The Svensson versus McCallum and Nelson Controversy Revisited in the BMW Framework

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THE SVENSSON VERSUS MCCALLUM AND NELSON CONTROVERSY REVISITED IN THE BMW FRAMEWORK

Peter Bofinger and Eric Mayer, Würzburg 2006

Summary

This note shows that the Svensson versus McCallum and Nelson controversy battled in the Federal Reserve Bank of St. Louis Review (September/ October 2005) can be mapped into a static version of a New Keynesian macro model that consists of an IS-equation, a Phillips curve and an inflation targeting central bank (e.g., Bofinger, Mayer, Wollmershäuser, (2006); Walsh (2002)). As a contribution to literature we supplement the controversy by a forceful graphical analysis. The general debate centers on the question by which notion monetary policy should be implemented. The two sides have fundamentally opposite views on this issue. Svensson argues for targeting rules as a notion of optimal monetary policy, whereas McCallum and Nelson promote simple instrument rules. In this note we systematically analyze these two categories of monetary policy rules. In particular we show that the rule discussed by McCallum and Nelson (2005) imposes different degrees of variability on the economy compared to a targeting rule when monetary policy falls prey to measurement error. To our opinion the hybrid Taylor rule developed by McCallum and Nelson contradicts the original idea of simple rules as a heuristic for monetary policy making and should be rebutted for practical reasons.

Introduction

In this note we review the controversy between Svensson versus McCallum and Nelson battled in the Federal Reserve Bank of St. Louis Review (September/ October 2005) in a static version of a New Keynesian model (Bofinger, Mayer, Wollmershäuser (2006), Walsh (2002)). Reduced to its fundamentals we analyze the controversy by showing the advantages

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and disadvantages of simple instrument rules versus a regime of optimal monetary policy. Additionally we highlight the question whether supposedly optimal rules are more likely in fact to be closer to optimal than well –designed simple instrument rules in the face of measurement error. As a contribution to literature we supplement the controversy by a forceful graphical analysis. The debate centers on the question by which strategy monetary policy should be implemented. Svensson ((2005), (2003)) strongly promotes so-called targeting rules. Generally speaking targeting rules can be considered as a policy regime that implements a linear combination of target variables. Often they are consolidated first-order conditions of the central bank’s optimization problem. Simple rules can never be as good as optimal monetary policy regimes that use all relevant information. Nevertheless simple rules have the advantage of being robust to model uncertainty as they are not-fine tuned towards a specific model (McCallum (1988)). Therefore, in the light of uncertainty on the true structure of the economy there is a case for simple rules (Levine, Williams (2003)).

In this note we will compare simple versus targeting rules to clarify the controversy. An important contribution of our note is the explicit comparison of Taylor-like instrument rules as proposed by McCallum and Nelson and optimal monetary policy regimes as proposed by Svensson in a static model. We argue that the rule proposed by McCallum and Nelson should be rebutted for practical reasons.

**Targeting rules versus Taylor rules in a simple framework**

Let us assume that the economy is characterized by the following static version of a New Keynesian macro model (Bofinger, Mayer, Wollmershäuser (2006), Walsh (2002)):

\[
\begin{align*}
1 & \quad y = a - br + \varepsilon_1 \\
2 & \quad \pi = \pi_0 + dy + \varepsilon_2.
\end{align*}
\]

Thus the demand side of the economy is governed by an IS-equation (1), where (a) denotes a positive constant and (r) is the real interest rate. The white noise term \(\varepsilon_1\) is composed of shocks hitting the demand side (e.g., fiscal spending shocks, preference shocks). The supply side of the economy is given by a New Keynesian Phillips curve, where \(\pi_0\) denotes the inflation target of the central bank and (y) measures the output gap. The white noise shock \(\varepsilon_2\) measures all cost push shocks hitting the economy (e.g., shocks to wages). This reduced-form model is well established in literature and applied in a dynamic version by Svensson (2005) as
well as by McCallum and Nelson ((2005), (2003)). The controversy centers around the issue by which monetary policy rule the model should be closed. Svensson insists that monetary policy in many inflation targeting countries can be described as if implementing targeting rules (Svensson (2005), p. 615):

“[…] the fact is that central banks normally do not use the fallback options of simple instrument rules of Taylor and McCallum, […]. Advanced central banks attempt to do better, to fulfill their objectives as well as possible, to optimize. I am advocating targeting rules as a better way to describe and prescribe this kind of monetary policy than the simple instrument rules.”

This view is categorically rejected by McCallum and Nelson who instead claim that actual central bank behavior of inflation targeting countries is best described as if implementing simple instrument rules (McCallum, Nelson (2005), p.602):

“Our second point concerns Svensson’s contention that actual central banks noted for their inflation-targeting regimes, including the Reserve Bank of New Zealand, the Bank of Canada, and the Bank of England, use in practice procedures that are more reasonably characterized by the notion of a targeting rule rather than an instrument rule. We have already mentioned that none of these central banks has publicly adopted an explicit objective function. But, furthermore, we find that descriptions of their policy procedures provided by officials and economists of these central banks read more like instrument rules than specific targeting rules.”

Thus the general debate not only centers around the question by which notion monetary policy should be implemented, but there is also a fundamental dissent by which notion monetary policy is actually implemented.

**Targeting rules**

If monetary policy is conducted according to the notion of a targeting rule as suggested by Svensson (2005), it will exploit its full knowledge on the transmission structure of the economy. Targeting rules which are directly derived from the central banks objective function are labeled as so called ‘strict targeting rules’. Let us assume that monetary policy is guided the following loss function:
Accordingly the central bank aims at stabilizing squared deviations of the inflation rate from the inflation target $\pi_0$ while equally having a concern for economic activity. If $\lambda > 0$ such preferences are defined as a policy of flexible inflation targeting; if $\lambda = 0$ we speak of strict inflation targeting or an inflation nutter (Svensson (1999)). The targeting rule can be derived by solving the following Lagrangian (Bofinger, Mayer, Wollmershäuser (2004)):

$$L = (\pi - \pi_0)^2 + \lambda y^2.$$  

where we have used that the IS-equation is a non-binding resource constraint from the perspective of the central bank. In other words, the use of the instrument (r) is not associated with any real costs (Walsh (2003), p.524). Taking the derivative with respect to the output gap (y) and the inflation rate $\pi$ we arrive at the following two first order conditions:

$$\xi = \frac{2\lambda y}{d}$$

$$\xi = -2(\pi - \pi_0).$$

Eliminating the Lagrange multiplier $\xi$ and solving the resulting expression for the inflation gap $(\pi - \pi_0)$ retrieves the targeting rule (see Svensson (2005) p. 618; McCallum, Nelson (2005), p. 603):

$$(\pi - \pi_0) = -\frac{\lambda}{d} y.$$  

Thus the targeting rule is directly derived from the central bank’s objective function subject to the Phillips curve. Therefore it can be interpreted as a high level specification of monetary policy, as it holds with equality if monetary policy is conducted optimally. Graphically the targeting rule (TR) can be depicted as a downward sloping curve in the $(\pi, y)$- space (see
Figure 1). If the central bank puts a higher weight on inflation (decreasing $\lambda$) the slope of the targeting rule flattens, whereas in the opposite case it will increase.

Figure 1: Targeting rule

Svensson strongly argues in favor of targeting rules as they can be interpreted as a step towards a more micro-founded perception of monetary policy. A development which, according to Svensson, has been taking place long ago in other branches of macroeconomic theory (Svensson (2005), p. 617):

“The consumption function can be seen as an instrument rule for consumption behavior, whereas the Euler condition […] can be seen as a targeting rule for consumption. When I argue for the adoption of targeting rules rather than instrument rules in modeling monetary policy, I am arguing for a development in the theory of monetary policy that already happened, a long time ago, in the theory of consumption”.

A micro-founded perception of targeting rules can also be given as follows. In equilibrium it will have to hold that the marginal rate of transformation (MRT) between the inflation gap ($\pi - \pi_0$) and the output gap ($y$) has to be equal to the marginal rate of substitution (MRS):

\[(8) \quad MRT = MRS.\]

The marginal rate of substitution is determined by the loss function (3) of the central bank, which trades off the goal variables by a factor of $\lambda$. The marginal rate of transformation is
embedded in the slope of the Phillips curve (d). According to equation (8) in equilibrium it will have to hold that:

\[
\left(\frac{\partial L}{\partial (\pi - \pi_0)}\right) = \frac{(\pi - \pi_0)}{\lambda_y} = -\frac{1}{d} \left(\frac{\partial PC}{\partial (\pi - \pi_0)}\right)
\]

Therefore as noted by Svensson (2005) the targeting rule might simply be interpreted as an efficiency condition between the marginal rate of substitution and the marginal rate of transformation.

Equations (1), (2) and (7) give a complete description of the economy under a regime of targeting rules. To evaluate the concrete implications of targeting rules we solve the model in terms of endogenous variables (\(y\), \(\pi\) and \(r\)) as functions of the exogenous variables \(\varepsilon_1\) and \(\varepsilon_2\):

\[
y = -\frac{d}{d^2 + \lambda} \varepsilon_2
\]

\[
\pi - \pi_0 = \frac{\lambda}{d^2 + \lambda} \varepsilon_2
\]

\[
r_{\text{opt}} = \frac{a}{b} + \frac{1}{b} \varepsilon_1 + \frac{d}{b(d^2 + \lambda)} \varepsilon_2.
\]

The reduced form system is described by the following characteristics. The reaction of the central bank to cost push shocks depends on preferences \(\lambda\). A central bank that only cares about inflation (\(\lambda = 0\)), requires a strong real rate response and, accordingly, a large output gap. With an increasing \(\lambda\) the real interest rate response declines. In equilibrium \((\varepsilon_1 = \varepsilon_2 = 0)\) the real interest rate will be given by the neutral real short-term interest rate \(r_0 = a/b\). Note that shocks hitting the demand side of the economy can be completely undone by adjusting the real interest rate by a factor of \((1/b)\), which is not preference dependent.

If the economy is confronted with a positive cost push shock \((\varepsilon_2 > 0)\), Figure 2(a) shows that the Phillips curve is shifted upwards. If the central bank decides to remain passive, we can see that at a constant real interest rate the output gap remains zero. The inflation rate increases
from $\pi_0$ to $\pi_B$ (point B). It is important to note that this requires an equivalent increase in the nominal rate because inflation has gone up. Alternatively the central bank can increase the real interest rate in order to keep inflation at its target level. In this case, it has to accept a negative output gap $y_A$ (point A). Of course, the central bank can also decide to target intermediate combinations of $(y)$ and $\pi$ that lie on the Phillips curve between points A and B. If it is guided by the specific targeting rule, it will opt for point C. By the very definition of a first-order condition this ensures that, for a given value of private sector expectations and thus any location of the Phillips curve, that the loss function (3) is minimized. Graphically, the optimal outcome is thus described by the intersection of the Phillips curve PC$_i$ with the specific targeting rule of the central bank. In the case of a demand shock we can see from Figure 2(b) that monetary policy is always able to maintain the bliss combination by adjusting the real interest rate ($r$) accordingly.

**Figure 2:** Monetary policy outcomes under a specific targeting rule

**McCallum and Nelson’s hybrid Taylor rule**

As noted, McCallum and Nelson (2003), (2005) have a different perception of the actual conduct of monetary policy. As a reference point they propose the following hybrid rule:

(13) $\quad r = r_0 + \mu \left( \pi - \pi_0 \right) + \frac{\lambda}{d} y.$
This rule is a hybrid rule as it combines elements of classical Taylor rules with elements of specific targeting rules typically assigned to optimal monetary policy. It resembles a Taylor rule as monetary policy reacts to deviations of the inflation rate from the inflation target \((\pi - \pi_0)\) and the output gap \(y\). It reminds us of a specific targeting rule as the deviation of the inflation target from the inflation rate versus the output gap is scaled by the optimal trade-off ratio \((\lambda/d)\). The intuition behind this rule is simple. If the central bank is off its reaction function \((\pi - \pi_0) + (\lambda/d)y \neq 0\) it should react with its instrument \((r)\). If \((\pi - \pi_0) + (\lambda/d)y > 0\) real interest rates need to be lowered as e.g., the inflation gap is smaller than the scaled output gap whereas in the opposite case interest rates are too lose given the state of the cycle.

**McCallum and Nelson’s rule as a simple instrument rules**

Let us first compare the hybrid Taylor with a targeting rule for small values of \(\mu\). For small values of \(\mu\) equation (13) becomes a simple instrument rule like the original Taylor rule (1993). At the heart of simple rules lies the notion that they are not derived from an objective function. More fundamentally they are not even based on a particular model. Instead the coefficients are chosen ad-hoc, based on the experiences and skills of the monetary policymakers. The most prominent version of a simple rule is the original Taylor rule (1993). According to this rule the actual real interest rate is defined as the sum of the equilibrium real interest rate \(r_0\) adjusted for the deviation of the inflation rate from the inflation target and the output gap. The relative weight attached to the gaps is determined by the coefficients \(e\) and \(f\):

\[
(14) \quad r = r_0 + e(\pi - \pi_0) + fy, \quad e,f > 0.
\]

If we assume for the sake of exposition that \((\lambda/d)\) is approximately one and set \(\mu\) equal to 0.5 the original Taylor rule (14) is just a special case of equation (13), where \(e=\mu\) and \(f=\mu(\lambda/d)\). Hence if monetary policy sets \(\mu\) sufficiently low we can evaluate the monetary policy outcomes of rule (13) by analyzing the Taylor rule (14). Under a regime of Taylor rules the economy is described by the equations (1), (2) and (14). The solution of the model in terms of endogenous variables \(y, \pi\) and \((r)\) as a function of exogenous variables \(\varepsilon_1\) and \(\varepsilon_2\) is given as follows:
Equations (15) and (16) show immediately that a simple rule is suboptimal compared to a targeting rule as demand shocks have an impact on the inflation rate and the output gap. Graphically the Taylor-rule translates into a downward sloping aggregate demand curve in the $(\pi,y)$ space.

\begin{align*}
(15) & \quad y = -\frac{be}{1+bf+dbe}\varepsilon_2 + \frac{1}{1+bf+dbe}\varepsilon_1 \\
(16) & \quad \pi = \pi_0 + \frac{d}{1+bf+dbe}\varepsilon_1 + \frac{1+bf}{1+bf+dbe}\varepsilon_2 \\
(17) & \quad r^{Taylor} = r_0 + \frac{ed+f}{1+bf+bed}\varepsilon_1 + \frac{e}{1+bf+dbe}\varepsilon_2.
\end{align*}

Figure 3: Monetary policy outcomes under a classical Taylor rule

If the economy is hit by a negative demand shock the IS-curve in the upper panel of Figure 3(a) shifts leftwards. In response to the decrease of the output gap from $0$ to $y^\prime$ the central bank lowers real interest rates – by moving along the MP($\pi_0$)-line – from $r_0$ to $r^\prime$, which leads
to the output gap $y'$. In the lower panel the aggregate demand curve has to shift. Its new locus is obtained by the fact that it has to go through a point, which is defined by the new output gap ($y'$) and the (so far) unchanged inflation rate $\pi_0$. The new equilibrium is reached by the intersection of the shifted aggregate demand curve with the unchanged Phillips-curve. It is characterized by an output decline to $y_1$ (which is less than $y'$) and an inflation rate $\pi_1$. The decline of the output gap from $y'$ to $y_1$ and the inflation rate to $\pi_1$ (instead of $\pi'$) is due to fact that the central bank additionally reduces the real interest rate, because the Taylor rule requires a lower real rate because of the decline in the inflation rate. In the upper panel this is reflected by a downward shift of the MP line, which intersects with the IS line at the same output level, which results from the intersection of the AD line with the Phillips curve in the lower panel. For a graphical discussion of a cost push shock we see that in the $y$-$\pi$ space (see Figure 3(b)), the Phillips curve is shifted upwards which increases the inflation rate to $\pi'$. In this case the Taylor rule requires a higher real interest rate which leads to a negative output gap $y_1$. The reduced economic activity finally dampens the increase of the inflation rate to $\pi_1$.

**Comparing simple versus specific targeting rule**

Let us now compare a targeting rule as proposed by Svensson with the rule proposed by McCallum and Nelson for small values of $\mu$. A simple rule like the Taylor rule which only reacts mechanically to inflation ($\pi$) and output ($y$) can generally not be as good as a rule that uses all relevant information on the concrete values of macroeconomic shocks $\varepsilon_1$ and $\varepsilon_2$. Within this simple framework we can identify the mechanisms that prevent simple rules from being as good as optimal rules. By comparing the coefficients in the reduced form instrument rules we see for the case of a demand shock that optimal and simple monetary policy rules can only be identical if the reaction coefficient in front of $\varepsilon_1$ in equation (12) and (17) are the same. Thus equating coefficients the following equation would have to hold:

$$\frac{1}{b + 1/(ed + f)} = \frac{1}{b}.$$  

(19)

Equation (19) however can only be true if either (e) or (f) go to infinity. Obviously for a parameterization as originally proposed by Taylor ($e=f=0.5$) this can never be the case. For a cost push shock both types of rules may lead to identical outcomes. Equating the reaction coefficients leads to:
Thus in the case of a cost push shock both types of rules may potentially yield identical results. In sum, considering the full universe of possible shocks simple rules are suboptimal compared to targeting rules. This might pose the question why to use simple rules at all! Following McCallum (1988) there is a strong argument for simple instrument rules in an uncertain environment. Empirical evidence seems to suggest that in an environment of model uncertainty simple rules perform on average better than a targeting rule taken from one model to another model (Levine and Williams, (2003)). McCallum and Nelson state (McCallum, Nelson (2005), p.599)

“Consequently, an attractive approach to policy design, promoted, for example, by McCallum (1988, 1999), is to search for an instrument rule that performs at least moderately well-avoiding disasters- in a variety of plausible models.”

**McCallum and Nelson’s rule as an optimal instrument rule**

The hybrid Taylor rule (13) as proposed by McCallum and Nelson can potentially lead to identical monetary policy outcomes as a regime of optimal monetary policy. A possible equivalence of a targeting rule as proposed by Svensson and the hybrid rule can be shown analytically by solving the IS-equation and the Phillips-curve in conjunction with the hybrid Taylor rule. Equations (1), (2) and (13) give a complete description of the economy. The implied reduced forms are given by:

\[
\lambda = \frac{d(1+bf)}{eb}.
\]

(20)

Thus in the case of a cost push shock both types of rules may potentially yield identical results. In sum, considering the full universe of possible shocks simple rules are suboptimal compared to targeting rules. This might pose the question why to use simple rules at all! Following McCallum (1988) there is a strong argument for simple instrument rules in an uncertain environment. Empirical evidence seems to suggest that in an environment of model uncertainty simple rules perform on average better than a targeting rule taken from one model to another model (Levine and Williams, (2003)). McCallum and Nelson state (McCallum, Nelson (2005), p.599)

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\[
y = \frac{d}{d+\mu b(d^2+\lambda)} \varepsilon_1 - \frac{bd}{d/\mu + b(d^2+\lambda)} \varepsilon_2
\]

(21)

\[
\pi = \pi_0 + \frac{d^2}{d+\mu b(d^2+\lambda)} \varepsilon_1 + \frac{d+\mu b\lambda}{d+\mu b(d^2+\lambda)} \varepsilon_2.
\]

(22)

\[
r = \frac{(d/\mu)\rho_0 + (d^2+\lambda)a}{(d/\mu) + b(d^2+\lambda)} + \frac{(d^2+\lambda)}{(d/\mu) + b(d^2+\lambda)} \varepsilon_1 + \frac{d}{(d/\mu) + b(d^2+\lambda)} \varepsilon_2.
\]

(23)
As claimed by McCallum and Nelson (2005) by comparing (21), (22) and (23) to (10), (11), and (12) it prevails that the ‘Svensson economy’ and the ‘McCallum and Nelson economy’ produce identical results in means and variances if we let \( \mu \) go to infinity. Although this might be analytically straightforward a much clearer cut intuition is given by the graphical exposition. Algebraically, the AD-curve can be derived by inserting the hybrid Taylor rule (13) into the IS-curve (1):

\[
(\pi - \pi_0) + \frac{d/\mu + b\lambda}{db} y = \frac{1}{d\mu} \varepsilon_1.
\]

As one can see quite easily for the case of \( \mu \) approaching infinity, the slope of the aggregate demand function converges towards \( (\lambda/d) \) which is identical to the slope of the targeting rule. Additionally from equation (13) we know that the slope of the hybrid Taylor-rule in the \((r;y)\)-space becomes vertical if \( \mu \) goes to infinity. Thus for this case the graphical analysis is identical to the case of optimal monetary policy. If the economy is hit by a negative demand shock \( (\varepsilon_1<0) \) (see Figure 4 (a)) the IS-curve is shifted to the left so that at a constant real interest rate \( (r_0) \) the output gap becomes negative \( (y<0) \).

**Figure 4**: Monetary policy outcomes under the hybrid Taylor rule

In the lower panel this translates into an inflation rate \( \pi_1 \) that would be below the central banks inflation target \( \pi_0 \). If monetary policy is conducted according to the hybrid Taylor-rule
(HR) as proposed by McCallum and Nelson (2005) the real interest rate will be lowered from \( r_0 \) to \( r_1 \) so that the output gap is closed and inflation is at its target level. Therefore demand shocks do not impose any costs on society if \( \mu \) goes to infinity. If the economy is confronted with a cost push shock \( \varepsilon_z > 0 \), the lower panel 4(b) shows that the Phillips curve will be shifted upward. If the central bank is guided by McCallum and Nelson’s (2005) hybrid Taylor rule (HR) it trades-off the inflation loss versus the output loss by a ratio of \( \lambda/d \). Graphically the optimal outcome is thus described by the intersection of the Phillips curve with the hybrid Taylor rule of the central bank.

Thus the analysis shows that the hybrid Taylor rule can change its nature depending on the size of \( \mu \). This can equally be shown with the help of efficiency frontiers. Under a specific targeting rule as proposed by Svensson (2005) the variance of the output gap \( y \) and the inflation gap \( \pi - \pi_0 \) are given as follows:

\[
\begin{align*}
\text{(25)} & \quad \text{Var}(y) = \left( \frac{d}{d^2 + \lambda} \right)^2 \text{Var}(\varepsilon_z) \\
\text{(26)} & \quad \text{Var}(\pi - \pi_0) = \left( \frac{\lambda}{d^2 + \lambda} \right) \text{Var}(\varepsilon_z).
\end{align*}
\]

In the case of the hybrid Taylor rule as proposed by McCallum and Nelson (2005) the variances are defined by the following expressions:

\[
\begin{align*}
\text{(27)} & \quad \text{Var}(y) = \left( \frac{d}{d + \mu b \left( d^2 + \lambda \right)} \right)^2 \text{Var}(\varepsilon_1) + \left( \frac{bd}{d/\mu + b \left( d^2 + \lambda \right)} \right)^2 \text{Var}(\varepsilon_2) \\
\text{(28)} & \quad \text{Var}(\pi - \pi_0) = \left( \frac{d^2}{d + \mu b \left( d^2 + \lambda \right)} \right)^2 \text{Var}(\varepsilon_1) + \left( \frac{d/\mu + b \lambda}{d/\mu + b \left( d^2 + \lambda \right)} \right)^2 \text{Var}(\varepsilon_2).
\end{align*}
\]

If we vary \( \lambda \) with step size 0.01, we can compute a stylized efficiency frontier for the case of an optimal monetary policy regime as suggested by Svensson (see Figure 5). The efficiency frontier divides the \text{Var}(y) and \text{Var}(\pi - \pi_0) plane in two regions. All points that lie below the line are not feasible. All points that lie above the frontier are feasible but not efficient. The line itself represents all feasible and efficient combinations of variances of the inflation gap.
and the output gap. The convex shape of the efficiency frontier reflects the trade-off induced by cost push shocks. A lower variance of the inflation gap (output gap) can only be realized at the cost of an increasing variance of the output gap (inflation gap). For increasing values of $\mu$ the feasible frontier generated by the hybrid Taylor rule converges towards the efficiency frontier generated by the targeting rule. For small values of $\mu$ the hybrid Taylor-rule shares important characteristics of simple rules, in particular that it is suboptimal, whereas for large values of $\mu$ it produces observationally equivalent outcomes as a targeting rule.

![Efficiency Frontiers](image)

**Figure 5**: Efficiency Frontiers

Therefore given the standard model described by the equations (1) and (2) in conjunction with a hybrid Taylor rule or a targeting rule McCallum, Nelson (2005, p.606) are correct to state:

"Thus, there appears to be little to choose from between targeting rules and instrument rules […]".

**Targeting rules or hybrid Taylor rules? The case of measurement error**

In a given New Keynesian model, where the interest rate impacts on inflation and output in the same period the two rules produce identical results in the limit if $\mu$ goes to infinity. Demand shocks do not impose any costs on society, whereas in the case of cost push shocks the central bank chooses its preferred stabilization mix, depending on its preferences $\lambda$ and the slope of the Phillips curve (d). Thus, the standard model is inappropriate to discriminate between targeting rules and a hybrid Taylor for large values of $\mu$. For small values of $\mu$ the question whether to implement simple or optimal policy largely depends on its knowledge of
the true structure of the economy. In an uncertain environment it seems reasonable to fall back on simple rules whereas in the case of full knowledge on the transmission structure targeting rules are superior. As a hybrid Taylor rule and a targeting rule produce identical outcomes one may ask: What is the fuzz all about? An additional argument of Svensson to discredit the rule discussed by McCallum and Nelson (2005) is to introduce measurement error. Let us assume that the central bank falls prey to measurement error and bases its interest rate decision upon a flawed information set. This means in particular that the central bank can only imperfectly observe private plans on consumption and pricing decisions. Accordingly an observational error $\varepsilon_3$ influences the interest rate setting behaviour of the central bank. Additionally let us assume that the inflation rate, the output gap, and the interest rate are all simultaneously determined. Accordingly the measurement error $\varepsilon_3$ becomes an exogenous variable. To compute the impact of measurement error one has to modify the rules as follows:

(29) \[ (\pi - \pi_0) + \frac{\lambda}{d} y + \varepsilon_3 = 0 \]

(30) \[ r = r_0 + \mu \left( (\pi - \pi_0) + \frac{\lambda}{d} y + \varepsilon_3 \right). \]

As beforehand the ‘Svensson economy’ and the ‘McCallum and Nelson economy’ are identical in outcomes for large values of $\mu$.

Extended for measurement errors the aggregate demand curves can be written as follows:

(31) \[ \pi = \pi_0 - \left[ \frac{1}{b \mu} + \frac{\lambda}{d} \right] y - \frac{1}{b \mu} \varepsilon_1 - \varepsilon_3 \]

(32) \[ \pi = \pi_0 - \frac{\lambda}{d} y - \varepsilon_3 \]

As $(\mu \lambda)$ is strictly positive the hybrid Taylor rule (31) will always be steeper in the $(y, \pi)$-space than the targeting rule (32). The relevance of measurement error for the comparative advantages of targeting rules versus hybrid Taylor rules prevails quite clearly in the graphical analysis (see Figure 6). Starting from the initial equilibrium $(\pi, 0)$ measurement error $\varepsilon_3$ moves the hybrid Taylor rule and the targeting rule by an identical shift. As the slope of the
hybrid Taylor rule is always steeper than the slope of the targeting rule; the inflation and output gaps are smaller than the corresponding gaps under a targeting rule.

Figure 6: Measurement error

This analysis shows that measurement error is no intrinsic argument for targeting rules. On the contrary the damage imposed by measurement error is larger under a targeting rule than under a hybrid Taylor rule. Depending on the absolute size of the measurement error this might even imply that the overall loss ranking between targeting rules and hybrid Taylor rules can be reversed.

**Measurement error and predetermined private plans**

If we modify the timing patterns of the economy, hybrid Taylor rules display central flaws. Let us assume that the private sector settles its labor and good markets contracts conditioned on the expected real interest rates $r^e$. So we have to modify the IS-equation as follows:

\[
y = a - br^e + \varepsilon_i.
\]

Then in the rational expectations equilibrium the private sector expects the measurement error of the central bank to equal zero. So the private sector expects a relation between the expected interest rate $r^e$, and its plans (and actual inflation and output gap), $\pi$ and $(y)$, given by

\[
r^e = r_0 + \mu \left( (\pi - \pi_0) + \frac{\lambda}{d} y \right),
\]
and chooses plans that fulfill (33) and (2). For given actual inflation and output gap equal to plans, the central bank then ex-post chooses the interest rate according to (30), in which case there is a large interest rate volatility for $\mu$, since the actual output and inflation then are given and don’t instantaneously respond to the interest rate. The solved model of endogenous variables in terms of exogenous variables for inflation and output is given as follows:

$$y = \frac{d}{d + b\mu + b\mu d^2} \varepsilon_1 - \frac{b\mu d}{d + b\mu + b\mu d^2} \varepsilon_2$$  \hspace{1cm} (35)$$

$$\pi = \pi_0 + \frac{d^2}{d + b\mu + b\mu d^2} \varepsilon_1 + \frac{d + b\mu \lambda}{d + b\mu + b\mu d^2} \varepsilon_2$$  \hspace{1cm} (36)$$

By assumption the measurement error does not influence the output or inflation gap. Nevertheless as the central bank is subject to measurement error the economy suffers under severe real interest rate volatility.

Figure 7: Measurement error and predetermined plans.

Figure 7 shows in the upper panel that the hybrid Taylor rule (HR) randomly drifts in the $(y;\tau)$-space, where the drawn bounds $\tau^{\text{min}}$ and $\tau^{\text{max}}$ might for the sake of graphical exposition be interpreted as the two standard deviation intervals given a normal distribution of $\varepsilon$. This leads Svensson to the conclusion (Svensson (2005), p. 621):
“Central bankers, beware of McCallum and Nelson’s instrument rule!”

Assuming more realistically that the economy is populated by firms that depend on financial intermediation the impact of instrument volatility might be detrimental for the financial industry if banks face costs in the adjustment of their loan portfolios (Hülsewig, Mayer, Wollmershäuser (2006)). Additionally a hybrid Taylor rule becomes completely impracticable for large values of \( \mu \) if there are lags between a shock and the interest rate reaction of the central bank. Figure 8 indicates to a certain extend, based on the actual data which interest rate the hybrid Taylor rule might have recommended.

\[ \text{Data: ECB’s AWM database, Economic data-fred; Calibration, } \mu=50, (\lambda/d)=1. \]

**Figure 8:** A hybrid Taylor rule for large values of \( \mu \).

Quite arguably the hybrid Taylor rule was not coined towards such an environment and economic agents are not accustomed to the rule, but the argument underlines that for large values of \( \mu \) the rule looses its character as a simple robust rule. To the defense of McCallum and Nelson they claim (McCallum and Nelson (2005), p. 630):

“[…] the rule \{ with a very large value of \( \mu \), \} is one that we say (explicitly) that we have not recommended \}. It was used in our 2004 paper as an implementation device; in our current paper, it serves to illustrate our analytical claim, namely, that our instrument rule (actually, class of rules) is usually superior in performance, with respect to Lars’s own criterion, to the targeting rule that it approximates.”
Concluding remarks

In this note we have reviewed the current controversy between Svensson versus McCallum and Nelson battled in the Federal Reserve Bank of St. Louis Review (September/ October 2005). We have shown that the debate can be mapped into a static version of a New Keynesian macro model (Bofinger, Mayer, Wollmershäuser (2006), Walsh (2002)). As a contribution to literature we have supplemented the controversy by a forceful graphical analysis. Reduced to its fundamentals we have analyzed the controversy by showing the advantages and disadvantages of simple instrument rules versus a regime of optimal monetary policy. Additionally we highlight the question whether supposedly optimal rules are more likely in fact to be closer to optimal than well –designed simple instrument rules in the face of measurement error. Simple rules can never be as good as optimal monetary policy regimes that use all relevant information. More concretely we have analyzed the hybrid Taylor rule as discussed by McCallum and Nelson. Analytically there is little too choose between a targeting rule as proposed by Svensson and a hybrid Taylor rule as suggested by McCallum and Nelson for large values of $\mu$. We can only discriminate between these two types of rules if we introduce measurement error. In the light of measurement error the arguments for one or the other rule are mixed. Under realistic assumptions on the frequency of private plans and the frequency by which the central bank might change its instrument the rule developed by McCallum and Nelson is likely to introduce a high degree of real interest rate volatility. Additionally the implementation of hybrid rules for large values of $\mu$ contradicts the original idea of simple rules. Simple rules where thought of as a heuristic that equipps policymakers with a robust device to set interest rates reasonably well in an environment when monetary policy is subject to uncertainty concerning the true structure of the economy. Therefore we conclude that McCallum and Nelson’s rule becomes completely impracticable in a richer model that incorporates observation and transmission lags. In such a context the implied real interest rate volatility might be detrimental for any kind of financial intermediation.
References


