A New Indicator Based on Neftçi’s Approach for Predicting Turning Points of the Euro-Zone Growth Cycle*

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Summary

"[...] in many situations a decision does not have to be made immediately, but can be delayed until additional information has been acquired. Sequential analysis seems particularly applicable to the problem of predicting turning points in the business cycle" (Palash and Radecki, 1985). Elaborating on this idea, we propose a new approach to predict cyclical turning points in the Euro-zone using the Neftçi’s approach. The output is a probability index for a forthcoming economic turning point.

1. Introduction

If there is a large literature on the analysis of economic fluctuations, there has been a renewed interest in cycle analysis and indicators in the past years due to the emergence and development of new innovative non-linear methods.

However, the identification of economic fluctuations in real time and the prediction of economic turning points raise various specific issues and may therefore involve different approaches. In short, the best model to explain the business cycle may not be the best model to predict. In a Bayesian framework, a priori information may be used to improve the performance of predictive indicators.

This paper presents a new approach to detect and predict turning points. Its validity is evaluated through an ex-post performance assessment. The Neftçi (1982) sequential probabilistic formula is applied to detect turning points in time series.1

In this paper we shall focus on a Euro-zone indicator. With the Economic and Monetary Union (EMU), European countries are facing a common monetary policy carried out by the European Central Bank (ECB) despite the heterogeneity of their economic performances. The objective of the ECB is to maintain the Euro-zone inflation under a fixed target of 2%. The economic heterogeneity is visible first at the level of real economic activity if we look at different statistics like GDP per capita, growth or potential growth. More recently, we have witnessed a divergence in the inflation patterns. Asymmetric inflation rates between EMU countries are a symptom of European structural disparities (market good or labor rigidities, sensibility to external inflation) and cyclical differences. Since the ECB has a unique inflation target for the Euro-zone, it appears useful to build a leading indicator of Euro-zone cyclical turning points, which should be helpful for any cyclical diagnosis of the Zone.

In our study, we chose to focus on the six major countries (Belgium, France, Germany, Italy, Netherlands and Spain) for the selection of the leading economic series. Their weight, 89.4% of total GDP in 20002, is sufficiently representative.

This paper is structured as follows. Section 2 presents a few concepts and the framework of our work. Section 3 deals with the Neftçi (1984) methodology and places it in relation to Hamilton (1989) Markov Switching Model. Sec-

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1 This approach has been developed in the Centre d’Observation Economique (COE) since 1997 to build turning point predictors for the growth cycles of France, Germany, Italy, the Eurozone and the United States. See Anas (1997) and Nguiffo-Boyom (1999) for detailed presentations.

2 At current prices and exchange rates (source: OECD).
Section 4 develops the COE leading indicator (IARC) methodology, with special attention to parameter estimation and aggregation procedure. Section 5 presents the empirical results and reviews the forecasting performance of the IARC.

2. Turning Point Detection

2.1 The concept of turning point

Turning points are events which need to be defined (see Burns and Mitchell, 1946; Okun, 1960). Two main types of turning points may be characterized (we shall focus on peaks):

- First, the turning point of the "classical" Business Cycle. It happens when the level of activity decreases sufficiently. This downward phase of the Business Cycle is generally called a recession (or hard landing).
- Second, the turning point of the Growth Cycle. It happens when the economic growth decreases below its trend growth rate (the so-called soft landing).

Sometimes, the growth rate decreases but remains above or close to the trend growth rate (very soft landing). In this case the economy slowdown is not sufficiently strong to impact on unemployment for example. During the Asian crisis, the French and US economies were affected in this way.

Business cycles may become less frequent in the future or even disappear because monetary policy may have become more efficient and the market rationally accounts for this regime shift. In any case, what is important is to prevent a soft landing from becoming a hard landing. As for the very soft landing, it has little impact on the aggregate economy.

In this paper we shall only focus on the turning points of the growth cycle.

2.2 The European cycle: measurement issues

Few studies (Artis and Zhang, 1997 and 1999) have tried to identify the origins of a European Business or Growth Cycle. The creation of the European Monetary System (EMS) should have accelerated the convergence of European economic cycles. However, in the 80’s, various asymmetric shocks weakened the correlation between the cycles in different European countries. Starting with the 1993 recession, we have witnessed a renewed increase in the correlation between European cycles as can be observed in figures 1 and 2.

3 The rule of three D’s may be useful to identify the recession: deepness, duration and diffusion.
4 The growth cycle is, by definition, equal to the deviation of GDP from trend.

![Figure 1](image-url)
There are similarities within country subgroups. The proximity of German and Italian cycles is well known while the French cycle seems quite idiosyncratic in the region. With the creation of the Euro in 1999, the correlation between cycles has probably increased. Nevertheless, the Asian shock did not produce the same effects on all countries. Germany and Italy experienced a soft landing while France experienced a very soft landing. It is also interesting to see how the decelerating American economy will affect the different Euro-zone countries.

Eurostat calculates the Euro-zone GDP. Various methods are available to identify the Euro-zone growth cycle. We chose the Band-Pass approach (Baxter-King filter, 1995) to identify the underlying trend. Unobserved Components Models (Harvey, 1985) may also provide turning point identification for both short and long term cycles. But this breakdown is more useful for ex-post studies than for predictions or real time estimation. Another class of identification methods involves regime switching models with fixed or variable transition probabilities (Hamiltonian models).

2.3 Detection

Detecting cycle turning points must be distinguished from dating or forecasting them.

- Dating: Even ex-post, the identification of turning points for business or growth cycles is not easy. Regarding the business cycles, the American NBER dating committee may wait until one year to identify the date of the last recession. A great number of series are analyzed for that purpose. A common convention is to identify recessions as two consecutive decreases in GDP quarterly growth. Another criterion for identifying cycles includes the observation crossing of the 50% threshold in Hamiltonian models. As yet, there is no commonly accepted criteria based on the movement intensity, the duration and the eventual crossing of the trend for growth cycles.

- Forecasting: Very few turning points forecasts are available. A rule is needed to extract the signal of a future cyclical turning point from the leading indicator (e.g. the Conference Board rule\(^5\)).

- Detecting: Detecting means looking for what is hidden. In this case the turning point has already occurred but is not known yet, because the series is not available or provisional and often volatile. Detection is a real time identification of the turning points. It is generally based on a coincident indicator.

\(^5\) Recession warnings are triggered when the annualised rate of change in the leading index falls below 3.5% while the diffusion index remains below 50% during a six-month span.
Neftçi’s formula is a useful tool for detecting swings in any variable. It also makes it possible to estimate the detection delay.

2.4 Using Neftçi’s formula to construct a leading indicator

Our aim was to devise a leading indicator of the growth cycle turning points. The probability of a turning point of the reference series is based on the probabilities of leading indicators turning points. An aggregation method has been designed to consider both type I errors (risk of a false signal) and type II errors (risk of missing the turning point).

We still need to define a measure of the intensity of a coming cyclical downturn. There are two possibilities. Either try to measure the movement intensity associated with the turning point, or we will build a leading indicator of Business Cycle turning points. We are currently working on the second approach. In this option, we focus on leading series crossing thresholds rather than on the detection of peaks and troughs.

3. Neftçi’s Formula

3.1 The framework

The objective is to predict cyclical turning points, which mark the beginning or end of a cyclical downturn. For this purpose, Neftçi (1982) developed a stochastic model for macroeconomic time series. It is based on the assumption that the series behaves differently depending on the downward or upward regime in which it evolves.

Let us consider the stochastic process \( \{X_t\}, t \in \mathbb{Z} \), where \( X_t \) represents observations on increments of the macroeconomic time series considered. According to the finite sample \( \{X_t\}_{t=1, \ldots, T} \) we will infer the occurrence or non-occurrence of a change in the economic regime.

Let \( Z \) (respectively \( Z' \)) be an integer-valued random variable denoting the date following a peak (respectively trough).\(^6\) Let us suppose \( Z = i \) or \( Z' = i \), which means that a turning point has appeared between dates \( i – 1 \) and \( i \). With the two following assumptions, we will be able to characterize the cumulative distribution function:

**Assumption 1:** The probability distribution of \( \{X_{i+j}\}, j = 0, 1, 2, 3, \ldots \) is different and independent of the distribution of \( \{X_{i-j}\}, j = 0, 1, 2, 3, \ldots \).

**Assumption 2:** The observations of \( X_t \) between and within regimes are independent.

If we consider that a peak has appeared between dates \( i – 1 \) and \( i \), or in other words \( \{Z = i, i < t\} \), then:

\[
P(\{X_0 \leq x_0, \ldots, X_i \leq x_i, \ldots\} = F^1(x_0, \ldots, x_i)\]

where \( F^1(.) \) and \( F^0(.) \) are the two cumulative distribution functions for the upward and downward regime, respectively.\(^7\)

The variable \( Z \) is not directly observable. Based on historical values of \( X_t \), we intend to determine at any date whether a turning point has already occurred \( Z \leq t \) or not yet occurred \( Z > t \).

Suppose that the forecaster has gathered some experience from the study of past turning points and has subjectively defined a priori probabilities. Let \( T_t \) be the a priori transition probability of the change from upward to downward regime,

\[
T_t = P(Z = t | Z > t - 1),
\]

and \( T_t' \), the a priori probability of the change from downward to upward regime

\[
T_t' = P(Z' = t | Z' > t - 1),
\]

Let us note \( \bar{x}_i = (x_0, \ldots, x_i) \) the historical values of \( X_t \) since the last trough. Given \( \bar{x}_i \), let us evaluate at any date \( t \) the probability of occurrence of a turning point in the recent past. Let \( P_t (P_t') \) denote the a posteriori probability of occurrence of a peak (trough) at or before date \( t \) based on observations of \( X_t \):

\[
P_t = P(Z \leq t | \bar{x}_i).
\]

Using Bayes’ rule, we get:

\[
P_t = \frac{P(\bar{x}_i | Z \leq t)P(Z \leq t)}{P(\bar{x}_i)},
\]

and by extension

\[
P_t = \frac{P(\bar{x}_i | Z \leq t)P(Z \leq t)}{P(\bar{x}_i | Z \leq t)P(Z \leq t) + P(\bar{x}_i | Z > t)P(Z > t)}
\]

(1)

Neftçi’s formula is recursively derived from (1).

For peaks \( t \geq 1 \):

\[
P_t = \frac{P_{t-1} + (1 - P_{t-1})T_t}{P_{t-1} + (1 - P_{t-1})T_t} F^1(x_t)
\]

\[
+ \frac{1 - P_{t-1})(1 - T_t)}{P_{t-1} + (1 - P_{t-1})(1 - T_t)} F^0(x_t)
\]

(2)

\(^6\) We suppose here that \( Z \) refers to the date of peaks. The results are diametrically symmetric for troughs. \( Z' \) refers to the date of troughs.

\(^7\) We adopt here the conventional notation for discrete values variables: capital letters for random variables and small letters for particular realisations.
where \( f^0(.) \) is the density function of \( X_t \) during a downward regime and \( f^1(.) \) during an upward regime, and where \( P_0 = 0 \).

We see from formula (2) that Neftçi’s formula allows us to compute the \emph{a posteriori} probability of occurrence of a turning point, incorporating current information into the posterior probabilities estimated over previous periods. As described in Niemira (1991): “[…] the Neftçi method accumulates probabilities from the start of the previous turning point. This particular dynamic characteristic of the Neftçi method is a major improvement over its predecessors.” This is an advantage over the Probit approach which has a poor dynamic contents and may therefore be less powerful if the lead times are unstable.

The Neftçi’s model appears to be a special case of the regime switching approach developed by Hamilton (1989) in which the observed \( X_t \) are assumed to depend on two different unobservable states.\(^8\) Suppose \( S_t = 0 \) stands for the downward regime and \( S_t = 1 \) stands for the upward regime.

By comparison with Neftçi’s notation:
\[
P_t = P(Z_{t-1} | X_t) = P(S_t = 0 | X_t)
\]

The underlying Neftçi’s model can be written as:
\[
\forall t, \: X_t = a^0 + a^1 S_t + \epsilon_t,
\]
where:
\[
\epsilon_t \sim iid (0, \sigma_0 + \sigma_1 S_t).
\]

Assuming a Normal distribution for \( \epsilon_t \) we get:
\[
X_t \sim N(a^0 + a^1 S_t, \sigma_0 + \sigma_1 S_t).
\]

The Transition probabilities are indicative of the persistence of the process. Hamilton (1989) assumed that these probabilities are constant over time. However recent works (see, for instance, Filardo, 1994) propose time dependent transition probabilities as a function of the age of the phase, or based on a leading indicator. In Neftçi’s approach, the transition probabilities are also non-constant and are estimated from past experience.

Assume that \( \{Y_t\} \) is a stochastic process where \( Y_t \) represents observations on increments of a macroeconomic coincident variable (GDP) and assume that \( \{X_t\} \) is a leading index. The underlying idea is that \emph{events starting a downturn in } \( Y_t \emph{ will be present in } \{X_t\} \emph{ before they show up in } \{Y_t\}. \] We need to introduce a decision rule so as to signal the forthcoming turning point in the macroeconomics coincident time series.

For this reason, we have to define a critical value for probabilities to minimize type I errors. Let \( \theta = 1 - \theta \) be the threshold beyond which we consider that a turning point has appeared in \( X_t \). Neftçi (1982) calls \( \theta \) the probability of a false alarm, while Niemira (1991) defines \( \theta \) as the confidence level.

### 3.2 Parameters estimation

The parameters of the probability distribution function of \( X_t \) are estimated over samples made up of upward and downward regimes. The \emph{a priori} transition probabilities denoted \( T^*_t \) and \( T_t \) must also be estimated.

The first step consists in an \emph{a priori} dating of the peaks and troughs of the growth cycle of \( X_t \). The data are split into upward and downward regimes in order to obtain two separate samples composed respectively of observations belonging to upward and downward regimes. The parameters of the probability density functions (\( f^0(X_t) \) and \( f^1(X_t) \)) and the \emph{a priori} probabilities (\( T^*_t \) and \( T_t \)) are computed dynamically.

A few assumptions are needed to estimate the parameters. We may suppose that the probability of a turning point is an increasing function of the age of the regime. In this case \emph{a priori} transition probabilities are duration dependent. Otherwise they will be constant. The probability density functions are estimated using an empirical distribution of \( X_t \) or by fitting a tabulate density function to observations of \( X_t \) in each regime. The threshold \( \theta \) has to be set to define the decision rule: a signal of imminent cyclical turning point is given when the probability \( P_t \) or \( P^*_t \) reaches \( 1 - \theta \). We define the detection delay as the time necessary to detect a turning point. This delay depends on the value of \( \theta \), whose choice also represents a compromise between the cost of a false signal and the cost of a too late signal. \( P_t(P^*_t) \) is initialized to 0 for the first observation, and more generally when a downward (upward) regime ends.

Neftçi (1982) uses the U.S. Department of Commerce’s composite index of leading indicators to test the recursive formula and to see if it could have predicted the 1974 recession and the 1979 slowdown. Parameters are estimated over the period 1948–1970 and sequential probabilities calculated from 1970. The density functions \( f^0(X_t) \) and \( f^1(X_t) \) are estimated by using the empirical frequency distribution of \( X_t \) during upward and downward periods. \emph{A priori} transition probabilities are approximated using a symmetric density with a mean of 50 months because of the duration dependence assumption. The \( \theta \) value is set at 0.1. Finally, Neftçi concludes that his decision rule is optimal to predict cyclical downturns.

Other empirical studies combine alternative assumptions as in Diebold and Rudebusch (1989), Niemira (1991)\(^8\)
and Artis et al. (1995), or use monetary and financial variables as leading indicators as in Palash and Radecki (1985). Therefore Neftçi’s approach has mainly been used to assess the relevance of signals obtained from a leading composite index. In other words, the sequential formula is a way to extract probabilistic signals from composite indices that are otherwise difficult to interpret.

The next section provides an original application of Neftçi’s formula to build a cyclical turning point leading indicator called IARC from the French acronym. This one is produced monthly and published\(^{10}\) by the COE for France, Germany, Italy, the Euro-zone and the Unites States.

4. The COE Leading Indicator

4.1 The COE approach

Classical leading composite indices are often constructed as a weighted average of normalized leading indicators. The rule for interpreting these composite indices (particularly to identify a signal) is either absent (OECD) or ad hoc (Conference Board). The Neftçi’s formula may be used to extract the signal of those composite indices (Artis et al., 1995; Diebold and Rudembusch, 1989).

Our approach is different. We start with the idea that the combination of statistical information is easier to perform in the space of probabilities than in the space of time series. Time series are often difficult to compare because of their different nature: opinion surveys or values, rates or levels, different frequencies and volatilities.

We therefore prefer to compute the probability of a future signal by using a set of leading indicators and find a way to aggregate the probabilities of their signals.

First of all, we need to select a set of leading indicators by evaluating their performance. The operational lead is defined as the theoretical lead minus the detection delay and the availability delay:

\[
OL = TL - DD - AD,
\]

where \(OL\) represents the operational lead, \(TL\) the theoretical lead, \(DD\) the detection delay and \(AD\) the availability delay.

The theoretical lead refers to the average observed lead between the turning point of the leading indicator and the corresponding coincident cyclical turning point. A substantial theoretical lead would not be useful if, at the same time, the detection delay were too long. For instance, we had to give up the boxboard new orders series traditionally considered as a leading series in France because the series turned out to be too volatile, implying a long detection delay.

Since we aim at forecasting the growth cycle turning points, we extracted the trend component of each non-stationary series by using the Hodrick-Prescott filter (more sensitive to a change of direction than the Baxter-King filter).

To apply the Neftçi formula we previously fitted a Normal density function to estimate \(f'(X_t)\) and \(f''(X_t)\). Niemira (1991) cites McCulloch (1975): "[…] once an expansion or recession has exceeded its historical minimum duration, the probability of a turning point is independent of its age. Therefore, a simple strategy would be to have probabilities built to the average duration and then held constant."

We have used this “simple strategy” to estimate the \(a\) priori transition probabilities in a static way because the Neftçi formula does not appear to be sensitive to a change in the estimation of \(T\) and \(T'\). Generally the estimate for monthly series is close to 5% which translates in an average constant duration of between 1.5 and 2 years. As in Artis et al. (1995), the value of \(\theta\) has been fixed to 0.05.

A false signal is a signal of a leading indicator without any coincident cyclical turning point within a predefined horizon. The false signal is what we will call the type I error. The other risk is to miss the coincident turning point, which we will call the type II error.

4.2 The aggregation procedure

We aim to aggregate the \(a\) posteriori probabilities obtained for each leading indicator from formula (2) and (3).

Suppose we selected \(N\) leading indicators \(X_t^k, k = 1, 2, \ldots, N\). For each of the \(N\) leading \(X_t^k\), we associate a latent variable \(X_t^k\) that equals 1 if a turning point of the series \(X_t^k\) has occurred before \(t\) and 0 otherwise. We define \(P_t^k = P(X_t^k = 1)\) as the Neftçi sequential probability of a recent turning point (assume it is a peak) of the \(k\)th the leading indicator. The question now is how to aggregate the probabilities \(P_t^k\). A first methodology would be a generalization of the diffusion index approach. The formula of the diffusion index when searching for a peak is:

\[
I_t(t) = \frac{1}{N} \sum_{k=1}^{N} X_t^k
\]

This diffusion index represents the percentage of leading indicators, which have reached a peak.

In practice, a good way to measure this is to compute the change over a six-month span.\(^{11}\) The decision rule is

\[\text{since we aim at forecasting the growth cycle turning points, we extracted the trend component of each non-stationary series by using the Hodrick-Prescott filter (more sensitive to a change of direction than the Baxter-King filter).}\]

\[\text{To apply the Neftçi formula we previously fitted a Normal density function to estimate } f'(X_t) \text{ and } f''(X_t).\]

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\[\text{This diffusion index represents the percentage of leading indicators, which have reached a peak.}\]

\[\text{In practice, a good way to measure this is to compute the change over a six-month span.}\]

\[\text{The decision rule is}\]

\[\text{9 In French, Indicateur Avancé de Retournement Conjoncturel (Leading Indicator of Cyclical Turning Points).}\]

\[\text{10 Also disseminated regularly on the COE website: www.coe.ccp.fr.}\]

\[\text{11 This diffusion index is used by the Conference Board as a way to estimate the extent of breadth of a particular cyclical movement.}\]
to detect when this change becomes negative. The formula becomes

\[ I_2(t) = \frac{1}{N} \sum_k 1_{(X^k_t < X^k_{t+H})} \]

We observe that each series in this index has the same weight. We consider that the mathematical expectation of the diffusion index is a better estimate:

\[ E[I_1(t)] = \frac{1}{N} \sum_k I_{k,1 \to H} \]

\[ = \frac{1}{N} \sum_k P(\bar{X}^k_t = 1) \]

\[ = \frac{1}{N} \sum_k P^k_t \]

We call this index the \textit{sequential probabilities diffusion index}. A shortcoming of this approach is to ignore the possible correlation between the sequential probabilities. We implicitly give the same weight to each leading indicator, which does not seem correct. Looking more closely to our goal (anticipating the peaks and troughs of the global economy), we introduce an implicit weighting scheme:

Consider a given forecast horizon \( H \) and define the random variable \( R_t \) equal to 1 if a cyclical turning point of the global economy occurs between \( t \) and \( t + H \) and 0 otherwise. For each leading indicator \( X^k_t \) the probability of an upcoming cyclical turning point can be developed by using the Bayesian formula as follows:

\[ P(R_t = 1) = P(R_t = 1 \mid \bar{X}^k_t = 1) \cdot P(\bar{X}^k_t = 1) + P(R_t = 1 \mid \bar{X}^k_t = 0) \cdot P(\bar{X}^k_t = 0) \]

(4)

The two risks\(^{12}\) associated with this approach are:

First, the risk of a false signal (or type I error) \( \alpha^k_t \), such as:

\[ \alpha^k_t = P(R_t = 0 \mid \bar{X}^k_t = 1) \]

Second, the risk of missing the cyclical turning point\(^{13}\) (or second type error) \( \beta^k_t \), such as:

\[ \beta^k_t = P(R_t = 1 \mid \bar{X}^k_t = 0) \]

Both risks are assumed to be constant over time, i.e.:

\[ \forall t, \alpha^k_t = \alpha^k \quad \text{and} \quad \beta^k_t = \beta^k \]

An estimate of \( P(R_t = 1) \) is \( P_k(R_t = 1) \) defined by:

\[ P_k(R_t = 1) = (1 - \alpha^k) \cdot P^k_t + \beta^k \cdot (1 - P^k_t) \]

\[ = \beta^k + (1 - \alpha^k - \beta^k) \cdot P^k_t \]

(5)

Note that \( P_k(R_t = 1) \) may reach 1 only if \( \alpha^k = 0 \), that is if the leading indicator did not produce any false signal in the past. Note also that \( P_k(R_t = 1) \) has a minimum value of \( \beta^k \) which represents the probability of missing a cyclical turning point. In practice, for any \( H \), estimation of \( \alpha^k \) and \( \beta^k \) is done over the past by confronting the chronology of leading indicator signals with the coincident cyclical dating. The probability of the type I error is the ratio of the number of false signals to the total number of leading indicator turning points, while the probability of the type II error is the ratio of the number of missed turning points to the total number of coincident cyclical turning points.

The Bayesian formula (5) can be generalized to \( N \) leading indicators but this multivariate formula becomes very complex and implies the estimation of too many parameters. However, it would be an effective way to take into account the correlations between leading indicators. The idea is to consider that we have \( N \) univariate estimates of \( P(R_t = 1) \). We can use a Probit approach to find the best weighted linear combination of those probabilities. However, this alternative results in non-significant coefficients at the 10% threshold. As a generalization of the diffusion index and considering we have a sample \( \{P_k(R_t = 1)\} \) of estimates of \( P(R_t = 1) \), we have decided to use a simple average of those estimates:

\[ \frac{1}{N} \sum_{k=1}^N [P_k(R_t = 1)] = \frac{1}{N} \sum_{k=1}^N \beta^k + (1 - \alpha^k - \beta^k) \cdot P^k_t \]

(6)

\[ = \bar{\beta} + \sum_{k=1}^N \frac{(1 - \alpha^k - \beta^k)}{N} \cdot P^k_t \]

(7)

Thus defined, this aggregation can be seen as the average of \textit{a posteriori} probabilities weighted by the probabilities of type I and type II errors. This formula has the advantage to be easily interpretable: values vary inside an interval which bounds are defined by the average probabilities of type I and type II errors: \([\bar{\beta}, 1 - \bar{\beta}]\); the inferior bound is reached if all \textit{a posteriori} probabilities equal 0 and the superior bound if all \textit{a posteriori} probabilities equal 1. The drawback of this formula is that the higher the average probability of type I error, the smaller the upper bound. Hence interpretation becomes more difficult, notably for cross-country comparisons.

For this reason, we decided to normalize this formula by dividing it by \( (1 - \bar{\alpha}) \) so that it would equal 1 as soon as \textit{a posteriori} probabilities equal 1. The final index we use (that we call IARC) is

\[^{12}\]The two risks are discussed at large in the seminal article of Okun (1960).

\[^{13}\]Either because the leading indicator missed the general economic turning point or because the signal was too late.
The last step consists in choosing a reading method in order to interpret movements of the IARC index. Now that the \textit{a posteriori} probabilities have been aggregated, a threshold for the index IARC has to be defined. We decided to take two empirically estimated thresholds into account over the period 1970–97. They are a way to make the understanding of the signal easier because we feel that a rough probability would be too difficult to interpret. The 60\% threshold corresponds to about 70\% chances of a true signal and the 80\% threshold is clearly a strong signal which can be interpreted as: “we forecast a turning point”.

- A turning point may occur in the next nine months when the IARC index reaches 60\%.
- A turning point will probably occur in the next three months when the IARC index reaches 80\%.

5. Application to the Euro-zone

5.1 Dating the growth cycle

Total GDP for the Euro-zone\textsuperscript{14} (see series in the appendix) was used as a basis to evaluate the growth cycle. Datation was based on three growth cycle estimates derived from the Hodrick-Prescott filter (HP filter), the Baxter-King band-pass filter (BP filter) and an Unobserved Components Model (UCM).

The last model is used in a simple univariate way,\textsuperscript{15} with a slowly moving smooth trend specification assuming the combination of a short and long-term cycle (periodicity of 3 and 12 years respectively):

\begin{align*}
\log (\text{GDP}) &= T_t + C_t + \varepsilon_t \\
T_t &= T_{t-1} + \beta_{t-1} \\
\beta_t &= \beta_{t-1} + \xi_t
\end{align*}

where $T_t$, $C_t$ and $\varepsilon_t$ represents respectively the trend, cyclical and irregular components.

The datation was based mainly on the peaks and troughs of the Baxter-King filter results.

\textsuperscript{14} Index base 100 = 1995 Q1. Eurostat data since 1995 Q1 and extrapolated backwards to 1970 by the authors for all Eurozone countries, except Greece, Ireland and Luxembourg (national volume indices are weighted by nominal GDP at prices and current PPP’s of 1995, according to OECD publication, 1999).

\textsuperscript{15} More sophisticated bivariate or multivariate models may improve the estimation.

Figure 3

<table>
<thead>
<tr>
<th>Euro-zone GDP: growth cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>BP filter</td>
</tr>
<tr>
<td>HP filter</td>
</tr>
<tr>
<td>UCM</td>
</tr>
</tbody>
</table>
5.2 Selection of leading indicators

Each leading indicator for the Euro-zone is a weighted average of member countries leading indicators. We selected five leading indicators: a stock market index, the spread between long and short term interest rates, the first factor of a principal component analysis of business survey results in the intermediate goods industry, manufacturer prices expectations concerning wholesale trade and the COE leading indicator of the American growth cycle (cf. figure 4).

For each series, a calculation of the operational mean lead is performed over the 1990–1999 period. The 80’s do not provide a good period for estimating the performance of indicators due to the relative heterogeneity of economic growth cycles in the zone. Criteria used for selecting the indicators are numerous. As outlined by the OECD, an economic rationale is needed to avoid taking only the statistical performance on a period of time into account. Also the series need to be available for a long period of time to allow for estimation. But the main selection criteria relates to the degree and stability of the operational lead as well as the low degree of first and second type risks. There is generally a trade-off: when the lead increases, the risks increase at the same time.

Tables 2 (lead if negative, lag if positive) and 3 show the good predictive performance of the financial indicators: the stock index and the interest rate spread. The probabilities of type I and type II errors in table 3 are rather low.

Table 1

<table>
<thead>
<tr>
<th>Cycle</th>
<th>Peak</th>
<th>Trough</th>
<th>Peak</th>
<th>Slowdown</th>
<th>Expansion</th>
<th>Cycle</th>
<th>Duration in months</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>Nov. 73</td>
<td>Aug. 75</td>
<td>Nov. 76</td>
<td>21</td>
<td>15</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Nov. 76</td>
<td>Nov. 77</td>
<td>Dec. 79</td>
<td>12</td>
<td>23</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Dec. 79</td>
<td>Jan. 83</td>
<td>Oct. 85</td>
<td>37</td>
<td>33</td>
<td>70</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Oct. 85</td>
<td>Apr. 87</td>
<td>Nov. 91</td>
<td>18</td>
<td>55</td>
<td>73</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Nov. 91</td>
<td>Aug. 93</td>
<td>Feb. 96</td>
<td>21</td>
<td>18</td>
<td>40</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Feb. 95</td>
<td>Nov. 96</td>
<td>Feb. 98</td>
<td>21</td>
<td>15</td>
<td>36</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Feb. 98</td>
<td>June 99</td>
<td>Nov. 00</td>
<td>16</td>
<td>17</td>
<td>33</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td></td>
<td></td>
<td></td>
<td>21</td>
<td>25</td>
<td>46</td>
<td></td>
</tr>
</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th>GDP datation</th>
<th>Euro-Stock</th>
<th>Euro-Spread</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TL</td>
<td>DD$^{(1)}$</td>
</tr>
<tr>
<td>Peaks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nov. 91</td>
<td>–6</td>
<td>6</td>
</tr>
<tr>
<td>Feb. 95</td>
<td>–12</td>
<td>4</td>
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<tr>
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<tr>
<td>Nov. 00</td>
<td>–9</td>
<td>4</td>
</tr>
<tr>
<td>Througths</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Aug. 93</td>
<td>–9</td>
<td>4</td>
</tr>
<tr>
<td>Nov. 96</td>
<td>–3</td>
<td>5</td>
</tr>
<tr>
<td>June 99</td>
<td>–10</td>
<td>2</td>
</tr>
</tbody>
</table>

$^{(1)}$ Estimated by Neftçi’s formula.
We observe two false signals related to the Euro-Stock in June 1990 for the peak and in December 1990 for the trough.

5.3 Performance

As shown in the following graphics, a strong signal is always emitted in the 3 months preceding the turning point.

It seems very probable that the Euro-zone upward growth cycle phase ended in the last quarter of 2000. A strong signal was given by the COE leading indicator in October 2000, anticipating a peak in the three following months (cf. figure 6). This was quite a good performance since at that time most economists were rather optimistic about economic growth in 2001 and did not anticipate any peak in the growth cycle.

The COE indicator is now used to anticipate the next trough. The current level (−27.9\textsuperscript{18} in March 2001) of the indicator does not allow to forecast any exit from the current slowdown during the following nine months (cf. figure 7).

5.4 Implication of the signal

When the peak is confirmed, it means that the Euro-zone growth rate decreases below its trend growth rate. If we consider the Baxter-King filter results, a 2.5% trend growth rate was estimated at the end of 2000. A Probit analysis also estimates the threshold at 2.4%. If we regress the annual change of quarterly GDP on recession phases, the best model is obtained with a lead of one or two quarters. If we regress the quarterly change, the best model is obtained with a centered 3 quarters moving average without delay. Consequently, we have to wait 4 to 5 months to check whether this implication has materialized.

5.5 Conclusion

In this paper, we showed that Neftçi’s criterion may be a useful way to select leading economic time series and to predict cyclical turning points of the Euro-zone in particular. However, assumptions are crucial and would need more investigation like duration dependence and asymmetry tests. Also, it could be beneficial to take the correlation between and within economic series into account. Finally, a business cycle leading indicator needs to be elaborated in order to inform on the intensity of a coming slowdown signaled by the growth cycle leading indicator.

\textsuperscript{18} The index varies from 0 to −100 when searching for a trough (with thresholds of −60 and −80).
References


Figure 6

IARC Eurozone (Search of next peak)

November 2000

99.8

Figure 7

IARC Eurozone (Search of next trough)

March 2001

-27.9


Ein neuer Indikator auf Basis des Ansatzes von Neftçi zur Vorhersage der Wendepunkte in Wachstumszyklen