The Real Exchange Rate in Open-Economy Taylor Rules: A Re-Assessment

February 20\textsuperscript{th}, 2016

Richard T. Froyen
Department of Economics
University of North Carolina
Chapel Hill, NC 27599-3305
USA

Alfred V. Guender
Department of Economics and Finance
University of Canterbury
Christchurch
New Zealand

Key Words: CPI, Domestic, REX Inflation Targeting, Taylor-Type Rules, Timeless Perspective, Real Exchange Rate.

JEL Code: E3, E5, F3

Abstract:

This paper re-examines the merits of including an exchange rate response in Taylor-type interest rate rules for small open economies. Taylor (2001) and Taylor and Williams (2011) express what has been the conventional view: inclusion of the real exchange rate will either add little or might negatively affect the rule’s performance. We argue that developments in the theory of optimal monetary policy for open economies taken together with increased instability in world financial markets warrant a re-examination of the issue. Examining three flexible inflation targeting strategies, we find that a small weight on real exchange rate stability in the loss function is sufficient to improve the performance of Taylor-type rules relative to optimal policy. Gains are substantial for domestic and REX inflation targets because a small weight on real exchange rate fluctuations inhibits the aggressive use of the policy instrument under optimal policy. As real exchange rate stability is a built-in feature of a CPI inflation objective, the gains under a CPI inflation target are considerably lower. A central bank that values real exchange rate stability and follows a Taylor-type rule should respond to the real exchange rate. Doing so reduces relative losses irrespective of the specification of the inflation objective. Only a complete disregard for exchange rate stability bears out the view that there is no substantive role for the real exchange rate in Taylor-type rules.

E-mail: froyen@email.unc.edu; Alfred.Guender@canterbury.ac.nz
Simple interest rate rules, often termed Taylor rules, have been used frequently to describe the implementation of monetary policy in closed economies. Before the onset of the Global Financial Crisis, central banks were typically viewed as responding to inflationary pressure and an overheating economy. The vagaries of financial markets since the crisis have, however, heightened the concern for financial stability. Greater awareness of the dangers of distorted asset prices and financial imbalances has produced more recent specifications of Taylor-type rules that include asset prices, interest rate spreads or credit aggregates. Their inclusion is meant to make monetary policy respond to signs of financial frictions.¹

A discussion of Taylor-type rules for small open economies raises the question of whether monetary policy should respond to the (real or nominal) exchange rate. In the past, the case for including an exchange rate argument in simple interest rate rules for monetary policy has been weak. With the exception of economies characterized by financial fragility or dominated by the foreign sector (e.g. Singapore), most of the literature supported the view of Taylor (2001, p.266) and Taylor and Williams (2011) that either there are small performance improvements from reacting to the exchange rate or that such reactions can make performance worse.² Two sets of developments within the past decade suggest the need to revisit this issue.

The first is the emergence of a new generation of open-economy macroeconomic models in which the real exchange rate plays a more fundamental role. The earlier generation of models that Taylor and Williams (2011) surveyed, for example, the models in Gali and Monacelli (2005) and Clarida, Gali and Gertler (1999, 2001, 2002) had the implication that optimal monetary policy in the open economy was isomorphic to policy in the closed economy. This suggested that the arguments in the Taylor rule for an open economy might not need to be extended beyond domestic inflation and the output gap.

¹ In fact, an extensive debate about the response of monetary policy to development in asset markets began at the turn of the millennium. Cecchetti et al (2000, 2002) were in favour of rules that respond to financial asset prices while Bernanke and Gertler (1999, 2001) were opposed. For a recent overview of the debate see Kaefer (2014).
² Empirical evidence cited by Taylor and Williams (2011) came from 1990s open economy econometric models. Their view also receives support from studies within New Keynesian models. Examples are Batini et al. (2003 and Leitemo and Söderström (2005). Garcia et. al. (2011) find that inclusion of the level of the real exchange rate in a Taylor-type rule increases the variability of inflation and the output gap. They do, however, find that smoothing the real exchange rate helps reduce financial volatility without adding to inflation or output variability.
The isomorphism is a distinctive feature of models where the Phillips curve has no direct real exchange rate channel. The real exchange rate affects domestic inflation only *indirectly* through its effect on the output gap. A later generation of models extends the role of the real exchange rate. Imported inputs (Monacelli (2013)), incomplete exchange rate pass-through (Monacelli (2005)), and concern about foreign competitiveness as a factor in firm pricing (Froyen and Guender (2015)) all suggest direct exchange rate effects in the Phillips curve. In models with a distinct real exchange rate channel in the Phillips curve the optimal target rule for monetary policy often contains the real exchange rate. Froyen and Guender demonstrate that whether or not the real exchange rate appears in the target rule depends on the definition of the inflation objective.

The recent literature also suggests that openness requires changes in the central bank’s objective function. Studies reach different conclusions on the appropriate inflation target for a small open economy. Allsopp, Kara and Nelson (2006) favor CPI inflation targeting. Kirsanova, Leith, and Wren-Lewis (2006), DePaoli (2009) and others support the earlier studies cited above in advocating a domestic rather than a CPI inflation objective. Froyen and Guender (2015), following Ball (1999), consider a real-exchange-rate-adjusted (REX) inflation target similar to core inflation measures employed by some central banks.

In a recent paper Monacelli (2013) asks whether monetary policy in an open economy is fundamentally different from that of a closed economy; for a variety of reasons in the newer generation of open economy models the answer is yes. This suggests the need to re-examine open-economy Taylor rules. Should they include the real exchange rate? Which inflation objective should be chosen? Are the answers to these questions inter-related?

The second set of developments concern changes in global financial markets following the financial crisis of 2007-2009. The policies of the Federal Reserve and other major central banks, consisting of large asset purchases and near zero policy interest rates, led to massive flows of capital to smaller open economies. Countries as diverse as Turkey, Brazil, South Africa, Malaysia, Mexico and Indonesia saw a substantial portion of their local currency government bonds purchased by foreign investors. Emerging markets experienced capital inflows to stocks and bonds averaging over $20 billion per month over 2010-2014. In many of these countries the result was a sharp appreciation of their currency. This led Guido Mantega, the finance minister
of Brazil, in 2010 to charge the Federal Reserve with waging a “currency war.” These open economies are now left vulnerable to sudden stops and falling currency values with capital flowing back to the United States as the Federal Reserve pursues “lift-off”, returning U.S. interest rates to more normal levels. Raghuram Rajan, Governor of the Indian central bank, has complained about the increased market volatility caused by the sharp swings in capital flows.\(^3\)

Because of the increased turbulence in world financial markets, central banks in emerging markets may find it necessary to add exchange rate stability to the list of policy goals. Support for this comes from inside international supervisory agencies. Olivier Blanchard et. al. (2010) argues that “[c]entral banks in small open economies should openly recognize that exchange rate stability is part of their objective function.” One would think that expanding policy goals beyond the dual mandate that underlies the standard Taylor rule suggests expanding the rule itself to include a real exchange rate objective.

The Global Financial crisis has sparked an extensive debate in the academic literature about the suitability of the New Keynesian framework for analyzing the connection between macroeconomic and financial stability.\(^4\) Woodford (2012) argues that the flexible inflation targeting framework is sound but needs to be expanded to take account of the threats posed by financial instability. The central bank’s objective function should include a term that captures financial distortions created by constraints on the behavior of financial intermediaries, households or firms. Vredin (2015) reviews the links between monetary policy and financial

---

\(^3\) Concerns about the potential disruptive effect of a change in US monetary policy in the foreign exchange and international capital markets have been expressed by the World Bank (2015), the IMF (Lagarde (2015)), and individual BRICS countries. Not only emerging or developing countries are worried about exchange rate instability. Concerns about an unjustifiably high exchange rate and the consequences of a free fall of the exchange rate should the Federal Reserve tighten its policy stance were raised by the Reserve Bank of New Zealand in 2014. In a sequence of statements released to financial markets over the July-September 2014 period, the Reserve Bank was openly critical of ongoing developments in the foreign exchange market. The concern about the Kiwi Dollar having become unjustifiably high and unsustainable culminated in the publication of a statement by the Governor where he warned of the dire consequences of an out-of-step exchange rate for the domestic economy. During the Global Financial Crisis in 2009 the Swiss National Bank took steps to stabilize the Swiss Franc vis-à-vis the Euro in a deflationary environment. It did so again in the wake of the Sovereign Debt Crisis in 2011 but decided to lift the floor on the exchange rate in January 2015.

\(^4\) See Leeper and Nason (2015) for an overview of the challenges faced by central bankers and academics in reconciling the achievement of financial stability with macroeconomic objectives in the current modelling framework. Smets (2014) reviews the conceptual frameworks of three different views on the objectives of central banks. Eichengreen et. al. (2011) provide an earlier assessment of necessary reforms of the macroeconomic policy framework.
stability and concludes that monetary policy rules should include indicators of financial instability because they are welfare-improving. The issues surrounding the inclusion of a financial stability measure in either the central bank loss function or in a Taylor rule overlap those concerning inclusion of the real exchange rate in the corresponding specifications for a small open economy. For such an economy, shocks to real exchange rates and foreign interest rates are major sources of financial instability.

This paper evaluates Taylor-type rules in an open economy framework where the central bank views exchange rate stability as an added but secondary objective. We consider three measures of inflation that can serve as the central bank’s inflation objective: domestic, CPI, and real-exchange-rate adjusted (REX) inflation. For each inflation measure we design a Taylor-type rule and compare its stabilizing properties to optimal policy from a timeless perspective which serves as a yardstick for measuring performance. The paper addresses several specific issues concerning the role of the real exchange rate in the various incarnations of a Taylor-type rule.

- Does a concern for exchange rate stability albeit small warrant a direct response to the real exchange rate in the Taylor rule?
- Do the gains or losses from inclusion of the real exchange rate in a simple Taylor rule depend on the central bank’s inflation objective?
- What is a proper weight on the real exchange rate in a simple Taylor rule?
- Under what conditions does a Taylor-type rule approximate optimal policy outcomes?

Our most important findings can be briefly summarized here. Even a small weight on real exchange rate stability is sufficient to affect materially the performance of Taylor-type rules relative to optimal policy. Gains are substantial particularly for domestic and REX inflation targets because even a small weight on real exchange rate fluctuations in the loss function inhibits the aggressive use of the policy instrument under optimal policy. As real exchange rate stability is a built-in feature of a CPI inflation objective, the gains under a CPI inflation target are considerably lower. A central bank that values real exchange rate stability to a degree and follows a Taylor-type rule should respond to the real exchange rate. Doing so reduces relative losses further irrespective of the specification of the inflation objective. Relative losses decrease still further if the central bank can optimize over the coefficients in the Taylor rule. Indeed,
relative losses hover around the 10 percent mark for all three inflation objectives. Only a complete disregard for exchange rate stability as an ultimate policy goal bears out the conventional view that there is no substantive role for the real exchange rate in Taylor-type rules.

The remainder of the paper is structured as follows. The next section introduces a New Keynesian model of a small open economy that serves as our frame of reference. This section also offers a brief analysis of flexible domestic, CPI, and REX inflation targeting under optimal policy from a timeless perspective. Section 3 compares the performance of various Taylor-type rules vis-à-vis optimal policy in two different scenarios, one where the central bank values exchange rate stability and the other where it does not. Section 4 discusses optimized Taylor rules and how they compare to optimal policy. A conclusion is offered in Section 5.

2. Inflation Objectives in a Small Open Economy

This section lays out a stylized model of a small open economy and discusses three alternatives choices for the inflation objective under flexible inflation targeting.

2.A. A Small Open Economy Model

The model consists of four equations which are briefly described below.\(^5\)

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + b(q_t - q_{t-1}) - \beta b (E_t q_{t+1} - q_t) + u_t
\]

\[
y_t = E_t y_{t+1} - a_1 (R_t - E_t \pi_{t+1}^{CPI}) + a_2 (q_t - E_t q_{t+1}) + a_3 (y_{t+1} - E_t y_{t+1}^f) + v_t
\]

\[
R_t - E_t \pi_{t+1} = R_{t+1}^f - E_t \pi_{t+1}^{CPI} + E_t q_{t+1} - q_t + \varepsilon_t
\]

\[
\pi_t^{CPI} = \pi_t + \gamma \Delta q_t
\]

where

\(\pi_t\) = the rate of domestic inflation

\(E_t \pi_{t+1}^{CPI}\) = the expected rate of CPI inflation

\(q_t\) = the real exchange rate

\(y_t\) = the output gap

\(R_t\) = the nominal rate of interest (policy instrument)

\(^5\) The derivation of the Phillips curve is explained in the appendix.
\[ R_t^f = \text{the foreign nominal rate of interest} \]
\[ E_t \pi_t^f = \text{the expected foreign rate of inflation} \]
\[ y_t^f = \text{the foreign output gap} \]

Lower case variables represent logarithms. All parameters are positive. The discount factor \( \beta \) is less than or equal to one.

Equation (1) is an open-economy Phillips curve that features a real exchange rate channel in addition to the standard output gap channel. Equation (2) is an open-economy IS relation with a real interest rate and a real exchange rate channel. A foreign output shock and an idiosyncratic domestic shock also affect the demand for domestic output.\(^6\) Equation (3) is the linearized uncovered interest rate parity (UIP) condition: apart from a stochastic risk premium \((\varepsilon_t)\) agents are assumed to trade in a frictionless international bond market. More formally, the stochastic disturbances are modeled as follows:\(^7\)

\[ u_t \sim N(0, \sigma_u^2) \quad v_t \sim N(0, \sigma_v^2) \quad \varepsilon_t \sim N(0, \sigma_{\varepsilon}^2) \]
\[ R_t^f \sim N(0, \sigma_{R_t^f}^2) \quad \pi_t^f \sim N(0, \sigma_{\pi_t^f}^2) \quad y_t^f \sim N(0, \sigma_{y_t^f}^2) \]

All foreign variables are exogenous independent random variables. Finally, equation (4) describes the relationship between the CPI inflation rate, the domestic inflation rate, the real exchange rate, and consumption openness \((\gamma)\) under perfect exchange rate pass-through.

B. The Choice of a Flexible Inflation Target by an Optimizing Central Bank

In the following subsections we consider three different flexible inflation targeting regimes. Each regime is associated with a particular definition of inflation. Domestic inflation, exchange-rate adjusted inflation or CPI inflation can serve as the inflation objective.

At the outset it is helpful to define a few simple concepts. A targeting regime is characterized by the objective function of the central bank. A flexible inflation targeting regime is one where the central bank is concerned about variables other than just the rate of inflation. An optimizing central bank practicing flexible inflation targeting minimizes its objective function

\(^6\) The derivation of the forward-looking IS relation from microeconomic foundations is explained in Guender (2006). A separate appendix, available from the authors, shows how the shocks that appear in the IS relation can be motivated.

\(^7\) The property that all shocks are white noise follows Woodford (1999). Its purpose is to show that gradual adjustment of the output gap, the rate of inflation, etc. and the policy instrument under policy from a timeless perspective is not exclusively tied to the presence of autocorrelated disturbances in the model.
subject to the constraint imposed by the model economy. Woodford’s (1999) policy from a
timeless perspective is the form of commitment the central bank adheres to.

A central bank with a dual mandate in practice is modelled in the academic literature as a
central bank engaging in flexible inflation targeting. Such a regime is typically specified in terms
of an objective function that consists of the squared deviations of the output gap and the
particular rate of inflation the central bank targets. As mentioned in the introduction, however,
recent experience and ongoing developments have cast some doubt on whether this convention
captures all objectives of a central bank in a small open economy. To articulate this point, we
view the mandate of a central bank as extending beyond a narrow focus on inflation and output
stability to include stability of the real exchange rate.

B.1. Targeting Domestic Inflation

In the first strategy we consider, the rate of inflation is defined in terms of changes in the
level of domestic prices. The explicit objective function that the central bank attempts to
minimize is given by:

\[ E_t \sum_{i=0}^{\infty} \beta^i [y_{t+i}^2 + \mu \pi_{t+i}^2 + \delta q_{t+i}^2] \]

(5)

\( \beta \) is the discount rate and \( \mu \) represents the relative weight the policymaker attaches to the
squared deviations of the rate of domestic inflation from target. In a similar vein, \( \delta \) is the relative
weight accorded to the squared deviations of the real exchange rate. 9

To reduce the dimension of the central bank’s domestic inflation targeting strategy to one
involving three choice variables, we need to take two additional steps. First, substitute for the
rate of CPI inflation in equation (2). Second, substitute the UIP condition into the IS equation.
The optimization problem can then be expressed as:

---

derive a utility-based welfare measure that includes squared deviations of domestic inflation and the output gap as
a special case. Using the same set-up as Gali and Monacelli (2005), Kirsanova et al (2006) show that relaxing
parameter restrictions leads to a more complex welfare criterion that includes the terms of trade. De Paoli (2009)
introduces a terms of trade externality to motivate the inclusion of the real exchange rate in the objective
function.

9 The target for the output gap, the rate of inflation, and the real exchange rate is zero, respectively.
\[
\min_{\pi_t, y_t, q_t} \sum_{i=0}^{\infty} \beta^i [y_{t+i}^2 + \mu \pi_{t+i}^2 + \delta q_{t+i}^2]
\]
\[\text{s. t}\]
\[\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + b(q_t - q_{t-1}) - \beta b(E_t q_{t+1} - q_t) + u_t \quad (6)\]
and
\[y_t = E_t y_{t+1} - a_1(R_t^f - E_t \pi_{t+1} + \epsilon_t) + (a_1(y - 1) - a_2)(E_t q_{t+1} - q_t) + a_3(y_{t}^f - E_t y_{t+1}^f) + v_t \]

Combining the first-order conditions yields the endogenous target rule. Under policy from a timeless perspective, the target rule is complex. The model is therefore solved numerically.

**B.2. Targeting CPI Inflation**

If the focus of policy rests on CPI inflation, then the policymaker minimizes
\[
\frac{1}{2} E_t \left[ \sum_{i=0}^{\infty} \beta^i \left[ y_{t+i}^2 + \mu \pi_{t+i}^2 + \delta q_{t+i}^2 \right] \right]
\]
subject to the constraint which is represented by the model economy. After rewriting the structure of the economy in terms of the CPI inflation rate, we can restate the policy objective as:

\[
\min_{y_t, \pi_t^{\text{CPI}}, q_t} \frac{1}{2} E_t \left[ \sum_{i=0}^{\infty} \beta^i \left[ y_{t+i}^2 + \mu \pi_{t+i}^{\text{CPI}}^2 + \delta q_{t+i}^2 \right] \right]
\]

subject to

\[\pi_t^{\text{CPI}} = \beta E_t \pi_{t+1}^{\text{CPI}} + \kappa y_t + (1 + \beta)(y + b)q_t - (y + b)q_{t-1} - \beta(y + b)E_t q_{t+1} + u_t \]
and

\[y_t = E_t y_{t+1} - a_1(R_t^f - E_t \pi_{t+1}^{\text{CPI}} + \epsilon_t) - (a_1(1 - y) + a_2)(E_t q_{t+1} - q_t) + a_3(y_{t}^f - E_t y_{t+1}^f) + v_t \quad (9)\]

As in the case of domestic inflation targeting the target rule for CPI inflation under policy from a timeless perspective proves complex and is therefore not reported. The variances of the endogenous variables are again determined by numerical solution.
B.3. Targeting “R(eal)-EX(change)-Rate-Adjusted” Inflation

This section introduces an alternative inflation target. This alternative target is domestic inflation stripped of the effects of changes in the real exchange rate. Defining the inflation objective in this way has some intuitive appeal as it transforms an open-economy Phillips curve into a closed-economy version.\textsuperscript{10} As a result, the exchange rate channel is effectively shut down and the monetary policy transmission mechanism works solely through the output gap.

Both the current and expected change in the real exchange rate appear on the right-hand side of the Phillips curve (equation (1)), which can be rewritten as

\[
\pi_t - b(q_t - q_{t-1}) = \beta(E_t \pi_{t+1} - b(E_t q_{t+1} - q_t)) + \kappa y_t + u_t
\]

Defining

\[
\pi_t^{REX} = \pi_t - b(q_t - q_{t-1})
\]

as the domestic rate of inflation purged of the real exchange rate effect allows us to rewrite the original open-economy Phillips curve as

\[
\pi_t^{REX} = \beta E_t \pi_{t+1}^{REX} + \kappa y_t + u_t
\]

Written in this form, equation (11) looks like the original Phillips curve. The only difference between equation (1) and equation (11) pertains to the definition of the rate inflation.

The remaining two equations of the model can be rewritten in terms of the real-exchange-rate-adjusted rate of inflation:

\[
y_t = E_t y_{t+1} - a_1(R_t - E_t \pi_{t+1}^{REX}) + (a_1(b + \gamma) - a_2)(E_t q_{t+1} - q_t) + a_3(y_{t+1}^f - E_t y_{t+1}^f) + \nu_t
\]

\[
R_t - E_t \pi_{t+1}^{REX} = R_t^f - E_t \pi_{t+1}^f + (1 + b)(E_t q_{t+1} - q_t) + \nu_t
\]

After substitution of equation (13) into equation (12) to eliminate the nominal interest rate, the optimization problem of the policymaker can be stated as:

\[
\min_{\pi_t^{REX}, y_t, q_t} E_t \sum_{i=0}^{\infty} \beta^i [y_{t+i}^2 + \mu \pi_{t+i}^{REX^2} + \delta q_{t+i}^2]
\]

s. t.

\textsuperscript{10} Ball (1999) calls his version of real-exchange-rate-adjusted inflation in a backward-looking model “long-run inflation.” It is defined as domestic inflation purged of the effect of the lag (not the change) of the real exchange rate.
\[ \pi_t^{\text{REX}} = \beta E_t \pi_{t+1}^{\text{REX}} + \kappa y_t + u_t \]

and

\[ y_t = E_t y_{t+1} - a_1(R_t - E_t \pi_{t+1} + \varepsilon_t) - (a_1(1 - \gamma) + a_2)(E_t q_{t+1} - q_t) + a_3(y_t - E_t y_{t+1}) + v_t \]

Solving the optimization problem yields the target rule under REX inflation targeting:

\[ \frac{\delta \Delta q_t}{a_1(1 - \gamma) + a_2} + \Delta y_t + \mu \kappa \pi_t^{\text{REX}} = 0 \quad (15) \]

It is evident that the systematic relationship between the target variables depends on demand-side characteristics of the model economy: the denominator of the coefficient on the change in the real exchange rate depends on \( \gamma, a_1 \) and \( a_2 \). Together with \( \delta \) these parameters determine the relative importance of the real exchange rate in the target rule.\(^{11,12}\) Combining equation (15) with equations (11) – (13) yields the solutions for the endogenous variables and the policy instrument.

3. EVALUATION OF SIMPLE INTEREST RATE RULES

This section evaluates the performance of Taylor-type rules in flexible inflation targeting regimes for each of the inflation objectives: domestic inflation, CPI inflation and REX inflation. Initially the targeting regime includes only the goals of inflation and output stabilization. No weight is put on the variance of the real exchange rate (\( \delta = 0 \) in equation (5) and the corresponding loss functions for the other inflation objectives). In this case a response to the real exchange rate in Taylor-type rules may still improve their performance because in the model in section 2 it has information content with respect to the other central bank objectives. Next, we allow for central

\(^{11}\) The target rule can also be written in level form as \[ \frac{\delta q_t}{a_1(1 - \gamma) + a_2} + y_t + \mu \kappa p_t^{\text{REX}} = 0. \] \( p_t^{\text{REX}} \) is the real-exchange-rate-adjusted price level.

\(^{12}\) Broadening the mandate of the central bank thus leads to a less parsimonious target rule. With \( \delta = 0 \), only two parameters, \( \kappa \) and \( \mu \), appear in the target rule: \[ \Delta y_t + \mu \kappa \pi_t^{\text{REX}} = 0. \] Barring the definition of inflation, the target rule is the same as in a closed economy framework.
bank concern with exchange rate volatility albeit with a small weight relative to the typical elements of the dual mandate ($\delta=0.1, \delta=0.2$).\textsuperscript{13}

To start, we evaluate the performance of Taylor’s original rule:

$$R_t = \tau_\pi \pi_t^i + \tau_y y_t$$  \hspace{1cm} (16)

$R$ is the interest rate; $\pi_t^i$ is the chosen inflation measure ($i$ = domestic, CPI or REX inflation); and $y$ is the output gap. The coefficients are set at the values suggested by Taylor: $\tau_\pi = 1.5$ and $\tau_y = 0.5$.

For each inflation objective we consider how the performance of this standard Taylor rule is affected by adding a response to the real exchange rate ($q$). The modified Taylor rule becomes

$$R_t = \tau_\pi \pi_t^i + \tau_y y_t + \tau_q q_t$$  \hspace{1cm} (17)

The value of $\tau_q$ is set at either 0.25 or 0.50.

Next, for each inflation objective we evaluate a variant of the rule where we change the relative weight on the inflation and output objectives. Finally, for reasons discussed previously, we consider the implications of central bank concern for real exchange rate volatility ($\delta>0$). In all cases the Taylor rule is evaluated relative to the optimal policy under commitment from the timeless perspective. Table 1 provides summary information about the parameter values and the distribution of the exogenous shocks used in the numerical calculations.

**A. Domestic Inflation**

1. The Standard Taylor Rule

Table 2 shows results for a domestic inflation target ($\pi$). Panel A is for the standard Taylor rule. In the first column the rule has only the inflation and output gap variables. In the second and third columns the rule includes the real exchange rate ($q$) with weight of 0.25 and 0.5, respectively. The cells in the table show the variances of all three inflation objectives, the output gap, the real exchange rate ($q$) as well as the policy instrument ($R$). Also shown are the value of the loss function (Loss) and the loss relative to the optimal policy outcome (Relative

\textsuperscript{13}The lower relative weight on the real exchange rate in the central bank’s loss function accords with Smets’ (2014) view whereby price (and output) stability dominate financial stability as final objectives.
Loss (%)),\textsuperscript{14} The loss under optimal policy is given in the first column of Panel C of Table 2—labelled TP for optimal policy from a timeless perspective. Panel C also shows the variances under the optimal policy of the same variables shown in Panel A.

The comparison of Panels A and C indicates that the loss under the Taylor rule exceeds that under the optimal policy by 75.9%. The general reasons for the greater loss are: first, that the simple rule responds only to the realized values of the target variables and not to the underlying shocks and second, the arbitrarily chosen response coefficients. The Taylor rule results in much higher output variance (0.92 compared to 0.11 under optimal policy) with only a modest advantage in the variance of domestic inflation (0.74 compared to 0.83).\textsuperscript{15}

Columns 2 and 3 of Panel A show results for the cases where the real exchange rate is included in the Taylor rule with weights of 0.25 and 0.50 respectively. The performance deteriorates somewhat with the inclusion of the real exchange rate. Loss relative to the optimal policy rises to 78.1% and then 88.2% as the weight on the exchange rate increases first to 0.25 and then to 0.5.

2. A Higher Weight on the Output Gap

A number of previous studies [e.g. Ball (1999)] have found that the performance of a simple interest rate rule such as equation (16) or (17) can be improved by increasing the relative weight on output. That this might be the case in our model is suggested by the fact that the main reason why the Taylor rules performance falls short of the optimal policy outcome is the high variance of the output gap. Panel B explores this possibility by increasing \( \tau_y \) from 0.5 to 1.0.

For equation (16), which excludes the real exchange rate, welfare loss relative to the optimal policy falls from 75.9% to 53.9%. The variance of output declines substantially with only a small rise in the variance of domestic inflation. In the case of equation (17) with the real exchange rate response included, increasing the weight on output is also welfare improving. It is still the case, however, that inclusion of the real exchange rate results in higher welfare loss.

\textsuperscript{14} The last four rows of the table can be ignored for the present.
\textsuperscript{15} Svensson (2003) criticizes Taylor rules for their inefficiency.
3. The Real Exchange Rate as Policy Goal

To this point the loss function gives weights only to inflation and the output gap stabilization ($\delta=0$ in equation (5)). The results in Table 2 suggest that concern for exchange rate volatility is important for the evaluation of the desirability of Taylor rules relative to the optimal policy. A comparison of the first column of Panel A with the first column of Panel C reveals that the variance of the real exchange rate ($q$) under the optimal policy exceeds that under the Taylor rule by 83% for the specification where there is no response to the exchange rate (equation (16)). The optimal policy is much more aggressive in pursuing the inflation and output goals as can be seen by the much higher variance of the interest rate under this policy. The result is a better outcome for the output goal though not for domestic inflation, but also much more volatility of the real exchange rate. From a welfare perspective, this higher variance of the real exchange rate is of no importance, however. As long as the variance of the real exchange rate is not included in the loss function the higher variance of the real exchange rate is not a consideration.

The last two rows of Panels A and B of Table 2 show the welfare loss from following a Taylor rule relative to the optimal policy when the real exchange rate is given a weight as a policy goal. The weight on the real exchange rate ($\delta$) is set at 0.1 or 0.2 times the weight on inflation ($\mu$). The weight on inflation and output are then adjusted such that the weights in the loss function sum to 2.0 as before.\(^\text{16}\) The comparison here is now to the losses under the optimal policy shown in the second and third columns of Panel C of the table. These compare with 1.0 for the weight on the variances of the output gap and inflation. Even for these relatively small weights on exchange rate stability, the loss from following a Taylor rule relative to the optimal policy is diminished considerably. For the standard Taylor rule with no response to the real exchange rate and with Taylor’s original coefficients on inflation and the output gap, the relative loss with $\delta=0.2$ is 42.8% compared to 75.9% percent. For the case where the coefficient on the output gap is higher (1.0 instead of 0.5) shown in column 1 of Panel B, the corresponding welfare loss diminishes from 53.9% to 28.4%.

Inclusion of real exchange rate volatility as a policy goal ($\delta>0$) also affects the desirability of including a response to the real exchange rate in the Taylor rule. For either weight

\(^{16}\) For all cases in this section the weights on inflation and the output gap are held equal. The method of calculating the scaled weights is explained in the notes accompanying Table 2.
attached to the output gap (0.5 or 1.0), it is now the case that adding the real exchange rate to the
Taylor rule reduces the loss relative to the optimal policy. For the larger weight on the real
exchange rate in the loss function (δ=0.2), a value of 0.50 for τ₉ reduces this loss from 42.8% to
29.9% for an output weight of 0.5 and from 28.4% to 20.3% for a weight of 1.0.

Panel D of Table 2 summarizes the way in which the losses under a Taylor rule relative
to the optimal policy change as we vary the weight on output (τᵧ), the weight on the real
exchange rate (τ₉) and the weight on the real exchange rate in the loss function (δ). The rule with
(τᵧ=1.0; τ₉=0.50; δ=0.2) has a loss of 20.3% relative to the optimal policy.

B. CPI Inflation

Table 3 presents results when CPI inflation (πᶜᵖⁱ) is the inflation objective..

1. The Standard Taylor Rule

Quantitatively, the losses from following a Taylor rule instead of the optimal policy are
smaller for CPI than for domestic inflation targeting. From the first column of Panel A, it can be
seen that for Taylor’s original rule the relative loss is 32.3% compared to 75.9% for domestic
inflation. The primary reason for the difference is that optimal policy is less aggressive when CPI
inflation is the target. The variance of the interest rate falls by more than 50% relative to that
under domestic inflation targeting. The interest rate is less variable because the resulting
volatility in the real exchange rate results in displacement in the CPI inflation. Even if the
variance of the exchange rate is given no weight in the central bank loss function, exchange rate
volatility is costly. Thus, for a CPI inflation target the loss under optimal policy from a timeless
perspective is greater and closer to the Taylor rule outcome. Output is more variable under the
Taylor rule but the difference is less marked than for the case of domestic inflation targeting.

A second pattern that differs for CPI relative to domestic inflation stabilization is that for
a standard Taylor rule it is now the case that including a small response to the real exchange rate
(τ₉=0.25) improves the rule’s performance; the relative loss falls from 32.3% to 28.8%. Increasing τ₉ to 0.5 raises this loss to 29.9%, still below the loss for the case of no response to the
real exchange rate.
2. A Higher Weight on the Output Gap

Panel B of Table 3 shows results for the case where the weight on output stabilization \((\tau_y)\) is increased from 0.5 to 1.0. As was the case with domestic inflation stabilization, this change reduces the relative loss from employing the Taylor-type rule. If we take the case where there is no real exchange rate response (Panel B, column 1), choosing the higher weight on output reduces the loss from 32.3\% to 22.3\%. From columns 2 and 3 of Panel B it can be seen that adding a response to the exchange rate results in a small improvement for \(\tau_q=0.25\) but a minor deterioration of performance for \(\tau_q=0.5\).

3. The Real Exchange Rate as a Policy Goal

If the central bank values real exchange rate stability \((\delta=0.1; \delta=0.2)\), policy from a timeless perspective becomes less aggressive. Output is moderately more variable and the real exchange rate is much less variable. The variance of CPI inflation is virtually unchanged. For comparison we begin with the standard Taylor rule \((\tau_\pi=1.5; \tau_y=0.5; \tau_q=0)\). As can be seen from the first column of Panels A and D, the relative loss from following the Taylor rule increases when the real exchange rate is given a weight in the loss function. The relative loss rises very slightly from 32.3\% to 32.5\% with a weight on the real exchange rate of 0.1 and to 37.3\% for a weight of 0.2. The optimal policy adjusts with the change but the Taylor rule is unchanged. Adding a real exchange rate response to the Taylor rule in this case, however, improves its relative performance substantially as can be seen from the second and third columns of Panels A and D of Table 3. With a response coefficient \((\tau_q)\) of 0.50 on the exchange rate, the relative loss for the Taylor rule falls from 37.3\% to 19.1\% for \(\delta=0.2\).

Panel B and the right-hand portion of Panel D of Table 3 show results with a higher weigh on output \((\tau_y=1.0)\) in the Taylor rule. These results parallel those for the standard Taylor rule. Adding the real exchange rate to the loss function with a Taylor rule that does not include a real exchange rate response causes the loss relative the optimal policy to increase (Panel B; column 1). When an exchange rate response is added to the Taylor rule, its relative performance improves substantially. With the higher weight on the real exchange rate in the loss function \((\delta=0.2)\) and the higher response coefficients on both output and the real exchange rate in the Taylor rule \((\tau_y=1.0\text{ and } \tau_q=0.50)\), the relative loss falls to 14.3\%.
C. REX Inflation

Table 4 presents results for the strategy of REX inflation ($\pi^{\text{REX}}$) targeting. As pointed out in Section 2, REX inflation targeting restores the isomorphism between optimal policy in the open economy and that in a closed economy. One result following from this property is that the optimal policy adjusts the interest rate to perfectly offset all demand-side shocks. Another implication is that the trade-off between output and inflation is least favourable under this inflation objective; reductions in inflation require the largest sacrifice in output. Optimal policy under this inflation objective thus results in a high degree of output stabilization. Moreover, there is an aggressive use of the interest rate instrument, resulting in a high variance of the real exchange rate if exchange rate stability is not a policy goal ($\delta=0$).

1. The Standard Taylor Rule

With Taylor’s original rule ($\tau_\pi=1.5; \tau_y=0.5; \tau_q=0$), the welfare loss from following a Taylor rule relative to the optimal policy is 99.5%, higher than with the other inflation targets (Panel A, Column 1)). The loss comes mostly from higher output instability. The variance of the output gap under the standard Taylor rule exceeds that under the optimal policy by a factor of 20. Adding a real exchange rate response worsens the performance of the standard Taylor rule (Panel A, columns 2 and 3). Loss relative to optimal policy rises to 102.2% for $\tau_q=0.5$.

2. A Higher Weight on the Output Gap

Panel B of Table 3 reports results where a higher weight is attached to the output gap ($\tau_y=1.0$) in the Taylor rule. As was the case with the other inflation targeting strategies, this change reduces the relative loss under the Taylor rule compared to the optimal policy. In this case the reduction is from 99.5% to 72.8% when no response to the real exchange rate is included in the rule. Including a response to the real exchange rate in this version of the Taylor rule again increases the relative loss.

3. The Real Exchange Rate as a Policy Goal

If real exchange rate stability is a policy goal ($\delta = 0.1$ or 0.2), optimal policy under the timeless perspective becomes considerably less aggressive (Panel C, columns 2 and 3). Now at the margin the gains in output or REX inflation stability do not justify the costs in terms of
increased real exchange rate volatility. With $\delta = 0.2$, the variance of the real exchange rate falls from 5.86 to 1.91. The variance of the policy instrument falls from 5.01 to 2.02. Optimal policy converges toward policy under the Taylor rule. The relative loss from following the standard Taylor rule falls substantially (Panel D). The fall is even more pronounced if a response to the real exchange rate is included in the Taylor rule. With a real exchange rate response coefficient of 0.5 and $\delta = 0.2$, the relative loss for the Taylor rule is 37.3% compared to 99.5% for the standard Taylor rule when real exchange rate is not a policy goal ($\delta=0$). As in previous cases putting a higher weight on the output gap ($\tau_y=1$ instead of 0.5) in the Taylor rule improves its performance. With $\delta=0.2$ in the loss function, this relative loss is reduced to 26.4% if the response coefficient to the real exchange rate ($\tau_q$) is 0.5.

D. Comparisons Across Inflation Targeting Strategies and Loss Functions

Table 5 presents comparisons of the relative losses that result from using a Taylor rule in place of the optimal policy from a timeless perspective across the three inflation targeting regimes. Tables 6 compares the gains from adding a real exchange rate response to various specifications of the Taylor rule for different specifications of the loss function.

1. Relative Losses across Inflation Targeting Regimes

Employing the standard Taylor rule instead of optimal policy results in substantial losses for each of the three inflation targets if only output and inflation appear in the loss function. This can be seen from the first row of panel A in Table 5. This results from the fact that the Taylor rule responds only to realized values of the target variables, not to the underlying shocks, and that the weights in the Taylor rule are set arbitrarily. In the case of targeting domestic or CPI inflation the optimal policy also takes advantage of the information content of the exchange rate.\(^17\)

The relative losses are largest for REX inflation targeting (99.5%) because in that case the optimal policy can, by responding to the demand-side shock directly, adjust the interest rate to offset its effect. The Taylor rule cannot. The relative loss under CPI inflation targeting is smallest (32.3%). For this inflation objective the optimal policy is least aggressive because

\(^{17}\) Froyen and Guender (2015) employ an open economy Phillips curve (Eq. 1) to show that the real exchange rate enters the target rule for both a domestic and CPI inflation objective under optimal discretionary policy.
adjusting the interest rate with consequent effects on the real exchange rate displaces CPI inflation; thus the optimal policy is closer to the standard Taylor rule for this objective. Losses with domestic inflation as the inflation objective fall in between (75.9%).

These relative losses result mainly from higher output variability under the Taylor rule. Thus in row 2 of Panel A we see that the performance of the Taylor rule is improved considerably when the weight on the output gap ($\tau_y$) in the rule is increased from 0.5 to 1.0. The gains are approximately 30% for each of the inflation objectives.

Panel B of Table 5 shows the relative losses from the Taylor rule when the variance of the real exchange rate ($q$) is also a policy goal. The case in the table is $\delta = 0.2$. The resulting scaled weights in the loss function ($\mu^\pi, \mu^y, \mu^q$) are (0.909, 0.909, 0.182); the weight on the real exchange rate is relatively small. For the results in lines 1 and 3 of Panel B, the Taylor rules are the same as those in Panel A. The interest rate settings are the same but loss is computed with the new weights. Moreover the optimal policy to which the Taylor rule is compared now is computed using these weights.

Relative losses from using the Taylor rules considered decline considerably with these alternative weights. The optimal policy becomes less aggressive when penalized for resulting real exchange rate volatility and comes closer to the behaviour of the corresponding Taylor rule. The relative ranking across the three inflation objectives remains the same. The range of the relative losses, however, is far more compact.

2. Gains from Adding an Exchange Rate Response to the Taylor Rule

Rows 2 and 4 of Panel B in Table 5 show the losses for the Taylor rule relative to optimal policy when a real exchange rate response is added to the rule ($\tau_q = 0.5$) and the variance in the real exchange rate given a weight in the loss function ($\delta = 0.2$). In Table 6 these losses are compared to the losses of the corresponding Taylor rules without the real exchange rate response (Panel B). Table 6 also shows the gain or loss from adding a real exchange rate response to the Taylor rules if the loss function gives a weight only to inflation and the output gap (Panel A).

In Table 6, if we confine ourselves to Panel A ($\delta = 0$), the results are consistent with Taylor’s (2001, p.266) description of previous studies: “they seem to be suggesting similar
conclusions, either that there are small performance improvements from reacting to the exchange rate or that such reactions can make performance worse.” Only in the case of CPI inflation targeting with the standard rule and Taylor’s original weights might the gain (10.8%) not clearly be “small.” In the cases of domestic and REX inflation targeting, performance of the Taylor rule deteriorates relative to the optimal policy when a real exchange rate response is added.

The situation is different when the real exchange rate variance is given a weight in the loss function. For each of the three inflation targets the gain is greatest for the higher real exchange rate response ($\tau_q = 0.5$). Across the three targeting strategies, the gains are highest for CPI inflation targeting: 48.8% for the rule with Taylor’s weight and 46.6% with the higher weight on the output gap ($\tau_y = 1$). Gains for the other two inflation targets are between 28% and 30%. All of these appear substantial. If an even smaller weight is chosen for real exchange rate variance ($\delta = 0.1$) such that the scaled weight (0.09524) comprises less than 5% of the sum total (2) of the weights in the loss function, adding an exchange rate response to the Taylor rule still appears desirable. For the rule with Taylor’s original weights, the gains are 33.5% for CPI inflation, 13.9% for REX inflation and 11.5% for domestic inflation.

What drives the results in Tables 5 and 6? With only inflation and the output gap as policy goals, optimal policy from a timeless perspective is very aggressive, resulting in high real exchange rate volatility. The Taylor rule is less aggressive. When the variance of the real exchange rate is also a policy goal, optimal policy takes volatility of the real exchange rate into account and becomes less aggressive. Relative losses from using the Taylor rule are generally lower. Moreover in this case a Taylor rule that smooths the exchange rate is preferred. When compared with the optimal policy, this rule results in losses of 14.3% for CPI inflation, 20.3% for domestic inflation, and 26.4% for REX inflation.\(^\text{18}\)

4. RESULTS WITH OPTIMIZED TAYLOR TYPE RULES

Taylor chose coefficients he believed were sensible but which were not tied to a specific model. Other papers have constructed Taylor-type rules with coefficients chosen to be optimal

\(^{18}\) The relative losses apply to the case where the central bank adopts a Taylor rule with $\tau_y = 1.0; \tau_q = 0.50$ and shows some concern for exchange rate stability ($\delta = 0.2$).
within a model. Some examples within an open economy context are: Garcia, Restrepo and Roger (2011), Leitemo and Söderström (2005), and Batini, Harrison and Millard (2001).

Consideration of losses with optimized Taylor-type rules compared to the optimal policy from a timeless perspective will provide an indication of the degree to which losses relative to optimal policy are due to the arbitrary choice of coefficients relative to the fact that Taylor rules respond to realized values of target variables, not to underlying shocks.

A. A Prior Adjustment

Previous studies of Taylor rules in both closed and open economy contexts have found that, when unconstrained, the optimal response coefficients in Taylor rules have been too high to be economically sensible. Previous studies have gotten around the problem by including the change in the interest rate (the policy instrument) as a cost in the policymaker’s loss function. There are valid reasons, such as a concern for financial stability, that justify interest rate stability as a policy objective. We follow this course. The resulting objective function for the central bank is given below Table 7. The real exchange rate only appears for \( \delta > 0 \). The weight on the interest rate stability argument (\( \varphi \)) is set at 0.1. The Taylor rules are unchanged from those in the previous section (equations (16) and (17)). The parameters of the rules are now chosen by joint optimization to minimize the new objective function. Because the objective function has changed the losses in this section are not strictly comparable with those in Section 3.

B. Losses from Optimized Taylor Rules

Panel A of Table 7 shows the parameter values of the optimized Taylor rules without the real exchange rate for each of the three inflation targets. The loss score and relative loss, measured by losses above that of optimal policy from a timeless perspective are also shown.

The first row of the table provides results with \( \delta = 0 \); the real exchange rate is given no weight the loss function. The optimized Taylor rules show a stronger response to the output gap than Taylor’s original rule (\( \tau_y = 0.5 \)) or the adjusted value used in the previous section (\( 1.0 \)) for

\( \text{From this point on we will simply refer to these as Taylor rules. Repetition of the qualifier is tedious.} \)

\( \text{On the issues relevant to interest smoothing as a policy goal of an optimizing central bank see Rudebusch (2002) and Söderström et. al. (2005).} \)
each of the three inflation targets. The response to inflation is smaller than Taylor’s value ($\tau_\pi=1.5$) for the domestic inflation or REX inflation target but higher for the CPI inflation target.

The pattern of losses relative to the optimal policy is the same as for the simple Taylor rules in the previous section. Losses for CPI inflation are smallest; those for REX inflation are largest with those for domestic inflation in between but closer to those for REX inflation. All the welfare losses relative to the optimal policy are smaller in Table 7A relative to those for the comparable rules in the previous section. This is due to choosing the optimal values of the response coefficients but we have noted the losses are not strictly comparable due to the inclusion of the interest rate stability argument in the objective function in this section.

The second and third lines of panel A provide results for the case where the central bank objective function includes stability of the real exchange rate as an objective. In the case of optimized Taylor rules, coefficients tend to decline as exchange rate volatility is given a larger weight. Optimal policy from a timeless perspective becomes less aggressive for the same reason with exchange rate volatility as an added policy goal. In the table the relative losses from using a Taylor rule are lower for the domestic and REX inflation target but higher for the CPI inflation target. We see presently that these losses are substantially reduced for all three inflation targets when the real exchange rate is added to the rule.

C. Optimized Taylor Rules with a Real Exchange Rate Response

Panel B of Table 7 shows results for optimized Taylor rules when the real exchange rate is added to the rule. The first row of the panel contains results for the case where the rule includes a response to the real exchange rate but the loss function does not give a weight to real exchange rate stability ($\delta = 0$). It is for this case that Taylor (2001) summarized the evidence as pointing to at best a small performance gain when a real exchange rate response is added to the rule. The table indicates no gain when domestic inflation is the target, a gain of 3.3% with the REX inflation target and 13.7% with a CPI inflation target.21 Only the last case might suggest need for a revision in Taylor’s summary of the evidence.

---

21 The gains are calculated in the same way as those reported in Table 6.
The situation is different when exchange rate stability is a policy goal even with a small weight ($\delta = 0.1, \delta = 0.2$). Results for these cases are shown in the second and third rows of Table 7B. The inclusion of exchange rate stability as a goal results in an increase in the optimal real exchange rate response in the Taylor rule. In a few cases the response to the real exchange rate exceeds the response to the inflation target. The gains to adding a real exchange rate response to the rule, measured by the reduction in loss relative to the timeless perspective, are substantial for each inflation targeting strategy. With the higher weight on exchange rate stability ($\delta = 0.2$) the gains are 41.4% for domestic inflation targeting, 58.9% for CPI inflation targeting, and 50.2% for REX inflation targeting. Even with the smaller weight ($\delta=0.1$), the performance of Taylor rules improves markedly with the addition of a response to the real exchange rate.

5 CONCLUSION

In our final exercise assessing the losses from using Taylor rules in place of optimal policy from a timeless perspective (the bottom line of Table 7B), the relative losses are small, approximately 10% for each of the inflation targeting strategies. This is for a rule with optimized responses to the target variables including the real exchange rate and for the case where exchange rate stabilization is a policy goal. These losses are substantially lower than in the first case we considered in Section 3. There the rule used Taylor’s original coefficients; there was no exchange rate response; exchange rate stabilization was not a policy goal. The relative losses to employing the Taylor rule were 75.9% for domestic inflation targeting, 32.3% for CPI targeting, and 99.5% for REX inflation targeting. Three factors are of importance in the improvement in our later examinations of the relative performance of the Taylor rule.

First, if real exchange rate stability is a goal, relative loss to a Taylor rule is reduced substantially for domestic and REX inflation targeting. The relative advantage of optimal policy for these strategies is that it is more aggressive, especially in stabilizing the output gap. But this advantage is achieved at the cost of increased real exchange rate volatility. If real exchange rate stability is a policy goal, this advantage is reduced. For CPI inflation targeting, exchange rate volatility affects the relevant inflation measure and is taken into account even without a real exchange rate goal in the loss function. Adding this goal does not reduce the relative loss from a
Taylor rule under this strategy. Thus the relative losses are therefore compressed across the three strategies.

Second, if real exchange rate stability is a policy goal, then adding a real exchange rate response to the Taylor-type rule substantially improves the rule’s performance relative to the optimal policy for each of the three inflation targeting strategies.

Third, within the model set out here, Taylor’s original coefficients are far from optimal. A simple adjustment of the output response (from $\tau_y=0.5$ to 1.0) improved the performance of the rule. If real exchange rate stability is a policy goal, then adding a response to the real exchange rate in the Taylor rule improves its performance. Finally, we have seen that using values of the coefficients in the Taylor rule chosen optimally for each inflation targeting strategy and including a real exchange rate response further reduces and compresses the welfare loss relative to optimal policy from the timeless perspective.

It may be argued that to employ coefficients that are chosen to be optimal within a specific model violates the spirit of the Taylor rule. With this view the losses from employing Taylor rules should be evaluated with those from Section 3. There, when openness is taken into account with a weight given to stabilizing the real exchange rate and an exchange rate response included in the rule, the losses are: 20.3% for domestic inflation targeting; 14.3% for CPI inflation targeting; and 26.4% for REX inflation targeting. These measures of losses reflect the first and second factors just listed that improve the performance of Taylor rules as openness comes into play.

Taylor and Williams (2011, p.829-30) echo Taylor’s (2001) pessimistic conclusion on the merits of including an exchange rate response in simple interest rate rules. The literature on open economies that they had to survey in this regard, however, dated from the 1990s. Most studies in their survey are focused on large economies and concentrate on the traditional elements of the dual mandate. Our results indicate that conclusions about both the losses from following a Taylor rule instead of the optimal policy from a timeless perspective as well as the

---

22 The losses are measured for $\tau_y=1$ instead of $\tau_y=0.5$ in Taylor’s original rule, an adjustment made in a number of previous studies.

23 See Bryant et al. (1993) and Henderson and McKibbin (1993). It should be noted, however, that several more recent studies, which we cite in the introduction, support their pessimistic conclusion.
merits of including an exchange rate response in the rule depend in important ways on the openness of the economy. A crucial feature is whether the central bank places a weight, albeit fairly small, on stability of the real exchange rate. If it does, as seems sensible given the current turbulence in world financial markets, the conventional pessimism concerning the usefulness of including a real exchange rate response in Taylor-type rules is unwarranted. This policy conclusion is borne out by our findings. In the open economy model we employ, the losses from employing exchange rate-augmented Taylor rules relative to optimal policy from a timeless perspective are substantially reduced.
References:


Appendix  Derivation of Open Economy Phillips Curve

The central building block for our open economy Phillips curve is the firm’s pricing equation. Recent evidence from surveys [Greenslade and Parker (2012) and Parker (2012)] as well as from micro data [Bunn and Ellis (2012a), (2012b)] indicates wide divergence in pricing behavior within and across sectors of the economy. These studies cast doubt on whether any one theory of price setting can adequately capture all the important features of firm pricing. In particular the popular specification of Calvo (1983) pricing supplemented by “rule of thumb” indexation appears to describe actual pricing behavior of only a small minority of firms. Given the heterogeneity of firm pricing behavior, any aggregate price setting equation will be an approximation aimed at capturing some central features of the process.

The pricing framework here emphasizes three elements, each of which receives support from surveys. First, we assume firms follow mark-up pricing influenced by the benchmark prices of competing domestic firms. Second, we assume that there is price stickiness due to menu costs. Menu costs include not just the physical costs of price changes but are also the result of implicit and explicit contracts as well as coordination problems. Finally, in the open economy the firm needs to be concerned with competitiveness at home and abroad. Thus the firm responds to exchange rate induced changes in the terms of trade.

We model these elements of optimal price setting within an extension of Rotemberg’s (1982) quadratic cost adjustment model of monopolistically competitive firms. In our open-economy version of the model an optimizing firm sets the price of output so that the cost function which consists of three components is minimized. The objective function of the typical firm \( j \) is:

\[
\min_{p(j)_t} \Omega(j)_t = E_t \sum_{i=0}^{\infty} \beta^i \left[ (p(j)_{t+i} - p(j)_{t+i})^2 + c (p(j)_{t+i} - p(j)_{t+i-1})^2 + a(q(j)_{t+i} - q(j)_{t+i-1})^2 \right]
\]

(18)

where:

\( \Omega(j)_t \) = the total cost of firm \( j \) at time \( t \)

\( p(j)_t \) = the price of the good produced by firm \( j \) at time \( t \)

\( p(j)_{t+i}^* \) = the optimal mark-up price for firm \( j \)

\( q(j)_t = p_t^f + s_t - p(j)_t \) = firm-specific terms of trade
\[ p_t^f = \text{the price of the foreign good in foreign currency} \]
\[ s_t = \text{the nominal exchange rate (units of domestic currency per unit of foreign currency)} \]
\[ \beta = \text{the constant discount factor} \]
\[ c = \text{the parameter that measures the costs of changing prices relative to the costs of deviating from the optimal mark-up price} \]
\[ a = \text{the parameter that measures the costs of changes in the firm’s terms of trade relative to the costs of deviating from the optimal mark-up price} \]
\[ E_t = \text{the expectations operator conditional on information available at time } t. \]

The first term within the brackets is the cost of being away from the optimal mark-up on cost, the price that the firm would charge in the absence of menu costs and foreign competition. This price, the specification of which is explained below, is denoted \( p(j)^* \). Menu costs are represented by the second term in brackets.

In an open economy firms are concerned about competitiveness abroad as well as at home. Define the terms of trade (or real exchange rate) as the domestic currency price of foreign output relative to the price of domestic output. The firm-specific terms of trade measure a representative firm’s price competitiveness vis-à-vis foreign competition. Changes in its terms of trade make it difficult for the firm to maintain its presence in established markets abroad and interfere with laying out roadmaps (product design, export strategies) for the future. To avoid changes in its terms of trade caused by sudden exchange rate movements or foreign price movements both of which are beyond its control, the firm is required to alter its price. Menu costs make this costly.

The firm sets the price of its output in domestic currency. Taking the first-order condition and running the expectations operator through it, we can characterize the relationship between past, current, and future prices as well as the current and expected change in the terms of trade as:\(^{24}\)

\[
p(j)_t - p(j)_{t-1} = \beta E_t(p(j)_{t+1} - p(j)_t) - \frac{1}{c}(p(j)_t - p(j)_t^*) - \frac{\beta a}{c}(E_t q(j)_{t+1} - q(j)_t) + \frac{a}{c}(q(j)_t - q(j)_{t-1}) \tag{19}
\]

\(^{24}\) From the definition of the firm-specific terms of trade and the fact that the firm cannot influence the price set by foreign competitors or the nominal exchange rate it follows that \( \frac{\partial q(j)_t}{\partial p(j)_t} = -1. \)
From equation (19) it is evident that the current and the expected change in the firm-specific terms of trade matter in setting the price at time \( t \). The greater \( a \) - the relative weight on the change in the terms of trade in the total cost function - compared to \( c \), the relative weight on costly price changes, the more current and expected changes in the terms of trade factor in the decision to change the price in the current period. As stated above, changes in the current nominal exchange rate and changes in the price charged by competing foreign firms are exogenous to the firm. Yet such changes affect a domestic firm’s terms of trade, i.e. its competitiveness. The only way that a domestic firm can counteract such pressure is to adjust its domestic price in such a way so that overall costs are minimized.

Next, consider the formation of the firm’s optimal mark-up price:

\[
p(j)_t = \hat{p}_t + \theta y(j)_t + \varsigma(j)_t \quad \varsigma > 0
\]  

(20)

where all variables are as previously defined. In addition:

- \( \hat{p}_t \) = the price charged by competing firms at time \( t \)
- \( y(j)_t \) = output produced (relative to potential) by firm \( j \)
- \( \varsigma(j)_t \) = a stochastic disturbance.

Under imperfect competition, \( p(j)_t \) is a mark-up over marginal cost. But marginal cost and output are positively related. Therefore we replace marginal cost with the output gap in (20).\(^{25}\) To capture the idea of a time-varying mark-up factor, we model it as a random element \( \varsigma(j)_t \) that enters into the process of setting the optimal mark-up price. The other important factor that influences the firm’s optimal mark-up price is the benchmark price set by competing firms. This price, denoted by \( \hat{p}_t \), equals the aggregate domestic price level \( p_t \).

Substituting equation (20) into (19) and aggregating over all firms yields equation (1) in the text, an open-economy Phillips curve where, in addition to the output gap and expected domestic inflation, the current and expected change in the real exchange rate affect current domestic inflation.

\(^{25}\) Here we follow Roberts (1995).
Table 1: Calibration of Model

The following values for the parameters and variances of the stochastic disturbances are used in the numerical calculations of the variances of the endogenous variables of the model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.643</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.85</td>
</tr>
<tr>
<td>$\eta^f$</td>
<td>0.75</td>
</tr>
<tr>
<td>$\gamma^f$</td>
<td>0.075</td>
</tr>
<tr>
<td>$\xi^f$</td>
<td>0.9</td>
</tr>
<tr>
<td>$a_1 = (1 - \gamma)\sigma$</td>
<td>0.45</td>
</tr>
<tr>
<td>$a_2 = \gamma \left((1 - \gamma)\eta + \eta^f\gamma^f\right)$</td>
<td>0.195</td>
</tr>
<tr>
<td>$a_3 = \gamma\xi^f$</td>
<td>0.27</td>
</tr>
<tr>
<td>$b$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma_j^2 = 1$ for $j = v, u, y^f, R^f, \epsilon, \pi^f$</td>
<td></td>
</tr>
</tbody>
</table>

$\gamma$ = degree of consumption openness

$\sigma$ = intertemporal elasticity of substitution of consumption

$\eta$ = elasticity of substitution between domestic and foreign consumption good

$\eta^f$ = foreign elasticity of substitution between foreign and domestic consumption good

$\gamma^f$ = degree of consumption openness abroad

$\xi^f$ = share of foreign consumption in foreign income
### Table 2: Domestic Inflation Target

<table>
<thead>
<tr>
<th>A.</th>
<th>Std. TR</th>
<th>Std. TR+.25q</th>
<th>Std. TR+.5q</th>
<th>B.</th>
<th>TR(τ_y = 1)</th>
<th>TR(τ_y = 1)+.25q</th>
<th>TR(τ_y = 1)+.5q</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0.9218</td>
<td>0.9103</td>
<td>0.9243</td>
<td>y</td>
<td>0.6704</td>
<td>0.6940</td>
<td>0.7286</td>
</tr>
<tr>
<td>π^REX</td>
<td>0.8939</td>
<td>0.9129</td>
<td>0.9266</td>
<td>π^REX</td>
<td>0.9108</td>
<td>0.9244</td>
<td>0.9348</td>
</tr>
<tr>
<td>π</td>
<td>0.7442</td>
<td>0.7763</td>
<td>0.8016</td>
<td>π</td>
<td>0.7870</td>
<td>0.8083</td>
<td>0.8262</td>
</tr>
<tr>
<td>π^CPI</td>
<td>0.9024</td>
<td>0.8082</td>
<td>0.7637</td>
<td>π^CPI</td>
<td>1.0167</td>
<td>0.9190</td>
<td>0.8632</td>
</tr>
<tr>
<td>R</td>
<td>1.2693</td>
<td>1.0411</td>
<td>0.9407</td>
<td>R</td>
<td>1.2612</td>
<td>1.0573</td>
<td>0.9541</td>
</tr>
<tr>
<td>q</td>
<td>2.8179</td>
<td>2.0138</td>
<td>1.5180</td>
<td>q</td>
<td>2.7400</td>
<td>2.0673</td>
<td>1.6194</td>
</tr>
<tr>
<td>Loss</td>
<td>1.6660</td>
<td>1.6866</td>
<td>1.7259</td>
<td>Loss</td>
<td>1.4574</td>
<td>1.5022</td>
<td>1.5548</td>
</tr>
<tr>
<td>Relative Loss (%)</td>
<td>75.9</td>
<td>78.1</td>
<td>82.2</td>
<td>Relative Loss (%)</td>
<td>53.9</td>
<td>58.6</td>
<td>64.1</td>
</tr>
<tr>
<td>Loss(q, δ=.09524)</td>
<td>1.8851</td>
<td>1.7981</td>
<td>1.7883</td>
<td>Loss(q, δ=.09524)</td>
<td>1.6490</td>
<td>1.6277</td>
<td>1.6350</td>
</tr>
<tr>
<td>Relative Loss (q, δ=.09524) (%)</td>
<td>46.2</td>
<td>41.7</td>
<td>40.9</td>
<td>Relative Loss (q, δ=.09524) (%)</td>
<td>30.0</td>
<td>28.3</td>
<td>28.9</td>
</tr>
<tr>
<td>Loss(q, δ=.1818)</td>
<td>2.0267</td>
<td>1.8992</td>
<td>1.8448</td>
<td>Loss(q, δ=.1818)</td>
<td>1.8229</td>
<td>1.7414</td>
<td>1.7077</td>
</tr>
<tr>
<td>Relative Loss (q, δ=.1818) (%)</td>
<td>42.8</td>
<td>33.8</td>
<td>29.9</td>
<td>Relative Loss (q, δ=.1818) (%)</td>
<td>28.4</td>
<td>22.7</td>
<td>20.3</td>
</tr>
</tbody>
</table>

Loss = V(y_t) + μV(π_t) For all cases considered μ = 1.

Loss (q, δ) = μ^yV(y_t) + μ^πV(π_t) + δV(q_t). For δ = 0.1μ^y and μ^y=μ^π the loss function becomes:

Loss (q, δ) = μ^yV(y_t) + μ^πV(π_t) + 0.1μ^yV(q_t). The sum of the weights must add up to 2.

Hence 2.1μ^y = 2 or μ^y = .9524. From this it follows that δ = .09524 = μ^y. Follow the same procedure for the case of δ = 2μ^y.

### C. TP | TP(q, δ=.09524) | TP(q, δ=.1818)
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0.1146</td>
<td>0.2104</td>
</tr>
<tr>
<td>π^REX</td>
<td>0.8392</td>
<td>0.8670</td>
</tr>
<tr>
<td>π</td>
<td>0.8537</td>
<td>0.8343</td>
</tr>
<tr>
<td>π^CPI</td>
<td>1.9618</td>
<td>1.3781</td>
</tr>
<tr>
<td>R</td>
<td>4.0318</td>
<td>2.5010</td>
</tr>
<tr>
<td>q</td>
<td>5.1574</td>
<td>2.8755</td>
</tr>
<tr>
<td>Loss</td>
<td>0.9472</td>
<td>1.2688</td>
</tr>
</tbody>
</table>

### D. τ_π = 1.5, τ_y = .5 | τ_π = 1.5, τ_y = 1
<table>
<thead>
<tr>
<th>δ / τ_q</th>
<th>0</th>
<th>.25</th>
<th>.5</th>
<th>0</th>
<th>.25</th>
<th>.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>75.9</td>
<td>78.1</td>
<td>83.2</td>
<td>53.9</td>
<td>58.6</td>
<td>64.1</td>
</tr>
<tr>
<td>.1</td>
<td>46.2</td>
<td>41.7</td>
<td>40.9</td>
<td>30.0</td>
<td>28.3</td>
<td>28.9</td>
</tr>
<tr>
<td>.2</td>
<td>42.8</td>
<td>33.8</td>
<td>29.9</td>
<td>28.4</td>
<td>22.7</td>
<td>20.3</td>
</tr>
</tbody>
</table>
Definitions:

Std. TR: $R_t = 1.5 \pi_t + 0.5 y_t$

Relative Loss = \( \frac{Loss^{TB}_t - Loss^{TP}_t}{Loss^{TP}_t} \times 100 \)
Table 3: CPI Inflation Target

<table>
<thead>
<tr>
<th>A.</th>
<th>Std. TR</th>
<th>Std.TR+.25q</th>
<th>Std.TR+.5q</th>
<th>B.</th>
<th>TR(τ_γ = 1)</th>
<th>TR(τ_γ = 1)+.25q</th>
<th>TR(τ_γ = 1)+.5q</th>
</tr>
</thead>
<tbody>
<tr>
<td>τ_y</td>
<td>0.9945</td>
<td>0.9613</td>
<td>0.9686</td>
<td>y</td>
<td>0.7432</td>
<td>0.7573</td>
<td>0.7865</td>
</tr>
<tr>
<td>τ_{REX}</td>
<td>0.8675</td>
<td>0.8978</td>
<td>0.9169</td>
<td>τ_{REX}</td>
<td>0.8961</td>
<td>0.9150</td>
<td>0.9283</td>
</tr>
<tr>
<td>π</td>
<td>0.7213</td>
<td>0.7672</td>
<td>0.7988</td>
<td>π</td>
<td>0.7737</td>
<td>0.8023</td>
<td>0.8241</td>
</tr>
<tr>
<td>τ_{CPI}</td>
<td>0.6759</td>
<td>0.6660</td>
<td>0.6714</td>
<td>τ_{CPI}</td>
<td>0.8019</td>
<td>0.7734</td>
<td>0.7616</td>
</tr>
<tr>
<td>R</td>
<td>1.0510</td>
<td>0.9992</td>
<td>0.9874</td>
<td>R</td>
<td>1.0019</td>
<td>0.9666</td>
<td>0.9531</td>
</tr>
<tr>
<td>q</td>
<td>2.5792</td>
<td>1.7463</td>
<td>1.2894</td>
<td>q</td>
<td>2.3746</td>
<td>1.7555</td>
<td>1.3655</td>
</tr>
<tr>
<td>Loss</td>
<td>1.6704</td>
<td>1.6272</td>
<td>1.6399</td>
<td>Loss</td>
<td>1.5451</td>
<td>1.5307</td>
<td>1.5482</td>
</tr>
<tr>
<td>Relative Loss (%)</td>
<td>32.3</td>
<td>28.8</td>
<td>29.9</td>
<td>Relative Loss (%)</td>
<td>22.3</td>
<td>21.2</td>
<td>22.6</td>
</tr>
<tr>
<td>Loss(q,δ=.09524)</td>
<td>1.8365</td>
<td>1.7162</td>
<td>1.6847</td>
<td>Loss(q,δ=.09524)</td>
<td>1.6977</td>
<td>1.6250</td>
<td>1.6045</td>
</tr>
<tr>
<td>Relative Loss (q, δ=.09524) (%)</td>
<td>32.5</td>
<td>23.8</td>
<td>21.6</td>
<td>Relative Loss (q, δ=.09524) (%)</td>
<td>22.5</td>
<td>17.3</td>
<td>15.8</td>
</tr>
<tr>
<td>Loss(q,δ=.1818)</td>
<td>1.9875</td>
<td>1.7969</td>
<td>1.7253</td>
<td>Loss(q,δ=.1818)</td>
<td>1.8364</td>
<td>1.7107</td>
<td>1.6556</td>
</tr>
<tr>
<td>Relative Loss (q, δ=.1818) (%)</td>
<td>37.3</td>
<td>24.1</td>
<td>19.1</td>
<td>Relative Loss (q, δ=.1818) (%)</td>
<td>26.8</td>
<td>18.1</td>
<td>14.3</td>
</tr>
</tbody>
</table>

Loss = V(τ_t) + μV(π_{CPI}^*) For all cases considered μ = 1. Loss (q,δ) = V(τ_t) + μV(π_{CPI}^*) + δV(q_t) See Table 2 for calculation of scaled weights.

C.         | TP      | TP(q,δ=.09524) | TP(q,δ=.1818) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0.6363</td>
<td>0.6752</td>
<td>0.7279</td>
</tr>
<tr>
<td>τ_{REX}</td>
<td>0.8035</td>
<td>0.8335</td>
<td>0.8550</td>
</tr>
<tr>
<td>π</td>
<td>0.6690</td>
<td>0.7137</td>
<td>0.7472</td>
</tr>
<tr>
<td>τ_{CPI}</td>
<td>0.6267</td>
<td>0.6198</td>
<td>0.6283</td>
</tr>
<tr>
<td>R</td>
<td>1.7628</td>
<td>1.6749</td>
<td>1.6262</td>
</tr>
<tr>
<td>q</td>
<td>2.3134</td>
<td>1.6017</td>
<td>1.1834</td>
</tr>
<tr>
<td>Loss</td>
<td>1.2629</td>
<td>1.3854</td>
<td>1.4480</td>
</tr>
</tbody>
</table>

D.         | τ_π = 1.5, τ_y = 1 | τ_π = 1.5, τ_y = 1 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>δ / τ_q</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>.25</td>
<td>.25</td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>.5</td>
</tr>
<tr>
<td>0</td>
<td>32.3</td>
<td>28.8</td>
</tr>
<tr>
<td>.1</td>
<td>32.5</td>
<td>23.8</td>
</tr>
<tr>
<td>.2</td>
<td>37.3</td>
<td>24.1</td>
</tr>
</tbody>
</table>

Relative Loss = \( \frac{Loss^{TR} - Loss^{TP}}{Loss^{TP}} \times 100 \)

Definitions:

\( \text{Std. TR} = R_t = 1.5\pi_{CPI}^* + 0.5y_t \)
### Table 4: REX Inflation Target

<table>
<thead>
<tr>
<th></th>
<th>Std. TR</th>
<th>Std.TR+.25q</th>
<th>Std.TR+.5q</th>
<th>B.</th>
<th>TR(τ_y = 1)</th>
<th>TR(τ_y = 1)+.25q</th>
<th>TR(τ_y = 1)+.5q</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0.9193</td>
<td>0.9053</td>
<td>0.9167</td>
<td></td>
<td>0.6623</td>
<td>0.6826</td>
<td>0.7154</td>
</tr>
<tr>
<td>π^{REX}</td>
<td>0.9023</td>
<td>0.9181</td>
<td>0.9300</td>
<td></td>
<td>0.9159</td>
<td>0.9278</td>
<td>0.9372</td>
</tr>
<tr>
<td>π</td>
<td>0.7558</td>
<td>0.7821</td>
<td>0.8043</td>
<td></td>
<td>0.7951</td>
<td>0.8127</td>
<td>0.8285</td>
</tr>
<tr>
<td>π^{CPI}</td>
<td>1.0243</td>
<td>0.8872</td>
<td>0.8162</td>
<td></td>
<td>1.1307</td>
<td>0.9969</td>
<td>0.9177</td>
</tr>
<tr>
<td>R</td>
<td>1.5963</td>
<td>1.2046</td>
<td>1.0161</td>
<td></td>
<td>1.5610</td>
<td>1.2219</td>
<td>1.0405</td>
</tr>
<tr>
<td>q</td>
<td>2.9496</td>
<td>2.1377</td>
<td>1.6202</td>
<td></td>
<td>2.9091</td>
<td>2.2065</td>
<td>1.7308</td>
</tr>
<tr>
<td>Loss</td>
<td>1.8216</td>
<td>1.8233</td>
<td>1.8467</td>
<td></td>
<td>1.5782</td>
<td>1.6104</td>
<td>1.6526</td>
</tr>
</tbody>
</table>

**Definitions:**

- Std. TR = R_t = 1.5π^{REX}_t + 0.5y_t
- Relative Loss = \left(\frac{Loss^{TR}_{t}-Loss^{TP}_{t}}{Loss^{TP}_{t}}\right) \times 100

Loss = V(y_t) + μV(π^{REX}_t)  For all cases considered μ = 1.

Loss (q, δ) = V(y_t) + μV(π^{REX}_t) + δV(q_t)  See Table 2 for calculation of scaled weights.

<table>
<thead>
<tr>
<th></th>
<th>TP</th>
<th>TP(q, δ = .09524)</th>
<th>TP(q, δ = .1818)</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0.0456</td>
<td>0.1408</td>
<td>0.2962</td>
</tr>
<tr>
<td>π^{REX}</td>
<td>0.8675</td>
<td>0.8886</td>
<td>0.9035</td>
</tr>
<tr>
<td>π</td>
<td>0.9473</td>
<td>0.9114</td>
<td>0.9058</td>
</tr>
<tr>
<td>π^{CPI}</td>
<td>2.5544</td>
<td>1.6954</td>
<td>1.3517</td>
</tr>
<tr>
<td>R</td>
<td>5.0124</td>
<td>2.7977</td>
<td>2.0169</td>
</tr>
<tr>
<td>q</td>
<td>5.8572</td>
<td>3.0888</td>
<td>1.9070</td>
</tr>
<tr>
<td>Loss</td>
<td>0.9131</td>
<td>1.2746</td>
<td>1.4371</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>TP</th>
<th>TP(q, δ = .09524)</th>
<th>TP(q, δ = .1818)</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>0.0456</td>
<td>0.1408</td>
<td>0.2962</td>
</tr>
<tr>
<td>π^{REX}</td>
<td>0.8675</td>
<td>0.8886</td>
<td>0.9035</td>
</tr>
<tr>
<td>π</td>
<td>0.9473</td>
<td>0.9114</td>
<td>0.9058</td>
</tr>
<tr>
<td>π^{CPI}</td>
<td>2.5544</td>
<td>1.6954</td>
<td>1.3517</td>
</tr>
<tr>
<td>R</td>
<td>5.0124</td>
<td>2.7977</td>
<td>2.0169</td>
</tr>
<tr>
<td>q</td>
<td>5.8572</td>
<td>3.0888</td>
<td>1.9070</td>
</tr>
<tr>
<td>Loss</td>
<td>0.9131</td>
<td>1.2746</td>
<td>1.4371</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>τ_π = 1.5, τ_y = .5</th>
<th>τ_π = 1.5, τ_y = 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ /τ_y</td>
<td>0</td>
<td>.25</td>
</tr>
<tr>
<td>0</td>
<td>99.5</td>
<td>99.7</td>
</tr>
<tr>
<td>.1</td>
<td>58.2</td>
<td>52.2</td>
</tr>
<tr>
<td>.2</td>
<td>52.5</td>
<td>42.4</td>
</tr>
</tbody>
</table>

Definitions:  Std. TR = R_t = 1.5π^{REX}_t + 0.5y_t

Relative Loss = \left(\frac{Loss^{TR}_{t}-Loss^{TP}_{t}}{Loss^{TP}_{t}}\right) \times 100
Table 5: Comparisons Across Inflation Targeting Strategies\textsuperscript{a}

A. Loss Function with Inflation and Output Gap

<table>
<thead>
<tr>
<th>Relative Loss %</th>
<th>Domestic</th>
<th>CPI</th>
<th>REX</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Standard TR</td>
<td>75.9</td>
<td>32.3</td>
<td>99.5</td>
</tr>
<tr>
<td>2. Standard TR ((\tau_y=1))</td>
<td>53.9</td>
<td>22.3</td>
<td>72.8</td>
</tr>
</tbody>
</table>

B. Loss Function Including Real Exchange Rate (\(\delta=0.2\))

<table>
<thead>
<tr>
<th>Relative Loss %</th>
<th>Domestic</th>
<th>CPI</th>
<th>REX</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Standard TR</td>
<td>42.8</td>
<td>37.3</td>
<td>52.5</td>
</tr>
<tr>
<td>2. TR with (\tau_q=0.5)</td>
<td>29.9</td>
<td>19.1</td>
<td>37.3</td>
</tr>
<tr>
<td>3. Standard TR ((\tau_y=1))</td>
<td>28.4</td>
<td>26.8</td>
<td>36.6</td>
</tr>
<tr>
<td>4. TR ((\tau_y=1)) with (\tau_q=0.5)</td>
<td>20.3</td>
<td>14.3</td>
<td>26.4</td>
</tr>
</tbody>
</table>

\textsuperscript{a} Entries in the table show the percentage by which the value of the loss function for policy by the Taylor rule exceeds that for the optimal policy from a timeless perspective.
<table>
<thead>
<tr>
<th></th>
<th>Domestic</th>
<th>CPI</th>
<th>REX</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loss Funct. with only</td>
<td>Taylor’s Weights</td>
<td>-2.9</td>
<td>10.8</td>
</tr>
<tr>
<td>inflation &amp; output gap</td>
<td>$\tau_y = 1$</td>
<td>-8.7</td>
<td>4.9</td>
</tr>
<tr>
<td><strong>B.</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Loss Funct. with inflation,</td>
<td>Taylor’s Weights</td>
<td>30.1</td>
<td>48.8</td>
</tr>
<tr>
<td>output gap, and r. exch. rate</td>
<td>$\tau_y = 1$</td>
<td>28.5</td>
<td>46.6</td>
</tr>
<tr>
<td>((\delta = 0.2))</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Gain or loss is measured as the change in the loss relative to the optimal policy under the timeless perspective. The real exchange rate response (\(\tau_q\)) is set at 0.25 or 0.5 depending on which minimizes relative loss. For example, for CPI inflation with the loss function containing only \(\pi^{\text{CPI}}\) and \(y\), in Table 3 we have: Relative Loss (\(\tau_q = 0\)) = 32.3% and Relative Loss (\(\tau_q = 0.25\)) = 28.8%. Thus the gain reported in the table is \((32.3-28.8)/32.3 = 10.8\%\).*
Table 7: Optimized Taylor Type Rules

A. Taylor Rule without the Real Exchange Rate

<table>
<thead>
<tr>
<th>Domestic Inflation Target</th>
<th>CPI Inflation Target</th>
<th>REX Inflation Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta )</td>
<td>( \tau_\pi^{(\ast)} )</td>
<td>( \tau_y^{\ast} )</td>
</tr>
<tr>
<td>0</td>
<td>0.924</td>
<td>2.428</td>
</tr>
<tr>
<td>0.1</td>
<td>0.82</td>
<td>1.816</td>
</tr>
<tr>
<td>0.2</td>
<td>0.85</td>
<td>1.525</td>
</tr>
</tbody>
</table>

B. Taylor Rule with the Real Exchange Rate

<table>
<thead>
<tr>
<th>Domestic Inflation Target</th>
<th>CPI Inflation Target</th>
<th>REX Inflation Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta )</td>
<td>( \tau_\pi^{\ast} )</td>
<td>( \tau_y^{\ast} )</td>
</tr>
<tr>
<td>0</td>
<td>0.928</td>
<td>2.442</td>
</tr>
<tr>
<td>0.1</td>
<td>0.818</td>
<td>2.413</td>
</tr>
<tr>
<td>0.2</td>
<td>0.819</td>
<td>2.606</td>
</tr>
</tbody>
</table>

1. Objective Function: \( E_t \sum_{j=0}^{\infty} \beta^j (y_{t+j}^{\ast} + \mu \pi_{t+j}^{\ast} + \delta q_{t+j}^{2} + \varphi \Delta R_{t+j}^{2}) \) where \( \pi \) is domestic, CPI, or REX inflation. \( \mu \) and \( \varphi \) is fixed at 1 and 0.1, respectively. The calculation of losses is based on the unconditional variances of the variables that appear in the objective function. Percentage loss is measured relative to optimal policy from a timeless perspective.

2. Numbers in bold represent cases where it was not possible to optimize freely over the policy parameters in the Taylor rule. Doing so violated the rank condition. Satisfying this condition ensures the existence of a determinate rational expectations equilibrium. To meet this condition, we instead performed a search over values of \( \tau_q \) to find the value which minimizes (along with the optimized coefficients on \( \tau_y \) and \( \tau_y^{\ast} \)) the objective function. This procedure had to be followed in case of a REX inflation target irrespective of the inclusion of the real exchange rate in the Taylor rule and in the case of a domestic inflation target when the central bank cares about real exchange rate fluctuations but does not respond to the real exchange rate in the Taylor rule. A superscripted asterisk denotes a freely optimized coefficient.