On the characterisation and measurement of the welfare effects of income mobility from an ex-ante perspective

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Abstract  The paper employs a rank-dependent formulation of the social welfare function with time-separable utilities to evaluate the economic consequences of income mobility from an ex-ante perspective. The resultant class of measures can be decomposed either in terms of structural and exchange mobility or in terms of vertical and horizontal mobility, thereby encompassing two of the main approaches in the literature. Illustrative results show that income mobility in the USA was both less pro-poor in absolute terms and more horizontally inequitable than in Germany, but that the latter did not translate into higher exchange mobility given higher levels of absolute inequality and the vertical stance of the growth process.

1. Introduction
The multifaceted nature of income mobility as a phenomenon has resulted in the emergence of several distinct strands in the literature exploring different aspects of the notion (see Fields and Ok (1999) for a survey). Thus, Shorrocks (1978, p.1016) argues that “interest in mobility is not only concerned with movement but also predictability – the extent to which future positions are dictated by the current place in the distribution.” This paper adds to a number of recent studies that have sought in a two-period setting to break down measures of overall income mobility into components due to various types of distributional transformation or change. In particular, both van Kerm (2004) and Ruiz-Castillo (2004) propose decompositions based on a distinction between ‘structural’ and ‘exchange’ mobility, originally derived from the sociology literature, where the former is identified with changes in the location and shape of the marginal income distribution and the latter with the permutation of a fixed set of income opportunities among individuals. Depending on the choice of measure, structural mobility may further be attributed to changes in mean income
due to growth in the size of the economy and to changes in the dispersion of incomes that are conventionally captured by measures of cross-section or snapshot inequality. Exchange mobility entails the reranking of individuals within the income distribution, which is generally regarded as a socially desirable phenomenon within the mobility literature on the grounds that it serves to equalise incomes over the longer term (see also Yitzhaki and Wodon, 2004; Fields 2005).

Mobility may also usefully be considered as a form of redistribution, which serves to emphasise the formal similarities with the analysis of the properties of tax and benefit schemes. In particular, Benabou and Ok (2001) argue that mobility is socially desirable if it attenuates the effects of initial income disparities on future income prospects, which will be the case if income changes are progressive in the sense that the ratio of expected income growth to initial income declines with the initial level of income. This judgement embodies a concern for vertical equity alone in that only the impact of mobility on individuals with differing initial levels of ‘wellbeing’ is considered. Benabou and Ok (2001) go on to propose a summary index of equalizing mobility that provides a measure a vertical redistribution formally equivalent to the Reynolds and Smolensky (1977) index of residual progressivity: the index will be positive if the poor receive a larger (smaller) share of the gains from income growth (or a smaller (larger) share of income losses if growth is negative) than of initial wellbeing. However it needs to be recognised that the direct application of this measure to income data will not fully capture the welfare implications of mobility, even from an ex-ante perspective, since it does not take account of the classical horizontal inequities that will arise because individuals with the same initial level of income will fare differently in the future if the income process is stochastic: if utility is concave in income then the expected utility of the distribution of future income prospects conditional upon initial income will be less than the utility of the expected value of the conditional distribution of income prospects. Moreover, as is explicitly acknowledged by Benabou and Ok (2001), if individuals are risk averse then the expected income growth rate conditional upon initial income will be similarly preferred to the prospect of facing a lottery yielding the same outcome on average. From either perspective, the conditional dispersion of income prospects will appear to be a socially undesirable phenomenon.

The main contribution of this paper is to provide a consistent framework for the measurement of mobility that can account for the reranking of individuals and classical horizontal inequities as distinct phenomena, even though both will usually arise from the
stochastic nature of the dynamic income process. More specifically, the paper proposes a class of ethical mobility measures that may be decomposed either in terms of structural and exchange mobility or in terms of vertical mobility, which is determined by the scale and distribution of income growth, and horizontal mobility, which reflects the inequity associated with the conditional dispersion of future income prospects. Welfare effects due to income mobility are initially evaluated from an ex-ante, risk-neutral perspective, in keeping with Benabou and Ok (2001), and expressed in terms of differences in equally distributed equivalent (ede) incomes. A simple extension of the measurement framework allows for risk aversion by assuming that individuals consider some certainty equivalent when evaluating the set of utility opportunities that they face. The resultant measures serve to both characterise and quantify the mobility process underlying the transition from the initial to the final distribution of incomes. Alternatively, they may be interpreted as embodying a concept of mobility that provides an ex-ante welfare comparison of the observed income distribution in the final period with a hypothetical immobile benchmark in which each individual receives the same income in the final period as in the initial period.

Following the integrative approach of Duclos et al. (2003) to the measurement of redistribution, the key to the methodology is the use of a class of time additive social welfare functions that includes both the rank-dependent S-Gini class of welfare functions and the utilitarian Atkinson social welfare function (Atkinson, 1970) as special cases. This dual functional structure provides fundamentally separate normative bases for the measurement of reranking and classical horizontal inequity effects. Rank dependence results in sensitivity of the mobility measures to how final incomes are distributed relative to individuals’ positions in the initial income distribution, with a positive value placed on exchange mobility if society is averse to relative deprivation such that the weights attached to individual utilities in the

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1 In principle, reranking may also arise from “income traps” in the growth schedule, i.e. if expected future incomes are a decreasing function of initial income over some range. The possibility of such “systematic” reranking can be ruled out empirically if, as is commonly assumed to be the case, the mobility process is strictly monotone in the sense of first order stochastic dominance (see e.g. Dardanoni, 1993; Benabou and Ok, 2001).

2 The idea of hypothetical benchmark structures is well established in the mobility literature. The benchmark in this paper is the same as in van Kerm (2004), since the intention is similarly to account for the whole of the change in the distribution of incomes over time. In contrast, Ruiz-Castillo (2004) abstracts from (relative) income growth by defining a benchmark in which each individual receives the same share of total income in the final period as in the initial period. Studies that focus solely on exchange mobility, such as Gottschalk and Spolaore (2002), employ a benchmark structure in which individuals are assigned the final period income associated with their rank in the initial period.
social welfare function decrease from poorest to richest. Non-linearity in incomes results in sensitivity of the mobility measures to the conditional dispersion of income prospects, with a negative value placed on classical horizontal mobility if utility is a concave function of income such that individuals are averse to inequality. This negative valuation is exacerbated once account is taken of the uncertainty of future incomes if individuals are also risk averse: in particular, it is no longer the case that the presence of exchange mobility will necessarily lead to a higher ex-ante valuation of welfare in the final period than the realised outcome.

Our measurement framework thus reveals a tension between the perceived value of reranking or reversals on the one hand and the predictability of future incomes or origin dependence on the other. This issue has previously been explored in the literature using a class of intertemporal welfare functions in which individual utilities are defined as a concave function of consumption levels in both periods. In particular, Atkinson and Bourguignon (1982) show that reranking will increase welfare if the positive value from aversion to multi-period inequality more than offsets any negative value due to aversion to intertemporal fluctuations in consumption. Gottschalk and Spolaore (2002) further show that once account is explicitly taken of uncertainty then the unpredictability of future incomes will serve to reduce welfare if individuals are more averse to risk than to either multi-period inequality or intertemporal fluctuations. However these parallel results are not directly comparable to those presented in this paper given the fundamental differences between the normative bases of the analyses.

The paper is organised as follows. Section 2 introduces the proposed measures to characterise and quantify mobility within a two-period framework in which the focus is on a single transition from an initial to a final state. In Section 3 we use our measures to compare mobility patterns in the USA and Germany. Section 4 concludes.

2. Measurement of mobility within a two-period framework
We follow much of the literature in focusing on a single transition between two periods and establish an ‘ethical’ basis for the measurement of mobility within this framework. Let $F(y_1, y_T)$ be the joint cumulative distribution function (cdf) for initial and final period incomes, $Y_i$ and $Y_F$ respectively, with support contained in the positive real orthant. Let $p_t = F_t(y_t)$ be the marginal cdf for period $t$ ($t=1,T$) incomes, where $p_t$ is the proportion of the population with an initial income less than $y_t$. The corresponding quantile function will be $Y_t(p_t) = F_t^{-1}(p_t)$ for
\[ p_t \in [0,1], \text{ which may loosely be thought of as the income of an individual with a (normalised) rank of } p \text{ in the period } t \text{ distribution. Hence, average income in period } t \text{ is } \bar{Y}_t = \int_0^1 Y_t(p_t) dp_t. \]

We employ a social evaluation function first proposed by Berrebi and Silber (1981) and more recently used by Duclos et al. (2003) to analyse redistributive effects in a continuous setting. This function allows us to define social value as the weighted average of individuals’ intertemporally separable utilities, where the weights are determined by individuals’ ranks in the income distribution. Thus welfare in period \( t \) evaluated on the basis of contemporaneous ranks \( p_t \) is equal to:

\[
W_t = \int_0^1 U(Y_t(p_t), \varepsilon) w(p_t, \nu) dp_t; \quad t=1, T
\]

where utility is given as an isoelastic function of income:

\[
U(y_t, \varepsilon) = \begin{cases} y_t^{\varepsilon}, & \text{when } \varepsilon \neq 1 \\ \ln (y_t), & \text{when } \varepsilon = 1; \quad \varepsilon \geq 0 \end{cases}
\]

and the rank-dependent weights are given by:

\[
w(p_t, \nu) = \nu(1-p_t)^{(\nu-1)}; \quad \nu \geq 1.
\]

Equation (1) includes the utilitarian Atkinson social welfare function (Atkinson, 1970) and the rank-dependent S-Gini class of welfare functions (see Yitzhaki, 1983) as special cases, when \( \nu = 1 \) and \( \varepsilon = 0 \) respectively. The ‘distributional judgement’ parameter \( \nu \) controls the rate at which the weights decrease from poorest to richest. Specifically, \( \nu = 2 \) leads to weights that decrease linearly with \( p \) from 2 to 0, as with the conventional Gini coefficient, whereas values greater (less) than 2 yield indices that give more (less) social weight to the utility of poorer individuals than implied by the conventional Gini. In the limit \( \nu = 1 \) and the social weights are independent of rank. The parameter \( \varepsilon \) is the well-known measure of relative inequality aversion introduced by Atkinson (1970). Smaller values of \( \varepsilon \) are associated with lower aversion to relative income inequality, with the limiting case \( \varepsilon = 0 \) implying inequality neutrality.

Following Atkinson (1970), let the \textit{ede} income of \( F_t(y_t) \) be the level of income per head which if equally distributed would give the same level of social welfare:

\[
\xi_t = U^{-1}(W_t) = \begin{cases} \left( \int_0^1 U(Y_t(p_t), \varepsilon) w(p_t, \nu) dp_t \right)^{\frac{1}{\varepsilon}}, & \text{when } \varepsilon \neq 1 \\ \exp \left( \int_0^1 U(Y_t(p_t), \varepsilon) w(p_t, \nu) dp_t \right), & \text{when } \varepsilon = 1 \end{cases} \quad \varepsilon \geq 0; t = 1, T;
\]
where $W_t$ is as defined in (1). Measuring mobility in terms of changes in *ede* income rather than welfare allows all indices to be expressed in monetary terms, thereby providing a direct indication of the underlying scale of resultant changes in the income distribution. Accordingly, the ‘cost of inequality’ is defined in absolute terms as the income per head which could be sacrificed with no loss of welfare if the remainder were equally distributed:

$$A_t = \bar{y}_t - \xi_t; \quad t = 1, T$$

(5)

where $\bar{y}_t \geq A_t \geq 0$ since $\bar{y}_t \geq \xi_t \geq 0$ by construction. For the limiting case of $v=0$, $\xi_t = W_t$ and $A_t = \int_{0}^{1} Y_t(p_t) [1 - w(p_t, v)] dp_t$ is identified as the absolute S-Gini coefficient.

Within this framework, the ex-ante evaluation of the incomes that individuals receive in the final period $T$ based on their rank positions in the initial period 1 yields:

$$W_X = \int_{0}^{1} \int_{0}^{1} U(Y_t(p_t), \varepsilon) w(p_t, v) dp_t dp_t$$

where $W_X$ will generally exceed $W_t$ as a result of the reranking of individuals within the income distribution. Let $\xi_X = U^{-1}(W_X)$ be the corresponding *ede* income. We proceed to define the following class of mobility measures:

$$M_N = (\xi_X - \xi_1)$$

(7)

which includes the second total mobility index proposed by Silber and Weber (2005) as a special case if $v=2$ and $\varepsilon=0$. In general, $M_N$ fully captures the impact of distributional changes from an ex-ante perspective if individuals are risk neutral. It is an ‘ex-ante’ measure in that the evaluation of welfare in both the initial and final periods is based on the social weights associated with individuals’ ranks in the initial income distribution. Following Dardanoni (1993), this asymmetric treatment can be justified on the grounds that the initially poor are disadvantaged to the extent that they face a worse lottery of future income possibilities than those who are better off, with the ‘distributional judgement’ parameter $v$ allowing for the calibration of the poverty focus of the evaluation (see Essama-Nssah, 2005). It is also possible in principle to employ individuals’ final period weights to evaluate mobility (see, for example, the third total mobility index proposed by Silber and Weber, 2005) but the forward-looking perspective is the more natural one when assessing the impact of mobility over time.

The risk-neutral mobility index $M_N$ may readily be decomposed to yield:

$$M_N = (\xi_X - \xi_1) = (\xi_T - \xi_1) + (\xi_X - \xi_T) = M_X + M_X$$

(8)
where $M_S$ and $M_X$ are identified respectively as structural and exchange mobility indices. The former is determined by changes in the marginal distribution of incomes whereas the latter arises from the permutation of a fixed set of income opportunities among individuals. Hence $M_S$ and $M_X$ are independent terms in that a change in structural mobility may occur without affecting the level of exchange mobility and vice versa.

The structural index $M_S$ is equal, if $v=2$ and $\varepsilon=0$, to the first total mobility index proposed by Silber and Weber (2005). More generally, $M_S$ may be seen to provide a ‘snapshot’ measure of total mobility that fully captures the welfare effects of income changes if movements of, but not within, the income distribution are of social concern. $M_S$ may be further decomposed, in the manner of van Kerm (2004) or Silber and Weber (2005), to give:

$$M_S = (\xi_T - \xi_1) + (\bar{y}_T - \bar{y}_1) + (A_T - A_T) = M_G + M_R;$$

where $M_G$ provides a ‘growth’ mobility index measuring the change in mean income over the period, and $M_R$ is a ‘redistributive’ mobility index$^3$ that measures the change in the per capita cost of inequality. $M_G$ is invariant to any redistribution of a given total income among the population whereas, for the limiting case of $\varepsilon=0$, $M_R$ is invariant to equal absolute (not proportionate) growth in all incomes.

The exchange mobility index $M_X$ is equal, if $v=2$ and $\varepsilon=0$, to $\bar{y}_T \cdot H_{A_T}$ where $H_{A_T}$ is the well-known Atkinson-Plotnick reranking index that is considered by Yitzhaki and Wodon (2004) as a measure of mobility in its own right. More generally, $M_X$ is equal to $A_T - A_X$, since $\bar{y}_X = \bar{y}_1$ by definition, from which it follows that the index will be non-negative as the concentration curve for period $T$ income, with individuals ranked by period 1 incomes, must lie on or above the Lorenz curve for period $T$ income (see Lambert, 2001, p.29). The index may thus be interpreted as a measure of the extent to which structural mobility overstates the disequalising effects of income growth, or understates its equalising effects, from an ex-ante perspective due to the reshuffling of individuals within the distribution. For any given change in cross-sectional inequality, social value will be maximised ex-ante by the complete inversion of the rank ordering of income between the two periods.

The risk-neutral mobility index $M_N$ may also be decomposed into vertical and (classical) horizontal mobility measures if mobility is considered as form of redistribution. Vertical mobility relates to how well individuals with different levels of initial income expect to fare over time, whereas horizontal mobility has to do with the divergence in the prospects

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$^3$ This term is identified as ‘structural’ mobility in Silber and Weber (2005)
of those with initially identical levels of income. The key to this alternative decomposition rests on the identification of expected period $T$ incomes conditional upon initial income (rank):

$$\bar{Y}_T(p_i) = \int_{Y_I(p_i)}^{Y_{f+1}(p_i)} Y_T(p_{T+1} | Y_I(p_i)) dp_{T+1},$$

(10)

where $Y_T(p_{T+1} | Y_I(p_i)) = F_T^{-1}(p_{T+1} | Y_I(p_i))$ is the conditional quantile function of final period incomes. The ex-ante evaluation of this conditional expected income distribution yields:

$$W_E = \int_{Y_{f-1}(p_i)}^{Y_{f+1}(p_i)} U(\bar{Y}_T(p_i), \epsilon) w(p_i, v) dp_i,$$

(11)

and the corresponding $ede$ income $\xi_E = U^{-1}(W_E)$. Hence:

$$M_N = (\bar{Y}_E - \bar{Y}_T) = (\xi_E - \xi_T) + (\xi_E - \xi_T) = M_V + M_H$$

(12)

where $M_V$ and $M_H$ are identified respectively as vertical and horizontal mobility indices. The former is determined by conditional expected income changes whereas the latter is determined by the conditional dispersion of income changes. Hence $M_V$ and $M_H$ are independent terms in that a change in vertical mobility may occur without affecting the level of horizontal mobility and vice versa.

The vertical index $M_V$ may be further decomposed to show how vertical mobility is determined by both the scale and pattern of expected income growth across the income range:

$$M_V = (\xi_E - \xi_T) = (\bar{Y}_E - \bar{Y}_{f+1}) + (A_{V} - A_{E}) = M_G + M_E$$

(13)

where $\bar{Y}_E = \int_{Y_{f-1}(p_i)}^{Y_{f+1}(p_i)} Y_T(p_i) dp_i$ equals $\bar{Y}_T$ by definition and $A_{V} = \bar{Y}_E - \xi_E$. Equation (13) provides a counterpart to the decomposition of $M_S$ into growth and redistributive elements, with the common element $M_G$ providing a measure of expected income growth over the whole population as before. The vertical equity index $M_E$ provides a measure of the vertical stance of the mobility process and may be interpreted as an index of equalising opportunity in the sense of Benabou and Ok (2001). For the limiting case, $\epsilon=0$:

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4 Given that $Y_T = \bar{Y}_T(p_i) + e_T$ by definition, where $e_T$ is an error term with zero mean, one might also consider the parallel decomposition $M_N = (\xi_E - \xi_T) + (\xi_E - \xi_T) = M'_H + M'_V$, where $\xi_E = U^{-1}(W_E)$ and $W_S = \int_{Y_{f-1}(p_i)}^{Y_{f+1}(p_i)} U(Y_T(p_i) + e_T, \epsilon) w(p_i, v) dp_i dp_T$, or some ‘Shapley value’ average of the two (cf. van Kerm, 2004). Our preference for (12) is based on the inherent interest of $\bar{Y}_T(p_i) = E[Y_T | Y_I(p_i)]$ as a counterfactual and the connection that its use establishes with both Benabou and Ok (2001) and the broader literature on redistribution. In practice, empirical estimates of $M'_V$ and $M'_H$ were very similar to those reported for $M_V$ and $M_H$ (and would be identical for the limiting case of $\epsilon=0$).
which is the negative of the absolute S-concentration coefficient of conditional expected income growth calculated using initial year rankings. Thus $M_E$ will be positive (negative) if income growth is progressive (regressive) in absolute terms such that expected income gains are a decreasing (increasing) function of initial income, and will equal zero if the growth schedule is uniform. More generally, $M_E$ will depend on both the distribution and scale of expected income growth.

The horizontal index $M_H$ will be non-positive reflecting the ex-ante welfare loss due to the classical horizontal inequity associated with the condition dispersion of future income prospects. To see this point note that (6) may be re-written as:

$$W_X = \int_0^1 \left( \int_0^1 U \left( Y_T \left( p_T \mid Y_T(p_i) \right), \varepsilon \right) dp_T \right) w(p_i, \varepsilon) \, dp_i = \int_0^1 \bar{U}_T \left( p_i, \varepsilon \right) w(p_i, \varepsilon) \, dp_i;$$

(15)

where $\bar{U}_T \left( p_i, \varepsilon \right)$ is expected utility in period $T$ conditional upon rank in period 1. $\zeta_X$ will exceed $\zeta_X$ if the utility function is concave and $M_H$ will therefore be non-positive. Thus the index may be interpreted as a measure of the extent to which vertical mobility overstates the positive effects of income growth because it fails to allow for the inequity manifest in the divergent prospects of those with initially identical incomes. $M_H$ will capture both the inefficiency and the unfairness manifest in such inequity given the specification of the welfare function (cf. Broome, 1989).

Hence, if individuals are risk neutral, total mobility may be seen from an ex-ante perspective as the sum either of structural and exchange mobility or of vertical and horizontal mobility. To see the link between these alternative decompositions it is useful to express period $T$ utilities $U_T(\varepsilon)$ as a function of individuals’ ranks in the initial income distribution:

$$U_T(\varepsilon) = \bar{U}_T(p_i, \varepsilon) + u_T; \quad \forall p_i$$

(16)

where conditional mean utility $\bar{U}_T(p_i, \varepsilon)$ is implicitly defined in (15) and $u_T$ is a ‘disturbance’ term with zero mean at each rank in the initial distribution. It follows that if the mobility process was non-stochastic then period $T$ incomes would be perfectly predictable from initial income (rank) and hence $U_T(\varepsilon)$ would equal $\bar{U}_T(p_i, \varepsilon) = U_T(\bar{Y}_T(p_i), \varepsilon)$ with $u_T = 0$ for all individuals. In these circumstances $M_S = M_S = M_T$ and $M_S = M_H = 0$ if, as will
usually be the case, $\bar{U}_T(p_1, \varepsilon)$ is an increasing function of initial income (rank),\(^5\) which provides a counter-factual benchmark that can be used to show how both exchange and classical horizontal mobility arise from the stochastic nature of the mobility process. Specifically, we note that $M_N$ is invariant to noise in the mobility process, since $W_X$ is shown in (15) to depend on conditional mean not actual utilities in period $T$, but that this is not true of $M_S$ unless $\nu=1$ nor of $M_V$ unless $\varepsilon=0$. In general, higher conditional dispersion in utility prospects will be associated both with an increase in cross-sectional inequality in the final period offset by a rise in exchange mobility due to greater reranking and with more progressive income growth offset by an increase in (classical) horizontal mobility due to the lower predictability of future incomes. Exchange and horizontal mobility may thus be seen as two sides of the same coin. But, whereas the persistence of inequality over time is often held to be more tolerable if accompanied by higher levels of exchange mobility, the pursuit of an absolute pro-poor growth strategy\(^6\) will be less successful if accompanied by higher levels of horizontal mobility.

We further note that $M_N$ will not fully capture the full economic consequences of the unpredictability of future utilities if individuals are risk averse. We can extend our measurement framework if $0 \leq \varepsilon \leq 1$ to take account of risk aversion by considering the certainty-equivalent values of utility in period $T$. Thus, following the approach taken in Gottschalk and Spolaore (2002), we assume that individuals in period 1 know both their current utility and the conditional density of utility outcomes in period $T$, and proceed to define certainty-equivalent utility as:

\[
\hat{U}(p_1, \gamma, \varepsilon) = \begin{cases} 
\left(\int_0^1 \left[ U\left(Y_T\left(p_T \mid Y_1(p_1)\right), \varepsilon\right)\right]^{-\gamma} dp_T\right)^{1/\gamma}, & \text{when } \gamma \neq 1 \\
\exp\left(\int_0^1 \log\left[U\left(Y_T\left(p_T \mid Y_1(p_1)\right), \varepsilon\right]\right) dp_T\right), & \text{when } \gamma = 1
\end{cases}
\]

\(5\) If $\bar{U}_T(p_1, \varepsilon)$ is decreasing over some range of initial incomes (see footnote 1 for discussion) then $M_N$ will equal $M_S$ but not $M_V$ if $U_T(\varepsilon) = \bar{U}_T(p_1, \varepsilon)$ for all individuals. In general, the extent of any systematic reranking may be estimated as the difference between the absolute concentration coefficients of final utility ranked by expected final utility and by initial utility. For a given change in cross-sectional inequality, the presence of systematic reranking will result in the mobility process being more progressive than it would otherwise have been.

\(6\) Growth is said to be pro-poor in absolute (relative) terms if it leads to a fall in absolute (relative) inequality, i.e. if the absolute (relative) benefits of growth received by the poor are more than those received by the non-poor (see Kakwani et al. 2004, for discussion).
where $\gamma$ is the well known Arrow-Pratt coefficient of relative risk aversion (CRRA), with lower values of the parameter implying lower aversion to risk and the limiting case of $\eta = 0$ indicating risk neutrality. Replacing $\bar{U}_T(p_1, \epsilon)$ in (15) by $\hat{U}_T(p_1, \gamma, \epsilon)$ yields:

$$W_A = \int_0^T \hat{U}_T(p_1, \gamma, \epsilon) w(p_1, v) \, dp_1$$

and the corresponding $ede$ income $\xi_A = U^{-1}(W_A)$. $\xi_A$ will be less than $\xi_T$ if $\gamma > 0$ and the mobility process is stochastic, implying that risk averse individuals will prefer the offer of the conditional expectation $\bar{U}_T(p_1, \epsilon)$ to the prospect of facing a lottery yielding the same outcome on average. Indeed, $\xi_A$ may even be lower than $\xi_T$ if the social value of exchange mobility is more than offset by the perceived costs of utility uncertainty. Accordingly, overall (risk-adjusted) mobility $M$ may be defined as:

$$M = (\xi_A - \xi_T) = (\xi_A - \xi_X) + (\xi_X - \xi_T) = M_A + M_N$$

which will be less than or equal to $M_N$, with the risk adjustment index $M_A$ providing a measure of the extent to which $M_N$ overstates the positive effects of mobility because it fails to allow for the uncertainty manifest in the divergent fortunes of those with initially identical incomes.\(^7\) And it is no longer unambiguously the case that the prospect of more exchange mobility will be preferable to less, holding the level of snapshot mobility $M_S$ constant (i.e. for a given change in the marginal utility distribution), as this now depends on whether or not the positive social value of reranking $M_X$ exceeds the perceived costs of the associated uncertainty $M_A$.

Finally, we note that all our mobility indices are expressed in monetary terms and are therefore not invariant to the choice of currency units. However all the indices may appropriately be normalised by the initial level of $ede$ income $\xi_1$, given that the measurement framework provides an evaluation of the mobility process from the standpoint of the initial distribution of income. Moreover $M_G$, which captures the effects of income growth, may suitably be expressed as a proportion of mean income in the initial period $\bar{y}_1$. Similarly $M_R$,

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\(^7\) Note that the treatment of “origin independence” in Gotschalk and Spolaore (2002) confounds a positive welfare effect due to what would be identified in this paper as the elimination of horizontal inequities (in final period consumption) with the negative value due to risk aversion per se. In our framework, the replacement of actual final period utilities by their conditional means has no effect on the ex-ante evaluation of welfare, as shown by (15), so $M_A$ captures the effects of risk aversion alone and is therefore non-positive by definition.
$M_S$, $M_E$ and $M_A$, which capture various aspects of the redistributive characteristics of the process, can be stated as a proportion of the per capita cost of inequality in the initial period $A_1$.

**Estimation of mobility indices**

Estimation of the mobility indices requires (sample weighted) observations on an identical sample at two points in time. $W_I$ and $W_T$ in (1) are estimated as weighted sums of sample utilities, where the utilities are calculated from (2) and the weights from (3) using the formula given in Lerman and Yitzhaki (1989) to obtain the normalised rank $p$ of each (sample weighted) observation. $M_S$, $M_G$ and $M_R$, as defined in (9), can then be calculated using the definitions of $\xi_t$ and $A_t$ in (4) and (5). $W_X$ in (6) is evaluated in similar fashion, cumulating final incomes over positions in the initial rather than the final income distribution, to yield an estimate of $\xi_X$ with which to calculate non-parametric estimates of $M_X$ and $M_N$ from (8).

Estimation of the remaining mobility indices is less straightforward as this requires knowledge of the conditional quantile function of period $T$ incomes $Y_T(p_T | Y_I(p_i))$ at each sample value of initial income. Following Duclos et al. (2003), one approach to this problem would be to estimate the conditional mean function $\bar{Y}_T(p_i) = E[Y_T | Y_I(p_i)]$ by non-parametric regression and then proceed to obtain a full characterisation of $Y_T(p_T | Y_I(p_i))$ under the assumption of conditional normality. However, the conditional normality assumption is unattractive in the current setting as it would further imply normality of the marginal distribution of final incomes. A more general approach is therefore adopted in which non-parametric regression techniques are used to estimate the Box-Cox model $y_T^{(\lambda)}(\lambda Y_T - 1)/\lambda = g(Y_I(p_i)) + \epsilon_T^{BC}$ with the conditional mean and quantile functions subsequently recovered under the assumption that $\epsilon_T^{BC}$ is normally distributed with zero mean and variance $\sigma_T^{2\epsilon_T(p_i)}$. The Box-Cox transformation $y_T^{(\lambda)}$ has been extensively used to induce normally distributed additive errors in regression models (see Cook and Weisberg, 1982), with the parameter $\lambda$ chosen using a grid search procedure so as to minimise the (sample

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8 Alternatively one might seek to estimate either a Singh-Maddala or a Dagum conditional income distribution in the manner of Biewen and Jenkins (2005). However, estimation of the three distributional parameters in each case as either linear or simple polynomial functions of initial income leads in both cases to estimates of the conditional mean income function that exhibited singularities within the observed income range. Conditional estimation of the (four-parameter) Generalized Beta of the second kind distribution failed entirely due to convergence problems.
weighted) residual sum of squares in the Zarembka (1968) scaled version of the model (see Spitzer, 1982). It follows that the conditional distribution of final period incomes will be approximately power normal, with the model encompassing normal and lognormal specifications as special cases when \( \lambda = 1 \) and \( \lambda = 0 \) respectively. The function \( g(Y_T(p_1)) \) is estimated using the variable span smoother of Sasieni (1998), which fits a local linear regression to the (sample weighted) observations on \( y_T^{(\lambda)} \) and \( y_I \) in the neighbourhood of each data point in the sample. The use of a local linear regression estimator may be expected to provide a reasonable approximation to \( g(Y_T(p_1)) \), and hence predictions of \( y_T^{(\lambda)} \), so long as the curvature of the unknown function is not excessive (Hastie and Loader, 1993), and also yields local estimates of the variance of the error term \( \sigma^2_{e_T(p_1)} \).

Given the Box-Cox model conditional quantile functions, estimates of \( \bar{Y}_T(p_1) \), \( \bar{U}_T(p_1, \gamma, \varepsilon) \) and \( \hat{U}_T(p_1, \gamma, \varepsilon) \) are obtained using (eighth order) Taylor-series approximations about the predicted values of \( y_T^{(\lambda)} \) for computational speed. These approximations then provide the basis for the calculation of \( M_N, M_V, M_H, M_G, M_E \) and \( M \) from (12), (13) and (19), using estimates of \( \xi_X, \xi_E \) and \( \xi_A \) based on (15), (11) and (18) respectively, and with an estimate of \( \bar{y}_e \) obtained by averaging over \( \bar{Y}_T(p_1) \). However a problem arises at this point in that, although the resultant set of mobility estimates are mutually consistent, they must be reconciled with our preferred non-parametric estimates of \( M_N \) and \( M_G \) from (8) and (9) in order to obtain a fully unified set of results. To do so, we first set \( M_N \) and \( M_G \) to their non-parametric values and then adjust \( M_H \) and \( M_E \) so that the sum of the two indices equals \( M_N \).

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9 In the empirical application, estimates of \( \lambda \) lie in the range 0.066-0.198 for the USA and 0.125-0.211 for Germany. Inspection of standardised normal probability plots confirms that the Box-Cox residual distributions are very close to normality with normal plot probability correlation coefficients (Filliben, 1975) in the range 0.994-0.997 for the USA and 0.993-0.998 for Germany. In contrast plots for normal (\( \lambda = 1 \)) and lognormal (\( \lambda = 0 \)) specifications reveal more pronounced departures of the residual distributions from normality with correspondingly lower correlation coefficients. Nevertheless, mobility estimates based on the lognormal model were very close to those reported in the paper, while those for the normal model were broadly similar but with somewhat larger (absolute) estimates of \( M_V, M_E, M_H \) and \( M_A \).

10 The accuracy of these approximations for the computation of the mobility indices was checked by numerical simulation of the conditional income distributions and found to be very high, even when proper allowance is made in the simulations for the truncation of the conditional distributions of \( y_T^{(\lambda)} \) at zero.
less $M_G$, with the necessary correction shared between $M_H$ and $M_E$ in proportion to the initial estimates of their magnitudes. $M_V$ is then recalculated as the sum of $M_G$ and the revised estimate of $M_E$. Finally, a revised estimate of $M$ is obtained as the sum of $M_N$ and $M_A$.

3. Empirical illustration

We apply our measurement framework to a comparative study of income mobility in the USA and Germany. The primary purpose of this study is to illustrate the use of our measures, but the application also makes a substantive contribution to the growing body of empirical literature exploring the contrasting inequality trends in the two countries (see, for example, Burkhauser and Poupart, 1997; Gottschalk and Spolaore, 2002; Massoumi and Trede, 2001; van Kerm, 2004; Jenkins and van Kerm, 2006). As is now well-established, relative inequality rose in both countries over the 1980’s and 1990’s, but by substantially more in the USA than in Germany. Our analysis allows us to assess both the nature of the income movements underlying these trends and their normative significance.

The empirical study is based on the Cross-National Equivalent File 1980-2005 (Frick et al., 2007), which contains equivalently defined variables for the US Panel Study of Income Dynamics (PSID) and the German Socio-Economic Panel (GSOEP). Our data for the USA covers the period 1980-97, where 1980 is the first year of PSID data in the CNEF and 1997 is the last year of data available on a consecutive annual basis. The German data cover the period 1984-2005, with 1984 being the first year of the GSOEP and 2005 being the latest year of GSOEP data in the CNEF, and include only observations from the original (pre-unification) West German samples. Following Jenkins and van Kerm (2006), income mobility is analysed over successive five year time spans so the US data allow eleven decompositions referring to 1981-86, 1982-87, …, 1991-96; and the German data fifteen decompositions for 1984-89 through 1999-2004.

Our measure of income for each individual is a three-year centred moving average of post-government annual real household equivalent income calculated using the modified OECD scale.\footnote{Using annual incomes rather than three-year moving averages yields broadly similar results, but with predictably higher levels of both exchange and horizontal mobility, resulting in...}

\footnote{Further information is available from the Cross-National Equivalent File (CNEF) website: http://www.human.cornell.edu/che/PAM/Research/Centers-Programs/German-Panel/cnef.cfm. The author bears sole responsibility for the further analysis and interpretation of the CNEF data employed in this study.}
mobility estimates due to transitory income shocks and measurement error (Solon, 2002). Incomes are deflated by the consumer price index so as to reflect real purchasing power. Observations with zero or negative incomes were dropped from all samples. Sample-specific outliers were also excluded in each decomposition using the procedure outlined in Jenkins and van Kerm (2006). Sample weights and other relevant aspects of survey design are taken into account in all calculations. In particular, we take into account that the income measure is common across individuals within a household in the estimation of the conditional distributions of final income and, furthermore, that it is correlated between panel interviews in the computation of bootstrap standard errors for all statistics.

Duclos et al. (2003) consider values of the relative inequality aversion parameter $\varepsilon$ between 0 and 1, and of the ‘distributional judgement’ parameter $\nu$ between 1 and 4. Table 1 presents illustrative results for the USA based on intermediate values of $\varepsilon=0.5$ and $\nu=2$, implying moderate levels of inequality aversion and concern for the poor. The cross-sectional statistics indicate that both mean income and absolute inequality rose in real terms in each 5 year sub-period between 1981 and 1996, with the rise in the former exceeding the latter in all sub-periods except for two that span the recession of the early 1990’s. Thus the analysis of structural mobility shows that the growth index $M_G$ is consistently positive and the redistribution index $M_R$ negative, with the overall index of snapshot mobility $M_S$ showing that welfare evaluated on a period-by-period basis rose in all but two of the five year spans. The positive association between the magnitudes of $M_G$ and $M_R$ suggests that the nature of economic growth in the USA was inherently disequalising, with both mean incomes and absolute inequality rising more rapidly during economic upswings.

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13 In the absence of both rank dependence ($\nu=1$) and inequality aversion ($\varepsilon=0$) then risk-neutral mobility $M_S$ will simply equal mean income growth $M_G$. Rank dependence by itself gives rise to redistributive and exchange mobility, with the size of both $M_R$ and $M_E$ increasing in the social weight attached to the fortunes of the initially poor. Holding the poverty focus constant and increasing the level of inequality aversion reduces the size of $M_S$ due to the concavity of the utility function. Conversely, inequality aversion gives rise to redistributive and horizontal inequity mobility, with both $M_R$ and $M_H$ increasing in the degree of inequality aversion as the inefficiency associated with both the unconditional and conditional dispersion of incomes rises. Holding the level of inequality aversion fixed and increasing the poverty focus of the evaluation reduces (increases) the size of $M_H$ if the conditional dispersion of incomes is greater (smaller) among the rich than the poor. Finally, the index of equalising opportunity $M_E$ will be more positive, or less negative, the higher the degree of inequality aversion and, hence, the more rapidly the marginal utility of income diminishes with income.
### Table 1. Real income mobility in the USA over successive 5 year spans, 1981-1996 (US Dollars at constant 1990 prices): $\gamma=0.5$, $\nu=2$.

<table>
<thead>
<tr>
<th>Period</th>
<th>81-86</th>
<th>82-87</th>
<th>83-88</th>
<th>84-89</th>
<th>85-86</th>
<th>86-91</th>
<th>87-92</th>
<th>88-93</th>
<th>89-94</th>
<th>90-95</th>
<th>91-96</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial: \emph{ede} income $\xi_i$</td>
<td>9986</td>
<td>10026</td>
<td>10213</td>
<td>10389</td>
<td>10573</td>
<td>10880</td>
<td>11161</td>
<td>11379</td>
<td>11516</td>
<td>11591</td>
<td>11900</td>
</tr>
</tbody>
</table>
| \begin{tabular}{c|c|c|c|c|c|c|c|c|c|c|c} \hline
\text{average income} $\overline{y}$ & 14514 & 14747 & 15225 & 15648 & 16159 & 16649 & 17304 & 17674 & 18076 & 18281 & 18861 \\
\hline
\text{absolute inequality} $A$ & 4528 & 4721 & 5012 & 5259 & 5586 & 5769 & 6143 & 6295 & 6560 & 6690 & 6961 \\
| | | | | | | | | | | | |
| Final: \emph{ede} income $\xi_f$ | 10,993 & 11,304 & 11,514 & 11,567 & 11,559 & 11,553 & 11,629 & 11,558 & 11,334 & 11,195 & 12,216 |
| \begin{tabular}{c|c|c|c|c|c|c|c|c|c|c|c} \hline
\text{average income} $\overline{y}$ & 16,834 & 17,408 & 17,827 & 18,205 & 18,245 & 18,378 & 18,488 & 18,555 & 18,494 & 18,476 & 19,709 \\
\hline
\text{absolute inequality} $A$ & 5,841 & 6,104 & 6,313 & 6,638 & 6,706 & 6,826 & 6,859 & 6,996 & 7,160 & 7,281 & 7,494 \\
| | | | | | | | | | | | |

\text{Risk neutral mobility} $M_x$ | 1,886 & 2,173 & 2,195 & 2,084 & 1,865 & 1,853 & 1,337 & 1,102 & 769 & 595 & 1,267 |
| | | | | | | | | | | | |

\text{Vertical mobility} $M_y$ | 2,308 & 2,590 & 2,645 & 2,555 & 2,323 & 2,009 & 1,843 & 1,648 & 1,402 & 1,243 & 1,781 |
| | | | | | | | | | | | |

\text{Growth} $M_c$ | 2,320 & 2,660 & 2,602 & 2,557 & 2,087 & 1,729 & 1,184 & 880 & 417 & 195 & 849 |
| | | | | | | | | | | | |

\text{Redistribution} $M_s$ | -1,313 & -1,383 & -1,301 & -1,379 & -1,120 & -1,057 & -716 & -701 & -600 & -591 & -332 |
| | | | | | | | | | | | |

\text{Structural mobility} $M_t$ | 1,007 & 1,278 & 1,301 & 1,178 & 967 & 672 & 469 & 179 & -182 & -396 & 316 |
| | | | | | | | | | | | |

\text{Exchange mobility} $M_f$ | 879 & 895 & 894 & 906 & 899 & 871 & 868 & 922 & 951 & 992 & 951 |
| | | | | | | | | | | | |

\text{Risk adjustment:} $M_a$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| | | | | | | | | | | | |

\text{Note: Bootstrap standard errors in parentheses based on 200 replications.}
However, the structural analysis of mobility does not take the movement of individuals within the income distribution into account and may therefore overstate social perceptions of the disequalising effects of income growth over the period. From an ex-ante risk-neutral perspective, welfare rose in each sub-period as indicated by the consistently positive values of the mobility index $M_N$. By implication, the prospect of the income distribution five years hence was consistently preferable to the current distribution, evaluated on the basis of individuals’ ranking in the current distribution. The level of exchange mobility $M_x$ was roughly constant over the whole period, suggesting that the processes leading to the reranking of individuals were largely unaffected by the strength of the economy (indeed, if anything, exchange mobility appears to be a mildly counter-cyclical phenomenon). This view is reinforced by consideration of the horizontal mobility index $M_H$ from the alternative decomposition of $M_N$. Thus $M_H$, which is determined by the conditional dispersion of income prospects over each five year time span, similarly lies within a relatively narrow range over the whole study period, showing little or no systematic variation over the economic cycle. We conclude that the predictability of future incomes in the USA was largely unaffected by prevailing economic conditions given that both exchange and horizontal mobility may be seen to arise from the stochastic nature of the mobility process.

The results of the alternative decomposition also show that the distribution of expected income opportunities in the final year of each sub-period was consistently equalising from an ex-ante welfare perspective, as indicated by the positive values of $M_E$, with income mobility more “progressive” in absolute terms during economic slowdowns. These findings appear to conflict with the evidence that cross-sectional inequality was increasing throughout the whole period, but may be reconciled once exchange and horizontal mobility are taken into account. Jenkins and van Kerm (2006) have explained in detail how (relative) pro-poor income growth can be accompanied by rising cross-sectional inequality due to the reshuffling of individuals within the income distribution over time. Measuring inequality in relative terms, they report that exchange mobility exceeded the rise in inequality in the USA throughout the 1980’s.\(^{14}\) In contrast, measuring inequality in absolute terms, we

\(^{14}\) In contrast to most of the mobility literature, Jenkins and van Kerm (2006) portray reranking as a negative phenomenon that exacerbates cross-sectional inequality compared to what it would be if the growth process did not induce any reshuffling of individuals in the income distribution. However this formulation of the issue in terms of the distinct redistributive effects of income growth progressivity and reranking is problematic in that progressivity is not invariant to exchange mobility – for example, in our framework, a simple permutation of period $T$ incomes induces offsetting changes in $M_E$ and $M_N$ for the limiting
find exchange mobility $M_E$ to have only exceeded the disequalising effects of redistributive mobility $M_R$ in the latter sub-periods spanning the slowdown of the early 1990’s. However, within our framework, costs of inequality may also arise from the classical horizontal inequities associated with the divergence in individual income changes about conditional mean growth rates. Thus income mobility would be $M_t$ rather than $M_E$, evaluated ex-ante on a risk-neutral basis, if all individuals received conditional mean incomes in the final year of each sub-period, where the former (but not the latter) exceeds $M_G$ in all sub-periods. Figure 1 displays the concentration curves of the utility changes associated with conditional mean income growth, with individuals ranked in ascending order of utility in the initial year, for 1981-86, 1986-91 and 1991-96. All three curves lie above the line of perfect equality indicating that opportunities for utility growth were consistently progressive in absolute terms from an ex-ante perspective. Moreover, the three curves lie ever further above the line of perfect equality indicating that the absolute progressivity of the mobility process was negatively associated with income growth which was weaker in each successive sub-period. Indeed, it is evident from the concentration curve for 1991-96 that the conditional mean income growth of many of those in the top income quintile in 1991 was negative over this five year time span.

Table 1 also reports the risk adjustment index $M_A$ for values of the relative risk aversion parameter $\gamma$ between 0 and 8 to cover the very wide range of CRRA values reported in the literature (see e.g. Kaplow, 2005; Meyer and Meyer, 2006). The results imply that the ex-ante welfare loss due to risk aversion would have exceeded the perceived benefits of exchange mobility, holding the level of snapshot mobility $M_S$ constant, if the CRRA had been case of $\varepsilon=0$ since cross-sectional inequality depends only on the marginal distribution of income. What one could say instead is that inequality will be higher in the future than would be the case if all future incomes were perfectly predictable from initial incomes, noting that $M_E$ is invariant to a change in the conditional dispersion of income prospects even though it is sensitive to the permutation of incomes. Indeed, if $\bar{Y}_T(p_t)$ is increasing then $M_E$ is simply the difference between the absolute Ginis of predicted and actual period $T$ incomes for $\varepsilon=0$.

If $\varepsilon$ is set equal to zero then mobility is deemed to be progressive (i.e. $M_E=M_S+M_R>0$ since $M_H=0$ in this case) in all sub-periods from 1985-90 onwards, providing a direct counterpart of the Jenkins and van Kerm (2006) results but with inequality measured in absolute rather than relative terms.

Note that $M_E$ cannot in general be written as a weighted sum of the utility changes associated with individual income movements. Nevertheless, the Figure does serve to corroborate the positive values obtained for $M_E$ by illustrating the progressivity of individual utility changes in absolute terms.
greater than about two. Overall, mobility would not have been perceived ex-ante as resulting in a loss of welfare unless the CRRA had been appreciably greater than four during the economic recovery of the early 1980’s, but this value drops to only about one in the slowdown of the early 1990’s. These results thus serve as a reminder that social perceptions of income mobility will be sensitive to attitudes to risk. In particular, whether exchange and overall mobility were perceived ex-ante to be desirable social phenomena or not would have depended on public attitudes to uncertainty.

Table 2 presents comparable results for Germany for the period 1985-2004, from which it emerges that the nature of the mobility process was broadly similar to that in the USA. First, income growth was disequalising in absolute terms, with cross-sectional income inequality actually falling (although not significantly) during the post-unification recession of the early 1990’s. However, Figure 2 shows that the trade-off between income growth and inequality was more favourable in Germany than the USA, with the lower trend line for
Table 2. Real income mobility in Germany over successive 5 year spans, 1985-2004 (Euro at constant 2001 prices): $\varepsilon=0.5$, $v=2$.

<table>
<thead>
<tr>
<th>Period</th>
<th>85-90</th>
<th>86-91</th>
<th>87-92</th>
<th>88-93</th>
<th>89-94</th>
<th>90-95</th>
<th>91-96</th>
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<th>97-02</th>
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<th>99-04</th>
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<tbody>
<tr>
<td>Initial:</td>
<td>ede income $\xi_i$</td>
<td>11001</td>
<td>11126</td>
<td>11429</td>
<td>11863</td>
<td>12116</td>
<td>12501</td>
<td>12805</td>
<td>12964</td>
<td>13074</td>
<td>12739</td>
<td>12945</td>
<td>12882</td>
<td>12551</td>
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<tr>
<td></td>
<td>(96)</td>
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<td>(142)</td>
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<td>(141)</td>
<td>(163)</td>
<td>(168)</td>
<td>(211)</td>
<td>(192)</td>
<td>(197)</td>
<td>(219)</td>
<td>(2225)</td>
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<tr>
<td></td>
<td>average income $\overline{y_i}$</td>
<td>14550</td>
<td>14786</td>
<td>15172</td>
<td>15710</td>
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<td>(137)</td>
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<td>(152)</td>
<td>(167)</td>
<td>(168)</td>
<td>(173)</td>
<td>(190)</td>
<td>(272)</td>
<td>(283)</td>
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<td>(268)</td>
<td>(284)</td>
<td>(297)</td>
<td>(266)</td>
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<tr>
<td></td>
<td>absolute inequality $A_i$</td>
<td>3549</td>
<td>3660</td>
<td>3742</td>
<td>3846</td>
<td>4003</td>
<td>4129</td>
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<td>(89)</td>
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<td>(92)</td>
<td>(97)</td>
<td>(98)</td>
<td>(108)</td>
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<td>(193)</td>
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<td>(136)</td>
<td>(147)</td>
<td>(197)</td>
<td>(189)</td>
<td>(167)</td>
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<tr>
<td>Final:</td>
<td>ede income $\xi_i$</td>
<td>12661</td>
<td>12827</td>
<td>12982</td>
<td>12966</td>
<td>12765</td>
<td>12733</td>
<td>12725</td>
<td>12746</td>
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<td>12990</td>
<td>13455</td>
<td>13310</td>
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<td></td>
<td>average income $\overline{y_i}$</td>
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<td>17202</td>
<td>17544</td>
<td>17446</td>
<td>17399</td>
<td>17279</td>
<td>17126</td>
<td>17290</td>
<td>17568</td>
<td>17762</td>
<td>18450</td>
<td>18550</td>
<td>18500</td>
</tr>
<tr>
<td></td>
<td>absolute inequality $A_i$</td>
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<td>4374</td>
<td>4562</td>
<td>4480</td>
<td>4634</td>
<td>4547</td>
<td>4391</td>
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<td></td>
<td>(100)</td>
<td>(96)</td>
<td>(110)</td>
<td>(89)</td>
<td>(99)</td>
<td>(104)</td>
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<td>(165)</td>
<td>(155)</td>
<td>(166)</td>
<td>(176)</td>
<td>(161)</td>
<td>(152)</td>
</tr>
</tbody>
</table>

| Structural mobility $M_s$ | 1659 | 1702 | 1552 | 1103 | 649 | 231 | -70 | -219 | -190 | 251 | 511 | 428 | 822 | 710 |
| Growth $M_G$ | 2275 | 2416 | 2373 | 1737 | 1280 | 649 | 14 | -296 | -241 | 293 | 712 | 680 | 1200 | 1182 |
| Exchange mobility $M_V$ | 969 | 1032 | 996 | 1008 | 989 | 915 | 891 | 833 | 873 | 894 | 1024 | 993 | 1086 |
| Risk neutral mobility $M_N$ | 2628 | 2733 | 2548 | 2110 | 1658 | 1221 | 845 | 672 | 643 | 1124 | 1405 | 1452 | 1815 | 1796 |
| Vertical mobility $M_V$ | 969 | 1032 | 996 | 1008 | 989 | 915 | 891 | 833 | 873 | 894 | 1024 | 993 | 1086 |
| Growth $M_G$ | 2275 | 2416 | 2373 | 1737 | 1280 | 649 | 14 | -296 | -241 | 293 | 712 | 680 | 1200 |
| Vertical equity $M_E$ | 658 | 646 | 510 | 698 | 721 | 895 | 1152 | 1289 | 1169 | 1124 | 987 | 1031 | 959 | 989 |

| Risk adjustment: $M_s$ | $\gamma=0$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $\gamma=0.5$ | -144 | -155 | -158 | -155 | -163 | -157 | -156 | -154 | -138 | -141 | -140 | -172 | -163 | -188 | -173 |
| $\gamma=2$ | -574 | -616 | -631 | -615 | -650 | -623 | -621 | -613 | -552 | -560 | -557 | -684 | -648 | -745 | -688 |
| $\gamma=4$ | -1144 | -1222 | -1259 | -1220 | -1288 | -1237 | -1233 | -1223 | -1101 | -1113 | -1107 | -1359 | -1281 | -1473 | -1362 |
| $\gamma=8$ | -2266 | -2398 | -2492 | -2397 | -2526 | -2431 | -2421 | -2418 | -2180 | -2192 | -2179 | -2672 | -2497 | -2864 | -2661 |

Note: Bootstrap standard errors in parentheses based on 200 replications.
Figure 2. Growth of mean incomes and cross-sectional inequality.

Figure 3. Changes in welfare due to vertical and horizontal mobility.

Figure 4. Changes in welfare due to equalising opportunities and mean income growth.
Germany indicating that a given proportionate change in mean income was associated with a smaller proportionate change in absolute inequality in Germany. That most points lie above the 45° line implies that relative inequality rose in virtually all sub-periods in both countries.

Second, the degree of unpredictability of future incomes appears largely insensitive to the state of the economy as indicated by the relatively stable values of $M_x$ and $M_H$. However, there is some evidence of pro-cyclicality, unlike the USA, perhaps in part as a result of stronger social insurance mechanisms to protect incomes during recessions. Figure 3 shows that the trade-off between vertical and horizontal mobility was more favourable in Germany than the USA, with the lower trend line for Germany indicating that a given proportionate change in $ede$ income due to the conditional mean growth of incomes was associated with a smaller proportionate loss due to the horizontal inefficiencies associated with the conditional dispersion of final year incomes. In the absence of horizontal mobility, $ede$ income in the final year of each sub-period would have been between 2.0% and 2.6% higher in Germany and between 3.3% and 4.1% higher in the USA from an ex-ante perspective. However, lower levels of horizontal mobility in Germany did not translate into comparably lower levels of exchange mobility, which averaged about 8% of initial year $ede$ income in both countries, because of lower levels of absolute inequality and more pro-poor income growth in absolute terms (see below).

Third, the index of equalising opportunity $M_E$ is consistently positive, with mobility more “progressive” in economic downturns. Figure 4 shows that the trade-off between vertical equity and mean income growth was more favourable in Germany than the USA, with the higher trend line for Germany at high levels of mean income growth indicating that a given proportionate change in $ede$ income due to mean income growth was associated with a larger proportionate rise due to vertical redistribution during economic upswings. What this shows is that the income growth process in Germany was generally more pro-poor in absolute terms than in the USA.

Finally we note that the threshold CRRA values at which the risk adjustment index outweigh the levels of exchange and overall mobility are higher in Germany than the USA, reflecting the greater predictability of future incomes.
4. Conclusion

The paper proposes a framework that can be used to both characterise and quantify the welfare effects of income mobility from an ex-ante, risk-neutral perspective. The resultant class of measures can be decomposed not only in terms of structural and exchange mobility but also in terms of vertical and horizontal mobility, thereby encompassing two of the main approaches to the economic evaluation of mobility currently found in the literature. We further show how the framework can be extended to take account of risk aversion by assuming that individuals consider some certainty equivalent when evaluating the set of utility opportunities that they face. All measures are expressed in monetary terms and thus provide a direct indication of the underlying scale of changes in the income distribution.

Our analysis reveals a tension between the perceived value of exchange mobility on the one hand and the horizontal inequities and uncertainty associated with the conditional dispersion of income prospects on the other. Given this tension, whether the stochasticity of the mobility process is seen as being desirable or not is likely to depend on the political culture of a society which in turn will shape the choice of policies that determine the future evolution of the income distribution. Alesina et al. (2004) argue that the greater tolerance of inequality that is evident amongst the poor in America compared to Europe can be explained by their perception that they are living in a society with higher opportunities for (upward) mobility. Conversely, more equal societies may be expected to put more weight on the predictability of incomes given that income differences between social classes are less pronounced. Adsera and Boix (2000) characterise continental European models of social democracy in terms of the conjunction of compressed wage differentials and high levels of social insurance compared to the USA, UK and Japan.

Our illustrative results show that levels of both structural and vertical mobility were directly linked to the strength of the economy in both the USA and Germany, but that levels of exchange and horizontal mobility, which are primarily determined by the predictability of future incomes, were much less sensitive to the economic cycle. The pattern of income mobility in the USA has been both less pro-poor in absolute terms than in Germany and more horizontally inequitable, but the latter did not translate into higher levels of exchange mobility given the vertical stance of the growth process and higher levels of absolute inequality. Further unreported results from a sensitivity analysis indicate that these broad comparative findings hold over the range of values of the relative inequality aversion and ‘distributional judgement’ parameters.
considered in Duclos et al. (2003). Thus, like the majority of studies on the issue (see, for example, Jenkins and van Kerm, 2006), we conclude that there has been less exchange mobility in the USA than Germany, but our measurement framework also provides fresh insight into why, contrary to popular belief, this may have been the case.

It would however be unwise to try to read too much into the empirical results. For a start, the analysis in each sub-period is based on a balanced panel and income differentials may be expected to have changed over the five year time span simply due to the ageing of the sample (Ayala and Sastre, 2002). Moreover, the estimates of the mobility indices may be biased due to non-random sample attrition over each sub-period (Frick et al., 2007). More generally, there is a need for more research into the determinants of individual income dynamics in order to better explain the sources of observed changes in cross-section or snapshot inequality over time.
References


