Measuring Changes in Multidimensional Inequality - An Empirical Application

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Abstract

During the past decade there has been a growing opinion of including more than an income perspective in the examination of inequality. As a result a broad theoretical literature on the subject of multidimensional inequality is present. This can mainly be divided into three different parts: item-by-item, non-aggregative and aggregative approach. However, there is hitherto no agreement over the measurement of inequality when each individual or household is characterized by a variety of attributes of wellbeing. In addition, there are less empirical examinations applying a multidimensional perspective to inequality. We apply three existing techniques, one from each of the mentioned strands of the theoretical literature, to the particular question of whether multidimensional inequality increased or decreased in Zambia between 1998 and 2004 using household indicators on expenditures, educational level, health status and land holdings. The purpose is to assess strengths and weaknesses of these theoretical developments in an empirical context and accordingly review their usefulness for measurement and policy analysis. Our examination points to that inequality comparisons taking interrelations between attributes into account are not always at odds with independent comparisons of different distributions. Consequently, if employing the item-by-item approach, at minimum, one should check the correlations between welfare distributions. The assessment of the aggregative approach show evidence of that different dimensions of wellbeing compensate and reinforce each other with respect to inequality in an empirical context. However, a majority of the results are very sensitive to the degree of substitution between attributes chosen. Sensitivity analyses and explicitness should thus accompany examinations of this kind. In applying a non-aggregative approach few combinations fulfill the required dominance conditions. Accordingly, generality and less imposed structure come at a cost. We conclude that the empirical usefulness of these existing techniques is reasonable as long as we stay aware of intrinsic weaknesses. Clearly, careful interpretations and analyzes involving more than one technique is constructive to portray multidimensional inequality.

Keywords: Multidimensional inequality, inequality indices, stochastic dominance, expenditures, education, health, land holdings, Zambia

JEL codes: D31, D63, I19, I29, Q15

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1. Introduction

During the past decades there has been a growing opinion in favor of including other dimensions than a monetary perspective in analyzing inequality. These concerns have received an increasing notice among economists and a broad theoretical literature on the subject of multidimensional inequality is present (c.f. Sen, 1973, Kolm, 1977, Atkinson and Bourguignon, 1982, 1987, Maasoumi, 1986 and Tsui, 1995, 1999). Generally the literature can be divided into three different parts. A first element of the literature applies an item-by-item approach, where comparisons over time or space are made independently for each dimension of interest. A second part applies an aggregative approach, where conclusions on inequality are established by the magnitude of different indexes based on an aggregation of multiple indicators. The third method concerns a non-aggregative approach. This part of the literature focuses on orderings rather than levels and use stochastic dominance techniques for analyzing changes in inequality. The lack of consensus about the measurement of inequality when an individual or a household is characterized by a variety of attributes of wellbeing remains.

Though a broad theoretical literature on multidimensional inequality comparisons is present, less attention has been paid to empirical research on the subject (Atkinson, 2000). Although it is common to assert that inequality is a multidimensional phenomenon, most empirical work on inequality applies a unidimensional monetary perspective. There is often an assumption in the economic literature that income inequality is closely related to other forms of inequality and thus can be used as a single proxy for the level of and changes in overall inequality (Maasoumi, 1999). When examinations use more than one indicator of wellbeing, analyses of inequality are generally made by applying an item-by-item approach.

Taking as a starting point that multidimensional inequality comparisons are ethically and theoretically attractive and that there hitherto is no pronounced agreement on how to assess inequality of several dimensions, there is an evident rationale to empirically examine existing methods on how to measure changes in multidimensional inequality. This is further justified by the limited number of empirical applications that are present. We apply three existing techniques, one from each of the above mentioned strands of the theoretical literature, to the particular question of whether multidimensional inequality increased or decreased in Zambia between 1998 and 2004. The purpose is to assess strengths and weaknesses of these theoretical developments in an empirical context and accordingly review their usefulness for measurement and policy analysis. A systematic use of data allow us to examine whether certain methods give specific indications to how multidimensional inequality changed over the time period and what implications imposed structures have to these conclusions.

This analysis takes four dimensions of economic inequality into consideration; consumption, education, health and land. These are four out of several dimensions of interest when studying inequality and household welfare, all of them well known from the poverty literature. As there are reasons to believe that economic conditions in monetary terms drive other aspects of living standards there are arguments for including a consumption variable in the study of inequality. On the other hand, non-monetary attributes as education and health may capture dimensions of a household's welfare that are non-tradable and thus not well proxied by consumption. In a development context it also seems relevant to include an asset perspective as land, not the least as land might contribute to household food security.

When examining inequality in the different attributes separately, we apply inequality indices customarily used in the univariate study of inequality. Concerning the second method we make use of a multidimensional index developed by Maasoumi (1986). In implementing the third technique, we employ sequential stochastic dominance conditions derived by Muller and Trannoy (2003) and Trannoy (2005) that follows the work of Atkinson and Bourguignon (1982, 1987).

The contribution of this paper is threefold. Firstly, the paper complements the growing theoretical literature on multidimensional inequality with an empirical application. Secondly, we apply a multidimensional perspective on inequality using household level data. Existing empirical examinations on multidimensional inequality are mainly introduced in a cross-country or regional based setting with aggregated data (Hirschberg et. al 1991, Lugo, 2004 and Quadrado et. al, 2001).[1] Thirdly, the paper provides indications on the development of inequality over the past six years in Zambia, not only with a monetary perspective but also including other dimensions of welfare.

Our examination points to that inequality comparisons taking interrelations between attributes into account are not always at odds with independent comparisons of different distributions. Consequently, if employing the item-by-item approach, at minimum, one should check the correlations between welfare distributions. The assessment of the aggregative approach show evidence of that different dimensions of wellbeing compensate and reinforce each other with respect to inequality in an empirical context. However, a majority of the results are very sensitive to the degree of substitution between attributes chosen. Sensitivity analyses and explicitness should thus accompany examinations of this kind. In applying a non-aggregative approach few combinations fulfill the required dominance conditions. Accordingly, generality and less imposed structure come at a cost. We conclude that the empirical usefulness of these existing techniques is reasonable as long as we stay aware of intrinsic weaknesses. Clearly, careful interpretations and analyzes involving more than one technique is constructive to portray multidimensional inequality.

This paper takes off with a brief discussion on concerns when examining inequality and the different dimensions of welfare here studied. Secondly we present the theoretical frameworks for comparisons of inequality in more than one dimension. With this foundation an empirical analysis on changes in multidimensional inequality using Zambian household data is performed, where the results from the different applications are discussed and contrasted. The paper ends with some concluding remarks.

[1] Justino (2004) performs an empirical application of different multidimensional inequality analysis methods on household data from Brazil and Vietnam. To our knowledge there are no examinations on multidimensional inequality on household level data in a sub-Saharan context. Although being a thorough review on different techniques, Justino (2004) does not include the test of sequential dominance presented in Trannoy (2005).

2. Inequality of what and between whom?

There are several dimensions of interest when studying inequality and household welfare, all of them well known from the literature on poverty.[2] These include income or consumption, education, health & nutrition, security, power, social inclusion and assets. The identification of relevant dimensions of welfare might involve numerous difficulties. First, it might be complicated to agree on what attributes that are of importance. Secondly, after agreement of what dimensions to include in an analysis, it is not always clear what the concepts or ideals of these attributes mean. Roemer (1996) claims that differences in welfare are acceptable when they are due to characteristics for which individuals can be deemed responsible. Likewise, Dworkin (1981) argues that society should not aim to equalize differences resulting from dissimilarities in tastes or preferences. Consequently, consensus over the choice of appropriate variables may not always be possible.

The first dimension examined in this paper, is consumption, which is the traditional variable used when studying magnitude and changes in economic inequality. The argument of using a monetary attribute in inequality analysis is clearly that there are reasons to believe that economic conditions drive other aspects of living standards and that a monetary indicator therefore does tell us what we need to know about wider aspects of wellbeing. On the other hand, although the permanent income hypothesis suggests that current consumption is related to lifetime welfare, capital market imperfections and other problems that are common in developing countries imply that it is of importance to include other indicators of welfare. Moreover, a monetary metric to measure inequality is satisfactory if it is able to catch relevant heterogeneity between households or individuals and their different situations (Ruggeri et. al 2003), but monetary inequality is ambiguous when households have different characteristics and needs (Maasoumi, 1999). We should also call to mind that a change in consumption inequality might be the result of bad outcomes in other dimensions of welfare, e.g. we might register a fall in monetary inequality due to an increase in HIV/AIDS related deaths among the poor or among the rich in a society. In addition, the wellbeing of a household might have dimensions that cannot be purchased (Duclos, Sahn and Younger, 2001).

In developing countries cultivable land is one of the most important physical assets for the majority of households (Appleton, 1996). Land might contribute to household welfare in that the growing of crops contributes to the household food security. In addition, land might increase a household's welfare by enable the household to receive credits since landholdings owned by a household might be used as collateral. Land holdings is a second variable here examined.

[2] There are a number of reasons why it is of relevance to study inequality per se. Firstly, inequality matters in its own right. There is a quite widely accepted ethical basis for being concerned that there is a reasonable degree of equality in a society (Sen, 1973, Rawls, 1971). In addition, as confirmed in an experimental context, there seems to exist such social preferences for equality (inequality aversion) that individuals are willing to take a personal cost for a final distribution of material outcome to get more equal. In other words, the utility of an individual includes his or her own earnings and a weight on other individuals' earnings (Camerer , 2001, Fehr and Schmidt, 1999). Further, inequality matters for growth as increasing empirical evidence point to that countries with high inequality achieve lower economic growth rates on average (Birdsall and Londoño, 1997, Deininger and Squire, 1998).Polarized income distributions within countries, rather than socio-economic resources per se, might also have a negative impact on well-being. Inequality in one dimension consequently may be welfare detrimental (Wilkinson, 1996).

An additional attribute of importance to household welfare might be education. In the literature it is often argued that access to a literate person in a household entails important externality effects and the educational level of the head of the household is likely to have an important welfare impact as other members of a household may benefit from having an educated person in the household (Basau and Foster, 1998). Furthermore, the educational status of a household is not necessarily constrained on the monetary level therein as public schooling might be offered for free and in a development context there has been an increased focus and assistance from donors during the past decades on targeting the poor in education as well as the health of the more disadvantaged (World Bank, 2006). This brings us to the last dimension of household welfare included in this inequality analysis, health status. The condition of health of household members, particularly members at work, might be of relevance to labor productivity and as a substitute for and complement to physical capital. Concerning distribution of health, large inequalities in health outcome imply a large loss in aggregate welfare since an individual's health status forms a critical component of his or her human capital (Makinen et al. 2000).

In this examination of multidimensional inequality, all welfare attributes correspond to outcomes in contrast to opportunities, i.e. what is measured is inequality of where households end up rather than where and how they begin. Much of the discussion on inequality in empirical work focuses on outcomes which are more easily observed and often can be measured with greater precision, e.g. income or consumption (McKay, 2002). On the other hand a substantial component of inequality reflects inequality in opportunities, with households being favored or disfavored according to if they e.g. live in urban or in rural areas. The question of inequality in outcomes is often said to deny the importance of individual choice and that it therefore is not appropriate to regard any inequality of outcome as evidence on injustice rather than the result of individual preferences. However, when the purpose is to measure inequality in several dimensions, it is not unreasonable to examine outcome variables, as heterogeneity between different households to some extent decreases with the number of dimensions included. Moreover, it is not always easy to distinguish inequality of opportunity from inequality in outcome in practice, since the former might be dependent on the latter.

Finally, as in much empirical work on unidimensional inequality, we examine multidimensional inequality between households. This perspective fails to take account of intra-household inequality which might clearly be an important issue. However, to allow for intra-household inequality we would need to consider attributes measured at the individual level, which in our case is not available.

3. Concepts and Theoretical Framework

3.1 An Item-by-item Approach

In applying an item-by-item approach to examining multidimensional inequality, each attribute of concern is regarded separately and no structure on the relations between different dimensions is introduced. Comparisons in inequality between units, groups and over time are performed by using measures frequently applied in unidimensional inequality analysis e.g. the Gini coefficient, General Entropy measures, the Atkinson index, or comparisons based on orderings e.g. first or higher order stochastic dominance conditions using one variable at the time (Lugo, 2004, Justino, 2004).[3]

It is of importance to note that the item-by-item approach to some extent disregard multidimensionality since, by putting no structure on the relations between different dimensions studied, nothing is said about the degree of substitution of attributes to households or about the relative weight society puts on different attributes of welfare. As a result, by independent one-at-a-time analysis of welfare attributes, it is not possible to conclude whether there is joint incidence of inequality, i.e. any complementarities, along different dimensions of interest (Brandolini and D'Alessio, 2001). An attempt to capture what interdependencies that exists between different distributions of welfare when applying an item-by-item approach is generally to perform a cross-correlation analysis.

Although multidimensionality to some extent is overlooked there is a rationale to include the above mentioned techniques in an examination as indications on changes and magnitude of inequality of individual attributes can act as a frame of reference. In order to get a first picture of what has happened to inequality of different dimensions over time we will calculate the values of four different inequality indices, the Gini coefficient and three indices belonging to the Generalized Entropy class.

The Gini coefficient is a statistical measure of inequality that takes values between zero (0) and one (1), where 0 implies complete equality and 1 complete inequality. If there are n households in the population, μ is the mean household value of the attribute x studied and x_i and x_j denote the allocation of this attribute in household i and j respectively, the Gini coefficient (G) can be written as follows

$$G = \frac{1}{2n^2\mu}\sum_{i=1}^{n}\sum_{j=1}^{n}\left|x_i - x_j\right|$$

Consequently, we here measure an average of pair wise differences between the individual observations in a population, weighted by the overall population mean (Cowell, 1995). Blackorby and Donaldson (1978) derive the particular form that a social welfare function consistent with the Gini coefficient must take. This unidimensonal inequality measure satisfies the Pigou-Dalton principle which demands that a transfer from a poorer household to a richer leads to an increase in inequality and is most sensitive to differences in allocations about the middle of a distribution.

[3] For a thorough discussion of univariate inequality measurement techniques see e.g. Cowell (1995, 2000).

Measures belonging to the Generalized Entropy class of inequality indices have their foundation in information theory and follow an axiomatic approach to inequality measurement. These indices allow us to examine the stability of welfare rankings for different weightings by selecting different choices of a parameter α. By following the above notation members of the GE class are derived by

$$GE(\alpha) = \frac{1}{\alpha^2 - \alpha}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\mu}\right)^{\alpha} - 1\right] \qquad \alpha \neq 0,1$$

For the special cases when $\alpha \to 0$ and 1 the above general form becomes

$$GE(0) = \frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{\mu}{x_i}\right) \text{ and}$$

$$GE(1) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\mu}\right)\log\left(\frac{x_i}{\mu}\right) \text{ respectively.}$$

GE(0) corresponds to the mean-log deviation and is particularly sensitive to low values in the distribution.[4] GE(1) corresponds to Theil's inequality index which allots equal weight to all observations in the distribution, and the GE(2) place greater weight to differences in the upper tail of a distribution (Myles, 2002).

The above inequality measures range from zero (0) to infinity (∞) and higher values indicate higher levels of inequality (Cowell, 1995). Although the indices of the Generalized Entropy class are not based on a welfare theoretic approach, we note that these tools for distributional analysis are ordinally equivalent to the Atkinson measure, in turn directly derived from a social welfare function, when $\alpha = 1 - \varepsilon$. The parameter ε defines inequality aversion (Cowell, 2000). As the Gini coefficient, measures belonging to the Generalized Entropy class follow the Pigou-Dalton principle. In addition, both the Gini and the GE measures are symmetric and obey the axioms of continuity and invariance to scalar multiplication. However, it is only the GE family that fulfills the requirement of additive decomposability.

[4] GE(0) is also referred to as Theil's lower index

3.2 *An Aggregative Approach*

A second approach to analyzing inequality with respect to different dimensions includes a more explicit multidimensional framework. In this case direct aggregated composite indices of multidimensional inequality, which synthesize information on distributions of interest into a single real-valued measure, are derived. The principal critique regarding multivariate indices concerns that an aggregation procedure leads to loss of information, and that it is difficult to develop consensus axioms (Duclos et al, 2001). In addition, all aggregated indices have in common that it is impossible to reach any conclusion regarding the value of a particular measure if we do not take a stand in the aggregation phase regarding (1) weighting structure, (2) degree of substitution between attributes and (3) degree of inequality aversion inequality (Bourguignon, 1999). On the other hand, we here get a complete ordering of distributions since a scalar measure is received, which clearly is practical in empirical examinations.

In the literature there are indices developed by means of an axiomatic approach (Tsui 1995, 1999) as well as ones derived ad-hoc (Maasoumi, 1999, Bourguignon, 1999).[5] In the axiomatic approach, a decision is made on desirable axioms whereupon a multidimensional inequality measure that fulfills the chosen axioms is derived. Composite indices created using the second method are in some cases constructed in one-stage, but several indices of multidimensional inequality include a two-stage procedure where a welfare function first is used to aggregate attributes for each individual or every household, upon which welfare is summed across all units (Justino, 2004). Two multidimensional inequality indices presented in this literature is the Maasoumi index (1986) and a measure derived by Bourguignon (1999).

In contrast to the proposed axiomatic indices, both measures allow explicit exploration of included parameters, as the degree of substitution and the degree of inequality aversion, of importance when evaluating empirical applications. While the Bourguignon measure provides a more direct link with stochastic dominance criteria (c.f Atkinson and Bourguignon, 1982), the basis of the Maasoumi index is found in the theory of information as the general entropy family measures are used in both stages. To examine changes in multidimensional inequality in applying an aggregative approach, we make use of the Maasoumi index. This was one of the first indices proposed in the literature and is the measure that has been utilized the most in an empirical context. It has the advantage that it aggregates across attributes in the first stage and not across individuals (Lugo, 2005). In addition, the measure allows us to in a straightforward manner compare results from the examination of inequality by techniques of the item-by-item approach.

Consider *j=1,2,….m* dimensions of wellbeing represented by attributes and *i=1,2,…n* statistical units representing individuals, households etc. For each statistical unit there is a non-negative value X_{ij} for every m dimension and thus a welfare matrix $X = X_{ij}$. The first step in calculating the Maasoumi index of multivariate inequality is to aggregate the chosen welfare attributes into a summary wellbeing function for the i-th statistical unit, $S_i = S(X_{i1}, X_{i2},..., X_{im})$. Assume further the existence of a scalar function (e.g. a social welfare function) of the matrix X. Maasoumi (1986) here defines a multivariate

[5] For a review of the aggregative approach and an empirical application to multidimensional inequality between provinces in Argentina, see Lugo (2004).

generalization of the GE measures of divergence or closeness between the m densities corresponding to

$$D_\beta(S,X,w)=\sum_j^m w_j\left\{\sum_{i=1}^n S_i\left[\left(\frac{S_i}{X_{ij}}\right)^\beta-1\right]/\beta(\beta+1)\right\}\qquad \beta\neq 0,-1$$

When $\beta\to 0$ or -1, one obtains the following indicators

$$D_0(S,X,w)=\sum_j^m w_j\left[\sum_{i=1}^n S_i\log\left(\frac{S_i}{X_{ij}}\right)\right]$$
$$D_{-1}(S,X,w)=\sum_j^m w_j\left[\sum_{i=1}^n S_i\log\left(\frac{X_{ij}}{S_i}\right)\right]$$

where w_j are the weights allotted each welfare attribute and where the parameter β is the coefficient of substitution between the different welfare dimensions. This coefficient guarantees that changes in inequality not only take place due to changes in rankings, but also to changes in the dependence between various welfare attributes (see also Atkinson and Bourguignon, 1982). The above generalized multivariate measure is stated to obey the Pigou-Dalton transfer principle. In a multidimensional context this requires that a transfer of a given attribute from a less endowed household, in terms of another attribute, to a more endowed household should not register a fall in multidimensional inequality (Justino, 2004). Consequently, transferring income from a household with high educational level to a less educated household, both having the same level of income, should decrease multidimensional inequality.

Minimizing D_β with respect to S_i, such that $\sum_{i=1}^n S_i=1$, generates 'optimal' aggregation functions interpreted by Maasoumi (1986, 1999) as the wellbeing for the i-th unit

$$S_i\propto\begin{cases}\left(\sum_{j=1}^m w_j X_{ij}^{-\beta}\right)^{\frac{-1}{\beta}},\beta\neq 0,-1\\ \prod_j X_{ij}^{w_j},\beta\to 0\\ \sum_j^m w_j X_{ij},\beta\to -1\end{cases}$$

The composite welfare indicator S_i, that for strictly positive X_{ij} is defined for all different degrees of substitution, can be interpreted as a utility function of the CES type with an elasticity of substitution defined by $\sigma=1/(1+\beta)$ when $\beta\neq 0,-1$ (Silbert and Deutch, 2005). Different values for the coefficient of substitution give different degrees of curvature to the social indifference curves with respect to household attributes. Consequently, the value of β depends on the degrees to which equality with respect to attributes is valued (Jehle and Reny, 2001). As β increases, there is less and less substitution between attributes.

In the limit, when $\beta \rightarrow \infty$ and $\sigma \rightarrow 0$, the included dimensions of welfare are assumed to be perfect complements and accordingly there is no substitution between attributes. During these circumstances S_i will mirror the attribute with the lowest value for every statistical unit, i.e. the worst performer of the selected welfare dimensions for each household or individual. As a result the composite indicator here approaches a Rawlsian form, where social bias in favor of equality between attributes is absolute. The aggregate function here is of Leontief type with L-shaped contour curves.

With $\beta \rightarrow 0$ the welfare indicator corresponds to a Cobb-Douglas utility with unit elasticity with respect to different dimensions. On the other hand, when $\beta \rightarrow -1$ and $\sigma \rightarrow \infty$ S_i can be interpreted as a linear utility function of the *m* attributes. In this case, low levels in one dimension can be fully compensated by high levels in another and attributes are assumed to be perfect substitutes. Accordingly, this corresponds to social indifference to how household or individual welfare is distributed among attributes, and the composite indicator of every statistical unit corresponds to the weighted arithmetic mean of the different dimensions included. In the literature $\beta > -1$ is a common restriction which implies a non-negative elasticity of substitution. This guarantees that the well being function S_i is quasi-concave with respect to attributes (Lugo, 2005).

Thus, as described above, the first step to generate the Maasoumi index of multidimensional inequality consist in obtaining aggregation functions over desired household welfare attributes where multivariate welfare is composed of two parts, a weighted sum of attribute inequalities and an adjustment due to the covariation between the attributes.

The second step in the procedure is guided by the analysis of inequality in the univariate literature and includes a selection of measures of multidimensional inequality. Once again Maasoumi (1986) make use of the general entropy measures defined by Shorrocks (1980), here applied to the obtained S distribution. If first defining d_i as the population share corresponding to the i-th unit in the distribution, in general equal to 1/n, and S_i^* which is S_i divided by the total sum of the welfare function S_i over all units. By also introducing a parameter α, representing inequality aversion, with more sensitivity to dispersion in the lower part of a distribution the lower the α, Maasoumi (1986) presents the following two multidimensional inequality measures, corresponding to Theil's two inequality indices.

$$M_0(S) = \sum_{i=1}^{n} S_i^* \log\left[\frac{S_i^*}{d_i}\right]$$

$$M_1(S) = \sum_{i=1}^{n} d_i \log\left[\frac{d_i}{S_i^*}\right]$$

In other words, the generalized entropy family of inequality indices is here extended to the multidimensional approach when applied to the summary welfare function S_i. It can be shown that these measures obey the properties of symmetry or anonymity, continuity, invariance to scalar multiplication as well as additive decomposability.

3.3 A Non-aggregative Approach

Turning to the non-aggregative approach the goal is to order different states and the method does not generate a specific real valued measure on the degree of inequality of a distribution. This approach allows for joint distributional analysis and uses stochastic dominance analysis to make judgments about which distribution is the more equal (Savaglio 2005, Tsui, 1999). Although the inequality measures discussed above generally meet a set of desirable axioms it might be that these indices rank the same set of distributions in different ways, since they put different sensitivity to allocations in different parts of the distributions. The general idea belived using this method in comparisons of inequality is to verify whether an ordering of distributions can be considered to remain the same under a wide spectrum of indices.

With a stochastic dominance approach there is a possibility of agreement over classes of welfare functions of different dimensions of welfare, without having to specify the precise form of a social welfare function (Maasoumi, 1999). This makes it possible to avoid the criticism inherent in the use of multidimensional indices. Assumptions are made for classes of utility functions from which conditions of dominance of different orders are derived. There exist several orders of stochastic dominance and all have an ethical interpretation in the context of social welfare (Trannoy, 2005). If dominance is achieved, one distribution is unambiguously preferred to another. Therefore, the gains ensured by employing stochastic dominance come from the robustness of the obtained rankings.

The literature on stochastic dominance and inequality in a multivariate setting can be separated into two strands – one where different welfare attributes are assigned a *symmetric* role and one where attributes are assigned an *asymmetric* role. A number of papers have been devoted to the first vein e.g. Huang, Kira and Vertinsky (1978), Atkinson and Bourguignon (1982), and Koshevoy (1995, 1998). In particular, Atkinson and Bourguignon (1982) derive appropriate dominance criteria that determine the conditions under which one multivariate distribution is more equal than another. Capturing the perspective that there might exist complementarities or substitutability between different attributes and that one dimension of welfare can compensate or reinforce deprivation in another, later adopted in the development of multidimensional indices (as above), they introduce the intuition that multidimensional inequality not only depends on the dispersion in the distributions, but also on how strong correlation there is between different distributions (Weymark, 2005).

In a symmetric setting we do not put more interest in a distribution of one specific characteristic than in another. In other words, if having a monetary and a health objective, there is equal concern with inequality of income among the unhealthy as well as with inequality of health among the poor in monetary terms. Consequently there is an anonymity property with respect to different attributes and no priority is given to a certain perspective, akin to the anonymity property between individuals in a univariate context.[6] This symmetry of attributes is reflected in that an implementation of a dominance criterion does not change when we permute the rows of an allocation matrix (Trannoy, 2005, Justino, 2004).

[6] When one-dimensional inequality is examined the anonymity principle implies that permutations of incomes among people should not matter for inequality judgments (Cowell, 1995).

The second part of the theoretical literature on multidimensional stochastic dominance put more attention to one distribution within the joint-distribution. Thus, in this setting one attribute has a particular position – often due to its transferability. This specific attribute plays the role of a compensating variable since it is assumed to be able to compensate for deficiencies in other characteristics. The pioneering work in an asymmetric setting was presented by Atkinson and Bourguignon (1987). The focus is here on the measurement of income inequality while at the same time accounting for households' different needs. One attribute, representing needs, is used to partition the population into homogenous groups, while social welfare defined from the attribute of particular concern, income, is considered within the need groups and in the whole society. However, by this second type of multidimensional welfare analysis, frequently presented as the needs approach, the marginal distribution of needs is assumed to be fix. This means that the distribution of needs used to divide the population into homogenous groups is assumed to be the same over time or between countries. Although Jenkins and Lambert (1993) and Lambert and Ramos (2002) show that that the particular dominance criterion derived in the needs approach does not change considerably in the case of non fix margins, this approach only makes it possible to evaluate policy impacts on the income distribution, but not whether there has been a simultaneous change in the distribution of the second variable used in a discrete manner. In other words, in using this method the marginal distribution of needs does not matter in itself and does not affect the comparison of two multivariate distributions.

These concerns have initiated a recent development in the literature where attributes are treated asymmetrically. In this examination we will make use of a framework presented by Muller and Trannoy (2003) and Trannoy (2005), where the asymmetric character of need and income is put in a multidimensional structure akin to the one introduced by Atkinson and Bourguignon (1982) and where attributes are assumed to be substitutes.[7] To catch the intuition, an arrangement of ALEP substitutability utility functions[8] is used.

$$U_{ALEP} = \{u_1, u_2 \geq 0, u_{11} \leq 0, u_{22} \leq 0, u_{12} \leq 0\}$$

Following the work of Atkinson and Bourguignon (1982) the partial cross derivative is assumed to be non-increasing. This implies that the marginal utility of one attribute (the compensating) decreases with the level of another attribute (the compensated). In other words, a household's marginal utility of e.g. income is lower if a household is e.g well educated compared to if the household's educational level is low. This can be interpreted as a Pigou-Dalton principle in a multidimensional context since a monetary transfer from a richer household to a poorer one with the same educational level, should not result in an increase in multidimensional inequality (Justino, 2004). The framework developed by Muller and Trannoy (2003) consequently captures the idea that compensation is good for welfare. In addition to requiring a non increasing partial derivative, it is here demanded that the marginal utility of income decreases as a statistical unit gets richer in terms of money and that the marginal utility of education decreases as a statistical unit gets more educated.

On top of securing the idea of substitutability Muller and Trannoy (2003) captures the intuition that compensation seems even more appropriate if people in the lower tail of one distribution in a population also have a poor situation in terms of another attribute. Let x_1

[7] For detailed discussion on this issue, see Tsui (1999) and Bourguignon and Chakravarty (2003).

[8] ALEP stands for Auspitz-Lieben-Edgeworth-Pareto. The ALEP substitutability property was first proposed by Chipman (1977). The set contains increasing utility functions that all are concave in their arguments.

represent the income and x_2 the educational level of a statistical unit. It is now assumed that the third cross partial derivative is non-decreasing which means the difference in marginal utility of income among well educated households is lower compared to among households with low education.

$$U_{MTx_1} = \{u_1, u_2 \geq 0, u_{11} \leq 0, u_{22} \leq 0, u_{12} \leq 0, u_{112} \geq 0\}$$

In this subset income is the compensating variable of particular interest and education is the compensated. Specifically, this class is designed to capture the view that we are primarily interested in the distribution of income among the less educated.

Muller and Trannoy (2003) and Trannoy (2005) also introduce a set of conditions for the family of utility functions satisfying

$$U_{MTx_2} = \{u_1, u_2 \geq 0, u_{11} \leq 0, u_{22} \leq 0, u_{12} \leq 0, u_{221} \geq 0\}$$

where education now is the compensating attribute and income the compensated. In this case there is agreement that the poor in terms of money must have priority in constructing educational policies. In line with the asymmetric approach the two families of utility functions U_{MTx_1} and U_{MTx_2} are not anonymous with respect to the set of attributes. This makes them intermediate between two classes presented in Atkinson and Bourguignon (1982) where attributes conversely are treated symmetrically and where there is equal interest in the income distribution of the uneducated and the educational distribution of the poor.[9]

When the compensated attribute is discrete some of the assumptions presented in the framework by Muller and Trannoy (2003) and Trannoy (2005) is akin to what is introduced in Atkinson and Bourguignon (1987). In this situation it is assumed that the marginal valuation of the compensating attribute is different between diverse groups, and that it is possible to identify and rank households according to this valuation. This particular separation allow for diverse judgments of welfare for different partitions identified by a given characteristics different than income (Maasoumi, 1999). As above, welfare cannot decrease as a result of increasing incomes. Moreover, it is possible to identify what households with the same income level that would benefit the most from a monetary increase as the effect on social welfare of a given increase in income, is larger the needier the group that receives this money. The assumption on the third cross partial derivative now implies that differences in the marginal valuation of income between groups decreases as we move toward higher monetary levels (Chambaz and Maurin, 1998). This reflects less concern with differences in needs for higher income groups.

The obtained criterion for one multivariate distribution (A) to second order dominate another (B), both containing a marginal distribution of income (x_1) and education (x_2), in this setting consists of two conditions. One condition for the compensating attribute, and one for the compensated. When the distribution of the latter variable is discrete, the distribution of the compensating variable of particular concern has to meet a *sequential generalized Lorenz condition*. This is akin to the test provided in Atkinson and Bourguignon (1987). A sequential test is implemented by initially examining whether there is dominance in e.g. income for the neediest group. If one distribution dominates the other, the exercise is continued by adding

[9] Atkinson and Bourguignon (1982) present the following classes of utility functions:

$$U_1 = \{u_1, u_2 \geq 0, u_{12} \leq 0\} \quad U_2 = \{u_1, u_2 \geq 0, u_{11}, u_{22}, u_{12} \leq 0, u_{112}, u_{122} \geq 0, u_{1122} \leq 0\}$$

in the second neediest group and now testing for dominance of the income variable for these two groups combined and so forth until the total population is included (Bazen and Moyes, 2003). We should note that a sequential dominance approach demands weaker conditions compared to examining income distributions of different groups, characterized by their different needs, separately. This is the case since a negative distributional effect in one group can be offset by a favorable distributional effect in another as the groups gradually are cumulated. Concerning the second dominance criterion derived by Muller and Trannoy (2003), the distribution of the compensated variable in addition has to satisfy a *generalized Lorenz (GL) condition.*

The statistical tool to examine of the distribution of the compensated attribute is to check the GL curves of the compensated variable in the two multivariate distributions, (A) and (B). When primary concern is the monetary distribution of the least educated, the GL curve of x_2 in distribution (A) must not be below the corresponding curve in distribution (B). To able a test of second order dominance of the chosen compensating variable income, a definition of the absolute poverty gap is defined in a multivariate setting

$$PG\left(x_1 \mid x_2\right) = \frac{1}{n} \sum_{\{i \mid (x_{i1}, x_{i2}) \leq (x_1, x_2)\}} (x_1 - x_{i1})$$

This is the cumulated poverty gap for all statistical units with income x_{i1} below or equal to an income limit x_1 and with an educational level below or equal to x_2. If this gap is not larger in (A) compared to (B) for all choices of x_1 and x_2, distribution (A) income poverty gap dominates (B), i.e.

$$PG_A(x_1 \mid x_2) \leq PG_B(x_1 \mid x_2) \quad \forall x_1 \in X_1, \forall x_2 \in X_2 \qquad \leftrightarrow \qquad A \geq_{PGX1} B$$

In a corresponding way the educational poverty gap is defined

$$PG\left(x_2 \mid x_1\right) = \frac{1}{n} \sum_{\{i \mid (x_{i1}, x_{i2}) \leq (x_1, x_2)\}} (x_2 - x_{i2})$$

where x_{i2} is the educational level of an individual or a household and x_2 is equivalent to an educational poverty line. If the main concern is the educational distribution among the poor, the educational poverty gap in distribution (A) must be no higher than in distribution (B), i.e.

$$PG_A(x_2 \mid x_1) \leq PG_B(x_2 \mid x_1) \quad \forall x_1 \in X_1, \forall x_2 \in X_2 \qquad \leftrightarrow \qquad A \geq_{PGX2} B$$

The poverty gap dominance condition is implemented sequentially. Achieved poverty gap dominance for the compensating attribute implies that generalized Lorenz dominance is fulfilled for this variable as well. Consequently, Trannoy (2005) state the following conditions as sufficient to realize dominance for the multivariate distribution (A) over the corresponding distribution (B) according to the asymmetric classes U_{MTx_1} and U_{MTx_2}

$$A \geq_{GL} B \text{ and } A \geq_{PGX1} B \quad \rightarrow \quad A \geq_{U_{MTx_1}} B$$

$$A \geq_{GL} B \text{ and } A \geq_{PGX2} B \quad \rightarrow \quad A \geq_{U_{MTx2}} B$$

In a situation when the marginal distribution of the compensated attribute is given this framework boils down to the one presented in Atkinson and Bourguignon (1987).

4. Datasets and Variables

For the empirical application on multidimensional inequality, the Zambian Living Conditions Monitoring Survey 1998 (LCMS II) and the Zambian Living Conditions Monitoring Survey 2004 (LCMS IV) are used. These household surveys were conducted in November and December 1998 and in October 2004 to January 2005, respectively, by the Zambian Central Statistical Office (CSO, 1999 and 2005). This is the end of the hot dry season and the beginning of the hot wet season, a number of months prior to the harvest period. In addition to being conducted during the same period of the year the two surveys have similar questionnaires which make the surveys comparable. The CSO has been conducting household based LCMS in the country since 1996, all being independent surveys interviewing different households in each year. Consequently, the datasets are not a panel of households, but two representative cross-sections.

The two surveys were employed using similar survey design with multiple stage sample selection process. In 1998 the sampling frame was developed from the 1990 census of population and housing while sampling frame in 2004 was drawn from the census of population and housing carried out in 2000. The LCMS II and the LCMS IV have nationwide reporting and cover 16710 respectively 19340 households and 93 471 respectively 103 242 individuals in all 9 provinces and 72 districts in Zambia. Sample weights are applied in all calculations to correct for differential representation of the sample at national and sub-national levels. Our analysis is based on 16445 and 19179 households.[10]

The unit of analysis in this paper is the household. A household is defined as the head, the spouse, children, relatives and other dependents living in the household but also usual members who at the moment for the conduction of the survey were away visiting, hospitalized etc. Visitors are generally excluded, unless they have lived with the household for six months or more. Although the standard apparatus of welfare economics and measurement concerns the wellbeing of individuals, in a development context an individual's wellbeing often depends on the resources available to the household, the size and the structure and in what manner resources are shared within the household (Deaton, 1997).

Four different dimensions of household outcome welfare are examined: expenditures, education, health and land holdings.[11] In both surveys there is information on household monthly expenditures and on total land area under crop on household level. Data on years in education and on health status are reported on an individual level. Representing a monetary distribution, household monthly per adult equivalent expenditures is used in the empirical application. In other words, the monthly consumption of every household is divided by an adult equivalence scale, rather than total household size.[12] To adjust for inflation over the time period we use information on the Consumer Price Index in Zambia (CSO, 2005)

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[10] We drop 265 households in 1998 respectively 161 households in 2004 from the original samples since these units do not have complete information concerning one or more of the variables used in the analysis. This represents a drop of 1.6% and 0.8%, respectively, of the original samples.

[11] Education is not strictly continuous as this variable takes discrete values. In addition, health and education is bounded from above. This should not, however, violate the application of the various measures of inequality applied in this paper as these can be extended to discrete variables (see Cowell, 1995).

[12] Male and female adults older than 12 year each have a consumption weight of one and children's adult equivalent scale ranging between 0.36 and 0.76

Representing a first human capital distribution we use the maximum educational grade obtained by the head of the household ranging between 0 and 17 in the empirical application, with 17 corresponding to doctoral level. To examine inequalities in health we estimate a health distribution by using an ordered probit regression using a categorical subjective health variable as the dependent variable. Each observation in the resulting variable corresponds to household average health status. The estimation follows a procedure in the World Bank's technical note #3 (2005) on quantitative techniques for health equity analysis, further described in van Doorslaer and Jones (2003). For detailed information of this procedure and estimation results see appendix II. Naturally, the results including the health dimension of welfare in this examination should be interpreted with particular care. Representing an asset distribution we use information on household land area under crop in the last agricultural season measured in hectares

In applying the non-aggregative approach, the four welfare variables in turn are used as discrete partitions. The aim is to generate a division where groups are relatively homogenous so that it is reasonable to consider the household marginal valuation of a second welfare dimension, corresponding to the compensating attribute, to be similar within partitions. When employing the monetary distribution as the compensated attribute, households are divided into the groups extremely poor (households with a monthly per adult equivalent real expenditure of less than K32,681), moderately poor (households with a monthly per adult equivalent real expenditure higher than K32,681 but less than K47,187) and non-poor (monthly per adult equivalent real expenditure higher than K47,187) following the poverty lines derived by the Zambian CSO based on the minimum calorie intake per day per person (CSO, 2005). The distribution of education is split up into the groups no or primary education (0-7 years of schooling), secondary education (8-12 years of schooling) and higher education (more than 12 years of schooling) corresponding to levels of educational grade in the Zambian education system. The land variable is divided into three sub groupings according to the definitions landless, small scale (households cultivating land more or equal to 0.1 hectares but less or equal to 2 hectares of land) and medium and large scale cultivators (households cultivating more than 2 hectares of land).[13] This grouping relate to the criteria for rural stratification of households in Zambia used in the previous National Census of Agriculture (CSO, 1994). The estimated health distribution is divided into population quintiles each representing 20% of the population.

[13] To assure that empirical results are not driven by how we create the different partitions we will also use a discrete partition, based on the expenditure, the education and the land variable, where the dataset is divided in quintiles corresponding to 20% of the population, i.e. the same kind of partition used for the health variable. This division makes sense when splitting the population according to the monetary distribution. On the other hand this division is not as logic when studying the distribution of educational level since this variable in it self is discrete. Consequently, households with the same level of education might belong to different quintiles in this case.

5. Empirical Application

5.1 An Item-by-Item Approach

Monetary inequality in Zambia is generally stated to be high and the country is often referred to one of the most unequal societies in the world. Table 1 confirms that the level of monetary inequality in the country is high in an international context. As measured by Gini coefficients of consumption distributions the World Bank only identifies 5 countries out of 83 that are less equal (World Bank, 2006).[14] Although McCullough et. al (2001) in an examination of changes in monetary poverty and inequality in Zambia establish that the Gini coefficient somewhat decreased during the 1990's, the below table indicates that consumption inequality at the national level has slightly increased between the two survey periods here studied, with the Gini coefficient rising from 0.533 in 1998 to 0.544 in 2004.

Table 1 Unidimensional Inequality Measures

	Gini	GE(0)	GE(1)	GE(2)
1998				
Expenditures	0.533	0.533	0.596	1.661
Education	0.354	2.728	0.262	0.194
Health	0.127	0.030	0.027	0.025
Land	0.675	5.654	0.901	1.615
2004				
Expenditures	0.544	0.551	0.738	19.281
Education	0.322	2.079	0.215	0.162
Health	0.138	0.037	0.032	0.030
Land	0.698	5.981	1.174	15.491
	Change Gini (1998-2004) %			
Expenditures	2.17			
Education	-9.10			
Health	9.47			
Land	3.36			

Source: Author's calculations from LCMS II and LCMS IV

The three GE inequality measures parallel the pattern revealed by the Gini coefficient as they point to an increase in consumption inequality over the time period. This indicates that the point estimates of inequality portrayed by the Gini coefficients are robust with respect to different weighting. Concerning specific changes in the expenditure distribution between 1998 and 2004 we note that the GE(2) value for the monetary variable more than tripled between the two time periods. This indicates a sharp increase in the consumption growth of the very richest households relatively to the sample as a whole.

The three additional welfare dimensions, education, health and land, have not been examined as much as income or consumption from an inequality perspective. Cross country studies on Sub-Saharan Africa though state regional average education Gini coefficients to be 0.66 (World Bank, 2006).[15] In relation to this, the magnitude of inequality in education in Zambia is low and as can be understood by table 1 education inequality has decreased over

[14] These countries are the Central African Republic (0.61), Lesotho (0.63), Panama (0.55), South Africa (0.58) and Zimbabwe (0.57).

[15] The education Gini presented in World Bank 2006 is based on data on the number of years of education completed by interviewees.

the past six years. Measured in terms of the Gini coefficient, inequality in this dimension has been reduced by 9%. This fall in education inequality is further confirmed by additional inequality indices and seems to be a decreasing function of education as the more weight given to distances between educational outcomes in the lower end of the distribution, the higher the fall in inequality. Particularly the decrease in education inequality between 1998 and 2004 is large when studying GE(0).

Concerning the level of inequalities in terms of the Gini coefficient, land seems to be the more unequally distributed welfare attribute in Zambia and inequality in this dimension is also high by international standards. Cross country studies on Sub-Saharan Africa state a regional average land Gini coefficients of 0.62 (IFAD, 2001) and 0.5 (World Bank 2006).[16] Large numbers in the GE(0) and the GE(2) column reflects a situation with a small share of the total population being involved in large-scale farming that, although not many in terms of numbers, cultivates very large areas of land and major disparities in the bottom of the distribution, mirroring that several households do not cultivate any land. Irrespective of what indicator examined, inequality in landholdings seems to have increased over time. Above all, the GE(2) indices point to a major raise in inequality between 1998 and 2004, indicating that the increase in land inequality mainly has been located in the higher percentiles.

The lowest level of inequality is found when examining the dispersion in the health distribution. Accordingly the health status appears to be relatively homogenous among Zambian households. On the other hand, the health Gini coefficient increased by more than 9% between 1998 and 2004. This increase in inequality over time is once again confirmed by changes in the class of GE(α) measures. The augmented health inequality does not seem to be an outcome of larger disparities in certain percentiles of the distribution, as the percentage increase in the GE(0), the GE(1) and the GE(2) are of similar quantity.

A general conclusion from the above exercise on monetary and non-monetary inequalities, concerning the magnitude of inequality rather than changes over time, is that there are differences in the pattern of inequality across the various welfare distributions. The monetary distribution is the least unequal when putting extra weight to the observations in the lower part of the distribution and increases with the choice of α. Conversely, the GE inequality measures for the non-monetary distributions of education and health decreases with the choice of α, implying large differences in the lower end of these distributions. Given the impact of human capital on economic development, and assuming abilities to be normally distributed across the population, this result represents a loss in aggregate welfare.

To get a first indication on multivariate inequality, Spearman's rank correlation coefficients are calculated.[17] A coefficient value of 1 indicates that the rankings of the two distributions are perfect and -1 that rankings are reversed. If rankings are completely independent the correlation coefficient is 0.

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[16] Concerning the magnitude of land Gini coefficients presented by the World Bank (2006) there is no information whether these calculations are based on nationally representative surveys or based on information on the allocations of land among households involved in agricultural activities.

[17] There are several methods for calculating correlation, e.g. nonparametric Spearman correlation and the parametric Pearson correlation. The latter approach assume that both X and Y values are sampled from populations that follow a normal distribution while the former method is based on ranking the two variables of interest and so makes no assumption about the distribution of the values.

Table 2 Spearman Rank Correlation

1998					2004				
	Expenditures	Education	Health	Land		Expenditures	Education	Health	Land
Expenditures	1				Expenditures	1			
Education	0.457*	1			Education	0.349*	1		
Health	0.100*	0.245*	1		Health	0.118*	0.398*	1	
Land	-0.267*	-0.290*	-0.085*	1	Land	-0.200*	-0.329*	-0.253*	1

Source: Author's calculations from LCMS II and LCMS IV. H0, that the variables are independent, is rejected for all cases. * Significance at 5% level.

From a general point of view, the absolute values of the different correlation coefficients do not indicate very strong relationships between the different attributes. Although the null hypothesis of independence between the distributions of expenditures, education, health and land is rejected in all cases, the extent of overlap between the distributions generally seems to be rather low.[18] The strongest relationship appears between expenditures and education in 1998 with a correlation coefficient of 0.457. In the 2004 sample the rankings of the educational and health distribution is somewhat more similar than the ranking in the expenditure and the educational distribution, respectively.

Despite that expenditure or income distributions often is used as a proxy for the distributions of other welfare attributes, we find that the relationship between expenditures and the other two indicators, health and land, is relatively weak both in 1998 and in 2004. In examining the relation involving expenditures and health, the Spearman rank correlation coefficient is positive and about 10-12% correlated in ranking relationship. Concerning expenditures and land the relationship is negative. Moreover, the welfare indicator land is negatively correlated to all other variables, indicating that one of the variables decreases as the other increases. Weak relationships between the distribution of expenditure and other welfare distributions are recurrent in the empirical literature (Lovell et. al. 1994, Ramos and Silber, 2005). In addition, static comparisons of monetary and non-monetary inequality from African developing countries suggest that a lack of overlap between the different indicators is common (Sahn and Stifel, 2003).

According to the results in table 2, it is evident that rank correlation between different distributions of attributes has somewhat changed over time. In addition to a change in correlations between education and expenditures and education and health, indirectly mentioned by the above statement that the latter formation is more correlated than the former in 2004, the rank correlation between the land and the monetary distribution has increased over time. On the other hand the agreement between the rankings in the health and the land distribution has to some extent decreased.

The results from the item-by-item analysis of independent distributions gives support to the plan of broadening the study of inequality and to the importance of including other variables than just expenditures in examining inequality in general. Not only the magnitudes of monetary and non-monetary inequalities seem to be dissimilar, but there are also different trends in changes over time. In addition, theoretical motives point to the relevance of examining how potential complementarities between different dimensions of welfare affect conclusions on inequality. This is further supported empirically by the results from the rank correlation examination of distributions.

[18] The Pearson's test of correlation generates the same general result as above with the difference that the correlations are overall lower and that correlation between expenditures and land is positive, but insignificant.

5. 2 An Aggregative Approach

Before aggregating the different attributes of concern into a multivariate welfare function it is necessary to transform the chosen attributes to the same unit of measurement. The technique used in this application is based on the HDI approach (UNDP, 1995) and the following formula generates distributions with observations varying between 0 and 1.

$$S_{ij} = \frac{x_{ij} - \min x_{ij}}{\max x_{ij} - \min x_{ij}} \qquad \text{with } i=1,2\ldots n \text{ and } j=1,2,3,4$$

For the purpose of comparing the change in multidimensional inequality between 1998 and 2004, the min and the max values used in the normalization of the monetary, the educational, the health and the landholding distributions are identical.[19] There are additional transformations suggested in the literature and the kind of transformation might affect the results. E.g. Hirschbert et al (1991) select a variable standardization that generates both negative and positive values which often is problematic when measuring inequality.

As recommended in the literature the degree of substitution between the attributes, β , is set to be larger than -1 in the calculations of the distribution functions, which implies a positive elasticity of substitution. To include the case when attributes are seen as complements we also let β take positive values.[20] Concerning the possible choice of relative inequality aversion we operate with $\alpha = 0$ and $\alpha = 1$ which corresponds to Theil's first and second measure of inequality. In all cases we weight the four attributes equally ($w1=w2=w3=w4$).

For test of statistical inference, a bootstrap procedure was used to generate estimates of the standard errors of the M(0) and the M(1) for different aggregations and choices of β . The bootstrapped samples mimic the empirical distributions of the LCMS II and LCMS IV survey samples. That is, p households were randomly drawn, with replacement, where p is the number of household units in the surveys, respectively. Since the bootstrap sampling is done with replacement, each household may appear one or more times in a given bootstrap sample, or not at all. Multidimensional inequality indices are calculated for each bootstrap sample. The process is repeated 500 times. The standard deviation of the inequality measures over the 500 replications is a consistent estimator of the standard error of the inequality index (Efron and Tibshirani, 1993, Arndt et al, 2006). The point estimates of the Maasoumi indices reported are calculated from the original, non-bootstrapped sample.

Starting from a traditional viewpoint, the first examination of the Maasoumi inequality index includes expenditures and one of the three other attributes in turn. The results in the below table clearly point to that we gain additional insights by applying a multivariate technique compared to when examining inequality with a monetary perspective exclusively. For example, when measured by Theil's lower index, inequality with respect to household

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[19] The minimum value of the distribution of land is the same in the two years but the maximum value used in the transformation is from 2004. The minimum value of real expenditures is achieved in 1998 and the maximum in 2004. The educational and the health distribution are bounded by definition, from 0 to 17 and from 0 to 1, respectively.

[20] The maximum value of the degree of substitution here equals 20. For larger choices of beta the calculation of the aggregation function S_i is demanding as a result of observations of small magnitudes in the different welfare distributions. However, in testing for other values of beta larger than one, we find that the differences in magnitude of M(0) and M(1) are extremely small when the degree of substitution exceeds the value of 5.

expenditures only is lower than the composite indices when $\beta > 0$. These results demonstrate that ignoring important ranges of economic conditions as education and health here seems to result in underestimation of inequality if attributes are assumed to be complements, but overestimation if the elasticity of substitution is larger or equal to one. One realizes also that these inequality indices of several dimensions takes an increasing magnitude across higher values of β. When $\beta \leq 0$ we assume that more of one attribute can compensate a poor situation in terms of another and the level of multidimensional inequality should reasonably be relatively low. As we impose less substitution the indices captures the social bias in favor of equality between different attributes and we have to accept larger magnitudes of inequity.

Table 3 Maasoumi Multidimensional Inequality Index

		S1:Expenditures	S2: Expenditures & Education		S2: Expenditures & Health		S2: Expenditures & Land	
Zambia 1998								
M(0)	β = -0.99999		0.260 (0.006)		0.026 (0.000)		0.544 (0.023)	
M(0)	β = 0.000001		0.372 (0.007)		0.152 (0.003)		0.694 (0.022)	
M(0)	β =1/2		0.696 (0.022)		0.549 (0.023)		0.735 (0.018)	
M(0)	β =1		0.780 (0.031)		0.643 (0.039)		0.742 (0.017)	
M(0)	β=20	0.533	0.800 (0.034)		0.663 (0.047)		0.775 (0.018)	
Zambia 2004								
M(0)	β = -0.99999		0.214 (0.004)	***	0.032 (0.001)	***	0.748 (0.138)	**
M(0)	β = 0.000001		0.366 (0.009)		0.198 (0.006)	***	0.796 (0.018)	***
M(0)	β =1/2		0.811 (0.048)	**	0.698 (0.039)	***	0.801 (0.015)	***
M(0)	β =1		0.938 (0.082)	*	0.821 (0.074)	**	0.820 (0.016)	***
M(0)	β=20	0.551	1.012 (0.098)	**	0.881 (0.105)	**	0.877 (0.017)	***

Source: Author's calculations from LCMS II and LCMS IV
Standard errors in parentheses, estimated by bootstrapping with 500 replications.
*** Difference between the two periods significant at 1% level **Difference between the two periods significant at 5% level * Difference between the two periods significant at 10% level

Examining changes in multidimensional inequality over time, all but one combination of attributes in the above table point to that inequality increased between 1998 and 2004. As we will return to in section 5.3, the assessment including expenditures and land points to a significant increase in multidimensional inequality across all degrees of substitution. This is also true when a health dimension is included in the composite welfare function. The exception is found in the second column combining a monetary and an educational dimension of household welfare as we here find evidence of decreasing multidimensional inequality over time. With this arrangement M(0) in 2004 is significantly lower compared to in 1998 when we assume attributes to be perfect substitutes. On basis of these results we realize that the aggregative approach to some extent mirrors the decrease in educational inequality observed when examining attributes separately.

It is also apparent that changes in bivariate inequality, calculated as percentage, are of greater magnitude compared to the change in expenditure inequality over time. The aggregative method thus seems to reflect that relative changes in inequality for the attributes education, health and land independently are greater compared to the change in expenditure inequality alone. We also gain insight on that the degree of correlation between attributes is of relevance to the conclusion on changes in inequality. In all three S2 aggregations there is evidence that the technique reflects changes in rank correlations between attributes over time. The decrease in inequality between 1998 and 2004 when combining expenditures and education is of greater magnitude the higher the degree of substitution. This pattern follows from the declining correlation between these dimensions of welfare over the time period. Correspondingly, we find a larger percentage increase in inequality examining the bivariate distribution of expenditures and land when $\beta \leq 0$ compared to when the attributes are seen as complements. This probably mirrors the noted increase in correlation between expenditures and land 1998 to 2004.

The decreased dispersion between observations in the educational distribution between 1998 and 2004 and the compensational effect from education on both levels and changes in multidimensional inequality revealed in table 3, motivates an examination of another set of combinations of the four attributes with this particular variable as a baseline.[21] Table 4 includes inequality indices of three multivariate S functions, together with the GE(0) measure of the educational distribution S1.

The inclusion of the health variable generates a situation where inequality seems to have decreased over the time period, although health inequality increased between 1998 and 2004 according to the examination of independent attributes. This is true irrespective of the degree of substitution also when we perform a more detailed examination of the choice of β (table 4 b and 4c in Appendix I). Operating with high elasticities of substitution this robustness is somewhat unexpected as the rank correlation between education and health increased over time. As the preceding conclusion might be a result of our choice of inequality aversion we also operate with $\alpha = 1$, which allot equal weight to all observations. However, also in this setting the method generates the same outcome with significantly lower magnitude of inequality in 2004 compared to in 1998. Consequently, the above reasoning on how changes in correlations between distributions are reflected in the outcomes of the aggregative approach can not always be generalized. As a result, we realize that the effect of less dispersion between the observations in the educational distribution in the latter period can counterweigh both the increased spread in the health distribution as well as the increase in correlation between the attributes over moments in time.

When examining multidimensional inequality including education, health and expenditures, S3, we find increasing inequality across a majority of choices of β. Consequently, the effect generated by declining inequality in household educational status between 1998 and 2004 is no longer dominant once monetary outcome of households is taken into account. Interestingly there is no monotonous development with respect to changes in inequality across degrees of substitution in this setting. When $\beta = -1$ and β = ½ or greater, multidimensional inequality significantly increase over time. If instead letting the degree of substitution take an intermediate value, the conclusion is that inequality significantly decreases. Similar results appear when household welfare is assumed to depend on all four attributes. This is not in line with earlier findings using this aggregative approach presented in Maasoumi and Nickelsburg (1988) and in Lugo (2004) as they conclude that changes in multidimensional inequality using this method are robust with respect to choice of β.

[21] The results of the Maasoumi index for all possible combinations of S2 and S3 are available upon request.

Table 4 Maasoumi Multidimensional Inequality Index

		S1: Education	S2: Education & Health		S3: Education, Health & Expenditures		S4: Education, Health, Expenditures & Land	
Zambia 1998								
M(0)	β = -0.99999		0.047 (0.001)		0.047 (0.001)		0.047 (0.001)	
M(0)	β=-1/2		0.090 (0.002)		0.090 (0.001)		0.086 (0.002)	
M(0)	β =0.000001		0.207 (0.006)		0.260 (0.006)		0.460 (0.007)	
M(0)	β=1/2		0.216 (0.006)		0.624 (0.015)		0.923 (0.018)	
M(0)	β=1		0.222 (0.006)		0.762 (0.026)		0.962 (0.021)	
M(0)	β=20	2.728	0.253 (0.006)		0.799 (0.034)		0.998 (0.020)	
Zambia 2004								
M(0)	β = -0.99999		0.032 (0.001)	***	0.050 (0.001)	***	0.049 (0.001)	**
M(0)	β=-1/2		0.083 (0.001)	***	0.083 (0.001)	***	0.077 (0.001)	***
M(0)	β =0.000001		0.168 (0.004)	***	0.238 (0.004)	***	0.625 (0.008)	***
M(0)	β=1/2		0.175 (0.003)	***	0.708 (0.018)	***	0.919 (0.014)	
M(0)	β=1		0.181 (0.004)	***	0.880 (0.045)	**	0.975 (0.017)	
M(0)	β=20	2.079	0.209 (0.004)	***	0.990 (0.117)	**	1.032 (0.019)	*

Source: Author's calculations from LCMS II and LCMS IV

Standard errors in parentheses, estimated by bootstrapping with 500 replications.

*** Difference between the two periods significant at 1% level **Difference between the two periods significant at 5% level * Difference between the two periods significant at 10% level

Although not presented in the above table, changing the parameter of inequality aversion, so that $\alpha = 1$, and thereby allotting equal weight to all households in the aggregated distributions, neither change the results for S3 or S4 (table 4d in appendix I). Also in this setting changes with respect to equality are generally negative over time when monetary and a land perspective are assumed to contribute to household welfare together with education. Moreover, despite that the dispersion between households increased over time when examining three out of the four distributions independently, also here multivariate inequality is sensitive to the choice of β.

The non-monotonous development with respect to changes in multivariate inequality across β motivates a more detailed examination of the choice of degree of substitution. We here focus on the range where the difference in inequality over time hover between positive and negative outcomes when operating the S4 aggregation. Interestingly, as can be noted in the below table, the spans of β equaling (-0.7 - -0.2) and (0.2- 0.6) all point to that inequality decreased between 1998 and 2004. Thus, in this setting the method seems to capture exceptions in changes in multidimensional inequality when β is close to -1 and 0. This is particularly noteworthy since *these exact values* of degrees of substitution regularly is examined in the empirical literature on multidimensional poverty and inequality using the same kind of aggregation procedure (c.f. Maasoumi and Nicklesburg, 1988, Lugo, 2004, Deutsch and Silber, 2005). The results here suggest further examinations on the consequence of different choices of β for the inference on changes in inequality of several dimensions in the above literature.

Table 5 Maasoumi Multidimensional Inequality Index across ranges of different degrees of substitution (β)

S4: Expenditures, Education Health & Land	β = -0.99999	β=-0.9	β=-0.8	β=-0.7	β=-0.6	β=-0.5	β=-0.4	β=-0.3	β=-0.2	β=-0.1
	**	***		***	***	***	***	***	***	***
Zambia 1998 M(0)	0.047 (0.001)	0.052 (0.003)	0.057 (0.001)	0.065 (0.001)	0.074 (0.001)	0.086 (0.002)	0.102 (0.002)	0.122 (0.003)	0.158 (0.004)	0.237 (0.005)
Zambia 2004 M(0)	0.049 (0.001)	0.055 (0.001)	0.057 (0.001)	0.062 (0.001)	0.069 (0.001)	0.077 (0.001)	0.087 (0.001)	0.102 (0.002)	0.138 (0.002)	0.296 (0.005)
	β =0.000001	β=0.1	β=0.2	β=0.3	β=0.4	β=0.5	β=0.6	β=0.7	β=0.8	β=0.9
	***	***								
Zambia 1998 M(0)	0.460 (0.007)	0.694 (0.010)	0.806 (0.013)	0.864 (0.015)	0.899 (0.017)	0.923 (0.018)	0.938 (0.018)	0.947 (0.016)	0.954 (0.019)	0.959 (0.017)
Zambia 2004 M(0)	0.625 (0.008)	0.740 (0.010)	0.800 (0.011)	0.851 (0.013)	0.891 (0.014)	0.919 (0.014)	0.936 (0.016)	0.952 (0.016)	0.962 (0.016)	0.969 (0.018)

Source: Author's calculations from LCMS II and LCMS IV

Standard errors in parentheses, estimated by bootstrapping with 500 replications.

*** Difference between the two periods significant at 1% level **Difference between the two periods significant at 5% level * Difference between the two periods significant at 10% level

As emphasized by Anand and Sen (2003) the issues of specific degree of substitution between the various welfare attributes, the choice of inequality aversion and the different weights attributed to different variables, do not take a predominant role in the current theoretical literature. However, the choice of β appears to be of major importance when examining multidimensional inequality empirically in the above context. Three combinations of attributes generate the same conclusion on whether inequality including several dimensions increased or decreased over time across all ranges of degree of substitution. All other aggregations of welfare dimensions, (also combinations of attributes not presented in the text) point to the same conclusion, namely that inequality are sensitive to the particular assumption on the degree of substitution between dimensions of welfare. This emphasizes the importance of not drawing major conclusions from one or two point estimates but rather testing a range of values of β when examining multidimensional inequality by this approach.

Turning to the choice of inequality aversion, inference on changes in welfare does however not seem to be sensitive to whether we employ Theil's first or second index. The general conclusion on inequality changes over time outlined previously is invariant with respect to the choice inequality measure using this aggregative technique. These results confirm the findings in earlier empirical applications using this aggregate index to measure changes in multidimensional inequality. Maasoumi and Nickelsburg (1988) as well as Lugo (2004) point to that increments and declines in inequality over time periods are robust with respect to α .

5.3 A Non-aggregative approach

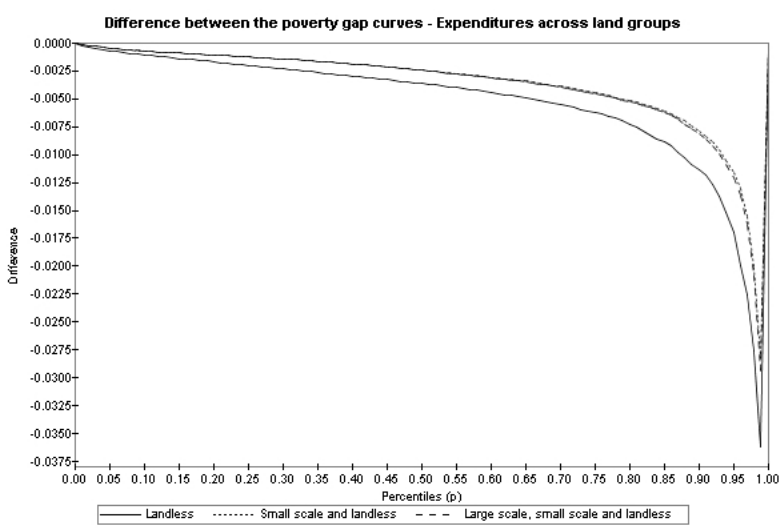
Turning to the examination of changes in multidimensional inequality applying an asymmetric non-aggregative approach, we first establish that with four welfare attributes there are consequently 12 possible combinations of variables to test across time in the chosen setting. Concerning the poverty gap dominance conditions there are three or five different distributional conditions respectively which must bee fulfilled for a distribution of a compensating variable to unambiguously dominate another, since the attributes used in a discrete manner takes three respectively five different values as shown in table 6.

Table 6 Combinations of Variables and Groupings

	Compensated variable			
Compensating variable	Expenditures	Education	Health	Land
Expenditures		No or primary, Secondary, Higher education	Quintiles	Landless Small scale Large scale
Education	Extremely poor Moderately poor, Non-poor		Quintiles	Landless Small scale Large scale
Health	Extremely poor Moderately poor, Non-poor	No or primary, Secondary, Higher education		Landless Small scale Large scale
Land	Extremely poor Moderately poor, Non-poor	No or primary, Secondary, Higher education	Quintiles	

Figure 1 illustrates how the distribution of expenditures of 1998 dominates the corresponding distribution of 2004 according to the poverty gap condition when partitioned by the three groupings representing different levels of households land holdings.[22] As the cumulated curves all take non-positive values this indicates that the expenditure poverty gap is lower in the first time period compared to the latter.

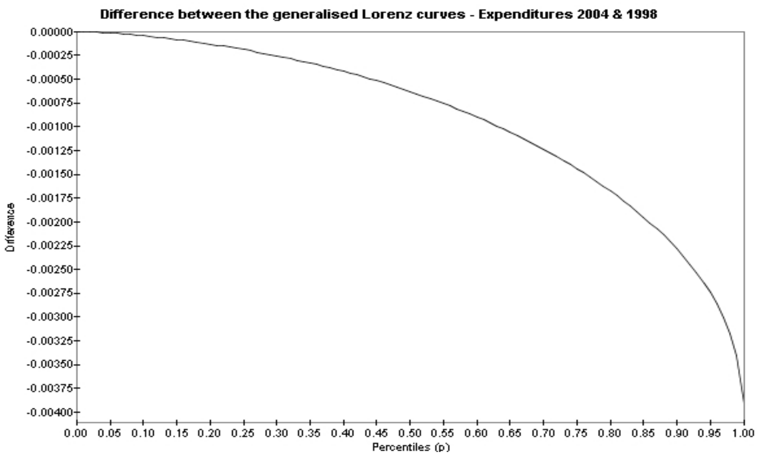
Figure 1



[22] For calculation of differences between poverty gap curves and generalized Lorenz curves and for the creation of the plotted graphs we have used the software DAD (Duclos et al.).

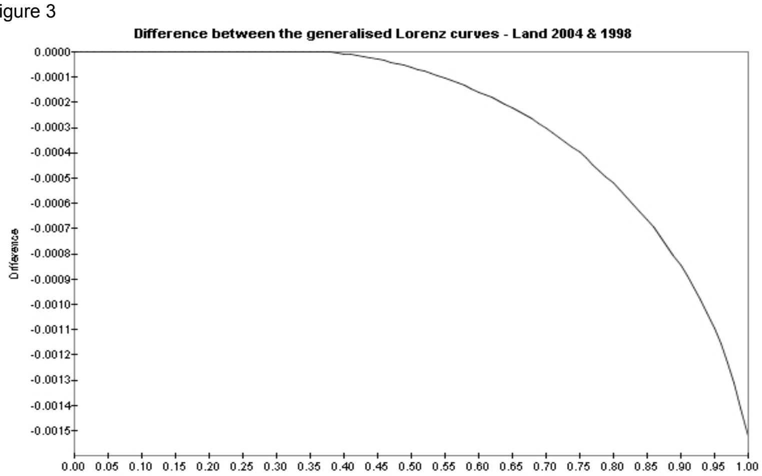
From this examination it is also apparent that the generalized Lorenz dominance criterion is fulfilled for the expenditure distribution, as this is an implication when dominance according to the poverty gap condition is satisfied. This result is moreover presented in figure 2 where it is clear that the difference between the generalized expenditure Lorenz curves in 2004 and 1998 is negative. In other words, the expenditure distribution of 1998 dominates the corresponding distribution in 2004 according to the generalized Lorenz requirement.

Figure 2



As multidimensional inequality only can be stated to be lower in on year compared to another if the Generalized Lorenz criteria for both marginal distributions of concern are fulfilled, we also implement this test for the distributions of land in 1998 and 2004. In studying figure 3 it is apparent that the generalized Lorenz condition of land holds since the difference between the two generalized Lorenz curves is equal or less than zero.

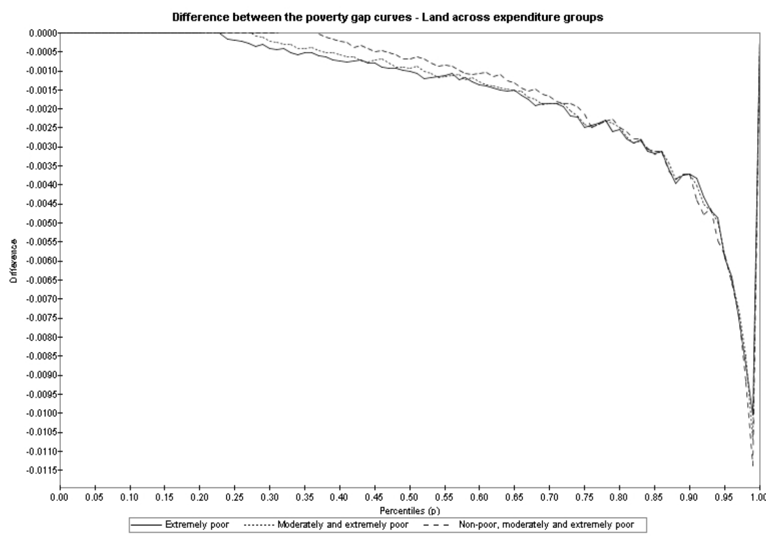
Figure 3



To assure that the above results on expenditures partitioned by land are robust we complement the graphical results by three tables that summarizes the value of the t-statistic of a conditional poverty gap dominance tests, when using a poverty line corresponding to the maximum value within the particular distribution. The results in table 9-11 in appendix I confirm that the expenditure poverty gap is larger in 2004 than in 1998 for all poverty lines, for all land partitions. As the differences are statistically significant for all test points we declare dominance, i.e. that the curves do not cross. Consequently we here come to an unambiguous conclusion concerning changes in inequality including a monetary and a land perspective. Multidimensional inequality in this setting, with expenditures as a compensating attribute, increased over the time period.

Turning to the analogous asymmetric class U_{MTx_2} where the distribution of land among the poor is of primary interest, we test for land poverty gap dominance across expenditure groups over time. As can be understood from the graphs in figure 4 this dominance condition is fulfilled for the group consisting of extremely poor households.

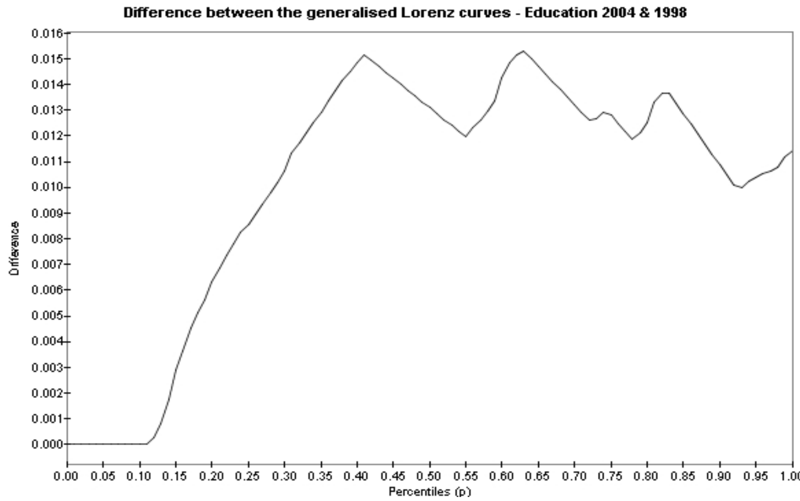
Figure 4



Furthermore the land poverty gap does not seem to be larger in 1998 compared to in 2004 for the two following cumulated expenditure groups, including moderately poor households and finally the total population. These results are confirmed when running the poverty gap dominance test (table 12-14 in appendix I). Together with the above presented result on generalized expenditure Lorenz dominance, this implies that multidimensional inequality has increased during the past six years also when land is set as an attribute compensating for low monetary levels.

The examination of changes in multidimensional inequality when including expenditures and education once again points to that the expenditure distribution in 1998 dominates the corresponding distribution in 2004 according to the poverty gap condition (figure 5 in appendix I). As a matter of fact we find that the income poverty gap condition is fulfilled for all educational groups separately, i.e. not cumulated. This is a stronger condition that corresponds to a situation where there is no agreement on how the marginal utility of income varies with the need of a household in terms of the second attribute education, i.e. no conformity on the second cross partial derivative (Atkinson and Bourguignon, 1987). This result imply that the mean expenditure of all education groups are not lower in 1998 than in 2004. On the other hand, in interpreting the table below, we find that the generalized educational Lorenz curve in 2004 dominates the corresponding curve in 1998. Consequently, we can not come to an unambiguous conclusion whether multidimensional inequality including a monetary and an educational viewpoint increased or decreased over the time period

Table 6



This outcome is moreover confirmed when studying the educational poverty gap across expenditure groups (figure 7 in appendix I) where it is apparent that the dominance condition does not hold in the first partition, as the curves showing the difference between educational poverty gaps takes positive as well as negative values.

Turning to the combination of expenditures and health, the results resemble of what is apparent in the expenditure and education setting. The income poverty gap is lower in 1998 than in 2004 for the cumulated health quintiles but the generalized Lorenz condition is not met for the attribute health as the two generalized health Lorenz curves cross at lower ends (figure 8 and 9 in appendix I). In addition, the health poverty gap condition when the distribution of health is partitioned on poverty groups is not fulfilled. Accordingly, this signals a conflict between the points of views captured by the dominance test.

The results of the different tests for the combinations land and education, land and health and education and health respectively are not included. Since the generalized Lorenz condition does not hold or point in different direction for one or more of the attributes in these combinations we conclude that there is no possibility to find an unambiguous conclusion concerning changes in multidimensional inequality including these attributes.[23]

The generality of the conclusions that can be drawn by using the stochastic dominance technique developed by Muller and Trannoy (2003) and Trannoy (2005) is attractive. However, as can be seen in this empirical application, such generality comes at a cost. When the cumulative density functions of a particular compensating distribution cross one or more times or when the generalized Lorenz condition does not hold for the attribute being compensated, there is no clear ordering. Consequently, by imposing less structure on how to exactly combine distributions compared to the procedure followed when applying an aggregative approach and the striving for ordinality to cardinality, we cannot tell whether inequality is lower in one year or the other in a majority of cases.

[23] All results in this section are confirmed using a different discrete partition of the variables. In this case the dataset is divided into quintiles corresponding to 20% of the population.

6. Concluding Remarks

Although there is no complete agreement on how to measure inequality of several dimensions of welfare, there are numerous reasons to consider inequality as a multidimensional phenomenon. Moreover, with a lack of consensus there is rationale to employ available methods to real data to gain knowledge on their strengths, weaknesses and usefulness in an empirical context and to contrast differing results. In this paper we have applied three of the existing techniques to the measurement of multidimensional inequality, using two Zambian sets of household data, to the particular question whether inequality of several dimensions increased or decreased during a period of six years.

Cardinal measures are clearly practical when examining changes in welfare as these exercises always generate a yes or a no to whether equality increased or decreased across time, regions or socio-demographic groups. The outcome of a scalar is accordingly an advantage of the item-by-item as well as the aggregative approach. When examining the attributes expenditures, education, health and land holdings separately, all distributions but education prove to have less dispersion at the first point in time compared to the second, for all inequality measures examined. However, this approach also unmask that correlations between different dimensions of welfare are not very strong, and somewhat changed over time. Accordingly, we here gets a first indication to that it is not necessarily the same households that experience a poor situation in one welfare dimension that also face an underprivileged position in another. This information has implications to the results generated in employing an aggregative and a non-aggregative approach, explicitly taking the interrelation between attributes into account.

In applying Maasoumi's multidimensional index we find evidence of why inequality measurement in any single attribute might be misleading as an overall measure. We observe that the increase in health inequality between 1998 and 2004 is neither evident nor tempered by the decrease in educational inequality during the same time period. Consequently, different dimensions of household welfare can compensate each other. Moreover, it is by no means certain that combinations of distributions that separately indicates increasing inequality over time or space necessarily point to that multidimensional inequality increases. To say the least, this implies that the use of one indicator of inequality alone, e.g. expenditures, may generate a rather incomplete picture. Moreover, as the interrelations between welfare dimensions are of importance to what conclusions to be drawn on changes in multidimensional inequality the strength of the clear-cut results from the item-by-item approach is somewhat dampened.

Concerning the empirical usefulness to measurement of multidimensional inequality using the aggregative technique it is of importance to note that a number of results are disturbingly sensitive to the degree of substitution between attributes. In a majority of cases the choice of β matters to what conclusions on changes in inequality can be drawn. Although our conclusions on changes in inequality are robust to the choice of inequality measure, the sensitivity of outcomes to a particular aggregation functions chosen, points to the importance of sensitivity analysis and explicitness regarding what particular assumptions are made when applying an aggregative approach.

With reference to the application of the non-aggregative approach, only the combination of land and expenditures fulfill the required dominance conditions both when expenditure is treated as a compensating attribute and land is compensated and vice versa. These results indicate that multidimensional inequality was unambiguously higher in 2004. Consequently, an agreement between the outcomes of the different techniques concerns the setting with land and expenditure with which there are straightforward results from all three approaches. Both distributions show signs of larger disparity according to all univariate inequality measures and the bivariate distribution examined by the Maasoumi index point to higher inequality in 2004 compared to 1998 for all degrees of substitution and choices of inequality aversion.

Regarding all other arrangements of attributes we cannot come to a clear-cut conclusion in using the non-aggregative approach. Thus, also in this application it is evident that combined arrangements of distributions that independently points toward increased dispersion do not identify a raise in multidimensional inequality. Furthermore, avoiding the computational complexity of aggregated measures of multidimensional inequality and allowing a less demanding structure comes at a cost, as we in a majority of cases cannot tell whether inequality is lower or higher in one time period or the other. On the other hand, one may argue that the non-aggregative technique that, at the same time as it seeks to avoid the arbitrariness implied by the choices we have to do concerning weighting, substitution and inequality aversion in calculating multidimensional indices, can take into account simultaneous impact from different welfare policies.

In reviewing three theoretical developments for measurement of changes in multidimensional inequality we find additional support to the claim in existing welfare literature that the analysis of inequality ought to multidimensional. A clear implication of our operationalization is that, at minimum, researchers and policymakers should check the correlations between welfare distributions. Bearing in mind that weak relationships between welfare distributions are recurrent, it is not unlikely that examinations combining and taking into account the interrelations between different household attributes, may lead to unexpected multivariate comparisons.

Our examination does not give a clear cut picture to whether multidimensional inequality, using four attributes of household welfare, increased or decreased in Zambia between 1998 and 2004. However, as declared by Sen (1997), the concept of inequality is ambiguous and rather than leaving analyzes of inequality of several dimensions undone, we should continue the research in this field. Clearly, more empirical applications employing existing techniques are needed as the gap between theoretical and empirical research in this field is substantial. The usefulness of the different techniques to measurement and policy analysis is reasonable given that we are aware of their intrinsic and weaknesses. Careful interpretations and analyzes involving more than one technique is constructive to portray multidimensional inequality.

Appendix I

Table 4b Maasoumi Multidimensional Inequality Index across ranges of different degrees of substitution (β)

S2: Education & Helath	β=0.1	β=0.2	β=0.3	β=0.4	β=0.5	β=0.6	β=0.7	β=0.8	β=0.9
Zambia 1998									
M(0)	0.209	0.211	0.213	0.214	0.216	0.217	0.218	0.220	0.221
M(1)	1.472	1.744	1.894	1.980	2.034	2.071	2.097	2.118	2.134
Zambia 2004									
M(0)	0.169	0.170	0.172	0.173	0.175	0.176	0.177	0.178	0.179
M(1)	1.410	1.655	1.777	1.844	1.885	1.913	1.933	1.949	1.961

Source: Author's calculations from LCMS II and LCMS IV

Table 4c Maasoumi Multidimensional Inequality Index across ranges of different degrees of substitution (β)

S2: Education & Helath	β=-0.1	β=-0.2	β=-0.3	β=-0.4	β=-0.5	β=-0.6	β=-0.7	β=-0.8	β=-0.9
Zambia 1998									
M(0)	0.199	0.173	0.139	0.111	0.090	0.076	0.065	0.058	0.052
M(1)	0.628	0.363	0.226	0.155	0.116	0.092	0.076	0.065	0.058
Zambia 2004									
M(0)	0.162	0.144	0.119	0.098	0.083	0.072	0.064	0.057	0.050
M(1)	0.547	0.306	0.193	0.138	0.107	0.088	0.075	0.064	0.058

Source: Author's calculations from LCMS II and LCMS IV

Table 4d Maasoumi Multidimensional Inequality Index

		S1: Education	S2: Education & Health	S3: Education, Health & Expenditures	S4: Education, Health, Expenditures & Land
Zambia 1998					
M(1)	β = -0.99999		0.055	0.052	0.052
M(1)	β=-1/2		0.116	0.114	0.109
M(1)	β = 0.000001		1.050	0.748	0.782
M(1)	β=1/2		2.034	1.614	3.010
M(1)	β=1		2.147	1.725	3.034
M(1)	β=20	0.262	2.265	1.741	3.537
Zambia 2004					
M(1)	β = -0.99999		0.055	0.055	0.055
M(1)	β=-1/2		0.107	0.105	0.097
M(1)	β = 0.000001		0.978	0.715	1.438
M(1)	β=1/2		1.885	1.765	5.855
M(1)	β=1		1.971	1.901	6.198
M(1)	β=20	0.215	2.068	1.933	6.396

Source: Author's calculations from LCMS II and LCMS IV

Figure 5

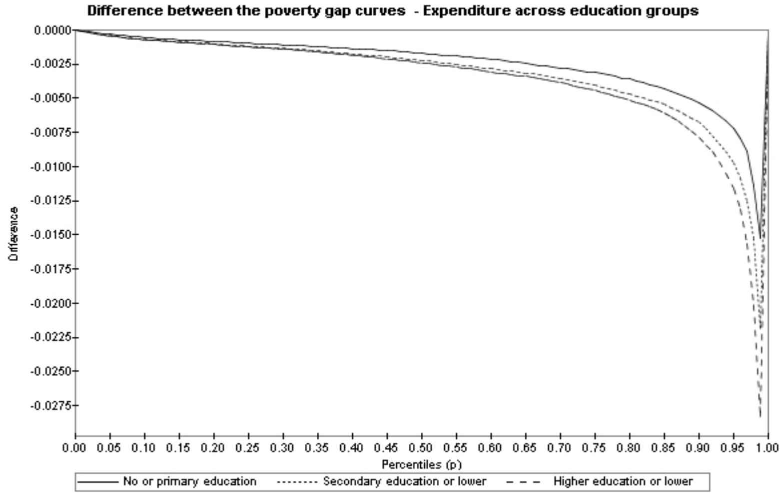


Figure 7

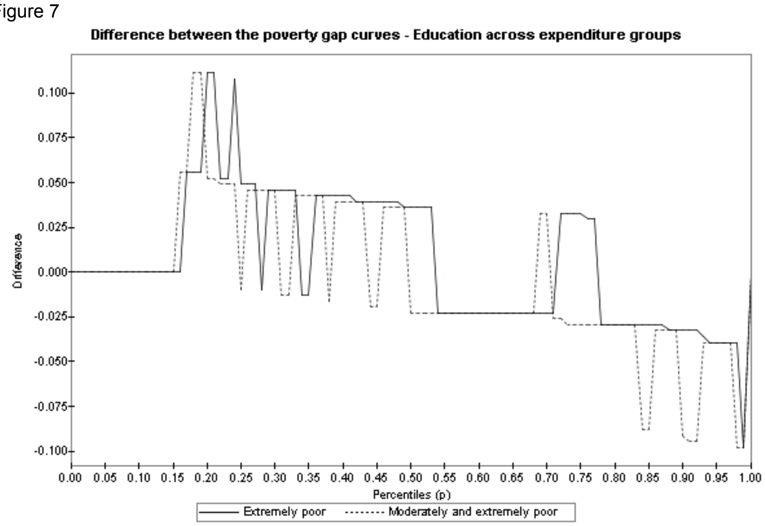


Figure 8

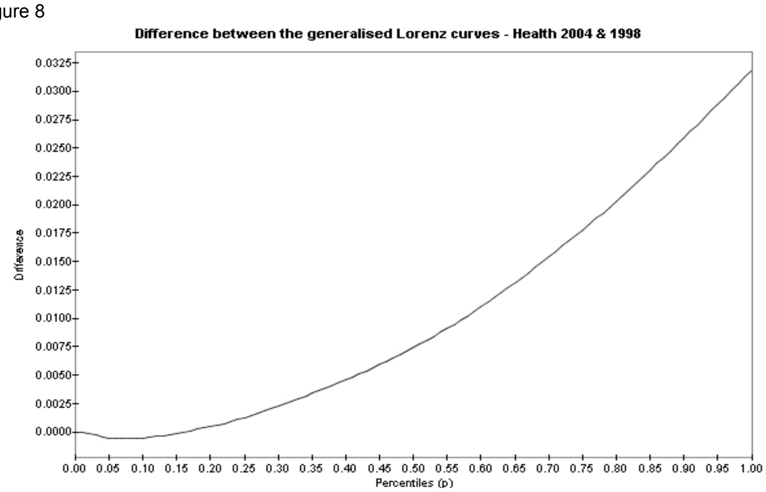
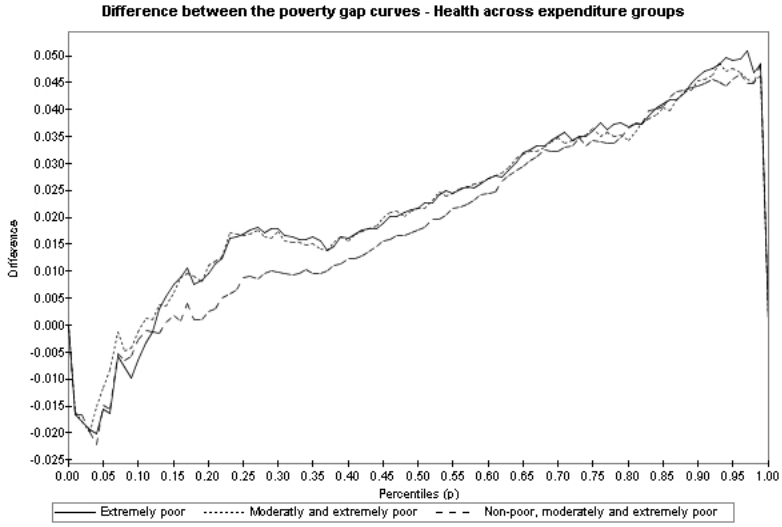


Figure 9



In the below exercise we run poverty gap dominance test for the expenditure distribution in 2004 versus the expenditure distribution in 1998 in three stages. In the first stage we test for expenditure poverty gap dominance when the sample consists of landless households. In the second stage the sample consist of all households cultivating less than 2 ha of land and in the final test all households are included. In a corresponding way we also test for land poverty gap dominance. In this case the first sample consists of extremely poor households; the second consists of households with monthly real per equivalent expenditure below the moderate poverty line. In the final test all households are included.

Table 9 Poverty Gap Dominance

Expenditures across Land groups - Landless

z	2004	1998	t-statistics
0.0005	0.0001	0	-50.73
0.0505	0.0496	0.0442	-62.68
0.1005	0.0996	0.0941	-56.41
0.1505	0.1496	0.1441	-54.07
0.2005	0.1995	0.1941	-52.18
0.2505	0.2495	0.2441	-50.41
0.3005	0.2995	0.2941	-49.3
0.3505	0.3495	0.3441	-48.72
0.4005	0.3995	0.3941	-48.15
0.4505	0.4495	0.4441	-47.56
0.5005	0.4995	0.4941	-46.93
0.5505	0.5495	0.5441	-46.25
0.6005	0.5995	0.5941	-45.55
0.6505	0.6495	0.6441	-44.82
0.7005	0.6995	0.6941	-44.08
0.7505	0.7495	0.7441	-43.31
0.8005	0.7995	0.7941	-42.54
0.8505	0.8495	0.8441	-41.76
0.9005	0.8995	0.8941	-40.98
0.9505	0.9495	0.9441	-40.2
1	0.999	0.9936	-39.43

Source: Author's calculations from LCMS II and LCMS IV

by using the program domsvy written by D.Stifel 2004

Table 10 Poverty Gap Dominance

Expenditures across Land groups - Small scale and Landless

z	2004	1998	t-statistics
0.0005	0.0001	0	-92.42
0.0505	0.0498	0.046	-76.22
0.1005	0.0998	0.0959	-67.13
0.1505	0.1498	0.1458	-62.82
0.2005	0.1998	0.1958	-61.02
0.2505	0.2498	0.2458	-59.76
0.3005	0.2998	0.2958	-58.87
0.3505	0.3498	0.3458	-58.39
0.4005	0.3998	0.3958	-57.91
0.4505	0.4498	0.4458	-57.39
0.5005	0.4998	0.4958	-56.84
0.5505	0.5498	0.5458	-56.24
0.6005	0.5998	0.5958	-55.61
0.6505	0.6498	0.6458	-54.95
0.7005	0.6998	0.6958	-54.27
0.7505	0.7498	0.7458	-53.55
0.8005	0.7998	0.7958	-52.83
0.8505	0.8498	0.8458	-52.08
0.9005	0.8998	0.8958	-51.33
0.9505	0.9498	0.9458	-50.56
1	0.9993	0.9953	-49.8

Source: Author's calculations from LCMS II and LCMS IV

by using the program domsvy written by D.Stifel 2004

Table 11 Poverty Gap Dominance

Expenditures across Land groups - Large scale, Small scale and Landless

z	2004	1998	t-statistics
0.0005	0.0001	0	-113.29
0.0505	0.0498	0.0461	-81.17
0.1005	0.0998	0.096	-70.46
0.1505	0.1498	0.1459	-65.74
0.2005	0.1998	0.1959	-63.3
0.2505	0.2498	0.2459	-61.33
0.3005	0.2998	0.2959	-59.65
0.3505	0.3498	0.3459	-59.15
0.4005	0.3998	0.3959	-58.66
0.4505	0.4498	0.4459	-58.2
0.5005	0.4998	0.4959	-57.89
0.5505	0.5498	0.5459	-57.56
0.6005	0.5998	0.5959	-57.21
0.6505	0.6498	0.6459	-56.83
0.7005	0.6998	0.6959	-56.44
0.7505	0.7498	0.7459	-56.02
0.8005	0.7998	0.7959	-55.58
0.8505	0.8498	0.8459	-55.13
0.9005	0.8998	0.8959	-54.66
0.9505	0.9498	0.9459	-54.18
1	0.9993	0.9954	-53.69

Source: Author's calculations from LCMS II and LCMS IV

by using the program domsvy written by D.Stifel 2004

Table 12 Poverty Gap Dominance

Land across Expenditure groups -Extremely poor

z	2004	1998	t-statistics
0.0015	0.0007	0.0003	-35.46
0.0515	0.0502	0.0421	-64.55
0.1015	0.1002	0.0917	-57.91
0.1515	0.1502	0.1416	-55.24
0.2015	0.2002	0.1916	-54.46
0.2515	0.2502	0.2416	-54.32
0.3015	0.3002	0.2916	-54.25
0.3515	0.3502	0.3416	-54.19
0.4015	0.4002	0.3916	-54.16
0.4515	0.4502	0.4416	-54.16
0.5015	0.5002	0.4916	-54.16
0.5515	0.5502	0.5416	-54.16
0.6015	0.6002	0.5916	-54.16
0.6515	0.6502	0.6416	-54.16
0.7015	0.7002	0.6916	-54.16
0.7515	0.7502	0.7416	-54.16
0.8015	0.8002	0.7916	-54.16
0.8515	0.8502	0.8416	-54.16
0.9015	0.9002	0.8916	-54.16
0.9515	0.9502	0.9416	-54.16
1	0.9987	0.9901	-54.16

Source: Author's calculations from LCMS II and LCMS IV

by using the program domsvy written by D.Stifel 2004

Table 13 Poverty Gap Dominance

Land across Expenditure groups - Moderately and Extremely poor

z	2004	1998	t-statistics
0.0015	0.0007	0.0004	-35.99
0.0515	0.0502	0.0423	-70.35
0.1015	0.1002	0.092	-63.62
0.1515	0.1502	0.1419	-61.12
0.2015	0.2002	0.1919	-60.34
0.2515	0.2502	0.2419	-60.22
0.3015	0.3002	0.2919	-60.16
0.3515	0.3502	0.3419	-60.1
0.4015	0.4002	0.3919	-60.08
0.4515	0.4502	0.4419	-60.08
0.5015	0.5002	0.4919	-60.08
0.5515	0.5502	0.5419	-60.08
0.6015	0.6002	0.5919	-60.08
0.6515	0.6502	0.6419	-60.08
0.7015	0.7002	0.6919	-60.08
0.7515	0.7502	0.7419	-60.08
0.8015	0.8002	0.7919	-60.08
0.8515	0.8502	0.8419	-60.08
0.9015	0.9002	0.8919	-60.08
0.9515	0.9502	0.9419	-60.08
1	0.9987	0.9904	-60.08

Source: Author's calculations from LCMS II and LCMS IV

by using the program domsvy written by D.Stifel 2004

Table 14 Poverty Gap Dominance

Land across Expenditure groups - Non-poor, Moderately and Extremely poor

z	2004	1998	t-statistics
0.0015	0.0008	0.0005	-36.96
0.0515	0.0503	0.0433	-77.83
0.1015	0.1002	0.0929	-69.58
0.1515	0.1502	0.1428	-65.81
0.2015	0.2002	0.1927	-63.76
0.2515	0.2502	0.2427	-62.75
0.3015	0.3002	0.2927	-62.02
0.3515	0.3502	0.3427	-61.37
0.4015	0.4002	0.3927	-60.9
0.4515	0.4502	0.4427	-60.45
0.5015	0.5002	0.4927	-60
0.5515	0.5502	0.5427	-59.51
0.6015	0.6002	0.5927	-59.01
0.6515	0.6502	0.6427	-58.49
0.7015	0.7002	0.6927	-58.05
0.7515	0.7502	0.7427	-57.62
0.8015	0.8002	0.7927	-57.18
0.8515	0.8502	0.8427	-56.78
0.9015	0.9002	0.8927	-56.43
0.9515	0.9502	0.9427	-56.07
1	0.9987	0.9912	-55.71

Source: Author's calculations from LCMS II and LCMS IV

by using the program domsvy written by D.Stifel 2004

Appendix II

Estimation of Health Distributions
Household surveys rarely include objective indicators on health, i.e. results from check-ups and examinations. On the other hand, there is often information on self-reported health in these kinds of surveys, such as an answer to the question how is your perceived health, which has ordered response categories. In both the Zambian LCMS II and the LCMS IV there are health sections where individuals of a household state whether they have been ill or not the past 14 days or if they are chronically ill which is ordered as (1), (2) and (3) where a lower value indicates the better health status .

Since the true scale of these responses most likely is not equidistant between categories, it is incorrect to simply score the different answers as 1, 2, 3 etc. To exploit the full range of categories in the question, without imposing the unrealistic assumption of equal distances between categories, it is possible to generate predictions of an underlying latent variable using an ordered probit regression model following the estimation procedure the World Bank's technical note #3 (2005) on quantitative techniques for health equity analysis, further described in van Doorslaer and Jones (2003). These predictions are then rescaled to a 0-1 interval to construct a health distribution.

An ordered probit model can be used to model a discrete dependent variable that takes ordered outcomes for each individual *i*, e.g. $y_i = 1,2,....m$. In technical terms we can express the model as

$$y_i = j \quad \text{if} \quad \mu_{j-1} < y_i^* \le \mu_j , \quad j=1, 2, 3$$

Where the latent variable y^* is assumed to be a function of socio-economic variables x

$$y_i^* = x_i\beta + \varepsilon_i \ , \qquad \varepsilon_i \sim N(0,1)$$

and $\mu_0 = -\infty, \mu_j \le \mu_{j+1}, \mu_m = \infty$.

The probability of observing a particular value of y is
$$P_{ij} = P(y_i = j) = \Phi(\mu_j - x_i\beta) - \Phi(\mu_{j-1} - x_i\beta)$$

where $\Phi(.)$ is the standard normal distribution function. With independent observations, the log-likelihood for the ordered probit model takes the form $\log L = \sum_i \sum_j y_{ij} \log P_{ij}$ where y_{ij} are binary variables equal to 1 if $y_i = j$. This can be maximized to give estimates of β and the unknown threshold values μ_j .Predictions of the linear index $x_i\beta$ can now be used as a measure of individual health after rescaling to the [0,1] interval.

In the Zambian case we estimate ordered probit models with data from the 1998 and the 2004 surveys with the above described categorical discrete health status indicator as a dependent variable. A number of models are estimated including different independent variables that might explain an individual's health status. If the dependent variable does not vary within every categories of an independent variable there will be a problem with estimation. Therefore we first control that it is possible to predict perfectly. The different models estimated are compared using scalar measures of fit. In the final models we include an urban rural dummy (urbrur), the size of the household (hhsize), a dummy for unprotected or protected water source (unprot_water) and dummies for sex and age of the individual (M0_4-K70_) as independent variables. These are all independent variables used in the literature. We do not include information on education level or income since the measures then become highly dependent on these variables, which might be a problem in the examination of multidimensional inequality. The estimation results are presented in table A.

The predictions of the linear indexes in 1998 and 2004 respectively generate a measure for every observation from which we calculate the household mean. Finally this household variable, z, is rescaled by the following formula

$$x_i = \frac{z_i - \min z}{\max z - \min z}$$

this is the final step in the construction of the two health distributions.

Table A Ordered Probit Estimates

	Health_1998		Health_2004
rururb	-0.097***		-0.107***
	[0.000]		[0.000]
household size	-0.032***		-0.021***
	[0.000]		[0.000]
poor	0.058***		0.080***
	[0.000]		[0.000]
unprot_water	0.058***		0.074***
	[0.000]		[0.000]
M0_4	1.126***		0.874***
	[0.000]		[0.000]
M5_9	-0.119*		-0.506***
	[0.093]		[0.000]
M10_14	-0.545***		-0.685***
	[0.000]		[0.000]
M15_19	-0.636***		-0.736***
	[0.000]		[0.000]
M20_24	-0.543***		-0.632***
	[0.000]		[0.000]
M25_29	-0.355***		-0.507***
	[0.000]		[0.000]
M30_34	-0.257***		-0.362***
	[0.001]		[0.000]
M33_39	-0.287***		-0.351***
	[0.000]		[0.000]
M40_44	-0.253***		-0.330***
	[0.002]		[0.000]
M45_49	-0.166**		-0.303***
	[0.042]		[0.000]
M50_54	-0.181**		-0.210***
	[0.034]		[0.007]
M55_59	-0.152*		-0.154*
	[0.090]		[0.056]
M60_64	-0.082		-0.079
	[0.394]		[0.364]
M65_69	0.147*		-0.05
	[0.094]		[0.583]
M70_	1.021***		0.186**
	[0.000]		[0.017]
K0_4	-0.127*		0.786***
	[0.072]		[0.000]
K5_9	-0.562***		-0.501***
	[0.000]		[0.000]
K10_14	-0.448***		-0.675***
	[0.000]		[0.000]
K15_19	-0.247***		-0.540***
	[0.001]		[0.000]
K20_24	-0.190***		-0.440***
	[0.009]		[0.000]
K25_29	-0.136*		-0.354***
	[0.067]		[0.000]
K30_34	-0.137*		-0.262***
	[0.072]		[0.000]
K35_39	-0.077		-0.180***
	[0.323]		[0.010]
K40_44	-0.091		-0.162**
	[0.263]		[0.022]
K45_49	-0.02		-0.141*
	[0.811]		[0.055]
K50_54	-0.04		-0.07
	[0.653]		[0.360]
K55_59	0.073		-0.104
	[0.419]		[0.191]
K60_64	0.088		0.079
	[0.355]		[0.342]
K70_	0.129		0.169**
	[0.164]		[0.032]
Observations	91572	91572	102643
Pseudo R2	0.1371	0.1371	0.1317

p values in brackets

* significant at 10%; ** significant at 5%; *** significant at 1%

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