Informational Control and Collusive Supervision

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Abstract

This paper studies optimal informational control in contracts under the threat of collusion and its implications for organizational form. I consider a principal-supervisor-agent model: The agent is privately informed about his costs to realize a project for the principal. The supervisor observes a signal that is informative of the agent's costs. The supervisor and the agent may collude. The principal sets a contract and designs the supervisor's signal without being able to observe it. I analyze the principal's trade-off between information elicitation and collusion prevention: A well-informed supervisor may provide good advice but can also organize collusion effectively. I study optimal signals and show that the principal wants to withhold information from the supervisor. Balancing upside potential and downside risk, the principal creates signal realizations of equal value that optimally incentivize the supervisor to participate in the contract. Given the optimal signal, the optimal contract can be implemented through delegation whereby the principal authorizes the supervisor to contract with the agent. I discuss implications for public procurement if corruption is an issue.

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1 Introduction

In many contractual relationships, a principal seeks advice from a supervisor before delegating a task to an agent. Empirical evidence suggests that collusion is a major problem in such situations: The manager of a division may overstate the difficulty of a project to her boss to increase the wages of her subordinates.[1] A board of directors may exaggerate the CEO's outside options to set a generous compensation package against the interests of shareholders.[2] Public procurement officers may misrepresent the costs of suppliers and pay overly high prices to the detriment of taxpayers.[3]

In each of these examples, the principal can influence what the supervisor knows about the agent. The head of an organization decides how frequently managers rotate between subdivisions. Shareholders choose the number and expertise of outside directors on the board. A government determines how much data a public procurement officer receives on the past performance of suppliers. In these instances, the principal controls the supervisor's information flow. Nevertheless, the principal lacks the expertise or the time to evaluate the information and needs to rely on the supervisor's advice.

Even though information can often be controlled, the existing literature on collusive supervision takes the information of the supervisor as given and studies the optimal monetary incentives a principal provides under the threat of collusion (Faure-Grimaud, Laffont and Martimort, 2003; Celik, 2009; Mookherjee, Motta and Tsumagari, 2015). The literature, therefore, does not provide any guidance for a principal who can use information and transfers to create incentives.

This paper closes the gap. The first main contribution is an analysis of how to optimally exert informational control under the threat of collusion. I show that the principal wants the supervisor to be only partially informed about the agent. As the second main contribution, this paper studies the implications of informational control for the principal's preferred form of organization. I

[1] Bertrand and Mullainathan (2003) provide empirical evidence of collusion between managers and workers that leads to higher wages

[2] Hallock (1997) and Fracassi and Tate (2012) document the positive effect of friendly boards on CEO compensation and the negative effect for shareholder value.

[3] Di Tella and Schargrodsky (2003) provide evidence of public procurement fraud in hospitals in Buenos Aires.

demonstrate that delegation is optimal under informational control. Thus – as long as the supervisor is optimally informed from the principal's perspective – the principal cannot do better than to authorize the supervisor to contract with the agent.

I derive these results in a principal-supervisor-agent model in which the agent can realize a project for the principal at a privately known cost. The supervisor observes a signal on the agent's costs. Moreover, the supervisor is protected by limited liability. The supervisor and the agent can collude. The principal has informational control over the supervisor, i.e., the principal determines what the supervisor learns about the agent's costs. Following the literature on mechanism design with collusion, the supervisor and the agent organize collusion with an enforceable side-contract that specifies side-payments and coordinates their behavior under the contract proposed by the principal. In the spirit of the literature on Bayesian persuasion, the principal exerts informational control by choosing an information structure that generates the supervisor's signal.

I analyze the principal's optimal combination of an information structure and a contract. The optimal information structure is shaped by a trade-off between information elicitation and collusion prevention. If the supervisor receives additional information, the agent's informational advantage over the supervisor decreases. As long as the supervisor shares her information truthfully, this is beneficial for the principal. However, it also reduces information asymmetry in the colluding coalition and therefore enables the supervisor and the agent to collude more effectively.

Due to this trade-off, the principal finds it optimal to withhold information from the supervisor. In particular, the principal designs the supervisor's signal such that all signal realizations are of equal value to the supervisor. Signal realizations with a high upside potential come with a high downside risk. This construction avoids bad signal realizations for which the principal would need to compensate the supervisor with high transfers. The supervisor and agent would then have a strong incentive to collude by reporting the bad signal realization to profit from the high compensation.

Equipped with the optimal information structure, I study the implications of informational control for the organization of the contractual relationship. I show that delegation is optimal as long as the principal chooses the optimal information structure. With optimal informational control, the principal can authorize the supervisor to contract with the agent and can achieve the same payoff as under the optimal centralized contract.

In contrast to delegation, under centralized contracting the principal can pay direct transfers to the agent. Thus, the principal has an additional instrument with which to fight collusion. If the principal increases the agent's transfer under the contract, it becomes harder for the supervisor to find a profitable side-contract as she needs to compensate the agent. Delegation is optimal if, and only if, the instrument of direct transfers is not valuable for the principal. The literature on collusive supervision shows that the value of the instrument depends on the information structure that generates the supervisor's signal. Whereas Faure-Grimaud et al. (2003) show that delegation is optimal, Celik (2009) arrives at the opposite result with a different assumption on the information structure.[4] I show that under the optimal information structure, the principal does not benefit from direct transfers to the agent. Thus, delegation is optimal and informational control substitutes direct control.

These results have implications for the design of guidelines for public procurement procedures – an area where policymakers and experts intensively discuss the role of information. In many developing countries, corruption in public procurement is facilitated by insufficient control of central authorities over local governments who engage in collusive activities with suppliers (OECD, 2016). International organizations such as the World Bank or the OECD promote digitalized procurement procedures (e-procurement) in which suppliers post required documents online and public authorities gather and access data about the past performance of suppliers (World Bank, 2006; OECD, 2016). The results of this paper point out that central governments can only benefit from e-procurement if the access of local governments to data about suppliers is carefully designed. In particular, it can be beneficial to withhold information from local governments to hamper their efforts to organize corruption.

The discussion on the link between information and corruption in public

[4]See the next section for more details on the difference between the information structures in Faure-Grimaud et al. (2003) and Celik (2009).

procurement is not restricted to developing countries. Traditionally, public procurement policies in the US and the EU reflect different approaches to data on the past performances of suppliers (Spagnolo, 2014). In the US, public procurement guidelines prescribe the use of such data to improve the long-term incentives for suppliers. In contrast, policymakers in the EU fear that their use may reduce the transparency of procurement procedures and conceal corruption. This paper contributes to this discussion and emphasizes that even if the procurement process itself cannot be made contingent on some data, procurement officers may use the knowledge gained from these data to organize collusion more effectively. Thus, it is important to manage what data are available to procurement officers.

If the access to data is managed well, central authorities can delegate the design of procurement contracts to local governments and procurement officers without aggravating the problem of corruption. In particular, this paper's results suggests that central authorities do not benefit from regulating the type of contracts that can be offered to suppliers.

In the next section, I discuss the related literature. Section 3 provides an example that illustrates the main results. In Section 4, I introduce the model. In Section 5, I characterize and analyze the optimal combination of an information structure and a contract. Section 6 presents a delegation game that implements the optimal centralized contract under the optimal information structure. Section 7 concludes the paper. All proofs can be found in Appendix A.

2 Related Literature

This paper builds on and contributes to two strands of literature. The first strand analyzes the implications of collusion under asymmetric information for mechanism design. The second studies optimal information design in games.

Laffont and Martimort (1997) are the first to model collusion as an enforceable side-contract under soft asymmetric information and to study its implications for mechanism design.[5,6] In this paper, I follow their approach. Thus, asymmetric information is the only friction in collusion formation.

More specifically, this paper contributes to the literature on collusive supervision with adverse selection.[7] This literature analyzes principal-supervisor-agent models. In contrast to this paper, the supervisor's signal is exogenously given. Most closely related are the works by Faure-Grimaud et al. (2003) and Celik (2009). Both papers study similar models of collusive supervision but differ in their assumption on the information structure that generates the supervisor's signal. Faure-Grimaud et al. (2003) assume that the agent has two types and the supervisor observes a binary signal. In this setup, they analyze the optimal contract and show that it can be implemented through delegation. Celik (2009) assumes that the agent has more than two types and that the supervisor's information is a partition of the agent's type space. Under these assumptions, he shows that delegation is suboptimal. In the present paper, the information structure is endogenously chosen by the principal. I show that the principal optimally selects an information structure under which delegation is optimal.

In contrast to Faure-Grimaud et al. (2003), Celik (2009), and this paper, Mookherjee and Tsumagari (2004) and Mookherjee et al. (2015) analyze models of collusive supervision in which the colluding coalition can enter a side-contract before accepting the contract offered by the principal. The participation decision of the agent and supervisor can therefore be part of the collusive agreement in the side-contract.[8] Both papers study optimal contracts under the threat of collusion and show that delegation is strictly suboptimal given their assumption on the timing.

This paper is also related to the literature on Bayesian persuasion and

[5]Collusion with symmetric information is studied in the seminal papers by Green and Laffont (1979) and Tirole (1986). Crémer (1996), McAfee and McMillan (1992), and Caillaud and Jehiel (1998) study collusion under asymmetric information in specific mechanisms.

[6]For a discussion on the damage of collusion for a principal, see Laffont and Martimort (2000) and Che and Kim (2006).

[7]Baliga and Sjöström (1998) and Laffont and Martimort (1998) study collusion and the optimality of delegation in a model with two productive agents and moral hazard (Baliga and Sjöström, 1998) or adverse selection (Laffont and Martimort, 1998).

[8]Further papers that study this form of collusion are Dequiedt (2007), Pavlov (2008), Che and Kim (2009), and Che, Condorelli and Kim (2014).

the optimal design of information in games. Kamenica and Gentzkow (2011) analyze a model of Bayesian persuasion in which a sender designs a signal by choosing an arbitrary information structure to influence the behavior of a receiver.[9] I follow their approach when modeling informational control.

Bergemann and Morris (2013, 2016), Taneva (2015), and Mathevet, Perego and Taneva (2016) study generally how a sender can design the information structure of a game to influence Bayes Nash equilibria played by a group of receivers.[10]

In an application of this approach to price discrimination, Bergemann, Brooks and Morris (2015) analyze the implications of a seller's information on a buyer's valuation for the possible distributions of profits between the seller and buyer. In the delegated contracting game studied in Section 6 of this paper, the supervisor and agent are in a similar situation as the seller and buyer in Bergemann et al. (2015). However, in this paper, there is a third party – the principal – who decides how much information the supervisor has on the agent and tries to extract profit from the supervisor and the agent.

Bergemann and Pesendorfer (2007) study the joint optimal design of information structure and auction format when the seller can disclose information to bidders. As in their paper, I analyze the joint design of a mechanism and an information structure which informs the supervisor and agent before they make their participation decision.[11]

Finally, this paper is connected to Ortner and Chassang (2015) and Ivanov (2010). Ortner and Chassang (2015) analyze a principal-monitor-agent model and show that corruption can be fought by introducing asymmetric information in the colluding coalition through the use of random transfers. In contrast to the present paper, it is therefore the terms of the contract and not the type

[9]See also Rayo and Segal (2010) for an early contribution, Ely, Frankel and Kamenica (2015) and Ely (forthcoming) for extensions to dynamic settings, Gentzkow and Kamenica (forthcoming, 2016) and Li and Norman (2015) for studies of competition between senders, and Kolotilin, Li, Mylovanov and Zapechelnyuk (2015) for the case of a privately informed receiver.

[10]See Alonso and Camara (2016), Bardhi and Guo (2016), and Chan, Gupta, Li and Wang (2016) for applications to voting.

[11]In this respect, my paper differs from Eső and Szentes (2007a,b) and Li and Shi (2015), who consider sequential information disclosure, where agents first decide whether to participate and then receive information, or from Bergemann and Wambach (2015) where agents receive new information sequentially.

of the agent over which there is asymmetric information. This implies that in their setting, the principal does not face a trade-off between information elicitation and collusion prevention. This trade-off is central to the analysis in this paper.

Ivanov (2010) studies informational control and delegation in the model of Crawford and Sobel (1982). The uninformed party can design the signal of the informed party and can decide whether to delegate decision-making to the informed party. Ivanov shows that informational control and direct control – the uninformed party's decision to keep decision rights – can be either substitutes or complements. This contrasts with the analysis under commitment in the present paper where informational control and direct control are shown to be substitutes.

3 An Illustrative Example

In this section, I illustrate the main results in an example: The central government P wants to implement a large infrastructure program to build new highways in the country. In some province where a new highway is planned, the company A can build the highway. Whereas A knows the costs of building the highway, P does not. The costs depend on problems that A encounters during construction and on A's specialization. Suppose for simplicity that A encounters exactly one of two problems – problem a or b – and is either specialized in solving one type of the problem or is an all-rounder. If the encountered problem is A's speciality, costs are low. If A encounters one type of the problem but is specialized in the other, costs are high. If A is an all-rounder, costs are intermediate, regardless of the problem. Each problem and each specialization is equally likely. Figure 1 summarizes this description and specifies the cost levels.

The benefit of the highway for P is 4. Without any further information, P's optimal price offer to A is the solution to the monopsony problem $\max_p \Pr(\text{costs} \leq p)(4-p)$. The price $p = 2$ is optimal and gives P an expected payoff of $\frac{4}{3}$.

Before offering a price to A, P can turn to the local government S. P can grant S access to data on the planned highway – such as blueprints prepared by

Figure 1: The determinants of A's costs

	problem a: $^1/_2$	problem b: $^1/_2$
a-specialist: $^1/_3$	1	3
b-specialist: $^1/_3$	3	1
all-rounder: $^1/_3$	2	2

Construction costs depend on the problem and A's specialization. Only one problem occurs and both are equally likely. A is either specialized in one type of problem or is an all-rounder. Each specialization is equally likely.

A – from which S can learn the problem type. Similarly, P may disclose data on past performances of A to S which allows S to evaluate A's specialization. The central government P cannot itself extract the information from the data due to the large number of concurrent projects in the country and its lack of knowledge on local circumstances such as ground conditions or wages. Thus, P relies on truthful reports of S about her findings. Due to existing agreements between the local government S and the central government P, P pays a contribution of 1 to the budget of S. S can only be held liable by P up to this value. S derives no benefit from the highway and maximizes the sum of tax revenue and bribes.

Without collusion (i.e. corruption) between S and A, P optimally discloses all data to S, asks S for advice, and makes a price offer to A. In this case, S is a disinterested party and reports the true costs to P. Therefore, P offers A a price equal to costs and receives the expected social surplus of $4 - \mathbb{E}[\text{costs}] = 2$.

With collusion this contract is prone to manipulation. In exchange for a side-payment, S promises A to always report high costs. A accepts to pay any side-payment up to the difference between costs and price. As S knows A's costs, she knows the maximal side-payment that A is willing to pay. Thus, S and A reach an effective collusive agreement and behave as a single player. Therefore P is back in the monopsony problem and makes an expected payoff of $\frac{4}{3}$. The damage of collusion is thus $\frac{2}{3}$ if S is perfectly informed.

An uninformed local government S cannot provide any helpful advice. However, perfect information enables S to organize collusion effectively, which again renders her advice useless to P. These extreme cases illustrate the trade-off P faces between information elicitation and collusion prevention. If P discloses additional data, the informational advantage of A over S shrinks. S may therefore provide better advice. At the same time, more information helps S to find a profitable collusive agreement with A. P needs to balance these two effects of disclosure.

Can P do better by withholding data from S? Suppose P allows S to learn only A's specialization. If A is an all-rounder, S knows that the costs are intermediate. If A is specialized, S infers that A's costs are high or low with equal probability. Furthermore, P commits to the following contract vis-à-vis S and A. If S reports to P that A is specialized, P offers a price $p = 1$ to A. If S reports that A is an all-rounder, P offers a price $p = 2$. P pays the contribution of 1 to S's budget if, and only if, A accepts the price offer. In case A accepts the low price offer $p = 1$, P pays a *bonus* of 1 to S.

Both S and A accept this contract. Suppose S reports her information truthfully to P. Then, S and A are as well off as without the contract. The price offer to A covers at most his costs. In case A is an all-rounder, P pays the contribution and S does not receive a bonus. In case A is specialized and rejects the low price offer, P withholds its contribution to S. In contrast, S receives the contribution and the bonus if A accepts the low price offer. S's expected payoff from accepting the contract is

$$\Pr(\text{costs} = 1|\text{specialized}) \times 1 + \Pr(\text{costs} = 3|\text{specialized}) \times (-1) = 0.$$

The contract is robust to collusion and incentivizes S to report A's specialization truthfully. Under the contract, S and A receive a total payment of 2 for the highway and make a payment of 1 to P[12] if the highways is not built. These payments are independent of S's report. A collusive agreement in which the highway is not built for low or intermediate costs reduces S and A's joint payoff. An agreement that induces the construction of the highway for high costs leads to a joint payoff of -1. Thus, the contract is collusion-proof.

[12]This payment is the loss of the contribution of P to S's budget.

Under the contract, P receives an expected payoff of

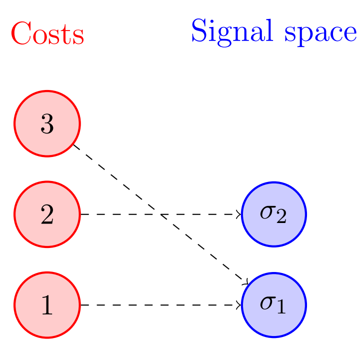
$$\Pr(\text{costs} = 3) \times 1 + \Pr(\text{costs} < 3) \times (4 - 2) = \frac{5}{3}.$$

Thus, it is optimal for P to withhold data from S. With informational control P recoups 50% of the damage from collusion.[13]

P can implement the contract through *delegation*. Thereby, P offers S a delegation contract under which S receives the contribution of 1 and a bonus of 2 if the highway is completed. P withholds the contribution if the highways is not built. In case S accepts the delegation contract, she offers A a price for the construction of the highway.

Under the delegation contract, S offers a price $p = 1$ if A is specialized and a price $p = 2$ if A is an all-rounder. S's expected payoff is zero in both cases and P receives an expected payoff of $\frac{5}{3}$. Given that P controls S's information, it can delegate direct control over A to S without losing money.

Figure 2: Information structure equivalent to learning A's specialization



The signal realization σ_1 is generated if costs are 1 or 3 and is equivalent to observing that A is specialized. The signal realization σ_2 is generated if costs are 2 and is equivalent to observing that A is an all-rounder.

If S evaluates A's specialization, her learning process can be represented as the *information structure* depicted in Figure 2. This information structure

[13]If S evaluates only the type of the problem, P's maximal expected payoff is also $\frac{4}{3}$. If the type of the problem is known, all cost levels of A are equally likely—as in the case where the type of the problem is unknown. Thus, a price of $p = 2$ remains optimal regardless of the type of the problem.

generates a signal realization σ_1 if the costs of A are high or low. The signal realization σ_2 is generated if the costs are intermediate. The signal realization σ_1 leads to the same posterior belief over the costs as if S learns that A is specialized. The signal realization σ_2 is equivalent to learning that A is an all-rounder. In the analysis of the general model, informational control is the choice of an information structure. This reduced-form approach allows me to model any learning process of S.

As will be shown in Section 5, the information structure of Figure 2 is the optimal information structure for P. Under this information structure and the contract specified above, P pays no rents to A and avoids profitable collusive agreements for S. The two signal realizations have the same value for S. Under the first signal realization σ_1, S either receives a bonus or loses P's contribution. Under the second signal realization σ_2, S neither receives a bonus nor loses the contribution. I show in section 5 that this is a general feature of an optimal information structure. Signal realizations with a higher upside potential come with a higher downside risk. This balance avoids bad signal realizations that S could falsely report to gain rents.

4 The Model

There are three players: the principal P ("it"), the supervisor S ("she"), and the agent A ("he"). P seeks to realize a project. It values the project by $v \in \mathbb{R}$. A can carry out the project for P at costs θ. A is privately informed about θ which belongs to the interval $\Theta = [\underline{\theta}, \overline{\theta}] \subset \mathbb{R}$. The costs θ are the realization of a random variable $\tilde{\theta}$ with distribution $F(\theta) = \Pr(\tilde{\theta} \leq \theta)$. The project is socially beneficial with a positive probability, i.e., $\Pr(\tilde{\theta} < v) > 0$. S might learn about A's costs θ by observing a signal σ. The signal is the realization of a random variable $\tilde{\sigma}$. The signal is also observed by A but not by P. S is protected by a limited liability, i.e., she can never incur losses that exceed the maximal loss $\ell \in [0, \infty)$.

Information structures

P has informational control and decides how S is informed of A's costs. Following the literature on Bayesian persuasion, I model informational control as the possibility to choose an information structure that specifies S's signal. An information structure is given by $I = (\Sigma, \mu)$ where Σ is a set of signal realizations with the generic element $\sigma \in \Sigma$ and $\mu \in \Delta(\Sigma \times \Theta)$ is a probability measure on the set of possible realizations of the costs and the signal. The measure μ induces a conditional distribution $G(\theta|\sigma) = \Pr(\tilde{\theta} \leq \theta|\sigma)$ and a marginal distribution $H(\sigma) = \Pr(\tilde{\sigma} \leq \sigma)$. These distributions are consistent with the unconditional distribution $F(\theta)$:

$$\int_{\Sigma} G(\theta|\sigma) dH(\sigma) = F(\theta).$$

Let $\mathcal{I}$ be the set of all information structures. Given some information structure I, $Supp(\mu) \subset \Sigma \times \Theta$ denotes the support of the random variable $(\tilde{\sigma}, \tilde{\theta})$.

Allocations

An allocation describes whether the project is realized and what transfers are paid from P to S and A. An allocation is given by $(x, t_S, t_A) \in \{0, 1\} \times \mathbb{R}^2$. Project realization is denoted by $x = 1$. The transfer from P to i is given by t_i with $i \in \{A, S\}$. The allocation (x, t_S, t_A) leads to payoffs of $t_A - \theta x$ for A, t_S for S, and $vx - t_A - t_S$ for P. S and A value their outside options at zero.

Collusion

Following the literature on collusion in mechanism design, collusion is modeled as an enforceable side-contract between S and A that coordinates their communication with P and specifies side-transfers. Thus, P cannot prohibit the exchange of transfers or communication between the agents. In contrast, the realization of the project is observable and contractible. S and A can coordinate their communication with P in order to influence the allocation. Their coordination is facilitated by the side-transfer τ. S proposes the side-contract to A in a take-it-or-leave-it offer. If the signal σ is not perfectly informative of the costs θ, the side-contract needs to incentivize A to report his costs truthfully.

Contracts and side-contracts

P offers S and A a contract

$$\beta = \left(x(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}), t_S(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}), t_A(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta})\right).$$

The contract assigns an allocation to any profile of reports $(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta})$ where $\hat{\sigma}_i$ is the report of $i \in \{S, A\}$ about $\sigma \in \Sigma$, and $\hat{\theta}$ is the report from A about $\theta \in \Theta$.

S offers A a side-contract

$$\gamma = \left(\rho(\breve{\theta}; \sigma), \tau(\breve{\theta}; \sigma)\right).$$

The reporting strategy $\rho : \Theta \times \Sigma \to \Sigma^2 \times \Theta$ specifies the communication of S and A with P. The side-transfer from S to A is determined by the function $\tau : \Theta \times \Sigma \to \mathbb{R}$. Under the side-contract, A's report to S about his costs is $\breve{\theta} \in \Theta$. For a signal realization $\sigma \in \Sigma$ and a report $\breve{\theta} \in \Theta$, the side-contract induces the allocation

$$\left(x(\rho(\breve{\theta}; \sigma)), t_S(\rho(\breve{\theta}; \sigma)) - \tau(\breve{\theta}; \sigma), t_A(\rho(\breve{\theta}; \sigma)) + \tau(\breve{\theta}; \sigma)\right).$$

The *null side-contract* is the side-contract under which S and A report their information truthfully without exchanging side-transfers. Thus, the null side-contract satisfies $\rho(\breve{\theta}; \sigma) = (\sigma, \sigma, \breve{\theta})$ and $\tau(\breve{\theta}; \sigma) = 0$ for all $\sigma \in \Sigma$ and $\breve{\theta} \in \Theta$.

Contracts and side-contracts take the form of direct mechanisms in which S and A directly report the signal realization and the costs. This is without loss of generality due to the collusion-proofness principle that I invoke and explain in Section 5.

Timing and equilibrium concept

The timing of the game is as follows.

t=0: P chooses an information structure $I \in \mathcal{I}$ and offers a contract β to S and A.

t=1: S and A observe I, β, and the realization of the signal σ. A furthermore observes θ.

t=2: S and A each accept or reject P's offer. If either of them rejects, both agents receive their outside option. Otherwise the game continues.

t=3: S offers a side-contract γ to A.

t=4: A accepts or rejects S's offer. If A accepts, the side-contract and the contract are executed. If A rejects, both agents play the contract non-cooperatively.

I focus on *perfect Bayesian equilibria* (PBE) with *passive beliefs*. In these equilibria, S does not update her belief about θ if A rejects the side-contract off the equilibrium path. This approach follows Laffont and Martimort (1997) and the concept of *weak collusion-proofness* in Laffont and Martimort (2000).

5 Optimal Informational Control and Contract

In this section, I analyze the optimal choice of information structure and contract under the threat of collusion. First, I state P's problem formally and present the benchmarks without supervision and without collusion. I then provide a solution to P's problem and analyze the properties of optimal information structures and contracts.

P's problem

P optimally chooses an information structure I and a contract β in order to maximize her expected payoff under the constraints that S and A want to participate in the contract, that S never incurs a loss greater than ℓ, that S and A report their private information truthfully to the contract, and that there does not exist a feasible side-contract which gives S a strictly higher payoff than to participate non-cooperatively in the contract. Thus, I invoke a collusion-proofness principle: Any payoff that P can achieve in an equilibrium where S and A collude through a non-trivial side-contract can also be attained in a collusion-proof contract. This approach follows Laffont and Martimort (1997) and Faure-Grimaud et al. (2003). In Appendix B, I provide a proof of the collusion-proofness principle along the lines of Faure-Grimaud et al. (2003).

Thus, P's problem is

$$\max_{I,\beta} \mathbb{E}\left[vx(\tilde{\sigma},\tilde{\sigma},\tilde{\theta}) - t_S(\tilde{\sigma},\tilde{\sigma},\tilde{\theta}) - t_A(\tilde{\sigma},\tilde{\sigma},\tilde{\theta})\right] \quad \text{subject to}$$

$$\mathbb{E}\left[t_S(\sigma,\sigma,\tilde{\theta})|\sigma\right] \geq 0, \tag{PC_S}$$

$$\mathbb{E}\left[t_S(\sigma,\sigma,\tilde{\theta})|\sigma\right] \geq \mathbb{E}\left[t_S(\hat{\sigma}_S,\sigma,\tilde{\theta})|\sigma\right], \tag{IC_S}$$

$$t_A(\sigma,\sigma,\theta) - \theta x(\sigma,\sigma,\theta) \geq 0, \tag{PC_A}$$

$$t_A(\sigma,\sigma,\theta) - \theta x(\sigma,\sigma,\theta) \geq t_A(\sigma,\hat{\sigma}_A,\hat{\theta}) - \theta x(\sigma,\hat{\sigma}_A,\hat{\theta}), \tag{IC_A}$$

$$t_S(\hat{\sigma}_S,\hat{\sigma}_A,\hat{\theta}) \geq -\ell; \tag{LL}$$

for all $(\sigma,\hat{\sigma}_S,\hat{\sigma}_A,\theta,\hat{\theta}) \in \Sigma^3 \times \Theta^2$ and subject to the collusion-proofness constraint

$$\mathbb{E}\big[t_S(\sigma,\sigma,\tilde{\theta})|\sigma\big] \geq \max_{\gamma} \mathbb{E}\left[t_S(\rho(\tilde{\theta};\sigma)) - \tau(\tilde{\theta};\sigma)|\sigma\right] \quad \text{subject to} \tag{CP}$$

$$t_A(\rho(\theta;\sigma)) + \tau(\theta;\sigma) - \theta x(\rho(\theta;\sigma)) \geq t_A(\sigma,\sigma,\theta) - \theta x(\sigma,\sigma,\theta), \tag{PC_A^γ}$$

$$t_A(\rho(\theta;\sigma)) + \tau(\theta;\sigma) - \theta x(\rho(\theta;\sigma)) \geq t_A(\rho(\breve{\theta};s)) - \tau(\breve{\theta};s) - \theta x(\breve{\theta};s), \tag{IC_A^γ}$$

$$t_S(\rho(\theta;\sigma)) - \tau(\theta;\sigma) \geq -\ell; \tag{LL^γ}$$

for all $(\sigma,\theta,\breve{\theta}) \in \Sigma \times \Theta^2$. A contract β is called *feasible* if it satisfies the constraints (PC_S), (IC_S), (PC_A), (IC_A), and (CP). A side-contract γ is called feasible if it satisfies the constraints (PC_A^γ) and (IC_A^γ). I denote P's expected payoff from an information structure I and a feasible contract β by $U_P(I,\beta)$. For a given information structure I, P's reduced problem of choosing an optimal contract is denoted by $\mathcal{P}_I$.

Benchmarks

Before I study P's problem, it is insightful to examine the following three benchmark cases: complete information, no supervision, and no collusion.

Complete information If θ is publicly observable, P implements the allocation $(x^*(\theta), t_S^*(\theta), t_A^*(\theta))$ where the project is realized whenever P's benefit exceeds A's cost, S receives no payment, and A is exactly compensated for his costs:[14]

$$x^*(\theta) = \mathbf{1}_{(\theta \leq v)}(\theta),\ t_S^*(\theta) = 0,\ \text{and } t_A^*(\theta) = \theta \cdot \mathbf{1}_{(\theta \leq v)}(\theta).$$

P achieves a payoff of $\max\{v-\theta, 0\}$. Its ex-ante expected payoff is the maximal expected social surplus

$$\overline{W} \equiv \int_{\underline{\theta}}^{v} (v - \theta) dF(\theta).$$

No supervision If P and A are the only players and no supervisor is available, P's problem is equivalent to that of a monopsonistic buyer. Thus, P optimally offers A the price

$$p^m = \arg \max_{p \in [\underline{\theta}, \overline{\theta}]} (v - p) F(p).$$

P can also achieve the monopsony payoff if S is present. For instance, P could choose an uninformative signal and make the contract independent of any report of S. Thus, the monopsony payoff is a lower bound on P's expected payoff. I denote the monopsony payoff by $\underline{W}$.

No collusion If collusion is not possible, the principal's problem is equivalent to problem $\mathcal{P}$ without the collusion-proofness constraint (CP). In this case, P can achieve an expected payoff equal to the maximal social surplus $\overline{W}$. This can be done by choosing a signal which perfectly reveals θ. If P pays S a constant transfer of zero independently of her report, S is willing to share her information with P and the limited liability constraint is satisfied. P can then offer A a price exactly equal to his costs as long as $\theta \leq v$. A cannot do better than to accept this offer and P receives a payoff of $\max\{v - \theta, 0\}$. Its expected payoff is therefore $\overline{W}$.

Outline of the analysis

In the remainder of this section, I characterize an optimal combination of information structure and contract. At first, I show that under any collusion-proof

[14] The indicator function $\mathbf{1}_A(x)$ satisfies $\mathbf{1}_A(x) = 1$ if $x \in A$ and $\mathbf{1}_A(x) = 0$ if $x \notin A$.

contract, the total sum of transfers to S and A depends only on the project realization decision. This simplification allows me to derive an upper bound on P's payoff which is independent of the information structure and the contract. I then present a combination of an information structure and a contract with which P can reach the upper bound.

Furthermore, I analyze the general properties of optimal information structures and contracts with respect to the informativeness of S's signal, the rents of S and A, and the efficiency of project realization. Finally, I provide comparative statics with respect to the maximal loss that S can incur.

Simplifying transfers

I start with an observation which considerably simplifies the structure of transfers that P pays to S and A in a collusion-proof contract.

Lemma 1. *Under any feasible contract β, the total sum of transfers depends only on the project realization decision:*

$$\begin{aligned} & x(\sigma, \sigma, \theta) = x(\sigma', \sigma', \theta') \\ & \qquad \Rightarrow \quad t_S(\sigma, \sigma, \theta) + t_A(\sigma, \sigma, \theta) = t_S(\sigma', \sigma', \theta') + t_A(\sigma', \sigma', \theta') \end{aligned}$$

for all $(\sigma, \theta), (\sigma', \theta') \in supp(\mu)$ and all information structures $I \in \mathcal{I}$.

The lemma implies that for any collusion-proof contract, the total transfer to S and A depends only on whether the project is realized or not. To see why this is the case, suppose there is a contract for which the two reporting profiles (σ, σ, θ) and $(\sigma', \sigma', \theta')$ both lead to the realization of the project but the sum of transfers is higher after the report $(\sigma', \sigma', \theta')$. S can then propose a side-contract which reports $(\sigma', \sigma', \theta')$ whenever the true types are (σ, σ, θ) and use the transfer τ to make A indifferent between participation in the side-contract and non-cooperative play of the contract. As the total sum of transfers is higher under the report $(\sigma', \sigma', \theta')$, the side-contract is strictly profitable for S. The original contract is therefore not collusion-proof.

It follows directly from Lemma 1 that for any feasible contract β, there exist two numbers $(T, r) \in \mathbb{R}^2$ such that the sum of transfers to S and A under

β can be expressed as a function of the project realization decision:

$$t_S(\sigma, \sigma, \theta) + t_A(\sigma, \sigma, \theta) = T + x(\sigma, \sigma, \theta)r.$$

I call (T, r) the *collective transfers* associated with the contract β. As the sum of transfers to the two agents can only depend on whether the project is realized, it can take two values at most. Thus, there exist two numbers, T and r, such that the sum of transfers is T if the project is realized and $T + r$ if the project is abandoned. The unconditional transfer T can be interpreted as a signing fee that P pays to the supervisor and agent upon acceptance of the contract. The transfer r can be interpreted as a bonus that is additionally paid contingent on the realization of the project. In terms of the collective transfers, P's expected payoff from a feasible contract is

$$(v - r)\mathbb{E}[x(\tilde{\sigma}, \tilde{\sigma}, \tilde{\theta})] - T.$$

Upper bound on P's payoff

The results from the previous paragraphs can be used to derive an upper bound on P's payoff for all information structures and feasible contracts. This upper bound is based on the social surplus and a lower bound on the joint payoff which S and A can secure through a side-contract.

Any contract β induces some ex-ante probability of project realization. This probability is given by $X = \mathbb{E}[x(\tilde{\sigma}, \tilde{\sigma}, \tilde{\theta})]$. The expected social surplus under this contract cannot be greater than the social surplus under a contract β' that reaches the same ex-ante probability of project realization X and minimizes the expected costs. Thus, under β', A realizes the project only if his costs are weakly below the cutoff

$$\theta^c(X) \equiv \min \{\theta \in \Theta : F(\theta) \geq X\}.$$

The expected social surplus from the contract β' is

$$W^*(X) \equiv \int_{\underline{\theta}}^{\theta^c(X)} (v-\theta)dF(\theta) - (F(\theta^c(X)) - X)(v - \theta^c(X))$$
$$= X(v - \theta^c(X)) + \int_{\underline{\theta}}^{\theta^c(X)} F(\theta)d\theta,$$

where the second line follows from integration by parts. Due to the participation constraints of S and A, $W^*(X)$ is an upper bound on P's expected payoff under any contract β with an ex-ante probability of production X.

The following lemma allows me to derive a second upper bound on P's payoff.

Lemma 2. *For any information structure I and any feasible contract β with $X = \mathbb{E}[x(\tilde{\sigma}, \tilde{\sigma}, \tilde{\theta})]$, the associated collective transfers $(T, r) \in \mathbb{R}^2$ satisfy i) $T \geq -\ell$, ii) $r \geq \theta^c(X)$, and iii) $T + r \geq \theta^c(X)$.*

The limited liability of S implies that she can never pay more than ℓ to P. When A decides whether to participate, he is informed about the signal realization σ and about his costs θ. This implies that A can always avoid negative payoffs by not participating in the contract. Point *i)* follows from these two observations.

A realizes the project for the cost level $\theta^c(X)$ after some signal realization σ^c. S and A need to receive a *bonus* of at least $\theta^c(X)$ to compensate A for the production costs. Otherwise, A either does not want to participate in the contract, or S and A can find a profitable side-contract where A does not produce if the costs are $\theta^c(X)$. This implies that point *ii)* needs to be satisfied.

Finally, point *iii)* is implied by the following argument. If the unconditional payment T is positive, point *ii)* implies *iii)*. If it is negative, the total payment to S and A still needs to exceed the cutoff type $\theta^c(X)$. Otherwise, S would have a negative expected payoff after the signal realization σ^c and would not participate in the contract.

Note that P's expected payoff from the contract β is

$$X(v - T - r) - (1 - X)T.$$

Together with Lemma 2 this implies that P's expected payoff from any contract with an ex-ante probability of project realization X is bounded from above by

$$W^{\circ}(X) \equiv X(v - \theta^{c}(X)) + (1 - X)\ell.$$

The upper bound $W^{\circ}(X)$ reflects that P needs to pay the coalition at least a transfer equal to the cutoff type, whenever the project is realized. If the project is not realized, P can extract at most the maximal liability of S.

P's expected payoff is bounded from above by the social surplus $W^{*}(X)$ and the function $W^{\circ}(X)$. The following proposition states this result formally.

Proposition 1. *For any information structure I and any feasible contract β with an ex-ante probability of project realization $X = \mathbb{E}[x(\tilde{\sigma}, \tilde{\sigma}, \tilde{\theta})]$, P's expected payoff cannot exceed the upper bound*

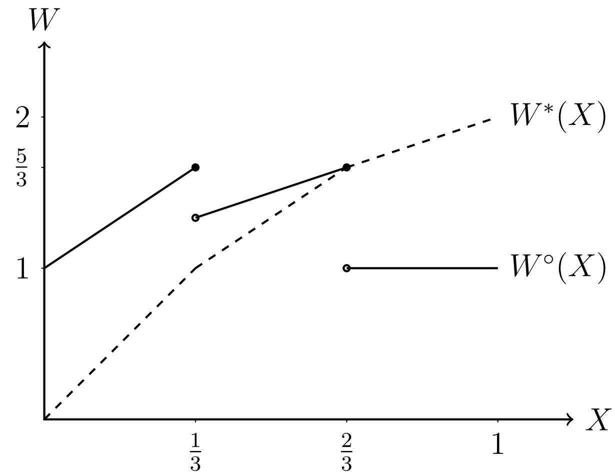
$$W(X) \equiv \min\left\{W^{*}(X), W^{\circ}(X)\right\}.$$

The proposition states that there exists an upper bound $W(X)$ on P's expected payoff which depends only on the ex-ante probability of project realization and is otherwise independent of the contract and the information structure. The minimal upper bound on the function $W(X)$ for any value of the ex-ante probability of project realization – given by $\sup_{X\in[0,1]} W(X)$ – constitutes a global upper bound on P's expected payoff for any information structure I and any feasible contract β.

Proposition 1 can be used to show that the partially revealing information structure and the contract proposed in the illustrative example in Section 3 are optimal. Figure 3 depicts the functions $W^{*}(X)$ and $W^{\circ}(X)$ for the illustrative example. The maximum of the two upper bounds is attained for $X = \frac{2}{3}$ where $W(\frac{2}{3}) = \frac{5}{3}$. The information structure and the contract described in the illustrative example give P an expected payoff of $\frac{5}{3}$ and are therefore optimal.

In Appendix A, I show that the upper bound $W(X)$ has a well-defined maximizer X^{c}. A global upper bound on P's expected payoff is therefore given by $W(X^{c})$. It follows that a combination of an information structure and a feasible contract is optimal if it gives P an expected payoff of $W(X^{c})$.

Figure 3: Upper bound on P's expected payoff in illustrative example



The dashed line is the first upper bound $W^*(X)$. The solid line is the second upper bound $W^\circ(X)$. The minimum of the two functions $W(X)$ is maximized at $X = \frac{2}{3}$.

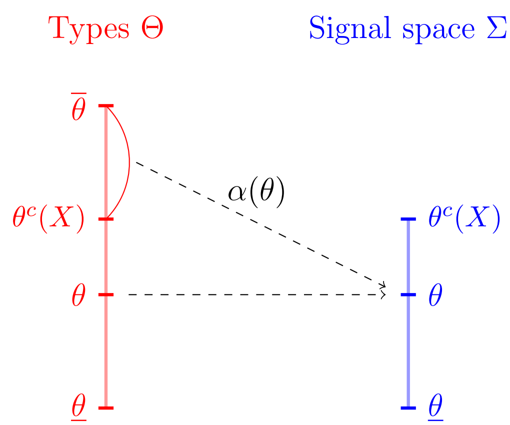
P can reach the upper bound

I now construct combinations of information structures and contracts with which P can implement any ex-ante probability of project realization X and achieve an expected payoff equal to the upper bound $W(X)$. P's optimal combination of information structure and contract implements the ex-ante probability of project realization X^c at an expected payoff $W(X^c)$.

The key idea behind the construction of the optimal combination of an information structure and a contract is to avoid rent payments to A without creating the possibility for S to earn high rents through collusion. For ease of exposition, suppose for now that the distribution $F(\theta)$ has a strictly positive density $f(\theta)$. All proofs in the appendix apply to the case where $F(\theta)$ exhibits mass points.

Consider an information structure as depicted in Figure 4. This information structure has a signal space equal to the cost-minimizing set of values of θ such that the project is realized with ex-ante probability X, i.e., $\Sigma = [\underline{\theta}, \theta^c(X)]$. If A's costs θ lie below the cutoff $\theta^c(X)$, the signal realization $\sigma = \theta$ is drawn. The remaining mass of non-producing types is distributed over all signal realizations in Σ. In particular, no type in $[\theta^c(X), \overline{\theta}]$ is more likely to generate a certain signal realization than any other type. The distribution of these types can therefore be described by a weighting function $\alpha : \Sigma \to \mathbb{R}_+$ which is positive and satisfies $\int_\Sigma \alpha(\sigma)d\sigma = 1$.

Figure 4: Weighted information structure



A weighted information structure has as many signal realizations as types below the cutoff $\theta^c(X)$. If costs take the value θ below the cutoff then the signal realization θ is generated. If costs take a value above the cutoff, a signal realization is generated according to the density $\alpha(\cdot)$ that depends only on the signal realization and not on the costs.

A weighting function $\alpha(\cdot)$ induces a cdf over costs conditional on a signal realization σ of

$$G(\theta|\sigma) = \begin{cases} 0 & \text{if} \quad \theta < \sigma \\ \frac{f(\sigma)}{f(\sigma)+(1-X)\alpha(\sigma)} & \text{if} \quad \theta \in [\sigma, \theta^c(X)] \\ \frac{f(\sigma)}{f(\sigma)+(1-X)\alpha(\sigma)} + \frac{(1-X)\alpha(\sigma)}{f(\sigma)+(1-X)\alpha(\sigma)} \frac{F(\theta)-F(\theta^c(X))}{1-F(\theta^c(X))} & \text{if} \quad \theta > \theta^c(X) \end{cases}$$

and a marginal cdf over signal realizations given by

$$H(\sigma) = \int_{\underline{\theta}}^{\sigma} (f(\sigma') + (1-X)\alpha(\sigma'))d\sigma'.$$

I denote an information structure with a characterizing weighting function of $\alpha(\cdot)$ by I_α and refer to it as a *weighted information structure.*

Such an information structure can be combined with a contract that induces project realization whenever S and A make the same report about the signal realization and this report coincides with A's report about his type:

$$x(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}) = \mathbf{1}_{(\hat{\sigma}_S = \hat{\sigma}_A = \hat{\theta} \leq \theta^c(X))}(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}). \tag{1}$$

The project is therefore realized whenever S and A both report the same signal realization and A reports having the lowest costs possible under the signal realization. Furthermore, the contract specifies a transfer to A that only compensates him for his costs in the event of his having the lowest cost level possible under the signal realization:

$$t_A(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}) = \hat{\theta} x(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}). \tag{2}$$

As the contract is required to be feasible, by Lemma 1, the transfer to S can be written as

$$t_S(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta}) = T + x(\hat{\sigma}_S, \hat{\sigma}_A, \hat{\theta})(r - \hat{\theta})$$

for the collective transfers (T, r).

Vis-à-vis A, the contract is equivalent to a price offer equal to the signal realization. Therefore, the contract satisfies A's participation constraint (PC_A) and his incentive compatibility constraint (IC_A). Under this contract, A never receives a rent.

On the equilibrium path after the signal realization σ, S receives an expected payoff of

$$U_S(\sigma) = T + \Pr(\theta = \sigma | \sigma)(r - \sigma) = T + \frac{f(\sigma)(r - \sigma)}{f(\sigma) + (1 - X)\alpha(\sigma)}.$$

From Lemma 2 it is known that $r \geq \theta^c(X)$. Thus, S has nothing to gain from misreporting the signal realization, as this would reduce her expected payoff to T. Her incentive compatibility constraint (IC_S) is therefore satisfied.

S's participation constraint (PC_S) is satisfied if S has a positive expected payoff after all signal realizations. P wants to set the fix payment T as low as possible. Thus, S's participation constraint has to be binding after the signal realization for which the expected gain from the project is smallest:

$$T = -\min_{\Sigma} \left\{ \frac{f(\sigma)(r-\sigma)}{f(\sigma)+(1-X)\alpha(\sigma)} \right\}.$$

For a given variable payment r, P wants to choose the weighting function $\alpha(\cdot)$ which maximizes S's minimal expected gain from project completion over all signal realizations. Thus, P would like to choose $\alpha(\cdot)$ such that S's expected gain from trade is constant across all signal realizations and would like to extract the expected gain through the fix payment T. If this is possible, P can avoid rent payments to S.

A form of the weighting function which leaves S's expected payoff $U_S(\sigma)$ constant across all signal realizations σ is given by

$$\alpha(\sigma) = \frac{f(\sigma)}{1-X} \cdot \left(\frac{r-\sigma}{C} - 1 \right).$$

This weighting function gives S an expected gain of $U_S(\sigma) = T + C$ for a given positive constant $C \in \mathbb{R}_+$.

If there exists a constant $C \leq \ell$ such that the weighting function α is well-defined then P can extract the whole expected social surplus for a given ex-ante probability of project realization of X without violating S's limited liability constraint (LL). This turns out to be possible for any ex-ante probability X where the function $W^*(X)$ is the stricter upper bound on P's expected payoff.

Lemma 3. *Consider the weighted information structure I_α and the contract β that are defined by the equations* (1), (2), *and*

$$T = -\frac{\int_{\underline{\theta}}^{\theta^c(X)} F(\theta) d\theta}{1-X}, \quad r = \theta^c(X) - T, \quad \textit{and} \quad \alpha(\sigma) = \frac{f(\sigma)}{1-X} \cdot \frac{\theta^c(X)-\sigma}{r-\theta^c(X)}.$$

If $W^*(X) \leq W^\circ(X)$, I_α is well-defined, β is feasible, and $U_P(I_\alpha, \beta) = W(X)$.

If the upper bound $W^\circ(X)$ is more restrictive, P can still find a constant C such that the weighting function $\alpha(\cdot)$ is well-defined. However, the constant has to be greater than the maximal loss ℓ. This is not feasible and implies that P has to leave an information rent to S. It turns out that it is optimal for P to set the weight of all signal realizations between some value $\check{\theta}(X)$ and $\theta^c(X)$

to $\alpha(\cdot) = 0$ and to choose for the remaining signal realizations in $[\underline{\theta}, \check{\theta}(X)]$ a weighting function which makes the expected payoff of S for these signal realizations constant:

Lemma 4. *Consider the weighted information structure I_α and the contract β that are defined by the equations* (1), (2), *and*

$$T = -\ell, \quad r = \theta^c(X) + \ell, \quad \text{and} \quad \alpha(\sigma) = \frac{f(\sigma)}{1 - X} \cdot \frac{[\check{\theta}(X) - \sigma]_+}{r - \check{\theta}(X)},$$

where $\check{\theta}(X) \in [\underline{\theta}, \theta^c(X)]$ is uniquely defined by the equation

$$\int_{\underline{\theta}}^{\check{\theta}(X)} F(\sigma) d\sigma = (1 - X)(r - \check{\theta}(X)).$$

If $W^\circ(X) \leq W^*(X)$, I_α is well-defined, β is feasible, and $U_P(I_\alpha, \beta) = W(X)$.

Under the information structures and the contracts defined in the two lemmas above, S cannot find a strictly profitable collusive agreement with A, i.e., the collusion-proofness constraint (CP) is satisfied. First, note that S has nothing to gain from a side-contract where the project is not realized even if A's costs lie below the cutoff $\theta^c(X)$. In this case, the colluding coalition loses a payoff of $r - \sigma$. Second, S does not gain from a side-contract where S and A misreport the signal realization and induce project realization if A's costs equal the true signal realization. In this case, the total expected payment to the coalition is the same with and without collusion and the probability of project realization does not change either. Finally, note that the collective transfers in both lemmas satisfy that the total payment equals the cutoff type if the project is realized, i.e., $T + r = \theta^c(X)$. This implies that it is not profitable for the colluding coalition to extend project realization to cost levels above the cutoff $\theta^c(X)$ where S and A would jointly incur a loss.

The insights from Lemmas 3 and 4 allow me to make the following statements.

Proposition 2. *For any ex-ante probability of project realization $X \in [0, 1]$ there exists an information structure I and a feasible contract β that implement X and give P an expected payoff equal to the upper bound $W(X)$.*

If $W^*(X^c) \leq W^\circ(X^c)$, P's problem is solved by the information structure and the contract defined in Lemma 3 for $X = X^c$. If $W^\circ(X^c) \leq W^*(X^c)$, P's problem is solved by the information structure and the contract defined in Lemma 4 for $X = X^c$.

For any ex-ante probability of project realization X, P can find an information structure and a contract with a payoff at the upper bound $W(X)$. P optimally implements the probability of project realization X^c that maximizes the upper bound function $W(X)$. Lemma 3 gives the optimal combination of information structure and contract in the case where the upper bound derived from social surplus $W^*(X)$ is more restrictive at X^c. If the upper bound $W^\circ(X)$ is more restrictive at X^c, Lemma 4 provides the optimal choice of P.

Properties of optimal informational control and contracts

In this section, I show how optimal information structures and contracts influence the informedness of S, rent payments, and the efficiency of project realization. Whereas the previous section characterized one solution to P's problem, the results derived here apply to any optimal combination of information structure and contract.

Proposition 3. *Any optimal combination of information structure and contract satisfies the following properties.*

- *S is partially informed about A's costs θ if $\ell > 0$.*
- *If $W^*(X^c) \leq W^\circ(X^c)$, neither S nor A receives a rent. If $W^\circ(X^c) \leq W^*(X^c)$, P leaves a rent to S and A.*
- *The project realization decision is inefficient if*

$$\ell < \overline{\ell} \equiv \begin{cases} \frac{\overline{W}}{1-F(v)} & \text{if} \quad v < \overline{\theta}, \\ \infty & \text{if} \quad v \geq \overline{\theta}. \end{cases}$$

P optimally chooses an information structure that gives S some but not all information on A's costs. With both extreme information structures – no information and full information – P makes at most the monopsony profit $\underline{W}$. With a partially revealing information structure, P can make a higher payoff.

P benefits from the presence of S due to the latter's ability to absorb losses. Under a partially revealing information structure, S is uncertain of A's behavior in both the contract and the side-contract. P exploits this uncertainty. In the optimal contract from the previous section, P demands an upfront payment from S. With these payments P extracts some of the rents that S earns through the side-contract with A. S is willing to enter the contract with P as she accepts reaching a break-even in expectation. Thus, S's uncertainty relaxes her participation constraint to the benefit of P.

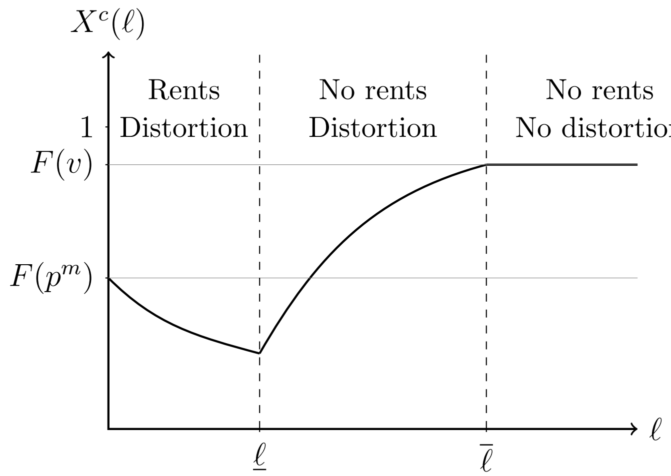
Rent payments to S and A depend on which of the two upper bounds $W^*(X)$ and $W^\circ(X)$ is more restrictive at the optimal ex-ante probability of project realization X^c. If $W^*(X)$ is more restrictive, the upper bound $W(X)$ equals the social surplus. In this case, neither S nor A receive a positive rent. If the second upper bound $W^\circ(X)$ is more restrictive, P cannot extract the complete social surplus. In the solution to P's problem presented in Lemma 4, S receives the remaining surplus as a rent. However, this solution is not unique. There exist other optimal combinations of an information structure and a contract for which the remaining surplus is split between S and A.

The project realization decision is distorted if the limitation on the liability of S is sufficiently strict. In this case, P implements an ex-ante probability of project realization that is smaller than the efficient probability $F(v)$. Efficiency is only achieved if S's maximal loss ℓ exceeds the expected maximal social surplus from project realization by a factor of $\frac{1}{1-F(v)}$. If the project has a high value for P, this factor is large. In this case, the distortion will most likely arise. Indeed, if P's value v exceeds the highest possible cost of A, then P can never extract the maximal social surplus.

Comparative statics

In this section, I analyze the influence of the maximal loss ℓ on rents, distortions, and the informedness of S in more detail. To this end, I assume that the cdf $F(\theta)$ is strictly increasing and differentiable on Θ with a decreasing inverse hazard rate $F(\theta)/f(\theta)$. This relatively mild assumption allows me to derive clear results on the effect of changes in the maximal loss ℓ. For a given maximal loss ℓ, $\theta^c(\ell)$ denotes the optimal cutoff value for project realization and $\Sigma(\ell)$ denotes the signal space chosen by P. The signal space $\Sigma(\ell)$ increases (decreases) in ℓ, if $\Sigma(\ell') \subset \Sigma(\ell'')$ for $\ell' < \ell''$ ($\ell' > \ell''$).

Figure 5: Optimal probability of project realization



For $\ell = 0$, the probability of project realization is the same as in the monopsony case. For $\ell < \underline{\ell}$, the probability is decreasing, P has to pay rents. For $\ell > \underline{\ell}$, neither S nor A receives a rent. For $\ell \in (\underline{\ell}, \overline{\ell})$, the probability is increasing. For $\ell \geq \overline{\ell}$, it is efficient. If there are certain gains from trade, i.e. $\overline{\theta} \leq v$, efficiency is never attained as $\overline{\ell} = \infty$.

Proposition 4. *Suppose $F(\theta)$ has a strictly positive density $f(\theta)$ and the inverse hazard rate $F(\theta)/f(\theta)$ is increasing. There exists a value of the maximal liability $\underline{\ell}$ with $\underline{\ell} < \overline{\ell}$ such that*

- *the probability of project realization is decreasing in ℓ for $\ell \in [0, \underline{\ell}]$, increasing in ℓ for $\ell \in (\underline{\ell}, \overline{\ell})$, and constant at the efficient level for $\ell \geq \overline{\ell}$.*
- *Total rent payments to S and A are decreasing in ℓ for $\ell \in [0, \underline{\ell})$ and zero for $\ell \geq \underline{\ell}$.*
- *the signal space $\Sigma(\ell)$ is decreasing in ℓ for $\ell \in [0, \underline{\ell}]$, increasing in ℓ for $\ell \in (\underline{\ell}, \overline{\ell})$, and constant for $\ell \geq \overline{\ell}$—given that P chooses the optimal information structure described in Proposition 2.*

The probability of project realization is not monotone in the maximal loss ℓ. Figure 5 depicts how the cutoff value of project realization changes with the maximal loss ℓ. For low levels of ℓ, an increase in the maximal loss reduces the probability with which the project is realized. For high levels of ℓ, the probability of project realization increases with ℓ.

For low levels of ℓ, P uses an increase in ℓ to reduce rent payments to S. This is optimally done by *deleting* the worst signal realizations and extracting a higher fixed payment (equal to ℓ) from better signal realizations. If ℓ reaches $\underline{\ell}$, P can avoid rent payments to S. Any increase of ℓ above $\underline{\ell}$ is therefore used by P to increase the probability of project realization and to extract the additional social surplus.

In contrast to the comparative statics of the probability of project realization and the total rents to S and A, the results for the information structure only hold if P selects the optimal information structure described in Proposition 2. Nevertheless, the comparative statics for this case generally show that S does not necessarily become better informed if she can absorb a higher loss. For low levels of ℓ, her informedness can decline as ℓ increases. Under the optimal weighted information structure in Proposition 2, the signal space is $\Sigma(\ell) = [\underline{\theta}, \theta^c(\ell)]$. The size of $\Sigma(\ell)$ changes one-to-one with $\theta^c(\ell)$ and is smallest for $\ell = \underline{\ell}$.

The size of the signal space is a rough measure of the informativeness of an optimal information structure. In order to state whether S is better informed for a maximal loss ℓ compared with a maximal loss ℓ', the two optimal information structures should be ranked in the sense of a Blackwell ordering. An information structure I is more informative in the sense of Blackwell than another information structure I', if the posterior beliefs that are reached through the different signal realization of I' can be replicated by sending S a garbling of the signal generated by I.

However, it is generally not possible to order the optimal information structures in the sense of Blackwell. Even if the signal space expands with a change in the maximal loss ℓ, the posterior belief after some signal realizations can become noisier. Therefore, the two signal structures pre- and post-change cannot be ordered unambiguously with respect to their informativeness. For the special case of a uniform distribution of $\tilde{\theta}$, Proposition 7 in the appendix shows that for low levels of ℓ, S's signal becomes less informative as ℓ increases.

6 Delegation

With optimal informational control, delegation is the best organization for P. Under delegation, P only contracts with S who in turn contracts with A. This form of organization can implement the optimal contract if P chooses an optimal information structure. P can give up direct control over A without a reduction in payoff as long as P exerts informational control optimally. Thus, informational control is a substitute for direct control.

The following timing defines the delegation game δ:

t=0: P chooses an information structure $I \in \mathcal{I}$ and offers a delegation contract (T, r) to S, where T is a transfer that P pays to S upon acceptance, and r is a price that S receives if the project is realized.

t=1: S and A observe I, (T, r), and σ, A observes θ.

t=2: S accepts or rejects the delegation contract. If S rejects, the game ends and the project is not realized.

t=3: If S accepts, she offers a price p to A.

t=4: A accepts the price and realizes the project or rejects the offer and the project is not realized.

In this delegation game, P contracts only with S and authorizes S to contract with A. I denote the reduced delegation game where the information structure I is exogenously fixed by δ_I.

If P chooses an optimal information structure, it can implement the optimal centralized contract in the delegation game.

Proposition 5. *If P exerts informational control optimally, delegation is an optimal form of organization. In particular, there exists a PBE of the delegation game δ where P chooses the information structure of Proposition 2 and the delegation contract defined by the collective transfers (T, r) associated with the optimal contract of Proposition 2. In this equilibrium, the project is realized* *with ex-ante probability X^c, and P attains the expected payoff $W(X^c)$. Following the choices of P at $t = 0$, the equilibrium of the continuation game starting at $t = 1$ is unique.*

Given an optimal information structure, P can delegate the interaction with A to S without a reduction of its payoff. Thus, informational control substitutes direct control. Moreover, the equilibrium of the continuation game starting after the first stage at which P selects the information structure and the delegation contract is unique. The optimal contract does not have this property, which renders delegation an attractive form of organization from the perspective of implementation.[15]

The proposition can be proved based on the following Lemma.

Lemma 5. *For any information structure I, the solution to P's reduced problem $\mathcal{P}_I$ can be implemented in the reduced delegation game δ_I if A receives no rent on the equilibrium path, i.e., $t_A(\sigma,\sigma,\theta) - \theta x(\sigma,\sigma,\theta) = 0$ for all $\sigma, \theta \in \Sigma \times \Theta$.*

If P delegates contracting with A to S, one instrument which is present under the centralized contract is forgone. This instrument consists of providing rents to A. These rents make it harder for S to find a side-contract that is acceptable to A. In particular, P can relax the collusion-proofness constraint (CP) by tightening A's participation constraint in the side-contract (PC_A^γ). However, this comes at the cost of providing rents to A. In other words, P may channel additional rents to A in order to make it expensive for S to *bribe* A into joining the colluding coalition.

If P can jointly choose the information structure and the contract, P does not make use of this instrument: Under the optimal combination of an information structure and a contract identified in Lemmas 3 and 4, A never receives a rent. P does not find it optimal to make it more expensive to bribe A. It follows that P is as well off under delegation as under the optimal contract.

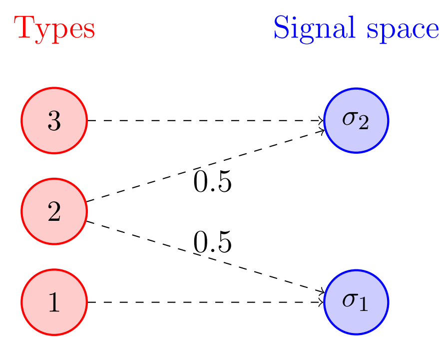
Under the optimal information structure and delegation contract S sets a price that gives her a strictly higher payoff than all other prices. Thus, the reduced delegation game for which the optimal information structure is exogenously fixed uniquely implements the outcome of the optimal contract.

[15]See also Faure-Grimaud et al. (2003) who stress this feature of delegation.

Delegation can be suboptimal for suboptimal information structures

Delegation is not optimal for any information structure. For a given suboptimal information structure, P can be harmed by giving up direct control over A. I show this by revisiting the illustrative example.

Figure 6: Suboptimal information structure and suboptimal delegation



Information structure with suboptimal delegation consists of two signal realizations. If costs are low, the signal realization σ_1 is generated. If costs are high, the signal realization σ_2 is generated. If costs are intermediate, each signal realization is generated with equal probability.

Suppose S's signal is generated by the information structure depicted in Figure 6. This information structure consists of two signal realizations σ_1 and σ_2, where σ_1 is always send for $\theta = 1$, σ_2 is always send for $\theta = 3$, and both signal realizations are send with equal probability for $\theta = 2$.

Given this information structure, P strictly prefers the optimal contract over delegation. P even prefers to ignore S and to contract directly with A instead of authorizing S to contract with A.

Proposition 6. *Suppose that θ is uniformly drawn from $\Theta = \{1, 2, 3\}$, $v = 4$, and $\ell \geq 0$. Given the information structure depicted in Figure 6, delegation is strictly suboptimal.*

The information structure in Figure 6 features one good and one bad signal realization. The presence of the bad signal realization harms P as it needs to ensure S's participation following the bad signal realization. For instance, consider the case where P sets a delegation contract under which the project is always realized. This implies that P receives the payoff $-T + 4 - r$. After the signal realization σ_2, S offers A a price of 3 to ensure that the project is realized. The payoff of S after this signal realization is therefore $T + r - 3$. S only accepts the delegation contract after the signal realization σ_2 if $T + r \geq 3$, which implies that P's payoff cannot exceed 1. As I show in the proof of this proposition, P can never earn a higher payoff than 1 for any delegation contract (T, r). In contrast, P can earn an expected payoff of $\frac{4}{3}$ if it directly offers the monopsony price to A. Delegation is therefore strictly suboptimal under the suboptimal information structure depicted in Figure 6.

7 Conclusion

This paper provides an analysis of the optimal use of informational control in a model of collusive supervision. I study a principal-supervisor-agent model in which the agent can realize a project for the principal at a privately known cost. The supervisor observes a signal about the agent's costs and can collude with the agent. The principal has informational control and designs the supervisor's signal without being able to observe it.

The first main contribution of this paper is an analysis of optimal combinations of a signal and a contract. The optimal design of the signal is driven by a trade-off between information elicitation and collusion prevention: The more informative the supervisor's signal, the smaller the informational advantage of the agent over the supervisor. This benefits the principal if the supervisor shares her information truthfully. However, a more informative signal reduces the information asymmetry in the colluding coalition and enables the supervisor to organize collusion with the agent more effectively to the detriment of the principal. The principal's optimal signal provides the supervisor with partial information about the agent's costs. Moreover, the principal designs the optimal signal such that all signal realizations are of equal value to the supervisor: upside potential is balanced by downside risk. This construction avoids bad signal realizations for which the principal would need to pay high transfers to compensate the supervisor.

As the second main contribution this paper studies the implications of informational control on the organization of contractual relationships. Given the optimal signal, I show that the principal can authorize the supervisor to contract with the agent and can still receive the same payoff as in the optimal contract. Thus, delegation is an optimal form of organization if the principal exerts informational control in the best possible way.

These results speak to the policy discussion on the differences between public procurement guidelines in the US and the EU. Traditionally, public procurement rules in the US have encouraged the use of data on the past performances of suppliers for the award of new contracts. In contrast, the use of such data is very restricted in the EU due to the fear that the transparency of procurement process may be reduced which would facilitate corruption. The results of this paper show that effective countermeasures against corruption should not only restrict the use but also the availability of data to procurement officers.

International institutions such as the World Bank or the OECD advocate the use of digitalized procurement procedures (e-procurement) as an antidote against corruption. E-procurement gives a central government much more control over the information a local government or procurement officer receives than traditional forms of procurement that necessitate personal contact between procurement officers and suppliers. Central governments should carefully choose the information that e-procurement systems make available to procurement officers. In particular, they should bear in mind that more information provision may facilitate corruption.

Appendix

A Omitted proofs

Proof of Lemma 1

Toward a contradiction, suppose there exists a contract β which satisfies all constraints. Moreover there exist signal realization-cost-pairs $(\sigma', \theta'), (\sigma'', \theta'') \in supp(\mu)$ such that

$$x(\sigma', \sigma', \theta') = x(\sigma'', \sigma'', \theta'') \text{ and}$$
$$t_S(\sigma', \sigma', \theta') + t_A(\sigma', \sigma', \theta') < t_S(\sigma'', \sigma'', \theta'') + t_A(\sigma'', \sigma'', \theta'').$$

Consider now the side-contract $\gamma = (\tau, \rho)$ which is defined as $\tau(\theta; s) = 0$ for $(\sigma, \theta) \neq (\sigma', \theta')$, $\tau(\theta'; \sigma') = t_A(\sigma'', \sigma'', \theta'') - t_A(\sigma', \sigma', \theta')$, $\rho(\theta; \sigma) = (\sigma, \sigma, \theta)$ for $(\sigma, \theta) \neq (\sigma', \theta')$, and $\rho(\theta'; \sigma') = (\sigma'', \sigma'', \theta'')$. This side-contract is feasible by construction, because (PC_A) implies (PC_A^γ) and (IC_A) implies (IC_A^γ). Furthermore

$$\begin{aligned}
&\mathbb{E}\left[t_S(\rho(\theta; \sigma')) + \tau(\theta; \sigma') | \sigma'\right] \\
&\quad = \mu(\theta \neq \theta' | \sigma') \mathbb{E}\left[t_S(\sigma', \sigma', \theta) | \sigma', \theta \neq \theta'\right] \\
&\qquad\qquad + \mu(\theta' | \sigma') \left(t_S(\sigma'', \sigma'', \theta'') + t_A(\sigma'', \sigma'', \theta'') - t_A(\sigma', \sigma', \theta')\right) \\
&\quad > \mu(\theta \neq \theta' | \sigma') \mathbb{E}\left[t_S(\sigma', \sigma', \theta) | \sigma', \theta \neq \theta'\right] + \mu(\theta' | \sigma') t_S(\sigma', \sigma', \theta') \\
&\quad = \mathbb{E}\left[t_S(\sigma', \sigma', \theta) | \sigma'\right].
\end{aligned}$$

Thus, (CP) is not satisfied for $\sigma' \in \Sigma$, which gives a contradiction. □

Proof of Lemma 2

$T \geq -\ell$ follows from the argument presented in the first paragraph after the lemma. It remains to show that $r \geq \theta^c(X)$ and $T + r \geq \theta^c(X)$.

Let σ^c be the signal realization after which the type $\theta^c(X)$ realizes the project. If $X = 1$, the expected payoff of S after the signal realization σ^c cannot be higher than $T + r - \theta^c(1)$. Thus, S's participation constraint is only satisfied after the signal realization σ^c if $T + r \geq \theta^c(1)$.

Suppose now that $X < 1$. Thus, there exist $(\sigma_0, \theta_0) \in \Sigma \times \Theta$ such that $x(\sigma_0, \sigma_0, \theta_0) = 0$. I first show that $r \geq \theta^c(X)$. Toward a contradiction suppose that $r < \theta^c(X)$. Consider now the side-contract γ which is defined as

$$\rho(\hat{\theta}; \sigma) = \begin{cases} (\sigma_0, \sigma_0, \theta_0) & \text{if } (\sigma, \hat{\theta}) \in \{\sigma^c\} \times (r, \theta^c(X)], \\ (\sigma, \sigma, \hat{\theta}) & \text{otherwise}, \end{cases} \quad \text{and}$$

$$\tau(\hat{\theta}; \sigma) = \begin{cases} t_A(\sigma^c, \sigma^c, \hat{\theta}) - t_A(\sigma_0, \sigma_0, \theta_0) & \text{if } (\sigma, \hat{\theta}) \in \{\sigma^c\} \times (r, \theta^c(X)], \\ 0 & \text{otherwise}. \end{cases}$$

This side-contract always gives A the same payoff as playing the contract non-cooperatively. It follows that the side-contract is feasible. S receives under the side-contract the same payoff as under the contract if $(\sigma, \theta) \notin \{\hat{\sigma}\} \times (r, \theta^c(X)]$. If $(\sigma, \theta) \in \{\hat{\sigma}\} \times (r, \theta^c(X)]$, S's payoff under the side-contract exceeds her payoff from the contract by $\theta - r > 0$. Thus, the contract β is not collusion-proof and it follows that $r \geq \theta^c(X)$.

S's expected payoff after the signal realization σ^c can be written as

$$\begin{aligned} &\mathbb{E}[x(\sigma^c, \sigma^c, \tilde{\theta})|\sigma^c](T + r) + (1 - \mathbb{E}[x(\sigma^c, \sigma^c, \tilde{\theta})|\sigma^c])T - \mathbb{E}[t_A(\sigma^c, \sigma^c, \tilde{\theta})|\sigma^c] \\ &\leq \mathbb{E}[x(\sigma^c, \sigma^c, \tilde{\theta})|\sigma^c](T + r - \theta^c(X)) + (1 - \mathbb{E}[x(\sigma^c, \sigma^c, \tilde{\theta})|\sigma^c])T \end{aligned}$$

where the second line follows from the fact that (PC_A) implies

$$\mathbb{E}[t_A(\sigma^c, \sigma^c, \tilde{\theta})|\sigma^c] \geq \mathbb{E}[x(\sigma^c, \sigma^c, \tilde{\theta})|\sigma^c])\theta^c(X).$$

It follows from (PC_S) that

$$\mathbb{E}[x(\sigma^c, \sigma^c, \tilde{\theta})|\hat{\sigma}](T + r - \theta^c(X)) + (1 - \mathbb{E}[x(\sigma^c, \sigma^c, \tilde{\theta})|\sigma^c])T \geq 0.$$

If $T \geq 0$, this is satisfied and $T + r \geq \theta^c(X)$. If $T < 0$, this can only be satisfied if $T + r \geq \theta^c(X)$. □

Proof of Proposition 1

The proof follows from the arguments in the main text. □

Characterization of the maximizer of $\mathbf{W}(\mathbf{X})$

I show that the maximization problem $\max_{X\in[0,1]} W(X)$ has a solution. In order to characterize the maximizer, denote the intersection point of $W^*(X)$ and $W^\circ(X)$ by $X^\dagger$. As the functions $W^*(X)$ and $W^\circ(X)$ may not necessarily intersect, the intersection point may also describe the point where one function jumps above the other. The intersection point is defined by

$$X^\dagger \equiv \Big\{X \in [0,1]: \quad W^\circ(X') > W^*(X') \text{ for } X' < X, \quad W^\circ(X') < W^*(X') \text{ for } X' > X\Big\}. \tag{3}$$

Furthermore, it is helpful to define the following constrained maximizer of the function $W^\circ(X)$

$$X^\circ(X) \equiv \max\left\{\arg\max_{X'\in[X,1]} W^\circ(X')\right\}. \tag{4}$$

The following Lemma characterizes the ex-ante probability X^c at which the upper bound on P's payoff attains its maximum.

Lemma 6. *The intersection point $X^\dagger$ is uniquely defined by equation (3) and $X^\circ(X)$ is well-defined for all $X \in [0,1]$ by equation (4). Moreover, a solution to the optimization problem $\max_{X\in[0,1]} W(X)$ exists and is given by*

$$X^c = \begin{cases} X^\dagger & \text{if} \quad X^\dagger < F(v) \text{ and } W^*(X^\dagger) > W^\circ(X^\circ(X^\dagger)) \\ X^\circ(X^\dagger) & \text{if} \quad X^\dagger < F(v) \text{ and } W^*(X^\dagger) \leq W^\circ(X^\circ(X^\dagger)) \\ F(v) & \text{if} \quad X^\dagger \geq F(v). \end{cases}$$

Proof. Note that $W^\circ(X) - W^*(X) = (1-X)\ell - \int_{\underline{\theta}}^{\theta^c(X)} F(\theta)d\theta$ is strictly decreasing in X. Thus, $X^\dagger$ is uniquely defined.

Next, observe that $\theta^c(X)$ is an increasing and left-continuous function in X as $F(\theta)$ is a cdf and is therefore increasing and right-continuous. It follows that $W^\circ(X)$ is left-continuous and makes a downward jump at any point where $W^\circ(X)$ is discontinuous, i.e., $\lim_{X'\nearrow X} W^\circ(X') \geq \lim_{X'\searrow X} W^\circ(X')$ for any $X \in [0,1]$. Thus, $\max_{X'\in[X,1]} W^\circ(X')$ has a solution for any $X \in [0,1]$ and $X^\circ(X)$ is well-defined.

As, $W^*(X)$ is a continuous function, it follows that $W(X)$ is a continuous function for $X < X^\dagger$ and a left-continuous function which exhibits only downward jumps for $X > X^\dagger$. At $X = X^\dagger$, either $W^*(X^\dagger) \geq W^\circ(X^\dagger)$ by continuity of $W^*(X)$ and left-continuity of $W^\circ(X)$. Thus, $W(X)$ is also left-continuous with only downward jumps. From this it follows that the solution to $\max_{X\in[0,1]} W(X)$ is well-defined.

If $X^\dagger \geq F(v)$, then for all $X \in [0,1]$, $W(X) \leq W^*(X) \leq W^*(F(v)) = W(F(v))$. Thus, $W(X)$ is maximized for $X = F(v)$ in this case. Suppose now that $X^\dagger < F(v)$. Note that $W(X) = W^*(X)$ for $X \leq X^\dagger$ and therefore $X^\dagger = \arg\max_{X\in[0,X^\dagger]} W(X)$. $X^\circ(X^\dagger)$ maximizes $W^\circ(X)$ on $[X^\dagger, 1]$. If $W^*(X^\dagger) > W^\circ(X^\circ(X^\dagger))$, then $W(X)$ is maximized for $X = X^\dagger$. If $W^*(X^\dagger) \leq W^\circ(X^\circ(X^\dagger))$, then $W(X)$ is maximized for $X = X^\circ(X^\dagger)$. Thus, X^c as defined in the Lemma maximizes $W(X)$. □

Proof of Lemma 3

To accommodate the possibility of mass points, I define

$$\Delta_F(\theta) \equiv F(\theta) - \lim_{\theta' \nearrow \theta} F(\theta') \tag{5}$$

and

$$f(\theta) = \begin{cases} F'(\theta) & \text{if} \quad F'(\theta) \text{ exists;} \\ \Delta_F(\theta) & \text{if} \quad \Delta_F(\theta) > 0; \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

I denote the set of mass points by $\Theta_D \equiv \{\theta \in \Theta : \Delta_F(\theta) > 0\}$. As $F(\cdot)$ is a cdf, there can be at most countably many points θ where $F(\cdot)$ is not differentiable.

It needs to be checked whether I_α is a well-defined information structure, whether β is a feasible contract, and whether P achieves an expected payoff of $W(X)$. This is the case if *i*) $\alpha(\cdot)$ is a well-defined weighting function, *ii*) the constraints (LL), (PC_S), (PC_A), (IC_S) and, (IC_A) hold, *iii*) S and A cannot profit from a joint misrepresentation of the signal realization σ, and *iv*) $U_P(I, \beta) = W(X)$.

Note at first that $\alpha(\sigma) \geq 0$ for all $\sigma \in \Sigma$. Furthermore,

$$\int_{\Sigma} \alpha(\sigma) d\sigma = \int_{\Sigma} \frac{f(\sigma)(\theta^c(X) - \sigma)}{(1-X)(r - \theta^c(X))} d\sigma = \frac{\int_{\underline{\theta}}^{\theta^c(X)} F(\sigma) d\sigma}{-(1-X)T} = 1,$$

where the second equality follows from integration by parts. Condition *i*) is thus satisfied and I_α is a well-defined information structure.

I now show that β is a feasible contract. β satisfies the limited liability constraint (LL):

$$\begin{aligned} T = -\frac{\int_{\underline{\theta}}^{\theta^c(X)} F(\theta) d\theta}{1-X} &\geq -\ell \\ \Leftrightarrow \quad (1-X)\ell - \int_{\underline{\theta}}^{\theta^c(X)} F(\theta) d\theta &= W^\circ(X) - W^*(X) \geq 0. \end{aligned}$$

Note that under the proposed contract, A faces a posted price that equals σ in equilibrium. Thus, (PC_A) and (IC_A) are satisfied. (IC_S) is also satisfied, because a unilateral misreport from S results in a payoff of $T < 0$. It remains to check whether (PC_S) is satisfied and this is indeed the case as

$$\begin{aligned} U_S(\sigma) &= T + \Pr(\theta = \sigma | \sigma)(r - \sigma) \\ &= T + \frac{f(\sigma)(r-\sigma)}{f(\sigma) + (1-X)\alpha(\sigma)} \\ &= T + r - \theta^c(X) = 0, \end{aligned}$$

where the third equality follows from the definition of $\alpha(\cdot)$ and the last equality follows from the definition of r. Thus, condition *ii*) is satisfied.

Next, it is shown that S cannot find a feasible and profitable side-contract. Under any such side-contract, S essentially needs to make a take-it-or-leave-it offer to A. As the support of the random variable $\tilde{\theta}$ conditional on the signal realization $\sigma = \theta$ is $\{\theta\} \cup (\theta^c(X), \overline{\theta}]$, a deviation implies that S offers a price $\theta' \geq \theta^c(X)$. S's payoff under such a deviation takes the form $T + q(r - \theta)$ where $q \in [0, 1]$ is the probability of project realization under the deviation.

Note that

$$T + q(r - \theta) \leq T + q(r - \theta^c(X)) = (1-q)T \leq 0.$$

Thus, S cannot find a profitable and feasible side-contract and condition $iii)$ is satisfied.

Finally, condition $iv)$ follows from the following calculation:

$$\begin{aligned}
U_P(I,\beta) &= X(v-r) - T \\
&= X(v-r-T) - (1-X)T \\
&= X(v-\theta^c(X)) + \int_{\underline{\theta}}^{\theta^c(X)} F(\theta)d\theta \\
&= W^*(X) \\
&= W(X).
\end{aligned}$$

□

Proof of Lemma 4

Consider $f(\cdot)$ as defined in the equations (5) and (6). It needs to be checked whether I_α is a well-defined information structure, whether β is a feasible contract, and whether P achieves an expected payoff of $W(X)$. This is the case if $i)$ $\alpha(\cdot)$ is a well-defined weighting function, $ii)$ the constraints (LL), (PC_S), (PC_A), (IC_S) and, (IC_A) hold, $iii)$ S and A cannot profit from a joint misrepresentation of the signal realization σ, and $iv)$ $U_P(I,\beta) = W(X)$.

Note at first that $\alpha(\sigma) \geq 0$ for all $\sigma \in \Sigma$. Furthermore,

$$\int_\Sigma \alpha(\sigma)d\sigma = \int_{\underline{\theta}}^{\check{\theta}(X)} \frac{f(\sigma)(\check{\theta}(X) - \sigma)}{(1-X)(r - \check{\theta}(X))} d\sigma = \frac{\int_{\underline{\theta}}^{\check{\theta}(X)} F(\sigma)d\sigma}{(1-X)(r-\check{\theta})} = 1,$$

where the second equality follows from integration by parts and the third equality follows from the definition of $\check{\theta}$, Condition $i)$ is thus satisfied and I_α is a well-defined information structure.

I now show that β is a feasible contract. β satisfies the limited liability constraint (LL) as $T = -\ell$.

Under the contract β, A faces a posted price that equals σ in equilibrium. Thus, (PC_A) and (IC_A) are satisfied. (IC_S) is also satisfied because a unilateral misreport from S results in a payoff of $T = -\ell \leq 0$. It remains to check whether (PC_S) is satisfied. For $\sigma \in [\check{\theta}(X), \theta^c(X)]$, the expected payoff of S after the signal realization σ is given by

$$\begin{aligned} U_S(\sigma) &= T + \Pr(\theta = \sigma|\sigma)(r - \sigma) \\ &= T + r - \sigma \\ &= -\ell + r - \sigma \\ &= \theta^c(X) - \sigma \geq 0. \end{aligned}$$

For $\sigma \in [\underline{\theta}, \check{\theta}(X))$ S's expected payoff is given by

$$\begin{aligned} U_S(\sigma) &= T + \Pr(\theta = \sigma|\sigma)(r - \sigma) \\ &= T + \frac{f(\sigma)(r - \sigma)}{f(\sigma) + (1 - X)\alpha(\sigma)} \\ &= T + r - \check{\theta}(X) \\ &= \theta^c(X) - \check{\theta}(X) \geq 0. \end{aligned}$$

where the third equality follows from the definition of $\alpha(\cdot)$ and the last equality follows from the definition of r. Thus, condition $ii)$ is satisfied.

Next, it is shown that S cannot find a feasible and profitable side-contract. Under any such side-contract, S essentially needs to make a take-it-or-leave-it price offer to A. As the support of the random variable $\tilde{\theta}$ conditional on the signal realization $\sigma = \theta$ is $\{\theta\} \cup (\theta^c(X), \overline{\theta}]$, a deviation implies that S offers a price $\theta' \geq \theta^c(X)$. S's payoff under such a deviation takes the form $T + q(r - \theta)$ where $q \in [0, 1]$ is the probability of project realization under the deviation. Note that

$$T + q(r - \theta) \leq T + q(r - \theta^c(X)) = (1 - q)T \leq 0.$$

Thus, S cannot find a profitable and feasible side-contract and condition $iii)$ is satisfied.

Finally, condition $iv)$ follows from the following calculation:

$$\begin{aligned}U_P(I,\beta) &= X(v-r)-T\\ &= X(v-r-T)-(1-X)T\\ &= X(v-\theta^c(X))+(1-X)\ell\\ &= W^\circ(X)\\ &= W(X).\end{aligned}$$

□

Proof of Proposition 2

The proof follows from the arguments in the main text. □

Proof of Proposition 3

From the analysis of the benchmarks, it follows that P can achieve at most the monopsony payoff $\underline{W}$ if S is either completely informed or completely uninformed. I show now that $W(X^c) > \underline{W}$ whenever $\ell > 0$. First, note that

$$\underline{W} = \max_{X\in[0,1]} X(v-\theta^c(X)) = \max_{X\in[0,1]} W^\circ(X)\big|_{\ell=0} = W^\circ(F(p^m))\big|_{\ell=0}.$$

For $\ell > 0$, $W(F(p^m)) > \underline{W}$ because $W^*(X) > W^\circ(X)\big|_{\ell=0}$ for any $X > 0$ and $W^\circ(X)$ is strictly increasing in ℓ. $W(X^c) \geq W(F(p^m))$ implies $W(X^c) > \underline{W}$. The optimal expected payoff can therefore never be achieved with a fully informative nor with a fully uninformative information structure if S can incur at least some losses. This proves the first bullet point.

The second bullet point follows from the observation that the total expected rents of S and A under the optimal combination of information structure and contract are given by

$$R = \max\left\{W^*(X^c) - W^\circ(X^c), 0\right\}.$$

It remains to prove the last bullet point, Production is inefficient if $\theta^c(X) \neq v$. From the characterization of X^c in Lemma 6 it follows that $\theta^c(X) < F(v)$

unless the intersection point $X^\dagger$ defined in equation (3) satisfies $X^\dagger \geq F(v)$. This is the case if and only if

$$W^*(F(v)) \leq W^\circ(F(v)) \Leftrightarrow W^*(F(v)) \leq (1-F(v))\ell \Leftrightarrow \ell \geq \frac{\overline{W}}{1-F(v)}.$$

□

Proof of Proposition 4

In the proposition, the following assumption is stated.

Assumption 1. *$F(\theta)$ has a strictly positive density $f(\theta)$ and $F(\theta)/f(\theta)$ is increasing.*

I prove the result along the following sequence of lemmas.

Lemma 7. *Under Assumption 1, $W^*(X)$ and $W^\circ(X)$ are both differentiable and strictly quasi-concave.*

Proof. Under the assumption, the inverse of the cdf $F^{-1}(X)$ exists and is differentiable. This implies that $\theta^c(X) = F^{-1}(X)$. $W^*(X)$ and $W^\circ(X)$ can therefore be written as

$$W^*(X) = X(v - F^{-1}(X)) + \int_{\underline{\theta}}^{F^{-1}(X)} F(\theta)d\theta \quad \text{and}$$
$$W^\circ(X) = X(v - F^{-1}(X)) + (1-X)\ell.$$

As $F^{-1}(X)$ is differentiable, both functions are differentiable.

I show now that both functions are strictly quasi-concave if $F(\theta)/f(\theta)$ is increasing. The first derivatives of both functions are given by

$$\frac{dW^*(X)}{dX} = v - F^{-1}(X) \quad \text{and}$$
$$\frac{dW^\circ(X)}{dX} = v - F^{-1}(X) - X(F^{-1})'(X) - \ell.$$

These first derivatives are both strictly decreasing and change their sign at most once from + to −. This is obvious for $\frac{dW^*(X)}{dX}$ as $F^{-1}(X)$ is strictly increasing. For $\frac{dW^\circ(X)}{dX}$, this follows from the assumption that $F(\theta)/f(\theta)$ is increasing: Define the strictly increasing function $\phi(\theta) \equiv \theta + F(\theta/f(\theta)$. As $F^{-1}(X)$ is strictly increasing, it follows that $\phi(F^{-1}(X))$ is strictly increasing. As

$$\phi(F^{-1}(X)) = F^{-1}(X) + X/f(F^{-1}(X)) = F^{-1}(X) + X(F^{-1})'(X),$$

$F^{-1}(X) + X(F^{-1})'(X)$ is strictly increasing. Thus, $W^*(X)$ and $W^\circ(X)$ are strictly quasi-concave. □

Lemma 8. *Under Assumption 1, $W(X)$ is maximized by $X = X^c(\ell)$ which is given by*

$$X^c(\ell) = \begin{cases} X^\circ(\ell) & \textit{if} \quad X^\dagger(\ell) \leq X^\circ(\ell); \\ X^\dagger(\ell) & \textit{if} \quad X^\dagger(\ell) \in (X^\circ(\ell), F(v)); \\ F(v) & \textit{if} \quad X^\dagger(\ell) \geq F(v). \end{cases}$$

Proof. The intersection point of $W^*(X)$ and $W^\circ(X)$ depends on ℓ and is denoted by $X^\dagger(\ell)$. The maximizer of $W^\circ(X)$ also depends on ℓ and is denoted by $X^\circ(\ell)$.

$X^c(\ell)$ is specified in its general form in Lemma 6. Here, it simplifies to the expression above due to the following three observations:

First, recall the definition of $X^\circ(X)$ and note that due to the strict quasi-concavity of $W^\circ(X)$, it holds that

$$X^\circ(X) = \begin{cases} X^\circ(\ell) & \text{if} \quad X \leq X^\circ(\ell), \\ X & \text{if} \quad X > X^\circ(\ell). \end{cases}$$

Second, the continuity of $W^*(X)$ and $W^\circ(X)$ implies that the intersection point $X^\dagger(\ell)$ satisfies $W^*(X^\dagger(\ell)) = W^\circ(X^\dagger(\ell))$. Finally, for all $\ell \geq 0$, it holds that $X^\circ(\ell) < F(v)$. This follows from the fact that

$$\frac{dW^\circ(F(v))}{dX} = v - F^{-1}(F(v)) - F(v)(F^{-1})'(F(v)) - \ell < 0.$$

□

Lemma 9. *Under Assumption 1, $X^\circ(\ell)$ is decreasing in ℓ, $X^\dagger(\ell)$ is increasing* *in ℓ, and $X^\circ(\underline{\ell}) = X^\dagger(\underline{\ell})$ for some $\underline{\ell} \in (0, \overline{\ell})$.*

Proof. $X^\circ(\ell)$ is implicitly defined by the first-order condition

$$\frac{dW^\circ(X^\circ(\ell))}{dX} = v - F^{-1}(X^\circ(\ell)) - X^\circ(\ell)(F^{-1})'(X^\circ(\ell)) - \ell = 0.$$

As $F^{-1}(X)$ is differentiable, $dX^\circ(\ell)/d\ell$ exists and satisfies

$$\frac{dX^\circ(\ell)}{d\ell} = -\frac{1}{\phi'(F^{-1}(X^\circ(\ell)))} \leq -1.$$

$X^\dagger(\ell)$ is implicitly defined by

$$W^\circ(X^\dagger(\ell)) - W^*(X^\dagger(\ell)) = (1 - X^\dagger(\ell))\ell - \int_{\underline{\theta}}^{F^{-1}(X^\dagger(\ell))} F(\theta)d\theta = 0$$

Thus, $dX^\dagger(\ell)/d\ell$ exists and is given by

$$\frac{dX^\dagger(\ell)}{d\ell} = \frac{1 - X^\dagger(\ell)}{X^\dagger(\ell) + \ell} \geq 0.$$

Note that $X^\circ(0) = F(p^m) > 0$ and $X^\dagger(0) = 0$. Thus, there exists some $\underline{\ell} \in (0, \infty)$ such that $X^\circ(\underline{\ell}) = X^\dagger(\underline{\ell})$. If $v < \overline{\theta}$, $X^\circ(\ell) < F(v)$ furthermore implies that $\underline{\ell} < \overline{\ell} \equiv \frac{W^*(F(v))}{1-F(v)}$, as $X^\dagger(\overline{\ell}) = F(v)$. □

Lemma 10. *Under Assumption 1, it holds that*

$$X^c(\ell) = \begin{cases} X^\circ(\ell) & \text{if} \quad \ell \in [0, \underline{\ell}); \\ X^\dagger(\ell) & \text{if} \quad \ell \in [\underline{\ell}, \overline{\ell}); \\ F(v) & \text{if} \quad \ell \geq \overline{\ell}. \end{cases}$$

$X^c(\ell)$ is decreasing for $\ell \in (0, \underline{\ell})$, increasing for $\ell \in (\underline{\ell}, \overline{\ell})$, and constant for $\ell \geq \overline{\ell}$.

Proof. Follows from the previous two lemmas. □

Lemma 11. *Under Assumption 1 and for any $\ell < \ell'$, it holds that*

- $\Sigma(\ell) \subset \Sigma(\ell')$ *for $\ell, \ell' \in [0, \underline{\ell}]$*

- $\Sigma(\ell') \subset \Sigma(\ell)$ *for* $\ell, \ell' \in [\underline{\ell}, \overline{\ell}]$
- $\Sigma(\ell) = \Sigma(\ell')$ *for* $\ell, \ell' > \overline{\ell}$

Proof. From Lemmas 3 and 4, it follows that $\Sigma(\ell) = [\underline{\theta}, F^{-1}(X^c(\ell))]$. As $F^{-1}(X)$ is increasing under Assumption 1, the result follows from the previous lemma. □

Lemma 12. *The total expected rents of* S *and* A *are positive and decreasing in* ℓ *for* $\ell < \underline{\ell}$ *and zero for* $\ell \geq \underline{\ell}$.

Proof. The total expected rents of S and A are given by

$$\begin{aligned} R(\ell) &\equiv [W^*(X^c(\ell)) - W^\circ(X^c(\ell))]_+ \\ &= \left[\int_{\underline{\theta}}^{F^{-1}(X^\dagger(\ell))} F(\theta)d\theta - (1 - X^\dagger(\ell))\ell\right]_+ . \end{aligned}$$

By Lemma 10 and the definition of the intersection point $X^\dagger(\ell)$, this expression is positive for $\ell < \underline{\ell}$ and zero for $\ell \geq \underline{\ell}$. For $\ell < \underline{\ell}$, it holds that

$$R'(\ell) = (X^c(\ell) + \ell) \cdot \frac{dX^c(\ell)}{d\ell} - (1 - X^c(\ell)).$$

By Lemma 10, this expression is negative. □

Proof of Proposition 5 and Lemma 5

Consider the continuation play of the game starting at $t = 1$ after P has set T and r and a weighted information structure I_α as defined in Lemma 3 and Lemma 4, respectively. In any PBE of the game, A accepts any price offer that lies weakly above his costs. After a signal realization σ, the collusion-proofness constraint (CP) in P's problem implies that a price $p = \sigma$ is optimal, as $t_A(\sigma, \sigma, \theta) - \theta x(\sigma, \sigma, \theta) = 0$.

Furthermore, this is the uniquely optimal price. Any price $p' < \sigma$ results in a payoff of $T < 0$. A price $p' \in (\sigma, \theta^c(X)]$ gives a payoff of $T + X(r - p') < T + X(r - \sigma)$ and any price $p' > \theta^c(X)$ gives a payoff $T + q(r - p') < T + q(r - \theta^c) < T + r - \theta^c = 0$ for some $q \in [0, 1]$. The continuation equilibrium at $t = 1$ is therefore unique and the ex-ante probability of project realization is X^c. As the delegation contract (T, r) equals the optimal collective transfers described in Lemmas 3 and 4, P attains an expected payoff of $W(X^c)$. $\square$

Proof of Proposition 6

Under delegation, P offers S a contract which prescribes an initial payment of T and an additional payment r contingent on project realization. After the signal realization σ_1, S offers A either a price $p = 1$ or a price $p = 2$. After the signal realization σ_2, S offers either $p = 2$ or $p = 3$. In any equilibrium, P's offer is accepted by S either after both signal realizations or only after the *good* signal realization σ_1. This gives six types of equilibria[16] which need to be considered. I show that in all six cases, P's equilibrium payoff under delegation is lower than 1. If P ignores S and contracts directly with A, she can achieve an expected payoff of $\frac{4}{3}$. Thus, delegation is strictly suboptimal.

In the following, $p(\sigma)$ denotes the price that S offers in equilibrium after observing the signal realization σ. I assume that ℓ is large enough such that the limited liability constraint $T \geq -\ell$ is never violated. Under this assumption, I show that P's expected payoff can never exceed 1. This implies that P's payoff is also lower than 1 for smaller values of ℓ.

At first, I consider the equilibria where S accepts P's offer after both signal realizations. In an equilibrium where S offers the prices $p(\sigma_1) = 1$ and $p(\sigma_2) = 2$, the expected payoffs of S are $U_S(\sigma_1) = T + \frac{2}{3}(r-1)$ and $U_S(\sigma_2) = T + \frac{1}{3}(r-2)$. As S optimally offers A a price of 2 for the signal realization σ_2, it needs to hold that $r \geq 2$. It follows that $U_S(\sigma_1) = U_S(\sigma_2) + \frac{1}{3}r > U_S(\sigma_2)$. P's expected payoff is

$$\begin{aligned}
&\Pr(\sigma_1)(-T + \Pr(\theta = 1|\sigma_1)(4 - r)) + \Pr(\sigma_2)(-T + \Pr(\theta = 2|\sigma_2)(4 - r)) \\
&\quad = \frac{1}{2}(-T + \frac{2}{3}(4 - r)) + \frac{1}{2}(-T + \frac{1}{3}(4 - r)) \\
&\quad = \frac{1}{2}(\frac{2}{3}(4 - 1) - U_S(\sigma_1)) + \frac{1}{2}(\frac{1}{3}(4 - 2) - U_S(\sigma_2)) \\
&\quad = \frac{4}{3} - \frac{1}{6}r - U_S(\sigma_2) \leq 1,
\end{aligned}$$

where the last inequality follows from the condition $r \geq 2$ and S's participation

[16] Not all six types of equilibria must necessarily exist.

constraint $U_S(\sigma_2) \geq 0$.

Consider now an equilibrium in which S offers A a price of 2 after both signal realizations, i.e. $p(\sigma_1) = p(\sigma_2) = 2$. This is optimal for S if

$$U_S(\sigma_1) = T + r - 2 \geq T + \frac{2}{3}(r-1) \Leftrightarrow r \geq 4 \text{ and}$$
$$U_S(\sigma_2) = T + \frac{1}{3}(r-2) \geq T + r - 3 \Leftrightarrow r \leq \frac{7}{2}.$$

Thus, we have a contradiction which implies that there does not exist an equilibrium with $p(\sigma_1) = p(\sigma_2) = 2$.

In the third type of equilibrium, A offers $p(\sigma_1) = 1$ and $p(\sigma_2) = 3$. This is optimal for S if

$$U_S(\sigma_1) = T + \frac{2}{3}(r-1) \geq T + r - 2 \Leftrightarrow r \leq 4 \text{ and}$$
$$U_S(\sigma_2) = T + r - 3 \geq T + \frac{1}{3}(r-2) \Leftrightarrow r \geq \frac{7}{2}.$$

As $U_1(\sigma_1) = U_1(\sigma_2) + \frac{1}{3}(7-r)$, P's payoff can be written as

$$\begin{aligned}
&\Pr(\sigma_1)(-T + \Pr(\theta = 1|\sigma_1)(4-r)) + \Pr(\sigma_2)(-T + 4 - r) \\
&= \frac{1}{2}(-T + \frac{2}{3}(4-r)) + \frac{1}{2}(-T + 4 - r) \\
&= \frac{1}{2}(\frac{2}{3}(4-1) - U_S(\sigma_1)) + \frac{1}{2}(4 - 3 - U_S(\sigma_2)) \\
&= \frac{3}{2} - \frac{1}{6}(7-r) - U_S(\sigma_2) \leq 1
\end{aligned}$$

where the last inequality follows from the participation constraint $U_1(\sigma_2) \geq 0$ and $r \leq 4$.

In an equilibrium where S offers $p(\sigma_1) = 2$ and $p(\sigma_2) = 3$, it holds that $U_S(\sigma_1) = T + r - 2$, $U_S(\sigma_2) = T + r - 3$, and $U_S(\sigma_1) = U_S(\sigma_2) + 1$. In this case,

P's expected equilibrium payoff can be written as

$$\begin{aligned}
&\Pr(\sigma_1)(-T+4-r)+\Pr(\sigma_2)(-T+4-r)\\
&\quad=\frac{1}{2}(-T+4-r)+\frac{1}{2}(-T+4-r)\\
&\quad=\frac{1}{2}(4-2-U_S(\sigma_1))+\frac{1}{2}(4-3-U_S(\sigma_2))\\
&\quad=1-U_S(\sigma_2)\leq 1
\end{aligned}$$

where the last inequality follows from the participation constraint $U_S(\sigma_2) \geq 0$.

I turn now to equilibria where S accepts P's offer only after the signal realization σ_1. In an equilibrium where S sets a price $p(\sigma_1) = 1$, P's expected payoff is

$$\Pr(\sigma_1)(-T+\frac{2}{3}(4-r))=\frac{1}{2}(\frac{2}{3}(4-1)-U_S(\sigma_1))\leq 1$$

where the last inequality follows from the participation constraint $U_S(\sigma_1)) \geq 0$. In an equilibrium with $p(\sigma_1) = 2$, P's expected payoff is

$$\Pr(\sigma_1)(-T+4-r)=\frac{1}{2}(4-2-U_S(\sigma_1))\leq 1$$

by the participation constraint $U_S(\sigma_1)) \geq 0$.

Thus, delegation is strictly suboptimal under the information structure depicted in Figure 6. □

Statement and Proof of Proposition 7

Proposition 7. *Suppose $\tilde{\theta}$ is uniformly distributed on $[0,1]$ and $v = 1$. If ℓ is low, S is becoming less informed as ℓ increases: For all ℓ, ℓ' with $0 \leq \ell < \ell' \leq \underline{\ell}$*

$$I_\alpha(\ell) \succ_{Blackwell} I_\alpha(\ell').$$

At first, I formally define a Blackwell ordering of information structures (Blackwell, 1951). This definition is based on the notion of a garbling introduced by Marschak and Miyasawa (1968).

Consider two information structures I_1 and I_2. Each information structure

$I_i = (\Sigma_i, \mu_i)$ has an associated conditional cdf $G_i(\theta|\sigma)$ for $i \in \{1, 2\}$. A *garbling* $\Gamma \in \Delta(\Sigma_1 \times \Sigma_2)$ is a joint cdf over the two signal spaces. Γ induces a cdf over Σ_i conditional on $\sigma_j \in \Sigma_j$ which I denote by $\Gamma_{ij}(\cdot|\sigma_j)$.

Definition 1 (Blackwell ordering). *$I_1 \succ_{Blackwell} I_2$ if there exists a* garbling Γ *such that*

$$G_2(\theta|\sigma) = \int_{\Sigma_1} G_1(\theta|z) d\Gamma_{12}(z|\sigma).$$

By Bayes' law

$$\Gamma_{12}(\sigma_1|\sigma_2) = \frac{\Gamma_1(\sigma_1)}{\Gamma_2(\sigma_2)} \Gamma_{21}(\sigma_2|\sigma_1)$$

where $\Gamma_i(\sigma_i) \equiv \int_{z_i \leq \sigma_i} \int_{\Sigma_j} d\Gamma(z_i, z_j)$.

I show now that there exists such a garbling for the information structures $I_1 = I(\ell_1)$ and $I_2 = I(\ell_2)$ with $0 \leq \ell_1 < \ell_2 \leq \underline{\ell}$. Both information structures are characterized by a weight function denoted by $\alpha_i(\cdot)$ for the information structure $I(\ell_i)$. I describe the garbling in terms of Γ_{21} which is illustrated in Figure 7.

Suppose S receives the signal realization σ which is generated through the combination of G_1 and Γ_{21} as depicted in Figure 6. S's belief over the type of A can then be described by the cdf

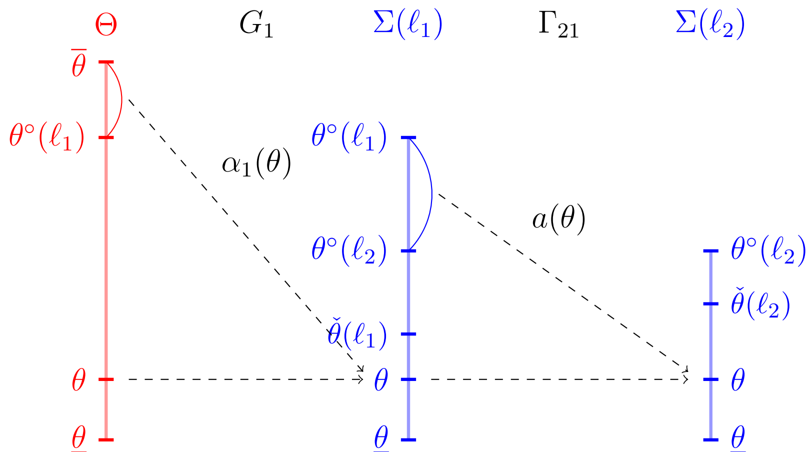
$$G^{\Gamma}(\theta|\sigma) = \begin{cases} 0 & \text{if} \quad \theta < \sigma \\ \frac{f(\sigma)}{f(\sigma)+(1-X_1)\alpha_1(\sigma)+(X_1-X_2)a(\sigma)} & \text{if} \quad \theta \in [\sigma, \theta^\circ(\ell_2)] \\ \frac{f(\sigma)}{f(\sigma)+(1-X_1)\alpha_1(\sigma)+(X_1-X_2)a(\sigma)} & \\ + \frac{(1-X_1)\alpha_1(\sigma)+(X_1-X_2)a(\sigma)}{f(\sigma)+(1-X_1)\alpha_1(\sigma)+(X_1-X_2)a(\sigma)} \frac{F(\theta)-F(\theta^\circ(\ell_2))}{1-F(\theta^\circ(\ell_2))} & \text{if} \quad \theta > \theta^\circ(\ell_2) \end{cases}$$

where $X_i = 1 - F(\theta^\circ(\ell_i))$.

This garbling Γ is characterized by the weighting function $a(\cdot)$. If there exists a weighting function $a(\cdot)$ such that $G^{\Gamma}(\theta|\sigma) = G_2(\theta|\sigma)$, then $I(\ell_1) \succ_{Blackwell} I(\ell_2)$. $G^{\Gamma}(\theta|\sigma) = G_2(\theta|\sigma)$ holds for

$$a(\theta) = \frac{(1 - F(\theta^\circ(\ell_2)))\alpha_2(\theta) - (1 - F(\theta^\circ(\ell_1)))\alpha_1(\theta)}{F(\theta^\circ(\ell_1)) - F(\theta^\circ(\ell_2))}.$$

Figure 7: Garbling



The information structure I_1 is more informative in the sense of Blackwell if I_2 can be replicated by a garbling of the signal generated by I_1. If the signal realization θ of I_1 is smaller than the cutoff $\theta^\circ(\ell)$, I_2 generates the signal realization θ. If it is higher than the cutoff, a signal realization of the signal space of I_2 is drawn according to the density $a(\cdot)$.

The equation defines a weight function if $i)$ $\int_{\Sigma_2} a(\sigma)d\sigma = 1$ and $ii)$ $a(\sigma) \geq 0$ for all $\sigma \in \Sigma_2$. $i)$ is satisfied as

$$\int_{\Sigma_2} a(\sigma)d\sigma = \frac{(1 - F(\theta^\circ(\ell_2))) \int_{\Sigma_2} \alpha_2(\sigma)d\sigma - (1 - F(\theta^\circ(\ell_1))) \int_{\Sigma_2} \alpha_1(\sigma)d\sigma}{F(\theta^\circ(\ell_1)) - F(\theta^\circ(\ell_2))}$$
$$= \frac{(1 - F(\theta^\circ(\ell_2))) - (1 - F(\theta^\circ(\ell_1)))}{F(\theta^\circ(\ell_1)) - F(\theta^\circ(\ell_2))} = 1$$

which follows from

$$\int_{\Sigma_2} \alpha_i(\sigma)d\sigma = \int_{\underline{\theta}}^{\check{\theta}(\ell_i)} \alpha_i(\sigma)d\sigma = 1.$$

$ii)$ is satisfied if $(1 - F(\theta^\circ(\ell_2)))\alpha_2(\theta) - (1 - F(\theta^\circ(\ell_1)))\alpha_1(\theta) \geq 0$. Using the definitions of T, r, $\alpha(\cdot)$, and $\check{\theta}$ from Lemma 4, this is equivalent to

$$f(\theta)\frac{\check{\theta}(\ell_2) - \theta}{\theta^\circ(\ell_2) + \ell_2 - \check{\theta}(\ell_2)} \geq f(\theta)\frac{\check{\theta}(\ell_1) - \theta}{\theta^\circ(\ell_1) + \ell_1 - \check{\theta}(\ell_1)}.$$

Thus, $I(\ell_1) \succ_{Blackwell} I(\ell_2)$ if $\frac{\check{\theta}(\ell) - \theta}{\theta^\circ(\ell) + \ell - \check{\theta}(\ell)}$ is non-decreasing in ℓ.

If $\tilde{\theta}$ is uniformly distributed on $[0, 1]$ and $v = 1$, it is easy to check from the definitions of $\theta^\circ(\ell)$ and $\check{\theta}(\ell)$ that $\theta^\circ(\ell) = 0.5(1-\ell)$ and $\check{\theta}(\ell) = 0.5(\sqrt{3}-1)(1+\ell)$. It follows that

$$\frac{\check{\theta}(\ell) - \theta}{\theta^\circ(\ell) + \ell - \check{\theta}(\ell)} = \frac{\sqrt{3} - 1}{2 - \sqrt{3}} - \frac{2\theta}{(\sqrt{3} - 1)(1 + \ell)}$$

which is increasing in ℓ.

Thus, $a(\cdot)$ is a weight function and $I(\ell_1) \succ_{Blackwell} I(\ell_2)$. □

B Collusion-proofness principle

In this appendix, I provide a proof for the collusion-proofness principle used in the analysis. This proof follows closely the arguments of Faure-Grimaud et al. (2003).

In general, a deterministic contract offered by P to S and A takes the form $(M_S, M_A, \overline{\beta})$ where M_i is the set of messages that i can send to P with $i \in \{S, A\}$ and $\overline{\beta}$ determines the allocation as a function of the messages sent by S and A:

$$\overline{\beta} = \Big(\overline{x}(m_S, m_A), \overline{t}_S(m_S, m_A), \overline{t}_A(m_S, m_A)\Big).$$

A deterministic side-contract $(M^{sc}, \overline{\gamma})$ consists of a set of messages M^{sc} that A can send to S and the collection of functions $\overline{\gamma} = \big(\overline{\gamma}_\sigma\big)_{\sigma \in \Sigma}$ where

$$\overline{\gamma}_\sigma = \Big(\overline{\rho}(m; \sigma), \overline{\tau}(m; \sigma)\Big)$$

determines messages that S and A send according to $\overline{\rho} : M^{sc} \times \Sigma \to M_S \times M_A$ and side-transfers paid from S to A according to $\overline{\tau} : M^{sc} \times \Sigma \to \mathbb{R}$.

By the standard revelation principle, one can restrict attention to such PBEs with passive beliefs in which S offers a direct side-contract that is always accepted by A and A reports his costs truthfully, i.e., $M^{sc} = \Theta$.

Consider a PBE with passive beliefs in which P offers a contract $\overline{\beta}^*$ and S offers a direct side-contract $\overline{\gamma}^*_d$ that is always accepted by A. I denote the equilibrium payoff of A by $\overline{u}_A(\sigma, \theta)$ and the payoff from playing the contract non-cooperatively by $\underline{u}_A(\sigma,\theta)$ with $\overline{u}_A(\sigma,\theta) \geq \underline{u}_A(\sigma,\theta)$. I want to argue that there exists another PBE with passive beliefs in which P offers the direct contract $\beta^* \equiv \overline{\beta}^* \circ \overline{\gamma}_d^*$, S offers the null side-contract γ_0, and A always accepts.

Given that P offers β^* and S offers γ_0, A accepts the contract and the side-contract and reports his costs truthfully. This follows from the fact that A finds it optimal to report truthfully and to accept the contracts in the PBE where $\overline{\beta}^*$ and $\overline{\gamma}_d^*$ are offered.

Furthermore, the null side-contract is optimal for S. Toward a contradiction, suppose that the null side-contract is suboptimal. Then there exists a side-contract γ^{**} that always gives A at least a payoff of $\overline{u}_A(\sigma,\theta)$ and gives S a strictly higher expected payoff than the null side-contract for some $\sigma \in \Sigma$. However, this implies that the side-contract $\overline{\gamma}_d^{**} \equiv \overline{\gamma}_d^* \circ \gamma^*$ is a profitable deviation from the side-contract $\overline{\gamma}_d^*$ if P offers the contract $\overline{\beta}^*$. This leads to a contradiction.

Thus, the null side-contract is optimal for S and P achieves the optimal expected payoff by offering β^*. □

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