Competitive Effects of Vertical Integration with Downstream Oligopsony and Oligopoly

Simon Loertscher and Markus Reisinger*

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Abstract

This paper analyzes the competitive effects of backward vertical integration by a partially vertically integrated firm that competes à la Cournot with non-integrated firms both on the upstream and on the downstream market. We find under general conditions that vertical integration is only anticompetitive if the ex ante degree of integration is high and the number of competitors is large. That is, for most cases vertical integration has a pro-competitive effect and leads to a price decrease of the final output. This result contrasts with previous findings of slightly different market structures, e.g. a dominant firm model. Therefore, the present model’s implications for the regulation of vertical mergers differ from the previous literature by suggesting a more permissive approach.

Keywords: Vertical Integration, Downstream Oligopsony, Downstream Oligopoly, Capacity Choice

JEL-Classification: D41.

*Emails: simonl@unimelb.edu.au and markus.reisinger@lrz.uni-muenchen.de Financial support through a Research Grant by the Faculty of Economics and Commerce at the University of Melbourne is gratefully acknowledged.
1 Introduction

Over the last two decades substantial progress has been made on identifying procompetitive and anticompetitive effects of vertical mergers. After the Chicago School critique on the relative aggressive enforcement policy on vertical mergers in the 1960’s, several theories have emerged that base the potential competitive effects of vertical integration on more solid ground. Yet, there is no general consensus under which conditions a merger is likely to benefit or harm consumers and decrease (or increase) final good prices. The elimination of double marginalization has been identified as a major efficiency gain from vertical integration. On the other hand, the merging parties may be inclined to raise the input prices to their rivals and thereby induce market foreclosure. As a consequence of this trade-off, vertical mergers are often judged by antitrust authorities and courts on a case-by-case basis. General conclusions under which conditions either effect dominates are not easily gained.1

Moreover, many models that identify different aspects of vertical mergers are not readily applicable for policy implications because they are unsatisfactory in two important aspects. First, merging parties often claim to merge because of efficiency gains in production that lead to cost reductions. This means that they not only avoid double marginalization but are also able to produce the output good in a different and more efficient way than without integration. This effect is not captured in several well-known models (e.g. Ordover, Saloner, and Salop, 1990; Hart and Tirole, 1990; Choi and Yi, 2000). Second, firms that have oligopolistic market power in the downstream market often also exert oligopsonistic market power when buying the intermediate goods. Yet, many important models do not take this into account explicitly (e.g. Salinger, 1988; Chen, 2001).

A notable exception is Riordan (1998) who considers a model with a dominant firm and a competitive fringe in the downstream market. To produce the final good, firms need a fixed input, termed capacity, that is competitively offered on an upward sloping supply curve. The dominant firm exerts market power both downstream and on the input market. The more capacity a firm owns the lower are its production costs of the final good. Therefore, the model is not open to the two criticisms above. If the dominant firm integrates backwards, it acquires more capacity and so produces more output. On the other hand, since demand of capacity increases, the price of capacity increases as well and so fringe firms are foreclosed. Riordan (1998) shows that the second effect always dominates and vertical integration is anticompetitive

1For recent surveys on the effects of vertical mergers, see Riordan (2008) and Rey and Tirole (2007).
in that it leads to a decrease in output and to an increase in the final good price. A drawback of
the model by Riordan (1998) is that it can only be applied under the rather rare circumstances
where the final good market is comprised of a dominant firm facing a competitive fringe.

The present paper provides an analysis of an oligopolistic downstream market in which
the structure of the model is close the one of Riordan (1998). We show that the conclusions
about the competitive effects of vertical integration differ substantially from the dominant firm
model. In an oligopolistic market structure vertical integration is only anticompetitive if the ex
ante degree of integration is high and the number of downstream competitors is large. Thus,
for many circumstances vertical integration is procompetitive. As a consequence, our policy
implications differ substantially from those in the dominant firm model. They are also clear
cut and can be easily checked in most cases of vertical integration. Especially, in industries
that exhibit almost no integration in the first place vertical integration is likely to be beneficial
to consumers. Competition authorities need only be wary if an already largely integrated firm
plans to integrate further.

The intuition behind our result is the following. If a firm integrates backwards it increases
its capacity and produces a higher amount of output due its lower production costs. Thus, there
is a positive direct effect of integration. As a consequence of that, the input price increases
and so its rivals lower their capacity and produce less. We show that in an oligopolistic market
the direct effect is larger than the indirect effects and, as a result, more capacity is used. The
countervailing effect is that the firm with a larger capacity uses this capacity less efficiently
than its rivals. The reason is that it has the ability to produce more and so can gain more
from a higher price. Thus, it is less inclined to expand its quantity and utilizes its capacity to
a smaller extent than smaller rival firms. This second effect is the driving force in the model
of Riordan (1998) and is the decisive effect why vertical integration is anticompetitive. This
contrasts with an oligopolistic market structure, where this effect is dominated by the direct
capacity expansion effect as long as the ex ante degree of vertical integration is not excessively
high. This can even go so far that monopolizing the downstream market is welfare enhancing
as long as the number of possible producers is not too large. For example, in a model with
linear demand and quadratic costs we show that a firm that is vertically integrated to such an
extent that all rivals are foreclosed and that thereby monopolizes the market produces a larger
quantity than an oligopoly that consists of no more than three firms downstream.

As mentioned, most of the literature on vertical integration is mainly concerned with the
trade-off between avoidance of double marginalization and foreclosure. Hart and Tirole (1990) or Ordover, Saloner, and Salop (1990), where no efficiency gains from vertical integration are present, are only concerned with the foreclosure motives. In Salinger (1988), Choi and Yi (2000) and Chen (2001) both effects are present but although the downstream market is comprised of an oligopoly or a duopoly, downstream firms have no market power in the intermediate good market. A recent model that incorporates both effects and additionally allows downstream firms to exert market power in the intermediate good market is Hendricks and McAfee (2007). In their model upstream firms can overstate their cost function while downstream firms can understate their revenue function in the output market. The intermediate good market clears by bringing the reports of both sides together, and the solution technique is similar to the supply function approach of Klemperer and Meyer (1989). Hendricks and McAfee’s model is more general than ours with respect to the upstream market because they allow for a general number of firms while we impose this market to be perfectly competitive. On the other hand, when analyzing vertical mergers they keep the downstream price fixed and suppose that the market structure consists of no vertical integration at the outset. They show that under these assumptions output always increases with vertical mergers. In contrast, in our model the downstream price is flexible and we show that a crucial variable to determine the competitive effects of vertical integration is the degree to which the industry is already integrated: Vertical integration is procompetitive as in Hendricks and McAfee (2007) if there is no ex ante integration but it may become anticompetitive if the existing degree of vertical integration is already high.

Our result that vertical integration is procompetitive if the industry is not yet characterized by a high degree of vertical integration seems also consistent with recent empirical evidence and with court decisions. For example, Hortaçsu and Syverson (2007) study vertical integration in the cement and ready-mixed concrete industries during 1963 to 1997. During this time the extent of vertical integration increased heavily in both industries. Yet, in each market there is still a large number of firms present and no firm had a dominant position. Hortaçsu and Syverson (2007) find that in both industries quantities increased and prices decreased steadily during this time which is consistent with our findings.

A different example in which vertical integration is considered to be procompetitive is given by a recent decision of the Competition Commission (CC) in the UK. In October 2006 the CC cleared a merger between English Welsh & Scottish Railway Holdings (EWS) and Marcroft Engineering. EWS is the largest operator of rail freight haulage services in Great Britain,
while Marcroft is the largest supplier of rail freight wagon maintenance. Both companies have a market share in their respective markets of more than 50 percent and so the potential for foreclosure is relatively high. Yet, the CC cleared the merger with the lowest remedy that was proposed by the Office of Fair Trading, namely the divestiture of only some parts of Marcroft’s outstation business. Previous models would conclude that such a merger is very likely to be welfare reducing because it almost monopolizes the industry. In light of the predictions of our model, such a merger would be welfare reducing only if there is a large number of downstream competitors and the market would shift from being relatively competitive to being dominated by one firm. But in the haulage market there are only two competitors present (Freightliner and Direct Rail Services). As a consequence, the CC decision is consistent with the predictions of our model that, although the market is dominated by the integrated firm after the merger, the efficiency gains may well dominate foreclosure motives in this case.

The remainder of the paper is organized as follows. Section 2 lays out the model. Section 3 presents the solution to the model and gives conditions under which vertical integration is procompetitive or anticompetitive. In Section 4 we derive some explicit solutions for the linear-quadratic case. Section 5 concludes. Longer proofs and derivations are in the Appendix.

2 Model

The following model is adapted from Riordan (1998) with some minor differences with respect to timing. We also assume that the vertically integrated firm has no inherent cost advantage in production (i.e. \( \gamma = 0 \) in his notation). The main difference is, of course, that we assume a downstream oligopoly in contrast to his assumption of a dominant firm and a competitive fringe.

We study a dynamic game with complete information. There are two types of firms, one vertically integrated firm, which we index by \( I \) and \( N \geq 1 \) non-integrated firms. A typical non-integrated firm is indexed by \( j \).²

All firms produce a homogenous good sold à la Cournot on the downstream market, where the inverse demand function is \( P(Q) \) with \( P' < 0 \), where \( P \) is the market clearing price for aggregate quantity \( Q = q_I + \sum_{j=1}^{N} q_j \). To ensure existence and uniqueness of equilibrium in the quantity stage, we also impose the sufficient condition \( P'' \leq 0 \).

²We use lower case index letters mainly, though not exclusively to denote non-integrated firms. The index \( j \) is exclusively used for non-integrated firms.
Total costs of producing \( q_i \) units are
\[
c_i(q_i, k_i) = k_i C\left(\frac{q_i}{k_i}\right)
\] (1)
for all firms \( j \in I, 1, \ldots, N \), where \( k_i \) is firm \( i \)'s production cost reducing capacity chosen in stage 1 and \( c_i(.) \) exhibits constant returns to scale in \( q_i \) and \( k_i \) and increasing marginal costs in \( q_i \), with \( i = I, 1, \ldots, N \). That is, doubling \( k_i \) and \( q_i \) will double \( c_i \) while \( C' \geq 0 \) and \( C'' > 0 \).³

Firm \( I \)'s initial capacity endowment, i.e. its ex ante (or exogenous) degree of vertical integration, is \( \underline{k} > 0 \). The competitive inverse supply function of capacity is \( R(K) \) with \( R' > 0 \) and
\[
K \equiv k_I + \sum_{j=1}^{N} k_j.
\]

The timing is as follows.

• **Stage 1: The Capacity Stage.** The ex ante degree of vertical integration \( \underline{k} \) is exogenously given and common knowledge. All firms \( i \) simultaneously choose their level of capacity \( k_i \). Firm \( I \) buys \( k_I - \underline{k} \) at the market price \( R(k_I + \sum_{j=1}^{N} k_j) \). Observe that \( I \) has the opportunity to sell undesired capacity, which occurs if \( k_I < \underline{k} \).

• **Stage 2: The Quantity Stage.** Having observed all the capacity levels \( \mathbf{k} = (k_I, k_1, \ldots, k_N) \) all firms choose simultaneously their quantities \( q_I, q_1, \ldots, q_N \). A Cournot-Walras auctioneer announces the market clearing price \( P(Q) \) and the game ends, where
\[
Q \equiv \sum_{j=1}^{N} q_j + q_I.
\]

We focus on subgame perfect equilibria that are symmetric, i.e. on equilibria where the non-integrated firms play the same strategies. The quantity subgame following the choice of capacity \( \mathbf{k} \) is called the \( \mathbf{k} \)-subgame.

3 General Equilibrium Properties

The following results obtain under general assumptions.

3.1 The Quantity Stage (Stage 2)

We begin with a simple result that has several important implications.

**Lemma 1** Every \( \mathbf{k} \)-subgame has a unique equilibrium.

**Proof:** The first order condition for a profit maximum in the \( \mathbf{k} \)-subgame for firm \( i \) is
\[
P'(Q) + P'(Q)q_i = C'_i \left(\frac{q_i}{k_i}\right) .
\] (2)

³This type of cost function was introduced by Perry (1978); see also Hendricks and McAfee (2007).

⁴To save on notation, here and in the following we abbreviate \( C\left(\frac{q_i}{k_i}\right) \) by \( C_i \).
Note that at this stage there is no intrinsic difference between an integrated and a non-integrated firm. It is also readily checked that the second order condition is satisfied. Moreover, if the quantities of all firms other than \( i \) are zero, then \( i \) optimally sets a positive quantity, while its optimal quantity is zero if the aggregate quantity of all other firms is sufficiently large. Finally, the slope of the reaction function of firm \( i \) is negative but larger than \(-1\). Therefore, an equilibrium (in pure strategies) is guaranteed to exist and to be unique. ■

We denote by \( Q(\mathbf{k}) \) the aggregate equilibrium quantity given any vector of capacities \( \mathbf{k} \) and by \( q_i(\mathbf{k}) \) the corresponding equilibrium quantity of firm \( i \) with \( i = I, 1, \ldots, N \).

**Corollary 1**

\[
q_h(\mathbf{k}) > q_l(\mathbf{k}) \iff k_h > k_l.
\]

**Proof:** Suppose to the contrary that \( k_h > k_l \) but \( q_h \leq q_l \). But then the right hand side of (2) is strictly smaller for \( h \) than for \( l \) because \( C_i \) is convex in \( q_i \) and decreasing in \( k_i \) while the left hand side is is (weakly) larger for \( h \) than for \( l \), which is a contradiction. Conversely, if \( q_h > q_l \), the left hand side of (2) is smaller than the right hand side. Since \( C_i \) is convex, \( k_h \) must be bigger than \( k_l \). ■

**Corollary 2**

\[
k_i = 0 \Rightarrow q_i(\mathbf{k}) = 0 \quad \text{and} \quad \lim_{k_i \to 0} \frac{q_i(\mathbf{k})}{k_i} > 0.
\]

**Proof:** Because \( C_i \) is strictly convex, \( C_i' \) is invertible and equation (2) can be written as

\[
\frac{q_i}{k_i} = C_i'^{-1}(P(Q) + P'(Q)q_i)
\]

or equivalently as

\[
q_i = k_iC_i'^{-1}(P(Q) + P'(Q)q_i).
\]

Now \( k_i = 0 \Rightarrow q_i = 0 \) follows directly from (5). Observe that the inverse \( C_i'^{-1}(\cdot) \) is strictly increasing and zero if and only if its argument is zero. Since all firms that produce a positive quantity face positive marginal costs, \( P(Q) > 0 \) holds in equilibrium. Moreover, since as \( k_i \) approaches zero \( q_i \) approaches zero,

\[
\lim_{k_i \to 0} \frac{q_i}{k_i} = \lim_{q_i \to 0} C_i'^{-1}(P(Q) + P'(Q)q_i) = C_i'^{-1}(P(Q)) > 0.
\]

■

**Corollary 3** \( \frac{q_h(\mathbf{k})}{k_h} \) decreases in \( k_h \).
**Proof:** If \( k_h > k_l \) then we know from the last Corollary that \( q_h > q_l \). But this implies that the left hand side of (2) is smaller for \( h \) than for \( l \). As a consequence, the right hand side must be smaller as well. Since \( C_i \) is convex it follows that \( \frac{q_h}{k_h} < \frac{q_l}{k_l} \). The only if part can be proved by following the steps in the opposite direction. ■

**Corollary 4**

\[
\frac{q_i(k)}{k_i} > \frac{dq_i(k)}{dk_i} \quad \forall i \in \{I, 1, ..., N\}
\]

**Proof:** Due to Corollary 3, \( \frac{d(q_i/k_i)}{dk_i} < 0 \) holds. That is, \( \frac{d(q_i/k_i)}{dk_i} = \frac{(dq_i/dk_i)k_i - q_i}{k_i^2} < 0 \), which in turn implies \( \frac{dq_i}{dk_i} - \frac{q_i}{k_i} < 0 \). ■

The following Lemma states that all own effects of capacity on quantity are positive and all cross effects are negative, which is very intuitive.

**Lemma 2**

\[
\frac{dq_i(k)}{dk_i} > 0 \quad \text{for all} \quad i \quad \text{and} \quad \frac{dq_m(k)}{dk_i} < 0 \quad \forall i \neq m.
\]

The proof of the second part is rather long and therefore relegated to the Appendix. The first part is already implied by Corollary 1.

### 3.2 The Capacity Stage (Stage 1)

We now move on to the first stage of the game, the capacity choice game. We now assume that \( P'' \) and \( C''' \) are small compared to \( P', C'', R' \) and \( R'' \), so that we can safely ignore them.\(^5\)

Throughout the paper, we focus on stable equilibria.\(^6\) Next we establish the very intuitive result that the integrated firm is the largest firm (where size is measured in units of capacity) and that its capacity increases in \( k \).

**Proposition 1**

\[
\frac{dk_I}{dk} > 0 \quad \text{and} \quad \frac{dk_j}{dk} < 0 \quad \forall j \in \{1, ..., N\}.
\]

The proof is in the appendix. Proposition 1 has the following corollary:

**Corollary 5** \( k^*_I(k) > k^*_j(k) \Leftrightarrow \bar{k} > 0 \).

Corollary 3, Lemma 2 and Proposition 1 immediately imply the following result:

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\(^5\)Notice that the model is still slightly more general than a linear quadratic model because we still allow that \( P' \) and \( C'' \) differ if the arguments of these functions differ. We only require the difference to be small.

\(^6\)A model that has a unique stable equilibrium is the model with linear demand and quadratic cost functions analyzed in Section 4 below.
Proposition 2  \( \frac{q_I}{k_I} < \frac{q_j}{k_j} \) and \( q_I > q_j \).

Proposition 2 implies that the integrated firm uses capacity less efficiently than a non-integrated firm, which replicates the finding of Riordan (1998), which is an empirical prediction that may be tested independently.\(^7\)

The following proposition states a necessary and sufficient condition for vertical integration to be procompetitive. Some additional notation is useful. Denote by \( k^*(k) = (k_1^*(k), k_2^*(k), \ldots, k_N^*(k)) \) the vector of capacity choices in a stable equilibrium and define

\[
B(k) = - \frac{C''_I q_I (C''_j - k_j P')}{N C''_J k_j (C''_I - k_I P')} |k^*(k)|. \tag{9}
\]

Accordingly, we denote by \( \frac{dk_j}{dk_I} |_{k^*(k)} \) the change in \( j \)'s equilibrium level of capacity after an (exogenous) change in \( k \), which induces \( I \) to adjust its level of capacity to the new equilibrium level and all non-integrated firms other than \( j \) to adjust their capacities as well.\(^8\)

Proposition 3

\[
\frac{dQ(k^*(k))}{dk} > 0 \Leftrightarrow \frac{dk_j}{dk_I} |_{k^*(k)} > B.
\]

The proof is in the Appendix.

This proposition is important in several respects. The expression for \( B \) gives us a measure against which we can compare the equilibrium adjustment \( \frac{dk_j}{dk_I} |_{eq.} \) to find out if vertical integration is procompetitive. The next Lemma describes some aspects of \( B \).

Lemma 3

\[ B > - \frac{1}{N} \quad \text{and} \quad \frac{dB}{dk} > 0. \]

The proof can be found in the Appendix.

The first part of Lemma 3 implies that vertical integration is anticompetitive if each of the non-integrated firm reduces its capacity by \( 1/N \) as an effect of further integration by the

\(^7\)Hortaçsu and Syverson (2007) also find that integrated firms operate more efficiently which, at first glance, contradicts with our results. Yet, for simplicity in our model we do not consider efficiency differences between downstream firms that do not stem from different degrees of integration. Our model can be easily extended to allow for those differences and one can show that a firm with a cost advantage has to gain most from integration. This is then consistent with the findings of Hortaçsu and Syverson (2007) who show that more productive firms are more likely to vertically integrate and that productive differences do not stem from the integrated structure.

\(^8\)Notice that \( \frac{dk_j}{dk_I} |_{k^*(k)} \) is not the slope of a reaction function. The slope of a reaction function, which we denote as \( \frac{dk_j}{dk_I} |_{k^*(k)} \), gives the change in the optimal capacity level of firm \( j \), \( dk_j \), after a change of \( dk_I \) in firm \( I \)'s capacity, keeping the capacity levels of other firms other than \( j \) (and \( I \)) fixed. To stress the difference even more, \( \frac{dk_j}{dk_I} |_{k^*(k)} \) is also not the slope of the reaction function at equilibrium levels of capacity.
integrated firm. The intuition behind this result is the following. If the capacity reduction of each non-integrated firm would indeed by $1/N$, the capacity expansion of the integrated firm would be fully offset and so the overall amount of capacity stays the same. Capacity is only shifted from the non-integrated firms to the integrated one. But since we know that firm $I$ uses its capacity less efficiently than the other firms, vertical integration must be capacity reducing.

The second result implies that with a growing ex ante degree of vertical integration (increasing $k$), the absolute value of the reaction of the non-integrated firms must smaller and smaller for further vertical integration to be procompetitive. The reason is that the larger the integrated firm, the less efficient is its use of capacity. So if it integrates further, the increase in capacity has a less and less positive impact on final output. As a consequence, the reduction in capacity of the non-integrated firms must be smaller for vertical integration to be still procompetitive.

### 3.3 Two Extreme Cases as Benchmarks

So far we determined the threshold for $\frac{d k_j}{d k_I}$ such that vertical integration is procompetitive if $\frac{d k_j}{d k_I} > B$. But we have not yet determined the expression for $\frac{d k_j}{d k_I}$ in equilibrium. In the following we make some progress on this part.

The profit function of a non-integrated firm $j \in \{1, ..., N\}$ is given by

$$
\Pi_j(k) = P(Q(k))q_j(k) - k_jC_j \left( \frac{q_I(k)}{k_I} \right) - k_I R \left( k_I + \sum_{h=1}^{N} k_h \right). \tag{10}
$$

The first order condition after maximizing with respect to $k_j$ can be written as

$$
\frac{\partial \Pi_j}{\partial k_j} = P'q_j \left( \frac{dq_I}{dk_j} + (N - 1) \frac{d q_i}{dk_j} \right) - C_j + C'_j q_j k_j - R - k_j R' = 0, \tag{11}
$$

with $i \neq j, I$. To determine $\frac{d k_j}{d k_I} |_{eq.}$ we may invoke the Implicit Function Theorem to get

$$
\frac{\partial^2 \Pi_j}{\partial k_j^2} d k_j = - \left( \frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} + (N - 1) \frac{\partial^2 \Pi_j}{\partial k_j \partial k_i} \frac{d k_i}{d k_I} \right) d k_I.
$$

Since in equilibrium $k^*_I = k^*_j$ the last equation can be written as

$$
\frac{d k_j}{d k_I} |_{eq.} = - \frac{\frac{\partial^2 \Pi_j}{\partial k_j \partial k_I}}{\frac{\partial^2 \Pi_j}{\partial k_j^2} + (N - 1) \frac{\partial^2 \Pi_j}{\partial k_j \partial k_i}}. \tag{12}
$$

We can now state the result on the extreme case where the ex ante degree of vertical integration is zero.
Proposition 4 For any finite \( N \) there exists a \( k^*(N) > 0 \) such that for all \( k < k^*(N) \), vertical integration is procompetitive.

The proof is relegated to the appendix.

In order to look at the other extreme case where \( k \) is so high that \( k_j = 0 \), \( j \in \{1, ..., N\} \), we need to determine the terms in (12) more explicitly, which we do in the Appendix. In Proposition 4 we analyzed the case when \( k \) is close to zero such that \( k_j \) was close to \( k_I \). To proceed we now look at the opposite case where \( k \) is so high that the resulting \( k_I \) in equilibrium is large enough so that \( k_j = 0 \), for all \( j \in \{1, ..., N\} \). In the proof of the proposition the following definition is useful. We define \( \bar{k} \) as the ex-ante degree of vertical integration at which \( k_j = 0 \) and therefore \( q_j(k^*)(\bar{k}) = 0 \), i.e. if \( k = \bar{k} \), only the integrated is active and the market is monopolized.

Proposition 5 For \( k > k^{**}(N) \) and \( N > N^{**} \), vertical integration is anticompetitive.

This result is of some interest. It shows that even if rival firms are completely foreclosed by the integrated firm this is not necessarily welfare reducing because the integrated firm has accumulated so much capacity that it finds it optimal to produce a large amount of output.

So far, we have determined the consequences of vertical integration in the extremes when either vertical integration is zero or when it is so large that only the integrated firm is active. In the first case vertical integration is procompetitive as long as \( k < k^* \) while in the second case it is either also procompetitive if \( N \) is small or anticompetitive if \( k > k^{**}(N) \) and \( N > N^{**} \).

What is still missing is a complete characterization of the equilibrium for “intermediate” values of \( k \). That is, keeping \( N \) fixed one would like to know whether vertical integration is always anticompetitive when \( k \) exceeds \( k^* \). Put differently, the question is whether as \( k \) varies \( \frac{dk_j}{dk_I} \Big|_{k^*(\bar{k})} \) and \( B(\bar{k}) \) cross either only once in which case \( k^* = k^{**} \) or never in which case vertical integration is always procompetitive or multiple times. We strongly believe that, depending on \( N \), they either cross once or never. However, a major obstacle for proving this conjecture rigorously is that \( \frac{dk_j}{dk_I} \Big|_{k^*(\bar{k})} \) is a rather complicated expression that is likely to increase in \( k \) just as does \( B(\bar{k}) \) (see Lemma 3), so that the result depends on the relative magnitude of the not so nice derivatives of \( \frac{dk_j}{dk_I} \Big|_{k^*(\bar{k})} \) and \( B(\bar{k}) \).
4 Equilibrium in a Linear-Quadratic Model

In this section we consider a simple yet tractable model where the demand function is linear and the cost functions are quadratic. To be specific, we assume that

\[ P(Q) = 1 - Q, \quad C_i(q_{ik}) = \frac{1}{2} \left( \frac{q_{ik}}{k_i} \right)^2 \]

for all \( i \in \{ I, 1, ..., N \} \), and \( R(K) = K \). The quadratic cost function implies that the marginal cost function is linear.

4.1 The Quantity Stage (Stage 2)

Solving the quantity setting stage yields equilibrium quantities of

\[ q_I(k) = \frac{k_I(1 + k_j)}{1 + 2k_I + (N + 1)k_j + (N + 2)k_jk_I} \]  \hspace{1cm} (13)

and

\[ q_j(k) = \frac{k_j(1 + k_I)(1 + k_i)}{1 + (N + 2)k_Ik_jk_i + (N + 1)k_Ik_j + (N + 1)k_jk_i + 3k_Ik_j + 2(k_I + k_j) + Nk_i}, \]  \hspace{1cm} (14)

where \( i \in \{ 1, ..., N \}, i \neq j \).

4.2 The Capacity Stage (Stage 1)

We can then turn to the first stage the capacity choice game. For example, the profit function of the integrated firm in this case can be written as

\[ \Pi_I(k) = \left( 1 - \frac{k_I + Nk_j + (N + 1)k_Ik_j}{1 + 2k_I + (N + 1)k_j + (N + 2)k_jk_I} \right) \frac{k_I(1 + k_j)}{1 + 2k_I + (N + 1)k_j + (N + 2)k_jk_I} \]

\[ - \frac{k_j(1 + k_I)^2}{2(1 + 2k_I + (N + 1)k_j + (N + 2)k_jk_I)^2} - (k_I - \bar{k}) \left( k_I + \sum_{j=1}^{N} k_j \right). \]

After differentiating and deriving the first order conditions, one can solve the model numerically, namely for any \( \bar{k} \) and \( N \) we can calculate the equilibrium capacities \( k_I^* \) and \( k_j^* \). We can use this to derive the following result.

**Proposition 6** In the linear-quadratic model, \( \frac{dQ}{dk} > 0 \) for \( N \leq 2 \), i.e. vertical integration (even to monopoly) is procompetitive.

**Proof:**

[NOTICE: IN THIS PROOF, \( \bar{k} \) IS MISSING EVERYWHERE.] In the linear-quadratic model, equation (9) can be written as

\[ B(k) = -\frac{1 + k_j(2 + k_j)}{N(1 + k_I(2 + k_I))}. \]
Thus, if vertical integration yields a monopoly and \(k_j = 0\) we have

\[ B(k) = -\frac{1}{N(1 + k_I(2 + k_I))} \]  

(15)

Calculating \(\frac{dk_j}{dk_I}\) for the case \(k_j = 0\) gives

\[ \frac{dk_j}{dk_I} |_{k_j=0} = -\frac{2 + k_I(7 + 12k_I + 8k_I^2)}{(1 + N)(2 + 3k_I)(1 + 3k_I(1 + k_I))}. \]  

(16)

Subtracting (15) from (16) gives

\[ -\frac{Nk_I(2 + 13k_I + 28k_I^2 + 30k_I^3 + 8k_I^4)}{(1 + N)(2 + 3k_I)(1 + 3k_I(1 + k_I)))N(1 + k_I)^2}. \]

We can use this expression and check if it is greater than zero at the equilibrium value of \(k_I\) at which \(k_j = 0\). For the case of \(N = 2\) we get that \(k = 0.41655\) induces \(k_I = 0.32305\) which yields \(k_j = 0\). With \(N = 3\) and \(k_I = 0.32305\) the expression is \(0.00115 > 0\) and so vertical integration to monopoly is procompetitive. On the other hand, for \(N \geq 3\), vertical integration yields a negative expression. ■

The above result might look surprising at first glance because it implies that one large firm is better for consumers than an oligopoly consisting of three firms. Yet, the result can be explained by the fact that our model does explicitly take into account efficiency gains in production that go beyond pure avoidance of double marginalization. To push its competitors out of the market, a firm must acquire a large amount of capacity with which it will produce a quantity that is larger than the oligopoly quantity. Thus, our model implies a more permissive handle of vertical integration than previous ones.

5 Conclusions

In this paper we show that vertical integration with a downstream oligopoly and an increasing upstream supply curve is procompetitive under a fairly wide circumstances. Whether it is procompetitive or anticompetitive depends on the degree of ex ante vertical integration. Only if this degree is high and the number of downstream competitors relatively large will vertical integration reduce final goods output and increase consumer prices. Otherwise, vertical integration lowers prices and thus benefits consumers. These predictions differ starkly from the dominant firm model. Consequently, it would be misleading to extrapolate these results to models with a different, and in some sense more general, downstream market structure.

An interesting extension of the model, suggested by Rey and Tirole (2007), is to endogenize \(k\). Our model makes clear predictions under which vertical integration is procompetitive, yet
it does not determine the optimal, or equilibrium, degree of vertical integration, assuming that
the vertically integrated firm has to pay to acquire capacity. Such an analysis is not possible
in the dominant firm model because there the dominant firm is the only one who can acquire
capacity at the upstream market. The present model with downstream oligopoly provides a
framework that permits an analysis of the question whether the amount of capacity acquired
in equilibrium is indeed small enough for vertical integration to be procompetitive. This is
something we plan to analyze in the near future.

A Appendix

A.1 Proof of Lemma 2

Let $i \neq j, j \neq I$ and $i \neq I$. Totally differentiating (2) with respect to $k_i$ yields

$$P' \frac{dQ}{dk_i} + P' \frac{dq_i}{dk_i} + P'' q_i \frac{dQ}{dk_i} = -C'' q_i \frac{1}{k_i} \frac{dq_i}{dk_i}. \quad (17)$$

Notice that $\frac{dQ}{dk_i}$ can be written as $\frac{dQ}{dk_i} = \frac{dq_I}{dk_i} + \sum_{j \neq i} \frac{dq_j}{dk_i} + \frac{dq_i}{dk_I}$, which under the symmetry assumption that $k_j = k$ for all $j \neq i, I$ becomes

$$\frac{dQ}{dk_i} = \frac{dq_I}{dk_i} + (N - 1) \frac{dq_j}{dk_i} + \frac{dq_i}{dk_I}. \quad (18)$$

Therefore, (17) can be written as an equation that depends on the three variables $\frac{dq_j}{dk_i}$, $\frac{dq_i}{dk_I}$ and $\frac{dq_I}{dk_I}$, which we wish to determine.\(^9\)

Totally differentiating the first order condition for firm $j$, which is analogous to (2), with
respect to $k_i$ yields

$$P' \frac{dQ}{dk_i} + P' \frac{dq_j}{dk_i} + P'' q_j \frac{dQ}{dk_i} = C'' q_j \frac{1}{k_j} \frac{dq_j}{dk_i}. \quad (19)$$

and analogously differentiating the first order condition for $I$ with respect to $k_i$ yields

$$P' \frac{dQ}{dk_i} + P' \frac{dq_I}{dk_i} + P'' q_I \frac{dQ}{dk_i} = C'' q_I \frac{1}{k_I} \frac{dq_I}{dk_i}. \quad (20)$$

The unique solution to the system of three equations (17), (19) and (20), which are linear in
the three unknowns $\frac{dq_j}{dk_i}$, $\frac{dq_i}{dk_I}$ and $\frac{dq_I}{dk_I}$, is

$$\frac{dq_I}{dk_i} = \frac{C'' q_I k_I (P' + P'' q_I)}{\eta k_I} < 0 \quad \text{for} \quad i \neq I, \quad (21)$$

$$\frac{dq_j}{dk_i} = \frac{C'' q_j [(P'' - P' k_I) (P' + P'' q_j)]}{\eta (C'' - P' k_I)} < 0 \quad \text{for} \quad i \neq j \quad (22)$$

\(^9\)On top of that, we also need to determine $\frac{dq_I}{dk_I}$ and $\frac{dq_I}{dk_I}$, which we do in a second step.
and
\[
\frac{dq_i}{dk_i} = \frac{C_i'' q_i [(P')^2 k_i k_1 (N + 1) + P'(P'' k_i k_1 (q_i + (N - 1)q_i) - 2C_i'' k_i - C_i'' k_i N)]}{\eta k_i (C_i'' - P'' k_i)} \\
+ \frac{C_i'' q_i [C_i'' C_i'' - P'' (C_i'' k_i q_i + (N - 1)C_i'' k_i q_i)]}{\eta k_i (C_i'' - P'' k_i)} > 0,
\]

where
\[
\eta \equiv \{(P')^2(N + 2)k_1 k_i + P'(P'' k_i k_1 (q_i + Nq_i) - C_i' k_i (N + 1) - 2k_i C_i'') \\
+ C_i'' C_i'' - P''(C_i'' q_1 k_i + C_i'' q_1 k_i N)\} > 0.
\]

So this completes the first step.

Totally differentiating the first order condition (2) for firm \( I \) and \( i \) with respect to \( k_I \) yields, respectively,
\[
P' \frac{dQ}{dk_I} + P' \frac{dq_i}{dk_I} + P'' q_i \frac{dQ}{dk_I} = -C_i'' q_i k_i + C_i'' \frac{1}{k_i} \frac{dq_i}{dk_I},
\]
and
\[
P' \frac{dQ}{dk_I} + P' \frac{dq_i}{dk_I} + P'' q_i \frac{dQ}{dk_I} = C_i'' \frac{1}{k_i} \frac{dq_i}{dk_I},
\]
where under symmetry
\[
\frac{dQ}{dk_I} = \frac{dq_i}{dk_I} + N \frac{dq_i}{dk_I}.
\]

Using (26) to replace \( \frac{dQ}{dk_I} \) in (24) and (25) yields a systems of two linear equations in the two unknowns \( \frac{dq_i}{dk_I} \) and \( \frac{dq_i}{dk_I} \). The solution is
\[
\frac{dq_i}{dk_I} = \frac{C_i'' q_i [P'' q_i + P']}{\nu} < 0
\]
and
\[
\frac{dq_I}{dk_I} = -\frac{C_i'' q_i [k_i (P' q_i (N + 1) + P'' Nq_i) - C_i'']}{\nu} > 0,
\]
where
\[
\nu \equiv k_I \{(P')^2 k_i k_1 (N + 2) + P'[P'' k_i k_1 (q_i + Nq_i) \\
- C_i' k_i (N + 1) - 2C_i'' k_i] + C_i'' C_i'' - P''[C_i'' q_1 k_i + C_i'' q_1 k_i N]\} > 0.
\]

Summarizing, we therefore have shown that all own effects are positive and all cross effects are negative.
A.2 Proof of Proposition 1

The profit function at the capacity stage for the integrated firm is

$$\Pi_I(k) = P(Q(k))q_I(k) - k_I C \left( \frac{q_I(k)}{k_I} \right) - (k_I - k) R \left( k_I + \sum_{j=1}^{N} k_j \right). \tag{30}$$

Using the envelope theorem and dropping arguments, the capacity first order condition for the integrated firm is

$$P' dQ - I dC + k_I C_I q_I - C_I' q_I k_I - R - (k_I - k) R' = 0, \tag{31}$$

where $Q_{-I}$ is the aggregate equilibrium quantity of all firms other than $I$, i.e. $Q_{-I} = \sum_{j=1}^{N} q_j$.

The profit function by a representative non-integrated firm $j$ is given by

$$\Pi_j(k) = P(Q(k))q_j(k) - k_j C \left( \frac{q_j(k)}{k_j} \right) - k_j R \left( k_I + \sum_{i=1}^{N} k_i \right) \tag{32}$$

and the first order condition is

$$P' dQ - j dC + C_j q_j - C_j' q_j k_j - R - k_j R' = 0. \tag{33}$$

Differentiating (31) and (33) with respect to $k$ yields

$$\frac{\partial^2 \Pi_I}{\partial k^2} \frac{dk_I}{dk} + N \frac{\partial^2 \Pi_I}{\partial k_I \partial k_j} \frac{dk_I}{dk} + \frac{\partial^2 \Pi_I}{\partial k_I \partial k} = 0$$

and

$$\frac{\partial^2 \Pi_j}{\partial k^2} \frac{dk_j}{dk} + (N - 1) \frac{\partial^2 \Pi_j}{\partial k_j \partial k_i} \frac{dk_j}{dk} + \frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} \frac{dk_I}{dk} = 0.$$

Using the fact that in equilibrium $\frac{dk_I}{dk} = \frac{dk_j}{dk}$, we get

$$\frac{dk_j}{dk} = \frac{\frac{\partial^2 \Pi_I}{\partial k_I \partial k_j}}{\frac{\partial^2 \Pi_I}{\partial k_j^2} + (N - 1) \frac{\partial^2 \Pi_j}{\partial k_j \partial k_i} - N \frac{\partial^2 \Pi_j}{\partial k_j \partial k_I}} \tag{34}$$

and

$$\frac{dk_j}{dk} = -\frac{\frac{\partial^2 \Pi_I}{\partial k_I \partial k_j}}{\frac{\partial^2 \Pi_I}{\partial k_j^2} + (N - 1) \frac{\partial^2 \Pi_j}{\partial k_j \partial k_i} - N \frac{\partial^2 \Pi_j}{\partial k_j \partial k_I}} \tag{35}.$$

Since we can ignore terms with $P''$, the six terms that determine the fractions of the last...
equations can be written as

\[
\frac{\partial^2 \Pi_j}{\partial k_j^2} = P' \frac{dq_j}{dk_j} \left[ \frac{dq_i}{dk_j} + (N-1) \frac{dq_i}{dk_j} \right] + P' q_j \left[ \frac{d^2 q_i}{dk_j^2} + (N-1) \frac{d^2 q_i}{dk_j^2} \right] \\
+ C''_j \frac{q_j}{k_j^2} \left( \frac{dq_j}{dk_j} - \frac{q_j}{k_j} \right) - 2R' - k_j R'',
\]

\[
\frac{\partial^2 \Pi_j}{\partial k_j \partial k_i} = P' \frac{dq_i}{dk_i} \left[ \frac{dq_i}{dk_j} + (N-1) \frac{dq_i}{dk_j} \right] + P' q_j \left[ \frac{d^2 q_i}{dk_j^2} + (N-2) \frac{d^2 q_i}{dk_j^2} \right] \\
+ C''_j \frac{q_j}{k_j^2} \left( \frac{dq_j}{dk_i} - R' - k_j R'' \right),
\]

\[
\frac{\partial^2 \Pi_I}{\partial k_I^2} = P' \frac{dq_I}{dk_I} N \frac{dq_j}{dk_I} + P' q_I N \frac{d^2 q_j}{dk_I^2} \\
+ C''_I \frac{q_I}{k_I^2} \left( \frac{dq_j}{dk_I} - q_I \right) - 2R' - (k_I - k) R'',
\]

\[
\frac{\partial^2 \Pi_I}{\partial k_I \partial k_j} = P' \frac{dq_I}{dk_I} N \frac{dq_j}{dk_I} + P' q_I \left[ \frac{d^2 q_j}{dk_I^2} + (N-1) \frac{d^2 q_j}{dk_I^2} \right] \\
+ C''_I q_I \frac{dq_j}{dk_I} - R' - (k_I - k) R'',
\]

\[
\frac{\partial^2 \Pi_I}{\partial k_I \partial k} = +R'.
\]

From the proof of Lemma 2 we know all the expressions containing first derivatives. From these expressions we can determine the expression for the ten second derivatives that are present in the last equations. These expressions are the following:

\[
\frac{d^2 q_I}{dk_I^2} = \frac{2}{k_I \left[ -k_I k_j (2 + N) (P')^2 + \left( 3k_I C''_j + 2k_I (1 + N) C''_I \right) P' - 2C''_I C''_I \right]} > 0,
\]

\[
\frac{d^2 q_j}{dk_j^2} = -\frac{2}{k_I \left[ -k_I k_j (2 + N) (P')^2 + \left( 3k_I C''_j + 2k_I (1 + N) C''_I \right) P' - 2C''_I C''_I \right]} > 0,
\]

\[
\frac{d^2 q_i}{dk_i d k_I} = \frac{P' C''_j C''_I \left( q_i C''_I - P' k_j (-q_I + q_j (N + 1)) \right)}{k_j \left[ (P')^2 k_I k_j (2 + N) - (2k_I C''_I + k_j (1 + N) C''_I) P' + C''_I C''_I \right]^2},
\]

\[
\frac{d^2 q_i}{dk_j d k_I} = \frac{\left( P')^2 C''_I C''_I \left( -k_I k_j (q_I + q_j) P' + C''_I q_j k_I + C''_I q_k k_j \right)}{k_I (C''_I - k_j P') \left( k_I k_j (N + 2) (P')^2 - (1 + N) k_I C''_I + 2k_I C''_I P' + C''_I C''_I \right)^2} > 0,
\]
\[
\begin{align*}
\frac{d^2q_k}{dk_jdk_i} &= 2\frac{(P')^2(C''_j)^2q_j(C''_j - P'k_j)}{k_j \left( (P')^2k_jk_i(2 + N) - (2k_jC''_j + k_j(1 + N)C''_j)P' + C''_jC''_j \right)^2} > 0, \\
\frac{d^2q_k}{dk_jdk_i} &= 2\frac{(P')^2(C''_j)^2q_j(C''_j - P'k_j)^2}{k_j(2 + N) - (2k_jC''_j + k_j(1 + N)C''_j)P' + C''_jC''_j} > 0, \\
\frac{d^2q_j}{dk_jdk_i} &= \frac{P'^2C''_jC''_j \left( P'(k_jq_jNC''_j + k_jC''_j(2q_l - q_j)) - (P')^2k_jk_i(-q_j + q_1(N + 1)) - C''_jC''_jq_j \right)}{k_j \left( (P')^2k_jk_i(2 + N) - (2k_jC''_j + k_j(1 + N)C''_j)P' + C''_jC''_j \right)^2} < 0, \\
\frac{d^2q_j}{dk_jdk_i} &= \frac{(P')^2C''_jC''_j \left( C''_jq_jk_i + C''_jq_jk_i - P'k_jk_i(q_l + q_j) \right)}{k_j \left( (P')^2k_jk_i(2 + N) - (2k_jC''_j + k_j(1 + N)C''_j)P' + C''_jC''_j \right)^2} > 0.
\end{align*}
\]

and
\[
\frac{d^2q_j}{dk_j^2} = 2\frac{(P')^2k_jq_jC''_j}{k_j \left( (P')^2k_jk_i(2 + N) - (2k_jC''_j + k_j(1 + N)C''_j)P' + C''_jC''_j \right)^2} > 0.
\]

Inserting the respective expressions into \(\frac{dk_j}{dk_k}\) and \(\frac{dk_j}{dk_k}\), simplifying and noting that \(P' < 0\), \(C''_j > 0, k \in \{j, I\}\), \(R'' > 0\) and \(R'' > 0\) yields the desired result.

**A.3 Proof of Proposition 3**

We have that \(\frac{dQ(k^*(k))}{dk} > 0 \iff \frac{dq_1(k^*(k))}{dk} + N\frac{dq_1(k^*(k))}{dk} > 0\). This can be rewritten as
\[
\left( \frac{dq_1(k^*(k))}{dk} + N\frac{dq_1(k^*(k))}{dk} \right) \frac{dk_j}{dk} + N \left( \frac{dq_1(k^*(k))}{dk} + \frac{dq_1(k^*(k))}{dk} + (N - 1)\frac{dq_1(k^*(k))}{dk} \right) \frac{dk_j}{dk} |_{k^*(k)} > 0.
\]

Rearranging gives
\[
\frac{dk_j}{dk} |_{k^*(k)} = \frac{\frac{dk_j}{dk} |_{k^*(k)}}{\frac{\frac{dk_j}{dk} + N\frac{dq_1}{dk}}{\frac{dq_1}{dk} + \frac{dq_1}{dk} + (N - 1)\frac{dq_1}{dk}}}
\]

Substituting equations from the proof of the quantity stage yields
\[
\frac{dk_j}{dk} |_{k^*(k)} = \frac{C''_j\frac{dq_1}{dk}(C''_j - k_jP')}{NC''_j\frac{dq_1}{dk}(C''_j - k_jP')} = B(k).
\]
A.4 Proof of Lemma 3

From Corollary 3 we know that \( \frac{\partial k}{\partial \epsilon} \) decreases in \( k_h \) and Proposition 1 says that \( k_I > k_j \) and \( \frac{dk_I}{d\epsilon} > 0 \). Therefore, we can write \( k_j = a(k)k_I \), with \( 0 \leq a(k) \leq 1 \), \( q_j/k_j = b(k)(q_I/k_I) \) with \( b(k) > 1 \) and \( C''_j = d(k)C''_I \) with \( d(k) > 1 \). Since \( dk_j/dk_I < 0 \) (because of the assumption that a unique pure strategy equilibrium exists), it follows that \( a' < 0 \), \( b' > 0 \) and \( d' > 0 \).

Substituting these into (9) yields

\[
B = -\frac{C''_I k_I (C''_j - k_j P')}{NC''_j k_j (C''_I - k_I P')} = -\frac{1}{N} \frac{d(k)C''_I - a(k)k_I P'}{b(k)d(k)(C''_I - k_I P')} > -\frac{1}{N},
\]

where the inequality follows because \( b(k)d(k) > 1 > a(k) \geq 0 \) holds, so that the second fraction is positive but less than 1. Dropping all arguments, one gets

\[
\frac{dB}{dk} = -\frac{1}{N} a'dkI + abdkI + a'bdkI - d^2b'C''_I > 0.
\]

A.5 Proof of Proposition 4

Preliminaries. Dividing \( \frac{dk_j}{d\epsilon} \) by \( \frac{dk_I}{d\epsilon} \) from Proposition 1 yields

\[
\frac{dk_j}{dk_I} \bigg|_{k^*(\epsilon)} = \frac{\frac{dk_j}{d\epsilon}}{\frac{dk_I}{d\epsilon}} \bigg|_{k^*(\epsilon)} = -\frac{\xi - \phi(R' + k_jR'')}{\psi - \phi((N + 1)R' + Nk_jR'')},
\]

with

\[
\xi = k_j q_j q_I C''_j C''_I \times
\]

\[
\left[ (P')^3 k_j^3 k_I \left( 3N + 2 - \frac{2q_I}{q_I} \right) - (P')^4 k_j^2 \left( C''_j k_I \left( 4N + 8 - \frac{5q_I}{q_I} \right) + C''_I k_j (3N - 1) \right) \right.
\]

\[
+ (P')^3 k_j C''_I \left( C''_j k_I \left( N + 8 - \frac{4q_I}{q_I} \right) + C''_I k_j (4N + 1) \right) \] - \[
- (P')^2 (C''_j)^2 \left( C''_j k_I \left( 2 - \frac{q_I}{q_I} \right) + C''_I k_j (N + 3) \right) + P'(C''_j)^3 C''_I \left< 0,
\]

\[
\psi = k_I q_j^2 C''_I \left[ -(P')^6 k_j^3 k_I (N+2)(3N+2) + (P')^5 k_j^2 k_I (C''_j k_I (18 + 13N + 3N^2) + 2C''_I Nk_j (5 + 3N) - \right.
\]

\[
- (P')^4 k_j ((C''_j)^2 k_I^2 (16 + 6N + N^2) + C''_j C''_I k_j k_I (15 + 17N + 6N^2) + 2(C''_j)^2 (N + 1)k_j^2 (3N - 1)) + \right.
\]

\[
+ (P')^3 C''_j (2(C''_j)^2 k_I^2 (N + 1) + C''_j C''_I k_j k_I (9 + 4N + N^2) + 2(C''_j)^2 k_j^2 (3 + 4N + N^2)) -
\]

\[
\]

19
\[-(P')^2 (C'_{ij}''C''_{ji}(N + 1) + 2(C''_{ij})^2 k_j^2(5 + 2N + N^2)) + P'(C''_{ij})^3 (C''_{ij})^2(N + 1) \right] < 0, \ (45)\]

and

\[\phi = k_I k_j^2 (C''_{ij} - k_j P')^2 \left( (P')^2 k_j k_I (N + 2) - P' (2C''_{ij} k_I + C''_{ij} k_I (N + 1)) + C''_{ij} C''_{ij} \right)^2 > 0.\]

Assume now that ex ante there is no vertical integration (\(k = 0\)), so that \(k^*_I = k^*_j\), \(q_I(k^*(k)) = q_j(k^*(k))\), and \(C''_{ij} = C''_{ij}\). In this case

\[B(0) = -\frac{1}{N}.\]

On the other hand, in the expression for \(\frac{dk_j}{dk_I} \mid_{k^*(k)}\), we get

\[\xi = -q_j^2 P' (C''_{ij})^2 (C''_{ij} - k_j P')^2 (3(P')^2 k_j^2 N - P' k_j C''_{ij} N + (C''_{ij})^2),\]

\[\psi = -q_j^2 P' C''_{ij} (C''_{ij} - k_j P')^2 ((P')^3 k_j^2 (N + 2)(3N + 2) - (P')^2 k_j^2 C''_{ij} (10 + 3N^2 + 7N) + P' k_j (C''_{ij})^2 (6 + N^2 + 3N) - (N + 1)(C''_{ij})^3)\]

and

\[\phi = k_I k_j^2 (C''_{ij} - k_j P')^4 (C''_{ij} - (N + 2)k_j P')^2.\]

Subtracting \(B\) from \(\frac{dk_j}{dk_I} \mid_{k^*(k)}\) at \(k = 0\) yields that

\[\frac{dk_j}{dk_I} \mid_{k^*(k)} - B > 0,\]

if and only if

\[R' k_j^2 (C''_{ij} - k_j P')^4 (C''_{ij} - (N + 2)k_j P')^2 - q_j^2 C''_{ij} P'(C''_{ij} - k_j P')^2 (C''_{ij} - (N + 2)k_j P')(\nu C''_{ij} (3N + 1) - 4P' k_j C''_{ij} + (C''_{ij})^2) > 0.\]

But obviously the last inequality is always satisfied since \(P' < 0\) and \(C''_{ij} > 0\). Thus, at \(k = 0\) vertical integration is consumer welfare increasing. \(\blacksquare\)
A.6 Determining Second Order Partial Derivatives in (12)

Differentiating (11) with respect to \( A \),

\[
\frac{\partial^2 \Pi_j}{\partial k_j^2} = P' q_j \left[ \frac{dq_i}{dk_j} + (N - 1) \frac{dq_i}{dk_j} \right] + P'' q_j \left[ \frac{d^2 q_i}{dk_j^2} + (N - 1) \frac{d^2 q_i}{dk_j^2} \right]
\]

\[+ \] \[P'' q_j \left( \left\{ \frac{dq_i}{dk_j} \right\}^2 + 2(N - 1) \frac{dq_i dq_i}{dk_j dk_j} + (N - 1) \left( \frac{dq_i}{dk_j} \right)^2 \right] \]

\[+ \] \[P'' q_j \frac{dq_i}{dk_j} \left( \frac{dq_i}{dk_j} + (N - 1) \frac{dq_i}{dk_j} \right) \]

\[+ \] \[C'' q_j \frac{dq_i}{dk_j} \left( \frac{dq_i}{dk_j} - \frac{q_j}{k_j} \right) - 2R' - k_j R'', \tag{46} \]

\[
\frac{\partial^2 \Pi_j}{\partial k_j \partial k_I} = P' q_j \left[ \frac{dq_i}{dk_j} + (N - 1) \frac{dq_i}{dk_j} \right] + P'' q_j \left[ \frac{d^2 q_i}{dk_j dk_I} + (N - 1) \frac{d^2 q_i}{dk_j dk_I} \right]
\]

\[+ \] \[P'' q_j \left( \frac{dq_i}{dk_j} + (N - 1) \frac{dq_i}{dk_j} \right) \left[ \frac{dq_i}{dk_I} + \frac{dq_i}{dk_I} + (N - 1) \frac{dq_i}{dk_I} \right] \]

\[+ \] \[C'' q_j \frac{dq_i}{dk_j} \left( \frac{dq_i}{dk_I} - R' - k_j R'' \right), \tag{47} \]

\[
\frac{\partial^2 \Pi_j}{\partial k_I \partial k_i} = P' q_j \left[ \frac{dq_i}{dk_j} + (N - 1) \frac{dq_i}{dk_j} \right] + P'' q_j \left[ \frac{d^2 q_i}{dk_I dk_i} + (N - 1) \frac{d^2 q_i}{dk_I dk_i} \right]
\]

\[+ \] \[P'' q_j \left( \frac{dq_i}{dk_j} + (N - 1) \frac{dq_i}{dk_j} \right) \left[ \frac{dq_i}{dk_I} + \frac{dq_i}{dk_I} + (N - 1) \frac{dq_i}{dk_I} \right] \]

\[+ \] \[C'' q_j \frac{dq_i}{dk_j} \left( \frac{dq_i}{dk_I} - R' - k_j R'' \right). \tag{48} \]

We need these expressions for the following proof.

A.7 Proof of Proposition 5

Let \( \bar{k} = \bar{k} \), so that \( k_j^*/(\bar{k}) = 0 \) for all \( j \neq I \). Therefore, by Corollary 3, \( q_j = 0 \) as well. Also, as we let \( k_j \rightarrow 0 \) for all \( j = 1, \ldots, N \), we have

\[
\lim_{k_j \rightarrow 0, j \neq I} \frac{q_j}{k_j} = (C_j')^{-1}(P(q_I)) > 0,
\]

from Corollary 2. To simplify notation in the following we denote \((C_j')^{-1}(P(q_I)) \equiv \alpha\). The threshold value for \( B \) in the case of \( \bar{k} = \bar{k} \) can then be written as

\[
B(\bar{k}) = -\frac{C'' q_I \bar{k}_I}{\alpha(C_I' - k_I P')} N.
\]

On the other hand,

\[
\frac{dk_j}{dk_I} \mid_{k^*(\bar{k})} = -\frac{C'' \bar{k} P' \alpha}{(N + 1) \left[ \alpha^2 P'(C_I' - k_I P') + \beta \right]}, \tag{49}
\]
with $\beta \equiv R'k_I (2P'k_I - C''_I) < 0$. It follows that

$$\frac{dk_j}{dk_I} |_{k^*_I(k)} < B(k)$$

if and only if

$$- \left( -\frac{N}{1 + N} \right) \frac{C''_I (P' + \frac{\beta}{k_I})}{\alpha (C''_I - k_I P')} + \frac{\beta}{\alpha (C''_I - k_I P')} < - \frac{C''_I q_I k^*_I(k)}{\alpha (C''_I - k_I P')}.$$  (50)

If $N \to \infty$, $N/(1 + N) \to 1$. In this case the left hand side of (50) is smaller than the right hand side. To see this, notice that $- \frac{C''_I (P' + \frac{\beta}{k_I})}{\alpha (C''_I - k_I P')} + \frac{\beta}{\alpha (C''_I - k_I P')} = - \frac{C''_I q_I k^*_I(k)}{\alpha (C''_I - k_I P')}$ if and only if $\beta = 0$. But since $\beta < 0$, the left hand side is smaller than the right hand side. Conversely, if $N$ is small the right hand side can either be larger or smaller than the left hand side dependent on the exact functional forms. As a result we have that if $k = \bar{k}$ an increase in $k$ can either be pro- or anticompetitive. ■

References


