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Research Note on "International Consumption Risk Sharing and Monetary Policy"

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1 Introduction

This model analyzes the impact of monetary policy on international consumption risk sharing. To this end, the setup by Ghironi and Stebunovs (2008) is extended in two dimensions. First, to allow for international portfolio choices, cross-border trade of home and foreign equity is brought in. Second, to assign a non-trivial role to monetary policy, nominal price rigidities are introduced as in Bilbiie, Ghironi, and Melitz (2007).

The model features incomplete goods as well as incomplete asset markets. Frictions in goods markets are given by variable iceberg-type costs when shipping goods. Financial markets are incomplete as the set of available assets cannot span all the uncertainty induced by potential shock scenarios. In addition, financial markets are not fully integrated as engagement in asset markets is costly.

This research note gives technical details on the solution of the model. In the following section, the basic setup of the model as well as the main variables and equilibrium conditions of the model are briefly summarized. Section 3 presents the steady state.

2 Model summary

There are two countries, home and foreign. In each country, there is a unit mass of identical households. The households receive income from inelastically supplied labor as well

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as from engagement in financial markets. Portfolio holdings consist of a bond that is traded only domestically and home and foreign equity that may also be traded across borders. In each country, domestic equity is bundled in a mutual fund and issued in the respective market.

When holding home and foreign equity, households have to pay quadratic portfolio costs to financial intermediaries as in Ghironi, Lee, and Rebucci (2007). The purpose of these costs is twofold. First, they pin down the steady state portfolio allocation and solve the problem of indeterminacy of asset holdings in a non-stochastic environment. Second, they keep the response of the endogenous variables to transitory shocks stationary as they link portfolio positions to consumption dynamics. Since bonds are traded only domestically and holdings therefore are zero in the aggregate, there are no costs on bonds.

In both countries, there is a continuum of monopolistically competitive firms, each producing one differentiated good. At the beginning of each period, there are prospective entrants in both countries. When entering the home (foreign) market, firms face a sunk entry cost $f_{E,t}$ ($f^*_{E,t}$). As in Ghironi and Stebunovs (2008) and Bilbiie, Ghironi, and Melitz (2007), these entry costs result in an endogenously evolving number of firms. To keep the number of firms operating in both countries well-behaved, any firm, already operating in the market or just entering, may be hit by a "death" shock that forces market exit and occurs with probability δ.

Exporting involves iceberg-type trade costs τ (τ^*). So a fraction $\tau - 1$ ($\tau^* - 1$) gets lost when shipping goods. In addition to this goods market friction, households in each country exhibit home bias in consumption which is governed by the parameter $\phi > 0.5$. Hence, even though all goods are traded, despite melting iceberg costs, households in each country may decide to consume less of foreign brands.

There are nominal rigidities in both countries. When changing their prices, firms have to pay Rotemberg-style adjustment costs. As in Bilbiie, Ghironi, and Melitz (2007), these costs are interpreted as firms' marketing expenses when adjusting prices for their products. In this setup, this leads to heterogeneous markups of firms in each country with respect to the destination market of the product.

Table 1 presents the notation of key variables and parameters. Equilibrium conditions of the aggregate model for the home and foreign country are summarized Table 2.

Table 1: Notation summary

Endogenous variables	
C_t, C_t^*	Consumption
w_t, w_t^*	Wage rate
$\rho_{D,t}$, $\rho_{D,t}^*$	Price for domestic goods
$\rho_{X,t}$, $\rho_{X,t}^*$	Price for export goods
$\mu_{D,t}$, $\mu_{D,t}^*$	Markup for domestic goods
$\mu_{X,t}$, $\mu_{X,t}^*$	Markup for export goods
N_t, N_t^*	Number of firms
$N_{E,t}$, $N_{E,t}^*$	Number of entrants
d_t, d_t^*	Total profits
$d_{D,t}$, $d_{D,t}^*$	Profits from domestic activity
$d_{X,t}$, $d_{X,t}^*$	Profits from exporting
v_t, v_t^*	Firm value
$R_{B,t}$, $R_{B,t}^*$	Return on riskless bond
$R_{H,t}$, $R_{H,t}^*$	Return on home equity
$R_{F,t}$, $R_{F,t}^*$	Return on foreign equity
$\mathscr{P}_t$, $\mathscr{P}_t^*$	Financial wealth
$\alpha_{H,t}$, $\alpha_{H,t}^*$	Portfolio share on home equity
$\alpha_{F,t}$, $\alpha_{F,t}^*$	Portfolio share on foreign equity
$x_{H,t}$, $x_{H,t}^*$	Share in the home mutual fund
$x_{FH,t}$, $x_{F,t}^*$	Share in the foreign mutual fund
Q_t	Real exchange rate
Parameters	
β	Subjective time preference factor
γ	Relative risk aversion
θ	Elasticity of substitution
ϕ	Home bias in consumption
κ	Degree of price rigidity
τ, τ^*	Trade costs
$f_{E,t}$, $f_{E,t}^*$	Sunk entry costs
δ	Probability of market exit
$\xi_{H,t}$, $\xi_{H,t}^*$	Home financial market frictions
$\xi_{F,t}$, $\xi_{F,t}^*$	Foreign financial market frictions
ε_t, ε_t^*	Monetary policy shock
L, L^*	Labor supply
Z_t, Z_t^*	Aggregate labor productivity

Notes: All endogenous variables are defined in real terms. Foreign variables are denoted by an asterisk. Parameters are identical across countries if only one parameter is specified.

Table 2: Model Summary

Price indexes	$\phi N_t \left(\rho_{D,t}\right)^{1-\theta} + (1-\phi) N_t^* \left(\rho_{X,t}^*\right)^{1-\theta} = 1$ $\phi N_t^* \left(\rho_{D,t}^*\right)^{1-\theta} + (1-\phi) N_t \left(\rho_{X,t}\right)^{1-\theta} = 1$
Profits	$d_t = \frac{\phi}{\theta} \left(\rho_{D,t}\right)^{1-\theta} C_t + \frac{1-\phi}{\theta} Q_t \left(\rho_{X,t}\right)^{1-\theta} C_t^*$ $d_t^* = \frac{\phi}{\theta} \left(\rho_{D,t}^*\right)^{1-\theta} C_t^* + \frac{1-\phi}{\theta} Q_t^{-1} \left(\rho_{X,t}^*\right)^{1-\theta} C_t$
Domestic prices	$\rho_{D,t} = \mu_{D,t} \frac{w_t}{Z_t}$ $\rho_{D,t}^* = \mu_{D,t}^* \frac{w_t^*}{Z_t^*}$
Export prices	$\rho_{X,t} = \tau Q_t^{-1} \mu_{X,t} \frac{w_t}{Z_t}$ $\rho_{X,t}^* = \tau^* Q_t \mu_{X,t}^* \frac{w_t^*}{Z_t^*}$
Number of firms	$N_t = (1-\delta)\left(N_{t-1} + N_{E,t-1}\right)$ $N_t^* = (1-\delta)\left(N_{t-1}^* + N_{E,t-1}^*\right)$
Firm entry	$v_t = w_t \frac{f_{E,t}}{Z_t}$ $v_t^* = w_t^* \frac{f_{E,t}^*}{Z_t^*}$
Euler equations for bonds	$1 = E_t \left[\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \left(R_{B,t+1}\right)\right]$ $1 = E_t \left[\beta \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma} \left(R_{B,t+1}^*\right)\right]$
Euler equations for home equity	$1 = E_t \left[\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{R_{H,t+1}}{\left(1+\xi_{H,t}\alpha_{H,t+1}\mathscr{P}_{t+1}\right)}\right]$ $1 = E_t \left[\beta \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma} \frac{R_{H,t+1}^*}{\left(1+\xi_{H,t}^*\alpha_{H,t+1}^*\mathscr{P}_{t+1}^*\right)}\right]$
Euler equations for foreign equity	$1 = E_t \left[\beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \frac{R_{F,t+1}}{\left(1+\xi_{F,t}\alpha_{F,t+1}\mathscr{P}_{t+1}\right)}\right]$ $1 = E_t \left[\beta \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\gamma} \frac{R_{F,t+1}^*}{\left(1+\xi_{F,t}^*\alpha_{F,t+1}^*\mathscr{P}_{t+1}^*\right)}\right]$

Continued.

Table 2 continued.

Return on home equity	$R_{H,t+1} = (1-\delta)\frac{v_{t+1}+d_{t+1}}{v_t}$ $R^*_{H,t+1} = (1-\delta)\frac{Q_t}{Q_{t+1}}\frac{v_{t+1}+d_{t+1}}{v_t}$
Return on foreign equity	$R_{F,t+1} = (1-\delta)\frac{Q_{t+1}}{Q_t}\frac{v^*_{t+1}+d^*_{t+1}}{v^*_t}$ $R^*_{F,t+1} = (1-\delta)\frac{v^*_{t+1}+d^*_{t+1}}{v^*_t}$
Net foreign assets	$\alpha_{F,t+1}\mathscr{P}_{t+1} - Q_t\alpha^*_{H,t+1}\mathscr{P}^*_{t+1} = -\alpha_{H,t+1}\mathscr{P}_{t+1}$ $+Q_t\alpha^*_{F,t+1}\mathscr{P}^*_{t+1} + (\alpha_{H,t}R_{H,t} + (1-\alpha_{H,t})R_{F,t})\mathscr{P}_t$ $-Q_t\left(\alpha^*_{F,t}R^*_{F,t} + (1-\alpha^*_{F,t})R^*_{H,t}\right)\mathscr{P}^*_t$ $+w_tL - Q_tw^*_tL^* - C_t + Q_tC^*_t$
Labor market clearing	$L = \frac{\theta-1}{w_t}N_td + N_{E,t}\frac{f_{E,t}}{Z_t}$ $L^* = \frac{\theta-1}{w^*_t}N^*_td^* + N^*_{E,t}\frac{f^*_{E,t}}{Z^*_t}$

Notes: See Table 1.

The system comprises 25 endogenous variables: C_t, C^*_t, w_t, w^*_t, $\rho_{D,t}$, $\rho^*_{D,t}$, $\rho_{X,t}$, $\rho^*_{X,t}$, N_t, N^*_t, $N_{E,t}$, $N^*_{E,t}$, v_t, v^*_t, d_t, d^*_t, $R_{B,t}$, $R^*_{B,t}$, $\alpha_{H,t}$, $\alpha_{F,t}$, $\alpha^*_{F,t}$, $\alpha^*_{H,t}$, $\mathscr{P}_t$, $\mathscr{P}^*_t$, and Q_t. Ten of these variables are predetermined as of time t: the number of firms, N_t and N^*_t, the (gross) return on domestic bonds, $R_{B,t}$ and $R^*_{B,t}$, portfolio shares, $\alpha_{H,t}$, $\alpha_{F,t}$, $\alpha^*_{F,t}$ and $\alpha^*_{H,t}$, and financial wealth, $\mathscr{P}_t$ and $\mathscr{P}^*_t$.

3 Steady state

The model is solved around a symmetric non-stochastic steady state, where $f_E = f^*_E$, $\tau = \tau^*$, $L = L^*$, and $Z = Z^* = 1$. It follows that $Q = 1$. Steady state variables are denoted without time subscripts. Symmetry of the steady state implies that $C = C^*$, $w = w^*$, $N = N^*$, $N_E = N^*_E$, $v = v^*$, $d = d^*$, $R_B = R^*_B$, and $\mathscr{P} = \mathscr{P}^*$.

Prices and profits

In steady state, prices for domestic and export goods at home and abroad are given by

$$\rho_D = \mu_D w, \text{ and} \tag{1}$$

$$\rho_X = \tau \mu_X w. \tag{2}$$

Note that in a non-stochastic steady state, markups for domestic and export goods are identical and determined solely by the elasticity of substitution across goods: $\mu_D = \mu_X = \mu = \frac{\theta}{\theta-1}$, so that

$$\rho_X = \tau \rho_D. \tag{3}$$

The aggregate price index for the home country is given by

$$\phi N \rho_D^{1-\theta} + (1-\phi) N^* \left(\rho_X^*\right)^{1-\theta} = 1. \tag{4}$$

The symmetry assumption together with (3) imply:

$$N = \frac{\rho_D^{\theta-1}}{\phi + (1-\phi)\tau^{1-\theta}}. \tag{5}$$

Profits from domestic and export activity can be written as

$$d_D = \frac{\phi}{\theta} \rho_D^{1-\theta} C \tag{6}$$

$$d_X = \frac{1-\phi}{\theta} \rho_X^{1-\theta} C^*. \tag{7}$$

Using again (3), profits from exporting relative to profits realized at home are

$$\frac{d_X}{d_D} = \frac{1-\phi}{\phi} \tau^{1-\theta}. \tag{8}$$

As intuition would suggest, profits from activity abroad are smaller compared to domestic profits the larger are consumption home bias and iceberg costs, as $\theta > 1$. Total profits can then be expressed as

$$d = d_D + d_X = d_D \left(1 + \frac{1-\phi}{\phi} \tau^{1-\theta}\right). \tag{9}$$

Portfolio shares and returns

The solution for portfolio shares and returns follows the same steps as in Blank (2009). Euler equations for equity holdings of the home and foreign household are given by

$$v = \frac{\beta(1-\delta)}{1-\beta(1-\delta)+\xi_H \alpha_H \mathscr{P}} d \tag{10}$$

$$v = \frac{\beta(1-\delta)}{1-\beta(1-\delta)+\xi_H^* \alpha_H^* \mathscr{P}^*} d \tag{11}$$

for home equity and

$$v^* = \frac{\beta(1-\delta)}{1-\beta(1-\delta)+\xi_F \alpha_F \mathscr{P}} d^* \tag{12}$$

$$v^* = \frac{\beta(1-\delta)}{1-\beta(1-\delta)+\xi_F^* \alpha_F^* \mathscr{P}^*} d^* \tag{13}$$

for foreign equity. (10) together with (11) and (12) together with (13) imply

$$\frac{\xi_H}{\xi_H^*} \alpha_H = \alpha_H^* \tag{14}$$

$$\frac{\xi_F^*}{\xi_F} \alpha_F^* = \alpha_F . \tag{15}$$

Shares of home and foreign equity holdings sum to one (as bond holdings are zero in the aggregate):

$$\alpha_H + \alpha_F = 1 \tag{16}$$

$$\alpha_H^* + \alpha_F^* = 1. \tag{17}$$

Portfolio shares by home and foreign households are then given by

$$\alpha_H = \frac{\xi_H^*}{\xi_H + \xi_H^*} = \frac{\xi_F}{\xi_F^* + \xi_F}, \qquad \alpha_F = \frac{\xi_F^*}{\xi_F^* + \xi_F} = \frac{\xi_H}{\xi_H + \xi_H^*}, \tag{18}$$

$$\alpha_H^* = \frac{\xi_H}{\xi_H + \xi_H^*} = \frac{\xi_F^*}{\xi_F^* + \xi_F}, \qquad \alpha_F^* = \frac{\xi_F}{\xi_F^* + \xi_F} = \frac{\xi_H^*}{\xi_H + \xi_H^*}. \tag{19}$$

Note that households in both countries face the same *relative* access to financial markets, as $\xi_H = \xi_F^*$ and $\xi_F = \xi_H^*$. If portfolio costs for foreign households for holdings of foreign equity are zero, $\xi_H^* = 0$, then home holdings of home equity are zero, irrespective of the

size of friction for holding home equity, except for the special case where both scaling parameters are zero, $\xi_H = \xi_H^* = 0$, which leaves the home portfolio share indeterminate.

As in Ghironi, Lee, and Rebucci (2007), the financial fees help to pin down portfolio shares exogenously in steady state. However, the value of each asset position held in the portfolio is endogenous. In addition, even though net foreign assets are zero in steady state due to symmetry, gross foreign assets need not.

Financial wealth at home and abroad equals each other in steady state, $\mathscr{P} = \mathscr{P}^*$, and sum to world equity market capitalization

$$\mathscr{P} + \mathscr{P}^* = v\left(N + N_E\right) + v^*\left(N^* + N_E^*\right). \tag{20}$$

The positions held by home and foreign households in the home and foreign mutual funds are given by

$$\alpha_H \mathscr{P} = x_H\left(N + N_E\right)v, \qquad \alpha_F \mathscr{P} = x_F\left(N^* + N_E^*\right)v^*, \tag{21}$$

$$\alpha_H^* \mathscr{P}^* = x_H^*\left(N + N_E\right)v, \qquad \alpha_F^* \mathscr{P}^* = x_F^*\left(N^* + N_E^*\right)v^*, \tag{22}$$

so that portfolio shares of each asset are equal to the respective shares held in the mutual funds

$$\alpha_H = x_H, \qquad \alpha_F = x_F, \tag{23}$$

$$\alpha_H^* = x_H^*, \qquad \alpha_F^* = x_F^*. \tag{24}$$

The evolution of the number of firms in steady state is given by

$$N_E = \frac{\delta}{1-\delta} N. \tag{25}$$

equation (20) implies that financial wealth can be expressed in terms of the entry condition, $v = wf_E$, and the number of firms

$$\mathscr{P} = v\left(N + N_E\right) = \frac{wf_E}{1-\delta} N \tag{26}$$

The returns on home and foreign equity are equal in steady state and identical to the

return on the portfolio R

$$R_H = R_F = R = (1-\delta)\left(\frac{d}{v}+1\right). \tag{27}$$

Using the Euler equation (10), the return is given by

$$R = \beta^{-1}\left(1+\frac{\xi_H\xi_H^*}{\xi_H+\xi_H^*}\mathscr{P}\right). \tag{28}$$

Because both countries face the same relative access to financial markets, it must hold that

$$\frac{\xi_H\xi_H^*}{\xi_H+\xi_H^*} = \frac{\xi_F\xi_F^*}{\xi_F+\xi_F^*}.$$

Note that absent quadratic financial fees, no arbitrage would leave the portfolio composition indeterminate in a non-stochastic steady state. In this setup, however, these fees pin down the steady state portfolio shares as shown above. The steady state return on the riskless bond equals the return on equity net of transaction costs

$$R_B = \beta^{-1}.$$

Labor market and consumption

The equilibrium on the labor market in steady state is determined by labor demand of firms already operation in the domestic market and the market abroad, as well as labor demand of new entrants

$$\begin{aligned} L &= \frac{\theta-1}{w}Nd_D + \frac{\theta-1}{w}Nd_X + N_E f_E \\ &= N\frac{\theta-1}{w}d + N\frac{\delta}{1-\delta}f_E, \end{aligned} \tag{29}$$

using the steady state evolution of the number of firms given in (25). Multiplying both sides by the real wage rate and substitute the entry condition, $v = f_E w$, in the above expression yields

$$wL = (\theta-1)Nd + \frac{\delta}{1-\delta}Nv, \tag{30}$$

As aggregate bond holdings are zero, home household's budget constraint in steady

state can by written as

$$C = (R-1)\mathscr{P} + wL. \tag{31}$$

Now substituting equation (28) for the portfolio return, equation (26) for the portfolio and equation (30) for labor income into the budget constraint yields

$$C = \theta N d. \tag{32}$$

Model dynamics

The model is log-linearized around the steady state levels of the endogenous variables derived in the previous subsections. Due to the financial fees, the model may be centered around setups featuring different degrees of financial market integration. Several scenarios may be analyzed: technology shocks (changes in Z and Z^*), shocks to monetary policy (changes in ε and ε^*), changes in market regulation (changes in f_E and f_E^*), and shocks to financial market access (changes in ξ_H, ξ_F, ξ_F^*, and ξ_H^*).

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