A Regime Switching Unobserved Component Analysis of the CDX Index Term Premium

Giovanni Calice
School of Management, University of Southampton
Highfield, Southampton, SO17 1BJ, U.K.
Email: G.Calice@soton.ac.uk

Christos Ioannidis
Department of Economics, University of Bath
Claverton Down, Bath, BA2 7AY, U.K.

RongHui Miao
Department of Economics, University of Bath
Claverton Down, Bath, BA2 7AY, U.K.

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1Corresponding author
Abstract

Using an unobserved component Markov switching model we decompose the term premium, the difference in the spread between the 5-year and 10-year maturities, of the North American CDX investment grade index (CDX-IG). We explain the evolution of these components in relating them to monetary policy and stock market variables. We establish that the inversion of the CDX index credit curve is induced by sudden changes in the unobserved random walk component, which represents the evolution of the fundamentals underpinning the probability of default in the economy. The impacts of observed monetary policy and stock market factors on the unobserved components are more prominent during the credit crisis period.

KEY WORDS: Credit Default Swap Index, CDX, Markov Switching, State-Space, Variance Decomposition

JEL Classification: G01, G15, G21, G24
1 Introduction

The sub-prime mortgage crisis, unveiled in July 2007, has caused billions of dollars of losses in the credit markets as systemically important financial institutions had been forced to write off mortgages and related securities linked to credit derivatives instruments, like credit default swaps (CDSs) and collateralized debt obligations (CDOs). Great uncertainties filled almost every corner of the financial markets, which seriously interrupted its normal functioning [Taylor and Williams (2009)].

CDS indices are credit default products referenced to portfolios of single-named credits. Users of the most popular indices (Dow Jones CDX/iTraxx) include those who want to hedge against credit defaults of pooled entities and those who want to speculate.

It has been argued (see, amongst the others, Stulz (2009)) that the 2007/2008 subprime crisis was amplified through structured credit products-tranches trading. Consequently, if on the one hand, these instruments seem to have enriched the scope of investment strategies, on the other hand, their increased complexity depth have unduly induced instability in financial markets.

CDS indices are simply portfolios of single name default swaps, serving both as trading vehicles and as barometers of credit market conditions. These indices are responsible for the increased liquidity and popularity of tranching of credit risk. By buying protection on an index, an investor is protected against defaults in the underlying portfolio and makes quarterly premium payments to the protection seller. If there is a default, the protection seller pays par to the protection buyer.

The term premium of the CDX index, measured as the difference between the CDX 10-year and the CDX 5-year maturities, can be viewed as representing the uncertainty regarding corporate default. The CDX term premium can be interpreted as an early warning market indicator of improvement or deterioration in macroeconomic conditions.

If an investor perceives the difference between the 5-year index spread and 10-year index spread too steep, in other words, that the implied probability of default between 5 and 10 years is higher than that implied from fundamentals, but he/she expects the slope to flatten, then this investor could buy 5-year protection and sell 10-year protection on the CDX index. Finance theory suggests that the spread curves of companies with high credit quality should be upward sloping, whereas those of companies having very poor credit quality do exhibit negative slopes. For example, the credit risk of an AAA rated corporate bond should in general be positively correlated with its maturity, and hence the required yields slope upwards against its maturity. In cross-sectional space, the likelihood of credit quality deterioration should increase as rating lowers, which is to say that the required average yield
should increase with the downgrading of corporate bonds. However, the spread curve for a company on the brink of default (or with foreseeable immediate downgrade) would invert to trend negatively to reflect higher credit risk in the near future. As a result, the yield for such bond is very high for short maturities but relatively lower for longer maturities, which reflects investors’ view that it is still possible for this company to improve its credit quality for longer term maturities.

CDX curve trading has assumed enormous importance over the latest very turbulent period. Clearly, curves tend to flatten in periods of imminent higher default rates, and tend to be steep in periods of economic expansion.

Index curve trading is generally motivated on one or more of the following:

a) As a way of expressing market direction views with different risk-reward profiles.

b) Carry and roll-down reasons.

c) Hedging purposes - both cash and CDS underlying portfolios.

Many opportunities for trading curves on single-name CDS occur around forecasted or announced specific corporate actions. Such events change the perception of a company’s creditworthiness and the shape of the CDS curve also evolves. Curve trades can be more attractive than outright positions around events, thanks to the variation in the available payoff profiles.

In addition, macroeconomic conditions can trigger default events that affect the curves of not only specific entities but also of entire industries. Changes in consumer preferences, the monetary policy stance, and developments in the housing market are critical industry-wide events and market sentiment often transcend worries about profitability and focus instead on viability and the possibility of default of a specific firm. In this case, recovery expectations, following a higher default rate regime, become progressively important in determining the curve shape of the index, as it clearly tends to flatten.

In this paper, we investigate the dynamic behaviour of the CDX index term premium through time by using a Markov Switching Unobserved Component (MS-UC) model. It seems natural to consider an economic time series in terms of permanent and transitory components. The decomposition of a univariate time series into these two components is a primary tool for analyzing business cycles, with these two components often used as measurements of unobserved trend and cycle. In the econometric literature, several approaches have been proposed on how the univariate time series could be decomposed. A well-established

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1For more details, see Quantitative Credit Strategy (2008), “CDS Curve Trading Handbook 2008”
methodology is the unobserved components approach, postulated in separate contributions by Harvey (1985), Watson (1986) and Clark (1987), respectively.

In formulating an unobserved components model for econometric analysis, we depart from others working on the observable determinants of the CDS indices. Alexander and Kaeck (2008) and Byström (2006), for example, relate the CDS/CDX spreads to several observed variables (such as the slope of the yield curve, the stock market return and the volatility of the stock market), and analyse the significance of each observable variable in determining the CDS iTraxx Europe spreads using single-equation regression estimations. Our interest in this paper, however, is to study how the factors themselves (not the factor loadings) drive the dynamics of the term premium. Since the CDX index measures the economy-wide default probabilities (the higher the index value, the higher the probability of default on firms included in the index), the macroeconomic conditions, which can be encompassed by those fundamental factors, will be closely related to the CDX index value and its term premium.

In addition the inclusion of a transitory process that reverts back to its long-term mean quickly after a shock can be regarded as the shortly lived market misbehaviour when the market is subject to a high degree of uncertainty.

To characterize the observed patterns of volatility jumps on the CDX index term premium, we also impose on the innovation terms of such components a regime switching process, following two distinct first-order Markov chain variables.

This paper contributes to the rapidly growing literature on structured credit in its attempt to understand the evolution of the term premium of the CDS index market and its link to observed macroeconomic and financial information.

The current literature on CDS is primarily limited to the pricing with a large strand of it revolving around the key determinants of these contracts. First, there is an extensive literature on the driving forces of CDS premiums ranging from the model of Hull et al. (2004), Aunon-Nerin et al. (2002), which examine the relationship between CDS premiums and credit spreads, to more elaborate analyses by - amongst the others - Zhu (2006), Longstaff et al. (2005) and Blanco et al. (2005) which include also bond and equity markets measures.

Much of the research on credit markets has focused on corporate bond spreads and single-name CDS spreads. Despite a sizeable literature on credit risk empirical studies on CDS that involve the modeling of the entire credit curve are uncommon. A major reason for this is that data on the CDS spreads for a wide range of maturities have only recently become available. Consequently there is a paucity of empirical works regarding CDS indices, with studies focused mainly on the North America CDX index (CDX.NA.IG).

Our work is also closely related to two recent studies by Pan and Singleton (2008) and
Zhang (2008), who attempt to estimate default risk using the entire credit curve of sovereign CDS spreads.

To the best of our knowledge, Byström (2005, 2006) and Alexander and Kaeck (2008) are the early studies on CDS indices. In a correlation study of a sample of European CDS iTraxx indices for different industrial sectors, Byström (2005) finds a tendency for iTraxx spreads to narrow when stock prices rise, and vice versa. Furthermore, he finds that stock market reacts quicker than the iTraxx market to firm-specific information and the stock price volatility is significantly and positively related to the volatility of CDS spreads. Alexander and Kaeck (2008) use a Markov switching model to examine the determinants of the European CDS iTraxx index in two different regimes. They find the CDS market is sensitive to stock returns under ‘ordinary’ market conditions but extremely sensitive to stock volatility during turbulent periods. One recent paper by Bhar et al. (2008), which is mostly related to our paper, decomposes three European CDS iTraxx indices spreads into persistent and transitory components using the Kalman filter. They investigate these dynamics for two different maturities (5 and 10 years) and find that the transitory component is affected largely by stock market volatility whereas the persistent component is more sensitive to illiquidity. However, their sample period does not include the recent sub-prime mortgage crisis. Therefore, the dynamic behavior of these two components during crisis time remains still unexplained.

Blanco et al. (2005) analyze the relationship between investment grade bonds and CDS, and explore the determinants of CDS premia. They find that the theoretical relationship linking credit spreads and CDS premia holds reasonably well for most of the investment grade reference entities. In addition, they report that increases in interest rates and equity prices reduce CDS premia whilst a steeper-sloping yield curve has the opposite effect.

The paper’s main results are as follows. First, the inversion of the term structure curve is induced by sudden changes in the RW factor, which represents the evolution of the fundamentals underpinning the probability of default in the economy. Equally notable is that our findings show that the stationary factor, which represents short-lived uncertainties and turbulence in credit markets, spikes quite dramatically around the occurrence of tail risk events (e.g. Bear Sterns bailout and Lehman Brothers bankruptcy).

Second, the impacts of observed economic and financial variables to track the evolution of the unobserved factors are more prominent during the credit crisis period. Monetary policy shocks (measured as the conditional volatility of changes in the US effective federal fund rate (DFF)) and the changes in the slope of the yield curve both flatten the term structure curve by pushing down the CDS premia for the short-term maturities. Increases in equity market returns significantly reduce default risk over the short horizon, which translates
into a steepened CDX credit curve. Moreover, we find that the increased volatility of the equity market translates into curve flattening, and in the extreme, inversion as short term premia increase rapidly. This could just reflect direct evidence that the market largely expects economy-wide default events. In accordance with economic theory predictions, these variables are found to make a statistically significant contribution. Finally, the analysis of variance reveals that the influence of such variables is persistent over 15 periods, accounting for a small albeit significant fraction of the unobserved factors.

In sum, the empirical evidence strongly suggests that the direct impact of monetary policy is very modest on the RW factor but rather potent on the transitory factor. Developments in both the first and second moments of the equity market have a lasting influence on both factors, with changes in the slope of the yield curve making a significant contribution to the transitory component.

The paper is organized as follows. Section 2 presents the empirical methodology that decomposes the term-premium of the CDX index into the unobserved factors allowing for regime switching. Section 3 presents and discusses the data used in the estimation. The results are reported in Section 4 and Section 5 concludes.

2 Empirical Methodology

The econometric methodology employed in this paper is based on the statistical approach developed initially by Nervole et al. (1979) and developed further by Harvey (1989) and Harvey and Shephard (1993). The essential element of this methodology is to estimate a model which considers the observed time series as being the sum of permanent and stationary components. These components capture the salient features of the series that may be unobserved and are useful in explaining and predicting its time evolution. In such models the parameters are time varying and are the explanatory variables are functions of time. The state of the system represents the various unobserved components. The Kalman filter is employed as the most efficient means of updating the state as new information becomes available, in linear models.

Additional features in our model are the interrelation between the stochastic elements of each component and the endogenous shift of their volatility between regimes. This feature enables us to capture the occasional and recurrent endogenous regime switches of volatilities in time series. To understand why we assume regime shifts in the two components’ disturbance terms consider the evidence on the standard deviations for different time periods of the term premium presented in Table 1. For the period 2004/2007 there is a modest change

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2Details of the data used in this paper are described in section 3.
in the standard deviation of the term premium prior to November 2007 as new observations are added, and the mean of the term premium remains around 23 over the three year periods since August 2004. However once interest rates began to rise and housing prices started to drop in 2006-2007 in many parts of the US, the refinancing of mortgages (especially the sub-prime mortgages) became extremely difficult. Defaults and foreclosure on those mortgages increased dramatically, which brought the sub-prime mortgage industry to the edge of collapse, and hence generated considerable uncertainty in financial markets.

The standard deviation of the CDX term premium for the November 2007-July 2009 sub-sample jumps to 18.804, which is more than 6 times higher compared to the August 2004-November 2007’s estimate. Concurrently, the mean of the term premium fell to -16.913, suggesting that the later sub-sample might be experiencing a different regime in terms of both mean and volatility. By allowing for regime switches in volatility (and in mean) to take place endogenously, we do not explicitly set a switching threshold value but we allow for the data to decide endogenously when to switch to a different regime.

TABLE 1 ENTERS HERE

This additional dimension in the unobserved components space allows us to generate probabilities that each component of the term premium experiences either high or low volatility regimes through time. Although it complicates the estimation procedures - since additional filters must be employed to make inference on the hidden Markov chain process - allowing the two components to depend on different states of the economy provide us with an alternative approach to deal with the potential heteroskedastic variance in the daily CDX index series. The more conventional way of testing for financial time series heteroskedasticity is to consider ARCH-type volatility models, which allow constant unconditional volatility but time-varying conditional volatility. However, neglecting possible regime shifts in the unconditional variance, as shown in Lamoureux and Lastrapes (1990), would overestimate the persistence of the variance of a time series.

Although the latent variable model, like that constructed in system, is an effective tool in decomposing macro/financial variables into a number of unobservable factors, the usefulness of the model is still limited if we are unable to link the factors to a set of observable economic
variables. To overcome this problem, one may model the unobserved factors and observed variables together in a macro-finance setting (as suggested, for example, by Ang and Piazzesi (2003)). Our analytical approach here is instead as follows: we begin by filtering out the unobserved factors and then in a second step, we empirically estimate the relationship between the unobserved factors and a set of variables observed at the same frequency.

In this way, we test for the dependence of the factors on the relevant monetary policy (the changes in the DFF as well as the slope of the yield curve derived from the risk-free U.S. government treasury bonds) and stock market variables (which include the daily returns from the Standard & Poor’s 500 index and the GARCH (1,1) conditional volatility of the index returns).

The remaining subsections present our stylized model of analysis. We first show how to construct the two components that drive the evolution of the CDS term premium. We outline the state space representation of the system and our extension of modeling Markov switching disturbance terms.

2.1 Transitory and random walk components in state space representation

Let $X_{1,t}$ represent the transitory component that drives the term premium, and assume that $X_{1,t}$ is an Ornstein-Uhlenbeck process, whose dynamic evolution can be described by the stochastic differential equation

$$dX_{1,t} = k(\delta - X_{1,t})dt + \sigma_1 dZ_{1,t}$$

(1)

where $\delta$ is the target equilibrium or mean value supported by fundamentals; $\sigma_1 > 0$ is the scale of volatility that the exogenous shocks can transmit to the dynamics of $X_{1,t}$; $dZ_{1,t}$ is the standard Brownian motion with zero mean and unity variance that generate random exogenous shocks; $k > 0$ is the rate by which these shocks dissipate and the variable, $X_{1,t}$, reverts back to its mean. The Ornstein-Uhlenbeck process is an example of a Gaussian process that admits a stationary probability distribution and has a bounded variance. In contrast to the Brownian motion process that has constant drift term, the former allows for a drift term that is dependent on the current value of the process. If the current value of the process is lower than its long-term mean value, the drift term will be positive in order to bring the process back to its long-term mean value. If, on the other hand, the current value of the process is greater than its long-term mean value, the drift term will be negative in order to drag down the process back to its long-term mean value. In other words, this is a mean-reverting process. Setting $f(X_{1,t}, t) = X_{1,t}e^{kt}$ and applying the Ito’s Lemma to this
function, this leads to
\[ df(X_{1,t}, t) = k\delta e^{kt}dt + \sigma_1 e^{kt}dZ_{1,t} \]  
(2)
iintegrating both sides of Equation 2, we obtain
\[ X_{1,t} = X_{1,0}e^{-kt} + \delta (1 - e^{-kt}) + \sigma_1 \int_0^t e^{-k(t-s)}dZ_{1,s}, 0 \leq s \leq t \]  
(3)
where \( X_{1,0} \) is the initial value of the process and the first and the second moments are given by
\[
E(X_{1,t}) = X_{1,0}e^{-kt} + \delta (1 - e^{-kt}) \\
Var(X_{1,t}) = \frac{\sigma_1^2}{2k} (1 - e^{-2kt}) 
\]
(4)
The Ornstein-Uhlenbeck process is one of several widely used approaches to model stochastically interest rates, exchanges rates and stock prices. The advantages of its simple and tractable solutions, under continuous-time framework, have been embodied in many empirical asset pricing models. The econometric modeling, however, emphasizes the discrete-time representation of stochastic processes, Consequently, Equation 1 reduces to a discrete autoregressive order-1 (AR(1)) process, which evolves according to
\[ X_{1,t} = k\delta + (1 - k\Delta t)X_{1,t-1} + \sigma_1\Delta Z_{1,t} \]  
(5)
where \( \Delta t \) is the discrete increment in time that, for convenience, we deliberately set equal to one\(^3\). The standard condition for the above AR(1) process to be stationary constrains \(|1 - k\Delta t| < 1\), or equivalently, \(0 < k\Delta t < 1\).

Now, let \( X_{2,t} \) be the second component that drives the term premium. We assume that it follows a random walk process with deterministic drift as shown in Equation 6
\[ dX_{2,t} = adt + \sigma_2 dZ_{2,t} \]  
(6)
where \( a \) is the drift coefficient, \( \sigma_2 \) is the scaled volatility parameter and \( dZ_{2,t} \) is the standard Brownian motion that can be assumed to be either dependent or independent of \( dZ_{1,t} \). The discrete time version of Equation 6 yields:
\[ X_{2,t} = a\Delta t + X_{2,t-1} + \sigma_2\Delta Z_{2,t} \]  
(7)
where \( a\Delta t \) is a constant representing the drift. The RW process has long been a popular choice for modeling the price dynamics of financial assets. In continuous time financial

\(^3\)Since we are using daily data, setting the discrete increment in time to one unit imply that the estimated volatility parameters will represent the daily volatility rather than the conventional annual volatility.
models, the price of stocks and stock indexes are modeled as geometric Brownian motions. It is relatively straightforward to show that the geometric Brownian motion of the price dynamic is equivalent to a RW path followed by the logarithm of the price in discrete time. The efficient market hypothesis in fact states that the financial asset’s price follows a RW process, which literally assumes that the asset’s price at time is determined by the price at the previous time period and the instantaneous price impact of the new flow of information. Although a RW process, like the one described in Equation 7, has infinite unconditional mean and variance, the conditional mean and variance can be measured as:

\[
E_t (X_{2,t}) = a \Delta t + X_{2,t-1}
\]

\[
Var_t (X_{2,t}) = \sigma^2
\]

(8)

where the conditional expectation of the process at current time \( t \) depends not only on the observation at previous time period but also on the constant drift \( a \).

Given the two unobserved components, constructed using Equation 1 to Equation 7, we estimate the parameter space, as given by the system in Equation 9, with the dynamics of the two components in a Bayesian updating manner, namely the Kalman filter algorithm based on a State Space system. State space representation is usually applied on dynamic time series models that involve unobserved variables (see, e.g., Engle and Watson (1981), Hamilton (1994), Kim and Nelson (1989)). In our modeling, the fact that the two driving forces of the CDX index term premium - transitory and random walk components - are assumed to be unobserved state variables leads to the justification of using the state space representation. A typical state space model consists of two equations. One is a state equation that describes the dynamics of unobserved variables, which is shown below in Equation 9; and the other one is the measurement equation that describes the relation between measured variables and the unobserved state variables, as shown in Equation 10.

\[
\begin{bmatrix}
X_{1,t} \\
X_{2,t}
\end{bmatrix}
= \begin{bmatrix}
k \delta \\
0
\end{bmatrix}
+ \begin{bmatrix}
1 - k \Delta t & 0 \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
X_{1,t-1} \\
X_{2,t-1}
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_{1,t} \\
\varepsilon_{2,t}
\end{bmatrix},
\end{equation}

\[
\begin{bmatrix}
\varepsilon_{1,t} \\
\varepsilon_{2,t}
\end{bmatrix}
\sim N\left(\begin{bmatrix}
0 \\
0
\end{bmatrix}, \begin{bmatrix}
\sigma^2_1 & \sigma_1 \sigma_2 \rho_{12} \\
\sigma_2 \sigma_1 \rho_{21} & \sigma^2_2
\end{bmatrix} \Delta t\right)
\]

(9)

\[
Y_t = X_{1,t} + X_{2,t}
\]

(10)

In Equation 9, the covariance terms \( \sigma_1 \sigma_2 \rho_{12} \) and \( \sigma_2 \sigma_1 \rho_{21} \) will be zero under the assumption of independence between the two disturbance terms (the correlation between the two disturbance terms \( \rho_{12} \) - is zero).
In compact form, Equation 9 can be rewritten as

\[ X_t = C + F X_{t-1} + \Sigma_t, \]
\[ \Sigma_t \sim N(0, Q) \]  \hspace{1cm} \text{(11)}

where \( X_t = \begin{bmatrix} X_{1,t} \\ X_{2,t} \end{bmatrix}, \)
\( C = \begin{bmatrix} k \delta \\ a \Delta t \end{bmatrix}, \)
\( F = \begin{bmatrix} 1 - k \Delta t & 0 \\ 0 & 1 \end{bmatrix} \), \( \Sigma_t = \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \)
and
\( Q = \begin{bmatrix} \sigma_1^2 & \sigma_1 \sigma_2 \rho_{12} \\ \sigma_2 \sigma_1 \rho_{21} & \sigma_2^2 \end{bmatrix} \Delta t. \)

The measurement equation, as described by Equation 10, links linearly the term premium of the CDX index to the transitory and RW components. Rewriting this expression in a compact form, Equation 10 reduces further to give

\[ Y_t = H X_t \]  \hspace{1cm} \text{(12)}

where \( Y_t \) is the term premium series and \( H = \begin{bmatrix} 1 & 1 \end{bmatrix} \) represents the weights of two components in the term premium.

### 2.2 State Space model with Markov switching disturbances

An additional feature of our model is to allow each component’s disturbance term to depend on different states of the economy. In practice, we let the volatilities of the disturbance terms to switch between high and low volatility regimes. Formally, we assume that \( \sigma_1^2 \) and \( \sigma_2^2 \) in Equation 9 are driven by two discrete-valued, independent unobserved first-order Markov chain processes \( S_{1,t} = \{0, 1\} \) and \( S_{2,t} = \{0, 1\} \) given by

\[ \sigma_1^2 = (1 - S_{1,t}) \sigma_{1H}^2 + S_{1,t} \sigma_{1L}^2, \]
\[ \sigma_{1H}^2 > \sigma_{1L}^2 \]
\[ \sigma_2^2 = (1 - S_{2,t}) \sigma_{2H}^2 + S_{2,t} \sigma_{2L}^2, \]
\[ \sigma_{2H}^2 > \sigma_{2L}^2. \]  \hspace{1cm} \text{(13)}

When both \( S_{1,t} \) and \( S_{2,t} \) are zeros, the two components will be in the high volatility state as \( \sigma_1^2 = \sigma_{1H}^2 \) and \( \sigma_2^2 = \sigma_{2H}^2 \); similarly if both \( S_{1,t} \) and \( S_{2,t} \) equal 1, the two components will be in the low volatility state since \( \sigma_1^2 = \sigma_{1L}^2 \) and \( \sigma_2^2 = \sigma_{2L}^2 \). The two remaining scenarios then categorize situations where the first component is in the high volatility state while the second is in the low \( (S_{1,t} = 0, S_{2,t} = 1) \) and where the first component is in the low volatility state while the second is in the high \( (S_{1,t} = 1, S_{2,t} = 0) \). This is Markovian chain process which means that the current value of the process at time \( t \) depends only on its previous value at time \( t - 1 \). The likelihood for the process to remain at the previous value or change to the alternative depends on the transition probabilities from one state to the other, which
are shown below as
\begin{align*}
p_{1,00} &= \Pr [S_{1,t} = 0|S_{1,t-1} = 0] \\
p_{1,11} &= \Pr [S_{1,t} = 1|S_{1,t-1} = 1] \\
p_{2,00} &= \Pr [S_{2,t} = 0|S_{2,t-1} = 0] \\
p_{2,11} &= \Pr [S_{2,t} = 1|S_{2,t-1} = 1]
\end{align*}

(14)
equivalently, the two transition probability matrices for each disturbance term can be written as:
\begin{align*}
p_1 &= \begin{bmatrix} p_{1,00} & p_{1,01} \\ p_{1,10} & p_{1,11} \end{bmatrix}, p_2 &= \begin{bmatrix} p_{2,00} & p_{2,01} \\ p_{2,10} & p_{2,11} \end{bmatrix}
\end{align*}

(15)
where \( p_{q,j} = \Pr [S_{q,t} = j|S_{q,t-1} = i] \) with \( \sum_{j=1}^{2} p_{ij} = 1, \forall i \) and \( q = \{1, 2\} \).

The estimation of the transition probabilities as shown above, requires the choice of the appropriate functional forms of the probability functions that govern the Markov chain variables. Since the transition probabilities have to be bounded within \([0, 1]\) the usual choice is the adoption of the logistic transformation on the probability terms as
\begin{align*}
p_{1,00} &= \Pr [S_{1,t} = 0|S_{1,t-1} = 0] = \frac{\exp (d_{1,0})}{1 + \exp (d_{1,0})} \\
p_{1,01} &= 1 - p_{1,00} \\
p_{1,11} &= \Pr [S_{1,t} = 1|S_{1,t-1} = 1] = \frac{\exp (d_{1,1})}{1 + \exp (d_{1,1})} \\
p_{1,10} &= 1 - p_{1,11} \\
p_{2,00} &= \Pr [S_{2,t} = 0|S_{2,t-1} = 0] = \frac{\exp (d_{2,0})}{1 + \exp (d_{2,0})} \\
p_{2,01} &= 1 - p_{2,00} \\
p_{2,11} &= \Pr [S_{2,t} = 1|S_{2,t-1} = 1] = \frac{\exp (d_{2,1})}{1 + \exp (d_{2,1})} \\
p_{2,10} &= 1 - p_{2,11}
\end{align*}

(16)
where \( d_{1,0}, d_{1,1}, d_{2,0} \) and \( d_{2,1} \) are the unconstrained parameter. Appendix contains a detailed account of the estimation procedure used in this model.

3 Data

In this section, we describe the relevant CDs and the data series used in the study.

CDS transactions are by nature over-the-counter. As a result, the availability and quality of the data are not as dependable as those of exchange-based transactions. CDS data are mainly collected by large investment banks which only record the transaction that they are
involved in. Although professional data vendors are the primary source of CDS research data, the data suffer the following pitfalls: (1) as in Zhu (2006), data prior to 1999 are very limited; (2) the frequency of CDS transaction data is low, therefore, the usual instantaneous lead-lag analysis which use much higher frequency data may not be robust; (3) CDS prices are usually obtained as “quoted” prices which may not reflect the actual information contained in trading prices; (4) CDS data are truncated in the way that the majority contracts have a maturity of 5 years with nominal amount of $5 million or $10 million; (5) CDS data are usually unevenly spaced with many spurious observations in time series.

To circumvent the above mentioned limitations, we restrict our sample of analysis to the CDS tranche index market, specifically to the North American CDX investment-grade indices. In these indices, all 125 single-name credits have equal weights in the portfolio. The Dow Jones CDX IG five-year index is a basket of CDSs on 125 names for the U.S. investment-grade market. Each reference entity has a weight of 0.8%. We use a representative dataset of daily CDS prices for the North American CDX tranche index and focus our analysis on the most liquid segments of the CDX index market, which are the 5-year and 10-year maturities. The analysis is based on daily data spanning from 2004 to 2009. The main source of CDX data is Markit.

In our sample period (August, 5, 2004 to July, 23, 2009), as shown in Table 2 and Figure 1, the average premium is 87.3161 basis points for the CDX 5-year index and 95.9463 for the CDX 10-year index. The CDX 5-year index reaches its maximum spread (283 basis points) on September, 16 2008, which is the day after the announcement of Lehman Brothers default. Similarly, the CDX 10-year index reaches its maximum premium (251 basis points) on the same day, as it is reflected in the negative slope of the credit curve. Not only the volatility of the CDS indices increased dramatically as of July 2007, but also the levels of these indices started to rise significantly at the onset of the crisis. This unprecedented financial turmoil directly raised investors’ expectations on imminent future defaults of firms’ debts, especially of those financial firms heavily exposed to sub-prime-mortgage lending. From the second panel of Figure 1, we can see that the term premium of the CDX 10-year - 5-year index becomes negative at the start of 2008, suggesting an increase of investors’ concerns over short-term default risk. Nevertheless, market sentiment remains unchanged over long-term horizons.

TABLE 2 ENTERS HERE

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4An in-depth discussion of CDS data quality can be found in: Blanco et al. (2005).
Daily DFF rates are obtained from the St. Louis FRED database. Daily changes in the slope of the yield curve, calculated using 10-year and 1-year U.S. Treasury bond yields, are from Thomson Reuters®Datastream. S&P 500 index daily observations are obtained from Thomson Reuters®Datastream. VIX index data are from the Chicago Options Mercantile Exchange (CBOE).

The descriptive statistics of monetary policy and equity market condition variables are also presented in Table 2 and Figure 2 depicts their evolution over the sample period. To gauge the monetary policy uncertainty, we fit the changes in DFF rates to a GARCH(1,1) model, and then collect the conditional variance as a measure of the prevailing uncertainty associated to changes in the stance of monetary policy.

4 Empirical Results

Using the methodology described in section 2, we estimate three nested MS-UC models. Models 1, 2 and 3 are estimated with time-invariant transition probabilities. Parameter estimates and test statistics are reported in Table 3.

Model 1 is the most flexible model, in which the RW factor has a switching drift. In model 2, we restrict the RW factor to a single regime drift. The likelihood ratio test on Model 2 restriction to Model 1 suggests that the RW drift term switches sign from a positive into a negative although the standard error of the positive drift is too large to be considered
statistically significant. In model 3, we constrain the correlation parameter between the two components’ disturbance terms to zero. This restriction, according to the likelihood ratio test statistic, is rejected. On the basis of these tests we conclude that Model 1 constitutes our preferred specification for describing the time evolution of the term premium in this market.

Table 3 highlights the commonalities among the three estimated models. The RW component’s drift is close to zero, and the unconditional mean of the transitory component \((\delta)\) is negative for the whole sample period. The mean reverting speed \((k)\) is greater than 1, indicating that the transitory process quickly reverts to its long term target once it is below or above it.

TABLE 3 ENTERS HERE

The distinguishing feature of our model is that it allows us to decompose the term premium into two correlated driving components. The filtered RW and the transitory components with the associated probabilities of switching regimes are displayed in Figure 3. In the first part of the sample period, temporary volatile movements in the RW and transitory components are induced by severe shocks such as the GM and Ford downgrade events of May 2005. In the subsequent period, the credit market enjoyed a rapid growth in terms of both trading volumes and products innovation. During this time period, both the RW and transitory components stay in the low volatility regime, as reflected by a flat CDX credit curve. The frequent regime changes of the RW component start taking place in the aftermath of Countrywide’s bankruptcy. In fact, on August, 15 2007, Countrywide Financial, the largest mortgage lender in the United States, announced that the foreclosure and mortgage delinquencies had risen to their highest level since early 2002. Since then, because of that episode and of the events around the onset of the subprime mortgage crisis, the CDX index term premium exhibits a downward trend. Ironically, that event occurred just one month after the DJIA index hit its historical record level of 14,000 (on July, 19 2007).

On September, 15 2008, Lehman Brothers’ demise brought about a dramatic increase in stock market volatility and liquidity shortages in funding markets. As illustrated in Figure 3, the abrupt explosion of the transitory component into the high volatility regime on that
day captures investor confidence-induced downward spirals. Although the probability of the transitory component to stay in the high volatility regime accurately signal greater market volatility since Bear Stearn’s bailout, it is quite evident from inspection of this data that the probability for the RW component to switch into the high volatility regime is close to zero at the occurrence of extreme credit events, such as the Bear Stearn’s and Lehman Brothers’ default announcements. At first sight, this counter-intuitive result may be difficult to understand. It conveys the message that credit market uncertainties, as measured by the conditional variance of the term premium, decline significantly with the abrupt unveiling of tail risk events.

Figure 4 plots the conditional variance derived from model 1. Using the definition of Kalman filter provided in the previous section, the conditional forecast error variance is given by

\[ f_{t\mid t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} = HP_{t\mid t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} H' \]

which can be regarded as the implied conditional variance of the CDX term premium. Since this conditional variance depends on the two Markov chain processes, combining the filtered probabilities of states

\[ \sum_{S_{1,t-1}=0}^{1} \sum_{S_{1,t}=0}^{1} \sum_{S_{2,t-1}=0}^{1} \sum_{S_{2,t}=0}^{1} \Pr (S_{1,t-1} = i, S_{1,t} = j, S_{2,t-1} = i, S_{2,t} = j \mid I_{t-1}) \]

with Equation 17, we can calculate the conditional variance as a product of these two equations, based on the available information at time \( t - 1 \). From Figure 4 we observe that the conditional variance, in the immediate aftermath of Bear Stern’s bailout (14th March 2008), remains at low levels for a few days. What is particularly striking is that the probability for the RW component to switch into the high volatility regime falls back to a near-zero value. In the sub-sample period surrounding Lehman Brothers’ default, the conditional variance of the CDX term premium is not as elevated as in the Bear Stern’s bailout period, as a consequence, the probability of the RW component to switch into the high volatility regime initially falls back to near-zero value, but rebounds very rapidly in the subsequent days as investors begin to worry about the stability of other systemically important financial institutions. Our results show that the persistent co-movement between the high conditional variance of the CDX term premium and the probability of the RW component switching into the high volatility regime disappears during the financial crisis period. This suggests that the RW and the transitory components may behave differently depending on whether the financial system is experiencing a systemic crisis or not. In other words, the weight of each component on the term premium seems to change drastically over time.
To gauge how the composition weights of each component in the CDX term premium varies over time, we implement the following testing procedure. Initially, we project the transitory component on a constant and the changes of the RW component. The unexplained residuals from this regression (called transitory regression residuals) are then entered on the right-hand-side of subsequent Equation 18, in which we explain the changes in the term premium of the index. The additional regressor, in Equation 18, denotes the changes in the RW, which is orthogonal to the transitory regression residuals. Coefficients and are recursively estimated and plotted in Figure 5.

\[ dTP = C(1)dRW + C(2)\text{Tran residual} \tag{18} \]

From Figure 5, we detect the presence of a structural break occurring in March 2008, which corresponds distinctively to the period around the Bear Stearns’ rescue. During this period, the estimated coefficients decline significantly: C(1) from 1.16 to 1.08 and C(2) from 1.2 to 0.7. The decline of C(2), which measures the marginal effect of one unit adjustment in
the transitory component on the changes of the CDX term premium, is considerably greater compared with that of C(1). To understand the relative impacts of the two right-hand-side variables in Equation 18, whilst taking the structural break into account, we proceed to divide the sample into four sub-periods, as shown in Table 4, and estimate the same equations for each sub-sample using the procedures described above. The R-square statistics are reported below.

TABLE 4 ENTERS HERE

For the whole sample period (as shown in column 1, Table 4), the changes in the RW component roughly explains 14.6% of the variation in the transitory component. This proportion increases quite dramatically to almost 75% during the calm period (third column) but then drops to 16% at the time of turbulent market conditions (fifth column). Remarkably, the transitory regression residual together with the changes in the RW component explain almost 90% of the total variation in the changes of the index term premium, and it falls to 85% in the volatile period. The R-square statistics from this partitioning of the sample re-enforce the evidence from Figure 5 where the impact of the factors are measured by and diminishes.

As we previously suggested our further aim is to test for the economically meaningful relationship between the unobserved factors and a set of observed information that is available to both market participants and policy makers. Such link, if established, will add predictive ability to the model as the evolution of the factors will be conditional on data and will enhance the model’s analytical appeal.

At a conceptual level, the US DFF is the standard monetary policy tool available to the Fed to influence the short segment of the yield curve and hence, in turn, affects investors’ expectations on the movements of long-term interest rates. An increase in DFF signals the Fed’s reaction against the risk of rising inflation in the near future and will aggravate the external financing position of companies that rely heavily on short-term financing.

Companies’ borrowing depends largely on the market value of their net worth (financial and tangible assets). Asymmetric information between borrowers and lenders, would prompt lenders to set forth the abilities of borrowers to repay the debt, which will take the form of collateralizing their financial assets. Falling asset prices erode the value of collateral,
tightening credit and depressing demand. Through the so-called “credit channel”, the level of economic activity and the aggregate output will eventually shrink. If an adverse shock to the macro-economy is amplified by credit rationing, conditions in the real economy and in financial markets mutually reinforce each other, giving rise to a feedback loop which may lead to a deep recession. This self-reinforcing process, known as the “financial accelerator” (a term coined by Bernanke and Gertler (1989); Bernanke et al. (1996); Bernanke (1983, 1981)), operates in reverse during a downturn.

Variables like the GARCH volatility of changes in the US DFF rate and changes in the slope of the yield curve can be thought of as two measurement proxies of the uncertainty over the rates of interest. The former is a historical measure of the degree of uncertainty in the movements of the short-term interest rate, largely attributable to the Fed’s systematic response to aggregate current and future macroeconomic conditions. The latter is simply a forward expectation of how the short rate is expected to fluctuate over long-term horizons, largely driven by the market-wide expectations about the future path of monetary policy. The return series of the S&P 500 index can be regarded as an indicator of the changes in the collateral value of equity whilst the natural logarithm of the VIX index is a measure of forward uncertainty in the value of collateral.

To test whether the RW component and the transitory component are determined by monetary policy variables or/and other indicators of financial market conditions, we construct the following VAR (4) model

\[ Y_t = c + \Pi_1 Y_{t-1} + \Pi_2 Y_{t-2} + \Pi_3 Y_{t-3} + \Pi_4 Y_{t-4} + \varepsilon_t \]

(19)

where \( Y_t \) includes changes in the transitory (\( tran \)) and in the RW components (\( difrw \)), the observed GARCH volatility of changes in the effective US effective DFF rate, the changes in the slope of the yield curve (calculated as the difference between 10-year and 1-year US treasury bond yields), the Standard & Poor’s 500 index return, and the implied volatility of the Standard & Poor’s 500 index (the volatility index of the S&P’s 500 index) \( Y_t = [ffr.ch.GARCH_t, Slope101_change_t, sp500 rtn_t, logvix_t, difrw_t, tran_t]’ \), \( c \) is a (6 \( \times \) 1) vector of constants, \( \Pi_i \) are (6 \( \times \) 6) coefficient matrices and \( \varepsilon_t \) is an (6 \( \times \) 1) unobservable zero mean white noise vector process with invariant covariance matrix \( \Sigma \).

The generalised impulse response functions are presented below in Figure 6. In addition, Table 5 summarizes the generalised variance decomposition for the changes in the transitory and in the RW components.

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5The order of the VAR is established using the SBC criterion.
The impulse response functions in Figure 6 reveal that the instantaneous responses of changes in the RW component to increased monetary policy uncertainty although is positive is statistically insignificant. On the contrary, the response of the transitory component to such a shock is negative and persistent. This, when related to the term premium of the CDX index, suggests that monetary policy uncertainty would exert a modest upward impact on the prices for the shorter maturities CDS (5-year in our case) relative to those prevailing for longer maturities thus decreasing the term premium. For shocks from forward uncertainty (changes in the slope of the yield curve), the response of the term premium is negative. The change in the slope of the yield curve is taken as expectation of future increases in the short-term interest rate and such anticipation affects the 5-year CDS premia, leading to a current lower term premium.

As forecasting variables of the variation in the term premium of the CDX index, these two monetary policy variables appear to impact differently on the RW and the transitory components. As an illustration, Table 5 shows that the monetary policy variables explain about 1% of the variation in changes of the RW component after 15 lags. For the transitory component, however, this proportion increases to almost 8%.

Additionally, Figure 6 shows that the term premium of the CDX index depends positively on the stock market returns and negatively on the VIX volatility index. These results have a natural interpretation. On the one hand, a rise in stock market returns boosts the equity value of a company, making short-term maturities CDS less expensive compared to longer-term maturities. This, in turn, will contribute to a positive term premium. On the other hand, greater stock market volatility makes it difficult to assess the value of collateralization, which in turn leads to higher prices for the short-term maturities CDS. In terms of variance decomposition, the key finding is the presence of strong effects attributable to stock market
variables on both components. Noticeably, this influence is more pronounced on the evolution of the RW.

Focusing our attention on the first and second half of our sample, we can get further insights on how the two components react to shocks in both monetary policy and stock market variables, respectively.

**FIGURE 7 ENTERS HERE**

**TABLE 6 ENTERS HERE**

In the period preceding the crisis (see Table 6 and Figure 7), we find that the impact of policy shocks is more distinct on the RW factor accounting for 2% of its variation. Additionally, all the other variables in the information set do not appear to have a distinct influence on the variation of both factors.

However, over the post-crisis period (see Table 7 and Figure 8), the transitory component becomes sensitive to monetary and stock market shocks, with almost 20% of its variance attributed to these variables. At the same time RW is significantly exposed to shocks from current and lagged stock market returns whilst the influence of policy shocks remains almost unchanged.

**FIGURE 8 ENTERS HERE**
4.1 A robustness check

In the econometric literature on long memory and regime switching studies (see, e.g., Hamilton (1989)), the transition probability of switching from one regime to another is constant. This property, in turn, implies that the expected duration of each regime is constant. As suggested by Diebold et al. (1994), the transition probability may be dependent on a set of economic fundamental variables. Extending the state space Markov switching model to allow for time varying transition probabilities is straightforward in our case, and more importantly, it provides a robustness check on the specification of a constant transition probability model (i.e. model 1). To this end, we let the transition probabilities to depend the same set of observed variables used in the VAR above. This time-varying transition probability model is denoted as model 4.

Table 8 reports the estimation results and Figure 9 and Figure 10 display the filtered components and the conditional variance of the term premium with time-varying transition probabilities.

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6See appendix for the formulation of time-varying transition probability.
Although our estimated parameters in model 4 provide a similar set of relationships to those reported in model 1 and the log-likelihood value of model 4 appears improved, the marginal benefit of employing a time-varying transition probability model would be diminished when parsimonious modeling is considered. Specifically, when we compare the filtered transition probabilities from model 1 with those from model 4, there are only a few differences between the two models. As an illustrative comparison between all the models, we report in Table 9 the predictive power results of each model relating to the number of days needed for each component to switch into the high volatility regime.\footnote{We define a component has a high probability to switch to a high volatility regime if the transition probability in that day has a value greater than 0.5.}

The time-varying transition probability model (model 4) tends to provide a higher estimate of the probabilities to switch into the high volatility regime. Out of total 1169 observations on the transition probabilities, model 4 reports more days (14 and 20 days, respectively) that the two components switch into the high volatility regime. Such differences are statistically insignificant.

A constant probability model, as mentioned earlier, implies that the expected duration of one regime is constant over time. However, if a structural change occurred during the sample period, a constant probability model would be unable to simultaneously reflect the changes in the expected duration of a regime and the regime itself. In our model, for instance, a structural change occurs as the financial crisis unfolds. This may certainly undermine the ability of the constant probability model of forecasting a high volatility regime.

To show how this potential structural change would affect a constant probability model, we regress model 4’s transition probabilities on those of model 1’s and plot the recursive
estimates of the coefficient in Figure 11. If the parsimonious Model 1 constant probability is equivalent to Model 4’s time-varying transition probability, the coefficients would gradually approach unity indicating a proportional relationship between the two models. It is clearly evident that, despite a radical change in March 2008 (at the time of Bear Stearns’ default), both coefficients gradually approach unity. This finding therefore lends support to a constant probability model in terms of both parsimony and stability.

FIGURE 11 ENTERS HERE

5 Conclusion

In this article, we estimate an unobserved components Markov switching model to explain the evolution of the term premium of the most liquid CDS maturities for the North American CDX index.

We consider an appropriately specified Markov Switching Unobserved Components (MS-UC) model as a reliable measure of volatility dynamics of the CDX index spread curve and investigate the presence and significance of both monetary policy adjustments and stock market returns for the US economy over the sample period August 2004 - April 2009.

To the best of our knowledge, this is the first direct empirically based evidence that is brought on the evolution of the term premium of the CDS index market and its observed macroeconomic and financial determinants.

To capture the magnitude of uncertainty in the CRT market, we decompose the level of the CDX index term premium into two components. The first, the random walk component is assumed to capture the fundamental forces driving the term premium whereas the second, a stationary AR(1) process, represents short-lived anomalies in the market. Furthermore, we formulate a model with time-varying regime-switching probabilities and regime dependent components.

Our results suggest that the inversion of the curve around September 2008 is driven largely by abrupt moves in the RW factor, representing the evolution of the fundamentals under-pinning the probability of default in the economy. In addition, during much of the
period between 2004 and 2008, the stationary factor stays consistently in the low volatility regime and then in late 2008 and throughout the final part of the sample period, eventually switches into the high volatility regime.

Remarkably, the inclusion of observed economic and financial variables to predict the evolution of the unobserved factors does a relatively good job only during the 'crisis' period. These variables are found to make a statistically significant contribution that is consistent with economic theory. Indeed, we find robust evidence that the unprecedented monetary policy response by the Fed was effective as by reducing market uncertainty helped to steepen the curve of the index thereby mitigating systemic risk concerns. Conversely, heightened volatility conditions in the equity market leads to a very substantial flattening of the curve as short term spreads widen out.

Overall, our study shows that the direct impact of monetary policy is very modest on the RW factor but rather potent on the transitory factor. Developments in both the first and second moments of the equity market have a lasting influence on both factors, implying that changes in the slope of the yield curve make a significant contribution to the transitory component.

The evolution of the CDX index in all maturities is an important signal of the 'health' of the economy over the short and long run. Sudden inversions indicate sharp deterioration of the current economic conditions and increased probability of default. Such movements are triggered by both the evolving stance of monetary policy and developments in the equity markets that make a significant albeit modest contribution to their predictability.

This article is only a first step toward the development of a fully fledged consistent framework to gain greater insight in the dynamics of the CDX curve indices across different parts of the credit cycle and in the relationship between the shape of the term structure and macro/financial variables fluctuations.

Further research is warranted. Interesting possibilities for further research include the consideration of an extended number of maturities and of other index tranches, such the high-yield segment of the market. These extensions along with a complementing examination of liquidity risks and the risk of spillovers will enhance our understanding of the dynamics of such important markets, primarily from a systemic viewpoint.
References


Appendix: Estimation procedures

To estimate the state space Markov switching model, described in detail in the previous subsections, we use Kim’s filter (Kim (1994)), which is a numerical algorithm that combine the Kalman filter in estimating state space models and the Hamilton filter (Hamilton (1989)) in estimating Markov switching models. In the conventional derivation of the Kalman filter for an invariant parameter state space model, the goal is to make predictions of the unobserved state variables based on the current information set, denoted $X_{t|t-1} = E(X_t|I_{t-1})$, where $I_{t-1}$ represents all observed variables available at time $t - 1$. The mean squared error of the prediction, denoted as $P_{t|t-1}$, is $P_{t|t-1} = E((X_t - X_{t|t-1})(X_t - X_{t|t-1})'|I_{t-1})$. The Kalman filter algorithm then implements a sequence of Bayesian updating on the unobserved variable $X_t$ and the mean square error $P_t$ when observing a new data entry. The updated unobserved variable $X_{t|t}$, given the observation of information the set at time $t$, is formed as a weighted average of $X_{t|t-1}$ and new information contained in the prediction error, where the weight assigned to this new information is called Kalman gain. This prediction and updating process evolve over time and are conditional on the correctly estimated parameters of the model. As a result, Kalman filter will need to be initialized in the first place with some carefully chosen initial values. Then, the prediction errors and their variances, as the by-products of the prediction process, will be used to construct the log-likelihood function

$$L(\theta) = -\frac{1}{2} \sum \ln [(2\pi)^n |\omega_{t|t-1}|] - \frac{1}{2} \sum \psi_{t|t-1}^{-1} \psi_{t|t-1} (20)$$

where $\psi_{t|t-1}$ is the prediction error and $\omega_{t|t-1}$ is its conditional variance.

For our model, however, the Markov variables $S_{1,t}$ and $S_{2,t}$, as the additional unobserved variables in the state space system, would undoubtedly complicate the estimation procedures. The prediction and updating processes in the Markov switching state space system will now additionally depend on both the previous and current values of the Markov variables. Since we have two independent Markov chain processes in our model, for given realizations of the two Markov variables at times $t$ and $t - 1$ ($S_{1,t-1} = i, S_{1,t} = j, S_{2,t-1} = i$ and $S_{2,t} = j$, where
\[ i = \{0, 1\}, j = \{0, 1\} \]

the Kalman filter equations can then be represented as follows

\begin{align*}
X_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= C + FX_{t-1|t-1}^{S_{1,t-1}S_{2,t-1}} \\
P_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= FX_{t-1|t-1}^{S_{1,t-1}S_{2,t-1}} F' + \Sigma_{S_{1,t}S_{2,t}} \\
\eta_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= Y_t - HX_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}}S_{2,t} \\
P_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= HP_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} \Sigma_{S_{1,t}S_{2,t}} \\
X_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= X_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} + P_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}H' \Sigma_{S_{1,t}S_{2,t}} \\
P_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} &= P_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} - P_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}H'H P_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} \Sigma_{S_{1,t}S_{2,t}} 
\end{align*}

where \( X_{t|t-1}^{S_{1,t-1}S_{2,t-1}} \) is the value of \( X_{t-1} \) based on the information up to time \( t-1 \), given that \( S_{1,t-1} = i \) and \( S_{2,t-1} = j \); \( X_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} \) is the updated value of \( X_t \) based on the information up to time \( t \), given that \( S_{1,t-1} = i \), \( S_{1,t} = j \), \( S_{2,t-1} = i \) and \( S_{2,t} = j \); \( P_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} \) is the mean squared error of the unobserved \( X_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} \) given \( S_{1,t-1} = i \), \( S_{1,t} = j \), \( S_{2,t-1} = i \) and \( S_{2,t} = j \); \( \eta_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} \) is the prediction error of \( Y_t \) in the measurement equation, given the updated forecast of \( X_t \) as \( X_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} \) conditional on \( S_{1,t-1} = i \), \( S_{1,t} = j \), \( S_{2,t-1} = i \) and \( S_{2,t} = j \) based on the information up to time \( t \); \( P_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} \) is the conditional variance of the forecast error \( \eta_{t|t-1}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} \); \( X_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} \) and \( P_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} \) are the updated \( X_t \) and \( P_t \) based on the information up to time \( t \), given that \( S_{1,t-1} = i \), \( S_{1,t} = j \), \( S_{2,t-1} = i \) and \( S_{2,t} = j \).

Since each iteration of the Kalman filter produces a 4-fold increase in the number of cases to consider\(^8\), we reduce the 16 one-period posteriors \( X_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} \) and \( P_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}} \) into 4 by taking appropriate approximations at the end of each iteration. This is computed through Kim’s approximation procedures

\[
X_{t|t}^{S_{1,t}S_{2,t}} = \sum_{S_{1,t-1}=0}^{1} \sum_{S_{2,t-1}=0}^{1} \Pr (S_{1,t-1} = i, S_{1,t} = j, S_{2,t-1} = i, S_{2,t} = j | I_t) X_{t|t}^{S_{1,t-1}S_{1,t}S_{2,t-1}S_{2,t}}
\]

\[
\Pr (S_{1,t} = j, S_{2,t} = j | I_t)
\]

\[ (22) \]

\(^8\)We have 4 cases to consider in each iteration of the Kalman filter: (1) both transitory and random walk components are in high volatility regime; (2) transitory component is in the high volatility regime while the random walk component is in the low volatility regime; (3) transitory component is in the low volatility regime while the random walk component is in the high volatility regime; (4) both transitory and random walk components are in low volatility regime. Therefore, every new iteration, the first order dependence of the current Markov chain variable on its previous value leads to a 4-fold increase in the number of cases to consider.
\[
P_{t|t}^{S_1,t,S_2,t} = \frac{1}{S_{t-1}^0} \sum_{S_{t-1}^0} \left( P_{t|t}^{S_1,t,S_{t-1}^0,S_2,t-1} + (X_{t|t}^{S_1,t,S_2,t} - X_{t|t}^{S_1,t,S_{t-1}^0,S_2,t-1}) (X_{t|t}^{S_1,t,S_2,t} - X_{t|t}^{S_1,t,S_{t-1}^0,S_2,t-1})' \right)
\]

where the probability terms in the above two equations are obtained from Hamilton’s filter as

\[
\Pr(S_{t-1} = i, S_{t-1} = j, S_{t-1} = i, S_{t} = j | I_t) = \frac{\Pr(Y_t | S_{t-1} = i, S_{t-1} = j, I_{t-1})}{\Pr(Y_t | I_{t-1})}
\]

with

\[
\Pr(Y_t | S_{t-1} = i, S_{t-1} = j, S_{t-1} = i, S_{t} = j | I_{t-1}) = \frac{1}{\sqrt{(2\pi)^N |f_{t_{t-1}}^{S_1,t,S_{t-1},S_2,t-1}|}} \exp \left( -\frac{1}{2} \begin{pmatrix} \eta_{t_{t-1}}^{S_1,t,S_{t-1},S_2,t-1} \\ \eta_{t_{t-1}}^{S_1,t,S_{t-1},S_2,t-1} \end{pmatrix}' \begin{pmatrix} \eta_{t_{t-1}}^{S_1,t,S_{t-1},S_2,t-1} \\ \eta_{t_{t-1}}^{S_1,t,S_{t-1},S_2,t-1} \end{pmatrix} \right)
\]

\[
\Pr(Y_t | I_{t-1}) = \sum_{S_{t-1}=0}^{1} \sum_{S_{t-1}=0}^{1} \sum_{S_{t-1}=0}^{1} \Pr(Y_t, S_{t-1} = i, S_{t-1} = j, S_{t-1} = i, S_{t} = j | I_{t-1})
\]

and

\[
\Pr(S_{t-1} = i, S_{t-1} = j, S_{t-1} = i, S_{t} = j | I_{t-1}) = \Pr(S_{t} = j | S_{t-1} = i) \Pr(S_{t-1} = i) \Pr(S_{t-1} = i, S_{t-1} = i, S_{t-1} = i | I_{t-1})
\]

with

\[
\Pr(S_{t-1} = i, S_{t-1} = i, S_{t-1} = i | I_{t-1}) = \sum_{S_{t-1}=2}^{1} \sum_{S_{t-1}=2}^{1} \Pr(S_{t-1} = i, S_{t-1} = i, S_{t-1} = i, S_{t-1} = i | I_{t-1})
\]

At the end of each iteration, Equation 22 and Equation 23 are used to collapse 16 one-period posteriors \(X_{t|t}^{S_1,t,S_{t-1},S_2,t-1} \) and \(P_{t|t}^{S_1,t,S_{t-1},S_2,t-1} \) into 4 \(X_{t|t}^{S_1,t,S_2,t} \) and \(P_{t|t}^{S_1,t,S_2,t} \). As a by-product of the Hamilton filter, the approximate log likelihood function is given by

\[
L(\theta) = \sum_{t=1}^{T} \ln f(Y_t | I_{t-1})
\]

that will be maximized with respect to the parameter vector space

\[
\Theta = \{p_{1,0}, p_{1,1}, p_{2,0}, p_{2,1}, \delta, k, a, \sigma_{1,H}, \sigma_{1,L}, \sigma_{2,H}, \sigma_{2,L}, \rho_{12}\}.
\]

For the model with time varying regime switching probabilities, we employ the same set of observed variables which includes the GARCH volatility of changes in the daily DFF rates (ffr_ch_GARCHt), the changes of slope of the yield curve on risk-free U.S. government bonds.
treasury bonds (Slope101_change), the daily returns on the Standard & Poor’s 500 index (sp500 rtn) and the natural logarithm of the implied volatility on the Standard & Poor’s 500 index - the VIX index (logvix). To estimate the sensitivities of the transition probability to these variables, we redefine the logistic functions in Equation 16 as

\[
\begin{align*}
    p_{1,00} &= \Pr [S_{1,t} = 0 | S_{1,t-1} = 0, G_{t-1}] = \frac{\exp (\gamma_{0,00} + G'_{t-1} \gamma_{1,00})}{1 + \exp (\gamma_{0,00} + G'_{t-1} \gamma_{1,00})} \\
    p_{1,01} &= 1 - p_{1,00} \\
    p_{1,11} &= \Pr [S_{1,t} = 1 | S_{1,t-1} = 1, G_{t-1}] = \frac{\exp (\gamma_{0,11} + G'_{t-1} \gamma_{1,11})}{1 + \exp (\gamma_{0,11} + G'_{t-1} \gamma_{1,11})} \\
    p_{1,10} &= 1 - p_{1,11} \\
    p_{2,00} &= \Pr [S_{2,t} = 0 | S_{2,t-1} = 0, G_{t-1}] = \frac{\exp (\phi_{0,00} + G'_{t-1} \phi_{1,00})}{1 + \exp (\phi_{0,00} + G'_{t-1} \phi_{1,00})} \\
    p_{2,01} &= 1 - p_{2,00} \\
    p_{2,11} &= \Pr [S_{2,t} = 1 | S_{2,t-1} = 1, G_{t-1}] = \frac{\exp (\phi_{0,11} + G'_{t-1} \phi_{1,11})}{1 + \exp (\phi_{0,11} + G'_{t-1} \phi_{1,11})} \\
    p_{2,10} &= 1 - p_{2,11}
\end{align*}
\]

(25)

where \( G_{t-1} = [\text{ffr}_t, \text{ch}_t, \text{GARCH}_t, \text{Slope101}_t, \text{sp500 rtn}_t, \text{logvix}_t] \), and \( \gamma_s \) and \( \phi_s \) are the corresponding sensitivity measures of the \( G_{t-1} \) vector for each transition probability. The parameter vector space \( \Theta \), now, is extended to include \( \gamma_s \) and \( \phi \), which will be estimated by maximizing the likelihood function.
Table 1: Mean and standard deviation of CDX 5-year, CDX 10-year and term premium for different sample periods

<table>
<thead>
<tr>
<th>Sample Period</th>
<th>CDX 5-year</th>
<th>CDX 10-year</th>
<th>Term Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Std. Dev</td>
<td>Mean</td>
</tr>
<tr>
<td>03Aug04 ~ 09Nov05</td>
<td>52.648</td>
<td>6.814</td>
<td>76.072</td>
</tr>
<tr>
<td>03Aug04 ~ 29Nov06</td>
<td>46.896</td>
<td>8.071</td>
<td>69.764</td>
</tr>
<tr>
<td>03Aug04 ~ 27Nov07</td>
<td>46.832</td>
<td>11.379</td>
<td>69.836</td>
</tr>
<tr>
<td>28Nov07 ~ 27Jul09</td>
<td>159.322</td>
<td>50.501</td>
<td>142.401</td>
</tr>
<tr>
<td>03Aug04 ~ 17Nov08</td>
<td>68.402</td>
<td>42.749</td>
<td>84.499</td>
</tr>
<tr>
<td>18Nov08 ~ 27Jul09</td>
<td>197.643</td>
<td>43.071</td>
<td>162.763</td>
</tr>
<tr>
<td>03Aug04 ~ 27Jul09</td>
<td>68.402</td>
<td>42.749</td>
<td>84.499</td>
</tr>
</tbody>
</table>

Table 2: Descriptive statistics of the data series

<table>
<thead>
<tr>
<th>CDX5</th>
<th>CDX10</th>
<th>Term Premium</th>
<th>FFR_CH</th>
<th>GARCH1</th>
<th>SLOPE101CHANGE</th>
<th>SP500RTN</th>
<th>VIX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>87.3161</td>
<td>95.9463</td>
<td>8.6302</td>
<td>0.0248</td>
<td>0.0004</td>
<td>-0.0001</td>
<td>21.2356</td>
</tr>
<tr>
<td>Median</td>
<td>53</td>
<td>77</td>
<td>22</td>
<td>0.0036</td>
<td>0</td>
<td>0.0007</td>
<td>15.63</td>
</tr>
<tr>
<td>Maximum</td>
<td>283.3708</td>
<td>251.3626</td>
<td>1.2984</td>
<td>0.96</td>
<td>-0.93</td>
<td>-0.947</td>
<td>89.89</td>
</tr>
<tr>
<td>Minimum</td>
<td>29</td>
<td>54</td>
<td>-53.3332</td>
<td>0.93</td>
<td>-0.0011</td>
<td>0</td>
<td>9.89</td>
</tr>
<tr>
<td>Skewness</td>
<td>62.6127</td>
<td>41.3712</td>
<td>0.1011</td>
<td>0.1247</td>
<td>0.0156</td>
<td>12.8773</td>
<td></td>
</tr>
<tr>
<td>Kurtosis</td>
<td>1.1973</td>
<td>1.1911</td>
<td>-1.3317</td>
<td>8.1389</td>
<td>0.0143</td>
<td>-0.2528</td>
<td>8.976</td>
</tr>
<tr>
<td>Probability</td>
<td>281.28</td>
<td>284.8754</td>
<td>363.461</td>
<td>286622.9</td>
<td>6054.117</td>
<td>4277.339</td>
<td>1312.913</td>
</tr>
<tr>
<td>Observations</td>
<td>1169</td>
<td>1169</td>
<td>1169</td>
<td>1169</td>
<td>1169</td>
<td>1169</td>
<td>1169</td>
</tr>
</tbody>
</table>
### Table 3: Parameter estimations and likelihood ratio tests of Model 1, 2 and 3

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>Estimate</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>0.0426</td>
<td>0.0133</td>
<td>0.0413</td>
<td>0.0135</td>
<td>0.0062</td>
<td>0.01</td>
</tr>
<tr>
<td>$a_2$</td>
<td>-0.0144</td>
<td>0.0133</td>
<td>0.0117</td>
<td>0.0136</td>
<td>0.0132</td>
<td>0.0136</td>
</tr>
<tr>
<td>$k$</td>
<td>1.2771</td>
<td>0.0273</td>
<td>1.2708</td>
<td>0.0213</td>
<td>1.2618</td>
<td>0.0072</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-12.4312</td>
<td>0.4628</td>
<td>-10.57</td>
<td>0.1741</td>
<td>-13.7681</td>
<td>0.1521</td>
</tr>
<tr>
<td>$\sigma_{TH}$</td>
<td>0.4982</td>
<td>0.0141</td>
<td>0.5526</td>
<td>0.1375</td>
<td>0.4759</td>
<td>0.0291</td>
</tr>
<tr>
<td>$\sigma_{TL}$</td>
<td>0.3339</td>
<td>0.0137</td>
<td>0.3168</td>
<td>0.0366</td>
<td>0.5272</td>
<td>0.0085</td>
</tr>
<tr>
<td>$\sigma_{RH}$</td>
<td>0.7762</td>
<td>0.0454</td>
<td>0.7579</td>
<td>0.0367</td>
<td>0.7244</td>
<td>0.078</td>
</tr>
<tr>
<td>$\sigma_{RL}$</td>
<td>0.0972</td>
<td>0.0528</td>
<td>0.0607</td>
<td>0.1392</td>
<td>0.0565</td>
<td>0.0846</td>
</tr>
<tr>
<td>$\rho_{TH,RH}$</td>
<td>-0.612</td>
<td>0.163</td>
<td>-0.659</td>
<td>0.2114</td>
<td>-0.612</td>
<td>0.163</td>
</tr>
<tr>
<td>$\rho_{TH,RL}$</td>
<td>0.9956</td>
<td>0.055</td>
<td>1</td>
<td>0.002</td>
<td>0.9956</td>
<td>0.055</td>
</tr>
<tr>
<td>$\rho_{TL,RL}$</td>
<td>-1</td>
<td>0.0003</td>
<td>-1</td>
<td>0.0003</td>
<td>-1</td>
<td>0.0003</td>
</tr>
<tr>
<td>$\rho_{TH,RL}$</td>
<td>0.9998</td>
<td>0.0001</td>
<td>0.998</td>
<td>0</td>
<td>0.9998</td>
<td>0</td>
</tr>
<tr>
<td>$\rho_{TL,RL}$</td>
<td>0.9994</td>
<td>0</td>
<td>0.9995</td>
<td>0.0001</td>
<td>0.9996</td>
<td>0</td>
</tr>
<tr>
<td>$\rho_{TH,RH}$</td>
<td>0.9629</td>
<td>0.002</td>
<td>0.953</td>
<td>0.0025</td>
<td>0.9712</td>
<td>0.0036</td>
</tr>
<tr>
<td>$\rho_{TL,RL}$</td>
<td>0.9896</td>
<td>0.0003</td>
<td>0.993</td>
<td>0.0006</td>
<td>0.9868</td>
<td>0.0008</td>
</tr>
</tbody>
</table>

Log-likelihood value: -1513.19, -1499.11, -1534.56

LR test on model 2 to model 1: $a_2 = 0$

LR test on model 3 to model 2: $\rho_{TH,RH} = \rho_{TH,RL} = \rho_{TL,RH} = \rho_{TL,RL} = 0$

### Table 4: Adjusted R-square statistics for different sample periods

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Transitory</td>
<td>0.1455</td>
<td>0.19408</td>
<td>0.74604</td>
<td>0.5959</td>
<td>0.16263</td>
</tr>
<tr>
<td>dTP = C(1)dRW + C(2)Trans_residual</td>
<td>0.89509</td>
<td>0.98953</td>
<td>0.97828</td>
<td>0.95734</td>
<td>0.8506</td>
</tr>
</tbody>
</table>
Table 5: Generalised variance decomposition of the DIFRW and the TRAN (whole sample)

<table>
<thead>
<tr>
<th>Period</th>
<th>S.E.</th>
<th>FFR, CH, GARCH</th>
<th>SLOPE101, CHANGE</th>
<th>SP500RTN</th>
<th>LOGVIX</th>
<th>DIFRW</th>
<th>TRAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.018</td>
<td>0.631</td>
<td>0.437</td>
<td>5.362</td>
<td>2.122</td>
<td>76.649</td>
<td>14.798</td>
</tr>
<tr>
<td>15</td>
<td>1.018</td>
<td>0.634</td>
<td>0.443</td>
<td>5.376</td>
<td>2.141</td>
<td>76.594</td>
<td>14.812</td>
</tr>
</tbody>
</table>

Table 6: Generalised variance decomposition of the DIFRW and the TRAN in the pre-financial crisis period (05/08/04 to 28/11/07)

<table>
<thead>
<tr>
<th>Period</th>
<th>S.E.</th>
<th>FFR, CH, GARCH</th>
<th>SLOPE101, CHANGE</th>
<th>SP500RTN</th>
<th>LOGVIX</th>
<th>DIFRW</th>
<th>TRAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.434</td>
<td>3.731</td>
<td>4.16</td>
<td>3</td>
<td>1.57</td>
<td>8.576</td>
<td>78.962</td>
</tr>
<tr>
<td>15</td>
<td>0.435</td>
<td>3.78</td>
<td>4.162</td>
<td>3.079</td>
<td>1.631</td>
<td>8.546</td>
<td>78.802</td>
</tr>
</tbody>
</table>

Table 7: Generalised variance decomposition of the DIFRW and the TRAN after the financial crisis (29/11/07 to 23/07/09)

<table>
<thead>
<tr>
<th>Period</th>
<th>S.E.</th>
<th>FFR, CH, GARCH</th>
<th>SLOPE101, CHANGE</th>
<th>SP500RTN</th>
<th>LOGVIX</th>
<th>DIFRW</th>
<th>TRAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.433</td>
<td>0.868</td>
<td>1.285</td>
<td>7.632</td>
<td>6.533</td>
<td>69.551</td>
<td>14.131</td>
</tr>
<tr>
<td>15</td>
<td>1.431</td>
<td>0.878</td>
<td>1.305</td>
<td>7.693</td>
<td>6.626</td>
<td>69.319</td>
<td>14.178</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Period</th>
<th>S.E.</th>
<th>FFR, CH, GARCH</th>
<th>SLOPE101, CHANGE</th>
<th>SP500RTN</th>
<th>LOGVIX</th>
<th>DIFRW</th>
<th>TRAN</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.706</td>
<td>4.01</td>
<td>7.478</td>
<td>3.846</td>
<td>4.686</td>
<td>6.542</td>
<td>73.438</td>
</tr>
<tr>
<td>15</td>
<td>0.709</td>
<td>4.057</td>
<td>7.477</td>
<td>3.995</td>
<td>4.852</td>
<td>6.497</td>
<td>73.123</td>
</tr>
</tbody>
</table>

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Table 8: Parameters estimation (Model 4)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>Parameters</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>Parameters</th>
<th>Estimate</th>
<th>Std. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_1$</td>
<td>23.1908</td>
<td>146.512</td>
<td>$\hat{\varphi}<em>{f</em>{r,h}}$ARCH.RH</td>
<td>26.5293</td>
<td>14.1643</td>
<td>$\gamma_{f_{r,h}}$ARCH.TH</td>
<td>5.6997</td>
<td>0.4417</td>
</tr>
<tr>
<td>$a_2$</td>
<td>-0.0204</td>
<td>0.015</td>
<td>$\hat{\varphi}_{Slope,101,change,RH}$</td>
<td>-1.0128</td>
<td>1.4609</td>
<td>$\gamma_{Slope,101,change,TH}$</td>
<td>4.5673</td>
<td>1.0571</td>
</tr>
<tr>
<td>$k$</td>
<td>1.982</td>
<td>0.0129</td>
<td>$\hat{\varphi}<em>{sp</em>{500,rtn,RH}}$</td>
<td>-17.6</td>
<td>5.7265</td>
<td>$\gamma_{sp_{500,rtn,TH}}$</td>
<td>36.0553</td>
<td>14.4934</td>
</tr>
<tr>
<td>$\delta$</td>
<td>-13.1081</td>
<td>1.441</td>
<td>$\hat{\varphi}_{log,vix,RH}$</td>
<td>0.2457</td>
<td>0.0465</td>
<td>$\gamma_{log,vix,TH}$</td>
<td>0.1991</td>
<td>0.123</td>
</tr>
<tr>
<td>$\sigma_{TH}$</td>
<td>0.3057</td>
<td>0.1429</td>
<td>$\hat{\varphi}<em>{f</em>{r,h}}$ARCH.RL</td>
<td>3.5061</td>
<td>2.7154</td>
<td>$\gamma_{f_{r,h}}$ARCH.TL</td>
<td>-7.042</td>
<td>23.0827</td>
</tr>
<tr>
<td>$\sigma_{TL}$</td>
<td>0.0722</td>
<td>0.0726</td>
<td>$\hat{\varphi}_{Slope,101,change,RL}$</td>
<td>1.0609</td>
<td>0.4685</td>
<td>$\gamma_{Slope,101,change,TL}$</td>
<td>0.2412</td>
<td>48.393</td>
</tr>
<tr>
<td>$\sigma_{RL}$</td>
<td>1.0485</td>
<td>0.0582</td>
<td>$\hat{\varphi}<em>{sp</em>{500,rtn,RL}}$</td>
<td>-1.0949</td>
<td>4.5667</td>
<td>$\gamma_{sp_{500,rtn,TL}}$</td>
<td>-0.0204</td>
<td>76.5788</td>
</tr>
<tr>
<td>$\rho_{TH,RH}$</td>
<td>-0.9992</td>
<td>0.0056</td>
<td>$\hat{\varphi}_{log,vix,RL}$</td>
<td>-0.277</td>
<td>0.0099</td>
<td>$\gamma_{log,vix,TL}$</td>
<td>1.2044</td>
<td>4.7811</td>
</tr>
<tr>
<td>$\rho_{TH,RL}$</td>
<td>0.2463</td>
<td>2.00</td>
<td>$\rho_{TH,RH}$</td>
<td>-0.9241</td>
<td>0.0025</td>
<td>$\rho_{TH,RL}$</td>
<td>-0.9097</td>
<td>0.1126</td>
</tr>
<tr>
<td>$\rho_{TL,RH}$</td>
<td>-0.2463</td>
<td>2.00</td>
<td>$\rho_{TH,RL}$</td>
<td>-0.9241</td>
<td>0.0025</td>
<td>$\rho_{TL,RH}$</td>
<td>-0.9097</td>
<td>0.1126</td>
</tr>
</tbody>
</table>

Log-likelihood value: -1468.6198

Table 9: Number of days that each model predicts on the number of days each component switches into a high volatility regime

<table>
<thead>
<tr>
<th>Transitory component</th>
<th>RW component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of days</td>
<td>346</td>
</tr>
<tr>
<td>Total number of days</td>
<td>1169</td>
</tr>
<tr>
<td>Ratio</td>
<td>0.296</td>
</tr>
</tbody>
</table>
Figure 1: CDX-5 Year, CDX-10 Year and CDX Term Premium
Figure 2: Actual plots of DFF.CH.GARCH, SLOPE101.CHANGE, SP500RTN and VIX index
Figure 3: Transitory and RW components with the associated probabilities of switching into a high volatility regime
The conditional variance and the probability of transitory component to switch to high volatility regime

The conditional variance and the probability of Random Walk component to switch to high volatility regime

Figure 4: Conditional variance of the term premium with each component’s probability of switching into a high volatility regime
Figure 5: Recursive estimates of C(1) and C(2) from Equation 18
Figure 6: Generalised impulse response functions of the VAR model (whole sample)
Figure 7: Generalised impulse response functions of the VAR model in the pre-financial crisis period (05/08/04 to 28/11/07)
Figure 8: Generalised impulse response functions of the VAR model in the post-financial crisis period (29/11/07 to 23/07/09)
The term premium and the probability of transitory component switching to high volatility regime

The term premium and the probability of Random Walk component switching to high volatility regime

Figure 9: Transitory and RW components with the associated probabilities of switching into a high volatility regime (Model 4)
The conditional variance and the probability of transitory component switching to high volatility regime

The conditional variance and the probability of Random Walk component switching to high volatility regime

Figure 10: Conditional variance of the term premium with each component’s probability of switching into a high volatility regime (Model 4)
Figure 11: Recursive coefficient estimate