Political Economy of Commuting Subsidies

Berlin, September 2004
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Political economy of commuting subsidies∗

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September 28, 2004

Abstract

We study the political economy of commuting subsidies in a model of a monocentric city with two income classes. Depending on housing demand and transport costs, either the rich or the poor live in the central city and the other group in the suburbs. Commuting subsidies increase the net income of those with long commutes or high transport costs. They also affect land rents and therefore the income of landowners. The paper studies how the locational pattern of the two income classes and the incidence of landownership affects the support for commuting subsidies.

JEL classification: R14, R48.

Key words: commuting subsidies, voting, monocentric city.

1 Introduction

Many countries subsidise commuting to work. Germany and France, for example, allow taxpayers to deduct commuting expenses from their income tax liability. It is estimated that scrapping tax deductibility in Germany would raise about 5.5 billion Euro in revenue (Bach, 2003). Other countries such as Canada and the US do not allow for a special tax treatment of commuting expenses. However, even in those countries commuters may not

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pay their full costs since transport is subsidised in many other ways. Brueckner (2003) cites evidence that fares for public transit cover only 25 percent of capital and operating expenses in the US and 50 percent of operating costs in Europe, while user fees, gasoline taxes and the like cover about 60 percent of total outlays for highways in the US. Gomez-Ibáñez (1997) analyzes five US and European studies which arrive at a similar conclusion. However, total transportation costs including costs of congestion, air pollution and accidents exceed user payments by far (two till more than ten times as high).

In this paper, we use a model of a monocentric city with two income classes to study reasons for the existence of commuting subsidies. Individuals choose their location within the city, and depending on parameters, either the rich or the poor live in the central city while the other group lives in the suburbs. We then study the effect of commuting subsidies (which may be negative, i.e., a tax on commuting) on the groups’ equilibrium utility. At the heart of the model are the redistributive effects of the subsidy. While city residents pay for the subsidy through general tax revenue, the subsidy redistributes income between residents with long and short commutes as well as between renters and land owners. These redistributive effects form the basis for the political support for or against commuting subsidies (or taxes).

Our main results are as follows. When land is owned by absentee landlords, all city residents may benefit from commuting subsidies if these reduce average land rent. With full citizen landownership, however, it must be the case that one group of residents benefits at the expense of the other. When landownership is symmetric across income classes, we find that the rich generally benefit from commuting subsidies at the expense of the poor if the rich live in the suburbs and the poor in the city. The converse, however, is not necessarily true: If the rich live in the centre, they may still prefer subsidising commuting since in this case their transport costs must be sufficiently larger than that of the poor. Finally, if landownership is asymmetrically distributed, we find that if the rich own more land than the poor, they will tend to oppose commuting subsidies.

The literature on commuting subsidies has largely followed two different strands. First, there is a number of studies which analyse the welfare properties of commuting subsidies, for instance, Brueckner (2003), Richter (2004), and Wrede (2000; 2001; 2003). Brueckner (2003) uses a spatial model like ours and finds that subsidising transport is inefficient (see also Fujita, 1989). Subsidies may be efficient, however, in a second best world. In a
multicentric-metropolitan-area framework Wrede (2003) shows that commuting subsidies may be second-best efficient if Labor income is taxed and if some production factors are immobile. The basic argument is that the choice of working place would be distorted by the Labor income tax if commuting costs were not deductible from the income tax base. Zenou (2000) shows that commuting subsidies can be beneficial by reducing urban unemployment.

Second, there are positive papers which study the effect of subsidies on urban sprawl, which is generally found to be positive (Brueckner, 2003; Arnott, 1998). As far as we know, while the redistributive effects of subsidies for commuting have been mentioned by commentators and calculated by empirical analysts, there is no formal analysis of their effect in a spatial model. On the empirical side, Kloas and Kuhfeld (2003) use survey data from Germany and find that the tax deductibility of commuting expenses benefits high income individuals who have long commutes. If this is correct, why would the poor majority not scrap the subsidy? This paper aims to provide possible answers to this question.

Methodologically, our paper builds on the model of a monocentric city model with two income classes (see Wheaton, 1976; Hartwick et al., 1976). Moreover, we use a model with (partial) public land ownership, a case which was analysed by Pines and Sadka (1986). A model with two income groups and disagreement over public goods levels is presented by de Bartolome and Ross (2003).

The paper proceeds as follows. We introduce our model of a linear city in the next section. Section 3 introduces our voting framework. In section 4, we study how the results change if the city is circular. Section 5 analyses the effect of financing subsidies through income taxes. The last section offers some conclusions.

\footnote{Focusing on the consumption-leisure choice, Wrede (2000) and Richter (2004) analysed tax deductions for work related expenses within an optimal-taxation framework. Deductions for commuting expenses are possibly second-best efficient if household production (of time) generates non-taxable pure profits. Otherwise commuting expenses should probably be taxed.}
2 The model

2.1 The monocentric city

Our model of a monocentric city is the standard model with two income classes (see, e.g., Wheaton, 1976; Hartwick et al., 1976). We begin with the case of a linear city with unit width. All individuals travel to the central business district (CBD) to work. The (CBD) is located at zero and has zero length. There are two groups of residents indexed \( i = l, h \) who differ by income and transport cost. Income is denoted \( y_i \), and we assume group \( l \) is poor and group \( h \) rich, \( y_l < y_h \) (more below). There is a total of \( n \) residents and \( n_i \) individuals in group \( i \) where \( n_l > n_h \), so the poor form the majority in the city. The round trip commuting cost for an individual who lives \( r \) km from the CBD is \( t_i r \) for \( i = l, h \). Since part of commuting costs consist of the opportunity cost of time, we assume that \( t_h \geq t_l \), i.e., the rich have higher commuting costs than the poor.

Commuting costs are subsidized at rate \( \sigma \). Note that we allow for negative subsidies, i.e. taxes on commuting expenditures. Examples of commuting taxes would include road pricing, gasoline taxes etc in excess of the true costs of commuting. For simplicity, we assume that the subsidy covers a certain percentage of all commuting costs. This assumption could be questioned on the grounds that the rich have higher time costs, but only money costs of commuting are subsidized. However, in general, the rich choose more expensive (viz. faster) transport modes. Brueckner (2003) studies transport mode choice by different income classes in the same framework as ours. He shows that the rich opt for transport modes with higher money and lower time costs than the poor. Hence, it would be possible to endogenise the differing time and money costs of commuting and reach similar conclusions as we do now.

To balance the budget, a lump-sum tax of \( T \) is levied on each city resident.\(^2\) We assume that city residents vote on the level of the subsidy and lump sum tax prior to choosing their place of residence and consumption of goods and housing. We solve the model backwards and first analyse consumption and location choices and turn to the determination of the voting equilibrium in the next section.

\(^2\)The form of financing the subsidy has obvious distributional consequences. We return to this question below in Section 5.
Individuals have identical, strictly increasing and quasiconcave utility functions, \( u(x, z) \), defined over consumption \( x \) and housing (lot size), \( z \). We assume consumption and housing to be normal goods. Let the price of housing be \( q \). Let individual labour income be \( w_i \) and assume that an individual of type \( i \) receives a fraction \( \theta_i \) of average differential land rent (ADR). Letting \( \alpha_i \equiv n_i/(n_l+n_h) \) be group \( i \)'s population share, we require \( \alpha_l\theta_l+\alpha_h\theta_h = \theta \), where \( \theta_i \geq 0, i = l, h, \) and \( \theta \in [0,1] \), and \( 1 - \theta \) is the degree of absentee land ownership. Total income for an individual in group \( i \) is then \( y_i = w_i + \theta_iADR \). We assume \( w_l < w_h \) and \( y_l < y_h \), i.e., group \( l \) has lower labour income than group \( h \) and in addition it may or may not have lower rental income. Individuals are assumed to be perfectly mobile. In equilibrium, therefore, an \( i \)-type individual must attain the same utility level regardless of his or her place of residence.

We model the city as closed, i.e., the utility level \( u_i \) attained by an \( i \)-type citizen in equilibrium is endogenous, whereas the number of \( i \)-type residents, \( n_i \), is exogenous. Conversely, in an open city, \( u_i \) is exogenous while \( n_i \) is endogenous, where changes in fiscal parameters lead to migration responses of citizens from or to other cities.\(^3\) The open city model would be more appropriate if the goal of the analysis is the introduction of commuting subsidies by a single city. However, if we want to study the effect of a coordinated use of subsidies in a system of cities, the closed city model is more appropriate.

### 2.2 The urban equilibrium

The consumer chooses \( x \) and \( z \) to maximize utility subject to the budget constraint, \( x + qz = y_i - T - (1 - \sigma)t_r r \), or using the budget constraint in the utility function:

\[
\max_z u(y_i - T - (1 - \sigma)t_r r - qz, z).
\]

The first order condition for an interior solution is:

\[
-qu_x + u_z = 0,
\]

\(^3\)See Brueckner (1987) and Fujita (1989) for the distinction between the open and closed city models and the differing comparative statics.
where \( u_x \equiv \partial u / \partial x \) etc. Mobility implies that in equilibrium a household with income \( y_i \) must achieve a constant utility level, \( u_i \), regardless of his or her residence:

\[
u(y_i - T - (1 - \sigma)t_ir - qz, z) = u_i.
\]

The lot size (the ‘bid max’ lot size) which solves (2) and (3) is denoted \( z_i = z(y_i - T, (1 - \sigma)t_i, r, u_i) \). Condition (3) together with the optimality condition (2) also defines an individual’s bid rent function \( R(y_i - T, (1 - \sigma)t_i, r, u_i) \), i.e., the maximum rent an individual living at \( r \) would pay to achieve utility level \( u_i \). In order to ensure that individuals attain the same equilibrium utility regardless of location, the bid rent must vary with distance to compensate for varying transport costs. Differentiating (3), using the envelope theorem, gives:

\[
R_y i = \frac{1}{z_i} > 0, \quad R_r i = -\frac{(1 - \sigma)t_i}{z_i} < 0, \quad R_u i = -\frac{1}{z_iu_x} < 0,
\]

where \( R_y i \equiv \partial R(y_l - T, (1 - \sigma)t_l, r, u_l) / \partial y_l \) and so on.

Landlords are assumed to rent land at the agricultural land rent \( R_A \) and subsequently rent out to the highest bidder in the city. The land rent function is:

\[
\Phi(r, \cdot) = \max\{ R(y_l - T, (1 - \sigma)t_l, r, u_l), R(y_h - T, (1 - \sigma)t_h, r, u_h), R_A \}.
\]

Based on (4), we focus on two possible location patterns:

**Assumption 1 (PIC)** For all \( r, t_l z_l > t_h z_h \), where \( z_i \equiv z(y_i - T, (1 - \sigma)t_i, r, u_i) \). That is, the bid rent function of the poor is steeper than that of the rich.

**Assumption 2 (RIC)** For all \( r, t_l z_l < t_h z_h \). That is, the bid rent function of the poor is flatter than that of the rich.

Under Assumption 1, the bid rent function becomes flatter with rising income, since the income elasticity of housing demand is assumed to exceed the income elasticity of transport cost.\(^4\) This implies that the poor outbid the rich in the centre and the rich outbid the poor in the suburbs. Hence, the mnemonic PIC (poor in city). Conversely, under Assumption 2, the rich have steeper bid rent functions than the poor; hence, in this case the rich live

\(^4\)Since transport costs differ, the income elasticity of housing demand should be interpreted as the elasticity with respect to income net of transport cost.
in the centre and the poor in the suburbs (for this case we use the mnemonic RIC, rich in city).

In the following, we use the subscripts $c$ to denote city residents and $s$ for suburban residents, respectively. That is, Assumption 1 implies $c = l$, $s = h$, and conversely, Assumption 2 implies $c = h$, $s = l$.

Average differential land rent is then defined as:

$$ADR = \frac{1}{n} \left( \int_{0}^{r_1} R(y_c - T, (1 - \sigma)t_c, r, u_c)dr + \int_{r_1}^{r_2} R(y_s - T, (1 - \sigma)t_s, r, u_s)dr - R_A \right),$$

(5)

where $r_2$ is the city border and $r_1$ the border between the two groups, both of which are endogenous.

The equilibrium in our model can now be completely defined by the following conditions:

$$R(y_l - T, (1 - \sigma)t_l, r_1, u_l) = R(y_h - T, (1 - \sigma)t_h, r_1, u_h)$$

(6)

$$R(y_s - T, (1 - \sigma)t_s, r_2, u_s) = R_A$$

(7)

$$\int_{0}^{r_1} \frac{1}{z(y_c - T, (1 - \sigma)t_c, r, u_c)}dr = n_c$$

(8)

$$\int_{r_1}^{r_2} \frac{1}{z(y_s - T, (1 - \sigma)t_s, r, u_s)}dr = n_s$$

(9)

$$T = \sigma \hat{t},$$

(10)

where $\hat{t} \equiv \frac{1}{n} \left[ t_c \int_{0}^{r_1} \frac{r}{z(y_c - T, (1 - \sigma)t_c, r, u_c)}dr + t_s \int_{r_1}^{r_2} \frac{r}{z(y_s - T, (1 - \sigma)t_s, r, u_s)}dr \right]$ is average transport costs. Equation (6) is the condition that the bid rent of the poor equals that of the rich at the boundary between the two classes, $r_1$. Likewise, (7) defines the city border, where the bid rent of the suburbanites (either rich or poor) equals the agricultural land rent. Equations (8) and (9) are the market clearing conditions for the housing market, and equation (10) is the government budget constraint.

Using (7) and (4) in (8) and (9) and integrating gives:

$$(1 - \sigma)t_c n_c = - \int_{0}^{r_1} R_c(y_c - T, (1 - \sigma)t_c, r, u_c)dr$$

$$= R(y_c - T, (1 - \sigma)t_c, 0, u_c) - R(y_c - T, (1 - \sigma)t_c, r_1, u_c)$$

(11)

$$(1 - \sigma)t_s n_s = - \int_{r_1}^{r_2} R_s(y_s - T, (1 - \sigma)t_s, r, u_s)dr$$

$$= R(y_s - T, (1 - \sigma)t_s, r_1, u_s) - R_A.$$
Further, integrating (10) by parts, using (7), (6) and (4):

\[(1-\sigma)\tilde{t} = \frac{1}{n} \left( \int_0^{r_1} R(y_c - T, (1-\sigma)t_c, r, u_c) \, dr + \int_{r_1}^{r_2} R(y_s - T, (1-\sigma)t_s, r, u_s) \, dr - r_2 A \right) \]

that is, in a linear city with linear transport costs, average transport cost net of subsidies equals average differential land rent (Arnott and Stiglitz, 1981). With (13) and (10), the utility function can be rewritten as

\[u(w_i + (\theta_i - (1+\theta_i)\sigma)\tilde{t} - (1-\sigma)t_ir - qz, z)\].

Differentiating (14) gives the effect of the subsidy and average transport costs on individual bid rent:

\[R^i_{\sigma} = \frac{t_i r - (1+\theta_i)\tilde{t}}{z_i}, R^i_{t} = \frac{\theta_i - (1+\theta_i)\sigma}{z_i}, i = l, h.\]

The subsidy lowers net commuting costs by $t_ir$, but also increases the lump sum tax by $\tilde{t}$ and reduces average differential land rent by $\tilde{t}$. The net effect is therefore positive if $t_ir - (1+\theta_i)\tilde{t} > 0$. For instance, if $\theta_i = 0$ and $t_l = t_h$, the subsidy has a positive income effect (and hence, increases the bid rent) for all individuals who live farther from the CBD than average. The subsidy will also affect bid rent through its effect on average transport costs $\tilde{t}$. If $\tilde{t}$ increases by a unit, this increases the lump sum subsidy by $\sigma$ and changes average differential land rent by $(1-\sigma)$, which is positive as long as $0 \leq \sigma \leq 1$.

3 Voting

We now turn to the description of the voting game. Each individual votes for her preferred subsidy and lump-sum tax. Brueckner (2003) shows that an efficient allocation is characterized by zero commuting subsidies: taking account of landowner welfare, the equilibrium without subsidies is efficient. Hence, in the case where all land is owned by city residents, the subsidy redistributes between the income classes. However, when the degree of absentee landownership is high, it is possible that both groups benefit from commuting subsidies since part of the cost may be borne by absentee landowners.

Equations (6), (7), (11), (12) and (13) implicitly define the endogenous variables $r_1, r_2, \bar{u}_l, \bar{u}_h$ and $\tilde{t}$ as a function of the parameters, $\sigma, t_l, t_h, w_l, w_h, n_l, n_h, \theta_l, \theta_h$ and $\theta$. We will use the notation $R^{ij} \equiv R(y_j - T, (1-\sigma)t_j, r_i, u_j)$ and so on (where $r_0 = 0$).
The voter’s problem is to choose the subsidy rate which maximises her utility subject to the government budget constraint.

Before explicitly deriving expressions for the reaction of equilibrium utility levels to the subsidy, we will try to give a preview of the pertinent effects. Consider the indirect utility function:

\[ v(\Phi, M_i) \equiv v(\Phi, w_i + \theta_i ADR - (1 - \sigma)t_i r - \sigma \tilde{t}). \]  

Using Roy’s identity, we have

\[ \frac{dv(\Phi, M_i)}{d\sigma} = v_M(-z d\Phi/d\sigma + t_i r - (1 + \theta_i)\tilde{t} + (\theta_i - (1 + \theta_i)\sigma)\tilde{t}). \]  

We can discern the following effects on utility. First, the subsidy will influence the price of housing through income effects and the relocation of residents: A resident may benefit if the subsidy causes the price of housing to fall. Second, there is an income effect which is positive if \( t_i r > (1 + \theta_i)\tilde{t} \). If \( \theta_i = 0 \) and \( t_i = t_h \), this will be the case if the individual lives farther from the CBD than average. Otherwise, this income effect is positively influenced by \( t_i \) and negatively by \( \theta_i \) (for given \( \tilde{t} \)). Note the asymmetry of this income effect: Assumption 1 implies that the income effect will be positive (on average) for the rich since they have larger marginal transport costs and live farther from the CBD than the poor. Assumption 2, however, implies the poor live farther from the CBD than the rich but, since their marginal transport cost is lower, it is not clear that the income effect is positive for them. Third, income is also affected via the reaction of average distance: if average distance increases, the lump sum subsidy increases. Moreover, if the subsidy reduces average differential land rent, net income falls as long as \( \theta_i > 0 \) and this effect will be more severe the larger \( \theta_i \). Since \( \partial(ADR)/\partial\sigma = -\tilde{t} + (1 - \sigma)\partial\tilde{t}/\partial\sigma \), we would expect average differential rent to fall if average distance increases only moderately.

In order to view the difference in the changes between cases PIC and RIC, consider figure 1, which depicts the ‘first round effects’ of a commuting subsidy. Panel (a) depicts case PIC and panel (b) case RIC. For the sake of argument, assume \( \theta_i = 0, i = l, h \), and that \( r_1 \) equals the average distance, \( \tilde{r} \), in both cases. Define \( \hat{r}_j = \tilde{r}/t_j \) for \( j = c, s \) to be the distance where transport costs for a type \( i \) resident just equal average transport costs. PIC implies \( \hat{r}_s < \tilde{r} < \hat{r}_c \); conversely, RIC implies \( \hat{r}_c < \tilde{r} < \hat{r}_s \). This is shown in the figures, where bid rent functions rotate around points A and B. It is also obvious that the effect of this would be to decrease \( r_1 \) in case PIC and increase \( r_1 \) in case RIC. An increase in
Figure 1: Effect of commuting subsidies in case PIC (a) and RIC (b).

$r_1$ increases utility of city residents. This discussion shows that in case PIC group $l$ – the central city residents – is likely to oppose the subsidy while this is not necessarily true in case RIC, where the income effect for group $h$ who lives centrally may be positive and the boundary between the two groups may shift out.

Of course, these first round effects lead to further adjustments which then determine the reaction of equilibrium utilities, along with the effect on $r_1$ and $r_2$. Consider for instance case PIC. If $r_1$ were to decrease and $r_2$ to increase as suggested by the figure, the rich would have more space than necessary to house them.\(^5\) To restore equilibrium, rich utility would have to increase so that the bid rent of the rich shifts down and housing consumption increases. Similar arguments apply to the poor: If $r_1$ were to decrease, population density would fall in the central city, and to restore equilibrium, poor utility would have to fall which would then decrease housing consumption and increase density. However, it may be that in equilibrium $r_1$ increases and poor utility increases too, since the poor benefit from the fact that the rich move further out. In fact, this is what we find in some of our examples below.

\(^5\)This follows since $z_y < 0$: for given utility, increasing income leads to higher bid rent and hence to lower housing consumption. But this implies that for given $\bar{u}_h$ population density would decrease for all $r_1 < r < r_2$, while the rich ‘territory’ has increased.)
The model does produce some preliminary results which are of interest.

**Lemma 1** Increasing $\sigma$ decreases equilibrium land rent at the CBD (0) and at the border between rich and poor, $r_1$.

**Proof.** Differentiating (12) shows $d\Phi(r_1)/d\sigma = -t_sn_s$. Using this in (6) and substituting in (11) gives $d\Phi(0)/d\sigma = -t_sn_s - t_cn_c$. ■

The degree of landownership is a crucial determinant of preferences for commuting subsidies in our model. We first analyse commuting subsidies when only citizens are landlords, i.e. when $\theta = 1$. We then consider the other extreme of complete absentee landownership, i.e. $\theta = 0$.

For citizen-landownership we get the following fundamental result.

**Proposition 1** If $\theta = 1$, starting from $\sigma = 0$, sign $\partial u_l/\partial \sigma = -\text{sign} \partial u_h/\partial \sigma$.

**Proof.** Totally differentiating the equilibrium equation system (6), (7), (11), (12) and (13), using (4) and (15), evaluating at $\sigma = 0$ and simplifying gives:

\[
\frac{\partial u_c}{\partial \sigma} = -u_x^0 u_x^{1c} u_x^{1s} \int_{r_1}^{r_2} R_a dr \left\{ n_s^2 \theta_c t_s \left( z^{1s} t_c - z^{1c} t_s \right) + \tilde{t}(n_l + n_h)(1 - \theta_c) t_c 
- z^{0c} \left( n_s + n_c (1 - 2 \theta_c) t_c (n_c t_c + n_s t_s) + \theta_c (n_s^2 t_c^2 - n_c^2 t_s^2) \right) \right\} / \\
\left\{ u_x^{1s} (n_l + n_h) \int_{r_1}^{r_2} R_a dr \left( u_x^{1c} (t_c - \theta_c (n_c t_c + n_s t_s)) + \theta_c n_s t_s u_x^{0c} \right) 
+ \theta_c n_s t_s u_x^{0c} u_x^{1c} \int_{0}^{r_1} R_a dr \right\} 
\]

\[
\frac{\partial u_s}{\partial \sigma} = -\frac{\partial u_c}{\partial \sigma} \int_{r_1}^{r_2} R_a dr, 
\]

where we have used (from (15) and integration by parts, using (11) and (12)) $\int_{0}^{r_1} R_t = n_c (\theta_c - (1 + \theta_c) \sigma), \int_{r_1}^{r_2} R_t = n_s (\theta_s - (1 + \theta_s) \sigma)$, and $\int_{0}^{r_1} R_s + \int_{r_1}^{r_2} R_s = -\tilde{t}(\theta_c n_c + \theta_s n_s)$. Hence, using (4) we have sign($\partial u_s/\partial \sigma$) = -sign($\partial u_c/\partial \sigma$). ■

If there are no absentee landlords, there is an exact distributional antagonism between rich and poor (except in the knife-edge case where both groups vote for a subsidy rate of zero). Hence, if one group loses the other one benefits from a small subsidy. This
makes intuitive sense, since we know that in this case subsidies are inefficient (Fujita, 1989; Brueckner, 2003). However, we cannot say in general which group will benefit from the subsidy. To end up with less ambiguity, we consider uniform landownership in the next proposition.

**Proposition 2** If \( \theta_l = \theta_h = 1 \), starting from \( \sigma = 0 \), (i) we have \( \partial u_l / \partial \sigma < 0 < \partial u_h / \partial \sigma \) in case of PIC provided that \( t_h - t_l \) is not too large. (ii) In case of RIC, if \( z^0_l \geq z^{1_l} \), \( \partial u_h / \partial \sigma > 0 > \partial u_l / \partial \sigma \).

**Proof.** Setting \( \sigma = 0 \) and \( \theta_l = \theta_h = 1 \) in (18) gives:

\[
\frac{\partial u_c}{\partial \sigma} = u^0_c u^1_c u^1_s x u^1_c x \int_{r_1}^{r_2} R_a dr \left\{ n_s t_s (z^{1_c} t_s - z^{1_s} t_c) + z^0 c (t_c - t_s) (n_c t_c + n_s t_s) \right\} / \left\{ u^1_s x \int_{r_1}^{r_2} R_a dr (u^1_c x (t_c - t_s) + t_s u^0_c x) + t_c u^0_c x u^1_c x \int_0^{r_1} R_a dr \right\}.
\]

(20)

Since \( R_a < 0 \), the denominator in (20) is negative under RIC and also under PIC provided that \( t_h - t_l \) is not too large. Since \( t_s / z^{1_s} < t_c / z^{1_c} \), the numerator is positive under PIC since \( t_s \geq t_c \) which proves (i). Under RIC, the numerator is positive if \( z^0 c \geq z^{1_s} \). The result then follows directly from Proposition 1. ■

To understand the result, note from Lemma 1 that equilibrium land rent at the CBD, \( \Phi(0) \), falls by \( t_c n_c + t_s n_s \). This fall can be decomposed into the partial effects of \( \sigma, u_c \) and \( \tilde{t} \). Since \( R^0_c = -2 \tilde{t} / z^{0_c} < 0 \) and \( R_u < 0 \), the fall in land rent must be due to an increase in central city residents’ utility if \( R^0_c \tilde{t} \sigma \) is larger than \( 2 \tilde{t} / z^{0_c} - (t_c n_c + t_s n_s) \). Writing out the corresponding expressions leads to (20).

Given a uniform distribution of land among rich and poor citizens, rich voters benefit from (small) commuting subsidies while poor voters oppose them in case of PIC. This holds true even if rich voters live closer to the CBD provided that the income effect on housing demand is strong. If the rich rent relatively large flats, they occupy a large area and commute long distances.

However, if the rich have larger rental income than the poor, \( \theta_h > \theta_l \), the effect of the subsidy on income received from land rents acts to mitigate the preference towards commuting subsidies more for the rich class. We have no general result for this but we will illustrate the effect of differential landownership by means of an example below.
The next proposition describes the interior optimum in case of uniform land distribution and identical commuting costs.

**Proposition 3** Let $\theta_l = \theta_h = 1$ and $t_l = t_h = t$. Suppose that utility is concave in $\sigma$ for both groups so there exist unique interior optimum subsidy rates, $\sigma(y_i) \in (-\infty, +\infty)$ for $i = l, h$. Then the preferred subsidy rate of high income voters is strictly higher than the preferred subsidy of low income voters.

**Proof.** Totally differentiating the equilibrium equation system (6), (7), (11), (12) and (13) at $\theta_l = \theta_h = 1$ and $t_l = t_h = t$, using (4) and (15), gives:

$$
\frac{\partial u_l}{\partial \sigma} = u_x^l u_x^l \left\{ n_l (2\sigma - 1)u_x^l (z^1l - z^{1h}) \int_{r_1}^{r_2} R_u dr + (n_l + n_h)\sigma(z^{0l}(n_l + n_h)t - 2\tilde{t}) \right\} / \left\{ (n_l + n_h)\sigma u_x^{1l} + (2\sigma - 1)u_x^{0l}(u_x^1 (2\sigma - 1) \int_{r_1}^{r_2} R_u dr + u_x^{1h} \int_{r_1}^{r_2} R_u dr \right\}
$$

(21)

$$
\frac{\partial u_h}{\partial \sigma} = u_x^h \left\{ u_x^l n_h t(z^{1h} - z^{1l}) \left( u_x^{0l} (2\sigma - 1) \int_{r_1}^{r_2} R_u dr + (n_l + n_h)\sigma \right) + (n_l + n_h)\sigma u_x^{0l} \left( (n_l + n_h)t - 2\tilde{t} \right) \right\} / \left\{ (n_l + n_h)\sigma u_x^{1l} + (2\sigma - 1)u_x^{0l}(u_x^1 \int_{r_1}^{r_2} R_u dr + u_x^{1h} \int_{r_1}^{r_2} R_u dr \right\}
$$

(22)

Substituting from (21) in (22), we get

$$
\frac{\partial u_h}{\partial s} = u_x^h \left( \frac{\partial u_l}{\partial \sigma} + tu_x^l n_h (z^{1h} - z^{1l}) \right).
$$

(23)

Since $z$ is a normal good, $z^{1h} > z^{1l}$. Therefore, since the optimum subsidy rate of the poor fulfills $\partial u_l / \partial \sigma = 0$, we have

$$
\frac{\partial u_h(s(y_i, \cdot))}{\partial \sigma} > \frac{\partial u_l(s(y_i, \cdot))}{\partial \sigma} = 0
$$

(24)

and hence, by concavity, $s(y_h, \cdot)$ must be larger than $s(y_l, \cdot)$.

According to Proposition 3, concavity of utility functions in $\sigma$ would imply that the rich prefer higher subsidies than the poor as long as $\theta_h = \theta_l$. With identical marginal transport costs and landownership across groups, the rich live in the suburbs and prefer higher
commuting subsidies than the poor. Optimal subsidy rates may, however, be positive or negative.

Next, we analyse complete absentee land ownership, i.e. \( \theta = 0 \).

**Proposition 4** If \( \theta = 0 \), starting from \( \sigma = 0 \), (i) under PIC at least one group votes either for a (small) subsidy or a tax. (ii) If \( ADR < z^{bc}(\Phi(0) - R_A) \), all citizens prefer a (small) subsidy under PIC.

**Proof.** Totally differentiating the equilibrium equation system (6), (7), (11), (12) and (13) at \( \theta = 0 \) and \( \sigma = 0 \), using (4) and (15), gives:

\[
\begin{align*}
\frac{\partial u_c}{\partial \sigma} &= \frac{u_{cx}^c}{u_{cx}^c} \left( z^c(n_c t_c + n_s t_s) - \tilde{t} \right) \quad (25) \\
\frac{\partial u_s}{\partial \sigma} &= \frac{u_{sx}^s}{u_{sx}^c} \left( t_s \frac{\partial u_c}{\partial \sigma} + u_{c x}^c \left( \tilde{t}(t_s - t_c) + n_s t_s (z^s t_c - z^{1c} t_s) \right) \right). \quad (26)
\end{align*}
\]

(i) Under PIC, since \( t_c \leq t_s \) and \( t_c/z^{1c} > t_s/z^{1s} \), (26) is strictly positive if (25) is zero (and (25) is strictly negative if (26) is zero). (ii) Using (11) and (12), (25) is positive if and only if \( ADR < z^{bc}(\Phi(0) - R_A) \), in which case (26) is also positive under PIC. \( \blacksquare \)

With absentee land ownership, the efficient equilibrium (with \( \sigma = 0 \)) is certainly not supported by all citizens under PIC. Proposition 4 shows the ambiguous effect of commuting subsidies. From Lemma 1, equilibrium land rent at the CBD must fall by \( t_c n_c + t_s n_s = \Phi(0) - R_A \). However, we know from (15) that the partial effect of the subsidy is to decrease the bid rent of the resident who resides at the CBD by \( \tilde{t}/z^{bc} \). Since \( R_u < 0 \), the utility of inner city residents increases if and only if \( z^{bc}(\Phi(0) - R_A) > \tilde{t} = ADR \) (at \( \sigma = 0 \)). The Proposition also shows that there may be cases where both groups benefit from a subsidy. The intuition for this is that part of the costs of the subsidy is borne by absentee landlords in the form of lower land rents.

**Example.** In order to illustrate our results and provide further insight, we now present a numerical example. Suppose utility is of the Cobb-Douglas form

\[ u(x, z) = x^{1-\alpha} z^\alpha. \]

Solving for the Marshallian demand for \( z \) gives

\[ Z(\cdot) = \frac{\alpha M}{q}, \quad \text{where} \ M \equiv w + (\theta - (1 + \theta)s)\tilde{t} - (1 - \sigma)tr. \quad (27) \]
We also find the bid rent function and ‘bid max lot size’ (Fujita, 1989):

\[ R(r) = \alpha(1 - \alpha)^{\frac{1}{\alpha}} \bar{u}^{-\frac{1}{\alpha}} M^{\frac{1}{\alpha}}, \quad z(\cdot) = (1 - \alpha)^{\frac{\alpha - 1}{\alpha}} \bar{u}^{\frac{1}{\alpha}} M^{\frac{\alpha - 1}{\alpha}}. \]  

(28)

Since Cobb Douglas utility implies an income elasticity of housing demand of 1, PIC (RIC) will apply if the income elasticity of commuting costs is smaller (greater) than one.

We then solve for the endogenous variables \( \bar{u}_l, \bar{u}_h, r_1, r_2 \), and \( \sigma \) as functions of the exogenous parameters \( n_l, n_h, w_l, w_h, t_l, t_h, R_A \), and \( \theta, \theta_l, \theta_h \).

Our benchmark parameters are \( \alpha = 0.25, R_A = 0.1, n_h = 2, n_l = 2.5, w_h = 2, w_l = 1.4, \) and \( t_l = 0.1. \)

First, let \( t_h = 0.12 \) (which implies PIC). Then, as we already know, for \( \theta_l = \theta_h = 1 \), the rich benefit from subsidies while the poor lose. This case is depicted in Figure 2, where the dark curves shows the case without subsidy and the grey curves show the case \( \sigma = 0.3 \). The figure shows that \( r_1 \) and \( r_2 \) increase while average differential rent falls. This is, in fact, true for all the examples presented in this paper.

We then vary \( \theta \) and \( \theta_l \) (\( \theta_h \) is then given by \( \max\{0, \frac{n_h + w}{n_h} - \frac{n_l}{n_h}\theta_l\} \)). At \( \theta = \theta_l = \theta_h = 0 \), both groups benefit from subsidies. For \( \theta_l = \theta_h = \theta \), the rich always prefer higher subsidies than the poor, and further, the poor oppose commuting subsidies for \( \theta > 0.67 \).
In general this is the picture we get: for each $\theta$, the lower $\theta_l$, the less the rich benefit from commuting subsidies and the more the poor benefit because part of the cost is borne by the rich in the form of lower differential land rent. Clearly, this effect becomes stronger the larger $\theta$, i.e. the lower the percentage of land in absentee ownership.

Figure 3 shows how support for commuting subsidies depends on $\theta$ and $\theta_l$. In region R in the figure, there is no majority for subsidies (or alternatively, the majority votes for commuting taxes) since in this region the poor oppose subsidies. In the region marked PR, there is unanimous support for subsidies by the rich and poor. This occurs for low $\theta$ and $\theta_l$ below a certain threshold which is a function of $\theta$. In region P, the majority consisting of the poor support subsidies which are opposed by the rich who in the region of high $\theta$ and low $\theta_l$ oppose subsidies. The figure nicely illustrates Proposition 1: At $\theta = 1$ there is never unanimous support for positive subsidies.

Consider now the case where commuting costs differ such that we obtain case RIC. In particular, we assume $t_h = 0.15$. Figure 4 shows the support for commuting subsidies in this case. Interestingly enough, the region where there is a majority which supports positive commuting subsidies is smaller than in the PIC case. Intuitively, one might have thought that in case RIC, majority support for commuting subsidies is higher than in
case PIC because the majority lives far from the CBD. As the example shows, this is not necessarily the case.

4 Circular city

As is well known, the effects of income redistribution in a monocentric city hinge critically on the assumed shape of the city (see Fujita, 1989). Hence, the assumption of a linear city is not innocuous.

Suppose instead the city is circular with the amount of land available for residential use at distance $r$ from the CBD given by $\gamma r$, where $\gamma \leq 2\pi$. To simplify, we assume $t_l = t_h = t$ which implies PIC.

While the equilibrium conditions (6) and (7) are analogous here, (8) and (9) are now replaced by

$$n_l = \int_0^{r_1} \frac{1}{z(y_l - T, (1 - \sigma)t, r, u_l)^{\gamma r}} dr$$

$$= \frac{\gamma}{(1 - \sigma)t} \left( \int_0^{r_1} R(y_l - T, (1 - \sigma)t, r, u_l) dr - r_1 \Phi(r_1) \right)$$

Figure 4: Support for commuting subsidies in case RIC.
and

\[ n_h = \int_{r_1}^{r_2} \frac{1}{z(y_h - T, (1 - \sigma)t, r, u_h)} \gamma r dr \]  
\[ = \frac{\gamma}{(1 - \sigma)t} \left( \int_{r_1}^{r_2} R(y_h - T, (1 - \sigma)t, r, u_h) dr + r_1 \Phi(r_1) - r_2 R_A \right), \]  

where again (30) and (32) follow from (29) and (31) on integrating by parts.

Further, in a circular city with linear transport costs, average differential land rent is half of average transport costs (Arnott and Stiglitz, 1981):

\[ (1 - \sigma)\tilde{t} = 2ADR, \]  

where

\[ ADR = \frac{1}{n} \left( \int_{r_1}^{r_2} R(y_c - T, (1 - \sigma)t_c, r, u_c) \gamma r dr \right) \]  
\[ + \int_{r_1}^{r_2} R(y_s - T, (1 - \sigma)t_s, r, u_s) \gamma r dr - \frac{\gamma r_2 R_A}{2}. \]  

For citizen landownership, we can show the following result.

**Proposition 5** Let \( \theta = 1 \). Then, in a circular city, starting from \( \sigma = 0 \), sign \( \frac{\partial u_l}{\partial \sigma} = -\text{sign} \frac{\partial u_h}{\partial \sigma} \). Further, if \( \theta_l = \theta_h = 1 \), increasing \( \sigma \) benefits the rich and hurts the poor if and only if

\[ \frac{n_l}{n_h} < \frac{\Phi(0) - \Phi(r_1)}{\Phi(r_1) - R_A}. \]  

**Proof.** Differentiating (6), (7), (30), (32), and (33) and simplifying at \( \theta = 1 \) and \( \sigma = 0 \) gives:

\[ \frac{\partial u_s}{\partial \sigma} = -\frac{\partial u_c}{\partial \sigma} \int_{r_1}^{r_2} R_u \gamma r dr. \]  

If \( \theta_l = \theta_h = 1 \), the poor live closer to the CBD and \( \frac{\partial u_l}{\partial \sigma} \) can be written as

\[ \frac{\partial u_l}{\partial \sigma} = \left\{ \begin{array}{c}
2 (z^{1h} - z^{1l}) \int_{r_1}^{r_2} R_u \gamma r dr (n_l(\Phi(r_1) - R_A) - n_h(R(0) - \Phi(r_1)))u_x^{1l}u_x^{1h} \\
\gamma r_1(\Phi(0) - R_A) \left( u_x^{1l} \int_{r_1}^{r_1} R_u \gamma r dr + u_x^{1h} \int_{r_1}^{r_2} R_u \gamma r dr \right) \\
+ u_x^{1l}u_x^{1h} (z^{1l} - z^{1h}) \left( (\Phi(0) - \Phi(r_1)) \int_{r_1}^{r_1} R_u \gamma r dr \int_{r_1}^{r_2} R_u \gamma r dr \right) \\
+ (\Phi(r_1) - R_A) \int_{r_1}^{r_2} R_u \gamma r dr \int_{r_1}^{r_1} R_u \gamma r dr \end{array} \right\}. \]  

18
Since $R_u < 0$, $\Phi(0) > \Phi(r_1) > R_A$ and $z^{1h} > z^{1l}$, the numerator is negative. The sign of the numerator in (37) is therefore the same as the sign of $n_l(\Phi(r_1) - R_A) - n_h(R(0) - \Phi(r_1))$.

**Example.** We proceed with the same example as in the previous section. In addition, we assume that all land is available for residential use, $\gamma = 2\pi$. Figure 5 shows the support for commuting subsidies in the linear city (grey lines) and circular city (black lines) with otherwise identical parameters. Transport costs are assumed identical for both groups, $t_h = t_l = 0.1$.

Note that the threshold value above which the poor oppose subsidies is higher in the circular city than in the linear city. This illustrates the property of monocentric models that the poor city residents may benefit from redistribution to the rich suburbians if rich move further out to the periphery and land there is relatively abundant.\(^6\) On the other hand, the region where the rich oppose commuting subsidies is larger in the circular city case.

\(^6\)Fujita (1989) shows this property in the case of absentee landownership.
5 Financing by income tax

In this section, we look at how the method of financing the subsidy changes the distributional effect of commuting subsidies. Suppose that commuting subsidies are financed by income taxes instead of head taxes. Assume further that only wage income is taxed proportionally at rate $\tau$. Then a consumer’s net wage income is $(1 - \tau)w$. Government budget balance requires

$$\tau \bar{w} = \sigma \tilde{t},$$

(38)

where $\bar{w} = (n_l w_l + n_h w_h)/(n_l + n_h)$ is the average wage. Note that voting now effectively occurs over two variables, $\tau$ and $\sigma$, but through (38) the choice of $\sigma$ determines the income tax rate.

Using (38), utility can now be written

$$u \left( w_i (1 - \sigma \tilde{t}/\bar{w}) - (1 - \sigma)tr + \theta_i (1 - \sigma)\tilde{t} - qz, z \right),$$

(39)

and we have

$$R_i^\sigma = \frac{rt - (w_i/\bar{w} + \theta_i) \tilde{t}}{z_i}, R_i^\tilde{t} = \frac{\theta_i - \sigma (w_i/\bar{w} + \theta_i)}{z_i}.$$

(40)

Suppose that $\theta_i = 0$. Then, (40) shows that the effect of the subsidy on the individual bid rent is positive if and only if $r/\tilde{r} > w_i/\bar{w}$, where $\tilde{r}$ is average distance: A voter’s net income rises with the subsidy to the extent that his wage income is smaller than average or the distance of his residence from the CBD is larger than average. The following result shows the effect of commuting subsidies for citizen landownership.

**Proposition 6** Assume a linear city with income tax financing. Then, (i) if $\theta = 1$, starting from $\sigma = 0$, sign $\frac{\partial u_i}{\partial \sigma} = -\text{sign} \frac{\partial u_h}{\partial \sigma}$. (ii) Further, if $\theta_l = \theta_h = 1$, increasing $\sigma$ benefits the rich and hurts the poor if and only if

$$\frac{n_l + n_h}{n_h} < \frac{(z^{1h} - z^{1l})(n_l w_l + n_h w_h)}{(w_h - w_l)\tilde{t}}.$$  

(41)

**Proof.** Totally differentiating the equilibrium equation system (6), (7), (11), (12) and
(13) at $\theta = 1$ and $\sigma = 0$, using (40), gives:

$$\frac{\partial u_c}{\partial \sigma} = -\left\{ \int_{r_1}^{r_2} R_u t u_x^0 u_x^{ll} u_x^2 (n_l + n_h)^2 \tilde{z}_t w_l - (1 - \theta_l)(n_l + n_h)^2 z^{ll}(n_l w_l + n_h w_h) \right. \left. -\theta_l(n_h^2(z^{ll} - z^{lh}) + (n_l w_l + n_h w_h)\tilde{t}) \right\} / \left\{ \left( \theta_l \int_{r_1}^{r_1} R_u n_h u_x^0 u_x^{ll} + \int_{r_1}^{r_2} R_u (\theta_l n_h u_x^0 + (1 - \theta_l)(n_l + n_h)u_x^{ll})u_x^{hh} \right)(n_l w_l + n_h w_h) \right\},$$

(42)

which implies (i). Setting $\theta_l = \theta_h = 1$ in (42) implies (ii) since $R_u < 0$. ■

**Example.** We continue with our previous example (with identical commuting costs of 0.1) and introduce income taxes. The support for commuting subsidies is shown in figure 6, where the dark lines show the case of income taxes and the light lines the case of lump sum taxation. The figure shows that the region of majority support for commuting subsidies expands, whereas the region of opposition by the rich is also larger under income taxation than under lump sum taxation.

**Tax on land rent income.** How do the results change when land rent is included in the taxable income base? At first sight, one might think that this would increase the
progressivity of income tax financed commuting subsidies if the rich earn more land rent income than the poor. However, this is not true. In fact, while increasing $\sigma$ now raises the tax rate necessary to finance the subsidy by reducing aggregate land rent, it also reduces the taxable income of each individual taxpayer (if average land rent falls with the subsidy). For our example, it turns out that the support for or against commuting subsidies remains unchanged.

**Tax deductibility of commuting expenses.** One method of income tax financing is to deduct commuting expenses from the income tax base. This is the method chosen in several European countries. What would be the effect of this method in our model? Intuitively, with linear income taxes, this would not change very much: the subsidy is the same for all individuals with the same transport costs and is linear in transport costs, whereas the tax burden rises linearly with income. If, however, the income tax is progressive, then richer individuals pay more to finance government subsidies, but also receive higher subsidies from deducting commuting expenses, for two reasons: first, because of higher transport costs, and second because of their higher marginal tax rate. The problem in this instance becomes more complex, since not only the rate of deductibility but also the total progressive tax schedule must be chosen.

6 Conclusion

In this paper, we have analysed the political economy of commuting subsidies in a monocentric city with two income classes. The model has the feature that there is no efficiency reason for commuting subsidies: if one takes the welfare of city residents and absentee landlords into account, commuting subsidies are Pareto inefficient (Fujita, 1989; Brueckner, 2003). Hence, the existence of commuting subsidies can be explained by the redistribution between groups with different political clout.

Some interesting conclusions emerge from our model. If land is owned by absentee landlords, all city residents may benefit from commuting subsidies if these reduce aggregate land rents. Hence, commuting subsidies may, at one extreme, be supported by all city residents. In the case of citizen landownership, commuting subsidies create a distributional antagonism between city residents and suburbians. In case PIC, this implies the intuitively
sensible result that the rich suburbians benefit from commuting subsidies at the expense of poor city residents. The converse case, however, is not as simple: if the poor live in the suburbs, it is not immediate that they benefit at the expense of the rich since the latter have higher marginal transport costs. Therefore the income effect of commuting subsidies may, on average be positive for the rich even though they have short commutes. As a result, the rich may benefit from commuting subsidies even if they live in the central city.

Some possible extensions to the model suggest themselves. First, one might think of extending the model to more than two income groups. One could then study whether the median group benefits from commuting subsidies. This, however, would not change the basic message. However, one might also look at models where there is no perfect segregation of income classes across space. This could be due to heterogeneous preferences within income classes (for instance, families versus singles) or difference in public goods supply (de Bartolome and Ross, 2003). Finally, an interesting and relevant extension would be a model with different transport modes with differing subsidy rates. For instance, some countries allow commuters to deduct only the costs of public transport from their income tax. Since the rich choose faster and more expensive transport modes (Brueckner, 2003), this might offset possibly regressive effects of commuting subsidies.

References


