Spatial Econometrics
The basics

J.Paul Elhorst, University of Groningen, the Netherlands

- What is a spatial econometric model?
- What are spatial lags/interaction effects?
- What are spatial spillover effects?
- How to interpret the outcomes of a spatial econometric model?
- How to estimate a spatial econometric model?
- How to select the spatial weights matrix W and the right econometric model?
- How to deal with critique on spatial econometrics?
- Role of economic theory

Background material

- Elhorst J.P. (2014) Spatial Econometrics: From Cross-sectional Data to Spatial Panels. Springer, Heidelberg New York Dordrecht London, http://www.springer.com/economics/regional+science/book/978-3-642-40339-2
- Elhorst J.P. (2013) Spatial panel models. In: Fischer M.M, Nijkamp P. (eds.), Handbook of Regional Science, pp. 1637-1652. Springer, Berlin Heidelberg.
- Halleck Vega, S., Elhorst J.P. (2015) The SLX model. *Journal of Regional Science* 55(3): 339-363.
- Elhorst J.P., Halleck Vega S.M (2017) The SLX model: Extensions and the sensitivity of spatial spillovers to W, *Papeles de Economía Española* 152: 34-50.
- Halleck Vega S., Elhorst J.P. (2016) A regional unemployment model simultaneously accounting for serial dynamics, spatial dependence and common factors. *Regional Science and Urban Economics* 60: 85-95.
- Elhorst, J.P., Gross M., Tereanu E. (2018) Spillovers in space and time: where spatial econometrics and Global VAR models meet. European Central Bank, Frankfurt. Working Paper Series No 2134. https://www.ecb.europa.eu/pub/pdf/scpwps/ecb.wp2134.en.pdf?b33bf8d0dc4c5addae515ce126b98b7d.
- Yesilyurt M.E., Elhorst J.P. (2017) Impacts of neighboring countries on military expenditures: A dynamic spatial panel approach. *Journal of Peace Research* 54(6), 777-790.
- Burridge P., Elhorst J.P., Zigova K. (2017) Group Interaction in Research and the Use of General Nesting Spatial Models. In: Baltagi B.H., LeSage J.P., Pace R.K. (eds.) Spatial

Econometrics: Qualitative and Limited Dependent Variables (Advances in Econometrics, Volume 37), pp.223 – 258. Bingley (UK), Emerald Group Publishing Limited.

- LeSage, J.P.; Pace, R.K. The biggest myth in spatial econometrics. *Econometrics* 2014, *2*, 217-249.
- LeSage, J., Chih, Y.-Y. (2017) A matrix exponential spatial panel model with heterogeneous coefficients. Geographical Analysis. https://doi.org/10.1111/gean.12152

Spatial Econometrics – Cross-sectional dependence
vs.
Time series Econometrics

1. Two-way rather than one-way relationship: Unit A can affect unit B, and vice versa. The past can affect the future, but the future cannot affect the past.
2. Wide variety of units of measurement is eligible for modeling spatial/cross-sectional dependence: geographical, political and socio-economic variables.

Spatial econometric model

Linear regression model (**$Y=X\beta+\varepsilon$**) extended to include

Endogenous interaction effect (1): **ρWY**

- Dependent variable y of unit A ↔ Dependent variable y of unit B
- Y denotes an **N**×1 vector consisting of one observation on the dependent variable for every unit in the sample (i=1,…,**N**)
- **W** is an **N**×**N** nonnegative matrix describing the arrangement of the units in the sample

Exogenous interaction effects (K): **$WX\theta$**

- Independent variable x of unit A → Dependent variable y of unit B
- X denotes an N×**K** matrix of exogenous explanatory variables

Interaction effect among **error** terms (1): **λWu**

- Error term u of unit A ↔ Error term u of unit B

Linear spatial econometric model for cross-section data in vector notation

$Y=\rho WY+\alpha\iota_N+X\beta+WX\theta+u, \quad u=\lambda Wu+\varepsilon$

Y denotes an N×1 vector consisting of one observation on the dependent variable for every unit in the sample (i=1,…,N), ι_N is an N×1 vector of ones associated with the constant term parameter α, X denotes an N×K matrix of exogenous explanatory variables, with the associated parameters β contained in a K×1 vector, and $\varepsilon=(\varepsilon_1,...,\varepsilon_N)^T$ is a vector of disturbance terms, where ε_i are independently and identically distributed error terms for all i with zero mean and variance σ^2.

The total number of interaction effects in this model is K+2.

W is an N×N matrix describing the spatial arrangement of the spatial units in the sample. Usually, W is row-normalized.

1	2	3

Example: Netherlands – Belgium – France

Row-normalizing $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ gives $W=\begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$.

This is an example of a row-normalized binary contiguity matrix for N=3.

Let $Y = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$, then $WY = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ 2\frac{1}{2} \\ 2 \end{bmatrix}$. Similarly, let

$X = \begin{bmatrix} -1 & 1 \\ 1 & 0 \\ 2 & 5 \end{bmatrix}$, then $WX = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 \\ 1 & 0 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/2 & 3 \\ 1 & 0 \end{bmatrix}$.

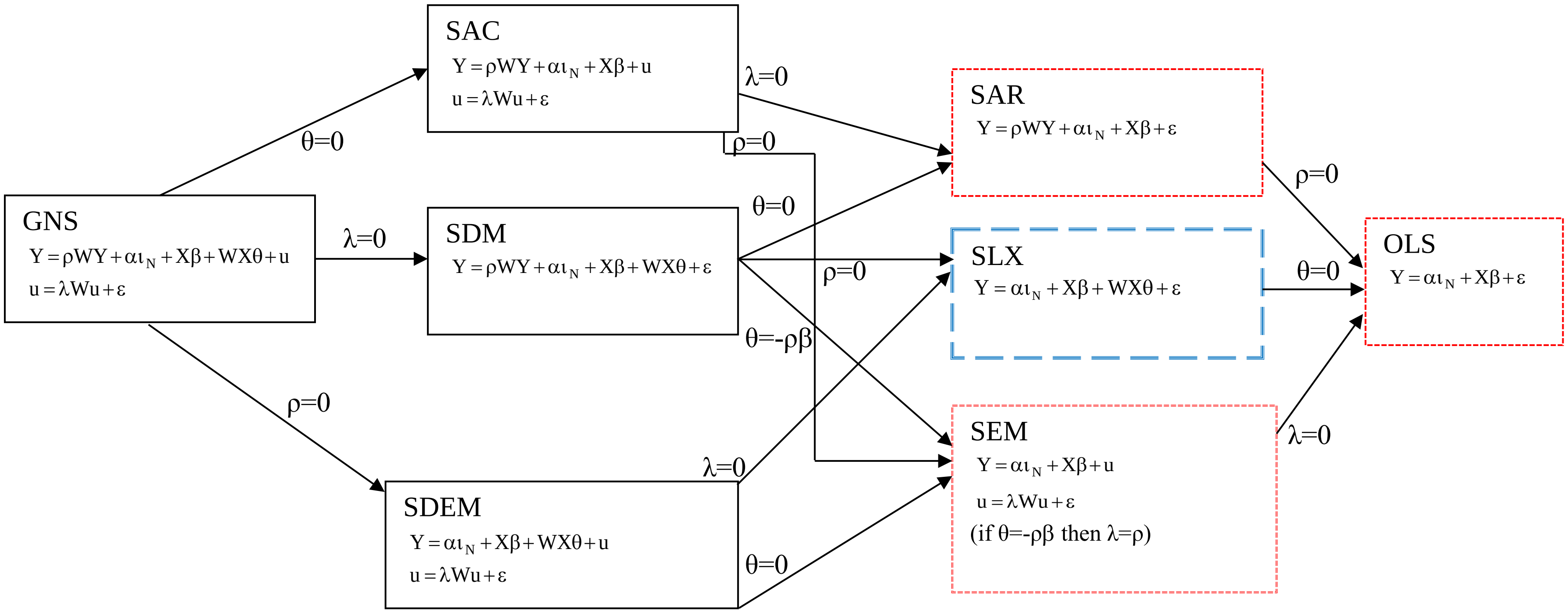
Finally, note that X may not contain a constant, since this constant and the corresponding WX variable

$WX = \begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ would be perfectly multicollinear.

Table 1. Spatial econometric models with different combinations of spatial interaction effects and their flexibility regarding spatial spillovers

Type of model	Spatial interaction effects	#	**Flexibility spillovers**
OLS, Ordinary least squares model	-	0	Zero by construction
SAR, Spatial autoregressive model	*WY*	1	Constant ratios
SEM, Spatial error model	*Wu*	1	Zero by construction
SLX, Spatial lag of *X* model	*WX*	K	Fully flexible
SAC, Spatial autoregressive combined model (SARAR)	*WY*, *Wu*	2	Constant ratios
SDM, Spatial Durbin model	*WY*, *WX*	K+1	Fully flexible
SDEM, Spatial Durbin error model	*WX*, *Wu*	K+1	Fully flexible
GNS, General nesting spatial model	*WY*, *WX*, *Wu*	K+2	Fully flexible

Figure 1. Comparison of different spatial econometric model specifications



Note: GNS = general nesting spatial model, SAC = spatial autoregressive combined model (SARAR), SDM = spatial Durbin model, SDEM = spatial Durbin error model, SAR = spatial autoregressive model (spatial lag model), SLX= spatial lag of X model, SEM = spatial error model, OLS = ordinary least squares model

Four generations of spatial econometric models

$Y=\rho WY+\alpha\iota_N+X\beta+WX\theta+u$, $u=\lambda Wu+\varepsilon$ **Cross-section data**

$Y_t=\rho WY_t+\alpha\iota_N+X_t\beta+WX_t\theta+u_t$, $u_t=\lambda Wu_t+\varepsilon_t$ Space-time data

$Y_t=\rho WY_t+X_t\beta+WX_t\theta+\mu+\alpha_t\iota_N+u_t$ **Spatial panel data**

μ: vector of spatial fixed or random effects
α_t: time period fixed or random effects (t=1,…,T)

$Y_t=\tau Y_{t-1}+\rho WY_t+\eta WY_{t-1}+X_t\beta+WX_t\theta+\mu+\alpha_t\iota_N+u_t$
Dynamic spatial panel data

$Y_t=\tau Y_{t-1}+\rho WY_t+\eta WY_{t-1}+X_t\beta+WX_t\theta+\Sigma_r\Gamma_r f_{rt}+u_t$
Common factors: cross-sectional averages or principal components

Interpretation estimation results:

Direct, indirect=SPATIAL SPILLOVER EFFECTS=MAIN FOCUS

Cross-section or non-dynamic spatial panel data model

$Y_t=\rho WY_t+X_t\beta+WX_t\theta+\mu+\alpha_t\iota_N+u_t$

Reduced form: $Y_t=(I-\rho W)^{-1}[X_t\beta+WX_t\theta+\mu+\alpha_t\iota_N+u_t]$

$$\left[\frac{\partial E(Y)}{\partial x_{1k}} \quad . \quad \frac{\partial E(Y)}{\partial x_{Nk}}\right]_t = \begin{bmatrix} \frac{\partial E(y_1)}{\partial x_{1k}} & . & \frac{\partial E(y_1)}{\partial x_{Nk}} \\ . & . & . \\ \frac{\partial E(y_N)}{\partial x_{1k}} & . & \frac{\partial E(y_N)}{\partial x_{Nk}} \end{bmatrix}_t = (I-\rho W)^{-1} \begin{bmatrix} \beta_k & w_{12}\theta_k & . & w_{1N}\theta_k \\ w_{21}\theta_k & \beta_k & . & w_{2N}\theta_k \\ . & . & . & . \\ w_{N1}\theta_k & w_{N2}\theta_k & . & \beta_k \end{bmatrix} =$$

$$= (I-\rho W)^{-1}(\beta_k I_N + \theta_K W)$$

Direct effect: Mean diagonal element (**or of different groups**)

Indirect effect: Mean row sum of off-diagonal elements

Problem: t-values of direct and indirect effects are bootstrapped

Note: Error terms ($\mu+\alpha_t\iota_N+u_t$) drop out due to taking expectations

Table Direct and spillover effects corresponding to different model specifications

Model	Direct effect	Spillover effect
OLS / SEM (*Wu*)	β_k	0
SAR (*WY*)/ SAC (*WY*, *Wu*) *	Average diagonal element of $(\mathbf{I}-\rho\mathbf{W})^{-1}\beta_k$	Average row sum of off-diagonal elements of $(\mathbf{I}-\rho\mathbf{W})^{-1}\beta_k$
SLX / SDEM WX / Wu	β_k	θ_k
SDM / GNS WY+WX/Wu	Average diagonal element of $(\mathbf{I}-\rho\mathbf{W})^{-1}[\beta_k+\mathbf{W}\theta_k]$	Average row sum of off-diagonal elements of $(\mathbf{I}-\rho\mathbf{W})^{-1}[\beta_k+\mathbf{W}\theta_k]$

* **Ratio between the spillover effect and the direct effect in the SAR/SAC model is the same for every explanatory variable**.

Two further properties: global and local spillover effects

Indirect effects that occur if ρ=0 (of WY) are known as ***local spillover effects***

$$\begin{bmatrix} \frac{\partial E(y_1)}{\partial x_{1k}} & . & \frac{\partial E(y_1)}{\partial x_{Nk}} \\ . & . & . \\ \frac{\partial E(y_N)}{\partial x_{1k}} & . & \frac{\partial E(y_N)}{\partial x_{Nk}} \end{bmatrix} = \begin{bmatrix} \beta_k & w_{12}\theta_k & . & w_{1N}\theta_k \\ w_{21}\theta_k & \beta_k & . & w_{2N}\theta_k \\ . & . & . & . \\ w_{N1}\theta_k & w_{N2}\theta_k & . & \beta_k \end{bmatrix} = \beta_k I_N + \theta_k W.$$

This is local because the indirect effects only fall on spatial units for which the elements of W are non-zero. Local spillovers go together with dense(r) W matrix.

Indirect effects that occur if ρ≠ 0 (of WY) are known as ***global spillover effects***

$$\begin{bmatrix} \frac{\partial E(y_1)}{\partial x_{1k}} & . & \frac{\partial E(y_1)}{\partial x_{Nk}} \\ . & . & . \\ \frac{\partial E(y_N)}{\partial x_{1k}} & . & \frac{\partial E(y_N)}{\partial x_{Nk}} \end{bmatrix} = (I - \rho W)^{-1} \beta_k = (\mathrm{I} + \rho \mathrm{W} + \rho^2 W^2 + \rho^3 W^3 + \ldots) \beta_k$$

This is global because the indirect effects fall on all units; even if W contains many zero elements, $(I-\rho W)^{-1}$ will not. Global spillovers tend to go together with sparse(r) W matrix; due to the higher-order terms $\rho^g W^g$ ($g>1$) locations farther away are reached anyway even if they are not directly connected.

Dynamic spatial panel data model with FE, RE or CF

$Y_t = \tau Y_{t-1} + \rho W Y_t + \eta W Y_{t-1} + X_t \beta + W X_t \theta$ + error terms

Short-term (ignore τ and η)

$$\left[\frac{\partial E(Y)}{\partial x_{1k}} \quad \text{L} \quad \frac{\partial E(Y)}{\partial x_{Nk}} \right]_t = (I - \rho W)^{-1} [\beta_k I_N + \theta_k W].$$

Long-term (set $Y_{t-1} = Y_t = Y^*$ and $WY_{t-1} = WY_t = WY^*$)

$$\left[\frac{\partial E(Y)}{\partial x_{1k}} \quad \text{L} \quad \frac{\partial E(Y)}{\partial x_{Nk}} \right] = [(1-\tau)I - (\rho + \eta)W]^{-1} [\beta_k I_N + \theta_k W].$$

Generally, it is hard to find significant spillovers since they depend on so many parameters (3 short term, 5 long term); many empirical studies do not recognize this.

Empirical illustration: Cigarette Demand in the US

Baltagi and Li (2004) estimate a demand model for cigarettes based on a panel from 46 U.S. states (N=46)

$$\log(C_{it}) = \alpha + \beta_1 \log(P_{it}) + \beta_2 \log(Y_{it}) + \mu_i (\text{optional}) + \lambda_t (\text{optional}) + \varepsilon_{it},$$

where C_{it} is real per capita sales of cigarettes by persons of smoking age (14 years and older). This is measured in packs of cigarettes per capita. P_{it} is the average retail price of a pack of cigarettes measured in real terms. Y_{it} is real per capita disposable income. Whereas Baltagi and Li (2004) use the first 25 years for estimation to reserve data for out of sample forecasts, we use the full data set covering the period 1963-1992 (T=30). Details on data sources are given in Baltagi and Levin (1986, 1992) and Baltagi et al. (2000). They also give reasons to assume the state-specific effects (μ_i) and time-specific effects (λ_t) fixed, in which case one includes state dummy variables and time dummies for each year.

We have reasons to believe that spatial interaction effects need to be included in this model!

BOOTLEGGING

- The main motivation to extend the basic model to include spatial interaction effects is the so-called **bootlegging effect**; consumers are expected to purchase cigarettes in nearby states, legally or illegally (smuggling), if there is a price advantage.
- This smuggling behavior is a result of significant price variation in cigarettes across US states and partly due to the disparities in state cigarette tax rates. Baltagi and Levin (1986, 1992) incorporate the minimum real price of cigarettes in any neighboring state as a proxy for the bootlegging effect.
- A limitation is that this proxy does not account for cross-border shopping that may take place between other states than the minimum-price neighboring state (Baltagi and Levin, 1986). This can be due to smuggling taking place over longer distances by trucks since cigarettes can be stored and are easy to transport (Baltagi and Levin, 1992) or due to geographically large states where cross-border shopping may occur in different neighboring states.
- To take this into account, other studies have extended the model to explicitly incorporate spatial interaction effects. However, while the specification originally adopted by Baltagi and Levin (1992) resembles the SLX model but then with only one exogenous interaction effect (price), applied spatial econometric studies have either included: (i) endogenous interaction effects, (ii) interaction effects among the error terms or (iii) a combination of endogenous and exogenous interaction effects.

Basic findings

TABLE Model comparison of the estimation results explaining cigarette demand

	OLS	SAR	SEM	SLX	SAC	SDM	SDEM	GNS	GNS2
ln(**P**)	-1.035	-0.993	-1.005	-1.017	-1.004	-1.003	-1.011	-1.020	-1.017
	(-25.63)	(-24.48)	(-24.68)	(-24.77)	(-24.49)	(-24.60)	(-24.88)	(-25.40)	
ln(**I**)	0.529	0.461	0.554	0.608	0.557	0.601	0.588	0.574	0.575
	(11.67)	(9.86)	(11.07)	(10.38)	(10.51)	(10.33)	(10.57)	(11.02)	
W × ln(**C**)		0.195			-0.013	0.225		-0.481	-0.400
		(6.79)			(-0.22)	(6.85)		(-7.01)	
W × ln(**P**)				-0.220		0.051	-0.177	-0.645	-0.555
				(-2.95)		(0.62)	(-2.24)	(-5.97)	
W × ln(**I**)				-0.219		-0.293	-0.168	0.079	0.053
				(-2.80)		(-3.70)	(-2.12)	(0.85)	
W × **u**			0.238		0.292		0.229	0.628	0.550
			(7.26)		(4.73)		(6.95)	(14.60)	
R^2	0.896	0.900	0.895	0.897	0.895	0.901	0.897	0.873	
Log-likelihood	1661.7	1683.5	1687.2	1668.4	1687.2	1691.4	1691.2	1695.1	

Notes: t-values are reported in parentheses; state and time-period fixed effects are included in every model, **W** = pre-specified binary contiguity matrix

- In this case including endogenous interaction effects (SAR model) implies that state cigarette sales directly affect one another, which is difficult to justify. The resulting global spillovers would mean that a change in price (or income) in a particular state potentially impacts consumption in all states, including states that according to **W** are unconnected.[1] Pinkse and Slade (2010, p. 115) argue that an empirical problem like this is insightful precisely because it is difficult to form a reasonable argument to include endogenous interaction effects even though they are easily found statistically. Given the research question of whether consumers purchase cigarettes in nearby states if there is a price advantage, this example points towards a local spillover specification such as the SLX model rather than a global spillover specification.
- The model is aggregated over individuals since the objective is to explain sales in a particular state, as in Baltagi and Levin (1986, 1992), Baltagi and Li (2004), Debarsy et al. (2012), and Elhorst (2014), among others. If the purpose, on the other hand, is to model individual behavior (e.g., the reduction in the number of smokers or teenage smoking behavior) then this is better studied using micro data.

[1]This implies that e.g., price changes in California would exert an impact on cigarette consumption even in states as distant as Illinois or Wisconsin.

TABLE Model comparison of the estimated direct and spillover effects on cigarette demand

	OLS	SAR	SEM	SLX	SAC	SDM	SDEM	GNS
Direct effects								
ln(**P**)	-1.035	-1.003	-1.005	-1.017	-1.004	-1.016	-1.011	-0.999
	(-25.63)	(-25.10)	(-24.68)	(-24.77)	(-24.47)	(-24.84)	(-24.88)	(-25.43)
ln(**I**)	0.529	0.465	0.554	0.608	0.556	0.594	0.588	0.594
	(11.67)	(10.18)	(11.07)	(10.38)	(10.56)	(10.88)	(10.57)	(10.35)
Spillover effects								
ln(**P**)		-0.232		-0.220	0,010	-0.215	-0.177	-0.122
		(-5.63)		(-2.95)	(0.17)	(-2.39)	(-2.24)	(-1.89)
ln(**I**)		0.107		-0.219	-0.006	-0.200	-0.168	-0.155
		(5.51)		(-2.80)	(-0.20)	(-2.30)	(-2.12)	(-2.16)

W = pre-specified binary contiguity matrix

Basic findings (W pre-specified, fixed!!!)

1. The direct effects produced by the different models are comparable.
2. The spatial spillover effects produced by the different models differ widely.
3. For various reasons the OLS, SAR, SEM, SAC and GNS models need to be rejected.

The OLS model is outperformed by other, more general models.

The spillover effects of the SEM model are zero by construction, while the results of more general models show that the spillover effect of the price and income variable are significant.

The SAR and SAC models suffer from the problem that the ratio between the spillover effect and the direct effect is the same for every explanatory variable. Consequently, the spillover effect of the income value variable gets a wrong and significant sign.

Often, the GNS model is overparameterized, as a result of which the t-values of the coefficient estimates have the tendency to go down (not here!, see Burridge et al. (2017) for better example), or its parameters are difficult to reproduce using Monte Carlo simulation experiments (see columns GNS2), probably due to multicollinearity issues. **Also the opposite signs of W*ln(C) and W*u are difficult to interpret (Note: this is a general problem of SARAR model approaches).**

In sum, only the SLX, SDM and SDEM models, i.e., models that include WX variables, produce acceptable results.

However, it is not clear which of these three models best describes the data. Even though they produce spillover effects that are comparable to each other, both in terms of magnitude and significance, this is worrying since these models have a different interpretation (global or local).

Furthermore, the price spillover effect has a negative sign rather than the expected positive sign; **could it be that W=binary contiguity matrix is wrong?**

The spatial weight matrix W

The specification of W is of vital importance:

1. The value and significance level of the interaction parameter depends on the specification of W.
2. **Importantly, the direct and indirect effects are sensitive for fundamental changes of W only, not for small changes, see Biggest Myth paper of LeSage and Pace (2014).**
3. The specification of W should follow from the theory at hand. In principle, different theories imply different W. However, although one may look to economic theory for guidance, it often has little to say about the specification of W. Therefore, empirical researchers often investigate whether the results are sensitive to the specification of W.

$$W = \begin{pmatrix} w_{11} & \cdot & w_{1N} \\ . & \cdot & . \\ w_{N1} & \cdot & w_{NN} \end{pmatrix}.$$

The row elements of a weights matrix display the impact *on* a particular unit *by* all other units, while the column elements of a weights matrix display the impact *of* a particular unit *on* all other units

Spatial weights matrices most often used in empirical research in spatial econometrics:

1. p-order **binary contiguity** matrices (if p=1 only first-order neighbors are included, if p=2 first and second order neighbors are considered, and so on).
2. **Inverse distance** matrices (with or without a cut-off point) or **exponential distance decay** matrices.
3. q-**nearest neighbor** matrices (where q is a positive integer).
4. Block diagonal or **group interaction** matrices where each block represents a group of spatial units that interact with each other but not with observation in other groups.
5. **Leader** matrices.
6. Matrices based on **socio-economic variables.**

If W is endogenous rather than exogenous, consult Qu and Lee (2014) on endogenous spatial weight matrix in Journal of Econometrics.

Row-normalization

$$w_{ij}^{normalized} = \frac{w_{ij}}{\sum_{j=1}^{N} w_{ij}}.$$

Row-normalization is standard in spatial econometrics!

Normalization by largest characteristic root

As an alternative to row-normalization, one may divide the elements of W by its largest characteristic root, ω_{max}, $W^S = 1/\omega_{max} W$, which might be labeled as **matrix normalization** (This has the effect that the characteristic roots of W are also divided by ω_{max}, as a result of which $\omega^S_{max}=1$, just like the largest characteristic root of a row- or column-normalized matrix). The advantage of matrix normalization is that the mutual proportions between the elements of W remain unchanged. For example, scaling the rows or columns of an inverse distance matrix so that the weights sum to one would cause this matrix to lose its **economic interpretation** for this decay.

Two basic identification problems in applied econometric research:

1. **How to find the right/best spatial econometric model specification.**
2. **How to find the right/best specification of the spatial weights matrix.**

See also critique special theme issue Journal of Regional Science (2012, volume 52, issue 2).

Economic Theory: (1) Use an economic-theoretical model, if available, or develop it. (2) Do global spillover effects make sense from a theoretical viewpoint, SDM vs. SDEM, and related to this is W dense or sparse?

Three "statistical type" of solutions

1. **SLX approach**
2. **Bayesian comparison approach**
3. **CD-test and exponent α-estimator**

The SLX approach Halleck Vega and Elhorst (SLX model, 2015, JRS; 2017, PEE):

1. Only in the SLX, SDEM, SDM and GNS models can the spatial spillover effects take any value. The SLX model is the simplest one in this family of spatial econometric models.
2. In the SLX model *W* can easily be parameterized ($w_{ij} = 1/d^{\gamma}$).
3. There are *K* spatial lags *WX*, and only one *WY* and only one *Wu*, so it makes sense to focus on *WX* variables first.
4. The estimation of this model also does not cause severe additional econometric problems (such as endogeneity, regularity conditions).
5. The SLX model allows for the application of standard econometric techniques to test for endogenous explanatory variables.
6. The SLX approach starkly contrasts commonly used spatial econometric specification strategies and is a complement to the critique of spatial econometrics raised in a special theme issue of the Journal of Regional Science (Volume 52, Issue 2).

Parameterizing *W* in the SLX model

Inverse distance: $w_{ij} = \frac{1}{{d_{ij}}^{\gamma}}$

Negative exponential: $w_{ij} = \exp(-\delta d_{ij})$

Gravity type of function: $w_{ij} = \frac{P_i^{\gamma_1} P_j^{\gamma_2}}{{d_{ij}}^{\gamma_3}}$,

where *P* measures the size of units *i* and *j* in terms of population and/or gross product. **Preferably, theory should be the driving force behind *W*, the gravity type of model is such a theory.**

The spatial weights matrix of every exogenous spatial lag $W_k X_k$ may also be modeled as $w_{ijk} = \frac{1}{{d_{ij}}^{\gamma_k}}$; why should the distance decay effect be the same for every explanatory variable.

TABLE 4: SLX model estimation results for pre-specified and parameterized **W**, for all regressors treated as exogenous, for ln(**P**) treated as endogenous, and for both ln(**P**) and **W** × ln(**P**) treated as endogenous

	OLS, **W**=BC	Nonl. OLS, **W**=1/d$^{\gamma}$	2SLS, **W**=BC ln(**P**) endogenous[a]	2SLS, **W**=BC ln(**P**), **W**×ln(**P**) endogenous[b]	2SLS, **W**=1/d$^{\gamma}$ ln(**P**) endogenous[c]	2SLS, **W**=1/d$^{\gamma}$ ln(**P**), **W**× ln(**P**) endogenous[d]
	(1)	(2)	(3)	(4)	(5)	(6)
ln(**P**)	-1.017	-0.908	-1.334	-0.785	-1.246	-1.273
	(-24.77)	(-24.43)	(-16.63)	(-3.69)	(-16.32)	(-15.40)
ln(**I**)	0.608	0.654	0.579	0.576	0.591	0.502
	(10.38)	(15.39)	(9.63)	(6.81)	(13.34)	(10.59)
W × ln(**P**)	-0.220	0.254	-0.109	-3.067	0.192	0.898
	(-2.95)	(3.08)	(-1.36)	(-3.59)	(3.00)	(6.25)
W × ln(**I**)	-0.219	-0.815	-0.230	-0.901	-0.750	-1.068
	(-2.80)	(-4.76)	(-2.89)	(-4.09)	(-14.14)	(-12.79)
γ		2.938			3.141	3.322
		(16.48)			(11.11)	(15.24)
R^2	0.897	0.916	0.374	<0	0.484	0.421
Log-Likelihood	1668.4	1812.9				
F-test instruments ln(**P**)			100.54	102.60	110.13	106.77
			[0.00]	[0.00]	[0.00]	[0.00]
F-test instruments **W** × ln(**P**)				46.07		156.09
				[0.00]		[0.00]
χ^2-test exogeneity instruments			0.087	3.84	0.112	0.907
			[0.99]	[0.28]	[0.99]	[0.82]
t-test ln(**P**) residual			4.63	-0.67	5.14	4.50
t-test **W** × ln(**P**) residual				2.94		-1.33

Notes: See note to Table 2; coefficient estimates of **W** × ln(**P**) and **W** × ln(**I**) represent spillover effects. p-values of test statistics in squared brackets. Degrees of freedom of the F-test is (i) number of instruments and (ii) number of observations minus number of instruments and number of fixed effects. Degrees of freedom of χ^2-test is number of surplus instruments.

a. Instruments (+exog.var. in eq.): **W** × Population, Tax, **W** × Tax + ln(**I**), **W** × ln(**P**), **W** × ln(**I**).

b. Instruments (+exog.var. in eq.): **W** × Population, Tax, **W** × Compensation + ln(**I**), **W** × ln(**I**).

c. Instruments (+exog.var. in eq.): Tax, **W** × Compensation + ln(**I**), **W** × ln(**P**), **W** × ln(**I**).

d. Instruments (+exog.var. in eq.): **W** × Population,Tax, **W** × Compensation + ln(**I**), **W** × ln(**I**).

Distance decay effect

- The estimate of the distance decay parameter is 2.938 and also highly significant. This makes sense because only people living near the border of a state are able to benefit from lower prices in a neighboring state on a daily or weekly basis. If the distance decay effect at 5 miles from the border is set to 1, it falls to 0.130 at 10 miles, 0.040 at 15 miles, and 0.017 at 20 miles.
- People living further from the border can only benefit from lower prices if they visit states for other purposes or if smuggling takes place by trucks over longer distances.
- It explains why the parameterized inverse distance matrix gives a much better fit than the binary contiguity matrix; the degree of spatial interaction on shorter distances falls much faster and on longer distances more gradually than according to the binary contiguity principle (see Figure 2). This is corroborated by the R^2, which increases from 0.897 to 0.916, and the log-likelihood function value, which increases from 1668.2 to 1812.9.

Another important issue to address is whether or not cigarette prices are endogenous. Except for Kelejian and Piras (2014), previous spatial econometric studies based on Baltagi and Li's cigarette demand model did not treat price as being potentially endogenous. Although these studies argue or assume that price differences across states are largely due to state tax differences which are exogenously set by state legislatures, it is likely that demand has a feedback effect on price.

Therefore, we formally test whether price and prices observed in neighboring states may be considered exogenous. The advantage of the SLX model over other spatial econometric models is that non-spatial econometric techniques can be used for this purpose. It concerns the **Hausman test for endogeneity** in combination with tests for the **validity of the instruments** to assess whether they satisfy the **relevance and exogeneity criterions**. The methodology behind these tests is explained in many econometric textbooks; we used Hill et al. (2012, pp. 419-422).

Table 3 SLX model estimation results explaining cigarette demand and the parameterization of *W*

	BC	ID (γ=1)	ID	ED	ID γ_k's	ID Gravity
	(1)	(2)	(3)	(4)	(7)	(8)
Price	-1.017	-1.013	-0.908	-1.046	-0.903	-0.841
	(-24.77)	(-25.28)	(-24.43)	(-29.58)	(-24.49)	(-23.03)
Income	0.608	0.658	0.654	0.560	0.667	0.641
	(10.38)	(13.73)	(15.39)	(15.44)	(15.76)	(15.16)
W×Price	-0.220	-0.021	0.254	0.108	0.385	0.041
	(-2.95)	(-0.34)	(3.08)	(2.08)	(1.81)	(0.87)
W×Income	-0.219	-0.314	-0.815	0.129	-0.838	-0.372
	(-2.80)	(-6.63)	(-4.76)	(1.80)	(-5.21)	(-4.97)
$\gamma_{distance}$			2.938	0.467		2.986
			(16.48)	(9.99)		(11.50)
$\gamma_{distance\ price}$ in col.(7) and $\gamma_{own\ population}$ in col. (8)					5.986	-0.018
					(8.86)	(-0.41)
$\gamma_{distance\ income}$ in col.(7) and $\gamma_{population\ neighbors}$ in col.(8)					2.938	0.340
					(17.70)	(2.63)
R^2	0.897	0.899	0.916	0.896	0.917	0.923
LogL	1668.4	1689.8	1812.9	1666.9	1818.4	1868.0
Prob. SDM	0.5502	0.0000	0.0000	0.3536		
Prob. SDEM	0.4498	1.0000	1.0000	0.6464		

Notes: t-statistics in parentheses; coefficient estimates of *WX* variables in the SLX represent spillover effects.
[§] *W* matrix similar to the one used to model exogenous spatial lags *WX*.

Conclusion SLX modeling approach

- Instead of the common SAR and SEM models for an exogenously specified *W*, we propose to take the SLX model as point of departure using a *W* that is parameterized and to apply standard econometric techniques to test for endogenous explanatory variables.
- Parameterizing *W* is step forward since choice of cut-off point and/or limiting interval of distance decay parameter is restrictive.
- Test for endogenous explanatory variables is step forward since many X variables are not exogenous.
- The sign, magnitude, and significance level of the spillover effects are sensitive to both the specification of *W* and the spatial econometric model specification; the

SLX model helps to test different non-parameterized and parameterized specifications of W against each other.

- The claim made in many empirical studies that their results are robust to the specification of *W* should thus be more sufficiently substantiated. It might be that these studies mainly focus on the direct effects rather than the spatial spillover effects, which are generally the **MAIN FOCUS** in spatial econometric studies.

****

Bayesian comparison approach

To choose between SDM and SDEM, and thus between a global or local spillover model, as well as to choose between different potential specifications of **W**, a Bayesian comparison approach may be applied. This approach determines the **Bayesian posterior model probabilities** of the SDM and SDEM specifications given a particular **W** matrix, as well as the Bayesian posterior model probabilities of different **W** matrices given a particular model specification. These probabilities are based on the log marginal likelihood of a model obtained by integrating out all parameters of the model over the entire parameter space on which they are defined. If the log marginal likelihood value of one model or of one **W** is higher than that of another model or another **W**, the Bayesian posterior model probability is also higher. Whereas the popular likelihood ratio, Wald and/or Lagrange multiplier statistics compare the performance of one model against another model based on specific parameter estimates within the parameter space, the Bayesian approach compares the performance of one model against another model, in this case SDM against SDEM, on their entire parameter space. This is the main strength of this approach.

Inferences drawn on the log marginal likelihood function values for the SDM and SDEM model are further justified because they have the same set of explanatory variables, **X** and **WX**, and are based on the same uniform prior for ρ and λ. This prior takes the form $p(\rho)=p(\lambda)=1/D$, where $D=1/\omega_{max}-1/\omega_{min}$ and ω_{max} and ω_{min} represent respectively the largest and the smallest (negative) eigenvalue of the matrix **W**. This prior requires no subjective information on the part of the practitioner as it relies on the parameter space $(1/\omega_{min}, 1/\omega_{max})$ on which ρ and λ are defined, where $\omega_{max}=1$ if **W** is row-normalized. **Note: only available in Matlab.**

We find that the Bayesian posterior model probability for SDEM when W is specified as the parameterized distance matrix (this is testing for W only) is 1.0000 (Note the W used in error term is same parameterized distance matrix ≠ binary contiguity matrix). This is also what you expect from a theoretical viewpoint.

The Bayesian comparison approach has been applied successfully in:

(1) Firmino Costa da Silva D. , Elhorst J.P., Neto Silveira R.d.M. (2017), Urban and Rural Population Growth in a Spatial Panel of Municipalities, *Regional Studies* 51(6): 894-908.

Table. Comparison of model specifications and spatial weights matrices

W Matrix	Statistics	SDM	SDEM
Binary Contiguity	log marginal likelihood	3616.03	3611.80
	model probabilities	0.9855	0.0145
Inverse distance	log marginal likelihood	3444.87	3455.44
	model probabilities	0.0000	1.0000
K=6 nearest neighbors	log marginal likelihood	3613.06	3613.60
	model probabilities	0.3676	0.6324

Source: Firmino et al. (2017)

(2)Yesilyurt M.E., Elhorst J.P. (2017) Impacts of neighboring countries on military expenditures: A dynamic spatial panel approach. *Journal of Peace Research*, http://journals.sagepub.com/doi/full/10.1177/0022343317707569.

Table II. Simultaneous Bayesian comparison of model specifications and spatial weight matrices

Data	Model	W1	W2	W3	Enemy	W1 + Enemy	W1 + Superpower	W1 + Do-minance	W1 + Enemy + Superpowers	Row total
COW Static model	SAR	0.1449	0.0000	0.0000	0.0000	0.0224	0.1435	**0.2838**	0.0244	0.6190
	SDM	0.0054	0.0000	0.0000	0.0000	0.0026	0.0052	**0.0119**	0.0028	0.0279
	SEM	0.0625	0.0000	0.0000	0.0000	0.0098	0.0676	**0.1603**	0.0111	0.3113
	SDEM	0.0074	0.0000	0.0000	0.0000	0.0033	0.0073	**0.0202**	0.0036	0.0418
COW Dynamic model	SAR	0.2335	0.0000	0.0000	0.0000	0.0315	0.2547	**0.4171**	0.0350	0.9719
	SDM	0.0002	0.0000	0.0000	0.0000	0.0000	0.0002	**0.0004**	0.0000	0.0009
	SEM	0.0043	0.0000	0.0000	0.0000	0.0016	0.0056	**0.0118**	0.0018	0.0252
	SDEM	0.0004	0.0000	0.0000	0.0000	0.0001	0.0004	**0.0010**	0.0001	0.0020
WB/SIPRI Static model	SAR	0.0016	0.0061	**0.5162**	0.0020	0.0019	0.0015	0.0016	0.0019	0.5328
	SDM	0.0081	0.0004	**0.0520**	0.0294	0.0055	0.0072	0.0081	0.0047	0.1155
	SEM	0.0013	0.0044	**0.2510**	0.0040	0.0013	0.0013	0.0013	0.0013	0.2660
	SDEM	0.0084	0.0003	**0.0235**	0.0275	0.0053	0.0076	0.0086	0.0086	0.0857
WB/SIPRI Dynamic model	SAR	0.0382	0.0645	**0.2365**	0.0353	0.0390	0.0383	0.0381	0.0390	0.5288
	SDM	0.0001	0.0002	0.0259	**0.0287**	0.0001	0.0001	0.0001	0.0001	0.0551
	SEM	0.0411	0.0480	**0.0835**	0.0387	0.0382	0.0413	0.0411	0.0382	0.3701
	SDEM	0.0001	0.0002	0.0181	**0.0272**	0.0001	0.0001	0.0001	0.0001	0.0459

The highest probability in each row is in bold and the probabilities in each block sum to 1.
Source: Own calculations, based on LeSage (2014, 2015).

Cross-sectional dependence tests of Pesaran (2015) in Econometric Reviews

The CD test uses the correlation coefficients between the time-series for each panel unit, which for N regions results in N x (N-1) correlations between region *r* and all other regions, for *r*=1 to *N*-1. Denoting these estimated correlation coefficients between the time-series for region *r* to *j* as $\hat{\rho}_{rj}$, the Pesaran (2015, eq.10) **CD test** is defined as $\mathrm{CD} = \sqrt{2T/N(N-1)}\sum_{r=1}^{N-1}\sum_{j=r+1}^{N}\hat{\rho}_{rj}$, where *T* is the number of observations on each region over the observation period 1973-2013. This test statistic has the limiting *N*(0,1) distribution as *T* goes to infinity first, and then *N*. This implies that the critical values of this two-sided test are -1.96 and 1.96 at the five percent significance level.

The **local CD test (dependent on W)** takes the form $\sqrt{T/S}\sum_{r=1}^{N-1}\sum_{j=1}^{N} w_{rj}\hat{\rho}_{rj}$ (Moscone and Tosetti, 20009, eq.22), where *S* is the sum of the elements of the spatial weight matrix and thus equal to N^2.

α-estimator of Bailey et al. (2017) in Journal of Applied Econometrics

To test whether the strength of the found cross-sectional dependence, we apply the **exponent α-test of Bailey et al. (2015).**[2] This test statistic can take values on the interval (0,1] and measures the rate at which the variance of the cross-sectional averages tends to zero; $\alpha \leq 1/2$ points to weak cross-sectional dependence only and $\alpha = 1$ to strong cross-sectional dependence. Values in between indicate moderate to strong cross-sectional dependence and require additional research to discriminate between weak and strong cross-sectional dependence.

$$\alpha = 1 + \frac{1}{2}\frac{\ln \sigma_{\bar{x}}^2}{\ln(N)} - \frac{1}{2}\frac{c_N}{(N\ln N)\sigma_{\bar{x}}^2} - \frac{1}{2}\frac{\ln u_v^2}{\ln(N)}$$

---

2 Gauss code to calculate the CD and the α-tests are made available in an online appendix to their paper.

The first component is the dominating term, the second and third components are bias correction terms. These three components are added to the constant 1. Prior to any calculations, the data need to be standardized for each single unit in the sample, to get $x_{it} \equiv (x_{it} - \bar{x}_i)/\frac{1}{N}\sum_{i=1}^{N}(x_{it} - \bar{x}_i)^2$. It is to be noted that standardization is not required for the CD test since the pairwise correlation coefficients do change when the data are standardized.

The term $\sigma_{\bar{x}}^2$ in the first component is defined as $\sigma_{\bar{x}}^2 = \frac{1}{T}\sum_{t=1}^{T}(\bar{x}_t - \bar{x})^2$, where $\bar{x} = \frac{1}{T}\sum_{t=1}^{T}\bar{x}_t$. These expressions state that, firstly, the cross-sectional average ($\bar{x}_t$) needs to be determined in each time period, secondly, the overall average $\bar{x}$ over these T cross-sectional averages and, finally, the standard deviation $\sigma_{\bar{x}}^2$ of this overall average. Due to the standardization of the data $\sigma_{\bar{x}}^2 < 1$, as a result of which $\ln \sigma_{\bar{x}}^2 < 0$ and $1 + \frac{1}{2}\frac{\ln \sigma_{\bar{x}}^2}{\ln(N)} < 1$.

Testing for common-factors: CD-test and exponent α-estimator

Elhorst, J.P., Gross M., Tereanu E. (2018) Spillovers in space and time: where spatial econometrics and Global VAR models meet. European Central Bank, Frankfurt. Working Paper Series No 2134. https://www.ecb.europa.eu/pub/pdf/scpwps/ecb.wp2134.en.pdf?b33bf8d0dc4c5addae515ce126b98b7d.

Interplay between cross-section dependence, CF, weight structure and estimation

α can be estimated consistently only for $1/2 < \alpha \leq 1$. Use Pesaran's CD test to find out whether α is smaller or greater than ½.

α	**Cross section dependence**	**Weight structure**
0<α<0.5	weak	sparse: local, mutually dominant units
0.5<α<0.75	moderate	still quite sparse
0.75<α<1	quite strong	dense
1	strong	CS averages or PC (no weights involved)

A regional unemployment model simultaneously accounting for serial dynamics, spatial dependence and common factors†

Solmaria Halleck Vega and J. Paul Elhorst

Key words: Regional unemployment, strong and weak cross-sectional dependence, dynamic spatial panel models, the Netherlands

JEL classification: C23, C33, C38, R23

---

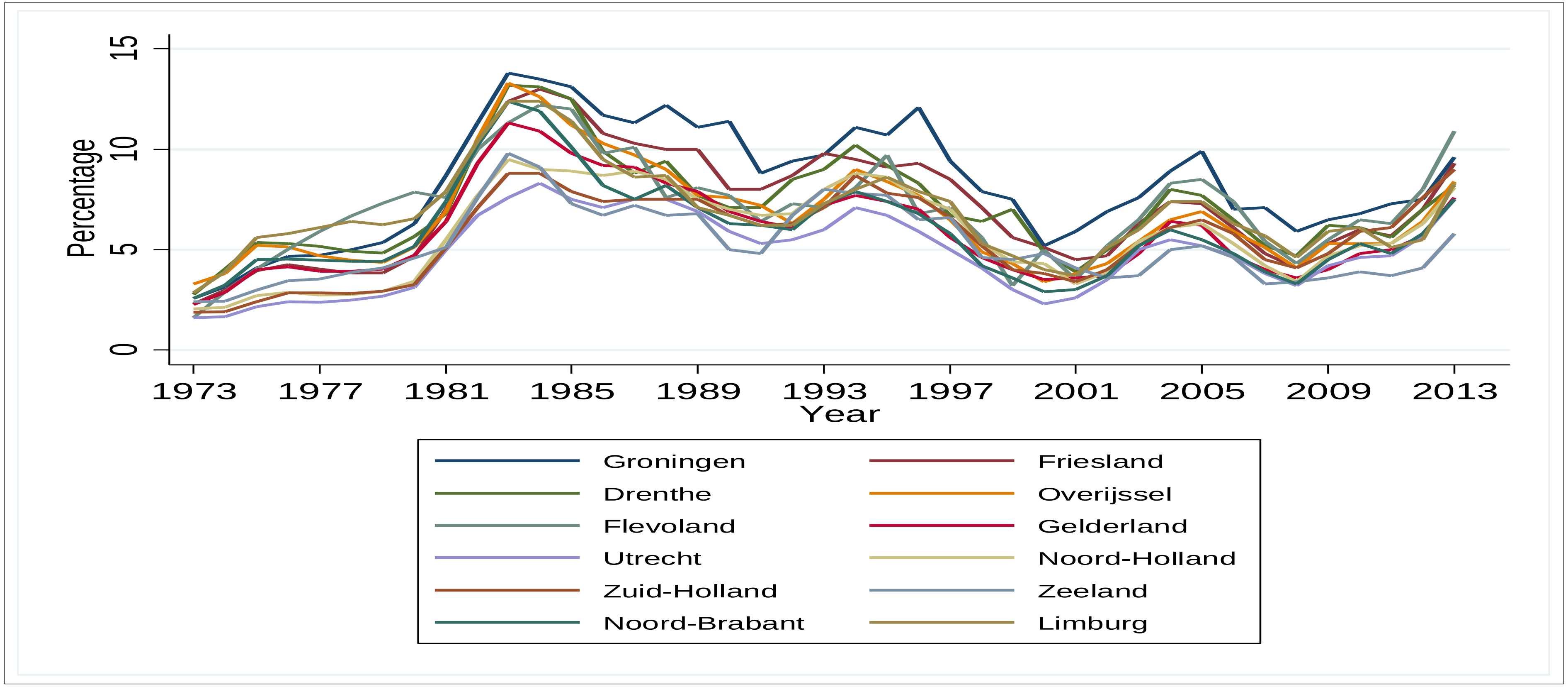
†Regional Science and Urban Economics 60 (2016) 85-95.

Regional unemployment rates tend to be strongly correlated over time: Serial dynamics.

Table 1. Regional unemployment correlations over time

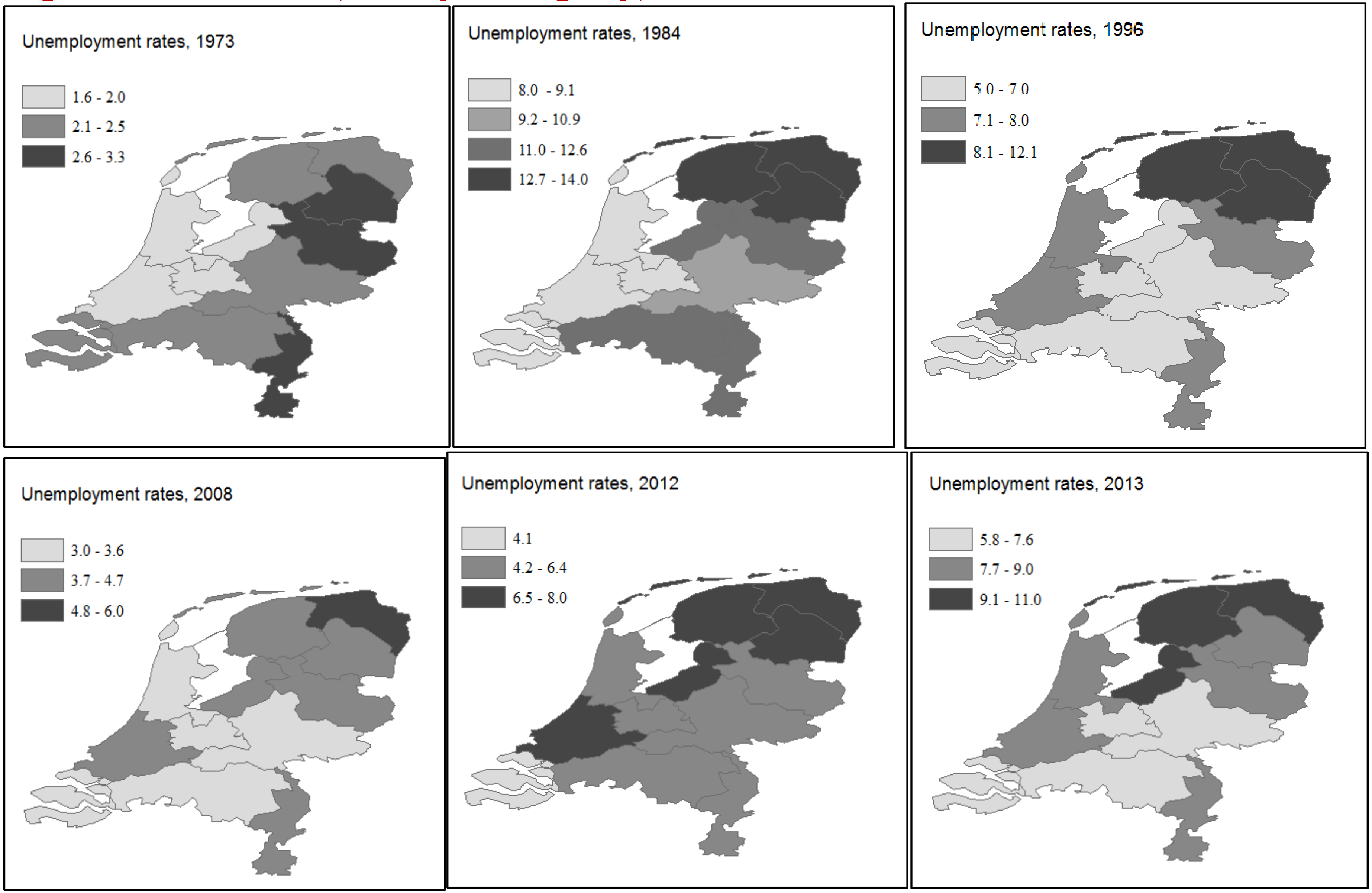
Year	1973	1974	1976	1981	1991	2001	2013
1973	1.00						
1974	0.82	1.00					
1976	0.67	0.95	1.00				
1981	0.45	0.73	0.83	1.00			
1991	0.34	0.43	0.41	0.67	1.00		
2001	0.37	0.35	0.36	0.57	0.73	1.00	
2013	-0.21	0.16	0.34	0.55	0.62	0.30	1.00

Regional unemployment rates in the Netherlands, 1973-2013



Regional unemployment rates parallel the national unemployment rate: Strong cross-sectional dependence. CD=46.31, α=1.008 (se=0.019)

Regional unemployment rates are correlated across space: Weak cross-sectional dependence. CD_{local}(Binary contiguity)=20.38



A unified methodology to simultaneously address the three key stylized facts, known as serial dynamics, strong and weak cross-sectional dependence.

To deal with these stylized facts, Bailey, Holly and Pesaran (2015, Journal of Applied Econometrics) propose a separation of two model stages: first, accounting for common factors (strong cross-sectional dependence) and second, accounting for spatial effects (weak cross-sectional dependence) and serial dynamics. However, it is more likely that weak and strong cross-sectional dependence are interdependent. *The impact of the national economy on its regions may affect the mutual structure among them, while a change in this mutual structure may affect the impact of the national economy.*

Unified approach

Stage 1: $u_{rt} = \gamma_{0r} + \gamma_{1r}\frac{1}{R}\sum_{j=1}^{R} u_{jt} \approx \gamma_{0r} + \gamma_{1r}u_{Nt}$ → $\hat{e}_{rt} = u_{rt} - \hat{\gamma}_{0r} - \hat{\gamma}_{1r}u_{Nt}$

Stage 2: $\hat{e}_{rt} = \alpha_0 + \alpha_1\hat{e}_{rt-1} + \alpha_2\sum_{j=1}^{R} w_{rj}\,\hat{e}_{jt} + \alpha_3\sum_{j=1}^{R} w_{rj}\,\hat{e}_{jt-1} + (\boldsymbol{\mu_r}) + (\boldsymbol{\lambda_t}) + \varepsilon_{rt}$

(Dynamic spatial panel data model without exogenous explanatory variables)

Elements of W specified as a binary contiguity matrix (1 share a common border, 0 otherwise)

Substitute 1 in 2

$$(u_{rt} - \gamma_{0r} - \gamma_{1r}u_{Nt}) = \alpha_0 + \alpha_1(u_{rt-1} - \gamma_{0r} - \gamma_{1r}u_{Nt-1}) + \alpha_2\sum_{j=1}^{R} w_{rj}\left(u_{jt} - \gamma_{0j} - \gamma_{1r}u_{Nt}\right) + \alpha_3\sum_{j=1}^{R} w_{rj}\left(u_{jt-1} - \gamma_{0j} - \gamma_{1r}u_{Nt-1}\right) + \varepsilon_{rt}$$

and rearrange terms

$$u_{rt} = \alpha_0 + \gamma_{0r} - \alpha_1\gamma_{0r} - \alpha_2\sum_{j=1}^{R} w_{rj}\,\gamma_{0j} - \alpha_3\sum_{j=1}^{R} w_{rj}\,\gamma_{0j}$$

$$+ \alpha_1 u_{rt-1} + \alpha_2\sum_{j=1}^{R} w_{rj}u_{jt} + \alpha_3\sum_{j=1}^{R} w_{rj}\,u_{jt-1}$$

$$+ \gamma_{1r}(1 - \alpha_2)u_{Nt} + \gamma_{1r}(-\alpha_1 - \alpha_3)u_{Nt-1} + \varepsilon_{rt}$$

$$u_{rt} = \underbrace{\alpha_0 + \gamma_{0r} - \alpha_1\gamma_{0r} - \alpha_2 \sum_{j=1}^{R} w_{rj}\, \gamma_{0j} - \alpha_3 \sum_{j=1}^{R} w_{rj}\, \gamma_{0j}}_{\mu'_r}$$

$$+ \alpha_1 u_{rt-1} + \alpha_2 \sum_{j=1}^{R} w_{rj} u_{jt} + \alpha_3 \sum_{j=1}^{R} w_{rj}\, u_{jt-1}$$

$$\underbrace{+ \gamma_{1r}(1 - \alpha_2)}_{\beta_{4r}} \boldsymbol{u_{Nt}} \underbrace{+ \gamma_{1r}(-\alpha_1 - \alpha_3)}_{\beta_{5r}} \boldsymbol{u_{Nt-1}} + \varepsilon_{rt}$$

The first composite term in the resulting equation is a heterogeneous constant, which can be accounted for by controlling for spatial (regional) fixed effects. The next three terms (second line) show that the regional unemployment rate at time *t* depends on its serially lagged value, spatially lagged value, and its value lagged both in space and time. In addition, the last two terms (last line) show that the regional unemployment rate also depends on the national unemployment rate at times *t* and *t*-1 with coefficients $\gamma_{1r}(1 - \alpha_2)$ and $\gamma_{1r}(-\alpha_1 - \alpha_3)$, respectively. They are accounted for by β_{4r} and β_{5r}.

Table Simultaneous approach to strong and weak cross-sectional dependence

Strong cross-sectional dependence					γ_{1r}		γ_{1r}	
	β_{4r}		B_{5r}		$\beta_{4r}/(1-\alpha_2)$		$B_{5r}/(-\alpha_1-\alpha_3)$	
Groningen	0.913	(0.114)	-0.675	(0.152)	1.034	(0.169)	**0.910**	(0.054)
Friesland	0.986	(0.112)	-0.736	(0.143)	1.118	(0.157)	0.991	(0.045)
Drenthe	1.020	(0.113)	-0.906	(0.145)	1.155	(0.156)	1.221	(0.035)
Overijssel	1.092	(0.111)	-0.919	(0.134)	1.237	(0.146)	1.238	(0.032)
Flevoland	0.925	(0.108)	-0.824	(0.131)	1.048	(0.162)	1.111	(0.035)
Gelderland	0.881	(0.109)	-0.726	(0.127)	0.998	(0.168)	0.978	(0.040)
Utrecht	0.696	(0.107)	-0.590	(0.128)	0.789	(0.196)	0.794	(0.054)
North-Holland	0.814	(0.108)	-0.636	(0.132)	0.922	(0.174)	0.857	(0.050)
South-Holland	0.764	(0.108)	-0.628	(0.126)	0.866	(0.181)	0.846	(0.048)
Zeeland	0.637	(0.111)	-0.550	(0.127)	0.722	(0.215)	0.740	(0.059)
North-Brabant	1.079	(0.106)	-0.949	(0.123)	1.223	(0.142)	**1.279**	(0.028)
Limburg	0.926	(0.113)	-0.839	(0.126)	1.050	(0.166)	1.130	(0.033)
Weak cross-sectional dependence								
α_1	**0.664**	(0.038)						
α_2	**0.118**	(0.059)						
α_3	0.079	(0.082)						
R^2	0.956							
Log-Likelihood	-362.3							

Notes: Standard errors are reported in parentheses; spatial fixed effects included. The bias corrected ML estimator developed in Yu et al. (2008) is applied.

CD = -0.020 and CD_{local} = -1.034 based on residuals of the model

Special case: Two-stage approach strong and weak cross-sectional dependence of Bailey et al. (2015):

$$\beta_{4r}/(1-\alpha_2) = \beta_{5r}/(-\alpha_1-\alpha_3) \text{ for } r = 1, \dots, R$$

$$\hat{e}_{rt} = u_{rt} - \hat{\gamma}_{0r} - \hat{\gamma}_{1r} u_{Nt}$$

$$\hat{e}_{rt} = \alpha_0 + \alpha_1 \hat{e}_{rt-1} + \alpha_2 \sum_{j=1}^{R} w_{rj}\, \hat{e}_{jt} + \alpha_3 \sum_{j=1}^{R} w_{rj}\, \hat{e}_{jt-1} + (\mu_r) + (\lambda_t) + \varepsilon_{rt}$$

Table. Strong cross-sectional dependence: First stage

	γ_{0r}		γ_{1r}	
Groningen	-0.069	(0.377)	**1.362**	(0.058)
Friesland	-1.418	(0.377)	1.341	(0.058)
Drenthe	0.004	(0.377)	1.160	(0.058)
Overijssel	-0.819	(0.377)	1.208	(0.058)
Flevoland	0.284	(0.377)	1.098	(0.058)
Gelderland	-0.940	(0.377)	1.108	(0.058)
Utrecht	-0.788	(0.377)	0.914	(0.058)
North-Holland	-0.781	(0.377)	1.063	(0.058)
South-Holland	-0.200	(0.377)	0.945	(0.058)
Zeeland	0.026	(0.377)	0.842	(0.058)
North-Brabant	-0.974	(0.377)	**1.119**	(0.058)
Limburg	0.603	(0.377)	1.011	(0.058)
R^2	0.918			
Log-Likelihood	-537.0			

Table. Weak cross-sectional dependence: Second stage

Panel B
(3)
0.001
(0.024)
0.643
(0.036)
0.147
(0.057)
0.054
(0.081)
No
No
0.455
-379.8

Notes: Standard errors are reported in parentheses.

A LR test whether these 12 coefficients in both columns are the same can be based on the log-likelihood function value of the simultaneous model (-362.3) and that of the de-factoring model including spatial fixed effects (-379.2). This yields 33.9 with 12 df and p=0.00, indicating that the two-stage model needs to be rejected in favor of the simultaneous model and that the national rate has a different impact on the provinces in time *t* compared to *t*-1.

Spatial panel dynamic approach: $\beta_{4r} = \beta_{5r} = 0$, but time-period fixed effects can partly (but not fully) mitigate the effects of omitting the national unemployment rate from the model, albeit $\gamma_{1r} = 1$ is unrealistic.

$$u_{rt} = \alpha_0 + \alpha_1 u_{rt-1} + \alpha_2 \sum_{j=1}^{R} w_{rj}\, u_{jt} + \alpha_3 \sum_{j=1}^{R} w_{rj}\, u_{jt-1} + \mu_r + \boldsymbol{\lambda_t} + \varepsilon_{rt}$$

Table. Weak cross-sectional dependence

	Panel A	
	(1)	(2)
α_0		
α_1	0.679	**0.716**
	(0.039)	**(0.037)**
α_2	**0.756**	**0.252**
	(0.025)	**(0.073)**
α_3	-0.460	-0.002
	(0.047)	(-0.038)
Spatial fixed effects	Yes	Yes
Time fixed effects	No	Yes
R^2	0.942	0.955
Log-Likelihood	-481.3	-368.9

Notes: Standard errors are reported in parentheses;

Time period fixed effects may partly cover common factors. However, it is important to note this is a homogeneous approach in the sense that it assumes that the impact of common factors is the same across regions, which is not likely to be the case in many applied settings.

-If time-period fixed effects are controlled for, the spatial autoregressive coefficient estimate is much lower with a value of 0.252 (0.073), that is, compared to the dynamic spatial panel data model without time-period fixed effects, while the lagged spatial autoregressive coefficient decreases considerably in magnitude and becomes insignificant. This confirms the importance of including time-period fixed effects to partly cover for the fact that regional unemployment rates tend to increase and decrease together along the national evolution of this variable over time.

-It also corroborates Lee and Yu's (2010a) finding that if this common effect is not taken into account and therefore not separated out from the local spatial interaction effects among the regions, the latter may be severely overestimated as is clearly shown by the numbers above.

-However, the impact of this common effect is assumed to be the same across regions, $\gamma_{1r} = 1$ for $r = 1, \cdots, N$, which is not realistic, especially considering the regional cyclical sensitivity and the common factor literature. This also follows from the CD and α tests applied to the residuals of this model: **CD = 2.052**, CD_{local} = 0.812, and α=0.722 with standard error 0.076. Local spatial dependence appears to be effectively covered, but **the CD test still provides evidence in favor of common factors,** while the α-test, despite its decrease when applied to the raw data, still points to moderate cross-sectional dependence.

- Conclusion: Common factors are to be preferred over time dummies.

-According to the two-step approach the impact of the national unemployment rate is strongest for Groningen with 1.362, while the elasticity amounts to 1.119 for North-Brabant.

-According to the simultaneous approach, we obtain 1.034 for Groningen and 1.223 for North-Brabant. Focusing on the coefficients in the last column of Table 4 which are more reliable, we obtain 0.910 for Groningen and 1.279 for North-Brabant.

-**Groningen is therefore less cyclically sensitive, while North-Brabant is more sensitive.** This change may reflect that the Northern provinces such as Groningen suffer more from long-standing disadvantage relative to the nation, such as structural unemployment, than the results of the two-stage approach suggest.

-The estimate of the contemporaneous spatial interaction effect (**weak cross-sectional dependence**) falls from **0.252** when ignoring strong cross-sectional dependence- but when accounting for spatial and time-period fixed effects-, to **0.147** when using the two-step approach, and finally to **0.118** when accounting for weak and cross-sectional dependence simultaneously.

Conclusion

To better explain the evolution of a cross-section of regional unemployment rates over time, a unified approach is needed that simultaneously accounts for both strong and weak cross-sectional dependence, as well as serial dynamics.

Otherwise the results will be biased!

Can be extended with X and WX variables.

Cross-sectional averages can be replaced by principal components.