Optimal Monetary Policy in a Monetary Union with Housing and Credit Market Heterogeneity

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Abstract
This paper develops a two–country DSGE model for a monetary union in which each country is populated by two types of households – savers and borrowers – and two types of production sectors – a consumption goods sector and a housing sector. Households trade nominal private debt in equilibrium, with the borrowers being subject to a collateral constraint, which is tied to the value of the stock of housing. The analysis focuses on the implications of housing and credit market heterogeneities for the design of optimal monetary policy of a common central bank. The results indicate that as long as the only heterogeneity between the two countries is the degree of nominal rigidity, the Benigno (2004) result according to which the common central bank puts a higher weight on stabilizing inflation in the country with a higher degree of nominal rigidity continues to hold. However, due to the introduction of collateralized household debt and borrowing constraints the effects of cross–country disparities in the degree of price rigidities on the volatility of inflation are amplified. While the volatility of inflation in the economy with a higher degree of price rigidity is almost as low as in the model without borrowers, inflation in the economy with more flexible prices becomes much more volatile. As a result, under optimal policy the central bank allows union–wide inflation to fluctuate more in response to productivity shocks than in an economy without credit constrained borrowers. In addition we find that housing and credit market heterogeneities have an impact on the central bank’s goal to stabilize inflation. Finally, the paper shows that even in the presence of a common productivity shock already small deviations of some of the credit market parameters from the symmetry assumption are sufficient to create sizeable reductions in the cross–country correlation of inflation rates.

JEL classifications: E32, E44, E52, F41, G10
Key words: optimal monetary policy, Ramsey planner, collateral constraints, monetary union, cross–country heterogeneity.

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1 Introduction

From an individual country’s point of view a monetary unification and/or an exchange rate peg represent trade-offs between the reduced ability to stabilize country specific shocks and an enhanced credibility of monetary policy, possibly resulting in a lower inflation outcome. While the latter is especially important for less developed countries which haven’t been able to achieve a sufficient level of central bank credibility, the benefits of joining a currency union for industrialized countries are less clear-cut and still not well understood. Some authors point to political economy considerations such as the higher independence of a common central bank from local governments.\(^1\) Others emphasize the avoidance of strategic exchange rate devaluations by adopting a common currency or the higher transparency about prices which might put competitive pressures forcing local governments to introduce structural reforms eliminating product as well as labor market distortions.\(^2\)

In contrast, and apart from any political economy considerations, a growing recent literature has analyzed the disadvantages of joining a monetary union within the context of New–Keynesian type models.\(^3\) These studies find that under quite general conditions, even in the face of small heterogeneities across countries, the adoption of a common currency is welfare reducing.\(^4\) The reason is simple. Structural cross–country differences with respect to the institutional framework, demography, tastes and the exposure to shocks usually lead to quantitatively as well as qualitatively different country specific cyclical fluctuations which in turn make country specific monetary policy interventions desirable. Equipped with only one instrument — a union–wide interest rate — the common monetary authority is no longer in the position to meet individual member states’ needs. Moreover, particular policy interventions may turn to be stabilizing for some, but quite destabilizing for other countries. Dealing with this heterogeneity is one of the main challenges the common central bank faces when conducting monetary policy. Some of the questions of major importance arising in this context are which variables should the central bank try to stabilize, is it sufficient to solely concentrate on area-wide averages or should the monetary authority assign a higher (above average) weight to the nominal and/or real stabilization of some states rather than others (of particular member states).

\(^1\)See for example Neumeyer (1998) and Beetsma and Giuliodori (2009) for an overview of related literature.

\(^2\)For example Eichengreen (1993), Frankel and Rose (1998), Baldwin (2006) and others.

\(^3\)Benigno (2004), Beetsma and Jensen (2005), Pappa and Vassilatos (2007), Galí and Monacelli (2008), Ferrero (2009) and others.

\(^4\)One important exception is the case of local currency pricing, also called pricing to market. Examples are Corsetti and Pesenti (2005) and Corsetti (2008).
In this paper we take a step towards answering some of these questions. It contributes to the literature on optimal monetary policy design in New-Keynesian DSGE models. Within one-sector closed-economy models without any financial frictions the welfare maximizing central bank typically faces a trade-off between stabilizing inflation and closing the gap between the actual and the efficient level of output. However, it is well known that this trade-off disappears if the steady state is efficient and the economy is not subject to shocks shifting the aggregate supply curve (cost push shocks). Benigno and Woodford (2005) show that this result can be extended to the case of a distorted steady state, e.g. due to monopolistic competition, as long as (however, only if) government spending and wage-markup shocks are neglected. The Ramsey plan of the monetary authority then calls for full inflation stabilization as a reaction to area wide technology shocks.

Benigno (2004) shows that this conclusion remains valid in a two-country monetary union with cross-country heterogeneity in the degrees of price rigidity as long as the productivity shocks hit both countries symmetrically. He shows that if the two countries share the same degree of nominal rigidity, the terms of trade are completely insulated from monetary policy and the optimal outcome is obtained by targeting a weighted average of the country-specific inflation rates. These weights coincide with the economic sizes of the countries. If the degrees of rigidity are different, the optimal plan implies a high degree of inertia in the inflation rate. But an inflation targeting policy in which higher weight is given to the inflation in the country with higher degrees of nominal rigidity is nearly optimal.

However, things become different if the economy is populated by two types of households — net savers and credit constrained net borrowers, the latter being more impatient. In a closed-economy model Monacelli (2009) shows that irrespective of the degree of price flexibility the borrowing constraint induces inefficient risk sharing between the different types of agents, inefficient movements in the demand for the good used as collateral and finally, inefficient fluctuations in relative prices. As borrowers exhibit preferences tilted towards current consumption, they do not act as full consumption smoothers. Two implications arise from

\footnote{The same conclusion is drawn by Aoki (2001) who sets up a closed-economy model consisting of two production sectors with different degrees of price rigidity.}

\footnote{His framework is inspired by Bernanke, Gertler, and Gilchrist (1996), Kiyotaki and Moore (1997), Iacoviello (2005) and Campbell and Hercowitz (2005), who emphasize that macroeconomic fluctuations are potentially amplified by financial frictions. Since lenders are likely to have little information about the creditworthiness of a borrower, they require borrowers to set forth their ability to repay, which may take the form of collateralizing their assets. A fall in the value of collateral assets induced by an initial shock limits the ability to borrow and consequently reduces economic activity.}
this feature. First, the capacity of borrowers to borrow depends on the develop-
ment of the collateral, i.e. the relative price of housing. Second, an additional
unit of housing provides an extra service as it allows expanding borrowing. If
debt contracts are signed in nominal terms, as usually assumed in the literature,
inflation fluctuations induce wealth effects which, in turn, (may) make the elim-
ination of the inefficiencies just mentioned possible. In such an environment the
central bank faces a trade–off between stabilizing inflation in order to minimize
price dispersion and allowing some inflation in order to tax the borrower or saver
and thus, avoid inefficient risk sharing and undesirable demand and relative price
movements. Thus, the introduction of nominal debt constitutes a motivation for
deviating from a price stability prescription.

This paper develops a two–country DSGE model for a monetary union in
which each country is populated by two types of households — net savers and
credit constrained net borrowers — and two types of production sectors — a
consumption goods sector and a housing sector. Households trade nominal private
debt in equilibrium, with the borrowers being subject to a collateral constraint,
which is tied to the value of the stock of housing. The central bank acts as
a Ramsey planner who seeks to implement monetary policy by maximizing the
welfare of all households in the currency union subject to the resource constraints,
the constraints describing the equilibrium in the private sector, and the constraints
resulting from all distortions that characterize both the long–run and the short–
run behavior of the area–wide economy.

The analysis focuses on the implications of housing and credit market hetero-
geneties for the design of optimal monetary policy of a common central bank,
such as the ECB.\footnote{As emphasized by Macleman, Muehlbauer, and Stephens (1998) and Calza, Monacelli, and
Stracca (2009) differences in housing and financial market institutions across the member states
of the European Monetary Union are still enormous. In particular mortgage lending remains
a predominantly domestic business activity, largely reflecting national traditions and cultural
factors as well as the institutional settings of the local banking sector. Typical loan–to–value
ratios vary from 50% in Italy to 90% in the Netherlands. But also house prices have shown
very different patterns across countries, from almost stable in Germany since the mid 1990s to
highly volatile in countries like Ireland and Spain.} The results of our paper indicate that as long as the only het-
erogeneity between the two countries is the degree of nominal rigidity, the Benigno
(2004) result according to which the common central bank puts a higher weight
on stabilizing inflation in the country with a higher degree of nominal rigidity con-
tinues to hold. However, due to the introduction of collateralized household debt
and borrowing constraints the effects of cross–country disparities in the degree of
price rigidities on the volatility of inflation are amplified. While the volatility of
inflation in the economy with a higher degree of price rigidity is almost as low
as in the model without borrowers, inflation in the economy with more flexible prices becomes much more volatile. As a result, under optimal policy the central bank allows union–wide inflation to fluctuate more in response to productivity shocks than in an economy without credit constrained borrowers. In addition we find that housing and credit market heterogeneities have an impact on the central bank’s goal to stabilize inflation. The higher, for example, the difference between the two countries regarding the share of borrowers, the lower the central bank’s weight on stabilizing inflation in the economy with the lower share (i.e. the less constrained economy). Finally, the paper shows that even in the presence of a common productivity shock already small deviations of some of the credit market parameters from the symmetry assumption are sufficient to create sizeable reductions in the cross–country correlation of inflation rates.

This paper is structured as follows. The next Section presents the model. In Section 3 the impact of housing and credit market heterogeneities on the design of optimal monetary policy is discussed. Section 4 summarizes the main results.
2 The Model

The New Keynesian DSGE model consists of two countries that form a currency union with a single central bank. The countries are labeled $H$ and $F$ and are of size $n$ and $1-n$. There is no possibility of migration across the countries.

Every country consists of two types of households: savers of measure $\omega$ and borrowers of measure $1-\omega$, who both consume and work. Firms are partitioned into final good producers and intermediate good producers. Final good producers operate in two sectors – a consumption goods sector and a housing sector – under perfect competition. Intermediate good producers in each sector produce differentiated goods under monopolistic competition. They have some market power over their own price, but face frictions as in Calvo (1983), which implies a staggered price setting. The continuum of intermediate good producers is indexed by $h \in [0, n]$ in the home country, and by $f \in [n, 0]$ in the foreign country.

As in Iacoviello (2005) and Monacelli (2009), we assume that the two types of households feature heterogenous preferences, with the borrowers being more impatient than the savers. Borrowers are constrained in their access to credit because they are obliged to deposit collateral, which is tied to the value of their existing stock of housing. In what follows, we present the home country block of the model only, since the foreign country block is laterally reversed. In case foreign country variables are used, these are denoted by an asterisk.

2.1 Savers

The representative saver in the home country maximizes the following utility function:

$$ E_0 \sum_{t=0}^{\infty} \beta^t \left[ \gamma \log(C'_t) + (1 - \gamma) \log(D'_t) - \frac{\varphi'}{1 + \eta} (N'_t)^{1+\eta} \right], $$

where $C'_t$ denotes an index of consumption goods, $D'_t$ is the stock of housing, $N'_t$ denotes hours worked, $\beta$ is the discount factor, $\gamma$ is the share of consumption goods, $\eta$ is the inverse elasticity of labor supply and $\varphi'$ is a parameter that indexes the preference for hours worked.\(^8\)

The index of consumption goods is composed of domestic consumption goods and foreign consumption goods given by:

$$ C'_t = \left[ \nu^t (C'_{ht})^{\frac{\nu-1}{\nu}} + (1 - \nu)^t (C'_{ft})^{\frac{\nu-1}{\nu}} \right]^{\frac{\nu}{\nu-1}}, $$

\(^8\)Notice that $N'_t$ reflects hours worked in the consumption goods sector and the housing sector, such that $N'_t = N'_{ct} + N'_{dt}$. 


where $C'_{Ht}$ is consumption of home produced consumption goods, $C'_{Ft}$ is consumption of foreign produced consumption goods, $\nu$ denotes the home bias in preferences and $b$ is the elasticity of substitution.\footnote{We assume that consumption goods are all tradable.}

The budget constraint of the representative saver in nominal terms is:

$$
P_{ct}C'_t + P_{dt}[D'_t - (1 - \delta)D'_{t-1}] + A'_t + \frac{\phi}{2}P_{ct}\left(\frac{A'_t}{P_{ct}}\right)^2 + B'_t = W_{ct}N'_t + W_{dt}N'_{dt} + R_{t-1}A'_{t-1} + R_{Ht-1}B'_{t-1} + \omega^{-1}\Gamma'_t, \quad (3)
$$

where $P_{ct}$ is the price index of consumption goods, $P_{dt}$ is the price index of housing, $A_t$ are bonds traded across countries, $B_t$ is the amount of credit lent to the group of borrowers, $\delta$ is the depreciation rate on the housing stock, $W_{ct}$ is the nominal wage in the consumption goods sector, $W_{dt}$ is the nominal in the housing sector and $\Gamma_t$ are profits from firms. We introduce a small quadratic adjustment cost of international bonds, whose relevance is of measure $\phi > 0$. These costs drive a wedge between the international gross interest rate $R_t$ and the domestic gross interest rate $R_{Ht}$.\footnote{The reason to include adjustment costs for international bonds is to achieve the stationarity of the financial system. See Schmitt–Grohe and Uribe (2003).}

For the representative saver the first–order conditions are given by:

$$
q_t = 1 - \gamma C'_t D'_t + \beta(1 - \delta)E_t\left[q_{t+1}\frac{C'_t}{C''_{t+1}}\right], \quad (4)
$$

where $q_t = P_{dt}/P_{ct}$ is the real housing price.

$$
1 = \beta E_t\left[\frac{C'_t}{C''_{t+1}}\frac{P_{ct}}{P_{ct+1}}\right] R_{Ht} \quad (5)
$$

$$
1 = \beta E_t\left[\frac{C'_t}{C''_{t+1}}\frac{P_{ct}}{P_{ct+1}}\right] R_t - \phi a'_t \quad (6)
$$

and

$$
\varphi'(N'_t)\gamma C'_t = \gamma w_t, \quad (7)
$$

where $w_t$ is the real wage, with $w_t = w_{ct} = w_{dt}$. Combining the expressions (5) and (6) gives:

$$
R_{Ht} = \frac{R_t}{1 + \phi a'_t}. \quad (8)
$$

The allocation of consumer goods expenditures between domestic and foreign produced goods is:

$$
C'_{Ht} = \nu \left(\frac{P_{Ht}}{P_{ct}}\right)^{-b} C'_t \quad (9)
$$
and
\[ C''_F = (1 - \nu) \left( \frac{P_{Ft}}{P_{ct}} \right)^b C'_t, \tag{10} \]
where \( P_{Ht} \) and \( P_{Ft} \) are the price indices of home and foreign produced consumption goods, respectively. The utility maximization problem of the representative saver in the foreign country is quite similar as we assume that the functional forms for preferences are identical across countries.

### 2.2 Borrowers

The representative borrower maximizes the utility function:
\[
E_0 \sum_{t=0}^{\infty} \nu^t \left[ \gamma \log(C''_t) + (1 - \gamma) \log(D''_t) - \frac{\varphi''}{1 + \eta} (N''_t)^{1+\eta} \right], \tag{11}\]
where \( C''_t \) is an index of consumption goods, \( D''_t \) is the stock of housing, \( N''_t \) is the amount of hours worked and \( \nu \) is the discount factor, for which we assume that \( \nu < \beta \). The index of consumption goods is given by:
\[
C''_t = \left[ \nu \left( C''_{Ht} \right)^{\frac{b-1}{b}} + (1 - \nu) \left( C''_{Ft} \right)^{\frac{b-1}{b}} \right]^{\frac{1}{b-1}}. \tag{12}\]

The budget constraint of the representative borrower expressed in nominal terms is:
\[
P_{ct} C''_t + P_{dt} [D''_t - (1 - \delta) D''_{t-1}] + B''_t = W_{ct} N''_ct + W_{dt} N''_dt + R_{Ht-1} B''_{t-1}. \tag{13}\]
Borrowers face a restriction on borrowing, which is given by the collateral constraint:
\[
R_{Ht} B''_t \leq (1 - \xi) E_t [P_{dt+1}(1 - \delta) D''_t], \tag{14}\]
where \( \xi \in (0, 1) \). The collateral constraint implies that borrowing is tied to the expected future value of the stock of housing after depreciation. According to Calza, Monacelli, and Stracca (2009) this type of constraint can be justified on the basis of limited enforcement. Since the borrower can possibly deny to repay the debt in case of default, requiring a collateral ex–ante serves as an insurance against that temptation. The parameter \( \xi \) indicates the share of the housing stock that cannot be used as collateral, which means that \( 1 - \xi \) provides a proxy for the loan–to–value ratio. Since this ratio can be interpreted as a direct measure of the flexibility of mortgage markets, we assume that \( \xi \) differs across countries.
For the representative borrower the relevant first--order conditions are summarized by:

\[
q_t = \frac{1 - \gamma C''_t}{\gamma D'_t} + (1 - \xi)(1 - \delta)\psi_tE_t[q_{t+1}\pi_{dt+1}]
+ v(1 - \delta)E_t \left[ q_{t+1} \frac{C''_{t+1}}{C''_t} \right]
\]

\[
1 = vE_t \left[ \frac{C''_t}{C''_{t+1}} \frac{P_{ct}}{P_{ct+1}} \right] R_{Ht} + \psi_t R_{Ht}
\]

\[\varphi''(N''_t)\psi''_t = \gamma w_t.\]  

Notice that \(\psi_t\) is the Lagrange multiplier associated with the collateral constraint, which is positive, since \(v < \beta\).

Finally, the allocation of consumer goods expenditures between domestic and foreign produced goods is:

\[
C''_{Ht} = \nu \left( \frac{P_{Ht}}{P_{ct}} \right)^{-b} \left( \frac{P_{ct}}{P_{ct+1}} \right)^{-b} C''_t,
\]

and

\[
C''_{Ft} = (1 - \nu) \left( \frac{P_{Ft}}{P_{ct}} \right)^{-b} C''_t.
\]

### 2.3 Aggregate Consumption, the Consumer Price Index, and Terms of Trade

Aggregate consumption by households is a weighted average of the corresponding consumption expenditures of each type of consumer, which is given by:

\[
C_t \equiv \omega C'_t + (1 - \omega)C''_t.
\]

Along similar lines we have:

\[
C_{Ht} \equiv \omega C'_{Ht} + (1 - \omega)C''_{Ht}, \quad \text{and} \quad C_{Ft} \equiv \omega C'_{Ft} + (1 - \omega)C''_{Ft}.
\]

The price index of consumption goods is:

\[
P_{ct} = \left[ \nu(P_{Ht})^{1-b} + (1 - \nu)(P_{Ft})^{1-b} \right]^{\frac{1}{1-b}}.
\]

The terms of trade are defined as:

\[
\tau_t = \frac{P_{Ft}}{P_{Ht}}.
\]
2.4 Final Good Producers

Final good producers operate in each sector $j = c, d$ under perfect competition. The technology to produce the aggregate final goods is given by:

$$Y_{jt} = \left( \frac{1}{n} \int_0^n Y_{jt}(h)^{\frac{\theta_j}{\theta_j - 1}} dh \right)^{\frac{\theta_j}{\theta_j - 1}},$$

(24)

for $j = c, d$, where $\theta_j$ is the elasticity of substitution.

Profit maximization by final good producers delivers the following demand functions for individual intermediate consumption goods:

$$Y_{ct}(h) = \frac{1}{n} \left( \frac{P_{Ht}(h)}{P_{Ht}} \right)^{-\theta_c} Y_{ct},$$

(25)

and individual intermediate housing services:

$$Y_{dt}(h) = \frac{1}{n} \left( \frac{P_{dt}(h)}{P_{dt}} \right)^{-\theta_d} Y_{dt}.$$

(26)

The corresponding price indices are given by:

$$P_{Ht} = \left[ \frac{1}{n} \int_0^n P_{Ht}(h)^{1-\theta_c} dh \right]^{\frac{1}{1-\theta_c}}$$

and

$$P_{dt} = \left[ \frac{1}{n} \int_0^n P_{dt}(h)^{1-\theta_d} dh \right]^{\frac{1}{1-\theta_d}}.$$  

(27)

2.5 Intermediate Goods Producers

Intermediate goods producers indexed by $h \in (0, n)$ produce differentiated goods under monopolistic competition. A generic firm $h$ in each sector $j$ has access to the technology:

$$Y_{jt}(h) = \exp(\varepsilon^{EMU}_t \varepsilon_t) N_{jt}(h)$$

(28)

for $j = c, d$, where $\varepsilon^{EMU}_t$ is a common technology shock and $\varepsilon_t$ is a technology shock that is country–specific. Both shocks are assumed follow an $AR(1)$ process in logs.\textsuperscript{11}

Profit by firm $h$ is given by:

$$\Pi_{jt}(h) = P_{it}(h) Y_{jt}(h) - W_{jt} N_{jt}(h),$$

(29)

where $i = H$ if $j = c$ and $i = d$ if $j = d$. The nominal marginal costs are:

$$MC_{ct}(h) = \exp(\varepsilon^{EMU}_t \varepsilon_t)^{-1} W_{ct}.$$  

(30)

\textsuperscript{11} Notice that we neglect sector–specific technology shocks.
and
\[ \text{MC}_{dt}(h) = \exp(\varepsilon_t^{EMU} \varepsilon_t)^{-1} W_{dt}. \] (31)

We assume that intermediate goods producers have some market power over their own product, but face price frictions as in Calvo (1983), which implies a staggered price setting. Only a fraction of firms \(1 - \alpha_j\) re-optimize their prices in each period, while the remaining fraction \(\alpha_j\) leaves their prices unchanged, where \(j = c, d,..\).

**Intermediate Consumption Goods Producers** Firms in the consumption goods sector that set their price optimally face the following maximization problem:

\[ E_t \sum_{i=0}^{\infty} \alpha_i^c Q_{i,t+i} Y_{ct+i}(h) \left[ \frac{\tilde{P}_{Ht}(h)}{P_{ct+i}} - \frac{\text{MC}_{ct+i}(h)}{P_{ct+i}} \right], \] (32)

subject to the demand equation:

\[ Y_{ct+i}(h) = \frac{1}{n} \left( \frac{\tilde{P}_{Ht}(h)}{P_{Ht+i}} \right)^{-\theta_c} Y_{ct+i}, \] (33)

where \(\tilde{P}_{Ht}(h)\) is the re-optimized price. Notice that \(Q_{i,t+i}\) is the stochastic discount factor that is equal to the intertemporal marginal rate of substitution of the representative saver, who owns the firms.

The optimal choice is given by:

\[ E_t \sum_{i=0}^{\infty} \alpha_i^c Q_{i,t+i} Y_{ct+i}(h) \left[ \frac{\tilde{P}_{Ht}(h)}{P_{Ht+i}} - \frac{\theta_c}{\theta_c - 1} \frac{\text{MC}_{ct+i}(h)}{P_{ct+i}} \right] = 0. \] (34)

The price index for domestic consumption goods evolves according to:

\[ P_{Ht} = \left[ (1 - \alpha_c)(\tilde{P}_{Ht})^{1-\theta_c} + \alpha_c(P_{Ht-1})^{1-\theta_c} \right]^{\frac{1}{1-\theta_c}}. \] (35)

**Intermediate Housing Producers** Firms in the housing sector face a similar maximization problem, and thus the optimal housing price \(\tilde{P}_{dt}\) and the evolution of the price index of housing \(P_{dt}\) have identical expressions.

### 2.6 Market Clearing Conditions

For the home country, the market clearing condition in the market for consumption goods is:

\[ Y_{ct} = nC_{Ht} + (1 - n)C_{Ht}^* + n\omega_0 \phi_t^2 \left[ \nu + (1 - \nu)\tau^{1-b} \right]^{1-b} \] (36),
where $C_Ht = \omega C_H't + (1 - \omega)C_H''t$ and $C^*_Ht = \omega^* C^*_{Ht} + (1 - \omega^*)C''_{Ht}$. The equilibrium in the market for housing services is given by:

$$Y_{dt} = n[\omega(D_t' - (1 - \delta)D_{t-1}'') + (1 - \omega)(D_t'' - (1 - \delta)D_{t-1}'')]$$

(37)

Hence, total output is:

$$Y_t = \tilde{p}_{yt}^{-1}Y_{ct} + \tilde{p}_{yt}^{-1}q_tY_{dt},$$

(38)

where $\tilde{p}_{yt} = P_{yt}/P_{ct}$, with $P_{yt}$ denoting the domestic output deflator.

For the foreign country, the analogous conditions are:

$$Y^{*}_{ct} = (1 - n)[\omega^*(D^{*}_{t} - (1 - \delta^*)D^{*}_{t-1}) + (1 - \omega^*)(D^{*'}_{t} - (1 - \delta^*)D^{*'}_{t-1})]$$

(39)

$$Y^{*}_{dt} = (1 - n)[\omega^*(D^{*}_{t} - (1 - \delta^*)D^{*}_{t-1}) + (1 - \omega^*)(D^{*'}_{t} - (1 - \delta^*)D^{*'}_{t-1})]$$

(40)

$$Y^{*}_{t} = (\tilde{p}_{yt}^*)^{-1}Y^{*}_{ct} + (\tilde{p}_{yt}^*)^{-1}q_tY^{*}_{dt},$$

(41)

where $C^{*}_{Ft} = \omega^* C^{*'}_{Ft} + (1 - \omega^*)C''_{Ft}$, $C_{Ft} = \omega C'_{Ft} + (1 - \omega)C''_{Ft}$, and $\tilde{p}_{yt}^* = P^{*}_{yt}/P^{*}_{ct}$.

The equilibrium in the labor market is characterized by the equality of labor supply and total hours worked:

$$N_{jt} = n[\omega N'_{jt} + (1 - \omega)N''_{jt}],$$

(42)

for $j = c, d$. Market clearing in the international bonds market is:

$$n\omega A^{'}_t + (1 - n)\omega^* A^{'*}_t P^{'*}_{ct}/P^{'*}_{ct} = 0,$$

(43)

while equilibrium in the national debt market is given by:

$$\omega B^{'}_t + (1 - \omega)B''_t = 0.$$  

(44)

Finally, the evolution of aggregate net foreign assets is:

$$n\omega A^{'}_t = n\omega R_{t-1}A^{'}_{t-1} + (1 - n)P^{*}_{Ht}C_{Ht} - nP_{Ft}C_{Ft}.$$  

(45)

### 2.7 EMU Aggregates

For total output in the EMU, we have:

$$Y_{t}^{EMU} = \tilde{p}_{yt}Y_{t} + \tilde{p}_{yt}^*Y^*_{t},$$

(46)

where $\tilde{p}_{yt} = P_{yt}/P^{EMU}_{ct}$ and $\tilde{p}_{yt}^* = P_{yt}^*/P^{EMU}_{ct}$. Finally, the area–wide inflation rate is given by:

$$\pi^{EMU}_{ct} = \kappa\pi_{ct} + (1 - \kappa)\pi^*_{ct},$$

(47)

where the parameter $\kappa$ denotes the weight.

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12 Notice that the second term on the left hand side of the expression is multiplied by the ratio home and foreign consumption price levels since $A^*_t$ is denominated in units of the foreign consumption good.
3 Optimal Monetary Policy

3.1 The Ramsey Problem

We proceed to analyze the role of the central bank in optimally conducting monetary policy. We assume that the central bank acts as a Ramsey planner who seeks to maximize the welfare of all households in the currency union subject to the resource constraints, the constraints describing the equilibrium in the private sector, and the constraints resulting from all distortions that characterize both the long-run and the short-run behavior of the area-wide economy.

We assume that the planner maximizes the following objective function:

$$W_0 \equiv n \left[ \omega \sum_{t=0}^{\infty} \beta^t U(C_t^*, D_t^*, N_t^*) + (1 - \omega) \sum_{t=0}^{\infty} \beta^t U(C_t^n, D_t^n, N_t^n) \right]$$

$$+ \left[ (1 - n) \left[ \omega^* \sum_{t=0}^{\infty} (\beta)^t U(C_t^{n*}, D_t^{n*}, N_t^{n*}) + (1 - \omega^*) \sum_{t=0}^{\infty} (\beta)^t U(C_t^{n**, D_t^{n**}, N_t^{n**}}) \right] \right].$$

Note that the individual utility functions of savers and borrowers are discounted by the same discount factor $\beta$. Otherwise the Ramsey planner would assign a falling (increasing) consumption path to the borrower (saver). As a result, the equilibrium would become dependent on the entire past and the recursive structure of the model would break down.\(^{13}\) In general, as there is uncertainty, the solution to the Ramsey problem is time inconsistent. To get rid of this problem, we follow Woodford (2003) and Benigno and Woodford (2005) and assume that the Ramsey planner is able to ex-ante commit to a pre-announced policy plan and view the welfare evaluation from a “timeless perspective”.

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\(^{13}\)See Monacelli (2009) and Andrés, Óscar Arce, and Thomas (2010) for a further discussion on this point.
3.2 Calibration

We set the parameters of the model on the base of quarterly evidence. We assume that in both countries a number of parameters are identical. The discount factors of savers are $\beta = \beta^* = 0.99$, while the discount factors of borrowers are $\upsilon = \upsilon^* = 0.98$. In the utility functions of households the weights of consumption goods $\gamma$ and $\gamma^*$ are equal to 0.85, respectively. The inverse of labor supply elasticity is $\eta = \eta^* = 1$.

We calibrate the elasticity of substitution between the intermediate goods to $\theta_c = \theta_d = \theta_c^* = \theta_d^* = 10.5$. Moreover, the elasticity of substitution between home and foreign goods is $b = b^* = 1.0001$, which implies that home and foreign goods are not perfectly substitutable. The size of each country is equal to 0.5. The adjustment cost parameters of international bond holdings $\phi$ and $\phi^*$ are 0.001, respectively. Finally, we calibrate the technology shock according to standard values assuming autocorrelation parameters of $\rho_\varepsilon = \rho_\varepsilon^* = \rho_\varepsilon^{EMU} = 0.9$. The standard error of the technology shocks is 1.

Table 1 summarizes the calibrated parameters, which remain unchanged throughout the various scenarios.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>discount factor savers</td>
<td>$\beta, \beta^*$</td>
<td>0.99</td>
</tr>
<tr>
<td>discount factor borrowers</td>
<td>$\upsilon, \upsilon^*$</td>
<td>0.98</td>
</tr>
<tr>
<td>inverse elasticity of labor supply</td>
<td>$\eta, \eta^*$</td>
<td>1</td>
</tr>
<tr>
<td>weight of consumption goods in utility function</td>
<td>$\gamma, \gamma^*$</td>
<td>0.85</td>
</tr>
<tr>
<td>elasticity of substitution between intermediate goods</td>
<td>$\theta_c, \theta_d, \theta_c^<em>, \theta_d^</em>$</td>
<td>10.5</td>
</tr>
<tr>
<td>elasticity of substitution between home and foreign goods</td>
<td>$b$</td>
<td>1.0001</td>
</tr>
<tr>
<td>share of domestic consumption goods in consumption index</td>
<td>$n$</td>
<td>0.8</td>
</tr>
<tr>
<td>size of country H</td>
<td>$n$</td>
<td>0.5</td>
</tr>
<tr>
<td>adjustment cost parameter of international bond holding</td>
<td>$\phi$</td>
<td>0.001</td>
</tr>
<tr>
<td>autocorrelation of technology shock</td>
<td>$\rho_\varepsilon, \rho_\varepsilon^*, \rho_\varepsilon^{EMU}$</td>
<td>0.9</td>
</tr>
<tr>
<td>standard error of technology shock</td>
<td></td>
<td>1%</td>
</tr>
</tbody>
</table>

As regards the steady state, we assume that labor supply of savers $N', N'^*$ and borrowers $N'', N''^*$ is equal to 1/3, which means that total hours worked amounts 8 hours during a typical working day.\textsuperscript{14} Accordingly, the steady state of total

\textsuperscript{14} Notice that in steady state, the values of the parameters $\varphi', \varphi'^*, \varphi''$ and $\varphi''^*$ are calculated to be consistent with this assumption.
output in the currency area $Y^{EMU}$ is also $1/3$, given that both countries are of equal size. Finally, all prices in steady state are identical, i.e. in the zero inflation steady state $P_c, P^*_c, P_d, P^*_d, P_H$ and $P^*_F$ are all equal to unity.

At last, we calibrate the remaining parameters using different values for each scenario. These parameters include the share of savers $\omega$ and $\omega^*$, the loan–to–value ratio $1 - \chi$ and $1 - \chi^*$, the Calvo parameters in the consumption good sector $\alpha_c, \alpha^*_c$ and the housing sector $\alpha_d, \alpha^*_d$, and the housing stock depreciation rate $\delta$ and $\delta^*$. The calibration of these parameters is described within each scenario.
3.3 A Monetary Union with Cross–country Homogeneity

As noted in the introduction to the current paper, in standard closed economy two-sector New Keynesian Models the central bank is simultaneously able to fully stabilize inflation and close the output gap provided both sectors are only subject to economy-wide technology shocks and government spending as well as markup shocks are neglected. However, things become different if the economy is populated by two types of households - net savers and credit constrained net borrowers, as described in section 2. Irrespective of the degree of price flexibility the borrowing constraint induces inefficient risk sharing between the different types of agents, inefficient movements in the demand for the good used as collateral and finally, inefficient fluctuations in relative prices. If debt contracts are signed in nominal terms, as usually assumed in the literature, inflation fluctuations induce wealth effects which, in turn, (may) make the elimination of the inefficiencies just mentioned possible. In such an environment the central bank faces a trade-off between stabilizing inflation in order to minimize price dispersion and allowing some inflation in order to tax the borrower or saver and thus, avoid inefficient risk sharing and undesirable demand and relative price movements. In fact, this is the main conclusion in Monacelli (2009) whose framework corresponds to the symmetric-country case with borrowing constraints and fully flexible house prices in our model. To describe the main mechanism we follow Monacelli (2009) and define the benchmark equilibrium as the one characterized by equally patient agents, without any borrowing constraints. As previously noted, in this environment the Ramsey planner reacts to economy wide technology shocks by fully stabilizing inflation. Furthermore, all resulting fluctuations are efficient.

In contrast, in a world with credit constraints an economy wide technology shock hitting all sectors symmetrically generates a gap between the borrowers’ and savers’ consumption levels since the former are more impatient than the latter. In the face of such a consumption gap the Ramsey planner has an incentive to deflate in order to tax borrowers’ net worth and thus, reduce their excessive willingness to consume. Additionally, the collateral motive on housing accumulation implies inefficiently large swings in housing demand, leading to deviations between the actual and the optimal response of the relative price $P_d/P_c$. This relative-price gap introduces a (is a) further incentive for the monetary authority to abandon the full-price-stability target. Absent any credit constraints, there will be no distortions with respect to the marginal utility of the housing stock and so, even in the case of sector-specific technology shocks, impatient households’ demand for housing won’t deviate from its efficient level. As a consequence, there won’t be any relative-price gap.\textsuperscript{15} Figure 1 displays the relation between the standard

\textsuperscript{15}See Monacelli (2009) for a further discussion on this point.
deviations of the sector and country specific inflation rates on the one hand and
the fraction of borrowers $\omega = \omega^*$ on the other, under the assumption that both
countries are only hit by common technology shocks. Obviously, a higher number
of borrowers (lower $\omega$ and $\omega^*$) implies more severe deviations from optimal risk
sharing and thus, makes inflation fluctuations less disadvantageous from the cen-
tral bank’s perspective. A very similar result is obtained in an \textit{ex ante} symmetric
two-country world in which only the foreign economy is subject to technology
shocks (see Figure 2). If there were no borrowers, the optimal reaction of the
monetary authority to a positive productivity disturbance in Foreign would be
a complete stabilization of both, aggregate (union-wide) consumption as well as
housing inflation.$^{16}$ However, in order to reduce the deviations between the actual
and the efficient response of the terms of trade, the home and foreign inflation
rates would be allowed to move in opposite directions. In particular, it would
be optimal to let each sectoral inflation at home increase by exactly the same
amount as the corresponding sectoral inflation in Foreign decreases. As a con-
sequence, the home and foreign inflation rates would exhibit identical but much
higher volatilities than in the common-shock case. The introduction of credit
constrained borrowers increases the central bank’s incentive to deflate in order to
tax this type of households. Therefore, the nominal interest rate is raised more
aggressively which in turn implies larger negative deviations of both foreign infla-
tions while the magnitude of the positive reaction of the sector specific inflations
at home decreases. Consequently, inflation volatility tends to be higher in Foreign
than in Home (see Figure 2).

Figure 1: Standard Deviation of Inflation after a Common Productivity Shock
(Annualized, in Percent)(Symmetric Two–Country Model with Collateralized
Household Debt and Borrowing Constraints)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1}
\caption{Standard Deviation of Inflation after a Common Productivity Shock
(Annualized, in Percent)(Symmetric Two–Country Model with Collateralized
Household Debt and Borrowing Constraints)}
\end{figure}

\textsuperscript{16}This is one of the main results in Benigno (2004). See also the discussion in subsection
3.4.1.
3.4 A Monetary Union with Cross-country Heterogeneity

3.4.1 Heterogeneity in the Degree of Nominal Rigidity

In an influential paper, Benigno (2004) investigates how monetary policy should be conducted in a two-country, general equilibrium model with monopolistic competition and price stickiness. He shows that if the two countries share the same degree of nominal rigidity, it is optimal to target a weighted average of the country-specific inflation rates. The weights coincide with the economic sizes of the countries and the implied average union-wide inflation rate is always zero. However, since the terms of trade are out of the central bank’s control, the resulting outcome is inefficient. If the degrees of rigidity are different, the optimal plan implies a high degree of inertia in the inflation rate. But an inflation targeting policy in which higher weight is given to the inflation in the country with higher degrees of nominal rigidity is nearly optimal.

The introduction of credit frictions again extends the number of trade-offs faced by the monetary authority: As noted above, in such an environment fluctuations in the inflation rate can be used to reduce the inefficiencies with respect to the resource distribution across different household types. Not surprisingly, in balancing between the different welfare goals after an union-wide technology shock the central bank will accept both, a drop in each sector’s inflation rate as well as some fluctuations in the terms of trade. Figure 5 displays the impulse responses to a common productivity shock under the assumption that the home country is characterized by a higher degree of price rigidity, $\alpha_c = \alpha_d = 0.75$ while $\alpha_c^* = \alpha_d^* = 0.5$. As can be seen, the main result of Benigno (2004) still holds - it is optimal to more strictly stabilize the sectoral inflation rates in the country exhibiting the higher degree of price rigidity. Since foreign prices are relatively
more flexible, their steady state deviation is stronger and more persistent than that of home prices. This, in turn, implies below average terms of trade during most of the adjustment process. The substantial change in the behavior of the Ramsey planner brought about by the introduction of borrowing constraints is also easily seen in Figure 3. It shows the standard deviation of the country and sector specific inflation rates for different fractions of borrowers $\omega$, $\omega^*$ and varying degrees of price rigidity abroad $\alpha_c^*$ and $\alpha_d^*$. Irrespective of the value of $\alpha_c^*$ and $\alpha_d^*$, raising the mass of impatient households from 0 to 50 percent increases the standard deviation of all inflation rates from zero to about 0.2 percent. Lower values of the Calvo parameters $\alpha_c^*$ and $\alpha_d^*$ lead to a slightly higher volatility of inflation (see Figure 3). The reason is twofold. On the one hand, everything else equal, the less pronounced the degree of price rigidity in one country, the lower the average level of price dispersion in the monetary union. On the other hand, from the point of view of the Ramsey planner the relative importance of the terms of trade gap is a decreasing function of any country’s degree of price stickiness.\(^{17}\) Hence, when the latter falls deviations from price stability become less disadvantageous.

Figure 3: Standard Deviation of Inflation after a Common Productivity Shock (Annualized, in Percent) (Two–Country Model with Heterogeneous Nominal Rigidity)

Notes: The lines without black dot refer to the Benigno (2004) model, i.e. $\omega = \omega^* = 1$. The lines with a black dot refer to the same model augmented with collateralized household debt and borrowing constraints, i.e. $\omega = \omega^* = 0.5$.

Benigno’s (2004) conclusions, too, survive in a world with collateralized household debt and borrowing constraints and asymmetric (country specific) technology shocks. It is again more important for the central bank to stabilize inflation in the country exhibiting a higher price stickiness, the home country in our case.

\(^{17}\)See Benigno (2004).
Figure 4: Standard Deviation of Inflation after a Foreign Productivity Shock (Annualized, in Percent)(Two-Country Model with Heterogeneous Nominal Rigidity)

Notes: The lines without black dot refer to the Benigno (2004) model, i.e. $\omega = \omega^* = 1$. The lines with a black dot refer to the same model augmented with collateralized household debt and borrowing constraints, i.e. $\omega = \omega^* = 0.5$.

The volatility of the union-wide inflation rate again increases with the reduction of the Calvo parameters in one of the countries (see the third panel in Figure 4). Finally, the relation between the volatilities of sectoral inflations and the degree of price rigidity in the foreign country is once again altered by the introduction of impatient borrowers (see Figure 4). As long as Foreign’s Calvo parameters $\alpha_d^* = \alpha_c^*$ are sufficiently high, the central bank’s incentive to deflate in order to dampen borrower’s consumption demand dominates. As a result, the volatilities of both countries’ inflation rates are higher than in a world with homogeneous households. However, once $\alpha_d^*$ and $\alpha_c^*$ have fallen below a certain value, the standard deviation of all inflation rates in the savers-borrowers world becomes lower compared to the homogeneous-households case considered by Benigno (2004). The reason is as follows. The foreign borrowers’ impatience induces large swings in foreign consumption demand and thus in relative prices which, in turn, work as an automatic stabilizer with respect to the terms of trade gap. The lower the price rigidity in Foreign the smaller the deviation between the actual and the efficient terms of trade path. In a world without borrowers the corresponding demand and price movements are far less pronounced with the consequence of a higher terms of trade gap. To close/reduce it, the central bank has to accept stronger fluctuations in inflation. Since this effect dominates for sufficiently low degrees of foreign price rigidity, inflation volatility in the economy with homogeneous households becomes higher than that in the world with credit constrained borrowers.
Figure 5: Responses to a Common Productivity Shock (Two-Country Model with Collateralized Household Debt, Borrowing Constraints, and Heterogeneous Nominal Rigidity, $\alpha_{c,d} = 0.75$ and $\alpha_{c,d}^* = 0.5$)

Notes: Inflation and output are expressed as percent deviations from their steady-state values. The nominal interest rate is measured in percent.
3.4.2 Housing Market Heterogeneity

The focus of the previous section was on the implications of credit constraints for the optimal conduct of monetary policy in a currency union under different assumptions about price stickiness. We now turn to the effects of cross-country heterogeneity with respect to important credit market parameters on the optimal behavior of the central bank. In particular, we vary the loan-to-value ratio $1 - \xi^*$ and the fraction of borrowers $\omega^*$ in Foreign while leaving the corresponding home parameters $1 - \xi$ and $\omega$ unchanged. In this Section we only consider the effects of common productivity shocks. This allows us to focus exclusively on the asymmetries which stem from cross–country differences in those structural parameters, which may have an impact on the degree of credit market frictions in our model.

Figures 6 and 7 show the volatility of country–specific and union-wide consumption goods and housing inflation as a function of the foreign loan-to-value ratio $1 - \xi^*$ and the foreign mass of savers $\omega^*$ respectively. The only exogenous driving force in this analysis is a common technology shock, hitting both countries and all sectors symmetrically. The main results can be summarized as follows. The introduction of credit market heterogeneity substantially alters the relative importance of the different stabilization goals faced by the central bank. The lower the loan–to–value ratio in the foreign economy, the higher the central bank’s weight on stabilizing inflation in the home (= the less constrained) economy. The higher the share of borrowers in the foreign economy (and hence the share of borrowers in the monetary union), the lower the central bank’s weight on stabilizing inflation in the home (= the less constrained) economy and the higher the volatility of all inflation measures (be it country–specific or union-wide).

What is the intuition behind these results? Let us take a look at Figure 6 first. First, a lower loan-to-value ratio $1 - \xi^*$ implies a more stringent collateral constraint and thus, less room for increasing consumption in the face of a positive technology shock. Second, changes in $1 - \xi^*$ alter the marginal rate of substitution between an additional unit of borrowing and an additional unit of labor supply. More precisely, a lower loan-to-value ratio induces borrowers to work more after a positive technology shock in order to (at least partly) compensate for the loss coming about with the tighter credit constraint. Both effects work have a negative effect on inflation, the first one by dampening consumption demand and the second one by increasing labor supply. As a consequence, after a positive common technology shock inflation decreases by more in the country with the lower loan-to-value ratio (Foreign in our case). Hence, lowering $1 - \xi^*$ while leaving $1 - \xi$ constant implies a tendency towards a larger inflation volatility and a higher overall degree of price dispersion. To combat the latter, the central bank has to engineer a fall in the nominal interest rate. In fact, as long as $1 - \xi$ is sufficiently
high, the price dispersion effect dominates and as Figure 6 reveals, the central bank is able to cope with it successfully. However, the heterogeneity with respect to the loan-to-value ratio also generates inefficient fluctuation in the terms of trade: Although the total factor productivity is identical in both countries there is a much weaker increase in consumption demand and at the same time a stronger increase in production in Foreign than in Home. Everything else equal, the lower $1 - \xi^*$ the larger the terms of trade gap. For low enough values of the loan to value ratio the welfare losses induced by this gap begin to dominate. Consequently, after a positive technology shock the central bank has to choose a higher interest rate and thus, accept a higher inflation volatility (see third panel in Figure 6).

Figure 6: Standard Deviation of Inflation after a Common Productivity Shock (Annualized, in Percent)

Now consider Figure 7. As can be seen, both sectoral inflations tend to fluctuate less in the country populated by a larger number of borrowers. The intuition is straightforward. Everything else equal, a higher number of borrowers leads to
positive reaction of consumption and housing demand in the face of an unexpected increase in total factor productivity. Therefore the drop in sectoral inflation rates turns to be smaller. Increasing the mass of impatient households in Foreign (lowering $\omega^*$) implies a more inefficient risk sharing on average. As described earlier, the central bank can reduce this sort of inefficiency by engineering an even more pronounced decrease (increase) after a positive (negative) technology shock. This kind of policy, however, leads to a higher volatility in the union-wide inflation rates (see third panel in Figure 7).
4 Conclusion

This paper develops a two-country DSGE model for a monetary union in which each country is populated by two types of households – savers and borrowers – and two types of production sectors – a consumption goods sector and a housing sector. Households trade nominal private debt in equilibrium, with the borrowers being subject to a collateral constraint, which is tied to the value of the stock of housing. The analysis focuses on the implications of housing and credit market heterogeneities for the design of optimal monetary policy of a common central bank.

The results indicate that as long as the only heterogeneity between the two countries is the degree of nominal rigidity, the Benigno (2004) result according to which the common central bank puts a higher weight on stabilizing inflation in the country with a higher degree of nominal rigidity continues to hold. However, due to the introduction of collateralized household debt and borrowing constraints the effects of cross-country disparities in the degree of price rigidities on the volatility of inflation are amplified. While the volatility of inflation in the economy with a higher degree of price rigidity is almost as low as in the model without borrowers, inflation in the economy with more flexible prices becomes much more volatile. As a result, under optimal policy the central bank allows union-wide inflation to fluctuate more in response to productivity shocks than in an economy without credit constrained borrowers. In addition we find that housing and credit market heterogeneities have an impact on the central bank’s goal to stabilize inflation. Finally, the paper shows that even in the presence of a common productivity shock already small deviations of some of the credit market parameters from the symmetry assumption are sufficient to create sizeable reductions in the cross-country correlation of inflation rates.
References


