Conditional Poverty Gap

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Abstract

We propose an easy way of evaluating prospective poverty reduction policies. Additionally we assess the response of a poverty gap to the income and inequality effects of a given policy. The paper contains a series of convenient analytical results useful for the evaluation of a wide variety of policy measures.

Introduction

In this paper we suggest a simple way of analyzing poverty reduction potential of the prospective policy measures.

The vast literature existing on the issue may be roughly divided in the two main streams. The first common way of assessing poverty reduction potential of the policies is that of microsimulations (see Datt and Jolliffe, 2005, for one of the recent examples). The second approach to the estimation of policy impacts is the application of poverty/income regressions (see e.g. Ravallion and Wodon, 1999). Clearly, the first approach has much more modelling- and predictive power then the second one. Its disadvantage, however, is a relative computational complexity and unavoidable *ad hoc* assumptions. The second approach, even though being relatively simple to implement, is mainly the ex-post analysis. This puts a certain limit on its predictive power for an arbitrary policy measure.

This paper develops the analytical toolbox that aims at combining the predictive power of microsimulations with the simplicity of implementation of the income regression. This is achieved through explicit consideration of the distributional properties of a poverty measure and those of a prospective policy.

The paper considers the parametric counterpart of the poverty gap as a relevant poverty measure.[1] The policy, in its turn, is formalized as a stochastic process controllable by a policy maker. Under common distributional assumption for both income and policy variables we derive analytical results that facilitate convenient assessment of the response of the poverty gap to any change in the parameters of the distribution of the policy variable. One particular advantage of our approach is that analytical solutions for all the effects never require any estimation beyond that of a simple wage regression . This provides an attractive alternative to microsimulations. Furthermore, our approach uses the available information about the distributions of the variables of interest much more widely, then it is done in the poverty-income regressions. This is another attractive side of the present contribution.

The paper is structured as follows. In Section 1 we consider the parametric poverty gap and formalize the policy. Here we also derive the income and inequality effects of the policy. Section 2 discusses inference and presents a small numerical example.

1 Parametric Poverty Gap

Poverty gap

$$G = n^{-1} \sum_{i=1}^{n} \left[1 - \frac{y_i}{z}\right] \mathbf{1}\left(y_i \leq z\right) \tag{1}$$

is a popular measure of the total shortfall of individual income behind the poverty line z. However, like any distribution-free measure, (1) remains silent about the quantitative response of poverty level to exogenous changes in the economic environment. This

[1] An extension to the squared poverty gap is equally possible.

relative disadvantage can be overcome by taking a suitable distribution for the earnings data and making it conditional on a set of variables of choice. We consider a parametric counterpart of (1)

$$G = \int_0^z \left[1 - \frac{y}{z}\right] f(y)dy = F(z) - \frac{1}{z}\int_0^z y f(y)dy, \tag{2}$$

in which $F(y)$ is the appropriate distribution of individual income Y.

1.1 Poverty gap and policy variable

Once dealing with poverty reduction measures the decision maker is rather concerned about the left tail of the earnings distribution. Therefore it would be natural to assume that income Y follows the lognormal distribution, $Y \sim \Lambda(y|\mu_Y, \sigma_Y^2)$. Assuming lognormality, one can demonstrate that (2) becomes

$$G\left(\mu_Y, \sigma_Y\right) = \Phi\left(\frac{\ln z - \mu_Y}{\sigma_Y}\right) - \frac{e^{\mu_Y + \sigma_Y^2/2}}{z}\Phi\left(\frac{\ln z - (\mu_Y + \sigma_Y^2)}{\sigma_Y}\right), \tag{3}$$

where $\Phi(x|\mu, \sigma^2)$ is the cumulative density of a normal variate.

This straightforward switch to the parametric formulation of the poverty gap is at the same time very important as it enables us writing down the conditional version of (3). The conditional version of (3), consequently, will help fully describing the response of G to any given measure of poverty reduction.

Assume that for some nonstochastic $\mathbf{x}$ there exists the log-linear dependence

$$\ln y = \mathbf{x}'\boldsymbol{\beta} + \varepsilon, \quad \varepsilon \sim N(0, \sigma_Y^2) \tag{4}$$

so that $E(\ln y|\mathbf{x}) = \mathbf{x}'\boldsymbol{\beta}$ and the distribution of Y given $\mathbf{x}$ is lognormal, $Y \sim \Lambda(\mathbf{x}'\boldsymbol{\beta}, \sigma_Y^2)$. Furthermore, define $\mu_{\mathbf{x}} \equiv \mathbf{x}'\boldsymbol{\beta}$. This leads to the conditional parametric version of the poverty gap in (1), namely:

$$G\left(\mu_{\mathbf{x}}, \sigma_Y|\mathbf{x}\right) = \Phi\left(\frac{\ln z - \mu_{\mathbf{x}}}{\sigma_Y}\right) - \frac{e^{\mu_{\mathbf{x}} + \sigma_Y^2/2}}{z}\Phi\left(\frac{\ln z - (\mu_{\mathbf{x}} + \sigma_Y^2)}{\sigma_Y}\right). \tag{5}$$

Now assume that within the conditioning set $\mathbf{X}$ there appears a certain random variable X_k, such that its' distribution $F(X_k \leq x_k|\boldsymbol{\xi})$ can be directly influenced by the policy maker. Let us call X_k a *policy variable*. We suggest that policy maker knows partial elasticity of earnings β_k with respect to this variable and has a full control over $F(x_k|\boldsymbol{\xi})$

through setting its' parameters $\boldsymbol{\xi}$. Altering the distribution of the policy variable of course induces changes in the distribution of income. These in turn invoke changes in the poverty gap. Thus, knowing the parametric form of the distribution of the policy variable and having got the result in (5), one can explicitly compute the reaction of the poverty gap to the respective policy changes. Rewrite $\mu_{\mathbf{x}} \equiv \mathbf{x}'\boldsymbol{\beta}$ as

$$\mu_{\mathbf{x}} = \underbrace{\mathbf{x}'_{-k}\boldsymbol{\beta}_{-k}}_{\equiv\mu} + x_k\beta_k = \mu + x_k\beta_k. \tag{6}$$

Substitution of (6) back into (5) underlines dependence of the poverty gap on the outcome of a given policy:

$$G\left(\mu_{\mathbf{x}}, \sigma_Y|\mathbf{x}\right) = \Phi\left(\frac{\ln z - (\mu + x_k\beta_k)}{\sigma_Y}\right) - \frac{e^{\mu+x_k\beta_k+\sigma_Y^2/2}}{z}\Phi\left(\frac{\ln z - (\mu + x_k\beta_k + \sigma_Y^2)}{\sigma_Y}\right). \tag{7}$$

So the last step to be made is to assume a certain parametric form for the distribution of the policy variable and represent (7) as an explicit function of the parameters of this distribution. This will provide us with the complete description of the reaction of the poverty gap to any given policy change.

At this point there are two ways to proceed. First, one can find the marginal distribution of income by integrating policy variable out of the conditional income distribution and then construct the poverty gap. Alternatively, one can initially construct the conditional poverty measure and then integrate out the policy variable. It is easy to show that for arbitrary income and policy distributions both ways lead to the same result. So the choice of either depends only on the simplicity of derivation given the specific distribution of the policy variable. Below we consider two such distributions.

Normally-distributed policy variable This is the case when policy maker has full control over mean and variance of policy variable X_k, while $X_k \sim N(\mu_k, \sigma_k^2)$. Integrating x_k out, one obtains

$$\begin{aligned} G(\mu_k, \sigma_k) = \Phi&\left(\frac{\ln z - (\mu + \beta_k\mu_k)}{\sqrt{\beta_k^2\sigma_k^2 + \sigma_Y^2}}\right) \\ &- \frac{e^{(\mu+\beta_k\mu_k)+\frac{1}{2}\left(\beta_k^2\sigma_k^2+\sigma_Y^2\right)}}{z}\Phi\left(\frac{\ln z - \left[(\mu + \beta_k\mu_k) + \left(\beta_k^2\sigma_k^2 + \sigma_Y^2\right)\right]}{\sqrt{\beta_k^2\sigma_k^2 + \sigma_Y^2}}\right), \end{aligned} \tag{8}$$

where G becomes now an explicit function of μ_k and σ_k.

Several possibilities for policy interventions can be considered here. For example, policy maker hopes reducing poverty by increasing the level of education. Assuming that years of education is a normally distributed variable he can, for instance, promote the overall increase in education level, by razing the mean μ_k and keeping the variance σ_k^2 intact. Alternatively, policy maker may pursue a targeted increase in education level of the less educated by razing the mean μ_k and narrowing the variance σ_k^2. Equation (8) readily provides quantitative response of the poverty gap to both policies.

Bernoulli-distributed policy variable This is the case of a binary response policy or a relative size of the treatment group often represented by a dummy variable. Assuming that $X_k \sim B(p)$ and integrating x_k out of (7) leads us to the parametric poverty gap

$$G(p) = \sum_{j=0,1} p^j (1-p)^{1-j} \times \left[\Phi\left(\frac{\ln z - (\mu + j\beta_k)}{\sigma_Y} \right) - \frac{e^{\mu + j\beta_k + \sigma_Y^2/2}}{z} \Phi\left(\frac{\ln z - (\mu + j\beta_k + \sigma_Y^2)}{\sigma_Y} \right) \right], \quad (9)$$

which is now an explicit function of the parameter p, the latter being directly controlled by the policy maker.

As an example one can take the economy where the fraction of workers employed in the formal sector of the labour market is equal to p. Believing that promotion of formal employment will reduce poverty, policy maker may target some new level $p^* > p$. The difference $G(p^*) - G(p)$ will then quantify the change of the poverty gap in response to the expansion of the formal sector.

Finally notice that for both Bernoulli and Normal cases, setting β_k equal to zero, which would mean absence of the policy variable in the set of covariates, will make both $G(\mu_k, \sigma_k)$ and $G(p)$ reduce to (5) with μ instead of $\mu_{\mathbf{x}}$.

1.2 Income and inequality effects of policy

Apart from quantifying the overall response of poverty gap to any given policy, it is important to know how much of it is attributable to the change in the mean income and how much of it is due to the change in the income inequality.

Once the distribution of income Y becomes a function of the parameters $\boldsymbol{\xi}$ of the policy distribution, so do the expected value and Gini coefficient of this distribution.

Consequently, any response of the parametric poverty gap to the policy can be represented by

$$dG = \frac{\partial G}{\partial E(Y)} \left[\frac{\partial E(Y)}{\partial \boldsymbol{\xi}'} d\boldsymbol{\xi} \right] + \frac{\partial G}{\partial \mathcal{G}_Y} \left[\frac{\partial \mathcal{G}_Y}{\partial \boldsymbol{\xi}'} d\boldsymbol{\xi} \right], \tag{10}$$

where $E(Y)$ is the mean and $\mathcal{G}_Y$ is the Gini coefficient of the income distribution. Consider again the Normal and Bernoulli cases.

Normally-distributed policy variable For the normally-distributed policy variable, the marginal distribution of income Y is a function of the parameter pair (μ_k, σ_k). Furthermore, we can show that income distribution is again lognormal. Therefore its mean and Gini coefficient of inequality are

$$E(Y; \mu_k, \sigma_k) = e^{(\mu + \beta_k \mu_k) + \frac{1}{2}\left(\beta_k^2 \sigma_k^2 + \sigma_Y^2\right)}, \tag{11a}$$

$$\mathcal{G}(Y; \mu_k, \sigma_k) = 2\Phi\left(\sqrt{\frac{1}{2}\left(\beta_k^2 \sigma_k^2 + \sigma_Y^2\right)}\right) - 1 \tag{11b}$$

respectively, where (11b) can be obtained using the corollary to Theorem 2.2 in Aitchison and Brown (1963), p.11.

Together with (10), the results in (11a)-(11b) show that poverty changes induced by changes in the mean of the income distribution can be achieved not only through varying the mean of the policy variable μ_k but also through altering the variance σ_k^2 of this policy variable. The inequality-related change in the poverty gap, in contrast, is a function of the variance of the policy variable exclusively. With the results above we can finally show that

$$\begin{aligned} dG(\mu_k, \sigma_k) = {} & \\ & \beta_k E(Y; \mu_k, \sigma_k) \left[\frac{\partial G(\mu_k, \sigma_k)}{\partial E(Y; \mu_k, \sigma_k)} \right] d\mu_k + \beta_k^2 \sigma_k E(Y; \mu_k, \sigma_k) \left[\frac{\partial G(\mu_k, \sigma_k)}{\partial E(Y; \mu_k, \sigma_k)} \right] d\sigma_k \\ & + \sqrt{2}\, \varphi\left(\sqrt{\frac{1}{2}\left(\beta_k^2 \sigma_k^2 + \sigma_Y^2\right)}\right) \frac{\beta_k^2 \sigma_k}{\sqrt{\beta_k^2 \sigma_k^2 + \sigma_Y^2}} \left[\frac{\partial G(\mu_k, \sigma_k)}{\partial \mathcal{G}(Y; \mu_k, \sigma_k)} \right] d\sigma_k, \end{aligned} \tag{12}$$

which provides the complete decomposition of the poverty gap. The first component in equation (12) represents the income-induced poverty change when income shifts are caused by the change of the mean of the policy variable. The second component is the income-induced poverty change caused by the alterations of the variance parameter of the policy. Finally the last component is the response of the poverty gap to the change in the income inequality induced by the variance shifts of the policy variable.

The expressions for $\partial G(\mu_k, \sigma_k)/\partial E(Y)$ and $\partial G(\mu_k, \sigma_k)/\partial \mathcal{G}_Y$ are obtained using equations (18), p.125, and (25), p.127, in Kakwani (1993) and are shown in the Appendix.

Bernoulli-distributed policy variable As above, we would like to learn how much of the change in the poverty gap is attributable to the policy-induced shift of the mean income and how much of this change can be attributed to the policy-induced dynamics of the income inequality. However the control variable is now Bernoulli distributed. It is possible to show that the expected value and the Gini coefficient for the marginal distribution of income become

$$\begin{aligned} E(Y;p) &= e^{\mu+\frac{1}{2}\sigma_Y^2}\left[e^{\beta_k}p + (1-p)\right], &\text{(13a)}\\ \mathcal{G}(Y;p) &= \frac{2}{e^{\beta_k}p+(1-p)}\left[\Phi\left(\frac{\sigma_Y}{\sqrt{2}}\right)\left[e^{\beta_k}p^2 + (1-p)^2\right] + p(1-p)\right.\\ &\quad \left.\times\left(e^{\beta_k}\Phi\left(\frac{1}{\sqrt{2}}\left[\sigma_Y + \frac{\beta_k}{\sigma_Y}\right]\right) + \Phi\left(\frac{1}{\sqrt{2}}\left[\sigma_Y - \frac{\beta_k}{\sigma_Y}\right]\right)\right)\right] - 1, &\text{(13b)} \end{aligned}$$

where (13b) is again obtained using the corollary to Theorem 2.2 in Aitchison and Brown (1963), p.11. Knowing the exact dependence of the expected value and the Gini coefficient on the policy parameter p we can decompose the poverty gap into

$$dG(p) = \frac{\partial G(p)}{\partial E(Y;p)}\left[e^{\mu+\frac{1}{2}\sigma^2}\left(e^{\beta_k}-1\right)\right]dp + \frac{\partial G(p)}{\partial \mathcal{G}(Y;p)}\left[\frac{\partial \mathcal{G}_Y}{\partial p}\right]dp. \tag{14}$$

The first component in this decomposition is the change of the poverty gap caused by the policy-induced shift in the mean income. The second component is the response of the poverty gap to policy-induced change in the income inequality. Expressions for $\partial G(p)/\partial E(Y)$, $\partial G(p)/\partial \mathcal{G}_Y$ and $\partial \mathcal{G}_Y/\partial p$ can be found in Appendix.[2]

In conclusion we also notice that for both Bernoulli and Normal cases, setting β_k equal to zero, which means the absence of the policy variable, will make both $E(Y;\boldsymbol{\theta})$ and $\mathcal{G}(Y;\boldsymbol{\theta})$ reduce to those of a lognormal distribution with conditional mean μ instead of $\mu_{\mathbf{x}}$.

[2] Again, the first two of them are computed using the results of Kakwani (1993).

2 Empirical Example

2.1 Inference

A particularly attractive feature of our analysis is the existence of analytical solutions for all the adjustments of the parametric poverty gap. Therefore, to conduct the inference one will never need anything beyond the consistent estimates of the linear regression (4). In the vast majority of cases these are just OLS estimates. Given the consistent estimates $\hat{\boldsymbol{\beta}}$ and $\hat{\sigma}_Y$ the consistent estimates of the poverty gap are

$$\hat{G}(\mu_k, \sigma_k) = n^{-1} \sum_{i=1}^{n} G(\mu_k, \sigma_k \,|\, \hat{\boldsymbol{\beta}}, \hat{\sigma}_Y, \mu_i)$$

for the Normal, and

$$\hat{G}(p) = n^{-1} \sum_{i=1}^{n} G(p \,|\, \hat{\boldsymbol{\beta}}, \hat{\sigma}_Y, \mu_i)$$

for the Bernoulli cases respectively. The same applies to decompositions in (12) and (14). The variance of all estimates can be either computed using delta method or bootstrapped.[3]

When performing the decompositions it is also reasonable to divide both parts of the equation by dG to get the interpretation in "shares" of each policy measure in the total change of the poverty statistic.

2.2 Application

Here is a small numerical illustration of the poverty reduction potential of different policy measures.

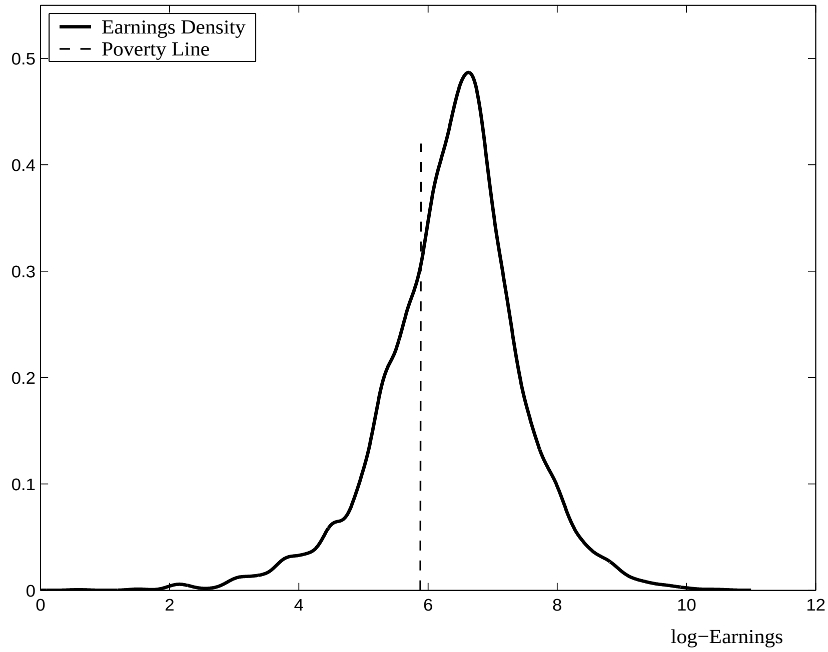
The data we use here are the Bolivian data, which is a cross-section of employed individuals with monthly individual earnings and other common characteristics.

For our illustration we set the poverty line equal to 360 Bolivianos per month, which corresponds to 1.5 USD per day. Figure 1 shows the distribution of log-earnings with vertical line being a poverty line. Summary statistics for selected variables is given in Table 1. According to the data, 27.7% of individuals are below the poverty line.

First we would like to see up to wich extent our parametric assumptions are consistent with the data. In Table 2 we report the estimates of the conventional nonparametric poverty gap (1) and the conditional parametric poverty gap (5).

[3] Bootstrap is the easier and therefore more advisable option.

Figure 1: Density of Monthly Wages



It turns out that the nonparametric poverty gap amounts to 0.1214, whereas the parametric one makes 0.1369. These numbers show that the discrepancy between the two measures is very small, so we can provide a reliable analysis using the log-normal form for the $F(y)$.

Table 1: Descriptive Statistics for Selected Variables

Variable	**Mean**	**Std.Deviation**	**Min**	**Max**
log-Earnings	6.3852	1.0532	0.5878	10.5214
Years of Education	9.1420	4.8558	0	17
Formal Market	0.3720	0.4834	0	1

Table 2: Poverty Measures

Poverty measure	Value
Headcount ratio	0.2770
Poverty gap	
– Nonparametric poverty gap	0.1214
– Parametric: G [Eq. (5)]	0.1369
– Parametric: $G(\bar{x}_k, s_k)$ [Eq. (8)]	0.1315

Furthermore, to learn about accuracy of the evaluation of policy experiments, in Table 2 we provide the parametric poverty gap (8) evaluated at the sample mean and standard deviation of a potential policy variable. Choosing years of education to be such variable we see that the difference between the conditional gap G and the measure $G(\mu_k, \sigma_k)$ evaluated mean and standard deviation of years of education is negligible. This completely justifies the normality assumption for the chosen policy variable and, most importantly, points at the high precision of the predictions.

Below we consider several sample poverty reduction experiments.

Investment in Education Using the positive significant dependence between monthly earnings and years of education (Table A.1) a policy maker might attempt reducing poverty via investing in human capital.

Assume one aims at 20% reduction of poverty gap [i.e. from current 0.1369 to 0.1046]. Once policy variable is normally distributed, three prospective measures can be taken right away. First, one can simply induce an overall increase in level of education in the population without changing the variance of years of education. Next, one can think of a targeted policy that will provide more education to less educated. Following this second path, policy maker will need to both increase the mean duration of education and reduce its variance. Increase in mean education will unambiguously shift the mean of the income distribution up. Contraction of the variance, however, will have both income reducing and inequality reducing effect, which is a consequence of lognormality (see equations 11a-11b). So it is not known which of these effects will dominate. Finally, one can aim at increasing the fraction of high-educated individuals in the society and not only raise the mean length of education but also increase its variance. This will correspond to widening the education gap, which will create more income inequality, though may boost the mean of the income distribution much higher than in the two previous cases if the income effect dominates.

Of course there is also a fourth option of pure variance change with no alteration of the mean length of education. This policy however is from the very beginning less effective, as pure variance shifts in any direction will always have the offsetting mean income and inequality effects on the poverty gap.

To achieve a 20% reduction in poverty gap through increasing years of education for everybody (i.e. variance of the education distribution remains unchanged) one needs to raise the level of education by three years. This is the Policy I shown in Table 3. By cross-validation we can find that in order to attain the same poverty reduction via educating low-skilled one has to increase the mean education by only 2.5 years, though simultaneously drop the standard deviation of the policy distribution by 2.5 years. This is Policy II. Finally the third policy will push up years of education by 3.5 and raise its standard deviation by 1.62. Approximate percentages of poverty gap changes reported in Table 3 are calculated using (12).

From Table 3 we see that among the three policies that cause identical change in poverty gap Policy III turns out to be the worst one because of the too strong negative inequality effect of the variance increase of the policy variable. Namely, the net variance effect will make almost -60% of the change in the gap, so an extremely big investment in overall increase of education should be made. Policy II, in contrast, is much more attractive, because educating low-educated has positive 20% net effect on poverty reduction due to substantially falling income inequality. Consequently, only moderate investment into overall increase of education (namely, increase by 2.5 years only) is necessary. The same applies to Policy I.

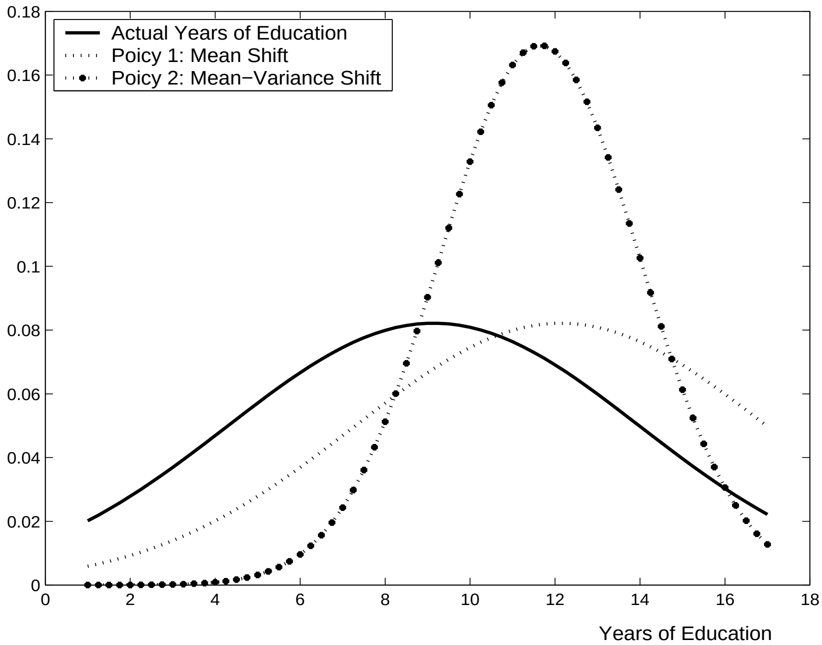
Further choice of either Policy II or Policy I depends both on costs of implementation and political economy considerations: For instance, simultaneous poverty and inequality reduction may find more approval than just plain poverty reduction.

Policy measures I and II are summarized in Figure 2. Solid line is the normal density curve fitted to the actual data.

Table 3: Alternative Poverty-Reduction Measures

	Policy Measure	% income effect of $d\mu_k$	% income effect of $d\sigma_k$	% inequality effect of $d\sigma_k$
I:	Overall increase of years of education: $G(\mu_k + 3, \sigma_k)$	100.0	0.0	0.0
II:	Targeted increase of education of low-educated: $G(\mu_k + 2.5, \sigma_k - 2.5)$	78.3	-9.9	31.6
III:	Targeted increase of education of of high-educated: $G(\mu_k + 3.5, \sigma_k + 1.62)$	159.0	25.5	-84.5

Figure 2: Investment in Public Education



Conclusions

We suggest a simple yet powerful toolkit that allows computing the reaction of poverty gap to exogenous changes in policy variables. Assuming that policy maker knows the costs of any prospective policy intervention, our analysis will provide him with the most effective poverty reduction strategy.

References

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Appendix

Partial elasticities of the parametric poverty gap

- *Normally-distributed policy variable*

$$\frac{\partial G(\mu_k,\sigma_k)}{\partial E\left(Y\right)} = -\frac{1}{z}\,\Phi\left(\frac{\ln z-\left[(\mu+\beta_k\mu_k)+\left(\beta_k^2\sigma_k^2+\sigma_Y^2\right)\right]}{\sqrt{\beta_k^2\sigma_k^2+\sigma_Y^2}}\right),$$

$$\frac{\partial G(\mu_k,\sigma_k)}{\partial \mathcal{G}_Y} = \frac{1}{\mathcal{G}_Y}\left(G(\mu_k,\sigma_k)+\Phi\left(\frac{\ln z-(\mu+\beta_k\mu_k)}{\sqrt{\beta_k^2\sigma_k^2+\sigma_Y^2}}\right)\left[\frac{e^{(\mu+\beta_k\mu_k)+\frac{1}{2}\left(\beta_k^2\sigma_k^2+\sigma_Y^2\right)}}{z}-1\right]\right).$$

- *Bernoulli-distributed policy variable*

$$\frac{\partial G(p)}{\partial E\left(Y\right)} = -\frac{1}{z\left[e^{\beta_k}p+(1-p)\right]}\left(\sum_{j=0,1}\left[p^j(1-p)^{1-j}\,e^{j\beta_k}\Phi\left(\frac{\ln z-(\mu+j\beta_k+\sigma_Y^2)}{\sigma_Y}\right)\right]\right),$$

$$\frac{\partial G(p)}{\partial \mathcal{G}_Y} = \frac{1}{\mathcal{G}_Y}\left[G(p)+\left(\frac{e^{\mu+\frac{1}{2}\sigma_Y^2}\left[e^{\beta_k}p+(1-p)\right]}{z}-1\right)\right.$$
$$\left.\times\sum_{j=0,1}p^j(1-p)^{1-j}\Phi\left(\frac{\ln z-(\mu+j\beta_k)}{\sigma_Y}\right)\right].$$

Finally, marginal change of inequality due to policy ($\partial\mathcal{G}_Y/\partial p$) in equation (14) is

$$\frac{\partial \mathcal{G}_Y}{\partial p} = -\frac{e^{\beta_k}-1}{e^{\beta_k}p+(1-p)}\left(\mathcal{G}_Y+1\right)+\frac{2}{e^{\beta_k}p+(1-p)}\left[2\Phi\left(\frac{\sigma_Y}{\sqrt{2}}\right)\left[e^{\beta_k}p-(1-p)\right]\right.$$
$$\left.+(1-2p)\left(e^{\beta_k}\Phi\left(\frac{1}{\sqrt{2}}\left[\sigma_Y+\frac{\beta_k}{\sigma_Y}\right]\right)+\Phi\left(\frac{1}{\sqrt{2}}\left[\sigma_Y-\frac{\beta_k}{\sigma_Y}\right]\right)\right)\right].$$

Estimation results

Table A.1: Estimation Results for Equation (4)

Variable	**Coeff.**	**(SE)** *	**z-Statistic**	**p-Value**
Intercept	3.2348	0.1259	25.6928	0.0000
Sex	0.3465	0.0268	12.9097	0.0000
Age	0.0868	0.0069	12.9098	0.0000
$Age^2/100$	−0.0971	0.0093	−10.4403	0.0000
Education	0.0534	0.0034	15.5916	0.0000
Hours	0.0101	0.0006	16.1705	0.0000
Spanish	0.1368	0.0335	4.0773	0.0000
Formal	0.3911	0.0330	11.8640	0.0000
Public	0.1295	0.0391	3.3132	0.0009
La Paz	0.0815	0.0259	3.1532	0.0016
Indigenous	−0.0977	0.0280	−3.4865	0.0005
		Obs.:		4696
		SE of Regression:		0.8773
		Adjusted R^2:		0.3061

* *White HCSE.*