Merger Efficiency and Welfare Implications of Buyer Power

Özlem Bedre-Defolie and Stéphane Caprice
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Abstract

This paper analyzes the welfare implications of buyer mergers, which are mergers between downstream firms from different markets. We focus on the interaction between the merger’s effects on downstream efficiency and on buyer power in a setup where one manufacturer with a non-linear cost function sells to two locally competitive retail markets. We show that size discounts for the merged entity has no impact on consumer prices or on smaller retailers, unless the merger affects the downstream efficiency of the merging parties. When the upstream cost function is convex, we find that there are “waterbed effects”, that is, each small retailer pays a higher average tariff if a buyer merger improves downstream efficiency. We obtain the opposite results, “anti-waterbed effects”, if the merger is inefficient. When the cost function is concave, there are only anti-waterbed effects. In each retail market, the merger decreases the final price if and only if it improves the efficiency of the merging parties, regardless of its impact on the average tariff of small retailers.

Keywords Buyer mergers, non-linear supply contracts, merger efficiencies, size discounts, waterbed effects.

JEL Classifications D43, K21, L42.

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1 Introduction

In recent years the grocery industry has undergone a dramatic consolidation. A substantial number of acquisitions have taken place between retailers serving different geographic markets, for example, the expansion by Sainsbury’s and Tesco in convenience store retailing in the UK. In addition to grocery retailers, in the last decade many cable network operators from different geographic markets have declared their interests to merge. For instance, in 2004 the Kabel Deutschland Group (KDG), which operates the former broadband cable network of Deutsche Telekom AG in all of Germany apart from three regions, proposed to acquire the network operators in those regions. Moreover, in 2005 Ish and Iesy, which were two cable network operators active in different local markets, merged and became Unity Media and in 2010 the German cable network operator Unitymedia was acquired by Liberty Global Europe Holding B.V. (LGE) of the Netherlands.

Such mergers between firms from different local markets do not raise any horizontal (anti-competitive) concerns, however, they could significantly affect the bargaining position or buyer power of the merging firms when purchasing inputs from a supplier. The supplier might then change its unit price to other buyers who could be the rivals of the merged entity in different markets. As a result, the buyer merger would modify the competitive conditions and consumer prices in the downstream markets. In particular, the effects of large grocery chains’ buyer power on consumers and on small independent retailers (e.g., convenience stores) have become one of the most controversial debate for anti-trust authorities and for academics.

The common view is that the exercise of buyer power by retailers may lower their purchasing costs and therefore lead to lower consumer prices. On the other hand, as argued by the UK Competition Commission, “the exercise of buyer power by the merged entity would have adverse effects on consumers and on small independent retailers.”

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1The concentration ratio of the five largest retailers (C5) in the 15 member countries of the EU is on average 50% (IGD European Grocery Retailing, 2005). The UK’s top four grocery retailers account for 65% of total retail sales (the Competition Commission, 2008, p.29). In the US, C8 was 17.5% in 2007, instead of 15.3% in 2002. See the US Census Bureau, Retail Trade, http://www.census.gov/econ/concentration.html
2See the groceries report prepared by the Competition Commission (2008).
3See the European Commission’s (EC) cases M.1832 (Ahold/ICA Förbundet/CANICA, 2000), M.2161 (Ahold/Superdiplo, 2000) and M.2604 (ICA Ahold/Dansk Supermarket, 2001).
4See the EC’s cases M.1904 (Carrefour/Gruppo GS, 2000), M.1960 (Carrefour/Marinopoulos, 2000), M.2115 (Carrefour/GB, 2000), M. 4522 (Carrefour/Ahold Polska, 2007) and M.5858 (Carrefour/Marinopoulos/Balkan JV, 2010).
5Bundesländer Hessen, Baden-Württemberg and North Rhine-Westphalia.
6‘Ish’ in North Rhine-Westphalian, Kabel Baden-Württemberg (KabelBW) and the Hessian cable operator ‘Iesy’. Bundeskartellamt (Germany’s Federal Cartel Office) prohibited the proposed takeover due to the KDG’s dominant position in Germany.
8See B7 – 22/05 and Iesy Repository/Ish, COMP/M.3674.
9The EC’s case M.5734.
11This goes back to Galbraith’s (1952) “countervailing power” argument.
effects on other, smaller, grocery retailers through the waterbed effect - that is, suppliers having
to charge more to smaller customers if large retailers force through price reductions which would
otherwise leave suppliers insufficiently profitable.\footnote{The Competition Commission’s report (2003), paragraph 2.218. See also Guidelines in the applicability of Article 81 of the EC Treaty to horizontal cooperation agreements (2001/C3/02), paragraph 126: “The primary concerns in the context of buying power are that ... it may cause cost increases for the purchasers’ competitors on the selling markets because either suppliers will try to recover price reductions for one group of customers by increasing prices for other customers ...”}

Besides affecting purchasing terms in the upstream (input) market, a merger might enhance the efficiency of some or all merging parties in the downstream market because the firms learn from each others’ management expertise\footnote{Farrell and Shapiro (1990).} improve their technologies by the diffusion of know-how, save costs from reallocating distribution across different stores, benefit from synergies, or save on costs of capital\footnote{Since larger firms usually have better access to the outside capital markets.}. In contrast, a merger might reduce the efficiency of the merging parties either because communication would be more costly within a larger firm\footnote{Bolton and Dewatripont (1994).} or due to the conflicting organizational cultures\footnote{Weber and Camerer (2003).}. When deciding whether to approve a merger, anti-trust authorities assess the efficiency gains from the merger against the possible anti-competitive effects of the merger\footnote{The EU Competition Law, Rules Applicable to Merger Control, 2010, pp. 186-187.}. It is therefore important to understand how the effects of a merger on efficiency interact with its potential anti-competitive effects.

This paper analyzes the implications of a “buyer merger” between two independent downstream firms on their rivals and consumers\footnote{We follow the literature by generating buyer power through an endogenous process of a merger between the firms active in different markets (like in Inderst and Wey, 2007; Inderst and Valletti, 2011). In this respect our paper differs from the earlier contributions by von Ungern-Sternberg (1996), and Dobson and Waterson (1997), who consider mergers between competing firms.}. Considering non-linear supply contracts, which appear to be widespread practices\footnote{Empirical studies find evidence that manufacturers and retailers use non-linear supply contracts in the markets for bottled water in France (Bonnet and Dubois, 2010) and for yoghurt in the US (Berto Villas-Boas, 2007). The supplier survey conducted by the GfK Group (2007), on the behalf of the Competition Commission, supports the use of complex non-linear supply contracts in the UK grocery market.} we show that when the supplier’s cost is convex, the buyer merger leads to size discounts for the merged entity, but these discounts are given through fixed transfers, and therefore have no impact on consumer prices or on the rival firms\footnote{This result is in parallel with a statement in the European Commission Guidelines in the applicability of Article 81 of the EC Treaty to horizontal cooperation agreements (2001/C3/02).}. However, when the merger generates some efficiency gains, we find that buyer power leads to “waterbed effects”, that is, higher average tariffs for the firms not involved in the merger (small firms). It also increases the total quantity in the final markets. On the other hand, if the merger deteriorates the efficiency of the merging firms, it could still be profitable due to the size discounts generated by the merger. In this case, we obtain the opposite results: a buyer merger leads to anti-waterbed effects for the other retailers and decreases the consumer surplus. If the supplier has a concave cost function, a profitable buyer merger is necessarily efficient and it also results in a higher total quantity in each
market, however it leads to “anti-waterbed effects” on the small retailers.\footnote{As long as retailers’ quantities are strategic substitutes.}

We focus on one upstream firm producing with a non-linear cost and supplying two competitive markets in which there are many retailers competing in quantities.\footnote{Main results would be valid if we considered a different timing leading to price competition. For instance, first, retailers simultaneously make two-part tariff contract offers to the supplier, second, retailers with accepted contracts compete à la (differentiated) Bertrand and then pay their tariffs to the supplier accordingly. See Bedre-Defolie and Caprice (2009) for this generalization.} In the case of convex upstream cost, for simplicity, we assume that retailers simultaneously make take-it-or-leave-it contract offers to the supplier\footnote{A more balanced bargaining power distribution in “contract equilibrium” (as defined in Crémer and Riordan, 1987) would lead to the same equilibrium quantities, but change only the profit sharing between the firms.} where the contracts determine a quantity and a tariff. The supplier then decides which offer(s) to accept. Finally, trade takes place according to accepted contracts. When the supplier has a concave cost, we assume sufficiently high bargaining power for the supplier and consider “contract equilibrium” as a solution to the negotiation between the supplier and each retailer, since otherwise the existence of equilibrium cannot be guaranteed.\footnote{See Segal and Whinston (2003).}

Our paper contributes to the literature analyzing the sources and implications of buyer power. Chipty and Snyder (1999) analyze the profitability of a buyer merger between retailers. They show that the effect of the merger on the merging entities’ buyer power (vis-à-vis one supplier) depend on the curvature of the industry surplus. For example, if the surplus function is concave (which might be due to a convex cost of the supplier), the buyer merger results in size discounts for the merging parties. Similar to Chipty and Snyder (1999), we model buyer power as an endogenous process which originates from a buyer merger due to the convexity of the supplier’s cost function and/or downstream efficiency generated by the merger. Different from Chipty and Snyder (1999), we analyze the implications of buyer power on retail competition, rival retailers and consumer prices by introducing downstream competition in each market.\footnote{Alternatively, in the setup of Chipty and Snyder, Smith and Thanassoulis (2011) introduce uncertainty on the supplier’s volume of sales at simultaneous negotiations with independent buyers. When the upstream cost is convex, they show that a larger buyer might pay a higher input price if the uncertainty is sufficiently high. In this case, with a high probability the supplier sells zero to the other buyers which increases its expected marginal cost when it negotiates with the large buyer.}

Alternatively, Katz (1987) models buyer power as a retailer’s ability to integrate backwards by paying a fixed cost. When the retailer gets larger, it could reduce the average cost of its alternative supply option and thereby get a better price from the supplier. Using the approach of Katz (1987), Inderst and Valletti (2011) allow all competing retailers to have access to a costly outside option and analyze the implications of a buyer merger on the wholesale prices offered by the main supplier, on retail competition and on final prices. They show that, since a buyer merger creates size asymmetry between the retailers, it leads to a lower wholesale price to the merged entity and higher wholesale prices to the other retailers, that is, waterbed effects. They find that waterbed effects are less significant when the fixed cost of the alternative supplier is lower. As a result, buyer power might increase the consumer surplus. This is found to be the case when the retail prices are strategic complements and the fixed cost of the alternative supplier is sufficiently low. Intuitively, in this
case the small retailers decrease their price as a reaction to a more efficient rival and increase their price due to waterbed effects, and the former effect dominates since the fixed cost is sufficiently low. The assumption of linear contracts seems to be critical for their results, since with two-part tariffs, as a reaction to the retailers’ asymmetry, the supplier might not change the wholesale prices, but might lower the fixed fee of the merged entity and increase the fixed fee of the other retailers.

Different from Inderst and Valletti (2011), we consider non-linear supply contracts. In our paper, the buyer merger creates size asymmetry between the ex-ante symmetric retailers and so generates a size discount (a lower average tariff) for the merged entity if the supplier’s cost is convex: a larger retailer negotiates less at the margin of the supplier’s cost. However, this does not lead to a larger average tariff for a small firm unless the merger increases the efficiency of the merging parties. In the case of an efficient merger, the merged entity sells more at each store and therefore lowers its average tariff even further. This changes the bargaining position of the other retailers and so leads to a higher average tariff for each small retailer, that is, there are waterbed effects due to the convexity of the supplier’s cost function. When quantities are strategic substitutes, the small firms sell less, competing against a more efficient rival, so the net effect of an efficient merger is not straightforward.\footnote{When quantities are strategic complements, small firms sell more as a result of an efficient merger, so the efficient merger always increases the total quantity.}

On the other hand, if the supplier’s cost is concave, the size of a retailer increases its average price, and so a buyer merger would be profitable only if it generates efficiency. In this case, the buyer merger results in a lower average tariff for the other retailers, that is, there are anti-waterbed effects. Despite paying a lower average tariff, we show that each small retailer sells less post-merger since they face a more efficient rival and quantities are strategic substitutes. As in the convex case, we find that the efficient merger increases the total quantity in each market.

Majumdar (2006), Chen (2003) and Bedre-Defolie and Shaffer (2011) provide analyses of waterbed effects in a context of ex-ante asymmetric retailers (a dominant or large retailer and competitive fringe or small firms). Majumdar (2006) shows that waterbed effects exist since the large retailer wants to own more stores to increase its rivals’ costs (the spot price for smaller retailers) as there are fewer small stores over which the upstream fixed cost can be spread.\footnote{Majumdar (2006) considers a large retailer which could contract with two perfectly competitive manufacturers outside of the spot market and, moreover, could appoint one or both of the manufacturers which then commit(s) to production by sinking a fixed cost.} Modeling buyer power as an exogenous bargaining strength of the dominant firm, Chen (2003) and Bedre-Defolie and Shaffer (2011) illustrate anti-waterbed effects on competitive fringe firms.\footnote{Bedre-Defolie and Shaffer (2011) show that by offering a lower wholesale price to the fringe firms, a supplier could increase its outside option and thereby capture a higher rent from a more powerful dominant retailer.} Chen (2003) finds that an increased buyer power lowers the final price, but the other papers conclude that the impact of buyer power on final prices is not clear.

The next section presents our benchmark model with a convex upstream cost. In Section 3 we solve for the equilibrium of the benchmark. Section 4 analyzes the implications of a profitable buyer merger. In Section 5 we extend our analysis to the case where the supplier has a concave...

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\footnote{Bedre-Defolie and Shaffer (2011) show that by offering a lower wholesale price to the fringe firms, a supplier could increase its outside option and thereby capture a higher rent from a more powerful dominant retailer.}
cost function. Finally, we discuss the case of strategic complementarity and conclude in Section 6.

2 The Model

We consider a vertical industry where one monopoly supplier sells its product to two locally competitive retail markets. In each local market $n$ identical retail stores resell the product of the supplier competing in quantities, where $n \geq 2$. The supplier’s cost of producing $Q$ units is $C(Q)$, which is assumed to be twice continuously differentiable, strictly increasing, $C'(Q) > 0$, and strictly convex, $C''(Q) > 0$. Retailers have constant marginal cost of retailing $c$. Let $Q_h$ denote the total quantity sold in local market $h$ ($h = 1, 2$). Suppose that for $\forall h$, the inverse market demand is given by $P(Q_h)$, which is twice continuously differentiable and downward sloping, $P'(Q_h) < 0$. We make the following assumption on the demand function,

Assumption 1. $P'(Q) + QP''(Q) < 0$ for any $Q$,

to ensure that the second-order condition for the existence and uniqueness of an equilibrium is satisfied.

The supply contracts are assumed to be quantity-forcing contracts. The contract of retailer $i$ specifies quantity $q_i$ to be delivered to retailer $i$ and money transfer $t_i$ to be made from retailer $i$ to the supplier, for $i = 1, ..., 2n$.

We assume that retailers have all the bargaining power vis-à-vis the supplier. Hence, retailers simultaneously make take-it-or-leave-it contract offers to the supplier who in turn decides whether to accept these offers or not. The retailers with accepted contracts buy the agreed quantity and pays its tariff. Retailers then re-sell the quantity they purchased to consumers. Retail competition is therefore à la Cournot, where quantities are determined by signed supply contracts between retailers and the supplier.

Let $T$ denote the total transfers made: $T = \sum_{i=1}^{2n} t_i$, and $Q$ denotes the total quantities sold: $Q = \sum_{i=1}^{2n} q_i$. Moreover, $T[i]$ (respectively $Q[i]$) refer to the sum of all transfers (quantities) except for the tariff paid (quantity sold) by retailer $i$. Similarly, $Q_h[i]$ denotes the sum of all quantities for local market $h$ except for the quantity sold by retailer $i$.

With this notation, the profit of the supplier is written as

$$\pi_U = t_i + T[i] - C\left(Q[i] + q_i\right),$$

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28 We extend the analysis to concave cost in Section 3.
29 This assumption is for simplicity. The main qualitative results would go through if we allowed for shared bargaining power in contract equilibrium where quantity-forcing contracts are determined by simultaneous and secret bilateral negotiations between the supplier and retailers, as in Allain and Chambolle (2011), for example. The proofs of this extension are available upon request from the authors.
30 We discuss the case of strategic complementarity in Section 6.
3 Equilibrium Contracts and Payoffs

Consider the contract choice of retailer $i$, taking the other contracts as given. Retailer $i$ chooses $(q_i, t_i)$ by maximizing its profit subject to the participation constraint of the supplier:

$$\max_{q_i, t_i} \left[ P(Q_{hi} + q_i) - c \right] q_i - t_i \quad \text{s.t.} \quad T_i + t_i - C(Q_i + q_i) \geq \pi_{U[i]} = T_i - C(Q_i)$$

where the disagreement payoff of the supplier with retailer $i$, $\pi_{U[i]}$, is the supplier’s profit when the negotiation with retailer $i$ fails. In equilibrium, the participation constraint of the supplier is binding, that is,

$$t_i^*(q_i) = C(Q_i + q_i) - C(Q_i)$$

since otherwise the retailer could increase its profits by raising $t_i$. In other words, in equilibrium, each retailer pays a tariff equal to its cost contribution. Plugging the equilibrium tariff (2) into the retailer’s profit, we re-write retailer $i$’s problem as

$$\max_{q_i} \left[ P(Q_{hi} + q_i) - c \right] q_i - \left[ C(Q_i + q_i) - C(Q_i) \right].$$

The first-order condition characterizes retailer $i$’s best reply quantity to the quantities of other retailers as,

$$P'(nq^* + q_i) q_i + P(Q_{hi} + q_i) = c + C'(Q_i + q_i)$$

In equilibrium, each retailer chooses a quantity which maximizes the bilateral profit with the supplier. Using the fact that the markets are symmetric, condition (3) yields the equilibrium quantity, $q^*_i = q^*$:

$$P'(nq^*) q^* + P(nq^*) = c + C'(2nq^*),$$

where the equilibrium quantity of local market $h$ and of the industry are respectively $Q^*_h = nq^*$ and $Q^* = 2nq^*$. Hence, each retailer pays

$$t^* = C(2nq^*) - C((2n - 1)q^*).$$

and earns

$$\pi^* = [P(nq^*) - c] q^* - [C(2nq^*) - C((2n - 1)q^*)].$$

Assumption 1 ensures that the second-order condition holds.
4 Buyer Merger and Size Discounts

We extend the model by introducing a single large retailer $L$ as a result of a merger between two independent stores. The merged entity ($L$) now operates two stores which are active in different retail markets and makes a quantity offer to the supplier to distribute it through its stores. A small retailer, $S$, operates only one store, and thus makes a quantity offer to the supplier to distribute it at that store.

Let $q_L$ and $q_S$ denote, respectively, the total quantity delivered by the large retailer and a small retailer (using symmetry, all small retailers buy the same quantity). We therefore have

$$Q_h = \frac{q_L}{2} + (n - 1)q_S, \quad Q = 2Q_h.$$

We allow for the possibility that the merger affects not only the size but also the downstream efficiency of the merging parties. Let $c + \mu$ be the marginal cost of retailing at each store of the large retailer. When $\mu = 0$, the merger has no effect on retail efficiency. However, when $\mu < 0$ (respectively $\mu > 0$), the merger improves (deteriorates) downstream efficiency. For instance, the merger could result in some economies of scale downstream or other types of synergies and/or increase the costs of communication and coordination within the merged entity. If the efficiencies generated by the merger are higher (lower) than the inefficiencies produced by the merger, we say that $\mu < 0$ ($\mu > 0$). For the small retailers the marginal cost of retailing is still equal to $c$.

The merged entity chooses $(q_L, t_L)$ by maximizing its total profit from operating two stores subject to the participation constraint of the supplier:

$$\max_{q_L, t_L} \left[ 2 \left( P\left(Q_{h[L]} + \frac{q_L}{2}\right) - (c + \mu) \right) \frac{q_L}{2} - t_L \right] \quad s.t. \quad t_L \leq C\left(Q_{h[L]} + q_L\right) - C\left(Q_{h[L]}\right) \quad (7)$$

After plugging the binding constraint into the problem, the optimality condition characterizes the best reply quantity of the large retailer to the other retailer’s quantities, $q_{BR}^L(Q_{h[L]}, Q_{h[S]}):

$$P'\left(Q_{h[L]} + \frac{q_L}{2}\right) \frac{q_L}{2} + P\left(Q_{h[L]} + \frac{q_L}{2}\right) = c + \mu + C'\left(Q_{h[L]} + q_L\right) \quad (8)$$

where $Q_{h[L]} = (n - 1)q_S$ and $Q_{h[S]} = 2(n - 1)q_S$. By the Implicit Function Theorem, observe that the large retailer sells more when the merger is more profitable,

$$\frac{\partial q_{L}^{**}}{\partial \mu} < 0.$$

The small retailers choose their contracts by solving problem (1), for $i = S$, and thus their best reply quantities are characterized by

$$P'\left(Q_{h[S]} + q_S\right) q_S + P\left(Q_{h[S]} + q_S\right) = c + C'\left(Q_{h[S]} + q_S\right) \quad (9)$$

By taking the total derivative of the first-order condition, we illustrate that the small retailer’s
equilibrium quantity decreases in the large retailer’s quantity,
\[
\frac{\partial q^*_S}{\partial q^*_L} = \frac{P''(Q_{h[S]} + q_S) q_S + P'(Q_{h[S]} + q_S) - C''(Q_{[S]} + q_S)}{P''(Q_{h[S]} + q_S) q_S + 2P'(Q_{h[S]} + q_S) - C''(Q_{[S]} + q_S)} < 0,
\]
due to Assumption 1 and the convexity of the cost. The solution to equations (8) and (9) characterizes the equilibrium quantities after the merger, \(q^*_L\) and \(q^*_S\). The profit of the large retailer and the small retailer are, respectively,
\[
\pi^*_L(\mu) = \left[ P(Q^h) - (c + \mu) \right] q^*_L - \left[ C(Q^*) - C(Q^* - q^*_L) \right],
\]
\[
\pi^*_S(\mu) = \left[ P(Q^h) - c \right] q^*_S - \left[ C(Q^*) - C(Q^* - q^*_S) \right].
\]

Comparing the optimality conditions before, (3), and after the merger, (8) and (9), gives us the following results:

**Lemma 1** When a buyer merger has no impact on retail efficiency, \(\mu = 0\), it does not change the equilibrium quantities sold at each store:
\[
\frac{q^*_L}{2} = q^*_S = q^*.
\]

Equilibrium transfers \(t^*_L\) and \(t^*_S\) are such that the large retailer pays a lower average tariff than the small retailers, which pay exactly the same tariff as before the merger:
\[
\begin{align*}
t^*_L(0) &= C(2nq^*) - C(2(n - 1)q^*) < 2t^*, \\
t^*_S(0) &= t^*.
\end{align*}
\]

Intuitively, the large retailer negotiates a larger quantity with the supplier which has a convex cost function, and thus has a higher incremental contribution to the industry profit than the small retailers. As a result, the large retailer pays a lower average tariff, that is, gets size discounts. Lemma 1 implies that the merging parties earn more than they would get if they were separated,
\[
\pi^*_L(0) = [P(nq^*) - c] 2q^* - [C(2nq^*) - C(2(n - 1)q^*)] > 2\pi^*.
\]
which proves the profitability of the merger:

**Corollary 1** A buyer merger, which has no impact on retail efficiency, \(\mu = 0\), is always profitable, since it brings size discounts. Size discounts for the large retailer alter neither equilibrium quantities nor the profits of the small retailers, that is, there is no waterbed effect:
\[
\pi^*_S(0) = \pi^*.
\]
Since we consider non-linear supply contracts and, in equilibrium, each contract is bilaterally optimal holding the others’ contracts fixed, the buyer merger results in a transfer of profits from the supplier to the large buyer without affecting quantities and retail prices. The parties always want to merge to increase their size and negotiate a better deal with the supplier. When the supplier has strictly increasing incremental production costs, a small buyer negotiates at the margin, where incremental costs are high. In contrast, if two (or more) small buyers merge, they account for a larger fraction of the supplier’s total sales, and thus negotiate less at the margin, thereby paying a lower price per unit.

When the buyer merger improves retail efficiency, $\mu < 0$, the parties always want to merge for two reasons: To extract discounts from the supplier and to benefit from the efficiencies generated by the merger. On the other hand, if the merger deteriorates downstream efficiency, $\mu > 0$, it might still be profitable. This would be the case, for example, when the inefficiency produced by the merger is low enough to be compensated by the gains from size discounts.

**Lemma 2** If the merger generates less efficiencies (or more inefficiencies), the equilibrium profit of the merged entity decreases, that is, $\partial_\mu \pi^{**}_L < 0$.

The merger’s efficiency (or inefficiency) affects the merged entity’s profit through two channels. First, when $\mu$ decreases, becoming more efficient (or less inefficient) increases its margin and so its profit. Second, when the merged entity becomes more efficient (or less inefficient), the rival retailers’ quantities change. Since retailers’ quantities are strategic substitutes, the small retailers sell less as a reaction to a more efficient (or less inefficient) merger, that is, $\frac{dq^*_S}{d\mu} > 0$. Moreover, the large retailer earns more when its rivals sell less,

$$\frac{\partial \pi^{**}_L}{\partial q^*_S} = (n - 1) \left[ P' (Q^*_h) q^*_L - 2 \left( C' (Q^{**}) - C' (Q^{**} - q^*_L) \right) \right] < 0,$$

because the inverse demand is decreasing and the upstream cost is convex. As a result, the large retailer’s profit decreases in $\mu$:

$$\frac{d \pi^{**}_L}{d\mu} = \frac{\partial \pi^{**}_L}{\partial q^*_S} \frac{dq^*_S}{d\mu} - q^{**}_L < 0.$$

Note that a buyer merger is always profitable if $\mu = 0$ (see Corollary 1) and that we have $\frac{d \pi^{**}_L}{d\mu} < 0$ from Lemma 2. By continuity, we show that

**Corollary 2** There exists $\tilde{\mu} > 0$ such that for any $\mu < \tilde{\mu}$, a buyer merger is profitable.

A buyer merger affecting downstream efficiency is going to change the equilibrium quantities due to the efficiency impact of the merger.

**Proposition 1** If a buyer merger generates downstream efficiency, $\mu < 0$, we have

$$q^*_S < q^* < \frac{q^{**}_L}{2} \quad \text{and} \quad Q^* < Q^{**},$$
Inversely, if a buyer merger produces downstream inefficiency, $\mu > 0$, we have:

$$\frac{q_{L}^{**}}{2} < q^{*} < q_{S}^{**} \text{ and } Q^{**} < Q^{*}.$$ 

**Corollary 3** As a result of a buyer merger affecting downstream efficiency, the total change in the small retailers’ quantities is lower than the change in the merging parties’ quantities:

$$2(n - 1)|q_{S}^{*} - q^{*}| < |q_{L}^{**} - 2q^{*}|.$$ 

When the merger improves the efficiency of the merging parties, each store of the merged entity has a competitive advantage against the small retailers and so sells more than before. This implies that each small retailer sells less after the merger. The first-order effect of the efficiency gains on the large retailer’s quantity dominates the second-order effect on the small retailers’ quantities. Hence, the total quantity increases after an efficient buyer merger. Symmetric intuition applies for an inefficient buyer merger.

Since the supplier has a non-linear cost function, the changes in quantities modify the average tariff paid by the small and the large retailers. The large retailer could further obtain more size discounts since it is asking for a larger quantity and the small retailers are buying a lower quantity. As a result, each small retailer negotiates more at the margin, and so pays a higher average tariff. To see this let $t_{S}^{*} (q)$ and $t_{S}^{**} (q)$ denote a small retailer’s transfer for a given $q$ when the other retailers sell their equilibrium quantities, respectively, before and after the merger. Before the merger a small retailer pays

$$t_{S}^{*} (q) = C \left(2q^{*} + (2n - 3)q^{*} + q\right) - C \left(2q^{*} + (2n - 3)q^{*}\right).$$

and after the merger it pays

$$t_{S}^{**} (q) = \left[C \left(q_{L}^{**} + (2n - 3)q_{S}^{**} + q\right) - C \left(q_{L}^{**} + (2n - 3)q_{S}^{**}\right)\right].$$

Consider for instance the changes after an efficient merger ($\mu < 0$). From Corollary 3 we have $q_{L}^{**} - 2q^{*} > (2n - 3) (q^{*} - q_{S}^{**})$. A small retailer pays more for any volume of sales $q$, $t_{S}^{*} (q) > t_{S}^{**} (q)$, because its cost contribution is higher when the other retailers’ total quantity is higher. Symmetrically, for an inefficient merger, a small retailer pays a lower average tariff post-merger since the other retailers’ total quantity is lower: $2q^{*} - q_{L}^{**} > (2n - 3) (q_{S}^{*} - q^{*})$.

This discussion illustrates the effects of a buyer merger on the average tariff of a small retailer. Since we consider non-linear supply contracts, we define a waterbed effect as an increase in the average tariff of a small retailer due to the merger. If the merger decreases the average tariff of a small retailer, we say that there are waterbed effects. The following lemma summarizes our results on waterbed effects:

**Lemma 3** As a result of an efficient buyer merger, each small retailer pays a higher average tariff for a given volume of sales, that is, there are waterbed effects. Conversely, an inefficient buyer merger results in a lower average tariff for a small retailer, that is, anti-waterbed effects.
The net effect of the merger on a small retailer also depends on how the merger changes its gross profit. Let $\pi^*_S (q)$ (respectively, $\pi^{**}_S (q)$) denote a small retailer’s profit for a given quantity $q$ when the other retailers sell their equilibrium quantities before the merger (respectively after the merger). Before the merger, a small rival retailer gets

$$\pi^*_S (q) = |P (q^* + (n-2)q^* + q) - c| q - t^*_S (q),$$

and after the merger it gets

$$\pi^{**}_S (q) = \left[ P \left( \frac{q^{**}_L}{2} + (n-2)q^{**}_S + q \right) - c \right] q - t^{**}_S (q).$$

As a result of an efficient merger, the gross profit of a small retailer decreases since the market price decreases due to the increase of the total quantity sold by its rivals. The reduction in the gross profit and the increase of the tariff lead to lower profit for the small retailer. Symmetrically, an inefficient merger increases the gross profit of a small retailer by increasing the final price. As a result of the price increase and the reduction in its tariff, the small retailer earns more.

**Proposition 2** As a result of an efficient (respectively inefficient) buyer merger, each small retailer earns less (respectively more) profit for a given volume of sales.

**5 Extension: concave upstream cost**

When the upstream cost is concave, there exists no Nash equilibrium in pure strategies if the retailers have all the bargaining power and make take-it-or-leave-it offers to the supplier. To avoid this problem of inexistence, we allow for distributed bargaining power between the supplier and retailers.

More specifically, we assume that supply contracts are determined by simultaneous and secret bilateral negotiations between the supplier and retailers. We look for a contract equilibrium such that there is no bilateral incentive for the supplier and any retailer to alter the terms of their contract. By definition, a contract equilibrium is immune to any bilateral deviation, holding other retailers’ supply contracts fixed. A contract equilibrium is therefore a vector of supply contracts, $(q^*, t^*)$, such that for all $i$, $(q^*_i, t^*_i)$ maximizes the bilateral profits $\pi_U + \pi_i$ of the upstream firm and retailer $i$, taking $(Q^*_i, T^*_i)$ as given.

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32 In a candidate equilibrium each retailer offers the cost contribution of its quantity to the supplier’s total cost of production, but then the supplier’s profit would be negative due to the concavity of the cost. See Segal and Whinston (2003).

33 Crémer and Riordan (1987) introduce the contract equilibrium concept in a setup where the interdependence between bilateral supply contracts is due to the non-linearity of the supplier’s cost and due to asymmetric information between the supplier and its independent customers. O’Brien and Shaffer (1992) redefine contract equilibrium in a setup where the interdependence between bilateral supply contracts comes from the downstream competition between retailers. Our contractual setup is a combination of these two setups since we look for a contract equilibrium of bilateral supply contracts negotiated simultaneously and secretly between locally competitive retailers and the supplier which has a concave cost function.
Since the upstream cost is concave, the quantities could be strategic complements. We therefore assume that the supplier’s cost is not so concave that the quantities are strategic substitutes:

**Assumption 1a.** \( P''(Q) q + P'(Q) - 2C''(2Q) < 0 \) for any \( q \leq Q \).  

Moreover, to ensure that the equilibrium profit of each retailer is decreasing in its rival’s quantity of sales, we assume that a change in a rival’s sales affects the revenue of the retailer more than its effect on the retailer’s cost contribution:

**Assumption 1b.** \( P'(Q) q < C'(2Q) - C'(2Q - 2q) \) for any \( q \leq Q \).

For example, a quadratic concave cost, \( C(Q) = aQ + bQ^2 \) with \( b < 0 \), and linear demand satisfy both Assumption 1a and 1b if \( b > \frac{P'(Q)}{2} \).

Consider a vector of contracts which simultaneously solve asymmetric Nash bargaining solutions between the supplier and each retailer. The disagreement payoff of the upstream firm with retailer \( i \), \( \pi_{U[i]} \), is the supplier’s profit when the negotiation with retailer \( i \) fails, taking the other supply contracts as given:

\[
\pi_{U[i]} = T[i] - C(Q[i]).
\]  

Each retailer’s disagreement payoff with the supplier is zero since there is no alternative supplier. The Nash bargaining problem between the supplier and retailer \( i \) is described by the disagreement points \( (\pi_{U[i]}, 0) \) and the relative (exogenous) bargaining power of the supplier vis-à-vis the retailer, \( \alpha \in (0, 1) \). Since the supplier and retailer \( i \) could share the gains from trade through fixed tariff \( t[i] \), at any Nash bargaining solution, the supplier and retailer \( i \) set \( q[i] \) to maximize their bilateral profits \( \pi_U + \pi_i \) taking the other supply contracts \( (Q[i], T[i]) \) as given. Hence, any asymmetric Nash bargaining equilibrium is a contract equilibrium with a particular distribution of rents (O’Brien and Shaffer, 1992). For a given \( \alpha \), the contract equilibrium \( (q^*, t^*) \) is the simultaneous solution to

\[
\max_{q[i], t[i]} (\pi_U - \pi_{U[i]})^\alpha \pi_i^{1-\alpha} \text{ for } i = 1, \ldots, 2n,
\]  

which is

\[
(q_i^*, t_i^*) = \arg \max_{q[i], t[i]} \left[ t[i] - \left( C(q_i + Q_i^*) - C(Q_i^*) \right) \right]^\alpha \left[ \left( P(q_i + Q^*_i) - c \right) q_i - t_i \right]^{1-\alpha}.
\]

Using the fact that all markets are symmetric, \( q_i^* = q^* \), the first-order conditions of problem (12) yield the optimal quantities,

\[
P'(nq^*) q^* + P(nq^*) = c + C'(2nq^*).
\]

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34 Under this assumption, the second-order conditions of the optimization problems are also satisfied.
35 Observe that this assumption is always satisfied for a convex cost. Moreover, for a concave cost where the concavity of the cost function decreases at larger quantities, that is, \( C''(.) < 0 \), Assumption 1a implies Assumption 1b.
36 Parameter \( \alpha \) captures any exogenous factor which affects the supplier’s relative bargaining power.
The optimal tariffs are used to share the bilateral profits with respect to the relative bargaining power:

\[ t^* = \alpha [P(nq^*) - c]q^* + (1 - \alpha) [C(2nq^*) - C((2n - 1)q^*)] \]. \tag{14}

Each retailer’s equilibrium payoff is equal to its share over the incremental contribution to the industry profit:

\[ \pi^* = (1 - \alpha) \left[ [P(nq^*) - c]q^* - [C(2nq^*) - C((2n - 1)q^*)] \right], \tag{15} \]

and the supplier earns

\[ \pi_U^* = 2n \{ \alpha [P(nq^*) - c]q^* + (1 - \alpha) [C(2nq^*) - C((2n - 1)q^*)] \} - C(2nq^*). \tag{16} \]

To ensure that the supplier’s equilibrium profit is non-negative, we define threshold \( \alpha \) at which \( \pi_U^* = 0 \) (see the Appendix for the explicit definition of \( \alpha \)) and assume that the supplier’s bargaining power is sufficiently high, \( \alpha \geq \alpha \).

As before, we introduce a single large retailer \( L \) as a result of a merger between two independent stores. Let \((q^*_{L}, t^*_{L})\) and \((q^*_{S}, t^*_{S})\) denote, respectively, the total quantity delivered and the tariff paid by the large retailer and a small retailer (given that, by symmetry, all small retailers buy the same quantity, and pay the same tariff) in a contract equilibrium. We therefore have

\[ Q^*_{h[S]} = \frac{q^*_{L}}{2} + (n - 1)q^*_{S}, \quad Q^*_{h[L]} = 2Q^*_{h[S]}. \]

For a given \( \alpha \), the contract equilibrium for a small retailer maximizes the joint profit of the supplier with the small retailer. The optimality condition yields to:

\[ P' \left( Q^*_{h[S]} + q_S \right) q_S + P \left( Q^*_{h[S]} + q_S \right) = c + C' \left( Q^*_{h[S]} + q_S \right). \tag{17} \]

where \( Q^*_{h[S]} = \frac{q^*_{L}}{2} + (n - 2)q^*_{S} \) and \( Q^*_{h[L]} = q^*_{L} + (2n - 3)q^*_{S} \). Similarly, the contract equilibrium for the merged entity leads to the optimality condition:

\[ P' \left( Q^*_{h[L]} + \frac{q_L}{2} \right) + P' \left( Q^*_{h[L]} + \frac{q_L}{2} \right) = c + \mu + C' \left( Q^*_{h[L]} + q_L \right). \tag{18} \]

where the large retailer’s cost is \( c + \mu \), \( Q^*_{h[L]} = (n - 1)q^*_{S} \) and \( Q^*_{h[L]} = 2(n - 1)q^*_{S} \). The simultaneous solution to equations (17) and (18) characterizes the equilibrium quantities for a contract equilibrium after the merger, \( q^*_{L} \) and \( q^*_{S} \).

The optimal tariffs are used to share the bilateral profits with respect to the relative bargaining power. The tariff paid by the large retailer and the tariffs paid by the small retailers are respectively:

\[ ^{37} \text{Compared to the case where the retailers have all the bargaining power, the pre-merger (respectively post-merger) equilibrium quantities are the same. In other words, bargaining power distribution in a contract equilibrium does not affect the equilibrium quantities, but only affect the equilibrium profit sharing between the firms.} \]
\[ t_L^*(\mu) = \alpha [P(Q_h^*) - (c + \mu)] q_L^* + (1 - \alpha) [C(Q^*) - C(Q^* - q_L^*)], \]  
\[ t_S^*(\mu) = \alpha [P(Q_h^*) - c] q_S^* + (1 - \alpha) [C(Q^*) - C(Q^* - q_S^*)]. \]  

The profits of the large retailer and the small retailers are respectively:

\[ \pi_L^*(\mu) = (1 - \alpha) [P(Q_h^*) - (c + \mu)] q_L^* - (c + \mu) q_L^* + (1 - \alpha) [C(Q^*) - C(Q^* - q_L^*)]. \]  
\[ \pi_S^*(\mu) = (1 - \alpha) [P(Q_h^*) - c] q_S^* - c q_S^* + (1 - \alpha) [C(Q^*) - C(Q^* - q_S^*)]. \]  

The profit of the supplier is,

\[ \pi_U^*(\mu) = \alpha [P(Q_h^*) - (c + \mu)] q_L^* + 2 (n - 1) [P(Q_h^*) - c] q_S^* + (1 - \alpha) [C(Q^*) - C(Q^* - q_L^*)] + 2 (n - 1) [C(Q^*) - C(Q^* - q_S^*)] - C(Q^*). \]  

As we did for the pre-merger case, we define threshold \( \alpha \) (in the Appendix) such that for \( \alpha \geq \alpha \) the supplier’s equilibrium profit is positive post-merger.

Hence, when the supplier’s cost is concave, the sufficient condition to ensure that the supplier earns non-negative profit in equilibrium is that

**Assumption 2.** \( \alpha \geq \text{Max}\{\alpha, \alpha\} \).

When a buyer merger has no impact on retail efficiency, i.e., \( \mu = 0 \), it is never profitable due to the concavity of the upstream cost function. More precisely, comparing the large retailer’s profits before and after the merger, \([15]\) and \([20]\), shows that

\[ \pi_L^*(0) = (1 - \alpha) [P(nq^*) - c] 2q^* - [C(2nq^*) - C(2(n - 1)q^*)] < 2\pi^*. \]

**Lemma 4** The equilibrium profit of the merged entity increases in the level of efficiency of the merger, i.e., \( \partial \mu \pi_L^{**} < 0 \).

By Assumption 1a retailers’ quantities are strategic substitutes, so the small retailers sell less as a reaction to a more efficient merger, that is, \( dq_S^*/d\mu > 0 \). Moreover, by Assumption 1b, the large retailer earns more when its rivals sell less,

\[ \frac{\partial \pi_L^{**}}{\partial q_S} = (1 - \alpha) (n - 1) [P'(Q_h^{**}) q_L^{**} - 2 [C'(Q^*) - C'(Q^* - q_L^{**})]] < 0 \]

As a result, the large retailer’s profit decreases in \( \mu \):

\[ \frac{d\pi_L^{**}}{d\mu} = \frac{\partial \pi_L^{**} \partial q_S^{**}}{\partial q_S} d\mu - (1 - \alpha) q_L^{**} < 0. \]

For \( \mu = 0 \), we previously showed that the merger is not profitable, \( \pi_L^{**}(0) < 2\pi^* \). By Lemma
we know that $\partial_{\mu} \pi_{L}^{*} < 0$. Since the pre-merger profit of a merging entity, $2\pi^{*}$, is constant in $\mu$, by continuity of the merged entity’s profit in $\mu$, we show that

**Corollary 4** There exists a threshold $\tilde{\mu} < 0$ such that for any $\mu < \tilde{\mu}$, the buyer merger is profitable.

The corollary implies that when the cost function is concave, the merger is profitable only if it improves efficiency. We moreover show that the merger changes the equilibrium quantities in the same way as an efficient merger in the case of convex cost (see Proposition 1).

**Proposition 3** If a buyer merger is profitable, we have $q_{S}^{* * } < q^{* } < \frac{q_{L}^{* * }}{2}$ and $Q^{* } < Q^{* * }$.

To analyze the merger’s impact on the equilibrium tariff of the small retailers, we define

$$t_{S}(q, x, y) = \alpha [P(x + q) - c]q + (1 - \alpha) [C(y + q) - C(y)],$$

for $q, x, y > 0$. We have $t_{S}(q, x, y)$ is decreasing in $x$ and also in $y$, since $P'(.) < 0$ and $C''(.) < 0$.

Before the merger, the transfer of a small retailer for a given quantity is equal to $t_{S}(q, x^{*}, y^{*})$ for $x^{*} = (n - 1)q^{*}$ and $y^{*} = (2n - 1)q^{*}$. After the merger, for the same quantity, the small retailer pays transfer $t_{S}(q, x^{**}, y^{**})$ for $x^{**} = Q_{h}^{* * } - q_{S}^{* * }$ and $y^{**} = Q^{* * } - q_{S}^{* * }$.

Since $Q_{h}^{* * } > Q_{h}^{*}$, $\frac{q_{S}^{* * }}{2} > q^{*}$ and $q_{S}^{* * } < q^{*}$ (from Proposition 3), we get $x^{**} = Q_{h}^{* * } - q_{S}^{* * } > Q_{h}^{* } - q^{*} = (n - 1)q^{*} = x^{*}$ and $y^{**} = Q^{* * } - q_{S}^{* * } > (2n - 1)q^{*} = y^{*}$. We thus show that $t_{S}(q, x^{**}, y^{**}) < t_{S}(q, x^{*}, y^{*})$ for any $q$, that is, the small retailer pays less post-merger.

**Lemma 5** As a result of a profitable buyer merger, each small retailer pays a lower average tariff for a given volume of sales, that is, there are anti-waterbed effects.

The lemma shows that the efficient merger increases the profit of a small retailer by reducing its average tariff. On the other hand, the efficient merger decreases each small retailer’s revenue by lowering the retail price in each local market. Hence, the net impact of the merger on a small retailer’s profit is not straightforward. Under Assumption 1b, we show that the negative effect of the merger dominates the positive effect:

**Proposition 4** As a result of a profitable buyer merger, each small retailer earns less for a given volume of sales.

### 6 Discussion and Conclusions

This paper analyzes the welfare implications of efficient or inefficient buyer mergers, which are mergers between retailers from different markets. More precisely, we focus on the interaction between merger efficiency and buyer power concerns in a setup where one manufacturer with a non-linear cost function sells its product to two locally competitive retail markets.
The European Commission has identified two potential concerns arising from buyer power: first, lower purchasing costs for powerful buyers might not be passed on to final consumers; second, there might be waterbed effects, that is, lower tariffs for powerful buyers might be at the expense of higher tariffs for less powerful buyers.

Our paper supports the first concern if the buyer merger has no efficiency effect. In this case, we formally show that even if a larger buyer obtains size discounts from the supplier, there is no pass-on of lower purchasing costs to consumer prices when supply contracts are non-linear. With regard to the second concern, we find that there are waterbed (respectively, anti-waterbed) effects if a buyer merger increases (respectively, decreases) the retail efficiency of merging parties and the upstream cost function is convex. When the cost function is concave, there are only anti-waterbed effects. The merger’s effect on efficiency is the only determinant of the implications for the consumer surplus. In each retail market, the merger decreases the final price if and only if the merger improves the efficiency of the merging parties, regardless of its impact on the average tariff of small retailers. Different from the literature, in our paper if a waterbed effect exists, it always increases the consumer surplus.

Besides the results summarized above, our analysis would have interesting implications for retail markets supplied by the same manufacturer and where the merged entity is not active, which we refer to as independent markets. When the upstream cost is convex, an efficient merger would lead to a higher retail price in each independent market through increasing the average tariff of each retailer in those markets. This means that, by contrast to the markets where the merged entity is active, waterbed effects lead to a higher retail price in each independent market. The opposite result holds if the merger is inefficient, in which case the merger decreases the price of each independent market due to anti-waterbed effects. The same mechanism applies when the supplier has a concave cost, in which case anti-waterbed effects originate from the efficiency of the merger.

We consider the case where quantities are strategic substitutes, since this was the most interesting scenario in our framework. In this case a small retailer sells less as a reaction to a more efficient rival (a store of the merged entity). When the upstream cost function is convex, this further deteriorates the bargaining position of a small retailer and so increases its average tariff, lowering its quantity even further. Hence, in this case it is not straightforward whether this negative effect of an efficient merger is dominated by the quantity expansion of the merged entity. However, if quantities were strategic complements, the impact of an efficient merger on the final prices would be straightforward since each small retailer sells more when its rival is more efficient.

Our work could be extended to deal with the long-run implications of buyer power on upstream investment, like in Inderst and Wey (2007), and Battigalli et al. (2007). Another promising research avenue is allowing for upstream competition (see, for instance, de Fontenay and Gans, 2007).
APPENDIX

Proof of Proposition 1:
We do the proof for \( \mu > 0 \). A symmetric argument would show the claim for \( \mu < 0 \).

**First step: \( Q^{**} < Q^* \)**
Before the merger, the first-order condition for equilibrium quantity \( q^* \) is given by

\[
P'(Q^*_h)q^* + P(Q^*_h) = c + C'(2Q^*_h).
\]

where \( Q^*_h = nq^* \).

Summing the condition for all retailers in a local market gives:

\[
P'(Q^*_h)Q^*_h + nP(Q^*_h) = nc + nC'(Q^*_h).
\]

(22)

After the merger, equilibrium quantities for the large firm (\( L \)), and a small firm (\( S \)) are respectively given by the first-order conditions (see (8) and (9)):

\[
P'(Q^{**}_h)\frac{q^{**}_L}{2} + P(Q^{**}_h) = c + \mu + C'(2Q^{**}_h),
\]

\[
P'(Q^{**}_h)q^{**}_S + P(Q^{**}_h) = c + C'(2Q^{**}_h).
\]

(23)

Summing the conditions for all retailers in a local market, we obtain

\[
P'(Q^{**}_h)Q^{**}_h + nP(Q^{**}_h) = nc + \mu + nC'(2Q^{**}_h).
\]

Since \( \mu > 0 \), we have \( nc + \mu > nc \) and comparing expressions (22) and (23), we then obtain:

\[
P'(Q^{**}_h)Q^{**}_h + nP(Q^{**}_h) - nC'(2Q^{**}_h) > P'(Q^*_h)Q^*_h + nP(Q^*_h) - nC'(2Q^*_h).
\]

From our assumptions, \( P''(Q)Q + P'(Q) < 0 \) and \( C''(Q) > 0 \), we deduce that \( P''(Q)Q + (n + 1)P'(Q) - 2nC''(2Q) < 0 \), which implies that \( P'(Q)Q + nP(Q) - nC''(2Q) \) is decreasing in \( Q \), we therefore have \( Q^{**}_h < Q^*_h \) and, \( Q^{**} < Q^* \) by multiplying by 2.

**Second step: \( q^* < q^{**}_S \)**
We show the claim by contradiction:

From our assumptions, \( P''(Q)Q + P'(Q) < 0 \) and \( P'(Q) < 0 \), we deduce that \( P''(Q)y + P'(Q) < 0 \) for any \( 0 < y < Q \) (for \( P''(Q) < 0 \), \( P''(Q)y + P'(Q) < 0 \) since \( P'(Q) < 0 \), and for \( P''(Q) > 0 \), \( P''(Q)y + P'(Q) < 0 \) since \( P''(Q)y + P'(Q) < P''(Q)Q + P'(Q) \) for any \( 0 < y < Q \) and \( P''(Q)Q + P'(Q) < 0 \) which implies that \( P'(Q)y + P(Q) \) is decreasing in \( Q \). Moreover, since \( C''(Q) > 0 \), we have \( P'(Q)y + P(Q) - C'(2Q) \) which is decreasing in \( Q \).
From \( Q_{h}^{**} < Q_{h}^{*} \), we then obtain:

\[
P'(Q_{h}^{**})y + P(Q_{h}^{**}) - C'(2Q_{h}^{**}) > P'(Q_{h}^{*})y + P(Q_{h}^{*}) - C'(2Q_{h}^{*}) \]

Suppose now that, \( q^{*} > q_{S}^{**} \), the result is that:

\[
P'(Q_{h}^{**})q_{S}^{*} + P(Q_{h}^{**}) - C'(2Q_{h}^{**}) > P'(Q_{h}^{*})q^{*} + P(Q_{h}^{*}) - C'(2Q_{h}^{*})
\]

since \( P'(Q)y + P(Q) - C'(2Q) \) is decreasing in \( y \) (\( P'(Q) < 0 \)).

Using the following first-order conditions:

\[
P'(Q_{h}^{*})q^{*} + P(Q_{h}^{*}) = c + C'(2Q_{h}^{*}), \quad \text{and}
\]

\[
P'(Q_{h}^{**})q_{S}^{***} + P(Q_{h}^{**}) = c + C'(2Q_{h}^{**}),
\]

we obtain

\[
P'(Q_{h}^{**})q_{S}^{***} + P(Q_{h}^{**}) - C'(2Q_{h}^{**}) = P'(Q_{h}^{*})q^{*} + P(Q_{h}^{*}) - C'(2Q_{h}^{*}),
\]

that is, we reach a result that contradicts the inequality at the beginning. Therefore, by contradiction we show \( q^{*} < q_{S}^{**} \).

**Third step:** \( \frac{Q_{h}^{*}}{2} < q^{*} \)

Inequalities \( Q_{h}^{**} < Q_{h}^{*} \) (from the first step) and \( q^{*} < q_{S}^{**} \) (from the second step) imply that \( \frac{Q_{h}^{*}}{2} < q^{*} \).

**Proof of Proposition 3**

We follow the same methodology as in the convex case (see the proof of the Proposition 1). The proof is now for \( \mu < 0 \).

**First step:** \( Q^{**} > Q^{*} \)

Before the merger, the first-order condition for equilibrium quantity \( q^{*} \) is given by

\[
P'(Q_{h}^{*})q^{*} + P(Q_{h}^{*}) = c + C'(2Q_{h}^{*}),
\]

where \( Q_{h}^{*} = nq^{*} \).

Summing the condition for all retailers in a local market give us:

\[
P'(Q_{h}^{*})Q_{h}^{*} + nP(Q_{h}^{*}) = nc + nC'(Q_{h}^{*}). \quad (24)
\]

After the merger, the equilibrium quantities for the large firm (L), and a small firm (S) are respectively given by the first-order conditions (see [18] and [17]):

\[
P'(Q_{h}^{**})q_{L}^{**} + P(Q_{h}^{**}) = c + \mu + C'(2Q_{h}^{**}),
\]

\[
P'(Q_{h}^{**})q_{S}^{**} + P(Q_{h}^{**}) = c + C'(2Q_{h}^{**}).
\]
Summing the conditions for all retailers in a local market, we obtain

\[ P'(Q_h^{**}) Q_h^{**} + nP(Q_h^{**}) = nc + \mu + nC'(2Q_h^{**}). \] (25)

Since \( \mu < 0 \), we have \( nc + \mu < nc \) and comparing expressions (24) and (25), we show that

\[ P'(Q_h^{**}) Q_h^{**} + nP(Q_h^{**}) - nC'(2Q_h^{**}) < P'(Q_h^{*}) Q_h^{*} + nP(Q_h^{*}) - nC'(2Q_h^{*}). \] (26)

From Assumption 1a for \( q = \frac{Q}{n} \), we have \( P''(Q) \frac{Q}{n} + P'(Q) - 2C''(2Q) < 0 \). Multiplying the both sides by \( n \) proves that \( P''(Q) Q + nP'(Q) - 2nC''(2Q) < 0 \) and therefore

\[ P''(Q) Q + (n + 1)P'(Q) - 2nC''(2Q) < 0, \]

since \( P'(Q) < 0 \). This implies that \( P'(Q) Q + nP'(Q) - nC'(2Q) \) is decreasing in \( Q \). As a result, inequality (26) proves that \( Q_h^{**} > Q_h^{*} \) and \( Q^{**} > Q^{*} \).

**Second step:** \( q^{*} > q^{**} \)

Assumption 1a, \( P''(Q) Q + P'(Q) - 2C''(2Q) < 0 \) for any \( q \leq Q \), and \( P'(Q) < 0 \) ensures that the function \( P'(Q) Q + P(Q) - C'(2Q) \) is decreasing in \( Q \).

Since \( Q_h^{**} > Q_h^{*} \) (from the first step), we have

\[ P'(Q_h^{**}) Q_h^{**} + P(Q_h^{**}) - C'(2Q_h^{**}) < P'(Q_h^{*}) Q_h^{*} + P(Q_h^{*}) - C'(2Q_h^{*}). \] (27)

Using the first-order condition of a small retailer before and then after the merger, respectively,

\[ P'(Q_h^{*}) q^{*} + P(Q_h^{*}) = c + C'(2Q_h^{*}), \]
\[ P'(Q_h^{**}) q^{**} + P(Q_h^{**}) = c + C'(2Q_h^{**}), \]

we obtain

\[ P'(Q_h^{**}) q^{**} + P(Q_h^{**}) - C'(2Q_h^{**}) = P'(Q_h^{*}) q^{*} + P(Q_h^{*}) - C'(2Q_h^{*}), \]

The latter together with inequality (27) imply that \( q^{*} > q^{**} \), since \( P'(.) < 0 \).

**Third step:** \( \frac{q^{**}}{2} > q^{*} \)

Inequalities \( Q_h^{**} > Q_h^{*} \) (from the first step) and \( q^{*} > q^{**} \) (from the second step) imply that \( \frac{q^{**}}{2} > q^{*} \).

**Proof of Proposition 4**

We define function

\[ \pi_S(q, x, y) = (1 - \alpha)(([P(x + q) - c] q - [C(y + q) - C(y)])], \]

for \( q, x, y > 0 \). We have \( \pi_S(q, x, y) \) is decreasing in \( x \), but is increasing in \( y \), since \( P'(.) < 0 \) and
\(C''(.) < 0\). Before the merger, for a given quantity \(q\), the small retailer earns \(\pi_S(q, x^*, y^*)\) for \(x^* = (n - 1)q^*\) and \(y^* = (2n - 1)q^*\), and after the merger, it obtains \(\pi_S(q, x^{**}, y^{**})\) for \(x^{**} = Q_h^* - q_S^*\) and \(y^{**} = 2Q_h^* - q_S^*\). Since we have \(x^{**} > x^*,\ y^{**} > y^*\), \(\pi_S(q, x, y)\) is decreasing in \(x\), but is increasing in \(y\), the effect of the merger on the profit of the small retailer is not straightforward.

To identify whether the small retailer’s profit decreases post-merger, we study the sign of \(\frac{d\pi_S(q,x^{**},y^{**})}{d\mu}\):

\[
\frac{d\pi_S(q, x^{**}, y^{**})}{d\mu} = (1 - \alpha) \left[ P'(x^{**} + q) q \frac{dx^{**}}{d\mu} - \left[ C'(y^{**} + q) - C'(y^{**}) \right] \frac{dy^{**}}{d\mu} \right].
\]

Our claim is that \(\frac{d\pi_S(q,x^{**},y^{**})}{d\mu} > 0\) and therefore the small retailer’s profit decreases post-merger.

By definition of the equilibrium quantities, we have \(y^{**} = 2x^{**} + q_S^{**}\). Using this, we rewrite the latter derivative

\[
\frac{d\pi_S(q, x^{**}, y^{**})}{d\mu} = (1 - \alpha) \left[ P'(x^{**} + q) q \frac{dx^{**}}{d\mu} - \left[ C'(y^{**} + q) - C'(y^{**}) \right] \frac{dy^{**}}{d\mu} \right].
\]

From Proposition 3, we have \(\frac{dx^{**}}{d\mu} < 0\) and \(\frac{dy^{**}}{d\mu} > 0\). Moreover, \(C'(y^{**} + q) - C'(y^{**}) < 0\) by the concavity of the cost. As a result, \(\frac{d\pi_S(q,x^{**},y^{**})}{d\mu} > 0\) if \(P'(x^{**} + q) - 2 \left[ \frac{C'(y^{**} + q) - C'(y^{**})}{q} \right] < 0\), which is the case by Assumption 1b since \(y^{**} + q = 2(x^{**} + q)\).

**Thresholds on \(\alpha\)**

We define the thresholds on \(\alpha\) above which the supplier’s equilibrium profit is positive. Before the merger, this threshold is \(\alpha\) such that

\[
\alpha = \frac{\frac{C(2nq^* - C(2nq^* - C((2n-1)q^*))}{P(nq^*) - C(2nq^* - C((2n-1)q^*))}}{\frac{C(2nq^*)}{P(nq^*) - C(2nq^* - C((2n-1)q^*))}}.
\]

Since the industry profit is non-negative in equilibrium, \(P(nq^*) - c \geq \frac{C(2nq^*)}{2nq^*}\), and the cost function is concave, \(\frac{C(2nq^*)}{2nq^*} > \frac{C(2nq^*) - C((2n-1)q^*)}{q^*}\), both the numerator and denominator are positive, and the numerator is lower. Hence, \(\alpha\) belongs to \((0, 1)\). Moreover, a more concave cost function increases the numerator of the latter fraction and decreases its denominator. As a result, \(\alpha\) increases in the concavity of the cost function for a given \(q^*\).

After the merger, this threshold is \(\alpha\) such that

\[
\alpha = \left( \frac{C(2Q_h^* - [(C(2Q_h^* - C(2Q_h^* - q_S^*)) + 2(n - 1)(C(2Q_h^* - C(2Q_h^* - q_S^*))])]}{(P(Q_h^*) - c)2Q_h^* - \mu q_L^* - [(C(2Q_h^* - C(2Q_h^* - q_L^*)) + 2(n - 1)(C(2Q_h^* - C(2Q_h^* - q_S^*))])} \right).
\]

Since the industry profit is strictly positive, \((P(Q_h^*) - c)2Q_h^* - \mu q_L^* - C(2Q_h^*) > 0\), and the numerator is positive by the concavity of the cost function, \(\alpha\) is between 0 and 1.
References


