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Structural Vector Autoregressions: Checking Identifying Long-run Restrictions via Heteroskedasticity[1]

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Abstract. Long-run restrictions have been used extensively for identifying structural shocks in vector autoregressive (VAR) analysis. Such restrictions are typically just-identifying but can be checked by utilizing changes in volatility. This paper reviews and contrasts the volatility models that have been used for this purpose. Three main approaches have been used, exogenously generated changes in the unconditional residual covariance matrix, changing volatility modelled by a Markov switching mechanism and multivariate generalized autoregressive conditional heteroskedasticity (GARCH) models. Using changes in volatility for checking long-run identifying restrictions in structural VAR analysis is illustrated by reconsidering models for identifying fundamental components of stock prices.

Key Words: Vector autoregression, heteroskedasticity, vector GARCH, conditional heteroskedasticity, Markov switching model

JEL classification: C32

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1 Introduction

An important problem in vector autoregressive (VAR) analysis is that the reduced form residuals are typically not the shocks that are of interest from an economic point of view. Determining shocks of economic interest is a main subject of structural VAR (SVAR) analysis. Proposals have been made how to identify shocks by placing restrictions on the impact effects, the long-run responses of the variables, or the signs of the impulse responses. More recently it has been discussed how to utilize changes in the volatility of the residuals or the variables for getting identifying information for the shocks. Important work on that topic is due to Rigobon (2003), Rigobon and Sack (2003), Normandin and Phaneuf (2004), Lanne and Lütkepohl (2008a), Bouakez and Normandin (2010) and Lanne, Lütkepohl and Maciejowska (2010). A survey of that literature appears in Lütkepohl (2013a).

In this study we first present a unifying framework for identifying structural shocks by imposing long-run restrictions possibly in combination with restrictions on the short-run effects. The latter are typically zero restrictions on the impact effects while long-run restrictions usually specify that specific shocks do not have a long-lasting or permanent effect on some of the variables whereas other shocks may have persistent effects. For example, in an important study in this context, Blanchard and Quah (1989) assume that demand shocks do not have permanent effects on output while the long-run impact of supply shocks is not restricted. Long-run effects of shocks are possible if some of the variables are integrated and, hence, have some persistence. In the Blanchard-Quah approach it is typically assumed that some variables appear in growth rates and the effects of the shocks on the levels or log-levels may be permanent. Thus, the variables enter in transformed stationary form in the VAR model. Such an approach is justified if there is no cointegration between the levels variables. For the case of cointegrated variables it is preferable to set up a vector error correction (VEC) model. King, Plosser, Stock and Watson (1991) propose an approach for imposing long-run restrictions for shocks in such cointegrated VAR models. It also allows for restrictions on the impact effects of shocks. As shown by Fisher and Huh (2014), other approaches of imposing long-run restrictions on SVAR models can be cast in this modelling framework as well. We first present the King et al. (1991) approach as a unifying framework for imposing long-run and short-run restrictions in SVAR models and then discuss the relation to the literature on identification via heteroskedasticity.

The long-run and short-run restrictions used for identifying structural shocks in VAR models are typically just-identifying. Hence, they can not be tested in a conventional framework. We show how changes in volatility can be used to obtain additional identifying information that can be used to test restrictions that are just-identifying in a conventional framework. To this end we give a brief survey of the main approaches that have been used for identifying SVARs through heteroskedasticity. Our review draws partly on Lütkepohl (2013a) but is more condensed and less technical. In contrast to Lütkepohl (2013a) we will pay special attention to VAR models with integrated and cointegrated variables.

Illustrative examples are presented from the literature that investigates the impact of fundamental shocks on stock prices. Since financial market series are often characterized by changes in volatility or conditional heteroskedasticity they have features that are an important precondition for applying the techniques developed in the related literature. Moreover, the identifying restrictions used in this context are sometimes on soft grounds so that bringing in alternative sources for identification is desirable.

This study is organized as follows. The general model setup is presented in Section 2. Specific models for changes in volatility that are useful for providing additional identifying information for shocks in SVAR models are reviewed in Section 3. Illustrations based on studies from the literature on the importance of fundamental shocks for stock price movements are discussed in Section 4 and conclusions are presented in Section 5.

2 SVARs with Integrated and Cointegrated Variables

We start from a K-dimensional reduced form VAR(p) model,

$$y_t = \nu + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t, \tag{1}$$

where ν is a $(K \times 1)$ constant term, A_j $(j = 1, \ldots, p)$ are $(K \times K)$ VAR coefficient matrices and u_t is a zero-mean white noise error term with nonsingular covariance matrix Σ_u, that is, $u_t \sim (0, \Sigma_u)$. Considering more general deterministic terms is easily possible. We avoid them in our basic model because they are of no importance for the structural analysis we are interested in.

The components of y_t may be integrated and cointegrated variables. For simplicity we assume that all variables are stationary ($I(0)$) or integrated of order one ($I(1)$). For our empirical example in Section 4 this assumption is general enough and it also covers the majority of examples in the related literature. Hence, it makes sense to avoid complications resulting from more general assumptions regarding the integration properties of the variables. If there are integrated variables, using the vector error correction (VEC) form of model (1) is often helpful. Assuming that there are r linearly independent cointegration relations, it can be written as

$$\Delta y_t = \nu + \alpha\beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_{p-1} \Delta y_{t-p+1} + u_t, \tag{2}$$

where Δ is the differencing operator such that $\Delta y_t = y_t - y_{t-1}$, η is a $(K \times 1)$ constant term, α is a $(K \times r)$ loading matrix, β is a $(K \times r)$ cointegration matrix and $\Gamma_1, \ldots, \Gamma_{p-1}$ are $(K \times K)$ coefficient matrices (see, e.g., Lütkepohl (2005) for the relation between the parameters in (1) and (2)).

SVAR analysis with homoskedastic reduced form residuals u_t assumes that the structural shocks, ε_t, are obtained from the reduced form residuals by a linear transformation $\varepsilon_t = B^{-1} u_t$. In other words, $B\varepsilon_t = u_t$, where B is such that the structural shocks are instantaneously uncorrelated, that is, $\varepsilon_t \sim (0, \Sigma_\varepsilon)$, and Σ_ε is a diagonal matrix. In fact, the structural variances are often normalized to one so that $\Sigma_\varepsilon = I_K$ and, hence, B is such that $\Sigma_u = BB'$. The matrix B is not uniquely determined by this relation. In fact, at least $K(K-1)/2$ further relations or restrictions are needed for uniquely identifying B.

Substituting $B\varepsilon_t$ for u_t in (1) or (2) the matrix B is easily recognized as the matrix of impact effects of the structural shocks. Thus, imposing restrictions on the impact effects constrains the elements of B directly. Exclusion restrictions are quite common in this context. In other words, a certain shock may be assumed to have no instantaneous effect on a particular variable. A recursive structure which implies a triangular B matrix is not uncommon in this framework. Exclusion restrictions are also often imposed on the instantaneous relations of the observed variables y_t. Such restrictions amount to restricting B^{-1}. Of course, a triangular matrix B^{-1} implies also a triangular matrix B.

Restrictions on the long-run effects of the shocks have also been used frequently for identifying B. To see how this is done it may be useful to consider the matrix of long-run effects of the reduced form errors from the Granger-Johansen representation (see Johansen (1995)) of the y_t corresponding to (2),

$$\Xi = \beta_\perp \left[\alpha'_\perp \left(I_K - \sum_{i=1}^{p} \Gamma_i \right) \beta_\perp \right]^{-1} \alpha'_\perp, \tag{3}$$

where $\beta_\perp$ and $\alpha_\perp$ are $(K \times (K-r))$ dimensional orthogonal complements of the $(K \times r)$ dimensional matrices β and α, respectively (see, e.g., Lütkepohl (2005, Chapter 9) for details). If the cointegration rank r is zero, the orthogonal complements matrices are simply replaced by $(K \times K)$ identity matrices so that the long-run effects matrix becomes

$$\Xi = \left(I_K - \sum_{i=1}^{p-1} \Gamma_i \right)^{-1}. \tag{4}$$

In each case the corresponding long-run effects of the structural shocks are given by ΞB. Since α and β have rank r if the cointegration rank is r, their orthogonal complements have rank $(K-r)$. Hence, Ξ also has rank $(K-r)$ and the same holds for ΞB because B is of full rank K. As a consequence, there can only be at most r shocks without any long-run effects because there can be at most r columns of zeros in ΞB. Generally, for a given Ξ matrix, restricting the product ΞB implies restrictions for B and, hence can help identify the structural shocks. Identification of the structural shocks in this framework is discussed by King et al. (1991) and an introductory account is given in Lütkepohl (2005, Chapter 9). Therefore we do not provide a full discussion here but just mention two specific issues for later reference that have to be taken into account: (1) Shocks without any long-run effect at all (with corresponding zero column in ΞB) need to be identified by other restrictions such as zero restrictions on the impact effects. Of course, if there is just one such shock, there is no need for further restrictions to identify that shock. (2) Since the rank of ΞB is $(K-r)$, it is in general not sufficient to identify all shocks by $K(K-1)/2$ restrictions on the matrix of long-run effects.

Because the case of a cointegration rank of zero is of specific interest, it may be worth discussing that situation in a little more detail here. As mentioned before, for $r=0$ the matrix of long-run effects ΞB is of full rank K and, hence, there cannot be shocks with no long-run effects at all, that is, there cannot be zero columns in ΞB. Thus, all K structural shocks have some long-run effects and it turns out that they can be identified via $K(K-1)/2$ suitable restrictions on ΞB. For example, restricting this matrix to be triangular is sufficient. Of course, if the cointegrating rank is zero, the VEC model (2) reduces to a VAR model in first differences,

$$\Delta y_t = \nu + \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_{p-1} \Delta y_{t-p+1} + u_t,$$

for which the accumulated long-run effects on the Δy_t are known to be $(I_K - \sum_{i=1}^{p-1} \Gamma_i)^{-1} B$. The accumulated effects on the first differences are just the long-run effects on the levels y_t. Considering the accumulated effects of a stationary system is of particular interest in this context because in a number of applied studies the restrictions and shocks are set up such that the accumulated long-run effects matrix is triangular. This case was first considered by Blanchard and Quah (1989) and in that case estimation of the structural parameters, i.e., the B matrix, is particularly easy (e.g., Lütkepohl (2005, Chapter 9)). We see another example in Section 4.

There is a large body of literature on long-run restrictions for identifying structural shocks in VARs. For example, there are proposals by Gonzalo and Ng (2001), Fisher, Huh and Summers (2000), and Pagan and Pesaran (2008). Fisher and Huh (2014) review that literature and discuss the relations between the various approaches. In the present context it is not important which approach is used for imposing long-run restrictions. They can all be combined with the identification procedures derived from time varying volatility discussed in the next section.

3 SVAR Models with Changes in Volatility

3.1 General Setup

So far we have discussed restrictions on the short- and long-run effects of shocks, that is, restrictions on B directly or via the restrictions on ΞB to make B unique for a given reduced form error covariance matrix Σ_u. Another way of getting a unique B matrix is available if there are two different covariance matrices, say Σ_1 and Σ_2, for example, if $E(u_t u_t') = \Sigma_1$ in the first part of the sample, say for $t = 1, \dots, T_1$, and $E(u_t u_t') = \Sigma_2$ in the second part of the sample ($t > T_1$). Then it is known from matrix algebra that there exists a matrix B and a diagonal matrix $\Lambda = \mathrm{diag}(\lambda_1, \dots, \lambda_K)$ such that

$$\Sigma_1 = BB' \quad \text{and} \quad \Sigma_2 = B\Lambda B'. \tag{5}$$

Using this B matrix to obtain structural shocks from the reduced form errors as $\varepsilon_t = B^{-1}u_t$, gives

$$E(\varepsilon_t \varepsilon_t') = \begin{cases} I_K, & t = 1, \dots, T_1, \\ \Lambda, & t > T_1. \end{cases}$$

These shocks satisfy the basic requirement of being instantaneously uncorrelated because Λ is diagonal. In fact, the matrix B is unique apart from changes in the signs and permutations of the columns if the diagonal elements of Λ are all distinct (Lanne et al. (2010)). In other words, if the latter condition holds, unique shocks are obtained by just imposing the basic requirement that they have to be instantaneously uncorrelated.

Note, however, that using the same transformation matrix B for the whole sample period, that is, for both volatility states, implies that the impact effects of the shocks are time-invariant and only the variances differ across states. All other requirements for uniqueness of the joint decomposition of the two covariance matrices do not affect the shocks substantively. The possibility for changing the sign of a column just means that we may consider negative instead of positive shocks and vice versa. Moreover, permuting columns of the B matrix just changes the ordering of the shocks and that can be chosen freely, as usual in SVAR analysis. Thus, fixing some ordering of the shocks and their sign is not restrictive.

Of course, the unique shocks obtained in this way may not be shocks of economic interest. That cannot be expected because no economics has been used in constructing them. However, if our minimal assumptions lead to unique shocks, then any further restrictions become over-identifying and, hence, testable. For example, if economic considerations suggest restrictions on the long-run effects of the shocks, these restrictions become testable in our framework even if they are just-identifying in a conventional setting and this is what makes it attractive to use changes in volatility in this context.

Of course, there may be more than two volatility regimes. If there are $M > 2$ regimes with corresponding covariance matrices $\Sigma_1, \dots, \Sigma_M$, the decomposition

$$\Sigma_1 = BB', \quad \Sigma_m = B\Lambda_m B', \quad m = 2, \dots, M, \tag{6}$$

with diagonal matrices $\Lambda_m = \text{diag}(\lambda_{m1}, \dots, \lambda_{mK})$ $(m = 2, \dots, M)$ may not exist. Hence, the decomposition (6) imposes testable restrictions on the covariance matrices. Therefore it can be checked with a statistical test whether the data are compatible with the decomposition and, hence, we can use B to transform the reduced form residuals into structural errors with time-invariant impact effects. Uniqueness of B (apart from ordering and sign) in this case follows if for any two subscripts $k, l \in \{1, \ldots, K\}$, $k \neq l$, there is a $j \in \{2, \ldots, M\}$ such that $\lambda_{jk} \neq \lambda_{jl}$ (Lanne et al. (2010, Proposition 1)). This identification condition has the advantage of being testable because, if there are M distinct volatility regimes, then the diagonal elements of the Λ_m matrices are identified and, hence, can be estimated consistently with a proper asymptotic distribution under common assumptions.

So far we have just considered finitely many volatility states. If conditional heteroskedasticity is generated by a GARCH process, then there is a continuum of conditional covariance matrices $\Sigma_{t|t-1}$. We discuss this case in more detail below and therefore just mention here that it can also be used for identifying shocks in SVAR models in much the same way as for finitely many covariance matrices. It should be clear by now that having two different covariance matrices is crucial for getting unique shocks. Having more covariance matrices is an advantage but not required for identification via changes in volatility.

3.2 Specific Models for Changes in Residual Volatility

Three main assumptions regarding the changes in volatility have been used in the SVAR literature: (1) exogenous changes in the unconditional residual covariance matrices in given time periods, (2) changes in the residual volatility generated by a Markov regime switching mechanism, and (3) volatility changes generated by a vector generalized autoregressive conditional heteroskedasticity (MGARCH) process. These three approaches will be presented formally in the following.

Assuming M exogenously determined volatility regimes, the reduced form residual covariance matrix in time period t can be represented as

$$E(u_t u_t') = \Sigma_t = \Sigma_m \quad \text{if} \quad t \in \mathcal{T}_m, \tag{7}$$

where $\mathcal{T}_m = \{T_{m-1} + 1, \dots, T_m\}$ $(m = 1, \dots, M)$ are M given volatility regimes. Here it is assumed that $T_0 = 0$ and $T_M = T$. The volatility change points T_m, for $m > 0$, are usually assumed known to the econometrician or they are determined with some preliminary statistical procedure. Rigobon (2003) considers this type of model of changes in the unconditional residual variance in his original article on identifying structural shocks in SVARs through heteroskedasticity. He also considers the possibility of misspecifying the change points. This type of model was also used in applications by Rigobon and Sack (2004), Lanne and Lütkepohl (2008a, 2008b) and Ehrmann, Fratzscher and Rigobon (2011).

If the reduced form error term is normally distributed, $u_t \sim \mathcal{N}(0, \Sigma_t)$, the model can be estimated by maximum likelihood (ML) (see, e.g., Lütkepohl (2005, Chapter 17) for details). If the errors are not normal, maximizing the Gaussian log-likelihood can be justified as a quasi ML estimation procedure or a GLS procedure may be used (see Lütkepohl (2013a)). In any case, if the VEC form of the VAR model is used, estimating the cointegration relations first from a reduced form VEC model and then keeping them fixed in the structural estimation may be useful. The estimators have standard asymptotic properties under common assumptions that can be used for inference in these models.

Another approach assumes that the volatility changes are generated endogenously within the model by a Markov regime switching (MS) mechanism. More precisely, the reduced form error term u_t is assumed to depend on a discrete, first order Markov process s_t $(t = 0, \pm 1, \pm 2, \dots)$ with M states, that is, $s_t \in \{1, \dots, M\}$ and transition probabilities $p_{ij} = \Pr(s_t = j | s_{t-1} = i)$, $i, j = 1, \dots, M$. This model was first considered in the present context by Lanne et al. (2010) who also propose a ML estimation procedure based on conditionally normally distributed u_t,

$$u_t | s_t \sim \mathcal{N}(0, \Sigma_{s_t}). \tag{8}$$

If the transition probabilities $p_{1j} = \cdots = p_{Kj} = \Pr(s_t = j)$, the state associated with period t is independent of the states of previous periods and we get a model with mixed normal residuals. In the present context such models are considered by Lanne and Lütkepohl (2010).

The MS model is in some sense rather restrictive because it does not allow the VAR coefficients to vary over time. Thus, it is more restrictive than the related models considered by Rubio-Ramirez, Waggoner and Zha (2005), Sims and Zha (2006) and Sims, Waggoner and Zha (2008), for example. The latter authors do not use the changes in volatility for identification of shocks, however, and, as we have discussed in Section 3.1, to use that device requires at least some time-invariance of the VAR coefficients. The M different covariance matrices corresponding to the different states of the Markov process are used for the identification of the shocks just as explained in Section 3.1. Notice that the model assumes conditional heteroskedasticity of the residuals. Moreover, it does not require the residuals to be in a particular state in each sample period but allows for the possibility that a particular period is in-between states by assigning a weighted sum of the states to it. Thus, the model can capture quite general forms of conditional heteroskedasticity as it is sometimes assumed to be present in financial time series.

ML estimation of these models is rather difficult, especially if there are many variables, lags and/or Markov process regimes. Herwartz and Lütkepohl (2011) discuss the related problems and present an EM algorithm based on Krolzig (1997). In practice the number of states for a suitable representation of the data will be unknown. Psaradakis and Spagnolo (2003, 2006) consider standard model selection criteria for selecting the number of Markov states and find that they work reasonably well. In contrast, standard testing procedures suffer from the problem that some parameters in the alternative model will not be identified if the null hypothesis imposes a reduced number of states. Hence, the usual tests have nonstandard properties (e.g., Hansen (1992), Garcia (1998)).

The model has been used in a number of applications. Examples are Lütkepohl and Netšunajev (2013), Herwartz and Lütkepohl (2011), Netšunajev (2013) and Velinov (2013) (see Lütkepohl (2013a) for a summary of some of that work). Moreover, Lanne and Lütkepohl (2010) apply their special case model with mixed normal residuals.

As mentioned earlier, the MS model can capture quite general forms of conditional heteroskedasticity. It may still be attractive in some situations to consider standard MGARCH models as alternatives if conditional heteroskedasticity is diagnosed. Normandin and Phaneuf (2004) propose these kinds of models for the SVAR context.

In their approach the reduced form errors u_t are assumed to be generated by an MGARCH process of the form

$$\Sigma_{u,t|t-1} = E(u_t u_t' | u_{t-1}, \dots) = B\Sigma_{\varepsilon,t|t-1}B', \tag{9}$$

where $\Sigma_{\varepsilon,t|t-1} = \mathrm{diag}(\sigma^2_{1,t|t-1}, \dots, \sigma^2_{K,t|t-1})$ is a diagonal matrix with

$$\sigma^2_{k,t|t-1} = \gamma_{k0} + \sum_{j=1}^{q} \gamma_{kj}\varepsilon^2_{t-j} + \sum_{j=1}^{s} g_{kj}\sigma^2_{k,t-j|t-j-1}, \quad k = 1, \dots, K. \tag{10}$$

In this setup the structural shocks, ε_t, are assumed to be instantaneously uncorrelated and have a diagonal MGARCH(q, s) structure. This type of model is sometimes called a generalized orthogonal GARCH (GO-GARCH) model and was proposed earlier by van der Weide (2002) and a closely related version is due to Vrontos, Dellaportas and Politis (2003). The standard model in the GARCH literature is actually an MGARCH(1,1) model and this is also the model considered by Normandin and Phaneuf (2004).

General identification conditions for this type of model are given by Sentana and Fiorentini (2001). They show that the structural shocks are identified (apart from sign changes and permutations) if the matrix $\Gamma'\Gamma$ is invertible, where Γ' is a matrix with kth row $(\sigma^2_{k,1|0}, \dots, \sigma^2_{k,T|T-1})$. Invertibility of $\Gamma'\Gamma$ means that the changes in volatility have to be sufficiently heterogeneous. For instance, at most one component can have constant conditional variances. If the identification conditions are satisfied and we choose $\varepsilon_t = B^{-1}u_t$ with the B matrix from (9), then the structural shocks are unique and the impulse responses, including the impact effects, are time-invariant.

Normandin and Phaneuf (2004) propose a two-step procedure for parameter estimation that fits a reduced form VAR(p) process in the first step and then estimates the GARCH and structural parameters by maximizing the corresponding Gaussian log-likelihood. Obviously, the estimation problem is a highly nonlinear optimization task that may pose a computational challenge for higher dimensional systems. Applications of the GARCH setup in SVAR analysis are reported by Normandin and Phaneuf (2004) and Bouakez and Normandin (2010).

In principle it is also possible to consider different types of MGARCH models instead of the GO-GARCH. For example, Weber (2010) and Strohsal and Weber (2012) consider a so-called SCCC MGARCH model. It does not ensure uncorrelated structural errors and may hence be regarded as problematic from a standard structural modelling point of view. The advantage of such alternative MGARCH models in the present context is not clear. Therefore we do not discuss them in more detail here.

4 Models for Stock Price Fundamentals

This section illustrates how the identification through heteroskedasticity technique can be used in practice. In particular, popular multivariate time series models dealing with stock price fundamentals are considered and their structural identification restrictions are tested so as to determine whether or not they are supported by the data. The empirical analysis in this section draws from the work of Velinov (2013) and extends it by examining impulse responses obtained from the structural models.

The topic of investigation is to what extent stock prices reflect their underlying economic fundamentals. This issue has naturally received a lot of attention in the empirical time series literature. In particular, many SVAR and SVEC models investigating this topic are based on the dividend discount model (DDM). The DDM claims that an asset's price is the sum of its expected future discounted payoffs (such as dividends). These pay-offs are necessarily linked to real economic activity such as real GDP, industrial production, company earnings and so on. An alternative view is that stock prices are to a large extent driven by speculation and, hence, their dependence on fundamentals is limited.

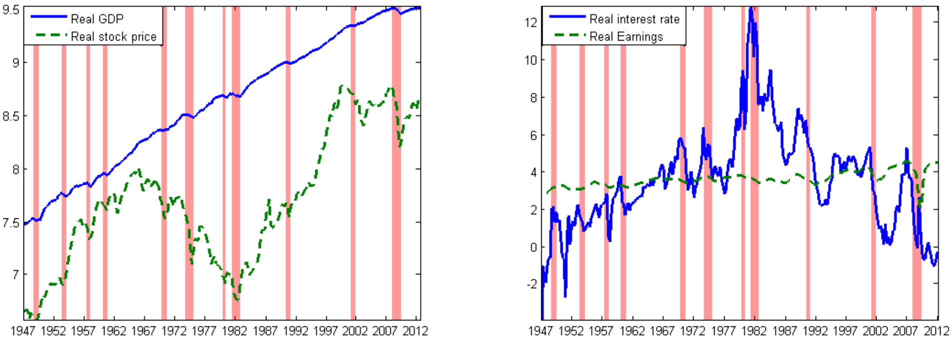
For illustrative purposes two of the models from Velinov (2013) are considered here. Both are trivariate models that have been used widely in the literature. The first model, called Model I in the following, consists of real GDP (Y_t), real interest rates (r_t) and real stock prices (s_t) ordered in that fashion. The second model, Model II, consists of real earnings (E_t) instead of real GDP and the other variables are the same. In other words, for Model I, $y_t = (Y_t, r_t, s_t)'$, while for Model II, $y_t = (E_t, r_t, s_t)'$.

Both models are related to the DDM and differ only in the proxy they use for real economic activity. They are both popular models. For instance, Model I is used in Lee (1995), Rapach (2001)[2], Binswanger (2004), Lanne and Lütkepohl (2010) and Jean and Eldomiaty (2010), while Model II is used in Binswanger (2004) and Jean and Eldomiaty (2010). They are also relatively simple models, consisting of only three variables, and will hence serve as a good illustration of the testing approach considered here.

The structural parameters in these models are identified by means of restrictions on the long-run effects matrix, denoted as ΞB as in Section 2. In particular, ΞB is always restricted to be lower triangular. In other words, the last shock has no long-run effects on real economic activity and the interest rate. Hence, many of the papers mentioned classify this as a non-fundamental shock. In contrast, only the first shock is allowed to have permanent effects on all the variables and in particular on the real GDP or real earnings variable and, hence, it is classified as being fundamental.

As was discussed in Section 2, the relevant (long-run) restrictions depend on the model used, i.e. whether it is in VEC or VAR form. Hence, before formally stating the relevant identification schemes to be considered, a brief

[2] In the case of Rapach (2001), Model I is a subset of a larger model, however, the identifying restrictions are the same.



(a) Y, s

(b) r, E

Figure 1: Data used with recession dates indicated by shaded bars.

note on the data and some basic diagnostic tests is in order to justify the model type.

4.1 Data and Model Specification

We use the same data as Velinov (2013). In other words, data on GDP, interest rates, stock prices and CPI are from the Federal Reserve Economic Database (FRED) whereas earnings data are from Robert Schiller's webpage.[3] The data is quarterly and ranges from 1947:I - 2012:III for Model I and until 2012:I for Model II, the slightly shorter range for Model II being due to missing observations on earnings for the last quarters of 2012. All variables are in real terms and in logs (except for the real interest rate series). The series are deflated by using the CPI inflation rate. Figure 1 plots the data used along with recession periods according to NBER dating marked by shaded bars.

Standard unit root tests indicate that all variables can be treated as $I(1)$. This is true even for the real interest rate series. Hence, Johansen (1995) trace tests are used to check for cointegration. There is substantial evidence of cointegration only for Model II, with a cointegration rank of one. Hence, Model I is set up as VAR in first differences, as in (1), while Model II is a

[3]Found at http://www.econ.yale.edu/ shiller/data.htm.

VEC of the form (2) with $r = 1$.

The traditional structural restrictions for identifying the shocks can be summarized as

$$\Xi B_{SVAR} = \begin{bmatrix} \star & 0 & 0 \\ \star & \star & 0 \\ \star & \star & \star \end{bmatrix} \quad \text{and} \quad \Xi B_{SVEC} = \begin{bmatrix} \star & 0 & 0 \\ \star & \star & 0 \\ \star & \star & 0 \end{bmatrix}, \tag{11}$$

for the SVAR, ΞB_{SVAR}, and the SVEC, ΞB_{SVEC}, case respectively. Here $\star$ denotes an unrestricted element. Note that the SVEC model has one more zero for its long-run effects matrix. This is because the rank of ΞB_{SVEC} in the VEC model is $K - r$, which is 2 in our case, hence a full column of zeros represents only 2 restrictions. Both identification schemes therefore give three independent restrictions. These just-identify the structural parameters in the traditional setup where volatility changes are ignored or at least not used for identification. Thus, in the traditional setup the restrictions cannot be checked with statistical tests. In the following it will be shown how changes in volatility can be used for checking the restrictions.

4.2 Checking Traditional Identification Restrictions via Changes in Volatility

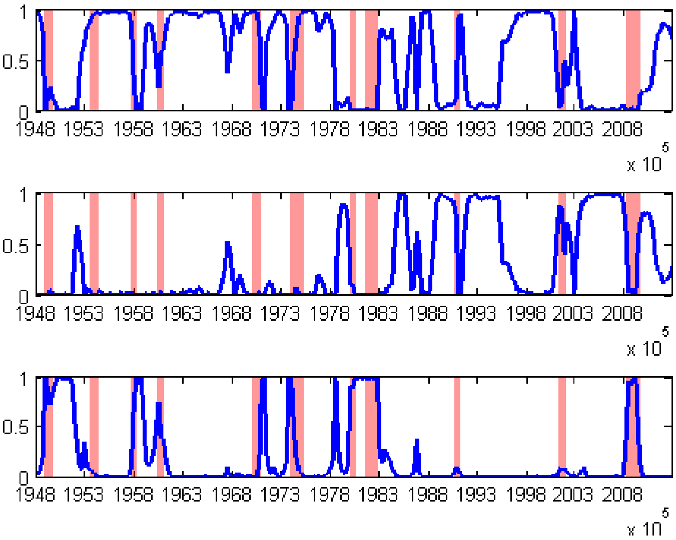
In both models changes in volatility are modelled via Markov processes with three states and the same VAR orders as in Velinov (2013), that is, Model I is a MS(3)-VAR(2) and Model II is a MS(3)-VEC model with two lagged differences of Δy_t in (2). Modelling changes in volatility by MS models makes sense here because it is quite flexible by allowing a number of different states and also mixtures of states. So it can capture conditional heteroskedasticity of quite general form. The three-state models have some support from standard model selection criteria and tests. The estimated state covariance matrices are given in Table 1 together with tests for the decomposition (6).

The variance estimates (the diagonal elements of the covariance matrices) tend to increase for each state, in particular for Model II. This means that each state captures slightly more volatile periods. Hence, the states can be classified as capturing periods of increasing volatility. The periods associated with each state can be seen in Figure 2, where smoothed state probabilities are plotted. In both models the first state captures mostly periods not associated with recessions while most recession periods are associated with States 2 and 3 which explains the higher volatility in these states. In fact, State 3 of Model II captures the last recession only, where apparently the volatility in our series was particularly high. Given the differences in volatility across the states, it makes sense to use the heteroskedasticity for identification purposes.

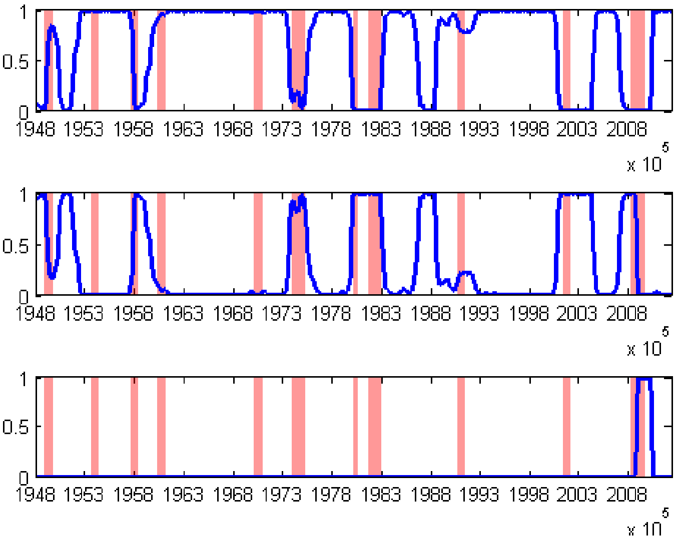
Since our approach relies on the covariance decomposition (6), the first question is whether such a decomposition is in line with the data. Using the likelihood ratio test from Lanne et al. (2010) for checking the decomposition gives the p-values in Table 1. They are both greater than the 5% critical level. Hence, we proceed by assuming that the data do not object to the decomposition in either of the two models. The next step is to check whether the decomposition can be used for identification of structural shocks.

To see whether the covariance decomposition in (6) is unique, we have to check the diagonal elements of the Λ_m matrices, that is, the relative variances, relative to State 1, for sufficient heterogeneity. The estimates are presented in Table 2. Even when accounting for the estimation uncertainty reflected in the standard errors, the estimated λ_{mk} are indeed quite heterogeneous. Hence, testing the identification conditions formally is plausible.

As noted in Section 3.1, uniqueness of the B matrix in (6) up to sign is ensured if all pairwise diagonal elements, $\lambda_{m,k}, m = 2, \dots, M, k = 1, \dots, K$, are distinct in at least one $\Lambda_m, m = 2, \dots, M$, matrix for a given pair. In other words, for our specific three-state models we have to check the null hypotheses given in Table 3. This can be done by a simple Wald test. Under standard assumptions the test distribution is asymptotically χ^2 with the number of degrees of freedom equal to the number of joint hypotheses tested. The relevant null hypotheses along with p-values of Wald tests are given in Table 3. They are all found to be rejected for Model I at conventional



(a) Model I



(b) Model II

Figure 2: Smoothed probabilities of State 1 (top), State 2 (middle) and State 3 (bottom) along with recession dates.

Table 1: Residual Covariance Matrices ($\times 10^3$) for 3-state MS-VAR Models

Model	I: $y_t = (Y_t, r_t, s_t)'$	II: $y_t = (E_t, r_t, s_t)'$
Σ_1	$\begin{bmatrix} 0.058 & - & - \\ 0.619 & 150.375 & - \\ 0.050 & -3.069 & 3.991 \end{bmatrix}$	$\begin{bmatrix} 0.329 & - & - \\ -0.287 & 226.053 & - \\ 0.096 & -3.015 & 2.129 \end{bmatrix}$
Σ_2	$\begin{bmatrix} 0.017 & - & - \\ 0.793 & 511.727 & - \\ 0.017 & 1.188 & 1.075 \end{bmatrix}$	$\begin{bmatrix} 1.872 & - & - \\ -1.483 & 1323.461 & - \\ 0.365 & -6.711 & 4.735 \end{bmatrix}$
Σ_3	$\begin{bmatrix} 0.218 & - & - \\ 4.231 & 2140.765 & - \\ 0.377 & 7.966 & 4.040 \end{bmatrix}$	$\begin{bmatrix} 343.388 & - & - \\ 250.287 & 11024.369 & - \\ 45.811 & 29.018 & 9.863 \end{bmatrix}$
Tests for state-invariant B		
p-value	0.065	0.600

significance levels. For Model II the first null hypothesis cannot be rejected at conventional levels, however. In other words, the B matrix obtained from the covariance decomposition may not be unique and, hence, the shocks are not identified purely by using the heteroskedasticity in the residuals. Given that two of the three identification tests clearly reject their respective null hypotheses, there is, however, some additional identifying information in the covariance decomposition that can be used to check the traditional identification restrictions given in (11). We will return to this issue shortly.

If the B matrix in (6) is unique up to sign, the restrictions in (11) can be tested by standard tests. Hence, let us for the moment ignore the possible uniqueness problem for Model II and use a likelihood ratio (LR) test for the restrictions in (11). The test has an asymptotic χ^2 distribution with 3

Table 2: Estimates and Standard Errors of Relative Variances of MS-VAR Models

Model	I: $y_t = (Y_t, r_t, s_t)'$		II: $y_t = (E_t, r_t, s_t)'$	
Parameter	estimate	standard error	estimate	standard error
λ_{21}	0.267	0.094	5.702	1.251
λ_{22}	0.277	0.089	5.925	1.311
λ_{23}	3.564	1.051	2.215	0.518
λ_{31}	0.845	0.252	1050.264	620.878
λ_{32}	3.777	1.004	48.731	36.057
λ_{33}	14.878	4.117	1.778	1.150

Table 3: p-values of Wald tests of Identification Conditions

H_0	Model I	Model II
$\lambda_{21} = \lambda_{22}, \lambda_{31} = \lambda_{32}$	0.011	0.263
$\lambda_{21} = \lambda_{23}, \lambda_{31} = \lambda_{33}$	0.000	0.014
$\lambda_{22} = \lambda_{23}, \lambda_{32} = \lambda_{33}$	0.001	0.016

degrees of freedom since in each case there are 3 independent restrictions if B is unique. The p-values of the tests based on $\chi^2(3)$ distributions are given in Table 4. Using standard significance levels, Model I cannot be rejected given a p-value of roughly 20%. On the other hand, Model II is clearly rejected at conventional significance levels. The latter result means that even though the B matrix is not unique, by using the heteroskedasticity of the residuals, the restrictions imposed are sufficient to reject the identification scheme for Model II. Note that nonuniqueness of B may imply that the actual number of degrees of freedom of the χ^2 distribution is lower than 3. Hence, the actual p-value may be even smaller than the one in Table 4. Thus, using the heteroskedasticity device, we can clearly reject the identification scheme for Model II and find some support for Model I.

Table 4: p-values of LR Tests for the Restrictions in (11)

Model I	Model II
0.207	1.045×10^{-7}

To summarize, so far we have tested the two structural identification schemes and have found that the one for Model I is supported, while the one for Model II is rejected. This means that the data do not object to the structural shocks for Model I which, hence, can be classified as in the literature as being fundamental and non-fundamental. In contrast, such shocks for Model II are rejected by the data. In the next subsection we will investigate the implications of ignoring the data evidence and labelling the first and second shocks as fundamental shocks 1 and 2, while the third shock is labelled as non-fundamental. These shocks will be used in an impulse response analysis.

4.3 Impulse Response Analysis

In this section we discuss the impulse responses obtained from the models presented in the foregoing. As usual, we determine confidence intervals around the impulse responses to assess the sampling uncertainty associated with these quantities. We use a fixed design wild bootstrap technique as proposed by Herwartz and Lütkepohl (2011) with reference to Goncalves and Kilian (2004) to construct confidence intervals for our impulse responses. More precisely, the series are bootstrapped as

$$y_t^* = \widehat{\nu} + \widehat{A}_1 y_{t-1} + \cdots + \widehat{A}_p y_{t-p} + u_t^*, \tag{12}$$

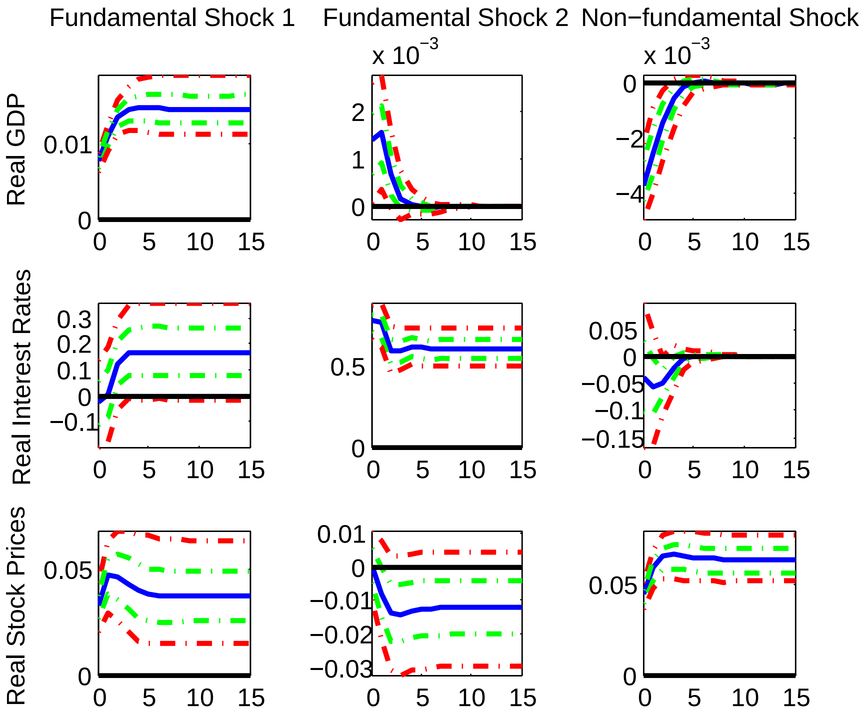
where $u_t^* = \psi_t \widehat{u}_t$ and ψ_t is a random variable, independent of y_t. It follows a distribution that is either 1 or -1 with a 50% probability. The VEC model is bootstrapped in a similar way, keeping lagged endogenous variables as in the original series. This technique usually generates slightly larger confidence bands than conventional bootstrapping techniques, however, it preserves the heteroskedastic properties of the residuals. The hats in (12) denote estimated coefficients.

Impulse responses for Models I and II are depicted in Figure 3. Confidence bands show the 95% and 68% individual bootstrap confidence intervals around the impulse responses. As was noted, for convenience, the shocks in both models are labelled in the same way, even though the identification scheme is rejected for Model II. Since the investigation is focused on stock price fundamentals we are particularly interested in the response of stock prices to the fundamental shocks.

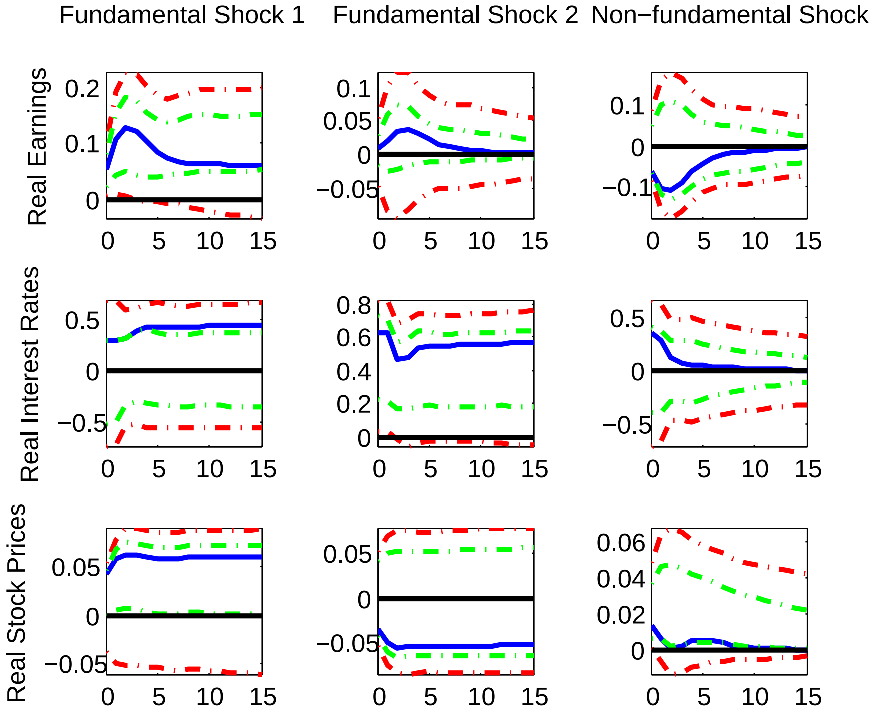
For Model I, the first fundamental shock has a positive and permanent impact on stock prices. Even the 95% confidence intervals are well away from the zero line so that a significant impact of the first fundamental shock on stock prices is diagnosed. For Model II, the impulse response function of stock prices to the first fundamental shock is still positive. Now the zero line is within the confidence band when a 95% level is used and it is at the lower bound when the 68% level is considered. It should be noted that this result is not merely due to using the fixed design wild bootstrap method. In fact, if the series is bootstrapped in a conventional way, the response is still not significantly away from zero at the 95% level.[4] Moreover, in Figure 3 it is seen that the second fundamental shock does not have a significant impact in Model II even if a 68% confidence level is used. Hence, if Model II is used for investigating the impact of fundamental shocks on stock prices, one may be led to conclude that fundamental shocks have little impact and that stock prices are mainly driven by speculation.

Thus, Models I and II lead to opposite conclusions and in a conventional analysis there is no statistical procedure to discriminate between them. In contrast, using the additional information from the changes in volatility allows us to discriminate between the two models. Clearly, Model II is rejected

[4]We have not explored the possibility to reduce the confidence bands by reduction techniques as discussed by Lütkepohl (2013b) because we are interested in the results obtained with the standard approach.



(a) Model I



(b) Model II

Figure 3: Impulse responses for models I and II with 95 and 68 percentile confidence intervals according to the fixed design wild bootstrap method.

by the data and, hence, we have a basis for deciding between the two models. This illustrates the virtue of using heteroskedasticity for testing identifying restrictions.

5 Conclusions

In this study we have considered long-run restrictions for identifying structural shocks in a VAR model. We have reviewed different approaches for checking restrictions that are just-identifying in a conventional setting by utilizing heteroskedasticity in the residuals, with special emphasis on the case where integrated and cointegrated variables are included in the models. The three main approaches for modelling changes in volatility that have been used in this context are exogenously generated changes in volatility, endogenous changes driven by a Markov process and volatility changes generated by MGARCH processes. We have briefly discussed the related techniques for identifying the shocks and, in particular, we have discussed how the additional identifying information obtained from heteroskedasticity can be used in a structural analysis.

To illustrate the identification through heteroskedasticity technique, two models investigating stock price fundamentals are considered. Both are trivarite models, based on the dividend discount model (DDM) and have widely been used in the empirical time series literature. The models labelled as I and II have different proxies of real economic activity. Model I uses real GDP, while Model II uses real earnings, the other variables being real interest rates and real stock prices ordered in that way. It is shown how the changes in volatility found in these models can be used for testing conventional identifying restrictions. Since the conventional restrictions are just-identifying, they cannot be formally tested in a standard setup and it turns out that the two models lead to quite different conclusions regarding the impact of fundamental shocks on stock prices. Model I indicates that they may have an important impact even in the long-run while no significant effect is found in Model II. Using the changes in volatility allows us to discriminate between the two models and it is shown that Model II is rejected by the data whereas support is found for Model I. In other words, support is found for the conclusion that stock prices are at least to some extent driven by fundamental shocks.

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