Informal Insurance and Income Inequality∗

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Abstract

This paper examines the effects of income inequality in a risk sharing model with limited commitment, that is, when insurance agreements have to be self-enforcing. In the context of the model of Ligon, Thomas, and Worrall (2002), numerical dynamic programming is used to examine three questions. First, we consider heterogeneity in mean income, and study the welfare effects when inequality together with aggregate income increases. Second, subsistence consumption is introduced to see how it affects consumption smoothing. Finally, income is endogenized by allowing households to choose between two production technologies, to look at the importance of consumption insurance for income smoothing.

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1 Introduction

In low-income village economies we often observe incomplete markets. Financial instruments or formal insurance contracts are often lacking. However, growing empirical evidence suggests that households enter into informal risk sharing arrangements and achieve some, though not perfect insurance. The questions are then, (i) how this partial insurance can be modeled, (ii) what are its implications for the consumption and welfare of households, and (iii) what policies are appropriate in this context. This paper considers a model where informal insurance is characterized by limited commitment, in other words, insurance arrangements have to be self-enforcing. This setting allows us to explain the observed partial insurance and shed some light on the mechanisms involved.

Examining informal risk sharing in the context of developing countries is important for two main reasons. On the one hand, people living in low-income, rural areas often face a huge amount of risk. Revenue from agricultural production is usually low and volatile, further, outside job opportunities are often lacking. On the other hand, financial instruments, or formal, legally enforceable insurance contracts are often not available to smooth consumption inter-temporally or across states of nature. The question is then, how can people in these kinds of environments somehow mitigate the effects of risk they face. Growing empirical evidence suggests that households achieve something better than autarky, but not quite perfect risk sharing (see the seminal paper by Townsend (1994), among many others), by transfers, gifts, quasi-credit, and the like among relatives, neighbours, or friends (see, for example, anthropological work by Platteau and Abraham (1987) and Platteau (1997)). This means that consumption reacts to idiosyncratic changes in income, but the variance of consumption is less than that of income.

Informal insurance is modeled in this paper by supposing that contracts have to be self-enforcing, because often no authority exists to enforce insurance agreements in poor villages in developing countries, while informational problems are less important. This approach yields partial insurance, which is consistent with empirical evidence. The model has a wide range of interpretations. In addition to thinking about households in a village, we may consider members of a family (Mazzocco, in press), an employee and an employer (Thomas and Worrall, 1988), or countries (Kehoe and Perri, 2002).

In this paper an infinite-time model is considered with risk-averse households, whose income follows some exogenous, discrete stochastic process, that
is common knowledge. I concentrate on insurance across states of nature, and ignore savings, or storage. I look for a constrained-efficient solution, maximizing a utilitarian social welfare function subject to resource constraints and enforcement constraints. That is, it is required that, for each household at every period and every state of the world, staying in the informal risk sharing contract be better than reverting to autarky. If income is independently and identically distributed (iid) or follows a Markov-process, we have the following important property characterizing the solution: the current ratio of marginal utilities between households, and therefore the consumption allocation, depends only on current income realizations and the ratio of marginal utilities in the previous period. In addition, unlike in the perfect risk sharing case, the allocation in the limited commitment solution depends not only on aggregate income, but also on its distribution. This is because individual income determines the utility a household may get were she in autarky, that is, her threatpoint, or bargaining power.

This paper examines the interaction of income inequality and self-enforcing risk sharing contracts. To do this, three types of simulation exercises are performed in the context of the model of risk sharing with limited commitment. In all cases I assume that only two households populate the village economy, and that each household’s income may take only two values, for clarity and computational ease.

First, we consider a “poor” household interacting with a “rich” one. The households have the same constant relative risk aversion (CRRA) utility function, and they differ in their mean income, while they face the same amount of risk in the sense that the coefficient of variation of their income process is the same. I perform a comparative statics exercise: while keeping the income process of the poor the same, the mean income of the rich is increased, thereby increasing inequality together with aggregate and per-capita income. Note that we do not expect this type of inequality to have any adverse effects, since what happens is just that in each state of the world we give more income to the rich, while leaving the income of the poor unchanged. However, for some reasonable parameter values, the poor is worse off when inequality together with per-capita income increases. This is in contrast with Genicot (2006), who emphasizes the possible positive effects of inequality, keeping aggregate income constant. Another difference from the present paper is that she restricts contracts to be static, which have been shown not to be constrained-efficient in the dynamic setting. The intuition behind my result is that the poor household’s relative bargaining
power decreases vis-a-vis the rich, thus she can secure smaller net transfers in the limited commitment solution. Another way of putting it is that the poor can provide less insurance to the rich as the later’s income increases, thus the rich does not value the contract much. The result warns of the possible adverse consequences of inequality for the poor even when per-capita income increases in the community, the reason being that the poor is more and more excluded from informal insurance arrangements.

Second, I take just one pair of income processes, but “subsistence consumption”, or, a “subsistence level” is added. In other words, I suppose decreasing relative risk aversion (Ogaki and Zhang, 2001). The effects of changes in the subsistence level is examined in this example. A higher subsistence level makes insurance more valuable for both agents, thus it may make perfect risk sharing self-enforcing. Still, the utility values should decrease as we increase subsistence consumption. However, the poor household’s expected welfare may increase. This happens when perfect insurance becomes self-enforcing. Further, here it is interesting to look at the properties of the consumption process, since income does not change. The consumption of the poor becomes less volatile as the subsistence level increases, but she has to sacrifice mean consumption to compensate the rich for the insurance she provides. However, mean consumption of the poor increases at one point, and this special point is once again when the subsistence level increases so that perfect risk sharing becomes self-enforcing.

Finally, in the last example economy, income in endogenized. In particular, the possibility to choose between two production technologies is introduced, to examine the consequences of lack of insurance for income smoothing (Morduch, 1995). A technology is described by the income process it generates. As in the first example, households have standard CRRA utility functions. We consider two types of heterogeneity, (i) the rich household has some exogenous wealth that yields a fixed revenue every period, and (ii) the rich is less risk averse than the poor\(^1\). Note that also in case (i), the rich behaves in a less risk averse fashion. Further, households may choose between two technologies, an “old”, safer technology with lower expected values, and a “new”, riskier, but more profitable technology. Two numerical examples are considered, where switching between the two technologies would only

\(^1\)In this last case, the terms “rich and “poor” are not really appropriate. Ex ante, households differ only in their risk preferences, and only because of the eventual difference in technology choice, the “rich” may end up having higher expected income.
occurs at the time of reverting to autarky. In both numerical examples one household chooses a different technology as a result of the availability of an informal risk sharing contract, in particular, she switches to the riskier technology with higher expected profits. This result illustrates the importance of consumption insurance for production choices, and the negative consequences high risk aversion may have on expected profits, for example when households living near the subsistence level are willing to bear very little risk.

The rest of the paper is structured as follows. Section 2 discusses some related literature. Section 3 outlines the model of risk sharing with limited commitment, and talks about some characteristics of the solution. An algorithm to numerically solve the model is described in the appendix. Section 4 presents simulation results to examine the interaction between informal risk sharing and income inequality. Section 5 concludes.

2 Related Literature

There is a growing literature on informal insurance in rural communities in developing countries. It has been recognized that even without formal contracts, households enter into risk sharing arrangements. In a world with complete information and perfect commitment, informal insurance would even achieve the first best, or full insurance, that is, the ratios of marginal utilities would stay the same in all states of nature and across time. This perfect risk sharing outcome can be imagined as the case where incomes are pooled in the village, and then redistributed according to some predetermined weights. A number of papers test the hypothesis of full insurance in low-income village economies (see Townsend (1994) for Indian villages in the semi-arid tropics, Grimard (1997) using data from Ivory Coast, Dubois (2000) on Pakistan, Dercon and Krishnan (2003a, 2003b) working with Ethiopian data, Laczo (2005) using Bangladeshi data, and Mazzocco and Saini (2006) for India, among others). Perfect insurance is rejected, but a remarkable amount of risk sharing is found. Thus a next step is to think about partial insurance, how and why households achieve something better than autarky, but not full insurance.

In modeling partial insurance we may relax the assumption of complete information or perfect commitment. Ligon (1998) introduces private information in a dynamic setting. He derives Euler-equation type reduced form restrictions to test the private information model against the alternatives of
full insurance and the permanent income hypothesis. Ligon (1998) finds that consumption in two of the three Indian villages examined is best explained by the private information model, while in the third village different households seem to belong to different regimes, but most of them are classified as belonging to the permanent income regime. Wang (1995) establishes some theoretical results for the model of risk sharing with private information, and provides an algorithm to compute the solution.

The second approach is to relax the assumption of perfect commitment, and instead require contracts to be self-enforcing. One may argue that this way of modeling partial insurance in small, rural communities is more appropriate, since households are able to observe what their neighbors are doing and shocks they face (crop damage, or illness for example), but there is no commitment device, like an independent authority, to enforce contracts. In addition, arguably this model is also appropriate when one thinks about risk sharing within the family, since husband and wife are free to end the contract, that is, they may divorce. Introducing lack of commitment extends the standard collective model of the household (Browning and Chiappori, 1998) in an interesting way (see Mazzocco (in press)). Another interpretation is long-term labour contracts, where both employer and employee may choose to end the contract in favour of an outside option (Thomas and Worrall, 1988). A further application concerns the interaction between two countries, since a country may default on its sovereign debt, facing possible exclusion from future international trade and financial contracts (see Kehoe and Perri (2002)). Schechter (2007) uses the model to explain the interaction between a farmer and a thief.

One-sided limited commitment is relevant for principal-agent models, for example in the case of a contract between an insurance company and an insured, where the insurance company (the principal) is fully committed, while the insured (the agent) is not. For empirical evidence on one-sided limited commitment see the work of Hendel and Lizzeri (2003) on life insurance, and Crocker and Moran (2003) on health insurance. Two-sided limited commitment is introduced in a dynamic wage contract setting by Thomas and Worrall (1988). A very important result they derive is that contracts are history dependent, that is, past outcomes influence today’s payoffs.

Kimball (1988) is the first to argue that informal risk sharing in a community may be achieved with voluntary participation of all members. He shows that for reasonable values of the discount factor and the coefficient of relative risk aversion, households could provide a substantial amount of
insurance to one another. Early contributions to modeling risk sharing with limited commitment include Coate and Ravallion (1993), who introduce two-sided limited commitment in a dynamic model, but they restrict contracts to be static. Their characterization of transfers is actually not optimal, once we allow for history-dependent contracts. On the other hand, Kocherlakota (1996) allows for dynamic contracts, and proves existence and some properties of the solution, but he does not give an explicit characterization. Early empirical evidence on dynamic limited commitment is provided by Foster and Rosenzweig (2001). They test the restriction that there is a negative relationship between the current transfer and aggregate past transfers, and they find some supporting evidence. Anthropological work by Platteau (1997) also points out the importance of limited commitment in informal risk sharing contracts. Charness and Genicot (2006) provide experimental evidence in support of the model.

Ligon, Thomas, and Worrall (2002) characterize and calculate the solution of a dynamic model of risk sharing with limited commitment. As a result, the authors are able to test in a structural manner the hypothesis of dynamic limited commitment against the alternatives of perfect risk sharing, autarky, and the static limited commitment model of Coate and Ravallion (1993). They find evidence in support of the dynamic limited commitment model, using data from Indian villages. In addition, Ligon et al. (2002) derive a number of theoretical properties of the solution. In particular, they look at the effect of changing the discount factor, relative income across different states of the world (or different riskiness of the environment), and the direct penalty faced by the household breaking the agreement. More risk raises the demand for insurance, while a higher discount factor and harsher penalties help to enforce more risk sharing.

Attanasio and Ríos-Rull (2000) examine the effects of the introduction of an aggregate insurance scheme in a world with informal insurance and lack of commitment. They show, by an example, that aggregate insurance might reduce welfare. The reason is that aggregate insurance crowds out informal insurance, because it raises the value of autarky, and in some cases it even crowds out more insurance than it provides. The authors also provide some

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2 In this paper, Ligon et al. (2002) assume no savings. In another contribution (Ligon, Thomas, and Worrall, 2000) the authors look at the effects of savings, and show, by an example, that the possibility to save may decrease welfare. In general it is difficult to allow savings in a model with limited commitment, since savings enter the enforceability constraints, and I will assume away savings as well.
suggestive empirical evidence on the crowding out of private transfers by public ones using data from Mexico, but their approach is reduced form, and they do not actually use the theoretical model to predict private transfers.

An important innovation of the above papers is the methodology used to calculate the solution of the problem. Ligon et al. (2002) use a Pareto-frontier approach to find the solution of the risk sharing with limited commitment model. Attanasio and Ríos-Rull (2000) and Kehoe and Perri (2002) apply a slightly different methodology, building on the work of Marcit and Marimon (1998). In this approach the social planner’s problem is examined. The problem is a difficult one, since future decision variables enter into today’s enforcement constraints, thus the problem is not recursive. However, the weights of households’ utilities in the social planner’s objective, equal to the ratio of marginal utilities in equilibrium, can be introduced as a co-state variable. With the new (co-)state variable the problem has a recursive structure. I use this later approach in this paper.

Some extensions of the model of risk sharing with limited commitment have been developed recently. Genicot and Ray (2003) consider possible deviations by a group of households in an informal risk sharing arrangement among $n$ households. The main message of their paper is that the stability of a risk sharing group with respect to deviations by a smaller group is a complex issue, and there is not much we can say in general. One interesting result is that stable groups are limited in size. Wahhaj (2006) introduces public goods, and shows that in this case, private consumption of a member may increase when the community experiences an adverse aggregate shock. He argues that this result is consistent with empirical evidence provided by Duflo and Udry (2003) on intrahousehold allocation in Cote d’Ivoire. Dubois, Jullien, and Magnac (2007) consider both formal and informal contracts. Formal contracts are short-term, so households may complement these by self-enforcing, informal ones. The authors use semi-parametric techniques to test the model, and find that it explains well the consumption of Pakistani households. Hertel (2007) considers both limited commitment and private information. As a simplification, one household receives a fixed income each period, while the second household’s income is stochastic, and its realization is her private information. The author shows that, with additional incomplete information, consumption adjusts slowly to income changes, while there still exists a unique nondegenerate stationary distribution of utilities.

The literature examining the relation between insurance and inequality includes Morduch (1994), who draws attention to the fact that lack of in-
surance may exacerbate poverty. In a simple model, he shows that a lack of consumption credit may lead the poor to forego risky, but profitable investment opportunities. Fafchamps (2002) summarizes some results concerning different concepts of inequality (income, wealth, cash-in-hand, consumption, and welfare) in environments that differ in the type of assets available and in risk sharing opportunities. He briefly talks about the limited commitment case as well, and states that, the more efficient risk sharing is, the more persistent poverty is, and that limited commitment as a departure from perfect risk sharing allows for social mobility. Furthermore, the author talks about the emergence of patronage in polarized societies, meaning that the rich provides insurance to the poor in exchange for net transfers from the poor on average. With positive returns to assets, patronage is transitory, because in the long run the poor also accumulates sufficient assets to self-insure. If returns are negative, patronage reinforces inequality in the short run, while in the long run all wealth is depleted.

Genicot (2006) examines similar issues as the present paper. The author considers the model of risk sharing with limited commitment as well. She argues that (i) in some cases wealth inequality may help risk sharing in the sense that perfect risk sharing is possible in a wider range of cases, and that (ii) total welfare may increase with inequality, keeping aggregate, or per-capita, wealth constant. On the modeling side, an important shortcoming of the paper is that it only considers static contracts, which have been proven not to be constrained-efficient in the dynamic case. The present paper allows for history-dependent contracts.

3 Modeling Informal Insurance

This section presents the basic model. First, we look at perfect risk sharing as a benchmark. Then limited commitment is introduced, requiring contracts to be self-enforcing. The context is a stochastic, dynamic framework with common beliefs, and egoistic, risk-averse households consuming a private, perishable good.

For the sake of clarity, let us consider a village, or community, of two households. Extending the model to $n$ households is straightforward. The

\footnote{Note that Fafchamps (2002) defines welfare inequality as the ratio of marginal utilities, so there is no social mobility in terms of welfare in the perfect risk sharing case by definition.}

\footnote{The theoretical properties can easily be extended in both the perfect risk sharing and}

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households live in an uncertain environment: their income realizations are unknown ex ante. Income realizations are common knowledge ex post. As a consequence, they might choose to insure through a formal or informal agreement against variation of incomes. Risk sharing can thus be defined as follows. “Any two [households] may be said to share risk if they employ state-contingent transfers to increase the expected utility of both by reducing the risk of at least one.” (Ligon, 2004)

In section 3.1, I describe the model of perfect risk sharing. Formally, households may sign an enforceable contract in period 0, in other words, we assume full commitment, and that income realizations are observable by both agents and verifiable by a third party. In section 3.2, households still observe the income realizations, but they cannot sign formal insurance contracts, only informal risk sharing arrangements are possible instead, meaning that at each period and each state of the world it is required that both households respect voluntarily the terms of the agreement.

3.1 Perfect Risk Sharing

The basic framework considers a dynamic model of risk sharing. I concentrate on insurance across state of nature, and assume no savings, or storage. Assume that the economy is populated by two infinitely-lived households, indexed 1 and 2. Their preferences are identical, and separable over time and across states of nature. The utility function $u()$ is defined over a private, perishable consumption good, $c$. $u()$ is monotone increasing, strictly concave (so households are risk averse), and twice continuously differentiable. Households live in an uncertain environment, and income of each individual $y_t$ follows some exogenous discrete stochastic process, that is common knowledge. In other words, beliefs about the distribution of the state of nature, or the “income state” (the vector (income of 1, income of 2)), are homogeneous. In mathematical terms, each agent $i$ seeks to maximize the following von Neumann-Morgenstern expected utility:

$$E_0δ^t u(c_t),$$

where $E_0$ is the expected value at time 0 calculated with respect to the probability measure describing the common beliefs, $δ ∈ (0, 1)$ is the discount the limited commitment case. The algorithm to compute the solution also logically extends to $n$ households, however, in the limited commitment case, computation time might be prohibitive with $n$ large, since we face the curse of dimensionality.
factor, and \( c_{it} \) is consumption of household \( i \) at time \( t \).

Let \( s_t \) (with a lower index \( t \)) denote the income state at time \( t \), and \( s^t = (s_1, s_2, \ldots, s_{t-1}, s_t) \) (with an upper index \( t \)) the history of income states up to \( t \). Let us first consider autarky as a benchmark. In autarky, each household consumes her own income in every state and every period, since there is no possibility to save or borrow. In this case, household \( i \) receives the following expected lifetime utility:

\[
\sum_{t=1}^{\infty} \delta^t \pi (s^t) u \left( y_i (s^t) \right),
\]

where \( \pi (s^t) \) is the probability of history \( s^t \) occurring, and \( y_i (s^t) \) denotes the income of individual \( i \) at time \( t \) when history \( s^t \) has occurred.

Now, suppose that households may sign an enforceable risk sharing contract. A risk sharing contract specifies transfers that may depend, a priori, on the whole history of income states \( s^t \). The timing is the following. At time 0, a risk sharing contract may be signed, then, at time 1 and each subsequent period, the income state is realized, transfers are made according to the contract, and finally, consumption takes place.

First, the properties of the contract are described, given that it is signed. Then, we examine under what conditions agents are ready to actually sign the contract at time 0, in other words, we look at the ex-ante participation constraints.

In the presence of complete information, that is, in each period each household perfectly observes the other household’s income realization, and under full commitment, ex-ante Pareto-optimal allocations can be found by considering the social planner’s problem. The problem faced by the social planner is to maximize a weighted sum of households’ lifetime utilities:

\[
\max_{\{c_{i}(s^t)\}} \sum_{i} \lambda_i \sum_{t=1}^{\infty} \sum_{s^t} \delta^t \pi (s^t) u \left( c_{i} (s^t) \right),
\]

where \( \lambda_i \) is the weight the social planner assigns to household \( i \), and \( c_{i} (s^t) \) denotes the consumption assigned to individual \( i \) by the social planner at time \( t \) when history \( s^t \) has occurred; subject to the resource constraint

\[
\sum_{i} c_{i} (s^t) \leq \sum_{i} y_{i} (s^t),
\]
for all histories $s^t$.

The Lagrangian is

$$
\sum_{t=1}^{\infty} \sum_{s^t} \delta^t \pi (s^t) \left[ \sum_i \lambda_i u_i (c_i (s^t)) + \gamma (s^t) \left( \sum_i y_i (s^t) - c_i (s^t) \right) \right]
$$

(5)

where $\delta^t \pi (s^t) \gamma (s^t)$ is the multiplier on the resource constraint at history $s^t$. Note that we can reverse the order of the summation signs because of two properties, (i) the linearity of the expected utility function, and because (ii) the social planner’s objective is additive in households’ lifetime utilities (utilitarian social welfare function).

The first order condition for household $i$, if history $s^t$ has occurred, is

$$
\lambda_i u_i' (c_i (s^t)) = \gamma (s^t)
$$

(6)

Combining the first order conditions for the two households at history $s^t$, we have:

$$
\frac{u_1' (c_1 (s^t))}{u_2' (c_2 (s^t))} = \frac{\lambda_2}{\lambda_1} \equiv x_0 = cste,
$$

(7)

where $x_0$ is the (initial) relative weight assigned to household 2.

Equation (7) indicates that the ratio of marginal utilities is constant across states and over time in the case of perfect risk sharing (Wilson, 1968). (7) is also called the Borch rule. Dividing the first order conditions across periods yields

$$
\frac{u_1' (c_1 (s^t))}{u_1' (c_1 (s^{t-1}))} = \frac{u_2' (c_2 (s^t))}{u_2' (c_2 (s^{t-1}))}, \forall s^t \supset s^{t-1},
$$

(8)

which means that the growth path of marginal utilities of all households is the same. Note that the expectations operator does not appear in this condition, which is the hallmark of full insurance.

Equations (7) and (8) give us the three major implications of efficient risk sharing in this framework. First, the (relative) Pareto weight $x_0$ is constant across time. Second, the consumption allocation at time $t$ depends only on income realizations at time $t$, and is independent of the history of income states $s^{t-1}$. Third and moreover, the consumption allocation, depends only on aggregate income, and is independent of the distribution of
income. Income pooling together with the constant relative weight determine the consumption of each agent, and assure *ex-ante* Pareto efficiency.

To summarize, the consumption allocation at time $t$, given the current income state $s_t$, only depends on aggregate income $y_1(s_t) + y_2(s_t)$, and the relative weight the social planner assigns to household 2, $x_0$, which pins down a point on the Pareto-frontier. Denote $c_i(s_t, x_0)$, $i = 1, 2$, the solution to (7) and (4), noting once again that the solution $c_i()$ only depends in $s_t$ and is independent of $s^{−1}$. $c_i(s_t, x_0)$ is called the *sharing rule*.

Clearly, an *ex-ante participation constraint* should also be satisfied, that is, at time 0 it must be that the expected lifetime utility for each household signing the contract is at least as high as in autarky. Technically, this implies that some points of the Pareto-frontier, or some $x_0$’s, cannot be attained under the risk sharing contract.

To introduce the participation constraints, we have to calculate each agent’s expected lifetime utility at the moment of contracting, and make sure that it is greater than the expected lifetime utility under autarky. Assuming that the income state follows a Markov-process allows us to express agents’ lifetime utility recursively. This is because with the Markov assumption, the current state $s_t$ tells us everything we need to know about the past. In mathematical terms, the conditional distribution of the income state at time $t + 1$ only depends on the realization of the income state at $t$, and not on the whole history.

The Bellman-equation can be written, when the state of the world is $s_t$, as:

$$U^\text{aut}_i(s_t) = u(y_i(s_t)) + \delta \sum_{s_{t+1}} \pi(s_{t+1} | s_t) U^\text{aut}_i(s_{t+1}),$$  \hspace{1cm} (9)

where $U^\text{aut}_i(s_t)$ is the value of the infinite consumption stream for household $i$ in autarky, or the lifetime utility, or the welfare of household $i$, given today’s state $s_t$, or, in other words, $U^\text{aut}_i()$ is the autarkic value function; and $\pi(s_{t+1} | s_t)$ is the conditional probability of state $s_{t+1}$ occurring tomorrow if state $s_t$ occurs today, which is common knowledge. $U^\text{aut}_i(s_t)$ can easily be found by successive iteration using the contraction mapping property of the Bellman-equation.

Suppose that the unconditional distribution of the income state at time 1 is known. Now, we may also compute the expected lifetime utility for agent $i$ at time 0, when the risk sharing contract may be signed. Ex ante, at time
0, the expected value of autarky for agent $i$, denoted $EU^\text{aut}_i$ is

\[ EU^\text{aut}_i = E_0 U^\text{aut}_i(s_1) \]

Let us now turn to calculating the lifetime utility of household $i$ in the case of perfect risk sharing, like we did for autarky. Assuming once again that the income process is Markovian, we have a recursive problem. The value function of agent $i$ at state $s_t$ and with weight $x_0$, in the case of perfect risk sharing, can be written recursively as

\[ U^\text{prs}_i(s_t, x_0) = u(c_i(s_t, x_0)) + \delta \sum_{s_{t+1}} \pi(s_{t+1} | s_t) U^\text{prs}_i(s_{t+1}, x_0), \]  

(10)

where $U^\text{prs}_i(s_t, x_0)$ is the value of the infinite consumption stream in case of full insurance, given today’s state $s_t$ and relative weight $x_0$. As the autarky utility, the value of perfect risk sharing can easily be found by successive iteration.

Finally, we may return to the ex-ante participation constraints. Given $x_0$ and the unconditional distribution of the income state at time 1, the expected value of the full insurance solution for agent $i$ is denoted $EU^\text{prs}_i(x_0)$, and is given by

\[ EU^\text{prs}_i(x_0) = E_0 U^\text{prs}_i(s_t, x_0). \]

So we require that, for $i = 1, 2$,

\[ EU^\text{prs}_i(x_0) \geq EU^\text{aut}_i. \]  

(11)

rules out for example that one agent makes a transfer to the other whichever income state occurs. For all $x_0$ such that (11) is satisfied, a contract ensuring perfect risk sharing is signed at time 0, and is implemented in all subsequent periods. For other $x_0$’s one agent prefers to stay in autarky, thus no insurance contact is signed.

3.2 Risk Sharing with Limited Commitment

In this section we consider the case when agents are unable to commit, and there is no authority to enforce risk sharing contracts either, building on Attanasio and Ríos-Rull (2000), Kocherlakota (1996), Ligon, Thomas, and Worrall (2002), and others. The objective (3) is maximized, subject to the
resource constraints (4), and additional enforcement constraints. At each time $t$, after each history $s^t$ (I speak about histories again, to write the basic model in a general form), and for all $i$, the following inequality must be satisfied:

$$
\sum_{r=t}^{\infty} \sum_{s^r} \delta^{r-t} \pi(s^r | s^t) u(c_i(s^r)) \geq U^\text{aut}_i(s^t),
$$

(12)

where $\pi(s^r | s^t)$ is the probability of history $s^r$ occurring given that history $s^t$ occurred up to period $t \ (r \geq t)$.

In words, (12) means that each household’s expected utility from staying in the informal risk sharing contract must be greater than her expected utility if she deviates and consumes her own income thereafter. This condition is based on the assumption that if one household deviates, the other household does not enter into any risk sharing with her any more. Note that reversion to autarky is the most severe subgame perfect punishment in this environment (Abreu, 1988). We might call reversion to autarky a trigger strategy, or the breakdown of trust. We may also call (12) an ex-post participation constraint, meaning that it requires each agent to voluntarily “sign” the contract after any realization of the history of states. Obviously, this is a stronger requirement than the ex-ante participation constraints that have to be satisfied in the case of perfect risk sharing.

Notice that adding the constraints (12) substantially complicates the analysis, because future decision variables enter into today’s enforcement constraints. Thus the problem at hand no longer has a recursive structure, even with a Markov-process assumption on incomes, and the whole history of states might matter. Following Marchet and Marimon (1998), Attanasio and Ríos-Rull (2000), and Kehoe and Perri (2002), I reformulate the problem. By adding additional co-state variables, in particular the relative weight in the social planner’s problem, or, in other words, the ratio of marginal utilities, the problem can be written in a recursive form.

Denoting the multiplier on the enforcement constraint of household $i$ by $\delta^t \pi(s^t) u_i(s^t)$, and the multiplier on the resource constraint by $\delta^t \pi(s^t) \gamma(s^t)$, when history $s^t$ has occurred, the Lagrangian is

$$
\sum_{i=1}^{\infty} \sum_{s^r} \delta^{r-t} \pi(s^r | s^t) \left[ \sum_i \lambda_i u(c_i(s^r)) \right] + \mu_i(s^t) \left( \sum_{r=t}^{\infty} \sum_{s^r} \delta^{r-t} \pi(s^r | s) u(c_i(s^r)) - U^\text{aut}_i(s^t) \right) + \gamma(s^t) \left( \sum_i y_i(s^t) - c_i(s^t) \right)
$$

(13)
The Lagrangian can also be written in the following form:

\[
\sum_{t=1}^{\infty} \sum_{i} \delta^t \pi(s^t) \left[ \sum_i M_i(s^{t-1}) u(c_i(s^t)) + \mu_i(s^t)(u(c_i(s^t)) - U_i^{out}(s^t)) + \gamma(s^t) \left( \sum_i y_i(s^t) - c_i(s^t) \right) \right]
\]

(14)

where \( M_i(s^t) = M_i(s^{t-1}) + \mu_i(s^t) \) with \( M_i(s^0) = \lambda_i \). In words, \( M_i(s^t) \) is the initial weight on agent \( i \) plus the sum of the Lagrange multipliers on her enforcement constraints along the history \( s^t \).

The first order condition with respect to \( c_i(s^t) \) is

\[
\delta^t \pi(s^t) M_i(s^t) u'(c_i(s^t)) - \gamma(s^t) = 0.
\]

(15)

We also have standard first order conditions relating to the resource and enforcement constraints, with complementarity slackness conditions. Combining the first order conditions (15) for the two households for history \( s^t \) at time \( t \), we have

\[
\frac{u'(c_1(s^t))}{u'(c_2(s^t))} = \frac{M_2(s^t)}{M_1(s^t)} = \frac{\lambda_2 + \mu_2(s^1) + \mu_2(s^2) + \ldots + \mu_2(s^t)}{\lambda_1 + \mu_1(s^1) + \mu_1(s^2) + \ldots + \mu_1(s^t)} \equiv x(s^t),
\]

(16)

where \( x(s^t) \) can be thought of as the relative weight assigned to household 2 when history \( s^t \) has occurred. Notice that, unlike in the perfect risk sharing case, where \( \mu_i(s^t) = 0, \forall i, \forall s^t \), in the case of limited commitment, the relative weight \( x(s^t) \) will vary over time and across states. We would like to keep \( x \) constant (as in first best), but when an enforcement constraint binds, we cannot do that. However, intuitively we will try to keep \( x(s^t) \), for all \( s^t \supset s^{t-1} \), as close as possible to \( x(s^{t-1}) \).

The relative weight \( x(s^t) \), defined in (16) is used as an additional co-state variable in order to rewrite the problem in a recursive form. This idea is due to Marrot and Marimon (1998). To do this, suppose once again that the state of the world with respect to income follows a Markov process, so that we may write \( \pi(s^t | s^{t-1}) = \pi(s_t | s_{t-1}) \). Still, the current income state \( s_t \) does not tell us everything we need to know about the past, only \((s_t, x_{t-1})\) does, where \( x_{t-1} \) is the relative weight inherited from the previous period. Denote \( x_t \) the new relative weight we have to find at time \( t \). We are looking for policy functions for the consumption allocation and the new relative weight, with support over the extended state space \((s_t, x_{t-1})\), that is, we want to know \( c_t(s_t, x_{t-1}) \), \( \forall i \), and \( x_t(s_t, x_{t-1}) \). At last, the value functions can be defined recursively as
\begin{equation}
V_i(s_t, x_{t-1}) = u(c_i(s_t, x_{t-1})) + \delta \sum_{s_{t+1}} \pi(s_{t+1} \mid s_t) V_i(s_{t+1}, x_t(s_t, x_{t-1})).
\end{equation}

We may also call \( c_i(s_t, x_{t-1}) \) the sharing rule. Note that since policies and values depend on \( x_{t-1} \), the contract is history dependent.

Numerical dynamic programming allows us to solve for the consumption allocation and lifetime utilities, given the income processes, utility functions and discount rates for the two households, and the initial relative weight in the social planner’s objective. The appendix explains how in details. The next section uses the algorithm to generate comparative static results to examine issues related to the interaction of inequality and informal risk sharing contracts.

What are the properties of the solution? First of all, it is easy to see that, if the discount factor \( \delta \) is sufficiently large, then the perfect risk sharing solution is self-enforcing for some \( x_0 \)'s (folk theorem), while if \( \delta \) is sufficiently small, there does not exist any non-autarkic allocation that is sustainable with voluntary participation. Now, suppose that there exists a non-autarkic solution, but the first best is not self-enforcing for any \( x_0 \).

The limited commitment solution can be fully characterized by a set of state-dependent intervals on the relative weight of household 2, or ratio of marginal utilities, \( x \), that give the possible relative weights in a given income state. Note that there is a one-to-one relationship between the relative weight and the consumption allocation, given the income state (see (16)). These are optimal intervals, meaning that they correspond to optimally chosen future promised utilities as well. Once we have found the intervals we know everything we can about the solution. Denote the interval for state \( s \) by \([\underline{x}_s, \overline{x}_s]\).

Suppose we have inherited some \( x_{t-1} \) from last period, and today the income state is \( s \). \( x_t \) is determined by the following updating rule:

\begin{equation}
x_t = \begin{cases} 
\overline{x}_s & \text{if} \quad x_{t-1} > \overline{x}_s \\
x_{t-1} & \text{if} \quad x_{t-1} \in [\underline{x}_s, \overline{x}_s] \\
\underline{x}_s & \text{if} \quad x_{t-1} < \underline{x}_s 
\end{cases}
\end{equation}

To see how this works, suppose that the two households are identical ex ante, \( u() = \log() \), and their income may only take two values, \( y^h \) (high) or \( y^l \) (low), with \( y^h > y^l > 0 \). There are four income states, \( hh, hl, lh, lh \),
and \( ll \), where the first argument refers to household 1’s income, and the second to household 2’s income. Suppose that the intervals overlap, except for states \( hl \) and \( lh \), so \( x^{hh}, x^{ll} > x^{lh} > 1 > x^{hl}, x^{ll} \). Take \( x_0 = 1 \), so the two agents have equal weights in the social planner’s objective. Now, suppose that at time 1 the state is \( hh \). In this case, \( x \) can be kept constant, because \( 1 \in [x^{hh}, x^{hh}] \), so \( x_1 = x_0 \), and no transfer is made. Suppose that at time 2 the state is \( lh \). We cannot keep \( x \) constant any more, because household 2 is not willing to share aggregate income equally (she would prefer to revert to autarky instead), her enforcement constraint is binding. We set \( x_2 = x^{lh} > x_1 \), agent 2 is making a transfer, but not as large as she would in the perfect risk sharing solution. Suppose that at time 3 the income state is \( hh \) once again. But, unlike at time 1, now we would like to set \( x_3 = x_2 > 1 \). We can do so, since we have supposed that the \( hh \) and \( lh \) intervals overlap. Notice that we are in a symmetric state, the incomes of the two households are equal, but household 1 is making a transfer to household 2, because of the history dependence of the contract. In this way household 1 partly reciprocates the transfer she got in period 2, so risk sharing with limited commitment has a quasi-credit element. Now, suppose that at time 4 the state is \( hl \). The best we can do is to set \( x_4 = x^{hl} \). Now household 1 is helping out household 2, who has a bad income realization. If at time 5 we are at a symmetric state again, agent 2 pays back some part of the “credit” she got the previous period. The credit of period 2 is forgotten for ever, what matters is only who was constrained last, thus we may say that the economy is displaying amnesia. Further, after a sufficient number of periods \( x \) only takes two different values, \( x^{lh} \) and \( x^{hl} \), and the consumption allocation converges weakly to the same distribution, independently of the initial \( x_0 \).  

\footnote{Take the numerical example from Ligon, Thomas, and Worrall (2002), that is \( y^l = 1 \) and \( y^h = 2 \), and suppose that the discount factor \( \delta = 0.95 \). Then the optimal intervals are \( [x^{hh}, x^{hh}] = [x^{ll}, x^{hl}] = [0.934, 1.070], [x^{hl}, x^{hl}] = [0.5, 0.961], \) and \( [x^{lh}, x^{ll}] = [1.041, 2] \), so the \( hl \) and \( lh \) intervals do not overlap, but both overlap with the interval for the symmetric states.  

\footnote{Note that, if perfect risk sharing is self-enforcing for some set of \( x \), denote this interval \( [x, \bar{x}] \), then it does matter which \( x_0 \) is chosen by the social planner. In particular, after a sufficient number of periods, with probability 1, the ratio of marginal utilities will be one of the following, in all periods and states: \( x_0 \) if \( x_0 \in [x, \bar{x}] \), \( \bar{x} \) if \( x_0 < x \), and \( \bar{x} \) if \( x_0 > \bar{x} \) (see Koche19lakota (1996)).}
4 Consequences and Sources of Income Inequality

This section examines the interaction of income inequality and self-enforcing risk sharing contracts in the context of the model presented in section 3. To do this, three types of simulation exercises are performed. In all cases I assume that only two households populate the village economy, and that each household’s income may take only two values. Households are allowed to be heterogeneous in either (i) the characteristics of their income process, (ii) some predetermined wealth, the returns of which are fixed each period, or (iii) their risk preferences.

The first example illustrates the possible adverse consequences of inequality on the welfare of the poor, even if per-capita income increases in the economy. The second example looks at the effects of changes in the subsistence level on consumption smoothing, and shows, for example, that as the subsistence level increases, both the mean and the volatility of the poor household’s consumption process decrease. The third example is an attempt to look at the effects of informal insurance on income smoothing. In particular, I show that (i) the availability of informal insurance may improve efficiency, in the sense that expected income increases, and that (ii) lack of wealth and/or higher risk aversion may prevent the poor from adopting a riskier, higher yielding technology.

All computations have been done using the software R (www.r-project.org).

4.1 How the poor can be worse off when the income of the rich increases

This section examines the consequences of inequality on the welfare of the poor, given that only the income of the rich changes. This means that we do not look at changes in inequality in the usual sense, but rather, inequality increases together with aggregate, and per-capita, income. This exercise is interesting because we put ourselves in a disadvantageous environment to find any adverse consequences for welfare. In particular, I fix the income of the poor and give some additional income to the rich in each state of the world. Therefore, if the poor is worse off in terms of welfare as a result, it
must somehow be due to the informal risk sharing arrangement.\footnote{Note that in autarky, the welfare of the poor does not change, while in the perfect risk sharing case, given $x_0$, the welfare of the poor increases as the income of the rich increases, provided that with the chosen $x_0$ the ex-ante participation constraints are still satisfied.}

Suppose that there are two households, a poor and a rich one. Both households have standard constant-relative-risk-aversion (CRRA) preferences,

$$u(c_{it}) = \frac{c_{it}^{1-\sigma}}{1-\sigma},$$

with identical coefficient of relative risk aversion ($\sigma_1 = \sigma_2 \equiv \sigma$). Both households discount the future with discount factor $\delta$. Note that a higher $\sigma$ increases the demand for insurance, while a higher $\delta$ helps enforcement, thus allows more risk sharing (see Ligon, Thomas, and Worrall (2002)).

The two households differ in their income process. The poor household receives $\underline{y} = 1.5$ or $\overline{y} = 2.5$, with equal probabilities, in each period. I perform a comparative statics exercise, changing the income process of the rich: starting from a situation close to equality, the rich getting $\underline{y} = 1.6875$ or $\overline{y} = 2.8125$, with equal probabilities as well, to a situation where she is ten times richer, that is, $y = 15$ or $\overline{y} = 25$, still with equal probabilities, and in each period. All along I keep the riskiness of the income process constant, meaning that its coefficient of variation stays the same. Note once again that in this way inequality increases together with per-capita income.

I use the algorithm outlined in the appendix to find the solution of the model given a set of parameter values\footnote{One also needs to choose the number of gridpoints, as the continuous variable $x$ is discretized. Here I take a grid of 500 intervals, considering the trade-off between precision and computation time. Computation time for each set of parameter values chosen is about 10 hours on a computer with a processor of 2 GHz and 1 GB RAM.}. The solution, that is, the constrained-efficient, informal contract, is given by a set of state-dependent intervals that tell us what ratios of marginal utilities are possible in each of the four states of the world. Once these intervals have been computed, I allow the economy to run for 1000 periods, that is, I generate a realization for the income state in each period, and let the contract tell us the consumption of the households. To calculate the lifetime utility of the poor, I take the last 900 periods. 100 periods is sufficient for the economy to reach the stable distribution of consumption, regardless of the initial relative weight, with probability very close to one. Finally, to compute the expected welfare of the poor, I redo the above simulation 1000 times. Each time I take $x_0 = 1$, that is, the social
planner would prefer and equal division of consumption and utilities in each period.

Take $\delta = 0.9$, and consider two different coefficients of relative risk aversion, high and low, with $\sigma^{\text{high}} = 2$ and $\sigma^{\text{low}} = 1.1$. Figures 1 and 2 show the expected lifetime utility of the poor as a function of inequality, for $\sigma^{\text{high}}$ and $\sigma^{\text{low}}$, respectively. Note that both welfare and inequality are measured on an ordinal scale here\(^9\), so only the slope is informative, the shape of the curves is not.

Figure 1 shows that for $\sigma = 2$ the welfare of the poor is increasing as we give more income to the rich. Remember that the income process of the poor does not change, thus in autarky she would be no better or worse off as inequality increases. However, with the two households interacting to share risk, the poor benefits from more per-capita income in the economy. Note that in this case, perfect risk sharing is self-enforcing for a small set of $x$’s for any level of inequality.\(^10\)

For $\sigma = 1.1$ (figure 2) we see something more surprising: the welfare of the poor is decreasing with increasing inequality and per-capita income, even if her income does not change. The intuition behind this result is the following. As the rich gets richer, her outside option becomes more attractive, or, her threatpoint, thus her bargaining power increases vis-a-vis that of the poor. A second point is that the poor can only make relatively small transfers, so the insurance the poor can provide becomes less valuable for the rich. These effects may outweigh the positive effect of higher per-capita income, thus the poor may be worse off. The negative effects are more pronounced for lower risk aversion. If households are highly risk averse, the outside option with no risk sharing is not very attractive even for the rich, and she values sufficiently the insurance the poor can provide.

To summarize, in the case of risk sharing with limited commitment, the poor may be more and more excluded from the informal insurance arrangement as the rich gets richer. This loss of insurance may cause a decrease in

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\(^9\)In particular, inequality = 1 is equality in fact, so when the rich earns $y = 1.5$ or $\overline{y} = 2.5$ as well (not represented), and inequality = 73 is the most polarized case, so when the rich is getting $y = 15$ or $\overline{y} = 25$, and in each step, 0.25 is added to the mean income of the rich, and the coefficient of variation is kept constant.

\(^10\)For example, for inequality = 2 the interval is [1.210, 1.308], or for inequality = 40 the interval is [30.161, 32.596]. Note further that, starting from $x_0 = 1$, $x$ always reaches the lower bound of these intervals, so we compute the upper bound for the welfare of the poor. The qualitative results do not change if we randomize over $x_0$. 

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Figure 1: The welfare of the poor as a function of inequality with $\sigma = 2$. Welfare is expected lifetime utility at the limited commitment solution, supposing that the economy has run for a sufficient number of periods to reach the stationary distribution of consumption. Inequality is measured on an ordinal scale, and it increases together with per-capita income. In particular, inequality increases by one unit as the mean income of the rich increases by 0.25, and the coefficient of variation is constant. The income process of the poor is kept constant. $\sigma$ is the coefficient of relative risk aversion, common to the two households. The non-smoothness of the curve comes from approximation error.
Figure 2: The welfare of the poor as a function of inequality with $\sigma = 1.1$. (See notes for figure 1.)
welfare for the poor. This result warns of the possible adverse consequence of growth in per-capita income for the welfare of the poor, when the poor do not receive any of the additional income.

Empirical evidence on the exclusion of poor households of risk sharing networks includes Townsend (1994), who finds that landless households are less well insured in one of the three Indian villages in the study. Jalan and Ravallion (1999) reject perfect risk sharing most strongly for the poorest households in their sample from rural China, and estimate that 40% of income shocks the poor face are passed onto current consumption (while for the richest households, only 10%). Santos and Barrett (2006) find direct evidence that the poorest households are excluded from social networks in Ethiopia, in particular, they do not receive transfers in case of a negative income shock.

4.2 Effects of changes in the subsistence level

This section performs another type of comparative statics exercise. In particular, keeping the income process of the two households fixed, I change the subsistence level, denoted $subs$. In this case, the utility function can be written as

$$ u(c_{it}) = \frac{(c_{it} - subs)^{1-\sigma}}{1 - \sigma}. $$

(19)

With $subs > 0$ preferences are characterized by decreasing relative risk aversion (DRRA). Note that the utility function (19) implies that the coefficient of relative risk aversion is $\sigma(\frac{c_{it}}{c_{it} - subs})$, which is decreasing in $c_{it}$ for $subs > 0$. Empirical evidence on the relevance of a subsistence level, or decreasing relative risk aversion, in the case of perfect risk sharing is provided by Ogaki and Zhang (2001).

A first, natural result is that as the subsistence level increases, perfect risk sharing may become self-enforcing. This is because insurance becomes more valuable for both households. The result follows from the fact that an increase in the subsistence level is equivalent to a decrease in some fixed revenue, or wealth. Thus with a higher subsistence consumption, households behave in a more risk-averse fashion.

To take a closer look at the effect of changes in the subsistence level on consumption smoothing, consider two households once again, a poor and a rich one. As in section 4.1, households differ in their mean income, while the coefficient of variation of the income processes is the same. The poor
gets $y = 6$ or $\bar{y} = 10$, with equal probabilities, in each period, and the rich household earns $y = 15$ or $\bar{y} = 25$, with equal probabilities as well, in each period. The subsistence level changes between 0 and 3.

Let us look at welfare first. Note that the welfare of both households should decrease as subsistence consumption increases. Figure 3 shows the expected lifetime utility of the poor, for $\sigma = 1.3$ and $\delta = 0.9$. The simulations are conducted as in section 4.1, except that now I keep the income processes constant, but increase subsistence consumption by 0.1 in each step.

Figure 3 shows that it may happen that an increase in the subsistence level actually increases the welfare of the poor.\footnote{This result is robust to changing the way in which we choose $x_0$, the initial relative weight. I have rerun the simulations choosing $x_0$ randomly among the gridpoints on $x$, and...} In particular this happens as subs...
Figure 4: Optimal intervals for the relative weight $x$ in the four income states increases from 1.1 to 1.2. This is exactly the change in the subsistence level that makes perfect risk sharing self-enforcing. Figure 4 shows the optimal intervals in the four income states. All the intervals overlap when $subs \geq 1.2$. Thus the intuition behind the increase of the poor household’s welfare is that her weaker bargaining power suddenly matters less, when perfect risk sharing becomes self-enforcing.

In this example preferences are changing actually, while income remains the same. Thus, it is interesting to look at the properties of households’ consumption process. The first panel of figure 5 shows that the mean containing the maximum value of $x$ on the grid, which is determined by the ratio of marginal utilities in autarky. With $x_0 = 1$, we get the upper bound on the welfare of the poor when perfect risk sharing is self-enforcing for some $x$, while with $x_0$ maximal, we find the lower bound.
Figure 5: Mean and standard deviation of the consumption process of the poor

Consumption of the poor is decreasing as the subsistence level increases, except for the moment when perfect risk sharing becomes self-enforcing. The second panel of this same figure shows that at the same time the consumption of the poor becomes less volatile. Thus we see that, as subsistence consumption increases, the poor “buys” more insurance from the rich by sacrificing expected income. As a result, the difference in expected consumption between the rich and the poor increases with the subsistence level.

Finally, let us look at the possible consumption values for the poor in the limited commitment solution as a function of the subsistence level (figure 6). First, the spread decreases as subsistence consumption increases. Second, when perfect risk sharing becomes self-enforcing, there is a discrete increase in the lowest consumption. Finally, note that even when perfect risk sharing is possible, consumption is not constant, only the ratio of marginal utilities. This is because only idiosyncratic risk is insured perfectly, the households would need a third party to insure against aggregate risk.

4.3 Consumption insurance and income smoothing

By way of a third type of examples, this section examines how (i) the possibility to share risk, (ii) the availability of wealth that yields a fixed revenue
Figure 6: Possible consumption levels for the poor
each period, and (iii) heterogeneous risk preferences, may influence the choice of production technology. A production technology is described by the income process it generates. We will see that lack of insurance, and/or lack of wealth, or higher risk aversion may lead to more income smoothing, and thereby a loss in efficiency.

The importance of consumption smoothing possibilities in income decisions in low-income economies has been recognized by Morduch (1994, 1995). He convincingly argues that lack of credit and insurance not only affects the ability of households to smooth consumption given income, but also has important consequences for production decisions. Households have to choose safer income generating technologies in order to avoid big income fluctuations, with which they would be unable to deal. This might cause considerable efficiency losses. However, Morduch (1995) does not formalize these ideas, while Morduch (1994) considers lack of consumption credit.

Rosenzweig andBinswanger (1993) provide some empirical evidence that people with lack of consumption smoothing instruments have to sacrifice expected profits for less volatile income. They look at the effect of weather variation on the mean and variance of farm profits using data from Indian villages, and find that mean profits decrease with weather volatility for poorer households, but not for the rich. Kurosaki and Fafchamps (2002) find evidence that crop choices of households in Pakistan depend on price and yield risk. Even though efficient risk sharing among households of the same village cannot be rejected, aggregate shocks are not insured, and risk attitudes do affect production choices.

Structural modeling of the case where only informal insurance is available for households to smooth consumption, and households take this into account when making production decisions, is thus an important problem. Here I aim to have some insights concerning the issue of income smoothing, setting up a general model, but solving only a special case.

In general, adding technology choice complicates substantially the problem at hand, because households may switch between the technologies in any state of the world and any time period, whether they stay in the informal risk sharing contract or revert to autarky. In other words, the choice of production technology to be used next period depends on the state of the world today. Below I construct two related examples, where at the constrained-efficient solution, a household chooses the same technology in all states of the world. The only switching, which is costless for simplicity, may occur when the household leaves the risk sharing contract, and stays in autarky.
thereafter. Even without solving the general model with possible switching at any time and state, switching has to be allowed when threatpoints are computed.

The timing is as follows. At time 0, each household chooses a technology. Note that each technology takes one period to yield some income (we may have agricultural production in mind, for example). At time 1, the state of the world, and incomes are realized first, according to the technology chosen at time 0. Then each household may decide to stay in the risk sharing arrangement, or deviate. In the first case, each household makes a payment to the other household as specified by the contract, consumption takes place, and finally, each household also decides which technology to use. In case one of the households deviates, no payments are made, each household consumes her income generated by the technology she chose one period before, and finally, each household chooses a technology, knowing that she will be in autarky in all future periods. At time 2 and thereafter, the same sequence of events follows as at time 1.

Let us now turn to the numerical examples. Suppose that two technologies are available in the economy, a safer one with lower expected income, which we call the “old” technology, and a riskier, “new” technology with higher profits in expectation. Both technologies yield an independently and identically distributed (iid) income process, with equal probabilities for each state. Once again, income of a household takes two values, $y^l$ (low) or $y^h$ (high), thus there are four income states. The old technology has the following payoffs: $y^l = 1.4$ or $y^h = 2.5$. The new technology yields $y^l = 1.2$ or $y^h = 2.9$, in each period. Households discount the future at the rate $\delta = 0.95$, and they both have utility functions of the CRRA form.

Now, let us look at two examples with different kind of heterogeneity among households. In example 1, the rich has some exogenous wealth that yields a fixed income every period, which is in addition to the stochastic income process from production, while the poor has no wealth. In example 2, households differ in their coefficient of relative risk aversion. I will call the less risk averse household the rich, and the more risk averse the poor, abusing terminology.

**Example 1.** Suppose that both households’ coefficient of relative risk aversion $\sigma = 1.5$. The poor household has no wealth, while the rich household possesses some assets that yield a sure revenue $w = 2$ each period. So the poor household’s income is $y(s_t)$, and the rich has $w + y(s_t)$. The social
Table 1: The welfare of the poor (no wealth) in autarky

<table>
<thead>
<tr>
<th>income \ technology</th>
<th>old</th>
<th>new</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>-29.765</td>
<td>-30.327</td>
</tr>
<tr>
<td>high</td>
<td>-29.340</td>
<td>-29.676</td>
</tr>
</tbody>
</table>

Table 2: The welfare the rich (some wealth) in autarky

<table>
<thead>
<tr>
<th>income \ technology</th>
<th>old</th>
<th>new</th>
</tr>
</thead>
<tbody>
<tr>
<td>low</td>
<td>-20.346</td>
<td>-20.323</td>
</tr>
<tr>
<td>high</td>
<td>-20.204</td>
<td>-20.108</td>
</tr>
</tbody>
</table>

planner’s objective is

$$\max_{\{c_1(s_t), c_2(s_t)\}} \sum_{t=0}^{\infty} \sum_{s_t} 0.95^t \pi(s_t) \left( \frac{c_1(s_t)^{-0.5}}{-0.5} + x_0 \frac{c_2(s_t)^{-0.5}}{-0.5} \right),$$

(20)

with $\pi(s_t) = 0.25$ for all $s_t$, subject to resource and enforceability constraints. I set $x_0 = 3$ in the social planner’s objective (20).

With these parameter values, in autarky the poor prefers to use the old technology, while the rich chooses the new technology. Tables 1 and 2 show the autarky values, or lifetime utilities, discounted to time 1, the poor and the rich get, respectively.

The values for the old technology are indeed higher for the poor household, and the lifetime utility the new technology gives is higher for the rich, for both low and high income today. From these values the threatpoints can be computed. If a household chose the technology optimal in autarky yesterday, just take the values from tables 1 and 2. When the household chose the other technology before, still participating in the risk sharing arrangement, but deviates to autarky today, she consumes the income realization from the other technology today, while tomorrow she receives the optimal autarky values above.

Now we can look at the limited commitment solution, using these threatpoints. I find a simple subgame perfect equilibrium (SPE) of this infinite game, supported by reversion to autarky, where both households choose the new technology in all periods and states. I compare the payoffs of a given technology choice, described by (technology choice of the poor, technology choice of the rich) with the payoffs of a one-sided deviation. Table 3 shows
lifetime utilities for the poor at the limited commitment solution, in the
four income states, described by (income of the poor, income of the rich).
Similarly, table 4 shows the values for the rich.

Table 3: The welfare of the poor (no wealth) with informal insurance

<table>
<thead>
<tr>
<th>state \ technology</th>
<th>(new,new)</th>
<th>(old,new)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(low,low)</td>
<td>-29.093</td>
<td>-29.293</td>
</tr>
<tr>
<td>(low,high)</td>
<td>-28.840</td>
<td>-29.081</td>
</tr>
<tr>
<td>(high,low)</td>
<td>-28.840</td>
<td>-29.126</td>
</tr>
<tr>
<td>(high,high)</td>
<td>-28.676</td>
<td>-28.946</td>
</tr>
</tbody>
</table>

Table 4: The welfare of the rich (some wealth) with informal insurance

<table>
<thead>
<tr>
<th>state \ technology</th>
<th>(new,new)</th>
<th>(new,old)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(low,low)</td>
<td>-20.172</td>
<td>-20.178</td>
</tr>
<tr>
<td>(low,high)</td>
<td>-19.997</td>
<td>-20.046</td>
</tr>
<tr>
<td>(high,low)</td>
<td>-19.997</td>
<td>-20.022</td>
</tr>
<tr>
<td>(high,high)</td>
<td>-19.883</td>
<td>-19.938</td>
</tr>
</tbody>
</table>

Example 2. Now, neither household has any wealth, but the poor household is more risk averse than the rich. The poor household’s coefficient of relative risk aversion \( \sigma_1 = 2.5 \), while \( \sigma_2 = 1.3 \). The social planner’s objective is

\[
\max_{\{c_1(s_t), c_2(s_t)\}} \sum_{t=0}^{\infty} \sum_{s_t} 0.95^t \pi(s_t) \left( \frac{c_1(s_t) - 1.5}{-1.5} + x_0 \frac{c_2(s_t) - 0.3}{-0.3} \right), \tag{21}
\]
with $\pi(s_t) = 0.25$ for all $s_t$, as before, subject to resource and enforceability constraints. I now set $x_0 = 0.5$ in the social planner’s objective (21).

With these parameter values, in autarky both households prefer the old technology. The autarky values for the poor household are shown in table 5, and for the rich in table 6.

The values for the old technology are indeed higher for both households for both low and high income. From these values we can calculate the threatpoints, similarly as for example 1. If a household chose the old technology in the previous period, just take the values for the old technology from the tables. When the household chose the new technology before, still participating in the risk sharing arrangement, but deviates to autarky today, she gets the payoff from the new technology today, while tomorrow she receives the old technology values above.

These threatpoints are used to find the constrained-efficient solution. Given the constrained-efficient informal risk sharing contract, the poor household chooses the old technology in all periods and states, while the rich produces using the new technology. Once again, I compare the payoffs of a given technology choice, described by (technology choice of the poor, technology choice of the rich) with the payoffs of a one-sided deviation. Tables 7 and 8 show the lifetime utility for the poor and the rich, respectively, in the four income states, described by (income of the poor, income of the rich), allowing households to enter into an informal risk sharing arrangement.

We see that (new,old) is preferred by the rich to (old,old), and the poor would rather use the old technology given that the rich uses the new. This second example shows as well that the availability of insurance to smooth consumption may indeed affect the choice of production technology. It also
Table 7: The welfare of the poor (high risk-aversion) with informal insurance

<table>
<thead>
<tr>
<th>state\technology</th>
<th>(old,new)</th>
<th>(new,new)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(low,low)</td>
<td>-5.497</td>
<td>-5.464</td>
</tr>
<tr>
<td>(low,high)</td>
<td>-5.339</td>
<td>-5.382</td>
</tr>
<tr>
<td>(high,low)</td>
<td>-5.377</td>
<td>-5.382</td>
</tr>
<tr>
<td>(high,high)</td>
<td>-5.292</td>
<td>-5.311</td>
</tr>
</tbody>
</table>

Table 8: The welfare of the rich (low risk-aversion) with informal insurance

<table>
<thead>
<tr>
<th>state\technology</th>
<th>(old,new)</th>
<th>(old,old)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(low,low)</td>
<td>-55.116</td>
<td>-55.122</td>
</tr>
<tr>
<td>(low,high)</td>
<td>-54.540</td>
<td>-54.741</td>
</tr>
<tr>
<td>(high,low)</td>
<td>-54.697</td>
<td>-54.741</td>
</tr>
<tr>
<td>(high,high)</td>
<td>-54.322</td>
<td>-54.493</td>
</tr>
</tbody>
</table>

demonstrates that higher risk aversion may cause more income smoothing, thus lower expected incomes. Further, we see a perverse affect of informal insurance in that it actually causes inequality in expected incomes. Notice that in autarky both households choose the same technology, and informal insurance allows the less risk averse household to become the “rich”. In terms of welfare, however, both households are better off if they share risk. This is trivially true, since, by definition, in the constrained-efficient solution both households must be at least as well off as in autarky, and they are strictly better off if some transfers occur in any state, which is the case here.

As mentioned already, the general solution of the model of risk sharing with limited commitment and technology choice is an interesting and difficult task for future research. The difficulty comes from the fact that in any period and any state of the world a household may decide to switch between the available technologies, based on the expected lifetime utility they provide given that the risk sharing contract is constrained-efficient. But the constrained-efficient transfers depend also on future technology choices. So we have to find the decision on technologies and the informal insurance contract simultaneously.
5 Concluding remarks

Empirical evidence from low income rural communities suggests the existence of informal insurance arrangements that achieve partial insurance. This paper has shown a way to model the observed partial insurance. In particular, we required risk sharing contracts to be self-enforcing. The numerical techniques developed to solve the model allow us to compute the allocation in parametrized economies. In this paper we have used these techniques to examine some issues related to income inequality. The importance of the possible effects shown by our examples, coming from the interaction of inequality and informal insurance contracts, is an empirical question.

However, our results warn of the possible adverse consequences of inequality on the welfare of the poor, even if an increase in inequality only means that the rich get richer. Further, we have seen that inequality may be reinforced without insurance, when the wealthy choose a more profitable technology, while the poor prefer the less risky, less efficient technology.

One direction for future theoretical work is to develop the model with more than one technologies. A first attempt is made in section 4.3 here. This extension would be a very important step, since in low-income economies production and consumption decisions are often intertwined, because of incomplete markets. It is not optimal for risk averse households to maximize their expected income, when financial instruments or insurance contracts are not available to smooth consumption inter-temporally or across states of nature.

The framework and methods discussed in this paper also allow us to examine the impact of some policy intervention in future work, for example a micro-insurance program, taking into account existing informal arrangements to share risk.
Appendix - Computation

We are interested in solving for the decision variables, consumption \( c_i(s_t, x_{t-1}) \), \( \forall i \), and the relative weight of household 2 \( x_t(s_t, x_{t-1}) \), and for the lifetime utility household \( i \) gets from her consumption stream being in the informal risk sharing arrangement \( V_i(s_t, x_{t-1}) \), \( \forall i \), given the state of the world today\( (s_t, x_{t-1}) \).

Define a grid over the continuous variable \( x \) for each value of \( s_t \). Denote \( X \) the set of gridpoints (I define the same points for all \( s_t \)). Guess a solution for the value functions, that is, guess \( V^0(s_t, x_{t-1}) \) for \( \forall i \) and each gridpoint. Unfortunately, the algorithm does not converge from any initial guess for the value functions, but the value of the perfect risk sharing case will do.\(^{13}\)

Now proceed to update the guess. Suppose we are at the \( n^{th} \) iteration. Let us look at gridpoint \((\tilde{s}_t, \tilde{x}_{t-1})\). Three cases have to be distinguished: (a) neither enforcement constraint binds, (b) the enforcement constraint for household 1 binds, or (c) the enforcement constraint for household 2 binds (the two constraints cannot bind at the same time, since only one of the two households has to make a transfer, and obviously the resource constraint always binds). We first suppose that neither enforcement constraint binds, that is, we try to keep \( x \) constant, then we see if we can do that or not.

(a) Neither enforcement constraint binds. This is the easy case, since \( x_t(\tilde{s}_t, \tilde{x}_{t-1}) = \tilde{x}_{t-1} \). So we only have to find \( c_1(\tilde{s}_t, \tilde{x}_{t-1}) \) and \( c_2(\tilde{s}_t, \tilde{x}_{t-1}) \), and we have two conditions: \( u'(c_1(\tilde{s}_t, \tilde{x}_{t-1}))/u'(c_2(\tilde{s}_t, \tilde{x}_{t-1})) = x_t(\tilde{s}_t, \tilde{x}_{t-1}) = \tilde{x}_{t-1} \) and the resource constraint \( c_1(\tilde{s}_t, \tilde{x}_{t-1}) + c_2(\tilde{s}_t, \tilde{x}_{t-1}) = y_1(\tilde{s}_t) + y_2(\tilde{s}_t) \). Replacing for \( c_2(\tilde{s}_t, \tilde{x}_{t-1}) \) from the resource constraint we have \( u'(c_1(\tilde{s}_t, \tilde{x}_{t-1}))/u'(y_1(\tilde{s}_t)) + y_2(\tilde{s}_t) - c_1(\tilde{s}_t, \tilde{x}_{t-1}) = x_t(\tilde{s}_t, \tilde{x}_{t-1}) = \tilde{x}_{t-1} \). Supposing logarithmic utility we can easily find closed form solutions. In sum, with \( u(c_i) = \log(c_i) \), we have the following updated policy functions:

\(^{12}\)Here I outline the algorithm for two households. There is no difficulty in extending the algorithm to \( n \) households theoretically. The state space has to include the vector of relative weights of length \( n-1 \). However, in terms of computation time we face the curse of dimensionality, and the computation time for \( n \) large, and computing the allocation and utilities with acceptable precision, is prohibitive.

\(^{13}\)Characterizing the convergence properties of the algorithm is left for future research. However, we know that the algorithm does not converge to the constrained-efficient solution from any initial guess for the value functions. For example, if we set the initial guess equal to the autarkic values, every iteration yields these same autarkic values. This is natural, since autarky is also a subgame perfect equilibrium (SPE).
\[ x_t(\tilde{s}_t, \tilde{x}_{t-1}) = \tilde{x}_{t-1} \]

\[ c_1(\tilde{s}_t, \tilde{x}_{t-1}) = \frac{y_1(\tilde{s}_t) + y_2(\tilde{s}_t)}{1 + x_t(\tilde{s}_t, \tilde{x}_{t-1})} \]

\[ c_2(\tilde{s}_t, \tilde{x}_{t-1}) = x_t(\tilde{s}_t, \tilde{x}_{t-1}) \frac{y_1(\tilde{s}_t) + y_2(\tilde{s}_t)}{1 + x_t(\tilde{s}_t, \tilde{x}_{t-1})}. \]

Now we have to check whether either of the enforcement constraints is violated. We can do this by verifying the weak inequality

\[ u(c_i(\tilde{s}_t, \tilde{x}_{t-1}))) + \delta \sum_{s_{t+1}} \pi(s_{t+1} | \tilde{s}_t) V_i^{n-1}(s_{t+1}, x_t(\tilde{s}_t, \tilde{x}_{t-1})) \geq U_i^{\text{aut}}(\tilde{s}_t). \tag{22} \]

Notice that we use \( V_i^{n-1}() \). If (22) is satisfied \( \forall i \), we set \( V_i^n(\tilde{s}_t, \tilde{x}_{t-1}) \) equal to the left hand side of (22), and we are done with gridpoint \((\tilde{s}_t, \tilde{x}_{t-1})\). If it is violated for household 1, we have to proceed to (b). If (22) is violated for household 2, we proceed to (c).

(b) The enforcement constraint for household 1 binds. Now we want to find \( c_1(\tilde{s}_t, \tilde{x}_{t-1}), c_2(\tilde{s}_t, \tilde{x}_{t-1}), \) and \( x_t(\tilde{s}_t, \tilde{x}_{t-1}) \), and we have three conditions: \( u'(c_1(\tilde{s}_t, \tilde{x}_{t-1}))/u'(c_2(\tilde{s}_t, \tilde{x}_{t-1})) = x_t(\tilde{s}_t, \tilde{x}_{t-1}) \), the resource constraint \( c_1(\tilde{s}_t, \tilde{x}_{t-1}) + c_2(\tilde{s}_t, \tilde{x}_{t-1}) = y_1(\tilde{s}_t) + y_2(\tilde{s}_t) \), and we know that household 1’s enforcement constraint is satisfied with equality, that is, \( u(c_1(\tilde{s}_t, \tilde{x}_{t-1}))) + \delta \sum_{s_{t+1}} \pi(s_{t+1} | \tilde{s}_t) V_i^{n-1}(s_{t+1}, x_t(\tilde{s}_t, \tilde{x}_{t-1})) = U_i^{\text{aut}}(\tilde{s}_t) \). In practice, since \( x \) is discretized, in general this equality will only be satisfied approximatively.

Let us look at the case \( u(c_i) = \log(c_i) \) once again. Now we can write an equation with only one unknown, \( x_t(\tilde{s}_t, \tilde{x}_{t-1}) \):

\[ \log\left( \frac{y_1(\tilde{s}_t) + y_2(\tilde{s}_t)}{1 + x_t(\tilde{s}_t, \tilde{x}_{t-1})} \right) + \delta \sum_{s_{t+1}} \pi(s_{t+1} | \tilde{s}_t) V_i^{n-1}(s_{t+1}, x_t(\tilde{s}_t, \tilde{x}_{t-1})) = U_i^{\text{aut}}(\tilde{s}_t). \tag{23} \]

Once again we use \( V_i^{n-1}() \), but we evaluate it at the gridpoints \((s_{t+1}, x_t(\tilde{s}_t, \tilde{x}_{t-1}))\) where the economy may end up next period. Since \( x \) is discrete, we cannot use standard techniques to find \( x_t(\tilde{s}_t, \tilde{x}_{t-1}) \). Instead, we look for \( x_t(\tilde{s}_t, \tilde{x}_{t-1}) \in X \) such that the left hand side of (23) is as close as possible to \( U_i^{\text{aut}}(\tilde{s}_t) \), provided that it is weakly greater. Once we have \( x_t(\tilde{s}_t, \tilde{x}_{t-1}) \), we can easily obtain the rest of the policies. In sum, for logarithmic utility, we have the policy updates
\[ x_t(\tilde{s}_t, \tilde{x}_{t-1}) = \text{(the solution of (23))} \]
\[
c_1(\tilde{s}_t, \tilde{x}_{t-1}) = \frac{y_1(\tilde{s}_t) + y_2(\tilde{s}_t)}{1 + x_t(\tilde{s}_t, \tilde{x}_{t-1})} \]
\[
c_2(\tilde{s}_t, \tilde{x}_{t-1}) = x_t(\tilde{s}_t, \tilde{x}_{t-1}) \frac{y_1(\tilde{s}_t) + y_2(\tilde{s}_t)}{1 + x_t(\tilde{s}_t, \tilde{x}_{t-1})}. \]

Finally, we can compute
\[
V^n_1(\tilde{s}_t, \tilde{x}_{t-1}) = \log(c_1(\tilde{s}_t, \tilde{x}_{t-1})) + \delta \sum_{s_{t+1}} \pi(s_{t+1} | \tilde{s}_t) V^{n-1}_1(s_{t+1}, x_t(\tilde{s}_t, \tilde{x}_{t-1})),
\]

or the left hand side of (23), and
\[
V^n_2(\tilde{s}_t, \tilde{x}_{t-1}) = \log(c_2(\tilde{s}_t, \tilde{x}_{t-1})) + \delta \sum_{s_{t+1}} \pi(s_{t+1} | \tilde{s}_t) V^{n-1}_2(s_{t+1}, x_t(\tilde{s}_t, \tilde{x}_{t-1})).
\]

Notice once again that we use \(V^{n-1}_i()\) on the right hand side.

(c) The enforcement constraint for household 2 binds. We proceed symmetrically as in (b).

Now we are done with gridpoint \((\tilde{s}_t, \tilde{x}_{t-1})\). We have to do the above steps at all other gridpoints as well. Then the \(n^{th}\) iteration is complete. We continue iterating until the policy and value functions converge given some convergence criterion. For example we stop iterating when
\[
|V^n_i(\tilde{s}_t, \tilde{x}_{t-1}) - V^{n-1}_i(\tilde{s}_t, \tilde{x}_{t-1})| < \epsilon, \forall i, \text{ for some small } \epsilon.
\]

To obtain actual numbers for the consumption allocation and the value functions, we have to specify the utility functions for the two households, their discount factor, the initial relative weight in the social planner’s objective, as well as the income processes. Using appropriate household survey data, all these can be estimated\(^{14}\), which allows structural testing of the model.

\(^{14}\)Except for the initial relative weight of household 2 in the social planner’s objective. But remember that the distribution of the consumption allocation is independent of the initial relative weight with probability 1, given that we are in the case of partial insurance, and the economy has been running for a sufficient number of periods.
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