Comparing Policies to Confront Permit Over-allocation*

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Abstract

Instability in cap-and-trade markets, particularly with respect to permit price collapses has been an area of concern for regulators. To that end, several policies, including hybrid price-quantity mechanisms and the newly introduced “market stability reserve” (MSR) systems have been introduced and even implemented in some cases. I develop a stochastic dynamic model of a cap-and-trade system, parameterized to values relevant to the European Union’s Emission Trading System (EU ETS) to analyze the performance of these policies aimed at adding stability to the system or at least at reducing perceived over-allocations of permits. Results suggest adaptive-allocation mechanisms such as a price collar or MSR can reduce permit over-allocations and permit price volatility in a more cost-effective manner than simply reducing scheduled permit allocations. However, it is also found that the performance of these adaptive allocation policies, and in particular the MSR, are greatly affected by assumed discount rates and policy parameters.

Keywords: cap-and-trade, market stability reserve, price collar, EU ETS

1 Introduction

Cap-and-trade policies have been used for decades as a means to control emissions from stationary sources and they appear to be the policy of choice in several recent efforts to control greenhouse gas (GHG) emissions.1 The virtues and negative aspects of cap-and-trade policies have been well documented in the economics literature. However, in recent

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1Recent cap-and-trade programs to regulate GHG emissions include the EU Emissions Trading Scheme (EU ETS), the Regional Greenhouse Gas Initiative in the Northeastern U.S., California’s AB 32 regulation, and New Zealand’s ETS. Several other regions have also applied carbon taxes, but the majority of regulated sources of GHG’s fall under a cap-and-trade program.
policy debates and in the popular press, price variability and in particular price collapses, have garnered much attention. The idea of price variability in cap-and-trade programs is, of course, not a new revelation. Analysis of quantity-based programs, such as a cap-and-trade policy, with uncertainty in marginal benefits and/or marginal abatement costs can be found in such classic works as Weitzman (1974), Roberts and Spence (1976), and Stavins (1996). Building on this work, several studies have also explored the impact of uncertainty on cap-and-trade systems and possible mechanisms to deal with it, such as banking and borrowing or price-quantity hybrid systems, in dynamic settings (e.g. Schennach (2000), Unold and Requate (2001), Fell and Morgenstern (2010), Fell et al. (2012b), Fell et al. (2012a)).

These aforementioned studies and others like it describe the relative trade-offs of various policy designs in an uncertainty modeling context and prescribe program features that could be applied from the program’s inception. However, recently, a more subtle debate has developed – how do we alter existing programs to deal with a situation in which there is an unexpected “over-supply” of emission allowances?

Several practical examples of this concern can be found in the SO$_2$ emissions trading program in the U.S., the Regional Greenhouse Gas Initiative (RGGI) in the Northeastern U.S., and the EU’s emission trading scheme for carbon (EU ETS). In the SO$_2$ trading program, U.S. Environmental Protection Agency (EPA) regulators determined there was a relative over-supply of permits given information about relatively high benefits from SO$_2$ abatement and the environmental problems created by interstate air pollution. The EPA

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An article in the *Economist* (2013) declared, “The ETS has long been a mess. ... Partly because recession has reduced industrial demand for the permits, and partly because the EU gave away too many allowances in the first place, there is massive overcapacity in the carbon market. The surplus is 1.5 billion-2 billion tonnes, or about a year’s emissions. Prices had already fallen from €20 ($30) a tonne in 2011 to €5 a tonne in early 2013.”
regulators sought to effectively reduce the supply of permits through the issuing of the Clean Air Interstate Rule (CAIR). Under the CAIR, which was set to begin in 2015, banked permits of vintage 2009 or earlier could be exchanged at a one-to-one ratio for compliance purposes, but firms would have to exchange two banked permits of vintage 2010 - 2014 for a unit of emissions. Furthermore, for permits of vintage 2015 and beyond, CAIR would have required firms to turn over 2.86 permits per unit of emissions.\(^3\) CAIR was eventually struck down by U.S. federal courts, but it does provide a possible policy, change the trading ratio of a permit, for dealing with a perceived over-supply.

The RGGI program, a CO\(_2\) emissions trading system among power plants in the Northeastern U.S. that began in 2009, also contains mechanisms to prevent over-allocation. First, RGGI has a price-floor in their permit auctions, which can serve to reduce the number permits entering the market. Second, given the permit banks acquired by the regulated entities over the first few years of the program, RGGI-participating states agreed to a considerable reduction in the planned cap in 2014 and beyond, reducing the planned 2014 cap by about 45 percent (165 million permits to 91 million) and having it decrease by 2.5 percent from 2014 through 2020.

Rather than being motivated by higher-than-expected benefits, EU regulators are concerned that permit prices are too low and therefore do not provide an adequate incentive for firms to undertake long-run investments needed to move to a low-carbon economy. EU regulators have thus proposed several plans to reduce, at least temporarily, the supply of allocated permits in an effort to prop-up EU ETS permit prices. There are two main ways that the European Commission (EC) regulators are proposing to reduce supply. The first

\(^3\)For more information on CAIR, see Fraas and Richardson (2010).
method is known as “backloading”. Under backloading, regulators will reduce the number of permits that will be available for auction in the next few auctions relative to what was initially planned. However, these permits that were removed from the near-term auctions will be offered several years later in subsequent auctions. Thus, this policy only temporarily removes permits from the market.

The other policy EC regulators are considering is a so-called “market stability reserve” (MSR) system. Under the MSR policy, a rules-based policy will be implemented such that if the total number of outstanding allowances (i.e., the cumulative bank of allowances held by firms) exceeds some threshold, then allocations in subsequent auctions will be reduced relative to the planned allocation. Likewise if the total number of outstanding allowances falls below some limit, additional permits beyond what was planned will be made available for sale in future auctions. The goal of the MSR is thus to reduce perceived permit “surplus” conditions and likewise to protect against system-wide permit shortages. Though not a stated goal of the EC regulators, such an adaptive permit supply system could also increase permit price stability.

In this paper, I develop a stochastic dynamic model of abatement cost to analyze various methods of dealing with over-supplies of permits. These methods include altering trading ratios, MSR policies, permit price collars, and policies aimed at completely vacating allocations in later years of the program are also considered. Using a model that is parameterized to the EU ETS, the relative effectiveness of these various policies in terms of expected abatement costs, permit price variability, and emissions variability are examined. These results are then used to make policy recommendations to the EC with regards to their efforts to increase permit market stability.
The results indicate that the MSR policy as proposed by the EC will likely reduce allocation primarily in the beginning years, leading to a near-term increase in permit prices, and does reduce permit price variation relative to the EU ETS in its current form. However, the price collar mechanisms modeled are found to achieve the same expected emission levels as the MSR but with less price variation and lower expected abatement costs than the MSR, albeit with higher volatility in cumulative emissions. The permit reduction policies, on the other hand, achieve the same expected cumulative emissions level as the MSR at a slightly higher expected abatement cost and larger variation in permit prices, but with virtually no uncertainty in achieved cumulative emissions.

Importantly, it is also shown that the effectiveness of the MSR in terms of removing perceived excess permits and promoting greater price supports is diminished as the regulated firms’ discount rate increases. In contrast, price collar mechanisms with a strict price-floor actually further reduce allocations with increasing discount rates and by definition prevent price collapses. Finally, in exploring parameterization sensitivity of the MSR it is found that the MSR designs that withdraw about as many permits in high-bank states as they inject in low-bank states do little to increase permit prices or decrease permit price variation relative to the EU ETS system as is.

The remainder of the paper is organized as follows. Section 2 describes the basic theoretic model that underlies the numeric modeling efforts and describes the various policy provisions. Section 3 describes the numeric modeling technique. In section 4, I describe the results from the simulation experiments under various parameterizations. Concluding remarks are given in the final section 5.
2 Basic Model Description

2.1 Individual Firm Model

In order to numerically solve the model, one must first determine the dynamic behavior that is consistent with a firm’s cost minimization problem. It is assumed that in each period \( t \) abatement for firm \( i \), with \( i = 1, \ldots, N \), is represented as \( a_{it} = \bar{q}_{it} + \theta_t - q_{it} \) where \( \bar{q}_{it} \) is the expected baseline or business-as-usual (BAU) emissions, \( \theta_t \) is the shock to BAU emissions assumed to be observed at the beginning of each period, and \( q_{it} \) is the chosen level of emissions in period \( t \), with \( q_{it} \geq 0 \). The abatement cost function is given as

\[
C_{it}(a_{it}) = C_{it}(\bar{q}_{it} + \theta_t - q_{it})
\]

with \( \frac{\partial C_{it}}{\partial a_{it}} = -\frac{\partial C_{it}}{\partial q_{it}} = -C^q_{it} > 0 \) and \( \frac{\partial^2 C_{it}}{\partial a^2_{it}} \geq 0 \). The firm can also purchase permits, \( x_{it} \), from an auction at a price, \( p_t \), that it takes as given.\(^4\) To begin, assume that for every unit of emissions the firm emits it must surrender one permit.

The model is dynamic in nature and this is expressed through several components. First, it is assumed that in each period \( t \) the firms may over-comply \( (q_{it} < x_{it}) \) and bank these unused permits for future use. Likewise, the firm may borrow permits from future allocations up to a limit, implying a minimum bank state \( B_{it}^{min} \) with \( B_{it}^{min} \leq 0 \).\(^5\) Thus, the banking

\(^4\)One could assume that permits purchased at auction could later be sold in a secondary market. However, given the information structure assumed here, namely that shocks are observed at the beginning of the period before permits must be purchased, a secondary market is unnecessary. Similarly, one could assume that some permits are given to the firm freely and that the firm could buy or sell permits at a price it takes as given in a secondary market. Again, given the information structure used here, such an set up would not alter the end dynamic efficiency conditions developed below. This was also effectively shown in Kling and Rubin (1997).

\(^5\)Banking and borrowing are allowed in the EU ETS, the system most closely examined here, under Directive 2003/87/EC. Banking now in the EU ETS is essentially unrestricted within and across compliance periods. Firms in the EU ETS can also freely borrow permits allocated within a compliance period and can borrow across compliance periods, though they are charged interest on those permits (see Chevallier (2010) for more details). Here, I simplify this set up by simply allowing borrowing up to a maximum borrowing level, in which permits can be borrow free of charge. Additionally, throughout the paper I use “minimum bank constraint” and “maximum borrowing constraint” as interchangeable phrases.
dynamics condition is given as:

\[ B_{it+1} = B_{it} + x_{it} - q_{it} \]  
\[ B_{it} \geq B_{t}^{min} \]  

(1)

It is also assumed that there is persistence in the common shock process, such that:

\[ \theta_t = \rho \theta_{t-1} + \epsilon_t \]  

(2)

with \( 0 < \rho < 1 \) and \( \epsilon_t \sim iid N(0, \sigma^2) \).\(^6\) Assume that \( \theta_t \) is sufficiently small such that \( \bar{q}_{it} - \theta_t > 0 \) for all values of \( \theta_t \). Finally, it is also assumed that total permits auctioned at time \( t \), \( y_t \), and \( \bar{q}_t \) all evolve under exogenously defined processes.

The firm’s dynamic cost minimization problem can be written as:

\[
\min_{q_{it}, y_{it}} \sum_{t=0}^{T} \beta^t E_t \left[ C_{it}(\bar{q}_{it} + \theta_t - q_{it}) + p_t x_{it} \right] 
\]  

(3)

subject to (1) and (2) and where \( \beta \) is the discount factor (\( \beta = \frac{1}{1+r} \) and \( r \) is the discount rate) and \( E_t \) is the expectations operator at time \( t \). The resulting Bellman equation for this minimization problem is then given as:

\[
V_{it}(B_{it}, \theta_t) = \min_{q_{it}, x_{it}} \left( C_{it}(\bar{q}_{it} + \theta_t - q_{it}) + p_t x_{it} \right) + \lambda_{it}(B_{t}^{min} - B_{it}) \]

\[ + \beta E_t [V_{it+1}(B_{it+1}, \theta_{t+1}) | q_{it}, x_{it}, \theta_t, B_{it}] \]

(4)

\(^6\)Note that with an assumed AR(1) process where \( \rho \neq 0 \), current shock values serve as the basis for future expected shocks. More formally, \( E[\theta_{t+1}|\theta_t] = \rho \theta_t \). Thus, there is an updating of expectations about future shock values with the observation of the current period’s shock.
subject to (1) and (2) and where $\lambda_{it}$ is the multiplier associated with the minimum bank constraint. The necessary first order conditions for the minimization of (4) are:

$$\frac{\partial V_{it}}{\partial q_{it}} = C_{it}^q - \beta E_t \left[ \frac{\partial V_{it+1}}{\partial B_{it+1}} \right] \geq 0; \quad q_{it} \geq 0; \quad q_{it} \frac{\partial V_{it}}{\partial q_{it}} = 0$$ (5)

$$\frac{\partial V_{it}}{\partial x_{it}} = p_t + \beta E_t \left[ \frac{\partial V_{it+1}}{\partial B_{it+1}} \right] = 0$$ (6)

$$\frac{\partial V_{it}}{\partial B_{it}} = \beta E_t \left[ \frac{\partial V_{it+1}}{\partial B_{it+1}} \right] - \lambda_{it}; \quad \lambda_{it} \geq 0$$ (7)

From (5) and (6) and assuming positive emissions, we have the result $-C_{it}^q = p_t$, marginal abatement costs equal the price of permits. Also under the assumption of positive emissions, from (5) and (7) the Euler equation for abatement costs can be written as:

$$C_{it}^q = \beta E_t \left[ C_{it+1}^q - \lambda_{it+1} \right] \rightarrow -C_{it}^q = \beta E_t \left[ -C_{it+1}^q + \lambda_{it+1} \right]$$ (8)

From the dynamic cost minimization condition in (8) we have an emissions decision criterion that can be applied in the numeric model described below. Equation (8) shows that when the maximum borrowing constraint is not binding the firm will choose emission and permit purchase levels to equate expected discounted marginal abatement costs across periods. However, with the minimum bank constraint, discounted marginal abatement costs, and therefore permit prices, are not always equated across time. Rearranging (8) and noting marginal abatement costs equal permit prices, we get the following relationship:

$$\frac{E_t(p_{t+1}) - p_t}{p_t} = r - \frac{\lambda_{it+1}}{p_t}$$ (9)
From (9) it can be seen that when the minimum banking constraint is not binding \((\lambda_{it} = 0)\) then expected permit prices rise at rate \(r\) - prices follow a Hotelling path. This is effectively a discrete time restatement of the result shown in Rubin (1996).\(^7\)

The final conditions necessary to solve the competitive equilibrium permit price, and thus emission and permit purchases, are the market clearing condition for the permits and a terminal period condition. The permit price \(p_t\) must vary each period to clear the market, formally such that \(\sum_{i=1}^{N} x_{it} \leq y_t\), where \(y_t\) is the total allocation of permits available for auction at \(t\). For the terminal condition, I assume no continuation value from holding a bank and that the firm cannot have a negative bank at the end of the regulation horizon \((B_{iT+1} \geq 0)\). Thus, the firm would cost minimize by driving the bank as close to zero as possible at the end of period \(T\). However, given the bank going into the last period and the shock received in the last period, the firm may not want to use all the remaining bank. More precisely, \(q_{iT} = \min(\bar{q}_{iT} + \theta_T, B_{iT} + x_{iT})\), so if the final period’s BAU emissions and/or shock is sufficiently large the firm will deplete its remaining bank plus permits purchased, but if \(B_{iT} > \bar{q}_{iT} + \theta_T\) a positive bank of \(B_{iT} - (\bar{q}_{iT} + \theta_T)\) will be left at the end of period \(T\). If the firm begins \(T\) with a negative bank \((B_{iT} < 0)\) it will have to purchase enough permits and reduce emissions sufficiently to make \(B_{iT+1} = 0\).\(^8\)

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\(^7\)This result has also been shown in other works and with some other relevant variations. For example, Yates and Cronshaw (2001) provide a modified Hotelling rule for permit prices for the case where emissions-regulated firms also face profit regulation. Note also that cost-efficient dynamic reallocation of permits may not lead to a socially optimal outcome particularly if damages are dependent on emission flows (e.g. Kling and Rubin (1997); Cronshaw and Kruse (1996)). As this study deals with a stock pollutant case, this emissions-flow dependent damages consideration is ignored.

\(^8\)I assume that \(|NB_{iT}^{min}| < y_T\), so there are a sufficient number of permits available in the final auction to ensure against the possibility that there are more permits owed than available in the last period.
2.2 Policy Scenarios

I now discuss possible policies that a regulator may impose upon firms in addition to the cap-and-trade regulation described above. The specific policies considered here are a so-called market stability reserve, a price collar, and permit reduction policies. Note, however, that these additional policies do not change the dynamic cost minimization conditions described above. That is, if one assumes that in a perfectly competitive environment firms take expected price paths and allocation paths as given, the dynamic cost minimization condition of equating expected discounted marginal abatement costs across periods will still exist regardless of the additional policies used. The additional policies will just alter the market clearing conditions.

2.2.1 Market Stability Reserve

As mentioned above, the market stability reserve (MSR) is a mechanism proposed by the EC that provides a “rules-based” policy to alter permit allocations in an effort to stabilize permit prices. More specifically, the proposed rule is written such that if at the end of year $t-2$ the number of permits held in circulations (i.e., the number of permits banked) exceeds 833 million then the number of permits in year $t$’s auction will be reduced by an amount equal to 12 percent of the cumulative bank held at the end of year $t-2$. Conversely if at the end of year $t-2$ the cumulative bank is 400 million allowances or lower then year $t$’s auction will be increased by 100 million allowances.9

Practically, because $B_{it}$ represents the bank at the beginning of period $t$ and is therefore equal to the bank at the end of period $t-1$, the allocation path of the model described

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9 For more information about how these policy parameters were derived see Gilbert et al. (2014).
above is altered such that:

\[ y_{t+1} = y^{S}_{t+1} - D^U_t \gamma B_t + D^L_t y^L \]  \hspace{1cm} (10)

In (10), \( y^{S}_{t+1} \) is the scheduled total number of permits to be auctioned in period \( t + 1 \) in the absence of the MSR, \( D^U_t \) is an indicator variable equal to one if the aggregate bank \( B_t \) is such that \( B_t \geq B^U \) and 0 otherwise, \( \gamma \) is a parameter that determines the reduction in the \( t + 1 \) auction (\( \gamma = 0.12 \) in the proposed legislation), \( D^L_t \) is an indicator equal to 1 if \( B_t \leq B^L \) and 0 otherwise, and \( y^L \) is the quantity of permits added to the \( t + 1 \) auction if the lower bank limit is triggered. Note again, if a competitive market is assumed, where the firms take the auction price as given, the MSR policy does not affect the firm’s dynamic cost minimization condition. Instead, the MSR just alters the market clearing condition \( \sum_{i=1}^{N} x_{it} = y_t \) by altering the aggregate number of permits to be auctioned.

### 2.2.2 Permit Price Collar

A permit price collar works in much the same way as an MSR, but instead of triggering permit injections or withdrawals based on the aggregate bank the trigger is based on the permit price. I consider two general forms of price collars in this analysis: a *hard* price collar and a *soft* price collar. Under a hard price collar, the regulator stands to limitlessly inject permits into the system to ensure the permit price does not exceed a given upper price limit or price ceiling, \( p^C_t \) and similarly has a standing offer to buy permits at a price \( p^F_t \) to support a price floor. Alternatively, for a soft price collar, the regulator has a limited number of permits, a permit reserve denoted as \( y^R_t \), to defend the upper price limit and also stands to purchase only a limited number of permits to support the price floor. Therefore, the permit
price may exceed $p^C_t$ if demand for permits exceeds the regular allocation plus the additional permits injected under the soft price collar or fall below the price floor if permit demand is sufficiently low. For both collar systems I assume there is a hard price floor, $p^F_t$, imposed on the permit auction and a soft price ceiling.\textsuperscript{10}

The price collar formulations also augment the auction allocation path. Formally, the auction schedule under the price collar is:

$$y_t = y^S_t + D^C_t y^C_t - D^F_t y^F_t \quad (11)$$

In (11) $y^S_t$ is again the number of permits scheduled to be auctioned in the absence of the price ceiling or floor being triggered. $D^C_t$ is an indicator variable equal to 1 if the permit price in the period $t$ auction would clear above $p^C_t$ in the absence of additional permits being added and 0 otherwise.\textsuperscript{11} Define $y^C_t$ as the additional allocation of permits needed to keep the permit price at exactly $p^C_t$. Thus, for the soft ceiling, $y^C_t = \min (y^C_t, y^R_t)$. Likewise, $D^F_t$ is an indicator variable equal to 1 if the permit price, in the absence of an allocation reduction, would drop below $p^F_t$. The variable $y^F_t$ then denotes the number of permits that must be removed from the total scheduled to be auctioned at $t$ in order to get the market to clear at $p^F_t$.

\textsuperscript{10}Using the language of Fell et al. (2012b), the soft collar modeled here is a non-symmetric soft collar - a collar that has a hard floor, but not a hard price ceiling.

\textsuperscript{11}Note that given the information schedule used here, the regulator could calculate the clearing price in the absence of any additional permit injections or withdrawals. In a more realistic setting, a regulator could announce a sequence of auctions, wherein the regulator stands to auction permits from its reserve in later auctions if in earlier auctions prices reached $p^C_t$. 

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2.2.3 Permit Reduction Policies

If the concern at the beginning of a regulation phase is that there has been an overallocation of permits, leading to insufficiently low permit prices and/or the possibility of excessive emissions, the regulator could simply reduce the number of permits in circulation at the outset of the phase. There are many ways in which a regulator could implement a permit reduction plan. For instance, similar to the CAIR policy in the U.S. SO$_2$ trading program, regulators could require firms to exchange more than one permit to cover a unit of emissions. This effectively just reduces the future allocation, so formally the new allocation for period $t$ is $y_t = s_t y^S_t$. In this setting, $s_t$ is an exchange ratio factor such that $0 \leq s_t \leq 1$.

Clearly, there is an infinite form this type of permit reduction system could take. I explore two specific forms of this type of regulation in the simulation analysis. First, I apply a constant trading ratio ($s_t = s \forall t$) in order to reduce allocation to a specific level. Second, all allocations beyond $t^*$ are vacated such that $s_t = 1 \forall t < t^*$ and $s_t = 0 \forall t \geq t^*$, where $t^*$ is determined in order to meet a pre-specified emissions target. From a discounted cost perspective, there are reasons to prefer the method that reduces total allocation by vacating allocations in the relatively distant future. However, if the goal is to increase prices regulators may prefer a strategy more like the first option that simply reduces each year’s allocation.

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12The regulator could also apply a trading ratio on banked permits held by the firms. However, this policy may be problematic politically to implement as it requires a regulator-created direct devaluation, at least in terms of implicit emissions, of an asset held privately. I therefore only consider alterations to the value of future allocations.
3 Numeric Model Description

Even with functional forms given for \( C_{it}(\bar{q}_{it} + \theta_{it} - q_{it}) \) an analytic solution to the cost minimization problem described above is not tractably solvable. I therefore employ a representative firm model, where the representative firm’s abatement cost function can be thought of as the aggregate cost function of the covered sectors, and numerically solve the problem. This aggregation also implies that the representative firm’s bank is equal to the covered sectors’ aggregate bank and that its allocation is the aggregate allocation, which further alleviates the need to consider inter-firm buying and selling of permits.

To begin, assume the following quadratic functional form for the marginal abatement cost (MAC):

\[
MAC_t(\theta_t, q_t) = b_{1t} (\bar{q}_t + \theta_t - q_t) + b_{2t} (\bar{q}_t + \theta_t - q_t)^2
\]  

The (12) was chosen over the more commonly used linear MAC based on results from Landis (2015). More specifically, using the well-cited PACE computable general equilibrium model, Landis (2015) solves the equilibrium for a cap-and-trade system under a series of different emissions caps. The resulting equilibrium prices under these different caps provide us with estimates of the marginal abatement costs at different abatement levels. Landis (2015) then fits continuous functions to each year’s estimated sets of marginal abatement costs and abatement levels. He showed that the fitted quadratic MACs much more accurately approximated the model-derived MACs than a linear functional form could.\(^{13}\)

It should also be noted that in this process of determining the MACs, Landis (2015) runs scenarios with no

\(^{13}\)Landis (2015) also provided other MAC approximations using higher-order polynomials. Some of these approximations performed marginally better, in terms of fit, than the quadratic MAC used here. However, these higher-order polynomials increase the computational complexity of the numeric methods employed in this study and, thus, I opted for the more easily implementable quadratic form.
emissions cap and the resulting emissions from these runs form the estimate of the expected BAU emissions $\bar{q}_t$.\(^{14}\)

Next the state space of the model is discretized. For the simple cap-and-trade model, the “state” at time $t$ is described by the realization $(B_t, \theta_t)$. Thus, $B_t$ and $\theta_t$ take $N_B$ and $N_\theta$ discrete values, respectively, and these discrete state spaces are constant for all $t = 1, ..., T$. For $\theta_t$, I followed the discrete approximation method for an AR(1) process given in Adda and Cooper (2003). This method also creates a $N_\theta \times N_\theta$ probability transition matrix, $P$, that gives the probabilities associated with moving from any given discrete value of $\theta_t$ to each of the discrete states in $\theta_{t+1}$.\(^{15}\) This probability transition matrix is then applied to form expectations of future MAC’s as described below.

The numeric model solves a policy function for the control variable, $q_t(B_t, \theta_t)$, that is consistent with the dynamic cost minimization condition given in (8). The policy function can be solved for through a backward recursion algorithm, starting at time period $T$ and working back to the beginning. $MAC_T$ can be determined using the terminal condition. Recall, because there is no continuation value of holding a bank of permits beyond $T$ and $B_{T+1} \geq 0$ then the bank should be pushed to as close to zero as possible in the last period. This process then determines $q_T(B_T, \theta_T) = \max(\min(\bar{q}_T + \theta_T, B_T + y_T), 0)$. Note given the discretized nature of $B_t$ and $\theta_t$ I can represent all $q_T(B_T, \theta_T)$ values in an $N_B \times N_\theta$ matrix, $q_T(B_T, \theta_T)$. Similarly, all $MAC_T(\theta_T, q_T(\theta_T, B_T))$ values can also be represented as an $N_B \times N_\theta$ matrix, $MAC_T(\theta_T, q_T(\theta_T, B_T))$. Given the probability transition matrix, $P$ is constant over the time span analyzed.

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\(^{14}\)The phrase “Business-as-usual” emissions may be somewhat misguided here. The EU currently has a cap-and-trade program (EU ETS) and is considering a change to that system, so one may consider BAU emissions as ones that keep on its current trajectory under the EU ETS. However, in this modeling framework, expected BAU emissions refer to emissions expected under no emissions restrictions.

\(^{15}\)I assume that the distribution of $\epsilon_t$ is constant over the time span analyzed and so $P$ is constant over the time span analyzed.
or negative emissions. These values are excluded in the program in the calculation of $q_{T-1}[MAC_T(\theta_T, q_T)] = MAC_T(\theta_T, q_T)P$. $E_{T-1}[MAC_T(\theta_T, q_T)]$ is also an $N_B \times N_\theta$ matrix, where the $(i, j)$ element is the expected MAC associated with being in the $i^{th}$ discrete bank state in period $T$ conditional on being in the $j^{th}$ discrete shock state in period $T - 1$. All of these matrices are stored before moving on to the next step.

Moving now to period $T - 1$, given a particular bank state value $B_{T-1}^i$ one can calculate the $q_{T-1}$ values associated with moving to each of the $N_B$ values in the vector $B_T$. Denote this vector of $q_{T-1}$ values as $q_{T-1}^i$. Given a particular shock state value $\theta_{T-1}^i$ and $q_{T-1}^i$ a vector of MAC’s can be calculated, denoted as $MAC_{T-1}^{ij}$. Given $\theta_{T-1}^i$, one also knows the expected MAC associated with being in each of the states in $B_T$, denoted as $EMAC_T^j$, where $EMAC_T^j$ is the $j^{th}$ column of $E_{T-1}[MAC_T(\theta_T, q_T)]$. Then, for the $B_{T-1}^i$ and $\theta_{T-1}^i$ values, one can solve for particular $q_{T-1}(B_{T-1}^i, \theta_{T-1}^i)$ among all values in $q_{T-1}^i$ as $q_{T-1}(B_{T-1}^i, \theta_{T-1}^i) = \min \left( |MAC_{T-1}^{ij} - \beta EMAC_T^j| \right)$. This process is repeated for all possible values of combinations of $(B_{T-1}^i, \theta_{T-1}^i)$ and therefore results in another $N_B \times N_\theta$ matrix, $q_{T-1}(B_{T-1}, \theta_{T-1})$. From $q_{T-1}(B_{T-1}, \theta_{T-1})$ one can also calculate $MAC_{T-1}(\theta_{T-1}, q_{T-1}(\theta_{T-1}, B_{T-1}))$ and $E_{T-2}[MAC_{T-1}(\theta_{T-1}, q_{T-1})]$.

This entire process is repeated for each time step, stepping back one period at a time, until the initial period is reached. The end result is $T$ policy function matrices $q_T(B_T, \theta_T)$, $q_{T-1}(B_{T-1}, \theta_{T-1})$, $\ldots$, $q_1(B_1, \theta_1)$. Given these policy function matrices I can run simulations from a given state starting point of $B_1$ and $\theta_1$. That is, given an initial $\theta_1$ I simulate a shock path $\theta_1, \theta_2, \ldots, \theta_T$ and given this shock path, along with $B_1$ and the policy function matrices, I can calculate time paths for emissions, bank levels, permit prices, and total

\[ \text{Note that some values in } q_{T-1}^i \text{ are not feasible in that they require emissions above BAU emissions or negative emissions. These values are excluded in the program in the calculation of } q_{T-1}(B_{T-1}^i, \theta_{T-1}^i) = \min \left( |MAC_{T-1}^{ij} - \beta EMAC_T^j| \right). \]
abatement costs. I simulate many, 15,000 to be exact, of these shock paths to form the basis of the simulation results presented below.

Adding the various policies alters the above-described algorithm in various ways. For the MSR, another state variable, $y_t$, must be tracked. More specifically, let us assume that of the $N_B$ bank states, $N_B^U$ are at or above $B^U$ and $N_B^L$ are at or below $B^L$. Then, given (10), the number of possible allocation states at time $t$ is $N_B^U + 2$ (one value for all $B^L < B_t < B^U$, one value for all $B_t \leq B^L$, and $N_B^U$ values for all $B_t \geq B^U$). Because the state is determined by the triplet $(B_t, \theta_t, y_t)$, there are $N_B \times N_\theta \times (N_B^U + 2)$ states possible at any time $t$. This alters the dimension of the policy function matrices $q_{T}^{MSR}(B_T, \theta_T, y_T)$, $q_{T-1}^{MSR}(B_{T-1}, \theta_{T-1}, y_{T-1})$, \ldots, $q_{1}^{MSR}(B_1, \theta_1, y_1)$ and one must consider the allocation state and the MSR allocation rule (10) when calculating the optimal emission level, but otherwise the algorithm described above works in essentially the same manner.

For the price collar mechanism, the algorithm is augmented by changing the calculation of the MAC associated with moving from a particular bank state at $t$ to a bank state at $t + 1$, given the shock state. More specifically, if the movement between bank states would cause the MAC to exceed the price ceiling in the standard cap-and-trade model, additional permits are added to the model to keep the MAC at the ceiling (up to the reserve limit $y_t^R$). Likewise, permits are removed if the MAC would fall below the price floor. I also collect the permit additions and permit withdrawals created by the price floor and ceiling in those modeling runs.

For the permit reduction policies, the change in the trading ratio or permit vacating rule is equivalent to changing the allocation path, so the algorithm described above can be used
as is, but under a different allocation path than those used for the standard cap-and-trade runs.

As noted above, the simulation model is parameterized with values relevant to the EU ETS. The modeling period is 2014 - 2050. Expected BAU emissions, $\hat{q}_t$, and the currently proposed permit allocation path, $y_t$, inclusive of the so-called “backloading” plan are given in the upper panel of Figure 1.\textsuperscript{17} The lower panel of this figure gives a plot of the MAC parameters $b_{1t}$ and $b_{2t}$. The remaining relevant parameters are given in Table 1.

4 Simulation Results

Under the base-case set of parameters I run several policy scenarios. First, I run the numeric model for the case with no additional stability mechanisms or allocation reduction policies. That is, a basic cap-and-trade scenario of the EU ETS is simulated with the regulations and allocations as they currently are given. This provides a reference case of prices, abatement costs, and emission levels, but is not intended to be directly compared to the policies discussed below as the “basic” policy has a different expected emission level and thus different environmental costs.

Next I run the model under the MSR, where the MSR has been designed to approximately align with policy guidelines set forth by the EC and as described above.\textsuperscript{18} Not surprisingly,

\textsuperscript{17}The “backloading” plan is scheduled to reduce the number of permits made available over the next several auctions in the EU ETS relative to what was originally scheduled and then to increase the quantity of permits by an equal amount in subsequent auctions.

\textsuperscript{18}The policy as designed here does not exactly match that set forth by the European Commission. In particular, the actual policy calls for an injection of permits drawn from a limited reserve. If the regulator does not have a sufficient number of permits in the reserve to supply the additional $y_L^r$ permits, it will supply whatever is left in the reserve. I model the reserve as limitless, thus the regulator can always inject the full $y_L^r$ permits when the bank drops below the lower limit.
applying this policy changes expected emissions over the period 2014 - 2050. The changing expected emissions make it difficult to compare this policy to other policies that may lead to other emission outcomes without further specifying an environmental damage function associated with emissions. Thus, to avoid the need to specify a damage function and introduce additional uncertainty, the remainder of the policy mechanisms modeled are designed such that these policies match the expected emissions of the MSR. These additional policies include two forms of a price collar mechanism - one where the initial width of the collar is 40€/mtCO₂ (i.e. \( p_{2021}^C - p_{2021}^F = 40 \)) and another where the initial width is 20€/mtCO₂.¹⁹

I assume that the price floors and ceilings for both collar types rise at the discount rate over time. Both collar policies are also modeled with a hard price floor, meaning the price permits generally never fall below \( p_t^F \), and a soft price ceiling where I assume the regulator will be willing to inject additional permits up to a limit of 10 percent of a given year’s stated allocation (i.e. \( y_t^R = 0.1 y_t \)).²⁰

The final two policies modeled are permit reduction policies. The first of these policies reduces all years’ allocations by a constant rate \( s \) in order to match the expected emissions from the MSR and the second reduction policy vacates the last several years’ allocations to hit the emissions target.²¹

A basic summary of the emissions and NPV of abatement costs from these policy simulations is given in Table 2. The “Policy” column identifies the policy modeled where “Basic” refers to the EU ETS policy as it currently is given (no additional market stability or permit

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¹⁹Just as with the MSR policy, I assume the collar policy would start in 2021.

²⁰Note that though I model this as a “hard” price floor if \( p_t^F > MAC(\theta_t, 0) \) then the permit price will clear below the price floor. Such an outcome is possible if a very low \( \theta_t \) value is realized.

²¹Note for the policy that vacates the final few years’ allocations, I also had to slightly adjust the allocation in the year prior to final years that receive no allocation in order to exactly match the expected emissions level of the MSR policy.
reduction policies), “MSR” refers to the MSR policy, “Collar - 40” refers to the price collar mechanism with a 40€/mtCO₂ initial width, “Collar - 20” refers to the price collar mechanism with 20€/mtCO₂ initial width, “Reduction - Const.” refers to the policy that reduces each year’s allocation by a constant rate, and “Reduction - End” refers to the policy that vacates the allocation for the last few years. The “Emissions” column gives the expected cumulative emissions of the policy (mean of the 15,000 simulation runs) over the 2014 - 2050 period. The bounds of the 95% confidence intervals (CI’s) of emissions are given in brackets below the mean value. The “NPV Costs” column gives the expected net present value of the abatement costs associated with each policy. Again, bounds of the 95% CI’s are given below in brackets. Note also, for the collar policies, the “NPV Costs” values are expected values associated with abatement costs only and do not reflect additional costs associated with permit purchases at the price ceiling.

This comparison of the scenarios shows clearly that the MSR under the assumed parameterization does lead to a reduction in the expected cumulative emissions relative to the basic cap-and-trade system. However, the variation in realized cumulative emissions increases considerably under the MSR relative to the basic case as noted by the increased CI’s in the MSR scenario. Indeed, cumulative emissions under the MSR, as well as in the price collar policies, did exceed cumulative emissions of the Basic case in several of the 15,000 simulations.

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22 I include 95% CI’s here instead of a variance measure because the distributions of the outputs are generally non-standard or at the very least non-symmetric about the mean values.

23 Note that the mean of the cumulative emissions for the “Basic” scenario can be less than the lower-bound of the 95% CI’s with uncertainty. It is possible that in the final year of the program the firm will have more permits banked than it can use if the shock in that year is very low. Thus, in this formulation, the firm over-complies or exactly complies with the stated allocation in the basic cap-and-trade scenario. This pulls the mean cumulative emissions below the cumulative allocation, but the realizations with cumulative emissions below the cumulative allocation occur in less than 2.5% of the 15,000 simulations. Hence, the mean is below the lower-bound of the 95% CI.
simulation runs. The expected abatement costs under the MSR also obviously rise as there is more abatement done on average. The variation in abatement costs is quite large for both the Basic and MSR policies.

In comparing the MSR to the other policies, it is found that both collar policies can achieve the same expected emissions at slightly lower abatement costs, with the Collar-20 having the lowest expected abatement costs. The MSR, as modeled, does have lower variation in realized emissions, however with the lower variation in emissions there is a corresponding higher variation in abatement costs. The permit reduction policies both have slightly higher expected abatement costs than the MSR with the “Reduction-End” policy leading to slightly lower expected abatement costs of the two reduction policies. The variation in costs for the reduction policies is much higher than that for the MSR or collar policies which is due to the lack of flexibility in the annual allocations.

As one may expect, the different policies also lead to considerably different price paths. Expected emission permit price paths (EUA prices in €/mtCO$_2$) of the six scenarios are plotted in Figure 2. The figure contains six panels, each plotting the given scenario’s expected EUA price as the solid line and the 95% CI’s, plotted as the dashed line, over the period 2014 - 2050.\textsuperscript{24} For the “Basic” case, the expected price path gradually increases over the entire period, as one would expect given the model description, ending at a price near 100€/mtCO$_2$. The prices for the Basic scenario are highly variable as can be seen from the width of the 95% CI’s. The MSR price plot shows a higher expected price path than the Basic scenario, as one would expect given there is more abatement on average under the MSR. The 95% CI’s of the expected price path are calculated by taking the mean price at each time period among the 15,000 simulations. The confidence intervals are calculated by taking the 2.5 and 97.5 percentile levels at each time period among the 15,000 simulations. Thus, the expected path and the confidence intervals do not correspond to any given simulations outcome, but are rather derived from the simulations.

\textsuperscript{24}The expected price path is calculated by taking the mean price at each time period among the 15,000 simulations. The confidence intervals are calculated by taking the 2.5 and 97.5 percentile levels at each time period among the 15,000 simulations. Thus, the expected path and the confidence intervals do not correspond to any given simulations outcome, but are rather derived from the simulations.
CI’s for the MSR case are also narrower than for the Basic case and are much more irregular, a feature due largely to the MSR design. For instance, the lower-bound of the 95% CI is decreasing until around 2020 and then begins increasing. This increase is due to the fact that the MSR policy begins in 2021 and for simulation runs where shocks are relatively low in the beginning periods banks will correspondingly be relatively high - high bank values trigger substantial permit allocation reductions under the MSR and subsequent price increases.

On the other hand, the upper bound of CI declines over 2040 - 2045. In these later years, the banks are drawn down and for simulation runs with high realized shock values (and correspondingly high prices) the bank minimum level, which is set at -1 GtCO$_2$ in all scenarios, is likely to be reached. Hitting the bank minimum restriction will cause expected prices to no longer grow at the discount rate (see equation (9)) and can make prices drop. In addition, low bank values also trigger permit injections under the MSR which can further drop the price. Prices eventually do rise again as the bank levels go from the bank minimum to zero over the last few years of the program.$^{25}$

For the price collar cases, the expected price path is very close to the lower-bound of the 95% CI, which for most time periods corresponds to the price floor level.$^{26}$ This outcome is necessary because prices from the simulations must often be at the price floor in order to reduce emissions enough to have the same expected cumulative emissions as the MSR policy.

Having the price floor high enough that it is binding in many periods also means that price

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$^{25}$When $B_t \neq B_{min}$ then $\lambda_t = 0$ and prices must be rising at the discount rate. Thus, prices rise in the last few periods because the firm is moving away from the maximum borrowing constraint.

$^{26}$The lower-bound of the confidence intervals does not always correspond to the price floor because it is possible to have a price lower than the price floor if $MAC(\theta_t, q_t = 0) < p^F_t$, an outcome that does happen given the parameterization used here. Similarly, the upper bound of the confidence intervals does not correspond to the price ceiling because I have modeled a “soft” price ceiling so prices may clear at levels above the ceiling if permit demand is high.
variation is considerably less in both collar scenarios relative to the other scenarios.

The reduction policies lead to expected price paths near that of the collar and MSR cases, but with much higher variation. The variation is particularly more severe on the low end of the spectrum, as both the “Reduction - Const.” and “Reduction - End” scenarios lead to lower-bound CI’s that are lower than that for the MSR case. This highlights a potential negative associated with fixed-allocation schemes, such as modeled in the Basic and permit reduction policies - if the system has no allocation adaptation mechanism built in then a series of negative shocks to BAU emissions can create situations with very low permit prices.

Finally, it should be noted that these price paths are plots of the average path. For any given simulation run, I find much less smooth permit price, bank, emission, and allocation paths. To highlight this, Figure 3 plots the permit price, bank, emission, allocation, and shock paths for a particular simulation run under the MSR policy. Note here jumps and drops in permit prices more in line with what we would expect from an actual market that is constantly updating abatement decisions based on shocks to the system are observed. Additionally, the difference in price paths across policies may also drive differences in investment in low-carbon technologies. This investment aspect is not modeled here, but has been studied in other policy comparison contexts (e.g. Phaneuf and Requate (2002); Zhao (2003)).

4.1 Sensitivity Analysis - Discount Rate Variations

The MSR triggers permit reductions and injections based on the size of the privately held bank. Banking is driven by the desire to temporally reallocate permits in order to remain on
the abatement cost-minimizing path which corresponds to the Hotelling permit price path. Therefore, the assumed discount rate plays a major role in determining the size of the bank and thus the role of an MSR in an emissions trading system. To highlight the importance of the discount rate, I model the MSR under various discount rates ranging from $r = 0.03$ to $0.15$.\footnote{A discount rate of 0.15 is quite high and is used primarily for expositional purposes. However, based on interviews, Schopp and Neuhoff (2013) note that carbon market speculators require rates of return near 15% to hold permits.}

Figure 4 plots the expected, privately-held bank under the MSR policy in the upper panel and the corresponding expected EUA price paths applying discount rates of 0.03, 0.07, 0.11, and 0.15.\footnote{By “privately-held” is meant to refer to the collective size of the bank held by the regulated firms or in the case of this modeling framework the bank held by the representative firm. Because the MSR can take permits out of the system, the regulator will also hold a bank equal to the net balance of permit withholding and permit injections.} As one would expect, higher discount rates lead to a lower expected bank held by the representative firm. The lower bank levels in the initial periods also mean that the initial abatement levels are lower with higher discount rates, leading to considerably lower allowance prices in the beginning periods as can be seen in the bottom panel of Figure 4.

Toward the end of the policy horizon, a few other features of the price paths are noticeable. First, near 2045, a flattening, and even a decrease for higher discount rates, of the expected EUA price path is seen. As noted above, this is due to the likelihood of the minimum bank constraint being binding near this period.\footnote{Note that while the expected plotted bank paths in Figure 4 do not hit the minimum bank level of -1 GtCO$_2$, there are many simulated paths that do have a binding minimum bank constraint which does affect the expected allowance price paths.} It can also be seen that the higher the discount rate, the higher the allowance prices in the final periods. This is as expected in the analysis of non-renewable resource problems under increasing discount rates, which is essentially how one can view a cap-and-trade policy. However, the difference in the price paths at the end of
the policy horizon across the discount rates is relatively small compared to the initial period
differences. This feature is due to the MSR design. The volume of withheld permits under
the MSR declines as the discount rate increases because bank levels are lower under high
discount rates. Also, the number of permits injected under the MSR increases as discount
rate increases because there are more periods when bank levels fall below $D^L$. As a result,
total expected emissions will increase as the assumed discount rate increases. This is shown
in the left panel of Figure 5, which plots expected cumulative emissions, the solid line, and
95% CI’s under the MSR policy for various discount rates.

Beyond the increasing discount rates leading to increased expected emissions under an
MSR, Figure 5 also shows that the variation in cumulative emissions is declining as the
assumed discount rate increases as noted by a narrowing of the 95% CI’s. This is as expected
because the withdrawal rates under the MSR are multiples of the bank size and thus with
generally lower banks being held with higher discount rates, bank variability decreases which
decrease permit withdrawal variability.

The impact of various discount rates is quite different for the other policies. For the permit
reduction policies, the issue is similar to what it would be in other standard non-renewable
resource cases - higher discount rates lead to lower initial abatement which means lower bank
levels and lower allowance prices, but greatly increase abatement levels and allowance prices
toward the end of the horizon and do not impact cumulative emission levels.

Varying the discount rate can have more involved effects under a collar policy. More
specifically, with higher discount rates, firms again have an incentive to reduce abatement
levels in the near term and increase them in later periods. However, with a price floor,
reducing near-term abatement increases the likelihood that the price floor will be binding.
Likewise, the increased abatement in the later periods increases the probability that permits will be injected to defend the price-ceiling. When the price collar is modeled as it is here, with a hard price floor and a soft price ceiling, the increasing discount rate *increases* the environmental stringency of the policy (i.e. lowers expected cumulative emissions) and avoids price collapses. This is displayed plainly in the right panel of Figure 5. However, this is not a general rule of price collars. It is possible that cumulative emissions could remain relatively flat or even increase if the system has a “symmetric collar” (i.e., equal support for the price ceiling and price floor) or a soft price floor and hard price ceiling. But, regardless of the impacts on cumulative emissions, it is clear that the collar mechanism at least partially defends against extremely low prices a system might experience with firms discounting at higher rates.

It should also be noted that this discussion of discount rates can also be framed in the sense of program uncertainty. More specifically, if firms assign some positive probability to the possibility that the cap-and-trade program will be discontinued in the next period, that programmatic uncertainty could be equivalently modeled in this framework as a problem with a higher discount rate. It is also noted that in Figure 5, the environmental stringency decreases as the discount rate increases. However, this is a general rule of price collars. It is possible that cumulative emissions could remain relatively flat or even increase if the system has a “symmetric collar” (i.e., equal support for the price ceiling and price floor) or a soft price floor and hard price ceiling. But, regardless of the impacts on cumulative emissions, it is clear that the collar mechanism at least partially defends against extremely low prices a system might experience with firms discounting at higher rates.

30 To be more concrete, assume that in each period the representative firm assigns a probability of \( \rho \) that the program will end. The cost minimization in (3) can be rewritten as 
\[
\min_{q_t, y_t} \sum_{t=0}^{T} \rho \beta^t E_t [C_{it}(\bar{q}_{it} + \theta_t - q_{it}) + p_t x_{it}]
\]
or, equivalently, as 
\[
\min_{q_t, y_t} \sum_{t=0}^{T} \beta^2 E_t [C_{it}(\bar{q}_{it} + \theta_t - q_{it}) + p_t x_{it}],
\]
with \( \rho \beta = \beta_2 \leq \beta \). The programmatic uncertainty thus has the same impact as lowering the discount factor or, equivalently, as increasing the discount rate.
uncertainty and price-collapses are avoided.

4.2 Sensitivity Analysis - MSR Parameterization

Because it appears that the MSR is a likely mechanism that the European Commission may implement to help support EUA prices, we also explored various parameterizations of the MSR design. A summary of a subset of this sensitivity analysis is given in Table 3. In this table, the expected emissions and NPV of abatement costs, with 95% CI bounds in brackets below, are shown for four different alternative MSR parameterizations and under two different discount rates, \( r = 0.03 \) and \( r = 0.07 \). The alternative parameterizations that are explored are a higher upper-bound bank triggering level (moving from \( B^{U} = 0.833 \) to \( B^{U} = 2.833 \)), a higher permit injection value (moving from \( y^{L} = 0.1 \) to \( y^{L} = 0.3 \)), a lower withdrawal rate (moving from \( \gamma = 0.12 \) to \( \gamma = 0.02 \)), and finally a parameterization that has both a higher injection rate (\( y^{L} = 0.6 \)) and a lower withdrawal rate (\( \gamma = 0.02 \)).

The general effects of these alternative parameterizations are as expected - expected cumulative emissions increase and expected abatement costs decrease as \( B^{U} \) increases, \( y^{L} \) increases, and/or \( \gamma \) decreases. While all parameterization changes have the expected impacts, the key parameter in terms of greatly altering outcomes is the withdrawal rate \( \gamma \). This is not unexpected given the numbers used here to simulate the EU ETS market. Bank levels are expected to be relatively high, well above \( B^{U} \), for many years, thus altering the withdrawal rate will have a large impact on the total allocation of permits over the 2014 - 2050 modeling horizon. On the other hand, because banks are relatively high under the values assumed here and because the draw-down in banks to levels below \( B^{L} \) occurs only at the last few periods
in expectation, large changes in $B^U$ and $y^L$ have relatively small impacts on allocations and thus on emissions and abatement costs.

Comparing the results across discount rates, one can see the general effects of the alternative parameterizations are the same. However, magnitudes of the effects are quite different across the discount rate comparisons. For example, moving from $\gamma = 0.12$ to $\gamma = 0.02$ increases expected cumulative emissions under the $r = 0.03$ scenario by about 25%, but for the case of $r = 0.07$ emissions jump by only about 14%. The expected cumulative emissions do not expand by as much when $\gamma$ falls for higher discount rate cases because bank levels are lower in general with higher discount rates, so reducing $\gamma$ has less of an impact on the reduction in allocation.

Plots of the expected EUA price paths and bank paths, along with 95% CI’s (dashed lines), for the alternative parameterizations are given in Figures 6 and 7, respectively. The price paths are essentially unchanged except for the alternatives that have lower withdrawal rates. For example, under the case of $r = 0.03$, $y^L = 0.6$ and $\gamma = 0.02$, cumulative emissions are near that of the “Basic” case and likewise the expected price path and 95% CI’s for that MSR are approximately the same as the Basic case. This implies MSR policies that inject about as many permits as they take out of the system do not have noticeable impacts on prices relative to a policy without an MSR.

The bank plots also are quite similar for the alternatives, except for those with lower $\gamma$ values. Indeed, the shapes of the expected bank paths change quite dramatically as withdrawal rates decline. Also, as noted above, one can see that bank sizes for the higher discount rate cases are lower in all settings relative to lower discount rate cases.

Finally, because uncertainty is the motivator for policy intervention, it is reasonable to
consider how sensitive the results are to assumptions about the variance of the shocks to BAU emissions. In Table 4, I present policy comparisons for the case where the variance of the shock to BAU emissions is halved relative to the baseline scenario, $\sigma^2 = 0.025$. With this lower variance level, expected costs of all of the emissions-reducing policies are lower relative to the higher variance baseline results. This is as expected given the form of the MAC function. The difference between the expected costs of the two forms of collar policies is also narrower in this setting which is expected given the collars are less likely to be binding in either form with lower shock variances. Overall, however, the general pattern of emissions and costs under the lower variance scenario follows that of the baseline results.

The simulation analyses overall provide several key insights. First, the MSR as modeled is effective at reducing permit allocations, increasing abatement levels, and, consequently, increasing permit prices relative to the emissions trading program as-is. However, the results also show that a price collar could similarly lead to permit reductions and permit price increases, albeit at lower expected abatement costs than the MSR. A second insight is that the MSR effectiveness is also quite sensitive to the discount rate applied. Specifically, higher assumed discount rates undermine the MSR’s ability to remove permits from the system and thus leads to overall lower permit prices. Lastly, the results highlight the sensitivity of the environmental, cost, and permit price to MSR design features, noting importantly that MSR systems designed to, over time, roughly balance the number of permits removed and injected does very little to increase permit prices and reduce price variability.

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31 It should be noted that at this variance level the average $\theta_t/\bar{q}_t$ ratio is about 0.11, which is approximately the same average $\theta_t/\bar{q}_t$ value used in similarly formed models of Fell et al. (2012b) and Fell et al. (2012a).
5 Conclusions

Stability in cap-and-trade systems has been and continues to be an area of concern for policy makers. In particular, the perceived problem of over-allocation has appeared to be more problematic for existing cap-and-trade programs than extreme high-cost outcomes. To that end, I have analyzed various policies that can ostensibly reduce permit price volatility and/or reduce the number of permits allocated at least in the relative near-term.

The results suggest that adaptive allocation policies, such as an MSR or permit price collar, can reduce permit over-allocations and stabilize prices in a manner that is more cost effective than policies that simply reduce permit allocations outright. However, these adaptive allocation policies are not without their issues. For example, I find that MSR becomes less effective at reducing over-allocations and reducing permit price volatility as the discount rate increases or equivalently as uncertainty about the existence of the program increases. This is due to the fact that increasing discount rates lowers bank levels and with lower bank levels the MSR takes fewer permits out of circulation. The higher discount rates also increase the number of periods, on average, that the bank is below the lower limit that triggers permit injections in the MSR and thus more permits are added to the system. Combined, these results suggest that the effectiveness, in terms of reducing over-allocation and permit price volatility, of the MSR could be increased if the regulators also made efforts to decrease uncertainty about the longevity of the program.

On the other hand, price collar mechanisms with hard price floors and soft price collars, as modeled here, actually become more environmentally stringent as the assumed discount rate increases. This occurs because higher discount rate cases lower near-term permit prices,
making the price floor binding more frequently which effectively reduces allocations. The permit price floor also protects against price crashes, an apparent increasing concern of policy makers.

Beyond the discount rate effects, there are other design parameters of price collars and MSR policies that also affect their effectiveness. Clearly, the levels of the price collars, as well as the number of permits regulators are allowed to reject or withhold to support the collars, will greatly affect the performance of the collars. For the MSR, the withdrawal rate, $\gamma$, is a very key parameter, particularly given that the EU ETS is currently in a state where they have a relatively large volume of banked permits. I find that if the withdrawal rate is set low, while simultaneously setting a high level of permit injections when bank levels are sufficiently low, such that expected cumulative emissions under the MSR are relatively the same as what would be with a regular cap-and-trade, then the MSR does relatively little to prop-up prices or reduce permit price volatility.

Overall, recommendations for the EC depend largely on what the goal of the intervention is for the EC. If the goal is to truly stabilize prices and remove some of the perceived over-allocation, results presented here suggest that a price collar, and in particular a price-floor could achieve these goals at the lowest expected abatement costs. If this type of hybrid mechanism is politically infeasible, then it appears that an MSR is better than simply reducing the permit allocations outright. Perhaps most importantly, the mere discussion of an intervention by the EC may signal to regulated entities that the EC is committed to the EU ETS and therefore regulatory uncertainty is reduced. This reduction in uncertainty works in much the same way as a reduction in the discount rate, which alone can increase permit prices in the near-term and increase the effectiveness of an MSR.
References


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<tbody>
<tr>
<td>$\rho$</td>
<td>0.9</td>
<td>$D^U$</td>
<td>0.833</td>
</tr>
<tr>
<td>$\sigma^2$</td>
<td>0.05</td>
<td>$\gamma$</td>
<td>0.12</td>
</tr>
<tr>
<td>$N_\theta$</td>
<td>101</td>
<td>$D^L$</td>
<td>0.4</td>
</tr>
<tr>
<td>$B_{min}$</td>
<td>-1</td>
<td>$y^L$</td>
<td>0.1</td>
</tr>
<tr>
<td>$N_B$</td>
<td>501</td>
<td>$y^R_t$</td>
<td>$0.1y_t$</td>
</tr>
<tr>
<td>$r$</td>
<td>0.03</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: $B_{min}$, $D^U$, $D^L$, $y^L$, and $y^R_t$ are in billions of permits.
Table 2: Scenario Comparison

<table>
<thead>
<tr>
<th>Policy</th>
<th>Emissions</th>
<th>NPV Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>58.89</td>
<td>478.38</td>
</tr>
<tr>
<td></td>
<td>[58.95, 58.95]</td>
<td>[79.51, 1387.55]</td>
</tr>
<tr>
<td>MSR</td>
<td>44.22</td>
<td>998.11</td>
</tr>
<tr>
<td></td>
<td>[32.44, 54.61]</td>
<td>[535.80, 1884.87]</td>
</tr>
<tr>
<td>Collar-40</td>
<td>44.22</td>
<td>936.09</td>
</tr>
<tr>
<td></td>
<td>[24.25, 58.45]</td>
<td>[846.99, 1402.69]</td>
</tr>
<tr>
<td>Collar-20</td>
<td>44.22</td>
<td>934.83</td>
</tr>
<tr>
<td></td>
<td>[24.15, 59.04]</td>
<td>[851.10, 1352.64]</td>
</tr>
<tr>
<td>Reduction - Const.</td>
<td>44.22</td>
<td>1096.06</td>
</tr>
<tr>
<td></td>
<td>[44.22, 44.22]</td>
<td>[296.97, 2566.18]</td>
</tr>
<tr>
<td>Reduction - End</td>
<td>44.22</td>
<td>1084.90</td>
</tr>
<tr>
<td></td>
<td>[44.22, 44.22]</td>
<td>[292.31, 2667.35]</td>
</tr>
</tbody>
</table>

Notes: Emissions are in billions of mtCO$_2$ and NPV Costs are in billions of €.

Table 3: MSR Sensitivity Analysis

<table>
<thead>
<tr>
<th>Policy</th>
<th>Emissions</th>
<th>NPV Costs</th>
<th>Emissions</th>
<th>NPV Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r = 0.03$</td>
<td>$r = 0.07$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B^U = 2.833$</td>
<td>45.89</td>
<td>919.41</td>
<td>51.27</td>
<td>283.17</td>
</tr>
<tr>
<td></td>
<td>[34.17, 56.07]</td>
<td>[473.95, 1795.87]</td>
<td>[39.76, 59.84]</td>
<td>[122.24, 646.29]</td>
</tr>
<tr>
<td>$y^L = 0.3$</td>
<td>46.15</td>
<td>886.11</td>
<td>52.78</td>
<td>253.02</td>
</tr>
<tr>
<td></td>
<td>[32.80, 58.52]</td>
<td>[511.94, 1555.86]</td>
<td>[39.17, 63.54]</td>
<td>[130.18, 550.49]</td>
</tr>
<tr>
<td>$\gamma = 0.02$</td>
<td>55.18</td>
<td>591.01</td>
<td>56.58</td>
<td>213.84</td>
</tr>
<tr>
<td></td>
<td>[53.31, 58.72]</td>
<td>[136.07, 1433.80]</td>
<td>[53.65, 60.22]</td>
<td>[46.52, 572.96]</td>
</tr>
<tr>
<td>$y^L = 0.6$, $\gamma = 0.02$</td>
<td>56.48</td>
<td>518.87</td>
<td>60.02</td>
<td>155.40</td>
</tr>
<tr>
<td></td>
<td>[53.32, 65.44]</td>
<td>[133.65, 1018.56]</td>
<td>[53.69, 70.79]</td>
<td>[43.95, 346.08]</td>
</tr>
</tbody>
</table>

Notes: Emissions are in billions of mtCO$_2$ and NPV Costs are in billions of €.
Table 4: Low Variance Sensitivity

<table>
<thead>
<tr>
<th>Policy</th>
<th>Emission</th>
<th>NPV Costs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic</td>
<td>58.95</td>
<td>419.77</td>
</tr>
<tr>
<td></td>
<td>[58.95, 58.95]</td>
<td>[115.17, 972.26]</td>
</tr>
<tr>
<td>MSR</td>
<td>45.10</td>
<td>910.6</td>
</tr>
<tr>
<td></td>
<td>[35.72, 53.18]</td>
<td>[610.23, 1424.65]</td>
</tr>
<tr>
<td>Collar-40</td>
<td>45.10</td>
<td>883.97</td>
</tr>
<tr>
<td></td>
<td>[30.33, 57.19]</td>
<td>[860.03, 1082.47]</td>
</tr>
<tr>
<td>Collar-20</td>
<td>45.10</td>
<td>883.65</td>
</tr>
<tr>
<td></td>
<td>[33.34, 57.15]</td>
<td>[859.63, 1077.06]</td>
</tr>
<tr>
<td>Reduction - Const.</td>
<td>45.10</td>
<td>960.52</td>
</tr>
<tr>
<td></td>
<td>[45.10, 45.10]</td>
<td>[372.86, 1903.00]</td>
</tr>
<tr>
<td>Reduction - End</td>
<td>45.10</td>
<td>957.49</td>
</tr>
<tr>
<td></td>
<td>[45.10, 45.10]</td>
<td>[376.94, 1867.56]</td>
</tr>
</tbody>
</table>

*Notes: All results are based on $\sigma^2 = 0.025$. Emissions are in billions of mtCO$_2$ and NPV Costs are in billions of €.*

Figures

Figure 1: Emissions and Parameters
Figure 2: Emission Permit Price Paths

Notes: All EUA Prices are in €/mtCO₂.

Figure 3: Outputs from an Individual Run with MSR
Figure 4: Private Bank and Emission Price Paths under MSR Policy

Notes: Private Bank values are in billions of permits and EUA Prices are in €/mtCO₂.
Figure 5: Cumulative Emissions and Discount Rates

Notes: Cumulative Emissions are in billions of mtCO₂.
Figure 6: EUA Price Path for MSR Design Sensitivity

Notes: EUA Prices are in €/mtCO₂. $B^U$ and $y^L$ are in billions of permits.
Figure 7: Banking Path for MSR Design Sensitivity

Notes: Bank, $B^{U}$ and $y^{L}$ are in billions of permits.