time,

\[
h_t = \frac{1}{\psi_{N(t-1)} + \delta' z_{t-1}} ,
\]

(5)

where \( \delta \) is a parameter vector. Equation (5) together with (2) constitute the ACH model.

Hamilton and Jordà (2002) suggest to combine the ACH with an ordered probit for the target change sizes. As these changes occur in discrete ordered steps, this seems a sensible econometric methodology. Although ordered probits are most frequently used in microeconometric applications involving cross sectional data, they have also been employed for the modeling of high frequency transaction data. Hausman, Lo, and MacKinlay (1992) is the classic reference. However, Russell and Engle (2005) argue that the dynamics of discrete transaction prices might be better captured by an explicit time series model specifically designed for discrete variables. This conjecture motivates the ACM methodology. Since the
ACM is formulated as a time series model for discrete variables. It seems natural to conjecture that it may also be better suited to model the dynamics of target size changes than the ordered probit.

In the following, we will show how the ACM methodology can be adapted to model the size of target changes occurring at infrequent event times. Let us first define a binary indicator $x_t$ which takes the value one if a target change occurs during week $t$ and is zero otherwise. Furthermore, denote by $y_t$ the size of the target change in $t$. $y_t$ is either zero for a week with no target change (if $x_t = 0$) or takes one of $k$ different ordered outcomes $s_1 < s_2 < ... < s_k$ (if $x_t = 1$). Let us further denote by $\pi_{jn}$ the probability that the $n^{th}$ target change is equal to $s_j$ and collect the complete set of $k$ probabilities in a vector $\tilde{\pi}_n = (\pi_{1n}, \ldots, \pi_{kn})'$. Since the columns of $\tilde{\pi}_n$ have to sum up to one, an arbitrary target change size, the $r^{th}$ category say, can be defined as a reference category. The probability of observing a target change in the reference category can then be calculated as $\pi_{rn} = (1 - \mathbf{1}'\pi_n)$ with $\mathbf{1}$ a $(k - 1) \times 1$ vector of ones. $\pi_n$ is a $(k - 1) \times 1$ vector that results from the deletion of the probability of the reference category, $\pi_{rn}$, from the vector $\tilde{\pi}_n$. To indicate the size of the $n^{th}$ target change it is convenient to introduce a $k \times 1$ vector $\tilde{x}_n$. Its $j^{th}$ element is equal to one if the size of the $n^{th}$ target change is equal to $s_j$, the other elements of $\tilde{x}_n$ are zero. Finally, define the $(k - 1) \times 1$ vector $x_n$ which results from deleting the $r^{th}$ element (indicating a target change size within the reference category) from $\tilde{x}_n$.

Adapting the ACM methodology to the present application, we allow for autoregressive dynamics of the size of the target changes and account for the impact of predetermined previous week variables, $w_{t-1}$, on the probabilities of observing one of the $k$ possible target change sizes:

$$\ell(\pi_{N(t)}) = A(x_{N(t)-1} - \pi_{N(t)-1}) + B\ell(\pi_{N(t)-1}) + D\mathbf{w}_{t-1}x_t,$$

where $A$ and $B$ are $(k - 1) \times (k - 1)$ parameter matrices. $D$ is a $(k - 1) \times m$ parameter matrix where $m$ denotes the number of predetermined variables. The logistic link function $\ell(\pi_{N(t)}) = \ln (\pi_{N(t)}/(1 - \mathbf{1}'\pi_{N(t)}))$ ensures that the resulting probabilities lie within the unit
The probabilities $\pi_{N(t)}$ can be recovered by computing

$$
\pi_{N(t)} = \frac{\exp[A(x_{N(t)} - \pi_{N(t-1)}) + B(\pi_{N(t-1)}) + Dw_{t-1}x_t]}{1 + \exp[A(x_{N(t)} - \pi_{N(t-1)}) + B(\pi_{N(t-1)}) + Dw_{t-1}x_t]} \quad .
$$

The term $(x_n - \pi_n)$ in Equation (6) can be interpreted as the innovation that is associated with the $n^{th}$ target change.

Combining Equations (5) and (6) constitutes the ACH-ACM model as an alternative MPP model for the federal funds rate target. Setting up the conditional likelihood function is straightforward. The probability of observing a target change of size $y_t$ conditional on $w_{t-1}$ and $x_t = 1$ can be written as $\tilde{x}_{N(t)}^'\tilde{\pi}_{N(t)}$. This implies that the joint density of target change indicator $x_t$ and target change size $y_t$ can be written as

$$
f(x_t, y_t|\Upsilon_{t-1}; \theta_{ACH}, \theta_{ACM}) = g(x_t|\Upsilon_{t-1}; \theta_{ACH})q(y_t|x_t, \Upsilon_{t-1}; \theta_{ACM}) = \{h_t\}^{x_t}\{1 - h_t\}^{(1-x_t)}\{\tilde{x}_{N(t)}^'\tilde{\pi}_{N(t)}\}^{x_t} \quad ,
$$

where we have collected the ACH parameters $\delta, \alpha, \beta$ in the vector $\theta_{ACH}$ and the vectorized ACM parameter matrices $A, B, D$ in $\theta_{ACM}$.

The ACH-ACM log-likelihood function is thus given by

$$
\mathcal{L}(\theta_{ACH}, \theta_{ACM}) = \sum_{t=1}^{T}\{x_t \ln(h_t) + (1 - x_t) \ln(1 - h_t)\} + \sum_{t=1}^{T}\tilde{x}_{N(t)}'\ln(\tilde{\pi}_{N(t)})x_t \quad .
$$

Maximization of (9) is equivalent to maximize separately the ACH part,

$$
\mathcal{L}_1(\theta_{ACH}) = \sum_{t=1}^{T}\{x_t \ln(h_t) + (1 - x_t) \ln(1 - h_t)\} \quad ,
$$

and the ACM part of the log-likelihood,

$$
\mathcal{L}_2(\theta_{ACM}) = \sum_{t=1}^{T}\tilde{x}_{N(t)}'\ln(\tilde{\pi}_{N(t)})x_t \quad ,
$$

if $\theta_{ACH}$ and $\theta_{ACM}$ have no parameters in common (see Engle, 2000).
4 Evaluation of Probability Forecasts of MPP Models

ACH-OP and ACH-ACM models lend themselves conveniently to an application of the density forecasting evaluation techniques revived and popularized by Diebold, Gunther, and Tay (1998) (henceforth DGT). Here we adapt their methodology to evaluate target forecasts of MPP models. Density forecast evaluation enjoys increasing popularity as it offers intuitive diagnostics to detect specification problems. However, before the tool can be applied in the context of the present paper two problems have to be addressed. First, DGT’s method is designed to evaluate density forecasts for continuous random variables which necessitates an adaption of the method to discrete random variables. Due to the discreteness of the target \( i_t \), ACH-ACM and ACH-OP models deliver probability forecasts, not density forecasts. Second, since the analytic computation of multi-period probability function forecasts delivered by MPP models is intractable we have to employ simulation techniques. We deal with both issues in the next two subsections.

4.1 Evaluation of One-Step Probability Forecasts

A density forecast is a probability density function defined for a one-step or \( \kappa \)-period ahead observation of the variable of interest, given the information at time \( t \). Since in the present application the variable of interest is a discrete random variable, our focus is on the evaluation of probability function forecasts instead of density forecasts. The ACH-ACM one-step probability function forecast is readily available as a byproduct of the construction of the likelihood function in (8)

\[
f(i_{t+1} | Y_t) = \begin{cases} 
  P(i_{t+1} = i_t | Y_t) = 1 - h_{t+1} \\
  P(i_{t+1} = i_t + s_j | Y_t) = h_{t+1} \pi_{jN(t+1)} & j = 1, 2, ..., k
\end{cases}
\]

(12)

For all other values of \( i_{t+1} \) the probability function is zero. The expression for the probability function in (12) is the same for both ACH-OP and ACH-ACM models with the only difference that the conditional probabilities \( \pi_{jN(t+1)} \) for the OP model originate from the ordered probit instead of the ACM model.

Before turning to the evaluation of probability forecasts, it is useful to briefly review the basic idea of DGT’s method to evaluate density forecasts. Assume for the moment that
the target is a continuous random variable and let us denote by \{f(i_t | \Upsilon_{t-1})\} a sequence of density forecasts and by \{p(i_t | \Upsilon_{t-1})\} the sequence of true densities. DGT show that the correct density is weakly superior to all other forecasts, i.e. will be preferred, in terms of expected loss, by all forecast users regardless of their loss functions. This suggests that forecasts can be evaluated by testing the null hypothesis that the forecasting densities are correct, i.e. whether
\[ \{f(i_t | \Upsilon_{t-1})\} = \{p(i_t | \Upsilon_{t-1})\} \tag{13} \]
At first sight, testing whether (13) holds appears infeasible because \(p(i_t | \Upsilon_t)\) is unobserved. However, the distributional properties of the probability integral transform,
\[ z_t = \int_{-\infty}^{i_t} f(u|\Upsilon_{t-1})du = F(i_t|\Upsilon_{t-1}) \tag{14} \]
provide the solution to this problem. The main contribution of DGT was to extend a classic result by Rosenblatt (1952) by showing that under the null hypothesis the distribution of the sequence of probability transforms \(\{z_t\}\) is iid \(U(0,1)\).

In the present application we cannot rely on iid uniformity of the probability integral transform (PIT). The reason is that the PIT theorem only holds for continuous random variables, i.e. for density function forecasts, but not probability function forecasts. To address this, we adopt a methodology proposed by Denuit and Lambert (2005) who derive a discrete analog of the PIT theorem. For notational convenience assume that \(s_{j+1} - s_j = c\) for \(j = 1, 2, ..., k - 1\).\(^8\) Transferring Denuit and Lambert’s (2005) results to the our application we ”continue” the discrete target value \(i_t\) by adding an independent uniformly distributed random variable with support \([-c, 0]\) viz,
\[ i_t^* = i_t + c(u_t - 1) \tag{15} \]
where \(\{u_t\}\) is an iid \(U(0,1)\) sequence. Denuit and Lambert (2005) show that the PIT of the continued variable \(i_t^*\) can be computed as
\[ z_t^* = F^*(i_t^*|\Upsilon_{t-1}) = F(i_t - c|\Upsilon_{t-1}) + f(i_t|\Upsilon_{t-1})u_t \tag{16} \]
\(^8\) Choosing the categorization as in (1) we have \(k = 5\) and \(c = 0.25\).
The discrete analog of the PIT theorem put forth by Denuit and Lambert (2005) states that $z_t^*$ is $U(0, 1)$ if the forecast probability function (12) is correctly specified. Having obtained the $z_t^*$ sequence it is possible to apply the diagnostic tools proposed by DGT to evaluate probability function forecasts of the target. Besides formal tests for iid uniformity this amounts to analyzing the shape of histograms and autocorrelation functions of the (continued) PIT sequence. We will come back to this issue in Section 6.

4.2 Evaluating Multi-Step Ahead Probability Forecasts

The continuation principle extends to multi-step ahead probability forecasts of ACH-ACM and ACH-OP models. Here the object of interest is the probability forecast $f(i_{t+\kappa}|\Upsilon_t)$ where $\kappa > 1$. Like for the one-step probability forecast, the evaluation of multi-period probability forecasts of MPP models focuses on assessing deviations of the continued PIT sequence

$$z_t^* = F(i_t - c|\Upsilon_{t-\kappa}) + f(i_t|\Upsilon_{t-\kappa})u_t$$

from $U(0, 1)$ uniformity and on analyzing the autocorrelation structure of $\{z_t^*\}$. However, because the analytic computation of the probability function $f(i_{t+\kappa}|\Upsilon_t)$ and the corresponding continued PIT sequence is numerically intractable, we adopt a simulation strategy of Hamilton and Jordà (2002).

The basic idea is to simulate future sample paths of an ACH-ACM process and rely on the law of large numbers (LLN) to deliver a consistent estimate of the probability forecast $f(i_{t+\kappa}|\Upsilon_t)$. For that purpose we first need a realization of $x_{t+1}$, the random variable which indicates whether a target change occurs at $t + 1$. Since the ACH model readily delivers $P(x_{t+1} = 1|\Upsilon_t) = h_{t+1}$ this can be obtained by drawing from a Bernoulli distribution with success probability $p = h_{t+1}$. Let us denote by $x_{t+1}^{(1)}$ the result of that draw. The superscript indicates that this is the first of many replications. If $x_{t+1}^{(1)} = 0$, the target is unchanged, $i_{t+1}^{(1)} = i_t$. If $x_{t+1}^{(1)} = 1$, we update the counting function by computing $N_{t+1}^{(1)} = N_t + 1$, and determine the size of the target change. This is done by drawing from the discrete distribution of the random variable $y_{t+1}$ determining the size of the target change. For the

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9 In the following we describe the simulation procedure that applies to the ACH-ACM. The procedure is analogous, but simpler for the ACH-OP. Within the simulation procedure one also needs forecasts of the exogenous variables $z$ and $w$. We assume that all future values of the exogenous variables are delivered by an appropriate forecast model.
ACM model, this probability distribution is given by \( \pi_{N(t)} \) (see Equation (7)). Having drawn the target change in \( t+1 \), \( y_{t+1}^{(1)} \), we add it to \( \lambda_t \) and obtain the simulated target value \( i_{t+1}^{(1)} \).

Iterating forward the ACH-ACM Equations (5) and (6) yields the probabilities \( h_{t+1}^{(1)} \) and \( \pi_{N(t+2)}^{(1)} \). These probabilities are used to simulate \( x_{t+2}^{(1)} \) and \( i_{t+2}^{(1)} \). This works as just described except that we condition on time \( t+1 \) simulated values instead of time \( t \) observed values.

The procedure is continued until \( t+\kappa-1 \). Conditioning on the simulated path of target values \( i_{t+\kappa-1}^{(1)}, i_{t+\kappa-2}^{(1)}, \ldots, i_{t+1}^{(1)} \) we can then compute, analytically just as in Equation (12), the one-step probability forecast

\[
f(i_{t+\kappa}^{(1)}|i_{t+\kappa-1}^{(1)}, \ldots, i_{t+1}^{(1)}, \mathcal{Y}_t) = \begin{cases} P(i_{t+\kappa}^{(1)} = i_{t+\kappa-1}^{(1)}|i_{t+\kappa-1}^{(1)}, \ldots, i_{t+1}^{(1)}, \mathcal{Y}_t) & = 1 - h_{t+1}^{(1)} \\ P(i_{t+\kappa}^{(1)} = i_{t+\kappa-1}^{(1)} + s_j|i_{t+\kappa-1}^{(1)}, \ldots, i_{t+1}^{(1)}, \mathcal{Y}_t) & = h_{t+1}^{(1)} \pi_{N(t+\kappa)}^{(1)} \end{cases}
\]

for \( j = 1, 2, \ldots, k \).

In order to obtain the \( \kappa \)-period ahead probability forecast \( f(i_{t+\kappa}^{(1)}|\mathcal{Y}_t) \) we need to remove the conditioning on the sample path. For this purpose we exploit that

\[
f(i_{t+\kappa}, \ldots, i_{t+1}|\mathcal{Y}_t) = f(i_{t+\kappa}|i_{t+\kappa-1}, \ldots, i_{t+1}, \mathcal{Y}_t) \cdot f(i_{t+\kappa-1}, \ldots, i_{t+1}|\mathcal{Y}_t) \]

and

\[
f(i_{t+\kappa}|\mathcal{Y}_t) = \sum_{i_{t+\kappa-1}} \cdots \sum_{i_{t+1}} f(i_{t+\kappa}|i_{t+\kappa-1}, \ldots, i_{t+1}, \mathcal{Y}_t) \cdot f(i_{t+\kappa-1}, \ldots, i_{t+1}|\mathcal{Y}_t)
= \mathbb{E}(f(i_{t+\kappa}|i_{t+\kappa-1}, \ldots, i_{t+1}, \mathcal{Y}_t))
\]

The computation of the multi-period probability forecast via Equation (20) is still computationally intractable. We address this problem by repeating the simulation of the sample paths described above \( M \) times. This delivers a vector sequence of \( M \) probability forecasts, each conditioned on the respective simulated sample path, \( \{f(i_{t+\kappa}^{(m)}|i_{t+\kappa-1}^{(m)}, \ldots, i_{t+1}^{(m)}, \mathcal{Y}_t)\}_{m=1}^{M} \).

Relying on a LLN and averaging over the \( M \) replications yields a consistent estimate of \( f(i_{t+\kappa}^{(1)}|\mathcal{Y}_t) \),

\[
\frac{1}{M} \sum_{m=1}^{M} f(i_{t+\kappa}^{(m)}|i_{t+\kappa-1}^{(m)}, \ldots, i_{t+1}^{(m)}, \mathcal{Y}_t) \to \mathbb{E}(f(i_{t+\kappa}|i_{t+\kappa-1}, \ldots, i_{t+1}, \mathcal{Y}_t)) = f(i_{t+\kappa}^{(1)}|\mathcal{Y}_t)
\]

(21)
A useful byproduct of this simulation strategy is the possibility to use the estimated probability forecast to produce multi-period point forecasts $\mathbb{E}(i_{t+\kappa}|Y_t)$ and conduct MSE comparisons between competing specifications as in Hamilton and Jordà (2002).

Figure 3 provides a graphical illustration of the simulation procedure. The left hand side of the picture depicts $M = 7$ simulated sample paths that reach different target values in $t + \kappa - 1$. Given the target value in $t + \kappa - 1$, only $k = 3$ different target values can be attained in period $t + \kappa$. The conditional probabilities for each of the possible target values in $t + \kappa$ can be computed via Equation (18). Some target values in $t + \kappa$, like $i^\circ$, may have non-zero probabilities in many replications, while some target values may be assigned non-zero probabilities only once, and some never. Summing the zero and non-zero probabilities for each possible target value over the $M$ replications and dividing by $M$ yields the estimate of the $t + \kappa$ period ahead probability forecast that is sketched on the right hand side of Figure 3.

Two more remarks concerning the multi-step probability forecast evaluation are in order. First, the simulation procedure just described is computer intensive, especially if one wants to analyze a variety of forecast horizons $t + \kappa$. Since the basic idea of the evaluation of probability/density forecasts is to test a sequence of PITs for iid uniformity, the $\kappa$-period probability forecast has to be computed for each observation $t = 1, 2, \ldots, T$. Second, the $\kappa$-step ahead PIT sequence in Equation (17) is not iid for $\kappa > 1$, but exhibits a MA($\kappa - 1$) autocorrelation structure. Thus, we follow DGT’s advice and partition the PIT sequence into sub-series for which we expect iid uniformity if the forecasts were correct. For instance, for correct 2-step ahead probability forecasts, the sub-series $\{z^*_1, z^*_3, z^*_5, \ldots\}$ and $\{z^*_2, z^*_4, z^*_6, \ldots\}$ should each be iid $U(0, 1)$, although the full series $\{z^*_t\}$ is not.
5 Estimation Results and Discussion

In this section we analyze the empirical performance of MPP models for the federal funds rate target. We first document a specification search and model selection procedure and present an interpretation of the estimation results of our selected specification. Second, we focus on the evaluation the forecast performance by a MSE comparison.

As outlined in the methodology section, estimation of the ACH and ACM parameters can be conducted separately if the two submodels do not have any parameters in common. Since we use the same data and since our focus is the model for the target change sizes, we adopt the same ACH specification as in Hamilton and Jordà (2002). Taking into account that the objective of the Federal Reserve changed (see Section 2), Hamilton and Jordà (2002) estimate separate ACH specifications for two subperiods. For the first subsample, covering the period from March 1, 1984 to November 23, 1989 the estimated ACH equation reads

$$
\psi_N(t) = 2.257 + 0.909 \tau_N(t-1) - 0.847 \psi_N(t-1) - 0.630 \text{FOMC}_{t-1},
$$

where FOMC$_t$ is a dummy variable that equals one if there was a FOMC meeting in $t$ and zero otherwise. The values in parentheses are standard errors. For the second subsample from November 30, 1989 to April 26, 2001 the estimated ACH equation is given by

$$
\psi_N(t) = 30.391 + 0.670 \tau_N(t-1) - 23.046 \text{FOMC}_t - 8.209 |\text{SP}_{t-1}|,
$$

where $|\text{SP}|$ is the absolute value of the spread between the six-month Treasury Bill rate and the effective federal funds rate.

Table 1 summarizes the estimation results for several ACM specifications. All are based on the $k = 5$ target change categories defined in Equation (1), so that target changes occur with fixed increments $c = 0.25$. For each ACM specification the third category (smallest absolute target changes within $(-0.125, 0.125)$ see Equation (1)) is chosen as the reference category. In accordance with Hamilton and Jordà’s ordered probit specification, we allow for an impact of the previous week’s spread on the size of the target change. The ACM variants
considered in the following are thus restricted versions of the general specification

\[ \ell(\pi_{N(t)}) = c + A(x_{N(t-1)} - \pi_{N(t-1)}) + B\ell(\pi_{N(t-1)}) + DSP_{t-1}x_t \tag{24} \]

\[
= \begin{pmatrix} c_1 \\ c_2 \\ c_4 \\ c_5 \end{pmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{24} & a_{25} \\ a_{41} & a_{42} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{54} & a_{55} \end{bmatrix}(x_{N(t-1)} - \pi_{N(t-1)}) + \begin{bmatrix} b_{11} & b_{12} & b_{14} & b_{15} \\ b_{21} & b_{22} & b_{24} & b_{25} \\ b_{41} & b_{42} & b_{44} & b_{45} \\ b_{51} & b_{52} & b_{54} & b_{55} \end{bmatrix}\ell(\pi_{N(t-1)}) + \begin{pmatrix} d_1 \\ d_2 \\ d_4 \\ d_5 \end{pmatrix}SP_{t-1}x_t \cdot x_t.
\]

Both OP and ACM are estimated using data on the sequence of target change events. These time series will contain much less observations than in Russell and Engle’s ACM application where transaction price changes were modeled. These data contain many thousands of events while our sample contains only 102 target changes. Hence, parsimony of the ACM specification is called for to avoid in-sample over-fitting and a deterioration of out-of-sample forecasts.

First, consider an ACM specification which leaves the parameter matrices \( c, A, \) and \( D \) unrestricted, but constrains the elements of \( B \) to zero. The vectors \( c \) and \( D \) are unconstrained (as in all the other specifications considered below). We refer to this specification as the \textit{unrestricted ACM}(1,0). It accounts for unconstrained responses of the state probabilities \( \pi_n = (\pi_{1n}, \pi_{2n}, \pi_{4n}, \pi_{5n})' \) after a target change shock, but rules out persistence of the state probabilities.

To provide a sensible parsimonious model Russell and Engle (2005) advocate ACM specifications that imply symmetries in the responses to previous target change shocks. R&E call a matrix \( A \) "response symmetric" if its elements are constrained in the following way:

\[
A = \begin{bmatrix} a_{11} & a_{12} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{24} & a_{25} \\ a_{41} & a_{42} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{54} & a_{55} \end{bmatrix} = \begin{bmatrix} a_1 & a_5 & a_8 & a_4 \\ a_2 & a_6 & a_7 & a_3 \\ a_3 & a_7 & a_6 & a_2 \\ a_4 & a_8 & a_5 & a_1 \end{bmatrix} \tag{25}
\]

Assuming a response symmetric matrix \( A \) in Equation (24) entails some a-priori plausible and testable symmetry assumptions. Specifically, Equation (25) implies that the effect of target change shock in the highest category on the probability that the subsequent target change is
maximally negative ($\pi_{1n}$) is identical to the effect of a target change shock in the smallest category, on the probability of observing a subsequent maximally large positive target change ($\pi_{5n}$). Accordingly, the effect of a target change shock in the second smallest category, on the probability of observing a subsequent positive and maximally large target change is also mirrored: It is the same as the effect of a target change shock in the second highest change size category on $\pi_{1n}$.

Persistence of the state probabilities can be allowed for by a non-zero parameter matrix $B$. For the sake of parsimony we focus our attention on diagonal $B$ matrices. Specifically, consider an ACM(1,1) specification that combines a response symmetric matrix $A$ with a diagonal, but otherwise unrestricted matrix $B$. Another variant where the diagonal matrix $B$ is also response symmetric, i.e. where

$$B = \begin{bmatrix} b_1 & 0 & 0 & 0 \\ 0 & b_2 & 0 & 0 \\ 0 & 0 & b_2 & 0 \\ 0 & 0 & 0 & b_1 \end{bmatrix}$$

is referred to as response symmetric ACM(1,1).

Finally, we consider two more parsimonious specifications. The diagonal ACM(1,1) restricts all off-diagonal elements in the matrices $A$ and $B$ equal to zero, but leaves the diagonal elements unrestricted. The most parsimonious specification (with a total of 12 parameters) considered here constrains $A$ and $B$ to be diagonal and response symmetric ($c$ and $D$ are unconstrained in all variants). We refer to it as symmetric diagonal ACM(1,1).

Table 1 summarizes the estimation results for seven ACH-ACM specifications which place certain restrictions on the parameter matrices in Equation (24). The table reports for each specification the number of parameters, the value of the maximized log-likelihood, Akaike information criterion, and a pseudo-$R^2$. For comparison the results for the ACH-OP are also reported.\(^{10}\) Table 1 is sorted in ascending order by AIC, so models that appear on top of the list are preferred by the AIC. Ordered probit and ACM are not nested, so likelihood ratio

\(^{10}\) Replicating H&J’s results the latent target change equation estimated by ML reads:

$$y_{N(t)} = \begin{cases} 2.545 & y_{N(t-1)} + 0.541 \cdot SP_{t-1} \cdot x_t \end{cases} \quad (0.426, 0.204)$$

The four threshold parameter estimates are not reported.
testing is not an option. However, the mean maximized likelihoods for ordered probit and ACM provide natural and well-interpretable goodness-of-fit measures. Recall that the target change sizes are discrete random variables. Hence, the likelihood contribution associated with each of the $N(T) = 102$ target change events is a conditional probability, and the maximized log-likelihood is a joint probability. Accordingly, the average maximized likelihood takes values between zero and one and can be interpreted as a pseudo-$R^2$.

For the ordered probit the value of this pseudo-$R^2$ is 0.30. The goodness-of-fit measure increases to 0.34 for the response symmetric ACM(1,1) and to 0.37 for the unrestricted ACM(1,0). The four other ACM specifications also provide an improvement of the goodness-of-fit compared to the ordered probit. However, when penalizing generous parameterizations according to the AIC, only the unrestricted ACM(1,0) and the response symmetric ACM(1,1) outperform the ordered probit.

Although these likelihood-based results are promising, they do not necessarily show the superiority of the ACM, because two arguments challenge this conclusion. First, the estimation of the model for the mark (OP or ACM) is carried out separately from the estimation of the duration model for the points, but for the target forecast both have to be combined. Hence, one could argue that although the isolated comparison of the models for the mark favors the ACM, the ACH-OP combination may be the more suitable MPP model. Second, as target forecasting that is the main objective of MPP models, likelihood-based comparisons may have limited relevance.

To address both issues, Table 2 compares the mean squared forecast errors (MSEs) of ACH-ACM target forecasts with H&J’s ACH-OP and a vector autoregressive model. This is an extension of the analysis reported in Table 7 in Hamilton and Jordà (2002). To ensure comparability of VAR and MPP model forecasts, two data related issues had to been taken into account. First, VARs typically do not include an equation for the target, but for the effective rate. Second, the VAR model is based on monthly data, while ACH-OP and ACH-ACM are estimated on finer granulated weekly data. To take these issues into account, the target forecasts of ACH-OP and ACH-ACM are compared with the effective rate when

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11 Both series evolve very closely (see Figure 1), as the target is the main factor determining the effective rate.
computing the MSE. Furthermore, to compute the MPP and VAR forecasts based on the same conditioning information, and do not give MPP models a head start, the MPP models’ forecasts are computed based on end-of-the calendar month information even if newer weekly data is available. As a matter of fact, this MSE comparison made a strong case for MPP modeling of the target. The first three columns in Table 2 show why. Although the ACH-OP model could use only a restricted information set, the MSE of its target forecast is considerably smaller than the MSE of the VAR. Averaged over the first three months the ACH-OP reduces the MSE of the VAR by 84%.

We extend this analysis by letting the ACH-ACM enter the competition. We focus on the results for the two AIC preferred specifications, the unrestricted ACM(1,0) and the response symmetric ACM(1,1), as well as a quite parsimonious specification, the diagonal ACM(1,1). Each is combined with the H&J’s ACH specification reported in Equations (22) and (23).12

Table 2 shows that both the ACH-response symmetric ACM(1,1) and the ACH-diagonal ACM(1,1) further improve the mean squared error of the ACH-OP target forecast at forecast horizons up to six months.13 The ACH-unrestricted ACM(1,0) outperforms the ACH-OP only at the forecast horizon of one month. This indicates that an ACH-ACM model employed for target forecasts should be parsimoniously specified. Furthermore, the results suggest that ACH-ACM specifications are especially useful for forecast horizons of up to one quarter of a year. Averaged over the first three months, the MSE of the ACH-ordered probit is reduced by 18% (ACH-diagonal ACM(1,1)) and 13% (ACH-response symmetric ACM(1,1)), respectively. Although this improvement is smaller than the 84% MSE reduction when replacing the VAR by the ACH-OP forecast, we take this as a promising result for the MPP model presented in this paper.

Table 3 shows that the response symmetric ACM(1,1) does well in terms of goodness-of-fit and delivers a good forecast performance. Furthermore, as shown in Table 3, the param-

\begin{equation}
SP_t = 0.129 + 0.228 i_t - 0.267 i_{t-1} + 0.723 SP_{t-1},
\end{equation}

with standard errors in parentheses.

12 The forecast of the MPP models is the (simulated) conditional expected target value for the respective horizon. The simulation techniques described in Section 4 are employed to compute these target forecasts.

13 To compute the forecast of ACH-OP and ACH-ACM we need forecast values of the spread (SP). For that purpose we rely on the following equation estimated by Hamilton and Jordà (2002)
eter estimates of this specification are quite plausible. First, the estimates of the state specific constants are higher for the "inner states" (medium size positive or negative target changes) which is in accordance with the unconditional empirical distribution depicted in Figure 2. Second, an increase in the spread of six month Treasury Bill rate positively impacts on the probability of observing a large target change (as in the ordered probit, too, see footnote 10). Third, the estimation results indicate persistence in the state probabilities which is largest in the categories indicating a medium size target change ($|y| = 0.25$). Large target changes ($|y| = 0.5$) are less persistent. Lastly, the estimates of the matrix $A$ are quite sensible: A target change shock in the largest size category ($1 - \pi_{5n-1}$) has a strong negative effect on the probability of observing a (large) subsequent negative target change and increases the probability of observing another large positive target change. Due to the response symmetry assumption these effects are mirrored for large negative target change shocks. Furthermore, a target change shock in the second highest category increases the probability of observing another positive target change, especially in the same category, while the probability of observing a subsequent negative target change is reduced.$^{14}$ Again this effect is mirrored for a negative target change shock in category two.

6 Probability Forecast Evaluation of MPP Models for the Target

Table 2 showed that the reduction of the MSE of the target forecast of vector autoregressive models produced by ACH-OP and ACH-ACM models is indeed stunning. However, an assessment of the overall quality of MPP forecasts for the federal funds rate target has not yet been delivered. This section addresses this issue by using the diagnostic tools for the evaluation of probability forecasts developed in Section 4. As in the case of density forecast evaluation this amounts to checking whether the (continued) PIT sequences at various forecast horizons are iid $U(0, 1)$. For that purpose we follow the advice of Diebold, Gunther, and Tay (1998) and focus the diagnostics on the assessment of density estimates and autocorrelograms of the continued PIT sequences computed at various forecast horizons. Diebold et al. point out that

$^{14}$ Again, the precise marginal effects on the state probabilities have to be computed from inverting the log-odds ratios. Due to the non-linearity of the model, the size of the marginal effects depends on the size of the shock.
tests of iid U(0,1) behavior, albeit readily available, are nonconstructive. When rejection of the null hypothesis occurs these tests provide no guidance about the reasons why. Visual inspection of density estimate and autocorrelogram of the $z^*$ sequence, however, assists in detecting particular forecast failures.

Using histograms instead of more sophisticated kernel density estimates to check uniformity of the continued PIT sequence has the advantage that the constraint that $z^*$ has support on the unit interval is easily imposed. Furthermore, confidence intervals under the null hypothesis of iid uniformity are straightforward to compute. Implementing this idea, Figures 4 (short term), 5 (medium term), and 6 (long term) depict the histograms of the continued PIT sequences at various forecast horizons, from one week (i.e. one step) to one year (52-step). In each figure the left panels depict the results for the ACH-ACM model while the right panels show the histograms for the ACM-OP model. To check for the independence of the $z^*$ Diebold, Gunther, and Tay (1998) recommend visual inspection of the autocorrelogram of the PIT sequence with the usual Bartlett confidence intervals. Accordingly, Figures 7 (short term) 8 (medium term), and 9 (long term) depict the autocorrelograms of the continued PIT sequences at various forecast horizons. As outlined in Section 4 we account for the correlation in the $z^*$ sequence computed for $h$-step forecasts forecasts by partitioning the original $z^*$ sequence into sub-series for which we expect iid uniformity. Table 5 augments the autocorrelograms with Ljung-Box statistics.

The histograms for the short term $z^*$ sequence (up to six weeks) depicted in Figure 4 do not hint at a violation of the iid U(0,1) hypothesis. The heights of the histogram bins fluctuate within the bounds of the 95 % confidence bounds both for the ACH-OP and the ACH-ACM model. In other words, the histograms fail to indicate flaws of the probability forecasts. For the mid-term horizon the shape of the histograms seems still acceptable for up to 12 weeks, but for the 16 week horizon departures from uniformity of the histograms become more pronounced (see Figure 5). Increasing the forecast horizon further as in Figure 6 the
histograms look distinctly non-uniform. More and more histogram bin heights fall outside the 95% confidence interval and the histograms take on a distinct U-shape. This indicates that ACH-OP and ACH-ACM models underestimate the occurrence of relatively large and small target values at longer forecast horizons.

A formal test for iid uniformity tells the same story.\textsuperscript{15} As can be seen from the $p$-values in Table 4, the null hypothesis of iid uniformity cannot be rejected up to a forecast horizon of 13 weeks at a 5% significance level. Clear rejection occurs at forecast horizons greater than 15 weeks.

The autocorrelograms depicted in Figures 7, 8, and 9 as well as the Ljung-Box test results reported in Table 5 suggest the same conclusion at least as far as the short and medium term forecasts are concerned. For the short term forecasts (up to six weeks) and, with a grain of salt, also for the mid-term forecast up to 12 weeks, the MPP models seem to capture the dynamics of the target quite satisfactory. For longer forecast horizons the autocorrelograms and the Ljung-Box test loose power to detect violations from the null hypothesis, as the number of observations used to compute the autocorrelations becomes quite small (recall that the autocorrelations have to be computed on the thinned sub-series).

Overall, these results suggest that MPP models for the federal funds rate target do a satisfactory job at forecasting the target at horizons up to three months. Beyond that forecast horizon, however, the forecast quality worsens such that the forecasts seem less valuable.

7 Conclusion

Forecasts of the federal funds rate target are of key interest for investors and financial institutions. Hamilton and Jordà’s (2002) finding that the quality of federal funds rate target

\textsuperscript{15} We adopt the test idea used in Bauwens, Giot, Grammig, and Veredas (2004). Their test statistic compares the absolute frequencies in the PIT histogram bins to what is expected if the data were iid $U(0,1)$. 

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forecasts can be substantially improved when the sequence of target changes is modeled as a marked point process (MPP) showed the importance of this econometric approach for a central issue in monetary economics. This paper introduced a new MPP model for the federal funds rate target which combines the autoregressive conditional hazard (ACH) put forth by Hamilton and Jordà (2002) and the autoregressive conditional multinomial (ACM) model developed by Russell and Engle (2005). From a methodological point of view the ACM model seemed a natural choice as it is formulated as a discrete-state time series model. This is precisely what is needed for the modeling of the evolution of the federal funds rate target.

Based on the data used in the seminal application in Hamilton and Jordà (2002) this paper documented that the ACH-ACM methodology qualifies as an interesting alternative to model and forecast the evolution of the federal funds rate target. The parameter estimates are sensible from an economic point of view, and the forecasting performance is encouraging. However, parsimony of the ACM specification is called for, as the data will contain only a relative small number of target change events. Accordingly, this paper advocated an ACM specification that imposes restrictions on the responses to previous target change shocks. The results showed that such a "response symmetric" ACH-ACM improves on the ACH-ordered probit methodology both in terms of information criteria and MSE of the target forecast. For a forecast horizon up to three months the MSE of the ACH-ACM reduces the MSE delivered by the ACH-OP by on average about 13%.

While the relative superiority of MPP models over vector autoregressive models when employed for forecasting the target makes a strong case for this econometric methodology, an analysis of the overall forecast quality had not yet been delivered. For that purpose this paper introduced techniques that can be used to evaluate probability forecasts delivered by MPP models for the federal funds rate target. The methodologies draw on the density forecasting techniques popularized by Diebold, Gunther, and Tay (1998). The results showed that MPP models are quite appropriate to predict target changes on a forecast horizon up to three months. However, the forecast quality deteriorates beyond this horizon. These results suggest that MPP models, and particularly a parsimoniously specified ACH-ACM model, are especially useful for predicting the federal funds rate target in the short and medium term.
References


Tables and Figures
The total number of parameters estimated for each model are reported in the column $n_{par}$. The column $\mathcal{L}_{max}$ contains the value of the maximized log-likelihood. The columns AIC and $R^2_{pseudo}$ report the Akaike information criterion and the pseudo-$R^2$ for the estimated models which are computed as

$$AIC = -2 \cdot \frac{\mathcal{L}_{max}}{N(T)} + 2 \cdot \frac{n_{par}}{N(T)}$$

$$R^2_{pseudo} = \exp \left( \frac{\mathcal{L}_{max}}{N(T)} \right)$$

with $N(T)$ as the total number of events. The models are ordered ascending by their AIC value.
<table>
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<tr>
<th>Forecast Horizon</th>
<th>VAR</th>
<th>ACH-OP</th>
<th>ACH-resp sym ACM(1,1)</th>
<th>ACH-diag ACM(1,1)</th>
<th>ACH-unrestr ACM(1,0)</th>
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<tr>
<td>1 month ahead</td>
<td>0.207</td>
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<td>0.029</td>
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<tr>
<td>3 months ahead</td>
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<td>0.181</td>
<td>0.164</td>
<td>0.155</td>
<td>0.196</td>
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<td>0.266</td>
<td>0.254</td>
<td>0.319</td>
</tr>
<tr>
<td>5 months ahead</td>
<td>1.501</td>
<td>0.380</td>
<td>0.379</td>
<td>0.368</td>
<td>0.458</td>
</tr>
<tr>
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<td>0.515</td>
<td>0.499</td>
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<td>1.401</td>
<td>1.590</td>
<td>1.578</td>
<td>2.005</td>
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</table>

Table 2: **Mean Squared Errors for 1- to 12-Months Ahead Forecasts.** The table shows the mean squared errors for the 1- to 12-months ahead forecasts obtained by the VAR model from Evans and Marshall (1998), the ACH-OP approach from Hamilton and Jordà (2002), and the ACH-ACM approach. The target forecast for the next month of the weekly models is based on the information available as of last week of the previous month. The table reports the squared difference between the forecast and the actual value of the effective federal funds rate.
Table 3: Estimation Results of the Response Symmetric ACM(1,1) Model. The table reports the maximum likelihood estimates of the response symmetric ACM (1,1) model for the period 1984-2001 using the data of Hamilton and Jordà (2002). The estimated specification,

\( \ell(\pi_N(t)) = c + A \cdot (x_N(t-1) - \pi_N(t-1)) + B \cdot \ell(\pi_N(t-1)) + D \cdot SP_{t-1} \cdot x_t \)

\( = \begin{pmatrix} c_1 \\ c_2 \\ c_4 \\ c_5 \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{24} & a_{25} \\ a_{41} & a_{42} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{54} & a_{55} \end{pmatrix} \begin{pmatrix} x_N(t-1) - \pi_N(t-1) \end{pmatrix} + \begin{pmatrix} b_{11} & b_{12} & b_{14} & b_{15} \\ b_{21} & b_{22} & b_{24} & b_{25} \\ b_{41} & b_{42} & b_{44} & b_{45} \\ b_{51} & b_{52} & b_{54} & b_{55} \end{pmatrix} \begin{pmatrix} \ell(\pi_N(t-1)) \end{pmatrix} + \begin{pmatrix} d_1 \\ d_2 \\ d_4 \\ d_5 \end{pmatrix} \cdot SP_{t-1} \cdot x_t \),

consists of a \((4 \times 1)\) constant vector \(c\), a martingale term \((x_N(t-1) - \pi_N(t-1))\) with a restricted response symmetric \((4 \times 4)\) coefficient matrix \(A\), a restricted response symmetric diagonal \((4 \times 4)\) coefficient matrix \(B\) for the autoregressive term \(\ell(\pi_N(t-1))\), and a \((4 \times 1)\) coefficient vector \(D\) for the spread between the six-month Treasury Bill and the federal funds rate. State 1 is associated with a 50 bp and State 2 with a 25 bp decrease and State 4 and State 5 with a 25 bp and a 50 bp increase of the target. Standard errors are reported in parentheses.
Table 4: P-values of a Test of iid Uniformity of the Continued PIT Series. For each horizon our methodology delivers a PIT sequence that is continued as suggested by Denuit and Lambert (2005), viz

\[ z_t^* = F(i_t - c|\Upsilon_{t-\kappa}) + f(i_t|\Upsilon_{t-\kappa})u_t. \]

The continuation of \( z^* \) is repeated in a monte carlo simulation by using 100 draws of \( \{u_t\} \) as in Jung, Kukuk, and Liesienfeld (2006). The Table reports the p-values (in %) for the test of iid U(0,1) of the \( z^* \)-sequence used in Bauwens, Giot, Grammig, and Veredas (2004). The first column is the forecast horizon \( \kappa \). The second column reports the p-values implied by the probability forecast of an ACH-response symmetric ACM(1,1) model. The third column reports the p-values implied by the probability forecast of the ACH-OP model.

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Table 5: **Ljung-Box Autocorrelation Test for the $z^*$ Sub-Series.** For each horizon our methodology delivers a PIT sequence that is continued as suggested by Denuit and Lambert (2005), viz

$$z^*_t = F(i_t - c|\Upsilon_{t-\kappa}) + f(i_t|\Upsilon_{t-\kappa})u_t.$$  

The continuation of $z^*$ is repeated in a monte carlo simulation by using 100 draws of $\{u_t\}$ as in Jung, Kukuk, and Liesenfeld (2006). Since the $\kappa$-step ahead PIT sequence (for $\kappa > 1$) exhibits a MA($\kappa-1$) autocorrelation structure, we follow DGT’s advice and partition the PIT sequence for each forecast horizon $h$ into $h$ sub-series. For instance, the two sub-series for a 2-step ahead probability forecasts are $\{z^*_1, z^*_3, z^*_5, \ldots\}$ and $\{z^*_2, z^*_4, z^*_6, \ldots\}$. Each sub-series should be iid $U(0,1)$, if the forecasts probability functions were correct. For forecast horizon $h$ we compute for each sub-series Ljung-Box statistics (for lag one and two) and take the average over the 100 monte carlo draws. The Table reports the maximal Ljung-Box statistic of the $h$ subseries. Column three contains the result for the ACH - response symmetric ACM(1,1) and column four for the ACH-OP. The fifth and sixth column contain the critical values at a 5%/h and 1%/h significance level.

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Figure 1: Effective Federal Funds Rate, Target, 6-month Treasury Bill rate and Target Changes. The Figure shows the time series data from 1984 until 2001 used in our application. The upper panel depicts the effective federal funds rate (FFR) in a dashed thin line that fluctuates around the federal funds rate target (solid bold line). The third line in the top panel (solid thin line) is the six-month Treasury Bill rate on the secondary market (TB6). All data are on weekly frequency. The difference between FFR and TB6 is the spread variable (SP = TB6 - FFR). The lower panel shows the associated changes of the federal funds rate target.
Figure 2: Frequency Distributions of Target Changes Before and After the Classification. The two panels depict the frequency distribution of the target changes covering the sample from 1984 until 2001. The upper panel shows the frequency distribution of the observed target changes before the classification according to Equation (1). The lower panel depicts the frequency distribution of the consolidated target changes after the classification.
Figure 3: Illustration of the Probability Forecast for the ACH-ACM and ACH-OP Models. The figure provides a graphical illustration of the simulation procedure. The left hand side of the picture depicts $M = 7$ simulated sample paths that reach different federal funds rate target values in $t + \kappa - 1$. Given the target value in $t + \kappa - 1$, only $k = 3$ different values can be attained in period $t + \kappa$. The conditional probabilities for each of the possible values in $t + \kappa$ can be computed via Equation (18). Some values in $t + \kappa$, like $\hat{i}$, may have non-zero probabilities in many replications, while some values may be assigned non-zero probabilities only once, and some never. Summing the zero and non-zero probabilities for each possible value over the $M$ replications and dividing by $M$ yields the estimate of the $t + \kappa$ period ahead probability forecast for the federal funds rate target that is sketched on the right hand side of the Figure.
For each horizon our methodology delivers a PIT sequence that is continued as suggested by Denuit and Lambert (2005), viz
\[ z_t^* = F(i_t - c|\Upsilon_{t-k}) + f(i_t|\Upsilon_{t-k})u_t. \]

The continuation of \( z^* \) is repeated in a monte carlo simulation by using 100 draws of \( \{u_t\} \) as in Jung, Kukuk, and Liesenfeld (2006). This Figure depicts the histograms for the continued probability integral transform (PIT) of the resulting 100 \( \{z^*\} \) sequences. The horizontal lines superimposed on the histograms are approximate 95% confidence intervals for the individual bin heights under the null that \( z^* \) is iid \( U(0,1) \). The left panels depict the histograms of the \( z^* \) series resulting from the forecast obtained by the ACH-response symmetric ACM(1,1) model the right panels show the histograms of the \( z^* \) series resulting from the ACH-OP forecasts.
Figure 5: Histograms of the Continued PIT for Mid-Term Horizons. For each horizon our methodology delivers a PIT sequence that is continued as suggested by Denuit and Lambert (2005), viz

$$z^*_t = F(i_t - c|\Upsilon_{t-k}) + f(i_t|\Upsilon_{t-k})u_t.$$ 

The continuation of $z^*$ is repeated in a monte carlo simulation by using 100 draws of $\{u_t\}$ as in Jung, Kukuk, and Liesenfeld (2006). This Figure depicts the histograms for the continued probability integral transform (PIT) of the resulting 100 $\{z^*\}$ sequences. The horizontal lines superimposed on the histograms are approximate 95% confidence intervals for the individual bin heights under the null that $z^*$ is iid $U(0, 1)$. The left panels depict the histograms of the $z^*$ series resulting from the forecast obtained by the ACH-response symmetric ACM(1,1) model the right panels show the histograms of the $z^*$ series resulting from the ACH-OP forecasts.
Figure 6: Histograms of the Continued PIT for Long-Term Horizons. For each horizon our methodology delivers a PIT sequence that is continued as suggested by Denuit and Lambert (2005), viz
\[ z_t^* = F(i_t - c|\Upsilon_{t-n}) + f(i_t|\Upsilon_{t-n})u_t. \]
The continuation of \( z^* \) is repeated in a monte carlo simulation by using 100 draws of \( \{u_t\} \) as in Jung, Kukuk, and Liesenfeld (2006). This Figure depicts the histograms for the continued probability integral transform (PIT) of the resulting 100 \( \{z^*\} \) sequences. The horizontal lines superimposed on the histograms are approximate 95 % confidence intervals for the individual bin heights under the null that \( z^* \) is iid \( U(0, 1) \). The left panels depict the histograms of the \( z^* \) series resulting from the forecast obtained by the ACH-response symmetric ACM(1,1) model the right panels show the histograms of the \( z^* \) series resulting from the ACH-OP forecasts.
Figure 7: Maximal Autocorrelations of the Continued PIT Sub-Series for Short-Term Horizons. For each horizon our methodology delivers a PIT sequence that is continued as suggested by Denuit and Lambert (2005), viz

\[ z_t^* = F(i_t - c|\Upsilon_{t-\kappa}) + f(i_t|\Upsilon_{t-\kappa})u_t. \]

The continuation of \( z^* \) is repeated in a Monte Carlo simulation by using 100 draws of \( \{u_t\} \) as in Jung, Kukuk, and Liesenfeld (2006). This Figure depicts the mean of the maximal autocorrelations for the continued PIT sub-series of the 100 \( \{z^*\} \) sequences. Since the \( \kappa \)-step ahead PIT sequence (for \( \kappa > 1 \)) exhibits a \( MA(\kappa - 1) \) autocorrelation structure, we follow DGT’s advice and partition the PIT sequence for each forecast horizon \( h \) into \( h \) sub-series. For instance, the two sub-series for a 2-step ahead probability forecasts are \( \{z_1^*, z_3^*, z_5^*, \ldots\} \) and \( \{z_2^*, z_4^*, z_6^*, \ldots\} \). Each sub-series should be iid \( U(0, 1) \), if the forecasts probability functions were correct. Therefore, the panel for the \( h^{th} \) forecast horizon plots the maximal autocorrelation of the \( h \) sub-series for eight lags. The horizontal lines superimposed on the autocorrelograms are Bartlett’s approximate 95% confidence intervals under the null that the sub-series of \( z^* \) is iid. The left panels depict the autocorrelation functions resulting from the forecast obtained by the ACH-response symmetric ACM(1,1) model the right panels show the autocorrelation functions resulting from the ACH-OP forecasts.
Figure 8: Maximal Autocorrelations of the Continued PIT Sub-Series for Mid-Term Horizons. For each horizon our methodology delivers a PIT sequence that is continued as suggested by Denuit and Lambert (2005), viz

\[ z^*_t = F(i_t - c|\Upsilon_{t-\kappa}) + f(i_t|\Upsilon_{t-\kappa})u_t. \]

The continuation of \( z^* \) is repeated in a monte carlo simulation by using 100 draws of \( \{u_t\} \) as in Jung, Kukuk, and Liesenfeld (2006). This Figure depicts the mean of the maximal autocorrelations for the continued PIT sub-series of the 100 \( \{z^*_t\} \) sequences. Since the \( \kappa \)-step ahead PIT sequence (for \( \kappa > 1 \)) exhibits a MA(\( \kappa - 1 \)) autocorrelation structure, we follow DGT’s advice and partition the PIT sequence for each forecast horizon \( h \) into \( h \) sub-series. For instance, the two sub-series for a 2-step ahead probability forecasts are \( \{z^*_1, z^*_3, z^*_5, \ldots\} \) and \( \{z^*_2, z^*_4, z^*_6, \ldots\} \). Each sub-series should be iid \( U(0, 1) \), if the forecasts probability functions were correct. Therefore, the panel for the \( h^{th} \) forecast horizon plots the maximal autocorrelation of the \( h \) sub-series for eight lags. The horizontal lines superimposed on the autocorrelograms are Bartlett’s approximate 95 % confidence intervals under the null that the sub-series of \( z^* \) is iid. The left panels depict the autocorrelation functions resulting from the forecast obtained by the ACH-response symmetric ACM(1,1) model the right panels show the autocorrelation functions resulting from the ACH-OP forecasts.
Figure 9: Maximal Autocorrelations of the Continued PIT Sub-Series for Long-Term Horizons. For each horizon our methodology delivers a PIT sequence that is continued as suggested by Denuit and Lambert (2005), viz

$$z_t^* = F(i_t - c|\Upsilon_{t-\kappa}) + f(i_t|\Upsilon_{t-\kappa})u_t.$$  

The continuation of $z^*$ is repeated in a monte carlo simulation by using 100 draws of $\{u_t\}$ as in Jung, Kukuk, and Liesenfeld (2006). This Figure depicts the mean of the maximal autocorrelations for the continued PIT sub-series of the 100 $\{z^*\}$ sequences. Since the $\kappa$-step ahead PIT sequence (for $\kappa > 1$) exhibits a $\text{MA}(\kappa - 1)$ autocorrelation structure, we follow DGT's advice and partition the PIT sequence for each forecast horizon $h$ into $h$ sub-series. For instance, the two sub-series for a 2-step ahead probability forecasts are $\{z_1^*, z_3^*, z_5^*, \ldots\}$ and $\{z_2^*, z_4^*, z_6^*, \ldots\}$. Each sub-series should be iid $U(0, 1)$, if the forecasts probability functions were correct. Therefore, the panel for the $h^{th}$ forecast horizon plots the maximal autocorrelation of the $h$ sub-series for eight lags. The horizontal lines superimposed on the autocorrelograms are Bartlett's approximate 95% confidence intervals under the null that the sub-series of $z^*$ is iid. The left panels depict the autocorrelation functions resulting from the forecast obtained by the ACH-response symmetric ACM(1,1) model the right panels show the autocorrelation functions resulting from the ACH-OP forecasts.