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Solving discretely-constrained MPEC problems with applications in electric power markets

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ABSTRACT

Many of the European energy markets are characterized by dominant players that own a large share of their respective countries' generation capacities. In addition to that, there is a significant lack of cross-border transmission capacity. Combining both facts justifies the assumption that these dominant players are able to influence the market outcome of an internal European energy market due to strategic behavior. In this paper, we present a mathematical formulation in order to solve a Stackelberg game for a network-constrained energy market using integer programming. The strategic player is the Stackelberg leader and the independent system operator (including the decisions of the competitive fringe firms) acts as follower. We assume that there is one strategic player which results in a mathematical program with equilibrium constraints (MPEC). This MPEC is reformulated as mixed-integer linear program (MILP) by using disjunctive constraints and linearization. The MILP formulation gives the opportunity to solve the problems reliably and paves the way to add discrete constraints to the original MPEC formulation which can be used in order to solve discretely-constrained mathematical programs with equilibrium constraints (DC-MPECs). We report computational results for a small illustrative network as well as a stylized Western European grid with realistic data.

1. Introduction

It is widely accepted that many of the national European Union (EU) electricity markets are characterized by market power of a single or only a few companies. In France, Electricité de France (EDF) has a market share of over 80%. In Germany, the two biggest players own together 55% of the generation capacities, the biggest four players have together 85% market share (Hirschhausen et al., 2007). These numbers show that there are still nearly monopolistic or oligopolistic market structures all over Europe. Moreover, these shortcomings in national competition also have an impact on competition within the entire EU electricity market, particularly because the national markets are separated from each other by limited transport capacities which might enable national players to use their domestic power against foreign competitors.

In light of these facts, several models and algorithms have been developed in order to simulate the outcomes of imperfect electricity markets. These models include a broad range of approaches using game theory. It can be stated that existing modeling efforts – discussed in the subsequent literature sections – have achieved some success but there is still room to handle larger-scale or more realistic models as might be found in the EU, North America or in other parts of the world.

At a first glance, it seems that one-stage Nash–Cournot models can be solved robustly for large-scale problems. If it comes to two-stage games involving either a mathematical program with equilibrium constraints (MPEC) or an equilibrium program with equilibrium constraints (EPEC), most algorithms are for the most part, still in the development stage when one considers large-scale or integer-constrained formulations.

In this paper, we present a new approach in order to solve two-stage Stackelberg games with one leader based on disjunctive constraints and linearization. We are able to replace the equilibrium constraints of the MPEC by integer restrictions in the form of disjunctive constraints (Fortuny-Amat and McCarl, 1981; Gabriel et al., 2007). Also, a bilinear objective function of an electricity market model stemming from the product of both price and generation variables is linearized using additional binary and continuous variables and new constraints. The end result is that the MPEC can be replaced by a mixed-integer linear program (MILP). This allows for a whole host of important applications such as: discrete generation levels, fixed cost problems involving binary variables, if-then logic relative to ramping constraints (Winston, 1994), discrete investment levels, and so on. A second advantage of using our method in this context is to be able to solve larger-scale problems in electric power markets than previously attempted. Lastly, we use a detailed formulation of the DC load flow model in order to model physical flows which facilitate a greater flexibility for changing the network

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topology. Our numerical results on two illustrative problems are promising.

The rest of this paper is organized as follows. In Section 2 we discuss the existing relevant literature concerning two-stage modeling and its applications in electricity markets. In Section 3 we present the general mathematical formulation for our two-stage problem. The upper level is a quadratic program and the lower level is a welfare maximization problem for an independent system operator (ISO). Next in Section 4, we describe numerical results of our approach for two illustrative examples. We conclude with summary remarks and future directions in Section 5.

2. Literature

As stated above, this paper discusses a new method for solving two-level planning problems with applications in electric power. The upper level involves generation decisions for the Stackelberg leader and the lower level depicts the rest of the market and the ISO problem. These two-level problems are known as Stackelberg games or more generally as mathematical programs with equilibrium constraints (MPEC) (Facchinei and Pang, 2003) which are NP-complete (Jeroslow, 1985). These problems have applications in a number of important planning areas such as: electric power management (Hobbs and Nelson, 1992; Hobbs et al., 2000); taxation and optimal highway pricing (Labbe et al., 1998), the government and the agricultural sector (Bard et al., 2000); chemical process engineering (Raghunathan and Biegler, 2003); engineering safety factors for a rubble mound breakwater and a bridge crane (Castillo et al., 2003); NOx allowances markets in electric power production (Chen et al., 2004); MPECs in traffic planning (Codina et al., 2006); network design in transportation (Gao et al., 2004); see the annotated bibliography (Vicente and Calamai, 1994) for related works.

Recently there has been a fair amount of research devoted to solving MPECs with continuous-valued variables. Some examples include: implicit nonsmooth approaches (Outrata et al., 1998), piecewise sequential quadratic programming (SQP) methods (Luo et al., 1996), and perturbation and penalization methods (Dirkse et al., 2002; Scholtes, 2001; Leyffer et al., 2006). As noted by Leyffer (2003), these approaches require more computational effort than standard nonlinear programming methods. However, within the last five years, there have been some algorithmic successes. For example, Fletcher and others (Fletcher et al., 2002; Fletcher and Leyffer, 2002; Fletcher and Leyffer, 2004) have shown that SQP methods have good numerical results as well as some advantageous convergence properties (Anitescu, 2000; Fletcher et al., 2002). Additionally, as noted by Leyffer (2003), interior-point methods while less reliable than SQP are still able to solve about 80% of the mathematical programs considered with complementarity constraints (MPCCs) using default settings. Further improvements can be gained by applying a relaxation of the complementarity constraints typically found from incorporating the lower-level optimality conditions in the upper-level (Liu and Sun, 2004; Raghunathan and Biegler, 2002; Ralph and Wright, 2004; De Miguel et al., 2004; De Miguel et al., 2005), or penalizing these constraints (e.g., PIPA, Luo et al., 1996; Hu and Ralph, 2004; Anitescu, 2004) but in the latter case, it has been shown that the standard PIPA method (Luo et al., 1996) can fail to converge (Leyffer, 2005) to a stationary point in some cases. Lastly one can also make use of a trust region approach applied to the nonlinear bilevel programming problem as described in Marcotte et al. (2001).

The proposed methodology by contrast can easily incorporate integer and continuous-valued variables. A few others have also considered methods for solving MPECs with integer restrictions. These other methods can be grouped into three categories: application-specific approaches, integer-programming methods, or nonlinear programming-based algorithms. Methods specific to applications include for example, a branch and bound version for a flow shop bilevel problem (Karlof and Wang, 1996), shortest path and transshipment algorithms for a modified network for special cases of a bilevel taxation and optimal high pricing formulation (Labbe et al., 1998), and a tree search approach (Scaparra and Church, 2008) to analyze critical infrastructure planning. Integer (or linear) programming methods include for example, a Grid Search Algorithm (Bard, 1983), a simplex-like method (Bialas and Karwan, 1984) or branch and bound (Bard, 1988; Moore and Bard, 1990; Bard and Moore, 1990; Karlof and Wang, 1996), Tabu Search (Wen and Huang, 1996), or genetic algorithm methods (Hejazi et al., 2002; Nishizaki et al., 2003). For discretely-constrained bilevel optimization problems, Moore and Bard (1990) point out that it is not always possible to get tight upper bounds using common relaxation methods and also two of three standard fathoming rules employed used in branch and bound cannot be fully used. Thus, the more traditional approaches may not work for these sorts of problems.

Barroso et al. (2006) develop a framework in order to simulate Nash equilibria in strategic bidding for short-term electricity markets which they denote as ‘binary expansion’ (BE). Our approach is similar to what they did, however, a major difference is the additional consideration of the physical transmission network in our model. Moreover, it is often argued that an EPEC formulation is the most appropriate way to model strategic behavior in short-term electricity markets as EPECs allow more than one strategic player. The justification for our MPEC approach, however, can be seen in other studies, e.g., one of the Center of European Economic Research (ZEW) in Germany (Nikogosian and Veith, 2009). They argue that in an increasing integrated market, the distribution of market shares changes which might lead to fewer or even only a single strategic actor. Furthermore, our formulation could include a simple form of a cartel.

3. General mathematical formulation

Before describing an application in power planning, we present a more stylized version of the problem at hand. The general form of the problem to be solved by the leader (e.g., strategic generator) is as follows where $x \in R^n$, $y \in R^m$ are respectively, the upper- and lower-level vectors of variables:

$$\min_{x,y} \left\{ \begin{pmatrix} d_x \\ d_y \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2} \begin{pmatrix} x \\ y \end{pmatrix}^T \begin{pmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \right\} \tag{1a}$$

$$\text{s.t. } A_1 y + B_1 x = b_1 \ (\beta_1) \tag{1b}$$

$$A_2 x = b_2 \ (\beta_2) \tag{1c}$$

$$A_3 x \le b_3 \ (\beta_3) \tag{1d}$$

$$A_4 y = b_4 \ (\beta_4) \tag{1e}$$

$$A_5 y \le b_5 \ (\beta_5) \tag{1f}$$

$$x_i \in Z_+ \ , i = 1, \ldots, n_1 \tag{1g}$$

$$x_i \in R, i = n_1 + 1, \ldots, n \tag{1h}$$

$$y \in S(x) \tag{1i}$$

$$y_j \ge 0, j = 1, \ldots, m_1 \tag{1j}$$

$$y_2 \text{ free}, \ j = m_1 + 1, \ldots, m \tag{1k}$$

where $A_1, A_2, A_3, A_4, A_5, B_1$ are matrices of suitable size conformal with the vectors x, y and right-hand sides b_1, b_2, b_3, b_4, b_5. The vectors d_x, d_y contain coefficients for x and y, and $M_{xx}, M_{xy}, M_{yx}, M_{yy}$ are the submatrices referring to the quadratic terms of the objective function. The objective function (1a) is quadratic in both the upper- and lower-level variables which in the particular power application described in Section 4 will involve pairwise products of variables (e.g., generation times price) as well as linear terms. Eq. (1b) is the set of joint constraints linking the upper- and lower-level variables with β_1 representing the dual variables to these constraints (similar notation for dual variables for the other constraints). Eqs. (1c) and (1d) are the constraints that only involve the upper-level variables x whereas Eqs. (1e) and (1f) are the counterparts for the lower-level variables y. Eqs. (1g) and (1h) indicate that a subset of the upper-level variables are integer-valued whereas constraint (1i) stipulates that y must be a solution to the lower-level problem given x. Lastly, the vector y is partitioned into a nonnegative subvector (y_1) and the remaining variables (y_2) free as shown in the last two constraints.

The lower-level problem will typically be either a convex, quadratic program whose necessary and sufficient Karush–Kuhn–Tucker conditions or a Nash–Cournot game can be expressed as a mixed linear complementarity problem (MLCP) (Facchinei and Pang, 2003) given as follows:

$$0 \le c_1(x) + M_{11}(x)y_1 + M_{12}(x)y_2 \perp y_1 \ge 0 \tag{2a}$$

$$0 = c_2(x) + M_{21}(x)y_1 + M_{22}(x)y_2 \quad y_2 \text{ free} \tag{2b}$$

where the dependence on the upper-level variables can be in the vector $c = (c_1(x)^T c_2(x)^T)^T$ and/or the matrix

$$M = \begin{pmatrix} M_{11}(x) & M_{12}(x) \\ M_{21}(x) & M_{22}(x) \end{pmatrix}.$$

Having a sufficiently large constant K, the complementarity conditions (2a)–(2b) can be converted to disjunctive constraints (Fortuny-Amat and McCarl, 1981; Gabriel et al., 2007) as

$$0 \le c_1(x) + M_{11}(x)y_1 + M_{12}(x)y_2 \le Kr \tag{3a}$$

$$0 \le y_1 \le K(1-r) \tag{3b}$$

$$0 = c_2(x) + M_{21}(x)y_1 + M_{22}(x)y_2 \quad y_2 \text{ free} \tag{3c}$$

where r is a vector of binary variables. In general finding a reasonable constant K may take trial and error. However, in specific instances such as the case study described below, a suitable value can easily be found; see Appendix A for further guidance on how to obtain such a constant.

Replacing Eq. (1i) by Eqs. (3a)–(3c) leads to the overall problem expressed in disjunctive form:

$$\min_{x,y} \left\{ \begin{pmatrix} d_x \\ d_y \end{pmatrix}^T \begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{2} \begin{pmatrix} x \\ y \end{pmatrix}^T \begin{pmatrix} M_{xx} & M_{xy} \\ M_{yx} & M_{yy} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \right\} \tag{4a}$$

$$\text{s.t. } A_1 y + B_1 x = b_1 \ (\beta_1) \tag{4b}$$

$$A_2 x = b_2 \ (\beta_2) \tag{4c}$$

$$A_3 x \le b_3 \ (\beta_3) \tag{4d}$$

$$A_4 y = b_4 \ (\beta_4) \tag{4e}$$

$$A_5 y \le b_5 \ (\beta_5) \tag{4f}$$

$$x_i \in Z_+, i = 1, \ldots, n_1 \tag{4g}$$

$$x_i \in R, i = n_1 + 1, \ldots, n \tag{4h}$$

$$0 \le c_1(x) + M_{11}(x)y_1 + M_{12}(x)y_2 \le Kr \tag{4i}$$

$$0 \le y_1 \le K(1-r) \tag{4j}$$

$$0 = c_2(x) + M_{21}(x)y_1 + M_{22}(x)y_2 \quad y_2 \text{ free} \tag{4k}$$

$$y_j \ge 0, j = 1, \ldots, m_1 \tag{4l}$$

$$y_2 \text{ free}, \ j = m_1 + 1, \ldots, m \tag{4m}$$

$$r_i \in \{0,1\}^{m_1}. \tag{4n}$$

In the next section we specialize this quadratic program with equilibrium constraints to an electric power market example.

4. Numerical example: strategic behavior in an electricity market

4.1. Literature

There are several recent models regarding strategic interactions in an electricity market with network constraints (Daxhelet and Smeers, 2001; Day et al., 2002; Neuhoff et al., 2005; Ventosa et al., 2005). Most of these models try to find a market equilibrium as a set of prices and quantities in terms of demand and output decisions of generators that intend to maximize their individual profits. Prices are then a result of the marginal cost of the last producing unit and the transmission fee due to congestion — and sometimes other factors, e.g. network losses. The resulting output and demand quantities lead to a certain utilization of transmission line capacities.

Strategic models can be mixed-complementarity problems (MCP), MPECs, or EPECs. Nash–Cournot models are often modeled as MCPs (e.g. Hobbs, 2001). For the purpose of including an oligopolistic market, it is often assumed that there are firms that act strategically and firms that act as price-takers (Neuhoff et al., 2005). Strategic firms decide first on their output decisions in order to maximize individual profits. These firms know that they influence the market equilibrium with their decisions. The output decision of the other fringe firms, however, is determined by the ISO whereas the quantities of the strategic firms are exogenous to the ISO problem. This type of problem is known as Stackelberg game where the strategic firms are the leaders and the independent system operator (ISO) is the follower. The ISO decides on the quantities of the fringe consistent with a pool-based power system.

A Stackelberg game with only one leader is an example of an MPEC. A Stackelberg game with several leaders is an EPEC (Ralph and Smeers, 2006). Ehrenmann (2004) states that for liberalized electricity markets, there are two types of models that lead to different types of strategic formulations. He distinguishes a centralized and a decentralized system. Within the centralized system, the energy and transmission markets are cleared simultaneously, whereas, in the decentralized system, there are separate markets for transmission and energy which require a different complementarity formulation of the problem; our case studies assume a centralized system.

4.2. Mathematical formulation

We next describe a specific form of Eqs. (4a)–(4n) specialized to strategic behavior in electric power markets where there is one dominant player at the upper-level and several competitive fringe players included in the lower-level ISO problem. The notation for the problem is shown first below.

Indices:

n, k	nodes in the network
k'	swing bus
l	line between n and k
f	firms in the market
s	firms acting strategically
j	competitive fringe
u	generation units
i	possible generation level of strategic generation

Sets in the problem:

F	set of all firms
L	set of all lines

N set of all nodes
U set of all generation units

Parameters:

a_n, b_n intercept and slope of linear demand functions (a_n, $b_n \geq 0$, $\forall n$)
c_{nfu} generation cost per MWh of firm f at node n for unit u ($c_{nfu} \geq 0$, $\forall n, f, u$)
$\bar{g}_{nfu}$ maximum generation capacity per MWh of firm f at node n for unit u ($\bar{g}_{nfu} \geq 0$, $\forall n, f, u$)
B network susceptance matrix $n \times k$
H network transfer matrix $l \times k$
lc_l physical line capacity limit of line l
sw_k swing bus vector, $sw_k = \begin{cases} 1 & \text{if } k = k' \\ 0 & \text{otherwise} \end{cases} \forall k$
K, $\bar{K}$, $\hat{K}$, $\tilde{K}$, $\check{K}$ positive constants used to replace complementarities by disjunctive constraints
M positive constant

Variables:

d_n demand at node n
g_{nfu} generation of firm f at node n for unit u
$q_{nsu,i}$, q_n^{λ} binary variables for the linearization of the objective function
$q_{nsu,i}^{v}$ variable for the linearization of the objective function
δ_k phase angle at node k
λ_n shadow price for energy at node n
$\bar{\mu}_l$ shadow price for transmission on line l in the forward direction
$\underline{\mu}_l$ shadow price for transmission on line l in the backward direction
β_{nju} dual variable of maximum generation constraint per unit u of fringe firm j at node n
γ_k dual variable for slack bus constraint
r_n, $\bar{r}_{nfu}$, $\hat{r}_l$, $\tilde{r}_l$, $\check{r}_{nfu}$ binary variables used to replace complementarities by disjunctive constraints

A similar approach as presented subsequently is also presented by Barroso et al. (2006). They develop a framework in order to simulate Nash equilibria in strategic bidding for short-term electricity markets which they denote as 'binary expansion' (BE). However, in contrast to Barroso et al. (2006), we include the physical transmission network into our model. Including transmission issues is often regarded as essential feature in order to find short-term market clearing prices for electricity markets (e.g. Stoft, 2002). In addition, our model allows for a quadratic subproblem of the MPEC whereas the model of Barroso et al. (2006) requires a linear subproblem.

We begin with a social welfare maximization model assuming perfect competition (Eqs. (5a)–(5h)) for an ISO as the lower-level problem and then describe the upper-level problem. Given a linear inverse demand function $a_n - b_n d_n$, the term $\left(a_n d_n - \frac{b_n d_n^2}{2}\right)$ describes the area below the (inverse) demand curve for region n, that is, the gross surplus. The production cost for region n is given by $(g_{nfu}c_{nfu})$ which is then subtracted from the first part after appropriately summing both terms.

In order to calculate the physical grid utilization, electric power market models mostly apply the DC load flow (DCLF) model (Neuhoff et al., 2005) in order to obtain the line flows resulting from a certain generation-load combination. There are basically two different characteristics of the DCLF. One characteristic is a network PTDF matrix (Christie et al., 2000; Delarue et al., 2007). A PTDF matrix contains factors that quantify the impact of an injection or withdrawal at a certain location on all lines within the network. The PTDF can be derived from the network transfer matrix H and network susceptance matrix B. However, one can directly use the product of network susceptance matrix B and the voltage angle δ (Eq. (5b)) and the product of the matrix H and the voltage angle δ (Eqs. (5c)–(5d)) in the mathematical problem (Schweppe et al., 1988; Stigler and Todem, 2005)

Table 1
Correspondence between general formulation of MPEC and specific numerical example formulation.

General form	Specific form
Eq. (1a)	Eq. (8a)
Eq. (1d)	Eq. (8b)
Eq. (1i)	Eqs. (8c)–(8j)
All other constraints	Vacuous

which leads to a greater flexibility, e.g., when considering models where B and H are not constant. In our approach, Eq. (5b) represents the energy balance at node n with $\sum_k (B_{nk}\delta_k)$ corresponding to the net injection/withdrawal which must match the net demand $d_n - \sum_f \sum_u g_{nfu}$. Inequalities (5c) and (5d) correspond to constraining the line flows determined by $\sum_k (H_{lk}\delta_k)$. Constraint (5e) provides an upper bound on generation relating to installed capacity and lastly Eq. (5f) defines a slack bus. The following ISO problem is the starting point for our modeling approach and is the precursor to the equilibrium constraints for the subsequent MPEC.

$$\min_{d_n g_{nfu} \delta_k} \left\{ \sum_n \left(-a_n d_n + \frac{b_n d_n^2}{2} \right) + \sum_n \sum_f \sum_u \left(g_{nfu} c_{nfu} \right) \right\} \tag{5a}$$

$$\text{s.t. } d_n + \sum_k (B_{nk}\delta_k) - \sum_f \sum_u g_{nfu} = 0, \forall n \ (\lambda_n) \tag{5b}$$

$$-lc_l + \sum_k (H_{lk}\delta_k) \leq 0, \forall l \ (\bar{\mu}_l) \tag{5c}$$

$$-lc_l - \sum_k (H_{lk}\delta_k) \leq 0, \forall l \ \left(\underline{\mu}_l\right) \tag{5d}$$

$$-\bar{g}_{nfu} + g_{nfu} \leq 0, \forall n, f, u \ \left(\beta_{nfu}\right) \tag{5e}$$

$$-sw_k \delta_k = 0, \forall k \ (\gamma_k) \tag{5f}$$

$$d_n \geq 0, \forall n \tag{5g}$$

$$g_{nfu} \geq 0, \forall n, f, u \tag{5h}$$

In order to write the problem in a way that we can apply our approach, we first write out the Karush–Kuhn–Tucker (KKT) conditions (6a)–(6h) of the problem (Castillo et al., 2002).

$$0 \leq -a_n + b_n d_n + \lambda_n \perp d_n \geq 0, \forall n \tag{6a}$$

$$0 \leq c_{nfu} - \lambda_n + \beta_{nfu} \perp g_{nfu} \geq 0, \forall n, f, u \tag{6b}$$

$$0 = \sum_n (B_{nk}\lambda_n) + \sum_l (H_{lk}\bar{\mu}_l) - \sum_l (H_{lk}\ \underline{\mu}_l) - \begin{cases} \gamma_k & \text{if } k = k' \\ 0 & \text{otherwise} \end{cases} \quad \delta_k(\text{free}), \forall k \tag{6c}$$

$$0 = d_n + \sum_k (B_{nk}\delta_k) - \sum_f \sum_u g_{nfu}, \lambda_n(\text{free}), \forall n \tag{6d}$$

$$0 \leq lc_l - \sum_k (H_{lk}\delta_k) \perp \bar{\mu}_l \geq 0, \forall l \tag{6e}$$

$$0 \leq lc_l + \sum_k (H_{lk}\delta_k) \perp \underline{\mu}_l \geq 0, \forall l \tag{6f}$$

$$0 \leq -g_{nfu} + \bar{g}_{nfu} \perp \beta_{nfu} \geq 0, \forall n, f, u \tag{6g}$$

$$0 = -sw_k \delta_k, \quad \gamma_k(\text{free}), \forall k \tag{6h}$$

Then the KKTs are replaced by disjunctive constraints (7a)–(7m) as described in Section 3. The purpose of this disjunctive form is to have mixed-integer linear constraints at hand.

$$0 \le -a_n + b_n d_n + \lambda_n \le K r_n, \forall n \quad (7a)$$

$$0 \le d_n \le K(1 - r_n), \forall n \quad (7b)$$

$$0 \le c_{nfu} - \lambda_n + \beta_{nfu} \le \overline{K}\,\overline{r}_{nfu} \quad (7c)$$

$$0 \le g_{nfu} \le \overline{K}\left(1 - \overline{r}_{nfu}\right), \forall n, f, u \quad (7d)$$

$$0 = \sum_n (B_{nk}\lambda_n) + \sum_l (H_{lk}\overline{\mu}_l) - \sum_l (H_{lk}\,\underline{\mu}_l) - \begin{cases} \gamma_k & \text{if } k' = k \\ 0 & \text{otherwise} \end{cases} \quad \delta_k(\text{free}), \forall k \quad (7e)$$

$$0 = d_n + \sum_k (B_{nk}\delta_k) - \sum_f \sum_u g_{nfu}, \quad \lambda_n(\text{free}), \forall n \quad (7f)$$

$$0 \le lc_l - \sum_k (H_{lk}\delta_k) \le \hat{K}\,\hat{r}_l, \forall l \quad (7g)$$

$$0 \le \overline{\mu}_l \le \hat{K}(1 - \hat{r}_l), \forall l \quad (7h)$$

$$0 \le lc_l + \sum_k (H_{lk}\delta_k) \le \tilde{K}\,\tilde{r}_l, \forall l \quad (7i)$$

$$0 \le \underline{\mu}_l \le \tilde{K}(1 - \tilde{r}_l), \forall l \quad (7j)$$

$$0 \le -g_{nfu} + \overline{g}_{nfu} \le \check{K}\,\check{r}_{nfu}, \forall n, f, u \quad (7k)$$

$$0 \le \beta_{nfu} \le \check{K}\left(1 - \check{r}_{nfu}\right), \forall n, f, u \quad (7l)$$

$$0 = -sw_k\delta_k, \quad \gamma_k(\text{free}), \forall k \quad (7m)$$

$$r_n, \overline{r}_{nfu}, \hat{r}_l, \tilde{r}_l, \check{r}_{nfu} \in \langle 0, 1\rangle \ \forall n, f, u, l$$

For the purpose of including strategic behavior in the problem, it is now assumed that the set of firms f is partitioned into two subsets. Index s corresponds to the firms that act strategically. Index j is for the firms that act as price-takers. It is assumed that the firms s decide first on their output decisions in order to maximize individual profits which means that their quantities are exogenous to the ISO problem. These firms know that they influence the market equilibrium with their decisions. The output decision of fringe firms j is determined by the ISO. The latter can be interpreted as a pool system. The entire problem is known as Stackelberg game where firms s are the leaders

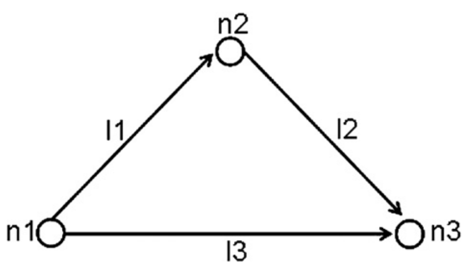

Fig. 1. Three-node example.

Table 2
Demand structure in the three-node network.

	n1	n2	n3
a_n	1	1	10
b_n	1	1	1

Table 3
Parameters of three-node network.

	Test 1	Test 2	Test 3	Test 4	Test 5
c_{n1su1} [€/MWh]	2	2	2	2	2
c_{n2su2} [€/MWh]	1	1	1	1	1
c_{n2j1u3} [€/MWh]	3	7	8	9	3
$\overline{g}_{n1su1}$ [MWh]	10	10	10	10	10
$\overline{g}_{n2su2}$ [MWh]	10	10	10	10	10
$\overline{g}_{n2j1u3}$ [MWh]	10	10	10	10	10
lc_{l1} [MW]	10	10	10	10	10
lc_{l2} [MW]	10	10	10	10	4
lc_{l3} [MW]	10	10	10	10	10

and the ISO (deciding on the quantities of firms j) is the follower. In our problem, we assume that only one player acts strategically resulting in the MPEC (8a)–(8j).

$$\min_{g_{nsu}, \lambda_n} \left\{ \sum_n \sum_s \sum_u (c_{nsu} - \lambda_n) g_{nsu} \right\} \quad (8a)$$

$$\text{s.t. } g_{nsu} - \overline{g}_{nsu} \le 0, \ \forall n, s, u \quad (8b)$$

$$0 \le -a_n + b_n d_n + \lambda_n \perp d_n \ge 0, \forall n \quad (8c)$$

$$0 \le c_{nju} - \lambda_n + \beta_{nju} \perp g_{nju} \ge 0, \forall n, j, u \quad (8d)$$

$$0 = \sum_n (B_{nk}\lambda_n) + \sum_l (H_{lk}\overline{\mu}_l) - \sum_l \left(H_{lk}\,\underline{\mu}_l\right) - \begin{cases} \gamma_k & \text{if } k = k' \\ 0 & \text{otherwise} \end{cases} \quad \delta_k(\text{free}), \forall k \quad (8e)$$

$$0 = d_n + \sum_k (B_{nk}\delta_k) - \sum_f \sum_u g_{nfu}, \lambda_n(\text{free}), \forall n \quad (8f)$$

$$0 \le lc_l - \sum_k (H_{lk}\delta_k) \perp \overline{\mu}_l \ge 0, \forall l \quad (8g)$$

$$0 \le lc_l + \sum_k (H_{lk}\delta_k) \perp \underline{\mu}_l \ge 0, \forall l \quad (8h)$$

$$0 \le -g_{nju} + \overline{g}_{nju} \perp \beta_{nju} \ge 0, \forall n, j, u \quad (8i)$$

$$0 = -sw_k\delta_k, \quad \gamma_k\ (\text{free}), \forall k \quad (8j)$$

The correspondence between Eqs. (8a)–(8j) and the more general problem (1a)–(1k) is shown in Table 1. As stated above the fringe firms' output decisions are determined by the ISO. Hence, the strategic generator takes into account their reaction in terms of the equilibrium problem (8c)–(8j) of the ISO within his profit maximization problem. One computational difficulty is the bilinear terms $\lambda_n g_{nsu}$ in the objective function. In order to deal with the bilinear objective function in Eqs. (8a)–(8j), we define valid generation levels for the strategic generation $\overline{g}_{nsu,i}$. One can think of this as selecting a discrete set of possible generation levels. We take, $q_{nsu,i}$ as indicator binary variables that equal 1 when the fixed generation level $\overline{g}_{nsu,i}$ is selected and zero otherwise. Also, q_n^{λ} is a binary variable for when the price $\lambda_n > 0$, and $q_{nsu,i}^{v}$ is a binary variable for the case when the variable $v_{nsu,i} > 0$ where

$$v_{nsu,i} = \begin{cases} \overline{g}_{nsu,i}\lambda_n & \text{if } q_{nsu,i} = q_n^{\lambda} = 1 \\ 0 & \text{otherwise} \end{cases}.$$

Table 4
Model results of the three-node network.

	Test 1		Test 2		Test 3		Test 4		Test 5	
	Comp	Strat	Comp	Strat	Comp	Strat	Comp	Strat	Comp	Strat
g_{n1su1} [MWh]	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	2.0	1.0
g_{n2su2} [MWh]	9.0	7.0	9.0	4.5	9.0	4.5	9.0	4.5	5.0	5.5
g_{n2j1u3} [MWh]	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
d_{n1} [MWh]	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
d_{n2} [MWh]	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
d_{n3} [MWh]	9.0	7.0	9.0	4.5	9.0	4.5	9.0	4.5	7.0	6.5
price_{n1} [€]	1.0	3.0	1.0	5.5	1.0	5.5	1.0	5.5	2.0	3.25
price_{n2} [€]	1.0	3.0	1.0	5.5	1.0	5.5	1.0	5.5	1.0	3.00
price_{n3} [€]	1.0	3.0	1.0	5.5	1.0	5.5	1.0	5.5	3.0	3.50
flow_{l1} [*MW*]	−3.0	−2.3	−3.0	−1.5	−3.0	−1.5	−3.0	−1.5	−1.0	−1.5
flow_{l2} [*MW*]	6.0	4.7	6.0	3.0	6.0	3.0	6.0	3.0	4.0	4.0
flow_{l3} [*MW*]	3.0	2.3	3.0	1.5	3.0	1.5	3.0	1.5	3.0	2.5

Table 5
Profit of strategic player in the three-node network.

		Test 1	Test 2	Test 3	Test 4	Test 5
profit_s	Comp	0.0	0.0	0.0	0.0	0.0
[€]	Strat	14.0	20.3	20.3	20.3	12.3

Taking into account linearization constraints for the bilinear terms (explained below) as well as the replacement of complementarity conditions by disjunctive constraints (previously described), we get the following mixed-integer linear problem for the MPEC in question.

$$\min_{d_n, g_{nfu}, r_n, \bar{r}_{nju}, \hat{r}_l, \tilde{r}_l, \check{r}_{nju}, \beta_{nju}, \gamma_k, \delta_k, \lambda_n, \bar{\mu}_l, \underline{\mu}_l} \left\{ \sum_n \sum_s \sum_u \left(c_{nsu} g_{nsu} - \sum_i v_{nsu,i}\right) \right\} \tag{9a}$$

$$0 \le \lambda_n \le M q_n^{\lambda}, \forall n \tag{9b}$$

$$g_{nsu} = \sum_i q_{nsu,i}\ \bar{g}_{nsu,i}, \forall n, s, u \tag{9c}$$

$$\sum_i q_{nsu,i} = 1, \forall n, s, u \tag{9d}$$

$$\begin{cases} q_{nsu,i}^{v} \le q_n^{\lambda}, \forall n, s, u, i \\ q_{nsu,i}^{v} \le q_{nsu,i}, \forall n, s, u, i \\ q_{nsu,i} + q_n^{\lambda} - 1 \le q_{nsu,i}^{v}, \forall n, s, u \end{cases} \tag{9e}$$

$$\begin{cases} v_{nsu,i} \le \bar{g}_{nsu,i} \lambda_n, \forall n, s, u, i \\ 0 \le v_{nsu,i} \le M q_{nsu,i}^{v}, \forall n, s, u, i \end{cases} \tag{9f}$$

$$0 \le -a_n + b_n d_n + \lambda_n \le K r_n, \forall n \tag{9g}$$

$$0 \le d_n \le K(1 - r_n), \forall n \tag{9h}$$

$$0 \le c_{nju} - \lambda_n + \beta_{nju} \le \bar{K} \bar{r}_{nju}, \forall n, j, u \tag{9i}$$

$$0 \le g_{nju} \le \bar{K}\left(1 - \bar{r}_{nju}\right), \forall n, j, u \tag{9j}$$

$$0 = \textstyle\sum_n (B_{nk} \lambda_n) + \sum_l (H_{lk} \bar{\mu}_l) - \displaystyle\sum_l \left(H_{lk}\ \underline{\mu}_l\right) - \begin{cases} \gamma_k & \text{if } k = k' \\ 0 & \text{otherwise} \end{cases} \quad \delta_k(\text{free}), \forall k \tag{9k}$$

$$0 = d_n + \sum_k (B_{nk} \delta_k) - \sum_f \sum_u g_{nfu}, \lambda_n(\text{free}), \forall n \tag{9l}$$

$$0 \le lc_l - \sum_k (H_{lk} \delta_k) \le \hat{K} \hat{r}_l, \forall l \tag{9m}$$

$$0 \le \bar{\mu}_l \le \hat{K}(1 - \hat{r}_l), \forall l \tag{9n}$$

$$0 \le lc_l + \sum_k (H_{lk} \delta_k) \le \tilde{K} \tilde{r}_l, \forall l \tag{9o}$$

$$0 \le\ \underline{\mu}_l \le \tilde{K}(1 - \tilde{r}_l), \forall l \tag{9p}$$

$$0 \le\ -g_{nju} + \bar{g}_{nju} \le \check{K} \check{r}_{nju}, \forall n, j, u \tag{9q}$$

$$0 \le \beta_{nju} \le \check{K}\left(1 - \check{r}_{nju}\right), \forall n, j, u \tag{9r}$$

$$0 =\ -sw_k \delta_k, \gamma_k(\text{free}), \forall k \tag{9s}$$

$$r_n, \bar{r}_{nju}, \hat{r}_l, \tilde{r}_l, \check{r}_{nju}, q_{nsu,i}, q_n^{\lambda} \in \{0, 1\}, \forall n, s, u, i, j$$
$$q_{nsu,i}^{v} \in [0, 1], \forall n, s, u, i$$

The logic of the constraints (9a)–(9e) is as follows:

1. By $0 \le \lambda_n \le M q_n^{\lambda}$, when $\lambda_n > 0$, since M is a suitably large positive constant, this means that $q_n^{\lambda} = 1$. Also, $q_{nsu,i} = 1$ corresponds to the ith discrete generation value $\bar{g}_{nsu,i}$ being selected via the constraints $\sum_i q_{nsu,i} = 1$, $g_{nsu} = \sum_i q_{nsu,i}\ \bar{g}_{nsu,i}$. Thus,

$$\begin{cases} q_{nsu,i}^{v} \le q_n^{\lambda} \\ q_{nsu,i}^{v} \le q_{nsu,i} \\ q_{nsu,i} + q_n^{\lambda} - 1 \le q_{nsu,i}^{v} \end{cases}$$

ensures that when both $q_n^{\lambda} = 1$ and $q_{nsu,i} = 1 \Leftrightarrow$ the binary indicator variable $q_{nsu,i}^{v} = 1$ since by the three constraints above, we have $1 \le q_{nsu,i}^{v} \le 1$. If one or both of q_n^{λ} and $q_{nsu,i} = 0$, then these constraints would force the nonnegative variable $q_{nsu,i}^{v} = 0$; see Williams (1999) for this and similar logic constraints.

2. The constraints

$$\begin{cases} v_{nsu,i} \le \bar{g}_{nsu,i} \lambda_n \\ 0 \le v_{nsu,i} \le M q_{nsu,i}^{v} \end{cases}$$

force $v_{nsu,i} \in [0, \bar{g}_{nsu,i} \lambda_n]$ when $q_{nsu,i}^{v} = 1$ and $v_{nsu,i} = 0$ when $q_{nsu,i}^{v} = 0$. Since the objective function has $-\sum_i v_{nsu,i}$, larger values of $v_{nsu,i}$ are always preferred. Thus, $q_{nsu,i}^{v} = 1 \Rightarrow v_{nsu,i} = \bar{g}_{nsu,i} \lambda_n$ so that part of the objective function matches the bilinear term exactly.

Table 6
Model statistics for three-node network.

	Tests 1–5
Problem size	comp $\left\{ \begin{array}{c} \text{52 continuous variables} \\ \text{0 discrete variables} \end{array} \right\}$
	strat $\left\{ \begin{array}{c} \text{2801 continuous variables} \\ \text{939 discrete variables} \end{array} \right\}$

Table 7
Computation times for three-node network.

		Test 1	Test 2	Test 3	Test 4	Test 5
Computation times	Comp	3 s	1 s	1 s	1 s	1 s
	Strat	15 s	26 s	31 s	33 s	17 s

3. Hence we see the following

$$\begin{aligned} &\lambda_n > 0 \text{ and } \bar{g}_{nsu,i} \text{ selected} \\ &\quad \Rightarrow q^v_{nsu,i} = 1 \\ &\quad \Rightarrow v_{nsu,i} = \bar{g}_{nsu,i}\lambda_n \end{aligned}$$

as desired. It therefore suffices to show that when either $\lambda_n = 0$ or $\bar{g}_{nsu,i}$ is not selected that this implies that $v_{nsu,i} = 0$. Clearly, $v_{nsu,i} \le \bar{g}_{nsu,i}\, \lambda_n$ forces $v_{nsu,i} = 0$ when $\lambda_n = 0$. On the other hand, if $\lambda_n > 0$ then $q^\lambda_n = 1$. But if $\bar{g}_{nsu,i}$ is not selected, then $q_{nsu,i} = 0$ so that in $\left\{ \begin{array}{l} q^v_{nsu,i} \le q^\lambda_n \\ q^v_{nsu,i} \le q_{nsu,i} \\ q_{nsu,i} + q^\lambda_n - 1 \le q^v_{nsu,i} \end{array} \right.$, we see that $q^v_{nsu,i} \le \min\{0,1\} = 0$ and $q^v_{nsu,i} \ge 0$, so that $q^v_{nsu,i} = 0$. Hence, by $0 \le v_{nsu,i} \le M q^v_{nsu,i}$, $v_{nsu,i} = 0$.

Hence, by transforming the mathematical problem (8a)–(8j) into (9a)–(9s), we manage to replace a bilinear term $\lambda_n g_{nsu}$ in the objective function by the linear term $\sum_i v_{nsu,i}$ and some more binary constraints. Steps 1–3 above describe how these additional constraints work. In particular, they ensure that the model is only able to pick one of the parameterized given output levels. Altogether, through this procedure a mixed integer nonlinear problem (MINLP) is replaced by a mixed-integer linear problem (MILP) which promises better numerical behavior.

4.3. How to compute the disjunctive constraints constants

An important issue that has to be mentioned is how to choose the constants $K, \bar{K}, \hat{K}, \tilde{K}$, and $\check{K}$ for the disjunctive constraints as well as the constant M in Eqs. (9b) and (9f). These K-constants are crucial for replacing KKTs by disjunctive constraints. However, there is no general formula how to obtain these values and they are specific to the application that is to be solved. As pointed out above, we assume a linear inverse demand function $p_n = a_n - b_n d_n$ per node with intercept a_n and slope b_n. Intercept a_n is the point of intersection of the linear demand function and the vertical price axis. Hence it is called the "prohibitive price" in economics as at this point demand d_n falls to 0. The point of intersection of the linear demand function and the horizontal quantity axis is referred to as the "market saturation" quantity D_n. That is the maximum quantity that consumers would buy at a nonnegative price.

Keeping these basics in mind, the derivation of the constant values is economic commonsense. The constants always refer to primal (quantity) and dual (price) information. The constant K for example refers to the demand function price information (9g) and demand quantity information (9h). Binary variables r_n are 0 as long as there are positive demands d_n. If demand d_n equals 0 for one of the nodes, its r_n can either be 0 or 1. Economically, zero demand means that the nodal price λ_n is greater than or equal to the prohibitive price a_n. For this result, it does not matter how big λ_n becomes as long as $\lambda_n \ge a_n$. It is economically reasonable to assume for example that $\lambda_n \le 2a_n$ which leads us according to Eq. (9g) to the first candidate for K: $K_1 = \max_n(a_n)$. Analogously, the second candidate for K can be found by looking at the inequality $d_n \le K(1 - r_n)$ for $r_n = 0$. A K for this case is identified by assuming that an economically meaningful demand quantity can not exceed the market saturation quantity of the entire system $K_2 = \sum_n(D_n)$. Thus, K can be defined as the maximum of these two values $K = \max(K_1, K_2)$. The other constants can be derived in a similar manner. Some advice on how to compute these disjunctive constraint constants in a more general setting for a linear program with upper-level variables appearing in the right-hand side of the constraints is provided in the Appendix A.

4.4. Computational results

4.4.1. Three-node network

The objective of this section is to apply to models of electric power markets. We start with a simple three-node example as depicted in Fig. 1[2] and then later describe the computational results for a fifteen-node network of the Western European grid. In both bases, the models were coded in GAMS and used the CPLEX solver.

[2] Note that the arrows define the forward direction for the line flows. If the values for the line flows are negative, the flow goes in the opposite direction.

Table 8
Demand structure in the fifteen-node network for Western Europe.

	n1	n2	n3	n4	n5	n6	n7
a_n	130	130	130	130	130	130	130
b_n	0.002	0.002	0.033	0.017	0.017	0.050	0.033

Table 9
Marginal costs per unit type in the fifteen-node network for Western Europe.

	Nuclear	Lignite	Coal	CCGT	Gas	Oil	Hydro	Pump
	u1	u2	u3	u4	u5	u6	u7	u8
Marginal Cost [€/MWh]	10	20	22	30	45	60	0	35

Table 10
Parameters of fifteen-node Western European network.

	Line capacity [MW]	Reactance [Ω]
l1	2970	12
l2	1840	69
l3	1840	43
l4	900	28
l5	1330	25
l6	1840	33
l7	1840	50
l8	1840	29
l9	640	61
l10	640	42
l11	940	34
l12	1840	31
l13	900	55
l14	1210	45
l15	270	156
l16	2760	22
l17	1840	27
l18	3330	38
l19	1280	11
l20	3330	41
l21	∞	46
l22	∞	46
l23	∞	46
l24	∞	46
l25	∞	46
l26	∞	46

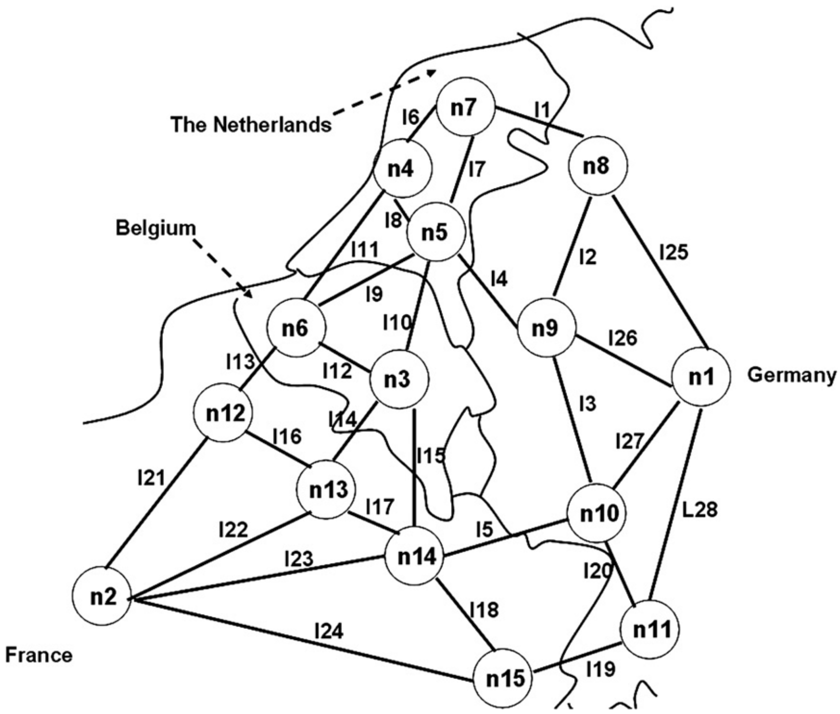

Fig. 2. Stylized network of the Western European grid. Source: Based on Neuhoff et al. (2005).

In order to run the test for the three-node example, the network parameters have to be defined. Two types of parameters can be distinguished. One type describes the topology of the network such as the characteristics of the lines, the information which nodes are interconnected, and so on.[3] The other type refers to the electricity market itself. The latter parameters describe the demand, supply, and respective locations within the network. As described in Section 4.2, we use a linear inverse demand function of the form $a_n - b_n d_n$ whose parameters a_n and b_n are displayed in Table 2. The demand structure is constructed such that there is a load center at node $n3$. The supply structure is specified by the marginal cost per plant and a maximum generation capacity for this plant. In order to carry out tests with the model, the supply side and the network parameters were varied for five different test cases (Table 3). We assume that the generation centers are located at nodes $n1$ and $n2$, thus there is an electricity transport required in order to balance demand and generation. For the first test cases (Tests 1–4 in Table 3), the network parameters are chosen in a way that congestion is not expected. For Test 5, there is congestion expected on line $l2$ as the line capacity for line $l2$ (lc_{12} in Table 3) is decreased from 10 to 4 in this scenario.

As our model is supposed to simulate the result of strategic behavior, we define a benchmark case against which the impact of this behavior ('strat') can be compared. The benchmark is the case of a perfect competition ('comp') model that is solved as a MCP (compare Eq. (6a)–(6h)). The results for Tests 1–4 shown in Tables 4 and 5 are easiest to follow. The strategic generator only produces with its cheapest plant $u2$ in both the perfect competition and the strategic gaming case. However, in perfect competition, this generator has a profit of 0 whereas the profit is positive for the strategic runs (Table 5). The strategic generator manages to increase its profit by holding back generation. This leads to a decrease in demand and production but maximizes its profit. However, its output decision is constrained by the cheapest plant of the fringe (Table 3). It cannot hold back too much capacity since otherwise the fringe would have an incentive to produce which would lower the profit of the strategic player. Accordingly, if the marginal cost of the cheapest fringe generator increases (Table 3), the strategic firm decreases its output to the point of maximum profit i.e., 4.5 MWh. Comparing the changes in strategic output and profits from Test 1 to Test 2 and from Test 2 through Test 4, it can be seen that the strategic result moves from producing 7 MWh to 4.5 MWh (Test 1 vs. Test 2). Furthermore, this level of 4.5 MWh is maintained from Test 2 to Test 4 (Table 4) albeit the marginal cost of the fringe still increases (Table 3). Hence, even increasing marginal cost of the fringe further does not impact the results. The same behavior can be observed for Test 5. However, the difference between Test 5 and the other test cases is that there is now network congestion. Hence, the strategic player cannot satisfy enough demand with its cheapest plant. By producing with its second (more expensive) unit, it can create counterflows and relieve congestion which in turn facilitates higher generation with the less expensive plant. The output decisions are chosen in the most profitable way (Tables 4 and 5). Lastly, the problem size and calculation times[4] are summarized in Tables 6 and 7.

4.4.2. Fifteen-node network

The second example is a more complex fifteen-node network representing a stylized grid of the Western European market based on Neuhoff et al. (2005). In order to obtain a linear inverse demand function, a reference demand and an elasticity for each node in the network were assumed. For a deeper discussion of the data and the methodology refer to Leuthold et al. (2008) and Leuthold et al. (2005). Demand data are based on UCTE[5]. The network aggregates data (Tables 8, 9 and 10) for Belgium, France, Germany, and the Netherlands with Germany and France represented by one node each ($n1$ and $n2$, respectively), Belgium by two nodes ($n3$ and $n6$) and the Netherlands by three nodes ($n4$, $n5$, and $n7$). Altogether, there are 15 nodes (Fig. 2) of which eight are auxiliary without supply and demand (nodes $n8$ to $n15$). These nodes are necessary for the adequate modeling of cross-border flows. We carry out four different tests. For each of these test runs, a different company is assigned the Stackelberg leader role. In Test_EDF, for example, the French company EDF is the Stackelberg leader and all other companies are fringe players. The same pattern applies for Electrabel (Ebel) of Belgium as well as for

[3] For this small three-node network, the reactance and resistance of all lines are taken to be equal.

[4] All tests were conducted on an Intel Xeon CPU E5420 (8 cores) with 16 GB RAM.

[5] www.ucte.org.

Table 11
Generation capacities (MW) of fifteen-node Western European network.

	u1	u2	u3	u4	u5	u6	u7	u8
n1.EON	7628	1125	6720	0	3536	2729	124	380
n1.RWE	6379	11,410	3334	2091	1978	765	0	150
n1.ENBW	4302	0	2683	915	132	319	0	0
n1.VAT	2031	7785	1627	415	554	1336	0	1544
n1.FriGER	0	833	14,600	4337	4434	368	1147	3030
n2.ENBW	0	0	0	0	0	0	150	0
n2.EDF	58,288	0	11,685	0	124	11,130	11,552	3408
n2.FriFR	0	580	4137	0	0	0	2679	0
n3.EBEL	2713	0	2474	350	460	373	0	1164
n3.FriBE	0	0	0	0	115	187	0	144
n4.EON	0	0	1040	0	828	0	0	0
n4.EBEL	0	0	602	0	0	0	0	0
n4.ESSENT	449	0	1696	0	460	0	0	0
n4.NUON	0	0	630	249	2506	0	0	0
n4.FriNL	0	0	0	0	1078	111	0	0
n5.ESSENT	0	0	0	0	1510	0	0	0
n5.FriNL	0	0	253	0	0	0	0	0
n6.EBEL	2618	0	1134	460	1306	1675	0	0
n6.FriBE	0	0	0	350	126	190	0	0
n7.EBEL	0	0	0	1705	2340	0	0	0
n7.FriNL	0	0	0	0	428	0	0	0

Table 12
Resulting profits in the fifteen-node Western European network without line constraints.

		Test_Ebel	Test_EDF	Test_EON	Test_RWE
Profit	Comp	72	1006	98	94
Leader [k€]	Strat	72	1111	98	94
Profit	Comp	1310	376	1296	1288
Fringe [k€]	Strat	1310	1032	1296	1288

EON and RWE of Germany in runs Test_Ebel, Test_EON, and Test_RWE, respectively. (Refer to Table 11 for the installed generation capacities per player.) However, it must be stated that the level of detail of the data and, particularly, of the network is too low in order to draw a conclusion for the real market. Hence, the subsequent results solely aim to show that the approach presented in this paper works for a medium-scale test network and we believe that our approach is a significant modeling advance.

Our numerical example shows that network effects have a significant impact on short-term market equilibria in electricity markets and cannot be neglected. Hence, we start by assuming that there are no network constraints within the entire network. For this case the prices are equal at each node within the network. Also, EDF alone does have the potential to lift the price above the competitive level by holding back production and thereby increase its own profit (Table 12). For all other test cases, the Stackelberg assumption for the respective other players that were tried as Stackelberg leaders does not have a profit-increasing effect.

However, the picture changes significantly if physical network constraints are included. The results of the fifteen-node example are then less intuitive than in case of the previous three-node network. Nonetheless, the outcome follows the same pattern. The Stackelberg leader is able to induce higher prices at the relevant nodes (Table 18), i.e., those nodes where it has significant production capacities, by holding back production (Tables 13–17). As could be observed in Test 5 of the three-node example, there is congestion in the larger network, too, evidenced by market prices that differ by node (Table 18). For Ebel, EDF, and RWE acting as the only strategic player is profitable (Table 13). Particularly for EDF, the potential to increase their individual profit is huge. Presumably, this is due to the fact that in our model EDF has a greater supply of low-cost generation capacity (Table 11), e.g., nuclear power plants which opens the potential to enact market power.

The important aspect is the issue of network congestion as mentioned earlier. The model is constructed in a way that EDF is the only player at node $n2$ representing France in a single node. Hence, competitive players would have to use the network in order to compete with EDF in node $n2$. However, by strategically choosing output decisions, EDF can influence flow patterns and reap the profits by itself. The situation is different in the case where competitors have generation capacities at the same nodes as the strategic player. In this case, the strategic player has to take into account output decisions of competitors at the nodes with several

Table 13
Resulting profits in the fifteen-node Western European network.

		Test_Ebel	Test_EDF	Test_EON	Test_RWE
Profit	Comp	82	140	121	94
Leader [k€]	Strat	150	1565	121	136
Profit	Comp	542	483	514	529
Fringe [k€]	Strat	513	547	514	915

Table 14
Strategic generation I.

	Test_Ebel				
	$g_{n3\text{Ebel}u1}$ [MWh]	$g_{n4\text{Ebel}u3}$ [MWh]	$g_{n6\text{Ebel}u1}$ [MWh]	$g_{n6\text{Ebel}u3}$ [MWh]	$g_{n7\text{Ebel}u4}$ [MWh]
Comp	2548	1000	3000	514	2000
Strat	2000	1000	500	2000	1000

Table 15
Strategic generation II.

	Test_EDF	
	$g_{n2\text{EDF}u1}$ [MWh]	$g_{n2\text{EDF}u7}$ [MWh]
Comp	54,209	14,000
Strat	31,000	14,000

Table 16
Strategic generation III.

	Test_EON				
	$g_{n1\text{EON}u1}$ [MWh]	$g_{n1\text{EON}u2}$ [MWh]	$g_{n1\text{EON}u3}$ [MWh]	$g_{n4\text{EON}u3}$ [MWh]	$g_{n4\text{EON}u5}$ [MWh]
Comp	8000	1000	7000	3000	303
Strat	8000	1000	1000	1000	0

Table 17
Strategic generation IV.

	Test_RWE	
	$g_{n1\text{RWE}u1}$ [MWh]	$g_{n1\text{RWE}u2}$ [MWh]
Comp	6000	11,000
Strat	6000	2000

Table 18
Resulting prices in fifteen-node Western European network.

	Test_Ebel		Test_EDF		Test_EON		Test_RWE	
	Comp	Strat	Comp	Strat	Comp	Strat	Comp	Strat
price_{n1} [€/MWh]	22.0	22.0	22.0	22.0	22.0	22.0	22.0	29.4
price_{n2} [€/MWh]	10.0	10.0	10.0	41.7	10.0	10.0	10.0	10.0
price_{n3} [€/MWh]	10.0	58.4	10.0	22.0	10.0	10.0	10.0	10.0
price_{n4} [€/MWh]	45.0	45.0	45.0	45.0	45.0	45.0	45.0	45.0
price_{n5} [€/MWh]	59.3	45.0	59.3	57.2	59.3	59.3	59.3	58.9
price_{n6} [€/MWh]	22.0	52.1	22.0	30.0	22.0	22.0	22.0	22.0
price_{n7} [€/MWh]	41.3	38.2	41.3	41.2	41.3	41.3	41.3	44.8

Table 19
Model statistics for fifteen-node Western European network.

		Test_Ebel		Test_EDF
Problem sizes	comp	582 continuous variables; 0 discrete variables	comp	582 continuous variables; 0 discrete variables
	strat	3309 continuous variables; 2006 discrete variables	strat	22029 continuous variables; 7406 discrete variables
		Test_EON		**Test_RWE**
Problem sizes	comp	582 continuous variables; 0 discrete variables	comp	582 continuous variables; 0 discrete variables
	strat	4389 continuous variables; 2726 discrete variables	strat	5109 continuous variables; 1766 discrete variables

Table 20
Computation times for fifteen-node Western European network.

		Test_Ebel	Test_EDF	Test_EON	Test_RWE
Computation times	Comp	2 s	2 s	2 s	2 s
	Strat	4 min	5 h	15 s	19 s

players. If the prices become too high, the competitive fringe companies still have an incentive to produce at marginal cost and cannot be excluded by network effects. Hence, in the case of the Ebel and RWE, the profit increase is less distinct due to a more competitive situation but also due to smaller power plant fleet of these players.

Furthermore, the results for EON seem to be surprising as EON is not able to increase individual profit (Table 13). However, this result is easy to explain: the network is structured such that that EnBW, RWE, and Vattenfall are aggregated within the same node as EON. Hence, withholding of EON does not have an effect as it can be entirely compensated by other competitors. The latter is of course strongly influenced by the simplified nature of the network representation.

Regarding computational issues, the calculation times vary significantly for the strategic cases. The computation times for Test_EON and Test_RWE are below 20 s. The Test_Ebel run takes about four minutes whereas it takes five hours for the Test_EDF case (Table 20). This is due to the fact that EDF has more production capacity. As we have to define feasible production levels for our approach, the number of discrete variables increases with the available production capacity of a player assuming intervals of the same size (Table 19). Another issue is that GAMS finds the final solution quite fast but it takes very long to verify the solution. Concerning the constants that are used in order to implement the disjunctive constraints, we carried out a sensitivity analysis to gauge the influence of the value of the *K*'s on the results. The impact seems not to be significant. Only if values too low are chosen for the *K*'s does the optimal solution change. However, if the *K*'s are chosen large enough changing the *K* values does not have an impact at all.

5. Conclusion

In this paper, we have presented a mixed-integer linear programming model for a Stackelberg game applicable to network-constrained industries. In order to do so, the equilibrium conditions of the associated MPEC have been converted to disjunctive constraints and a linearization of nonlinear terms has been used. This approach is then applied to two different test networks in electricity markets: a three-node example network and a fifteen-node model of the Western European grid. The results show that the approach works well and is promising for solving larger-scale models in the future.

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Appendix

Result that Shows that $q^{v}_{nsu,i} \in [0, 1]$ is Valid

We first show that the variable $q^{v}_{nsu,i}$ need not be specified as binary but rather constrained to be in the range [0, 1]. We present the result in a slightly more general setting.

Theorem 1. The solution set to (10) and (11) are the same where

$$z \leq x \tag{10a}$$

$$z \leq y \tag{10b}$$

$$x + y - 1 \leq z \tag{10c}$$

$$x, y \in \{0, 1\} \tag{10d}$$

$$z \in \{0, 1\} \tag{10e}$$

and

$$z \leq x \tag{11a}$$

$$z \leq y \tag{11b}$$

$$x + y - 1 \leq z \tag{11c}$$

$$x, y \in \{0, 1\} \tag{11d}$$

$$z \in [0, 1] \tag{11e}$$

Proof. Let $(\bar{x},\bar{y},\bar{z})$ be a solution to (10). Since $z \in \{0,1\} \Rightarrow z \in [0,1]$ and (10a)–(10d) exactly match (11a)–(11d) this means that $(\bar{x},\bar{y},\bar{z})$ is a solution to (11). Now let $(\hat{x},\hat{y},\hat{z})$ be a solution to (11). If $z \in \{0,1\}$ then we are done. Thus, assume that $z \in (0,1)$. But by (10a) and (10b) since $0 < z < 1 \Rightarrow x = y = 1$. Then by (10c) $z \geq 1$ which is a contradiction. □

How to Compute the Disjunctive Constraints Constants in the Case of a Linear Programming Subproblem

Having fixed values for the the upper-level vector x, a lower-level linear programming subproblem from before but with upper bounds y^{up} on the y variables added is of the form

$$\begin{aligned} & \min_y \; e^T y \\ & s.t. \; My \geq k - Nx \\ & \qquad y \leq y^{up} \\ & \qquad y \leq 0 \end{aligned} \tag{12}$$

The necessary and sufficient KKT conditions are to find vectors (y, z, w) such that

$$0 \leq e - M^T z + Iw \perp y \geq 0 \tag{13}$$

$$0 \leq My + Nx - k \perp z \geq 0 \tag{14}$$

$$0 \leq y^{up} - y \perp w \geq 0 \tag{15}$$

First note that all variables are nonnegative so that we need only consider upper bounds on y, z, w. Also, we have assumed that the upper level variable x is bounded above in the upper-level problem, a reasonable assumption given that this variable will relate to a physical quantity. Also, by design, we have $0 \leq y \leq y^{up}$ so that y is bounded from above. What about bounds on the "dual" variables z and w? First, suppose that there exists a positive constant C^w such that $0 \le w_i \le C^w$, $\forall i$. Depending on the application, the existence of positive constants C^w and C^y (which can be taken greater than or equal to y^{up}) may be a reasonable assumption. For example, bounds on the primal variables y are often employed if these variables relate to real, physical quantities (e.g., power generation). Bounds on the dual variables w are also reasonable in that they relate to the shadow price of capacity constraints on the y variables. Typically these prices correspond to the marginal cost of one more unit of capacity which itself should be bounded due to physical considerations. Having made the assumption of bounds on the y and w variables, the next result shows a reasonable condition to generate a bound C^z on the other dual variables z. This condition states that there must be at least one column of the matrix M with all positive entries.

Theorem 2. Suppose that there exists a column j of M such that $M_j^{\min} = \min_i \{M_{ij}\} > 0$. Then

$$z \le \min_{j:M_j^{\min}>0} \left\{ \left(\frac{ej + C^w}{M_j^{\min}} \right) \right\} = C^z, \text{ where } C^z > 0.$$

Proof. From (13) we see that for 1 the vector of all ones,

$$0 \le e - M^T z + Iw \Rightarrow M^T z \le e + Iw \le e + C^w 1$$

or

$$M_j^{\min} \sum_i z_i \le \sum_i M_{ij} z_i \le e_j + C^w, \forall j$$

where $M_j^{\min}$ is the minimum value of M_{ij} for column $_j$: Now if there is a column j where $M_j^{\min}>0$, we see that

$$\sum_i z_i \le \frac{e_j + C^w}{M_j^{\min}}$$

Moreover, this has to hold for each column j where $M_j^{\min}>0$ hence

$$\sum_i z_i \le \min_{j:M_j^{\min}>0} \left\{ \left(\frac{e_j + C^w}{M_j^{\min}} \right) \right\}$$

If $e_j<0$ then, without loss of generality, C^w can be taken sufficiently large so that $min_{j:M_j^{\min}>0} \left\{ \left(\frac{e_j + C^w}{M_j^{\min}} \right) \right\} = C^z > 0 \Rightarrow z_j \le C^z$ for all j in light of the fact that $z \ge 0$. □

Remark 1. *The condition that there exists a column j of M such that* $M_j^{\min} = \min_i \{Mi_j\} > 0$ *may be satisfied for a large class of matrices.*

Remark 2. *The other condition in (14) which involves the vector z namely,* $(My + Nx - k)^T z = 0$ *does not impose any additional bounds on z.*

Remark 3. *Getting an appropriate value for the disjunctive constraints constant is then straightforward given the above result. For example in*

$$0 \le e - M^T z + Iw \perp y \ge 0, \; M_{ij} \ge 0 \; \forall i, j$$

we can see that for row i we have

$$0 \le e_i - \sum_j \left(M^T\right)_{ij} z_j + w_i$$

$$\le e_i + \sum_j \max_i \left\{ \left(M^T\right)_{ij} \right\} C^z + C^w \le K_i$$

So we can take any value K_i *such that the above inequality holds. Then a valid value for the disjunctive constraints constant is* $K = \max \{\max_i \{K_i\}, C^y\} > 0$. *A similar line of reasoning applies for the other complementarity constraints* $0 \le My + Nx - k \perp z \ge 0$ *in light of the fact that both y and x are bounded variables.*

References

Anitescu, M., 2000. On solving mathematical programs with complementarity constraints as nonlinear programs. Preprint ANL/MCS-P864-1200, MCS Division. Argonne National Laboratory, Argonne, IL, USA.

Anitescu, M., 2004. Global convergence of an elastic mode approach for a class of mathematical programs with complementarity constraints. Preprint ANL/MCS-P1143-0404, MCS Division. Argonne National Laboratory, Argonne, IL, USA.

Bard, J.F., 1983. An efficient point algorithm for a linear two-stage optimization problem. Operations Research 31 (4), 670–684.

Bard, J.F., 1988. Convex two-level optimization. Mathematical Programming 40 (1), 15–27.

Bard, J.F., Moore, J.T., 1990. A branch and bound algorithm for the bilevel programming problem. SIAM Journal on Scientific and Statistical Computation 11 (2), 281–292.

Bard, J.F., Plummer, J., Sourie, J.C., 2000. A bilevel programming approach to determining tax credits for biofuel production. European Journal of Operational Research 120, 30–46.

Barroso, L.A., Carneiro, R.D., Granvile, S., Pereira, M.V., Fampa, M.H.C., 2006. Nash equilibrium in strategic bidding: a binary expansion approach. IEEE Transactions on Power Systems 21 (2), 629–638.

Bialas, W.F., Karwan, M.H., 1984. Two-level linear programming. Management Science 30 (8), 1004–1020.

Castillo, E., Conejo, A.J., Pedregal, P., Garciá, R., Alguacil, N., 2002. Building and Solving Mathematical Programming Models in Engineering and Science. John Wiley & Sons, New York.

Castillo, E., Conejo, A.J., Minguez, R., Castillo, C., 2003. An alternative approach for addressing the failure probability—safety factor method with sensitivity analysis. Reliability Engineering & System Safety 82, 207–216.

Chen, Y., Hobbs, B.F., Leyffer, S., Munson, T.S., 2004. Leader–follower equilibria for electric power and NOx allowances markets, preprint ANL/MCS-P1191-0804. Mathematics and Computer Science Division. Argonne National Laboratory, Argonne, IL USA, p. 60439.

Christie, R.D., Wollenberg, B.F., Wangensteen, I., 2000. Transmission management in the deregulated environment. Proceedings of the IEEE 88 (2), 170–195.

Codina, E., Garcia, R., Martin, A., 2006. New algorithmic alternatives for the O–D matrix adjustment problem on traffic networks. European Journal of Operational Research 175, 1484–1500.

Daxhelet, O., Smeers, Y., 2001. Variational inequality models of restructured electric systems. In: Ferris, M.C., Mangasarian, O.L., Pang, J.-S. (Eds.), Applications and Algorithms of Complementarity. Kluwer, Dordrech.

Day, C.J., Hobbs, B.F., Pang, J.S., 2002. Oligopolistic competition in power networks: a conjectured supply function approach. IEEE Transactions on Power Systems 17 (3), 597–607.

Delarue, E., Bekaert, D., Belmans, R., D'haeseleer, W., 2007. Development of a comprehensive electricity generation simulation model using a mixed integer programming approach. International Journal of Electrical, Computer, and Systems Engineering 1 (2), 92–97.

De Miguel, V., Friedlander, M.P., Nogales, F.J., Scholtes, S., 2004. An interior-point method for MPECs based on strictly feasible relaxations. Technical Report. London Business School, London, NW1 4SA, UK.

De Miguel, V., Friedlander, M.P., Nogales, F.J., Scholtes, S., 2005. A two-sided relaxation scheme for mathematical programs with equilibrium constraints. SIAM J. Optim. 16 (2), 587–609.

Dirkse, S.P., Ferris, M.C., Meerhaus, A., 2002. Mathematical programs with equilibrium constraints: automatic reformulation and solution via constraint optimization. Technical Report NA-02/11. Oxford University Computing laboratory. July.

Ehrenmann, A., 2004. Equilibrium problems with equilibrium constraints and their application to electricity markets. Dissertation. Fitzwilliam College, University of Cambridge, UK.

Facchinei, F., Pang, J.S., 2003. Finite-Dimensional Variational Inequalities and Complementarity Problems Volumes I and II. Springer, New York.

Fletcher, R., Leyffer, S., 2002. Numerical experience with solving MPECs as NLPs, numerical analysis report NA/210, Department of Mathematics, University of Dundee. Dundee, UK.

Fletcher, R., Leyffer, S., 2004. Solving mathematical programs with complementarity constraints as nonlinear programs. Optimization Methods and Software 19 (1), 15–40.

Fletcher, R., Leyffer, S., Ralph, D., Scholtes, S., 2002. Local convergence of SQP methods for mathematical programs with equilibrium constraints. Numerical Analysis Report NA/209. Department of Mathematics, University of Dundee, Dundee, UK.

Fortuny-Amat, J., McCarl, B., 1981. A representation and economic interpretation of a two-level programming problem. The Journal of the Operational Research Society 32 (9), 783–792.

Gabriel, S.A., Shim, Y., Conejo, A.J., de la Torre, S., Garcia-Bertrand, R., 2007. A Benders Decomposition Method for Discretely-Constrained Mathematical Programs with Equilibrium Constraints with Applications in Energy. in review, October.

Gao, Z., Sun, H., Shan, L.L., 2004. A continuous equilibrium network design model and algorithm for transit systems. Transportation Research Part B 38, 235–250.

Hejazi, S.R., Memariani, A., Jahanshahloo, G., Sepehri, M.M., 2002. Linear bilevel programming solution by genetic algorithm. Computers and Operations Research 29, 1913–1925.

Hirschhausen, C.v., Weigt, H., Zachmann, G., 2007. Price formation and market power in Germany's wholesale electricity markets. Electricity Markets Working Papers WP-EM-15b, Chair of Energy Economics and Public Sector Management. Dresden University of Technology, Dresden Germany. Available online: http://www.tu-dresden.de/wwbwleeg/publications/wp_em_15b_hirschhausen_weigt_zachmann_marktet_power_germany.pdf.

Hobbs, B.F., 2001. Linear complementarity models of Nash–Cournot competition in bilateral and POOLCO power markets. IEEE Transactions on Power Systems 16 (2), 194–202.

Hobbs, B., Nelson, S., 1992. A nonlinear bilevel model for analysis of electric utility demand-side planning issues. Annals of Operations Research 34, 255–274.

Hobbs, B.F., Metzler, C.B., Pang, J.S., 2000. Strategic gaming analysis for electric power systems: an MPEC approach. IEEE Transactions on Power Systems 15 (2), 638–645.

Hu, X., Ralph, D., 2004. Convergence of a penalty method for mathematical programming with complementarity constraints. Journal of Optimization Theory and Applications 123 (2), 365–390.

Jeroslow, R.G., 1985. The polynomial hierarchy and a simple model for competitive analysis. Mathematical Programming 32, 146–164.

Karlof, J.K., Wang, W., 1996. Bilevel programming applied to the flow shop scheduling problem. Computers and Operations Research 23 (5), 443–451.

Labbé, M., Marcotte, P., Savard, G., 1998. A bilevel model of taxation and its application to optimal highway pricing. Management Science 44 (12), 1608–1622.

Leuthold, F., Rumiantseva, I., Weigt, H., Jeske, T., Hirschhausen, C.v., 2005. Nodal pricing in the German electricity sector — a welfare economics analysis, with particular reference to implementing offshore wind capacities. Electricity Markets Working Papers WP-EM-08a, Chair of Energy Economics and Public Sector Management. Dresden University of Technology, Dresden Germany. Available online: http://www.tu-dresden.de/wwbwleeg/publications/ wp_em_08a_leuthold_ rumiantseva_weigt_et%20al_nodal_%20pricing_germany.pdf.

Leuthold, F., Weigt, H., Hirschhausen, C.v., 2008. Efficient Pricing for European Electricity Networks — The Theory of Nodal Pricing Applied to Feeding-In Wind in Germany. Utilities Policy 16 (4), 284–291.

Leyffer, S., 2003. Mathematical programs with complementarity constraints. SIAG/OPT Views-and-News 14 (1), 15–18 April.

Leyffer, S., 2005. The penalty interior point method fails to converge. Optimization Methods and Software 20 (4–5), 559–568.

Leyffer, S., Lopez-Calva, G., Nocedal, J., 2006. Interior methods for mathematical programs with complementarity constraints. SIAM Journal on Optimization 17 (1).

Liu, X., Sun, J., 2004. Generalized stationary points and an interior point method for mathematical programs with equilibrium constraints. Mathematical Programming 101 (1), 231–261.

Luo, Z.Q., Pang, J.S., Ralph, D., 1996. Mathematical Programs with Equilibrium Constraints. Cambridge University Press, Cambridge, United Kingdom.

Marcotte, P., Savard, G., Zhu, D.L., 2001. A trust region algorithm for nonlinear bilevel programming. Operations Research Letters 29, 171–179.

Moore, J.T., Bard, J.F., 1990. The mixed integer linear bilevel programming problem. Operations Research 38, 911–921.

Neuhoff, K., Barquin, J., Boots, M.G., Ehrenmann, A., Hobbs, B.F., Rijkers, F.A.M., Vázquez, M., 2005. Network-constrained Cournot models of liberalized electricity markets: the devil is in the details. Energy Economics 27, 495–525.

Nikogosian, V., Veith, T., Strategic Pricing and Competition in Retail Electricty Markets, Proceedings 6th Internationale Energiewirtschaftstagung an der TU Wien, February 11–13, 2009, Vienna, Austria. Available online: http://eeg.tuwien.ac.at/events/iewt/iewt2009/papers/1B_1_NIKOGOSIAN_V _P.pdf.

Nishizaki, I., Sakawa, M., Kan, T., 2003. Computational methods through genetic algorithms for obtaining Stackelberg solutions to two-level integer programming problems. Electronics and Communications in Japan, part 3 86 (6), 1251–1257.

Outrata, J., Kocvara, M., Zowe, J., 1998. Nonsmooth Approach to Optimization Problems with Equilibriuim Constraints. Kluwer Academic Publishers, Dordrecht.

Raghunathan, A., Biegler, L.T., 2002. Barrier Methods for Mathematical Programs with Complementarity Constraints (MPCCs). Technical Report, Carnegie Mellon University, Department of Chemical Engineering, Pittsburgh, PA 15213, December.

Raghunathan, A.U., Biegler, L.T., 2003. Mathematical Programs with Equilibrium Constraints (MPECs) in process engineering. Computers and Chemical Engineering 27, 1381–1392.

Ralph, D., Wright, S.J., 2004. Some properties of regularization and penalization schemes for MPECs. Optimizations Methods and Software 19 (5), 527–556.

Ralph, D., Smeers, Y., 2006. EPECs as Models for Electricity Markets, Invited Paper, 2006 Power Systems Conference and Exposition (PSCE), Atlanta, GA, USA.

Scaparra, M.P., Church, R.L., 2008. A Bilevel Mixed-Integer Program for Critical Infrastructure Protection Planning. Computers and Operations Research 35 (6), 1905–1923.

Scholtes, S., 2001. Convergence properties of regularization schemes for mathematical programs with complementarity constraints. SIAM Journal on Optimization 11 (4), 918–936.

Schweppe, F.C., Caramanis, M.C., Tabors, R.E., Bohn, R.E., 1988. Spot Pricing of Electricity. Kluwer, Norwell, Massachusetts.

Stigler, H., Todem, C., 2005. Optimization of the Austrian electricity sector (control zone of Verbund APG) under the constraints of network capacities by nodal pricing. Central European Journal of Operations Research 13, 105–125.

Stoft, S., 2002. Power Systems Economics. IEEE Press, Piscataway, NJ.

Ventosa, M., Baíllo, Á., Ramos, A., Rivier, M., 2005. Electricity market modeling trends. Energy Policy 33, 897–913.

Vicente, L.N., Calamai, P.H., 1994. Bilevel and multilevel programming: a bibliography review. Journal on Global Optimization 5, 291–306.

Wen, U.P., Huang, A.D., 1996. A simple Tabu search method to solve the mixed-integer linear bilevel programming problem. European Journal of Operational Research 88, 563–571.

Williams, H.P., 1999. Model Building in Mathematical Programming, fourth edition. John Wiley and Sons, LTD, New York.

Winston, W.L., 1994. Operations research: Applications and algorithms. Duxbury Press, Belmont, CA.