A Long Run Structural Macroeconometric Model for Germany

Elena Schneider∗ Pu Chen† Joachim Frohn‡

Faculty of Economics, University of Bielefeld, October 13, 2006

Abstract

The aim of this paper is to develop and estimate a core model for Germany with a transparent and theoretically coherent foundation. The modelling strategy includes a practical approach to long-run structural relationships suggested by economic theory and an otherwise unrestricted VAR model. Both domestic and foreign variables are used in the VAR approach: domestic and foreign real per capita output, domestic and foreign producer prices and nominal interest rates, the nominal effective exchange rate, the price of oil and the domestic real per capita stock of money. The data taken from the OECD Statistical Compendium and the IMF Data Base is adjusted quarterly and seasonally. It covers the period 1991Q1-2005Q4. Tests of restrictions on the long-run relations of the model are presented. The dynamic properties of the model are discussed using short-run estimation and impulse response analysis for the effects of an exogenous oil price shock on the endogenous variables of the model. This research was supported by a German Science Foundation grant (DFG): FR 2121/1-1 given to Pu Chen.

KEYWORDS: Long-Run Structural VAR, Macroeconomic Modelling, A Core Germany Model, Oil Price Shock.

JEL Classification: C32, E24

∗E-Mail: eschneider@wiwi.uni-bielefeld.de
†E-Mail: pchen@wiwi.uni-bielefeld.de
‡E-Mail: jfrohn@wiwi.uni-bielefeld.de
1 Introduction

The aim of the long run structural modelling by Garratt et al. (2000, 2002, 2003) is to develop a core model with transparent and theoretically coherent foundation. The Authors advanced the modelling framework of King et al.( 1991), Gali (1992), Mellander et al. (1992) and Crowder et al. (1999), and developed a long-run framework suitable for modeling a small open macroeconomy. Theirs new strategy offers a practical approach to relationships suggested by economic theory in an otherwise unrestricted vector autoregressive (VAR) model. The core model of UK includes transparent and theoretically founded long-run equations of the type exhibited by RBC. The long-run relations are derived from production, arbitrage, solvency and portfolio balance conditions. The five equation of core model does not describe a closed model like for example SDGE models.

For your empirical analysis Garratt et al. (2003) used a log-linear approximation of the five long-run equilibrium relationships set out. Additionally they introduced a 'long-run forcing' variable for the determination of output, the oil prices. 'Forcing’ variable is considered in the since that changes in oil prices have a direct influence on output, but they are not affected by the presence of disturbance terms, which measure the extent of disequilibria in the economy. Estimation of the parameters of the core model is based on a modified and generalised version of Johansen’s (1991, 1995) maximum likelihood approach to the problem of estimation and hypothesis testing in the context of vector error correction models (VECM)\(^1\).

In our paper we used the theoretical model from Garratt et al. (2003) and estimated a core long-run model for Germany. Using a VAR(1) model with unrestricted intercepts and treating the oil price variable as 'long-run forcing' we found five cointegrating relationships. With five cointegrating relations can be required five restrictions on each relationship to exactly identify them. Tests of this restrictions (over identifying) can be used to test rigorously the validity of the long-run restrictions implied by economic theory. Using Impulse Response Analysis would be simulated the effects of an exogenous oil price shock on the long-run relations and the endogenous variables of the model.

The plan of the paper is as follows: Section 2 describes a long-run theoretical framework for macromodelling for the small open economy, and derives testable restrictions on the long-run relations. This Section also outlines how the long-run theoretical relations are embodied in a VECM. Section 3 describes the empirical analysis underlying the construction of the core model and discusses the results obtained from testing its long-run properties. It will reported the estimates of short-run dynamics

\(^1\)e.g. Pesaran and Shin (2002) and Garratt et al. (2000)
and of impulse response functions for the oil price. Section 4 finishes the paper with some concluding remarks.

2 A Long Run Structural Macroeconometric Model

Like the core model for UK by Garratt et al. (2003) we modeled the Germany economy as a small open economy, subject to economic developments in the rest of the world. Hence in the VAR approach both domestic and foreign variables are used. The variables are domestic and foreign real per capita outputs \( (y_t, y^*_t) \), domestic and foreign producer prices \( (p_t, p^*_t) \) and nominal interest rates \( (r_t, r^*_t) \), the nominal effective exchange rate \( (e_t) \), the price of oil \( (p^o_t) \) and the domestic real per capita stock of money \( (h_t) \), all modeled in logarithms.

The underlying economic theory delivers five long-run relations or equilibrium conditions among these variables. This based on production, arbitrage, solvency and portfolio balance conditions, together with stock-flow and accounting identities. The first long-run relation is a purchasing power parity relation (PPP), based on international goods market arbitrage. PPP was modified by the effect of oil prices (cf. Chaudhuri and Daniel, (1998)). The second relation, a nominal interest rate parity relation, based on arbitrage between domestic and foreign bonds. Then an "output gap" relation derived from the neoclassical growth model, assuming common technological progress in production at home and overseas. Last relations are trade balance and real money balance relations, based on long-run solvency conditions and assumptions about the determinants of the demand for domestic and foreign assets.

The economic theory says nothing about the statistical characteristics of the variables, but once it is assumed that they are difference-stationary. The equilibrium relations become candidate cointegrating relations in the VECM representation.

2.1 A Framework for Long-Run Macromodeling

The macroeconometric modeling of a small open economy begins with a rigorous derivation of the long-run steady-state relationships expected to predominate between the main variables in the core model. The analysis includes arbitrage conditions and stock-flow equilibria. This long-run relationships correspond to many of the long-run properties of the RBC and large macroeconometric models. An approach to the derivation of long-run is to work directly with the arbitrage conditions.
**Production technology and output:**

We assume that, the aggregate output (in the long-run) is determined by following production function

\[ \frac{Y_t}{P_t} = F(K_t, A_t N_t) = A_t N_t F\left(\frac{K_t}{A_t N_t}, 1\right). \tag{1} \]

The constant returns to scale production function in labour (denoted by \(N_t\)) and capital stock (denoted by \(K_t\)) is linear homogenous in both variables. We analyze a real aggregate output \((Y_t/P_t)\), where \(P_t\) is a general price. \(A_t\) stands for an index of labour-augmenting technological progress. Technological progress assumed to be composed of a deterministic and a stochastic mean-zero components

\[ \ln(A_t) = a_0 + g_t + u_{at}. \]

The fraction of the population, which is employed at the time \(t\), assumed to be a stationary process

\[ N_t/POP_t = \lambda \exp(\eta nt). \]

Accordingly the real per capita output in logarithmic form \(y_t = \ln((Y_t/P_t)/POP_t)\) is given

\[ y_t = a_0 + g_t + u_{at} + \ln(\lambda) + \ln(f(\kappa_t)) + \eta_{nt}, \tag{2} \]

with \(\kappa_t\) denoted the capital stock per effective labour unit \((\kappa_t = K_t/(A_t N_t))\) and \(f(\kappa_t)\) is a well behaved function in the sense that is satisfies the Inada conditions.

Profit maximisation on the part of firms ensures that, in the steady-state the real rate of return \(\rho_t\) will be equal to the marginal product of capital \(\rho_t = f'(\kappa_t)\). The assumption, that the steady state distribution of the real rate of return will also be ergodic and stationary can be written

\[ 1 + \rho_{t+1} = (1 + \rho) \exp(\eta_{\rho,t+1}), \]

where \(\eta_{\rho,t+1}\) is a stationary process normalized so that \(E(\exp(\eta_{\rho,t+1}|I_t)) = 1\), and \(I_t\) is the publicly available information set at the time \(t\). This normalization ensures that \(\rho\) is in fact the mean of the steady state distribution of real returns, \(\rho_t\), given by \(E(f'(\kappa_{\infty}))\).

For the small and open economy it is reasonable to assume that, in the long-run, domestic technological process \(A_t\) is determined by the level of technological progress in the rest of the world \(A_t = \gamma A_t^* \exp(\eta_{at})\). There \(\gamma\) captures productivity differential based on fixed, initial technological endowments, \(A_t^*\) represents the level of foreign technological progress. Assuming, that per capita output in the rest of the world is also determined according to a neoclassical growth model, then we have

\[ y_t - y_t^* = \ln(\gamma) + \ln(\lambda/\lambda^*) + \ln(f(\kappa_t)/f^*(\kappa_t^*)) + \eta_{at} + (\eta_{nt} - \eta_{nt}^*) \tag{3} \]

**Arbitrage conditions:**

The set of arbitrage conditions to be considered in this paper are included in many macroeconomic models in one form or another. They are the (relative) Purchasing Power Parity (PPP), the Fisher Inflation Parity (FIP), and the Uncovered Interest Parity (UIP) relationships.
PPP is based on the presence of good market arbitrage. According to this the price of a common basket of goods will be equal in different countries, when these price is in a common currency. The domestic price is determined $P_t = E_t P^*_t \exp(\eta_{PPP,t})$, where $\eta_{PPP,t}$ is assumed to follow a stationary process capturing short-run variations in transport costs, information disparities, and the effects of tariff and non-tariff barriers. $E_t$ is the effective exchange rate, defined as the domestic price of a unit of foreign currency at the beginning of period $t$. An increase in the exchange rate represents a depreciation of the home country currency. In log-linear form we have

$$p_t = e_t + p^*_t + \eta_{PPP,t}$$

with $p_t = \log(P_t), p^*_t = \log(P^*_t)$ and $e_t = \log(E_t)$.

The FIP relationship include the equilibrium outcome of the arbitrage process between holding bonds and investing in physical assets. The nominal interest rate is determined $1 + R_t = (1 + E(\rho_{t+1})) \left(1 + \frac{E(P_{t+1}) - P_t}{P_t}\right) \exp(\eta_{FIP,t})$, where $(E(P_{t+1}) - P_t)/P_t$ is the inflation expectation. $\eta_{FIP,t}$ is the risk premium, capturing the effects of money and goods market uncertainties on risk-averse agents. As before, we assume that $\eta_{FIP,t}$ follows a stationary process with a finite mean and variance. In log-linear form we can written the FIP following

$$r_t = \ln(1 + \rho) + \ln \left(1 + \frac{\Delta P_{t+1}}{P_t}\right) + \eta_{FIP,t+1} + \eta_{\rho,t+1} + E(\eta_{P,t+1}) + E(\eta_{\rho,t+1}),$$

where $r_t = \ln(1 + R_t)$.

The third arbitrage condition is based on the UIP relationship, which captures the equilibrium outcome of the arbitrage process between holding domestic and foreign bonds. In this way any differential between interest rate across countries must be an adjustment by expected exchange rate changes to eliminate the arbitrage. For the Interest Rate Parity relationship we use the UIP equation in the form $(1 + R_t) = (1 + R^*_t) \left(1 + \frac{E(E_{t+1}) - E_t}{E_t}\right) \exp(\eta_{UIP,t+1})$. There $\eta_{UIP,t+1}$ is the risk premium associated with the effects of bonds and foreign exchange uncertainties on risk-averse agents. The IRP relationship in log-linear form can be written as

$$r_t = r^*_t + \eta_{\Delta e,t+1} + E(\eta_{\epsilon,t+1}) + \eta_{UIP,t+1}.$$  

**Output-expenditure flow identity and long-run solvency requirements**

\[\text{For the expectation we write } 1 + E(\rho_{t+1}) = (1 + \rho_{t+1}) \exp(E(\eta_{\rho,t+1})) \text{ and } E(P_{t+1}) = P_{t+1} \exp(E(\eta_{P,t+1})). \text{ We use also here, that the steady state distribution of the real rate of return will also be ergodic and stationary.}\]

\[\text{We assume, that the expectation for exchange rate is } E(E_{t+1}) = E_{t+1} \exp(E(\eta_{E,t+1})), \text{ where the expectations errors } E(\eta_{E,t+1}) \text{ follow stationary processes.}\]

5
In addition to the arbitrage condition, the economy is subject to the long-run solvency constraint obtainable from the stock-flow relationships. For the core model we use the following stock identities:

\[ D_t = H_t + B_t \]

with \( D_t \) is a net government debt, \( H_t \) is the stock of high-powered money, \( B_t \) is the stock of domestic bonds issued by the government; for the net foreign asset position of the economy \( F_t \),

\[ F_t = E_t B^*_t - (B_t - B^*_t) \]

where \( B^*_t \) is the stock of foreign assets held by domestic residents, \( B^*_t \) is the stock of domestic assets held by domestic residents; and \( L_t = D_t + F_t \) where \( L_t \) is the stock of financial assets held by the private sector.

For the output-expenditure identity we have \( Y_t = C_t + I_t + G_t + (E_t - I_t) \), where \( Y_t \) is gross domestic product, \( C_t \) are consumption expenditures, \( I_t \) investment expenditures, \( G_t \) government expenditure, \( E_t \) and \( I_t \) are expenditures on export and import.

In order to ensure the long-run solvency of the private sector asset/liability position, it is assumed \( L_{t+1}/Y_t = \mu \exp(\eta_{y,t+1}) \), where \( L_{t+1}/Y_t \) is a ratio of total financial assets to the nominal income level. The process \( \eta_{y,t+1} \) must be stationary, so that the \( L_{t+1}/Y_t \) is stationary and ergodic. This expression captures the idea that domestic residents are neither willing nor able to accumulate claims on, or liabilities to, the government and the rest of the world.

The modeling the equilibrium portfolio balance of private sector assets is like Branson’s (1977) Portfolio Balance Approach. The stock of financial assets held by private sector consist of the stock of high-powered money \( H_t \) plus the stock of domestic and foreign bonds held by domestic residents. It is specified two independent equilibrium relationships relating to asset demand: namely, those relating to the demand for hight-power money and for foreign assets:

\[
\frac{H_{t+1}}{L_t} = F_h \left( \frac{Y_t}{P_t} \frac{P_{t+1}}{P_t}, E_{t+1}(\rho_t), E_{t+1}(\rho^*_t), \frac{\Delta E_{t+1}(P_t)}{P_t}, t \right) \exp(\eta_{hl,t+1})
\]

with \( F_{h1} \geq 0, F_{h2} \leq 0, F_{h3} \leq 0, F_{h4} \leq 0 \).

\[
\frac{F_{t+1}}{L_t} = F_f \left( \frac{Y_t}{P_t} \frac{P_{t+1}}{P_t}, E_{t+1}(\rho_t), E_{t+1}(\rho^*_t), \frac{\Delta E_{t+1}(P_t)}{P_t}, t \right) \exp(\eta_{fl,t+1})
\]

with \( F_{f1} \leq 0, F_{f2} \leq 0, F_{f3} \geq 0, F_{f4} \geq 0 \).

In view of the IRP it is clear, that in the steady state, domestic and foreign bonds become perfect substitutes, and their expected rates of return are equal. Similar, given the FIP relationship the real rates of return on domestic and foreign bonds are equal to the (stationary) real rate of return on physical assets in the steady state. Hence, the asset demand relationships can be written equally as:

\[
\frac{H_{t+1}}{L_t} = F_h \left( \frac{Y_t}{P_t} \frac{P_{t+1}}{P_t}, r_t, t \right) \exp(\eta_{hl,t+1})
\]

6
\[
\frac{F_{t+1}}{L_t} = F_f \left( \frac{Y_t/P_t}{POP_t}, r_t, t \right) \exp(\eta_{lt,t+1})
\]

The solvency condition \( L_{t+1}/Y_t = \mu \exp(\eta_{gy,t+1}) \) combined with equation \( H_{t+1}/L_t \) now yields
\[
\frac{H_{t+1}}{Y_t} = \mu F_h \left( \frac{Y_t}{F_t}, r_t, t \right) \exp(\eta_{gy,t+1} + \eta_{hl,t+1})
\]

### 2.2 Econometric Formulation of the Core Model

For empirical purposes we employ a log-linear approximation of the five long-run equilibrium relationships set out in the previous section in (4), (6), (3), (7) and (5).

\[
(p_t - p_t^*) - e_t = a_{10} + \xi_{1,t+1}
\]
\[
r_t - r_t^* = a_{20} + \xi_{2,t+1}
\]
\[
y_t - y_t^* = a_{30} + \xi_{3,t+1}
\]
\[
h_t - y_t = a_{40} + \beta_{42} r_t + \beta_{43} y_t + \xi_{4,t+1}
\]
\[
r_t - \Delta p_t = a_{50} + \xi_{5,t+1}
\]

where \( p_t = \ln(P_t), p_t^* = \ln(P_t^*), e_t = \ln(E_t), y_t = \ln(Y_t/P_t), y_t^* = \ln(Y_t^*/P_t^*), r_t = \ln(1 + R_t), r_t^* = \ln(1 + R_t^*), h_t - y_t = \ln(H_{t+1}/P_t) - \ln(Y_t/P_t) = \ln(H_{t+1}/Y_t) \) and \( a_{50} = \ln(1 + \rho)^4 \).

We have allowed only for intercept in the equations. The disturbances \( \xi_{i,t+1} \) are related to the structural disturbance \( \eta_{hi,t+1} \) in the following manner:

\[
\xi_{1,t+1} = \eta_{PP,t} + a_{10}
\]
\[
\xi_{2,t+1} = \eta_{\Delta e,t+1} + E(\eta_{e,t+1}) + \eta_{UIP,t+1} - a_{20}
\]
\[
\xi_{3,t+1} = \eta_{nt} + (\eta_{nt} - \eta_{nt}^*)
\]
\[
\xi_{4,t+1} = \eta_{y,t+1} + \eta_{hl,t+1}
\]
\[
\xi_{5,t+1} = \eta_{FIP,t+1} + \eta_{\rho,t+1} + E(\eta_{P,t+1}) + E(\eta_{\rho,t+1}).
\]

These relationships can be difficult involved in identifying the effects of changes in particular structural disturbances \( \eta_{hi,t} \) on the dynamic behavior of the macroeconomy. Firstly, there are many more long-run structural disturbances than there are long-run reduced form disturbances; and secondly, there is no reason to believe that the \( \eta_{hi,t} \) are not themselves correlated.

The five long-run relations of the core model can be written more compactly as

\[
\xi_t = \beta^t z_{t-1} - a_0
\]

\(^4\text{We denote } \ln(H_{t+1}/P_t) \text{ by } h_t \text{ rather than } h_{t+1}. \text{ } H_{t+1} \text{ is the stock of hight powered money at the beginning of period } t+1.\)
where
\[ z_t = (p^o_t, r_t, y_t, \Delta p_t, p_t - p^*_t, e_t, h_t - y_t, r^*_t, y^*_t)' \]
\[ a_0 = (a_{10}, a_{20}, a_{30}, a_{40}, a_{50})' \]
and
\[ \beta' = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & -\beta_{42} & \beta_{43} & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \end{pmatrix} \]
(14)

Here, \( p^o_t \) is the logarithm of the oil price. A general specification for oil prices is given by
\[ \Delta p^o_t = \delta_0 + \sum_{i=1}^{s} \delta_i \Delta z_{t-i} + \varepsilon_{o,t} \]
where \( \varepsilon_{o,t} \) represents a serially uncorrelated oil price shock with a zero mean and a constant variance.

For the difference-stationary variable \( z_t \) the modelling strategy is now to embody \( \xi_t \) in an otherwise unrestricted VAR\((s - 1)\) in \( z_t \). The base of analysis is the following conditional error correction model
\[ \Delta z_t = b - \alpha \xi_t + \sum_{i=1}^{s-1} \Gamma_i \Delta z_{t-i} + \psi \Delta p^o_t + u_t \]
(15)
where \( b \) is an \( 9 \times 1 \) vector of fixed intercepts, \( \alpha \) is a \( 9 \times 5 \) matrix of error-correction coefficients, \( \Gamma_i \) are \( 9 \times 9 \) matrices of short-run coefficients, \( \psi \) is an \( 9 \times 1 \) vector representing the effects of changes in oil prices on \( \Delta z_t \), and \( u_t \) is an \( 9 \times 1 \) vector of disturbances assumed to be \( IID(0, \Sigma) \), with \( \Sigma \) being a positive definite matrix, and by construction uncorrelated with \( \Delta p^o_t \). With (13) we have
\[ \Delta z_t = c - \alpha \beta' z_{t-1} + \sum_{i=1}^{s-1} \Gamma_i \Delta z_{t-i} + \psi \Delta p^o_t + u_t \]
(16)
where \( c = b + aa_0 \) and \( \beta' z_{t-1} \) are the error correction terms.

For the estimation of the parameters of the core model for Germany (8) we use a modified and generalised version of Johansen’s (1991, 1995) maximum likelihood approach. With \( \lambda_{\text{max}} \) - and \( \lambda_{\text{trace}} \)-statistic we test for the number of cointegrating relations among the 9 variables in \( z_t \). We use the Akaike- and Schwarz-Information Criterion to selection the order of the underlying VAR model. We compute the LR-statistic for models with exact and over-identifying restrictions on the long-run coefficients. An exact identification in our model requires five restrictions on each of the five cointegrating
vectors, or a total of twenty-five restrictions on $\beta$. The economic theory as characterized in the matrix (14) defines forty-three restriction. Estimation of the model subject to all this restriction enables a test of validity of the over-identifying restrictions.

3 Estimation and Testing of the Model

The variables for the core model under consideration are $y_t, y_t^*, r_t, r_t^*, e_t, h_t - y_t, p_t, \hat{p}_t, p_t^*$ and $p_t^o$. A detailed description of these variables is given in Table (1). The data taken from the OECD Statistical Compendium and the IMF Data Base is adjusted quarterly and seasonally. It covers the period 1991Q1-2005Q4 (56 observations). Analog to paper by Garratt et al. (2003) we use the producer price indices to construct deviations between the domestic and foreign price levels in the PPP relationship. Instead of the retail price index we use the consumer price index to measure domestic inflation in the FIP relationship.

The Augmented Dickey-Fuller (ADF) test statistics for the levels and first differences of the core variables are reported in Table (2). The results suggest that it is reasonable to treat $y_t, y_t^*, r_t, r_t^*, e_t, h_t - y_t, p_t, p_t^*$ and $p_t^o$ as I(1) variables. For these variables the unit root hypothesis is rejected when applied to their first differences for all variables with level of significance 1% excepting $y_t^*$ and $r_t^*$. For this variables the unit root hypothesis is rejected with level of significance of 5%. When the tests are applied to levels of variables is no evidence with which to reject the unit root hypothesis. There is, however, an exception regarding the order of integration of the price variable $\hat{p}_t$. The application of the ADF tests to $\Delta \hat{p}_t$ not rejects the unit root hypothesis, but $\Delta \Delta \hat{p}_t$ is identified as a stationary variable. This corresponds, that $\hat{p}_t$ is a I(2) variable. However this variable exists according to the model description only as differences in the Fischer equation. Therefore the econometric conditions on variables are fulfilled.

Testing and Estimation of the Long Run Relations

The first stage of our modeling sequence is to select the order of the underlying VAR in these variables. Here we find that a VAR of order one appears to be appropriate when using the AIC and SIC as the model selection criterion. Using a VAR(1) model with unrestricted intercepts and treating the oil price variable, $p_t^o$, as weakly exogenous for the long-run parameters (or long-run forcing), we computed Johansens $\lambda_{trace}$ and $\lambda_{max}$ statistics. These statistics, together with their associated 90% and 95% critical values, are reported in Table (3).

The maximal eigenvalue statistic indicates the presence of just tree cointegrating relationships at the 95% significance level, which does not support our a priori expec-
tations of five cointegrating vectors. However, Cheung and Lai (1993) presented, that the maximum eigenvalue test is generally less robust to the presence of skewness and excess kurtosis in the errors than the trace test. We have evidence of non-normality in the residuals of the VAR model used to compute the test statistics. Therefore we think it is more appropriate to base our cointegration tests on the trace statistics. The trace test reject the null hypotheses that \( r = 0, 1, 2, 3 \) and 4 at the 5% level of significance but cannot reject the null hypothesis that \( r = 5 \). This is in line with our a priori expectations based on the long-run theory. Hence we proceed under the assumption that there are five cointegrating vectors.

With five cointegrating relations we require five restrictions on each relationship to exactly identify them. In view of the underlying long-run theory in the relations (8)-(12) we impose the following 25 exact-identifying restrictions on the cointegrating matrix:

\[
\beta' = \begin{pmatrix}
\beta_{11} & 0 & 0 & \beta_{14} & 1 & \beta_{16} & 0 & \beta_{18} & 0 \\
\beta_{21} & 1 & 0 & \beta_{24} & 0 & 0 & 0 & \beta_{28} & \beta_{29} \\
\beta_{31} & 0 & 1 & 0 & \beta_{35} & 0 & \beta_{37} & 0 & \beta_{39} \\
\beta_{41} & \beta_{42} & \beta_{43} & \beta_{44} & 0 & 0 & 1 & 0 & 0 \\
\beta_{51} & \beta_{52} & 0 & -1 & 0 & 0 & \beta_{57} & 0 & \beta_{59}
\end{pmatrix}
\] (17)

that corresponds to \( z_t = (p_{o}t, r_t, y_t, \Delta \tilde{p}_t, p_t - p^*_t, e_t, h_t - y_t, r^*_t, y^*_t) \). The first vector (the first row of \( \beta' \)) relates to the PPP relationship defined by (8) and is normalised on \( p_t - p^*_t \); the second relates to the IRP relationship defined by (9) and is normalised on \( r_t \); the third relates to the "output gap" relationship defined by (10) and is normalised on \( y_t \); the fourth is the money market equilibrium condition defined by (11) and is normalised on \( h_t - y_t \); and the fifth is the real interest rate relationship defined by (12), normalised on \( \Delta \tilde{p}_t \).

We have 20 unrestricted parameters in (17), and two in full restricted model, yielding a total of 18 over-identifying restrictions. In addition, working with a cointegrating VAR with unrestricted intercept coefficients, there are potentially five further parameters in the five cointegrating relationships. There are just 25 parameters to be freely estimated in the cointegrating relationships and provide a total of 18 over-identifying restrictions on which the core model is based and with which the validity of the economic theory can be tested. LR statistic for testing the eighteen over-identifying restrictions in the matrix (17) takes the value 70.86. According to Garrat et al. (2003) is the relevant critical values for the joint tests of the 25 over-identifying restrictions are 67.51 at the 10% significance level and 73.19 at the 5% level. Therefore we can not reject the over-identifying restrictions implied be the long-run theory in (4), (6), (3), (7) and (5).
The Vector Error Correction Model

The long-run relations, which incorporate all the restrictions suggested by the theory, are summarised below:

\[(p_t - p_t^*) - e_t = 0.186 + \hat{\xi}_{1,t+1}\]  
\[ (18) \]

\[r_t - r_t^* = 0.002 + \hat{\xi}_{2,t+1}\]  
\[ (19) \]

\[y_t - y_t^* = 0.134 + \hat{\xi}_{3,t+1}\]  
\[ (20) \]

\[h_t - y_t = -1.8409 + 72.32y_t + \hat{\xi}_{4,t+1}\]  
\[ (6.41) \]

\[ (21) \]

\[r_t - \Delta \tilde{p}_t = 0.006 + \hat{\xi}_{5,t+1}\]  
\[ (22) \]

The first equation (18), describes the PPP relationship and cannot reject this in the context of the core model. The co-movements of exchange rates and relative prices has been examined frequently in the literature. The empirical evidence on PPP appears to be sensitive to the data set used and the way in which the analysis is conducted. The finding here that PPP can be readily incorporated into the model is a useful contribution to this literature.

The second cointegrating relation, defined by (19), is the IRP condition. This includes an intercept, which can be interpreted as the deterministic component of the risk premium associated with bonds and foreign exchange uncertainties. Its value is estimated at 0.0021, implying a risk premium of approximately 0.8% per annum.

The third long-run relationship, given by (20), is the output "gap relationship" with per capita domestic and foreign output levels. This relationship indicates the moving the output levels in tandem in the long-run. This suggests that average long-run growth rate for Germany is the same as that in the rest of the OECD. The hypothesis advanced here, that \(y_t\) and \(y_t^*\) are cointegrated, is much less restrictive than the hypothesis considered in the literature that all pairs of output variables in the OECD are cointegrated.

For the money market equilibrium (MME) condition, given by (21), we could not reject the hypothesis that the elasticity of real money balances with respect to real output \((h_t - y_t)\) is equal to unity. Therefore (21) in fact represents a M0 velocity equation. The MME condition contains strong statistical evidence of a positive output per capita effect on real money balances.

Finally, the fifth equation, (22), defines the FIP relationship, where the estimated constant implies an annual real rate of return of approximately 2.2% per annum. Our results support the FIP relationship and again highlights the important role played by
the FIP relationship in a model of the macroeconomy which can incorporate interac-
tions between variables omitted from more partial analysis.

The short-run dynamics of the model are characterised by the error correction
specifications given in Table (4). The estimates of the $\alpha$-coefficients (also known as
the loading coefficients) show that the long-run relations make an weak contribution
in most equations. Two or maximal three coefficients are statistical relevant in the
$\alpha$-Matrix. The variables interest rate, income and inflation represent a statistically
significant set by long-run adjustment mechanism. Similarly to estimation of core
model for UK is the $\Gamma$-Matrix (known as the short run adjustment) weak assign. The
changes in domestic income, world interest rate and exchange rate from previous period
($t-1$) affect the changes in the other variables of the $z_t$ vector. The rate of increase the
oil price have had a statistical significant impact on the $\Delta(p_t - p^*_t)$ and on the change
der world interest rate $\Delta r^*_t$.

**Impulse Response Analysis**

The results of impulse response analysis are reported in Figure 1. Its shows, that
the oil price shock has a permanent effect on the level of the individual series. The oil
price shock effect on domestic output has finally the expected negative sign, reducing
the output by approximately 0.002% after 6 years. However foreign output raises by
approximately 0.001%, so that the output gap returns to its equilibrium. On impact
the oil price shock declines the domestic rate of inflation by 0.02%, and raises by 0.01%
after 1 quarter before gradually falling back close to zero after approximately 2 years.
Despite the higher domestic prices the oil price shock generate a appreciation of the
nominal exchange rate after 2.5 yares as can be seen from Figure 2. The oil price shock
is accomponied by increases in both domestic and foreign interest rates.

**4 Conclusions**

This paper provides an example of macroeconometric-modelling with an application to
the long-run structural VAR modelling approach. As basic framework for this paper is
used the core model for the UK by Garratt et al.(2003). The paper outlines a theoretical
framework for the long-run analysis of a small open macroeconomy; introduces a
practical approach to theory-based long-run relationships in an otherwise unrestricted
VAR; presents the estimates and the tests to construct a core macroeconometric model
of Germany. The description of approach to modelling starts with a presentation of
a set of long-run relationships between the macroeconomic variables such as interest
rate, output or exchange rate. These long-run relationships are based on production,
arbitrage, solvency and portfolio balance conditions, together with stock-flow and
accounting identities. Further these long-run relationships are embedded within an otherwise unrestricted VAR model in nine core variables, augmented appropriately by intercepts. The VAR model is estimated over the period 1991Q1-2005Q4, subject to the theory restrictions on the long-run coefficients using recently developed econometric techniques.

An important component of our modelling approach is the possibility for testing formally the validity of restrictions suggested by economic theory in the context of a complete macroeconomic model. The underlying economic theory provides five long-run relations or equilibrium conditions among the nine core variables of the macro-model. The statistical tests provided little evidence to reject this view. Under the assumption that there are five long-run relationships, we obtained a model with seven freely estimated parameters. Further, the likelihood ratio tests did not reject the overidentifying restrictions suggested by economic theory, so that we conclude that the estimated model is both theory and data consistent.

The second stage of our modeling sequence is to estimate the short-run dynamic and the analysis of the effects of exogenous shocks. We analyse the response on the levels of the model’s eight endogenous variables to the oil price shock. Among other results we find an evidence that an unexpected rise in oil prices increases domestic and foreign interest rates, has a moderate contractionary impact on real domestic output, increases the inflation rate and leads to an appreciation of the nominal exchange rates.

References


<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_t$</td>
<td>natural logarithm of the Germany real per capita GDP (GDP deflator) (2000=100).</td>
</tr>
<tr>
<td>$p_t$</td>
<td>natural logarithm of the Germany Producer Price Index (2000=100%).</td>
</tr>
<tr>
<td>$\tilde{p}_t$</td>
<td>natural logarithm of the Germany Consumer Price Index (2000=100%).</td>
</tr>
<tr>
<td>$r_t$</td>
<td>is computed as $r_t = 0.25\ln(1 + R_t/100)$, where $R_t$ is the 90 day Interbank discount rate per annum.</td>
</tr>
<tr>
<td>$h_t$</td>
<td>natural logarithm of the Germany real per capita M1 money stock (2000=100%), Germany share in M1 EMU (from 2003 without Cash)</td>
</tr>
<tr>
<td>$e_t$</td>
<td>natural logarithm of the nominal DM/Euro exchange rate, monthly average (2000=100%).</td>
</tr>
<tr>
<td>$y_t^*$</td>
<td>natural logarithm of the foreign (OECD) real per capita GDP (GDP deflator) (2000=100%).</td>
</tr>
<tr>
<td>$p_t^*$</td>
<td>natural logarithm of the foreign (OECD) Producer Price Index (2000=100%).</td>
</tr>
<tr>
<td>$r_t^*$</td>
<td>is computed as $r_t = 0.25\ln(1 + R^a_t/100)$, where $R^a_t$ is the weighted average 90 day interest rate per annum in the USA, UK, Japan and France</td>
</tr>
<tr>
<td>$p_t^o$</td>
<td>natural logarithm of oil price, measured as the Average Price in US$ per Barrel Oil.</td>
</tr>
<tr>
<td>$t$</td>
<td>time trend, taking the values 1, 2, 3,... in 1991Q1, · · · , 2005Q4 respectively.</td>
</tr>
</tbody>
</table>
Table 2: Augmented Dickey-Fuller Unit Root Test Applied to Variables in the Core Model; 1991Q1-2005Q4

<table>
<thead>
<tr>
<th>variable</th>
<th>for the levels</th>
<th>for the first differences</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t-statistik</td>
<td>lags</td>
</tr>
<tr>
<td>$y_t$</td>
<td>-2.43</td>
<td>4</td>
</tr>
<tr>
<td>$y^*_t$</td>
<td>-1.07</td>
<td>1</td>
</tr>
<tr>
<td>$r_t$</td>
<td>-1.90</td>
<td>2</td>
</tr>
<tr>
<td>$r^*_t$</td>
<td>-3.03</td>
<td>2</td>
</tr>
<tr>
<td>$e_t$</td>
<td>-2.45</td>
<td>4</td>
</tr>
<tr>
<td>$h_t - y_t$</td>
<td>-2.67</td>
<td>1</td>
</tr>
<tr>
<td>$p_t$</td>
<td>-2.08</td>
<td>2</td>
</tr>
<tr>
<td>$\hat{p}_t$</td>
<td>-3.18</td>
<td>5</td>
</tr>
<tr>
<td>$p^*_t$</td>
<td>-2.15</td>
<td>2</td>
</tr>
<tr>
<td>$p_t^p$</td>
<td>-1.63</td>
<td>6</td>
</tr>
<tr>
<td>$p_t - p^*_t$</td>
<td>-0.80</td>
<td>1</td>
</tr>
<tr>
<td>$\Delta \tilde{p}_t$</td>
<td>-2.57</td>
<td>4</td>
</tr>
</tbody>
</table>

Notes: The t-statistic are computed using ADF regressions with an intercept, a linear time trend and s lagged depended variables, when applied to the levels, and with intercept and s lagged first-differences of depended variable, when applied to the first difference. The order of augmentation in the Dickey-Fuller regressions chosen using the Akaike Information Criterion, with a maximum lag order of ten. The critical values for the t-Test: -4.12 (level of significance 1%) and -3.49 (level of significance 5%) for the levels. -3.55 (level of significance 1%) and -2.91 (level of significance 5%) for the differences. The critical values are from MacKinnon (1996).
Table 3: Cointegration Rank Statistics for the Core Model

\((y_t, y_t^*, r_t, r_t^*, e_t, h_t - y_t, p_t - p_t^*, \Delta \tilde{p}_t)\)

(A) Trace Statistic

<table>
<thead>
<tr>
<th>(H_0)</th>
<th>(H_1)</th>
<th>(Test\text{Statistic})</th>
<th>95% Critical values</th>
<th>90% Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r = 0)</td>
<td>(r &gt; 0)</td>
<td>325.08</td>
<td>199.12</td>
<td>192.80</td>
</tr>
<tr>
<td>(r = 1)</td>
<td>(r &gt; 1)</td>
<td>248.73</td>
<td>163.01</td>
<td>157.02</td>
</tr>
<tr>
<td>(r = 2)</td>
<td>(r &gt; 2)</td>
<td>177.41</td>
<td>128.79</td>
<td>123.33</td>
</tr>
<tr>
<td>(r = 3)</td>
<td>(r &gt; 3)</td>
<td>128.72</td>
<td>97.83</td>
<td>93.13</td>
</tr>
<tr>
<td>(r = 4)</td>
<td>(r &gt; 4)</td>
<td>87.87</td>
<td>72.10</td>
<td>68.04</td>
</tr>
<tr>
<td>(r = 5)</td>
<td>(r &gt; 5)</td>
<td>48.28</td>
<td>49.36</td>
<td>46.00</td>
</tr>
<tr>
<td>(r = 6)</td>
<td>(r &gt; 6)</td>
<td>22.93</td>
<td>30.77</td>
<td>27.96</td>
</tr>
<tr>
<td>(r = 7)</td>
<td>(r &gt; 7)</td>
<td>9.53</td>
<td>15.44</td>
<td>13.31</td>
</tr>
</tbody>
</table>

(B) Maximum Eigenvalue Statistic

<table>
<thead>
<tr>
<th>(H_0)</th>
<th>(H_1)</th>
<th>(Test\text{Statistic})</th>
<th>95% Critical values</th>
<th>90% Critical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(r \leq 0)</td>
<td>(r = 1)</td>
<td>76.35</td>
<td>58.08</td>
<td>55.25</td>
</tr>
<tr>
<td>(r \leq 1)</td>
<td>(r = 2)</td>
<td>71.32</td>
<td>52.62</td>
<td>49.70</td>
</tr>
<tr>
<td>(r \leq 2)</td>
<td>(r = 3)</td>
<td>48.69</td>
<td>46.97</td>
<td>44.01</td>
</tr>
<tr>
<td>(r \leq 3)</td>
<td>(r = 4)</td>
<td>40.85</td>
<td>40.89</td>
<td>37.92</td>
</tr>
<tr>
<td>(r \leq 4)</td>
<td>(r = 5)</td>
<td>39.59</td>
<td>34.70</td>
<td>32.12</td>
</tr>
<tr>
<td>(r \leq 5)</td>
<td>(r = 6)</td>
<td>25.35</td>
<td>28.72</td>
<td>26.10</td>
</tr>
<tr>
<td>(r \leq 6)</td>
<td>(r = 7)</td>
<td>13.40</td>
<td>22.16</td>
<td>19.79</td>
</tr>
<tr>
<td>(r \leq 7)</td>
<td>(r = 8)</td>
<td>9.53</td>
<td>15.44</td>
<td>13.31</td>
</tr>
</tbody>
</table>

Notice: The underlying VAR model is of order 1 and contains unrestricted intercepts, with \(p_t^*\) treated as exogenous I(1) variable. The asymptotic critical values are taken from Pesaran, Shin and Smith (2000).
Figure 1: Impulse Responses to Unit Price Shock

Response of IR to P_OIL

Response of Y to P_OIL

Response of INFLAT to P_OIL

Response of PPP to P_OIL

Response of EXR to P_OIL

Response of H_YSA to P_OIL

Response of IR_WORLD to P_OIL

Response of YA to P_OIL

Response to Cholesky One S.D. Innovations
Table 4: Reduced Form Error Correction Specification for the Core Model of Germany

(A) \( \alpha \)-Coefficients

<table>
<thead>
<tr>
<th>Equations</th>
<th>( \Delta p_t^c )</th>
<th>( \Delta r_t )</th>
<th>( \Delta y_t )</th>
<th>( \Delta (\Delta \hat{p}_t) )</th>
<th>( \Delta (p_t - p_t^c) )</th>
<th>( \Delta \epsilon_t )</th>
<th>( \Delta (h_t - y_t) )</th>
<th>( \Delta r_t^* )</th>
<th>( \Delta y_t^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \xi_{1,t} )</td>
<td>0.00 (0.00)</td>
<td>0.0006 (0.001)</td>
<td>0.011* (0.005)</td>
<td>0.029* (0.007)</td>
<td>0.005 (0.007)</td>
<td>0.059 (0.074)</td>
<td>0.043 (0.049)</td>
<td>-0.0003 (0.0006)</td>
<td>-0.005</td>
</tr>
<tr>
<td>( \xi_{2,t} )</td>
<td>0.00 (0.00)</td>
<td>-0.133* (0.034)</td>
<td>0.349* (0.191)</td>
<td>-0.385 (0.249)</td>
<td>-0.180 (0.233)</td>
<td>-2.167 (2.55)</td>
<td>2.11 (1.674)</td>
<td>0.010 (0.026)</td>
<td>-0.036</td>
</tr>
<tr>
<td>( \xi_{3,t} )</td>
<td>0.00 (0.00)</td>
<td>-0.014* (0.006)</td>
<td>-0.094* (0.028)</td>
<td>-0.140* (0.044)</td>
<td>-0.034 (0.041)</td>
<td>0.070 (0.449)</td>
<td>0.280 (0.295)</td>
<td>0.005 (0.005)</td>
<td>0.042</td>
</tr>
<tr>
<td>( \xi_{4,t} )</td>
<td>0.00 (0.00)</td>
<td>0.0005* (0.0001)</td>
<td>0.003* (0.0006)</td>
<td>0.001 (0.001)</td>
<td>0.002 (0.0009)</td>
<td>-0.004 (0.010)</td>
<td>-0.01 (0.006)</td>
<td>-0.0001 (0.0001)</td>
<td>-0.0002</td>
</tr>
<tr>
<td>( \xi_{5,t} )</td>
<td>0.00 (0.00)</td>
<td>-0.027 (0.031)</td>
<td>0.128 (0.144)</td>
<td>1.433* (0.223)</td>
<td>-0.026 (0.208)</td>
<td>0.736 (2.285)</td>
<td>0.99 (1.498)</td>
<td>-0.054* (0.023)</td>
<td>0.036</td>
</tr>
</tbody>
</table>

(B) \( \Gamma \)-Matrix

<table>
<thead>
<tr>
<th>( \Delta p_{t-1}^c )</th>
<th>( \Delta r_{t-1} )</th>
<th>( \Delta y_{t-1} )</th>
<th>( \Delta (\Delta \hat{p}_{t-1}) )</th>
<th>( \Delta (p_t - p_{t-1}) )</th>
<th>( \Delta \epsilon_t )</th>
<th>( \Delta (h_t - y_t) )</th>
<th>( \Delta r_t^* )</th>
<th>( \Delta y_t^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00 (0.00)</td>
<td>0.0006 (0.0008)</td>
<td>-0.009* (0.004)</td>
<td>-0.011 (0.006)</td>
<td>-0.015* (0.006)</td>
<td>0.030 (0.061)</td>
<td>-0.001 (0.040)</td>
<td>-0.0002 (0.0006)</td>
<td>-0.004</td>
</tr>
<tr>
<td>0.00 (0.00)</td>
<td>-0.106 (0.191)</td>
<td>-2.081* (0.892)</td>
<td>-2.535 (1.377)</td>
<td>-1.222 (1.288)</td>
<td>7.915 (14.1)</td>
<td>10.48 (9.257)</td>
<td>-0.108 (0.143)</td>
<td>0.124</td>
</tr>
<tr>
<td>0.00 (0.00)</td>
<td>0.022 (0.028)</td>
<td>-0.318* (0.131)</td>
<td>0.122 (0.205)</td>
<td>0.466* (0.189)</td>
<td>-1.285 (2.064)</td>
<td>-2.77* (1.355)</td>
<td>-0.006 (0.021)</td>
<td>0.061</td>
</tr>
<tr>
<td>0.00 (0.00)</td>
<td>-0.0009 (0.020)</td>
<td>0.096 (0.094)</td>
<td>0.039 (0.145)</td>
<td>-0.111 (0.135)</td>
<td>-1.055 (1.482)</td>
<td>0.329 (0.973)</td>
<td>-0.015 (0.015)</td>
<td>0.109</td>
</tr>
<tr>
<td>0.00 (0.00)</td>
<td>-0.002 (0.017)</td>
<td>-0.012 (0.081)</td>
<td>0.286* (0.126)</td>
<td>-0.081 (0.117)</td>
<td>-1.585 (1.286)</td>
<td>0.58 (0.844)</td>
<td>0.008 (0.013)</td>
<td>-0.043</td>
</tr>
<tr>
<td>0.00 (0.00)</td>
<td>0.001 (0.002)</td>
<td>-0.022* (0.010)</td>
<td>0.031* (0.015)</td>
<td>0.007 (0.014)</td>
<td>0.234 (0.157)</td>
<td>-0.09 (0.103)</td>
<td>0.001 (0.002)</td>
<td>-0.009</td>
</tr>
<tr>
<td>0.00 (0.00)</td>
<td>-0.004 (0.004)</td>
<td>0.005 (0.017)</td>
<td>0.018 (0.026)</td>
<td>-0.029 (0.024)</td>
<td>0.18 (0.265)</td>
<td>0.08 (0.174)</td>
<td>0.001 (0.003)</td>
<td>0.014</td>
</tr>
<tr>
<td>0.00 (0.00)</td>
<td>0.461* (0.202)</td>
<td>2.187* (0.942)</td>
<td>2.381 (1.454)</td>
<td>1.179 (1.36)</td>
<td>-28.35 (14.89)</td>
<td>-11.2 (9.772)</td>
<td>0.619* (0.151)</td>
<td>0.75</td>
</tr>
<tr>
<td>0.00 (0.00)</td>
<td>-0.012 (0.042)</td>
<td>0.133 (0.196)</td>
<td>0.496 (0.302)</td>
<td>-1.029* (0.283)</td>
<td>-1.676 (3.094)</td>
<td>-1.44 (2.03)</td>
<td>-0.015 (0.031)</td>
<td>0.075</td>
</tr>
</tbody>
</table>

(C) \( \psi \)-Coefficients and Constant

<table>
<thead>
<tr>
<th>( \Delta p_t^c )</th>
<th>( \Delta r_t )</th>
<th>( \Delta y_t )</th>
<th>( \Delta (\Delta \hat{p}_t) )</th>
<th>( \Delta (p_t - p_t^c) )</th>
<th>( \Delta \epsilon_t )</th>
<th>( \Delta (h_t - y_t) )</th>
<th>( \Delta r_t^* )</th>
<th>( \Delta y_t^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00* (0.00)</td>
<td>0.0008 (0.0007)</td>
<td>-0.003 (0.003)</td>
<td>0.009 (0.005)</td>
<td>0.021* (0.005)</td>
<td>0.008 (0.052)</td>
<td>0.00 (0.034)</td>
<td>0.001* (0.001)</td>
<td>0.005</td>
</tr>
<tr>
<td>0.00 (0.00)</td>
<td>-0.00203 (0.0003)</td>
<td>0.003 (0.001)</td>
<td>-0.003 (0.002)</td>
<td>0.00 (0.002)</td>
<td>0.00 (0.002)</td>
<td>0.03* (0.014)</td>
<td>0.00 (0.01)</td>
<td>0.005*</td>
</tr>
</tbody>
</table>

Notice: Standard errors are given in parenthesis; * indicates significance at the 10% level.