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Operational Conditions in Regulatory Benchmarking Models - A Monte Carlo Analysis *

Maria Nieswand Stefan Seifert †

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Abstract

Benchmarking methods are widely used in the regulation of firms in network industries working under heterogeneous exogenous environments. In this paper we compare three recently developed estimators, namely conditional DEA (Daraio and Simar, 2005, 2007b), latent class SFA (Orea and Kumbhakar, 2004; Greene, 2005), and the StoNEZD approach (Johnson and Kuosmanen, 2011) by means of Monte Carlo simulation focusing on their ability to identify production frontiers in the presence of environmental factors. Data generation replicates regulatory data from the energy sector in terms of sample size, sample dispersion and distribution, and correlations of variables. Although results show strengths of each of the three estimators in particular settings, latent class SFA perform best in nearly all simulations. Further, results indicate that the accuracy of the estimators is less sensitive against different distributions of environmental factors, their correlations with inputs, and their impact on the production process, but performance of all approaches deteriorates with increasing noise. For regulators this study provides orientation to adopt new benchmarking methods given industry characteristics.

JEL-Codes: L50, Q50, C63

Keywords: Monte Carlo Simulation, Environmental Factors, StoNEZD, Latent Class SFA, Conditional DEA, Regulatory Benchmarking

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1 Introduction

External factors outside the control of firms are a proximate cause of heterogeneous technologies.[1] When regulatory outcomes, such as price or revenue caps, rely on firm comparison, the heterogeneity of technologies requires an appropriate consideration for obtaining optimal solutions (Shleifer, 1985). If external factors are not considered, the regulatory outcomes, i.e., price or revenue caps, diverge from the optimum and can lead to welfare losses due to inefficiently low or high caps, or bankruptcies of the regulated firms.

For natural monopolies, the regulator aims to implement a cap scheme in which the regulated prices equal the average costs of firms, i.e., efficient costs.[2]. This is, however, only achievable if the regulator is fully informed about the technologies or costs, respectively. To mitigate the information asymmetries between regulated firms and regulators (see Laffont and Tirole, 1993), European regulators tend to combine price or revenue cap schemes with benchmarking techniques, particularly in the regulation of electricity distribution networks (for overviews see Agrell et al., 2013a; Haney and Pollitt, 2009; Jamasb and Pollitt, 2001).

Regulators employ benchmarking techniques to estimate production or cost functions based on observed data to approximate the unknown technologies, and thus, efficient cost levels. The estimated technology, or "frontier", and represents the efficient input-output-combinations measured in physical or monetary terms depending on the chosen approach. Once estimated, each firm can be compared to the frontier. A firm is considered efficient if it is on the frontier and inefficient otherwise. The distance of a firm's input-output-combination to this frontier is interpreted as inefficiency (and noise). Regulators use the estimated inefficiency to set the firms' production or cost targets, which are ultimately translated into the respective regulatory outcome.

Given that the technology and observed data are likely to be influenced by external factors, the firms' observed output (input) will also deviate from its maximum (minimum) due to the presence of external factors. Failing to control for external factors appropriately when estimating the frontier is likely to transfer to the firms' inefficiency estimates and penalize or favor firms and customers (see, e.g., Bjørndal et al., 2016, for an empirical investigation of Norwegian electricity distributors).

This paper assesses the ability of three frontier approaches to account for the influence of external factors in a production setting. We compare the non-parametric condi-

[1]In this paper, we use the terms environmental factor, environmental condition, operational condition, and z-variables interchangeably.

[2]In regulation economics this solution is referred to as the second-best solution (Laffont and Tirole, 1993; Armstrong and Sappington, 2007)

tional DEA (cDEA, Daraio and Simar, 2005, 2007b) with the parametric latent class SFA (LC-SFA, Orea and Kumbhakar, 2004; Greene, 2005) and the stochastic semi-nonparametric envelopment of z variables data (StoNEZD, Johnson and Kuosmanen, 2011). Specifically, we analyze their performance with respect to two criteria, i.e., the bias and the mean squared error, using Monte Carlo simulation.

Each of the three estimators has been specifically designed to control for heterogeneous technologies. They exhibit, however, very different approaches to incorporate external factors in the frontier estimation and differ considerably in their statistical characteristics. Each of the three estimators offers advantages depending on the setting in which they are used. To account for their different strengths and weaknesses, we construct a large number of different scenarios with variations in the sample sizes, in the distributions of noise and inefficiency, and in the impact of environmental factors on production possibilities. We construct the data generating processes (DGPs) to replicate real regulatory data with, e.g., high correlations among inputs, non-normal distributions of environmental factors and sample sizes including a few large firms.

While yet almost neglected by regulators, cDEA, StoNEZD, and LC-SFA are widely accepted in the scientific community. Compared to the standard benchmarking approaches used in regulation, the estimators come with the cost of limited empirical experience, higher complexity and often difficult implementation,[3] but offer better statistical properties and advanced capabilities to account for firms' operating environment.

Whether better statistical properties goes hand in hand with better estimation results remains open, as empirical evidence for most frontier estimators in terms of simulation results is limited to the original paper proposing an estimator. Studies of the newer semi-parametric approaches include Andor and Hesse (2014), who find StoNED to perform well in noisy settings compared to SFA and DEA, but also a stronger sensitivity of StoNED against an increasing number of explanatory variables and a general tendency to underestimate the true frontier. Krüger (2012) compares the semi-parametric order-m and order-α estimators with DEA, Free Disposable Hull (FDH) and SFA, but finds no advantage in using them for well behaved production settings and low levels of noise. Two other simulation studies are relevant to this study's focus on the impact of environmental factors in frontier estimation. Yu (1998) examines the frontier estimation in the presence of environmental variables and compares different SFA and DEA specifications. Results indicate a general advantage of SFA if the model is specified correctly, and that DEA performs quite well if the effect of the environmental variables is low. Similarly, Cordero et al. (2009) compare different ways to account for environ-

[3]From the regulators perspective, DEA and SFA are mature models that that provide well-known strength and weaknesses and ease of implementation and comprehensibility.

mental factors in DEA and find the four-stage model introduced by Fried et al. (2002) dominating other approaches.

To the best of our knowledge we are the first to provide a comparative study covering non-parametric, parametric, and semi-parametric approaches with a focus on the estimators' ability to account for environmental factors. Given the importance of controlling for external factors in regulatory schemes, they seem to be appealing alternatives for regulators to the methods in place. From a practitioners perspective, our analysis provides guidance to adopt new benchmarking methods given industry size and industry characteristics.

The remainder of this paper is structured as follows. Section 2 outlines and compares the different frontier estimators. Section 3 explains the simulation design and the different scenarios considered in the simulation. Section 4 presents the results, and section 5 concludes.

2 Methodology

The following section is based on a production model with $n(i = 1, ...n)$ decision making units (DMUs, i.e., firms). Each firm employs M inputs $x_{i1}, ..., x_{iM}$ with the firms input vector x_i to produce scalar output y_i. All firms have access to the same production technology with the production function $f(x)$ that gives maximum output for a given input level. All firms' actual output can deviate from the maximum due to random noise v_i, non-negative inefficiency u_i, and due to the influence of the firms' environments z_i (with $z_i = z_{i1}...z_{iL}$) with the impact $\delta = (\delta_1, ..., \delta_L)$. The multiplicative model is written as $y_i = f(x_i) * e^{\varepsilon_i}$ with $\varepsilon = \delta z_i + v_i - u_i$, which allows two equally valid interpretations: δz influences the location of the frontier and the attainable output for each firm depending on z ($y_i = f(x_i) * \delta z_i$), or δz_i influences the distance to the production function (Johnson and Kuosmanen, 2011).

2.1 Conditional DEA

Initially proposed by Cazals et al. (2002) in the order-m framework and further developed by Daraio and Simar (2005), Daraio and Simar (2007b), and Daraio and Simar (2007a), the conditional DEA is one extension of the standard DEA to incorporate environmental factors in performance evaluations. The approach aims to compare only units that operate under similar operational environments, i.e., the selection of the reference group for a particular observation is conditional on their z-variables. Conditional DEA does not rely on the separability condition between the output space and the space of z-variables (Simar and Wilson, 2007). Hence, the shape of the production set, and therefore, the frontier are allowed to be influenced by environmental variables. This frontier shift is exactly what regulators typically aim to compensate for.

The conditional DEA estimator computes efficiency scores based on an attainable production set that is conditioned on a set of z-variables, denoted as $\hat{\Psi}^z_{DEA}$. The statistical properties of this estimator are derived in Kneip et al. (2008), and its consistency is established in Jeong et al. (2010). Estimating $\hat{\Psi}^z_{DEA}$ implies the estimation of a non-standard conditional distribution function, where the production process is conditional to a particular level of z (Daraio and Simar, 2007b; Bădin et al., 2010). Since the latter requires the application of a smoothing technique, we conduct kernel estimation with an Epanechnikov kernel $K(\cdot)$ as suggested in Daraio and Simar (2005) and Daraio and Simar (2007b). The kernel is defined as

$$K_h = \frac{|z_i - z_k|}{h}, \tag{1}$$

where z_i is the vector of z-variables of the unit of interest i, z_k is the vectors of all other observations, and h is the vector of selected bandwidths. For each of the environmental variables a bandwidths is computed based on least squares cross validation following Hall et al. (2004), Li and Racine (2007) and Li and Racine (2008). The bandwidth selection procedure relies on estimating the conditional probability distribution function of y given a particular level of z.[4] Hall et al. (2004) emphasized that their proposed method assigns large smoothing parameters to components of z that are irrelevant for estimating the density of y. Therefore, the sizes of the selected bandwidths themselves already contain information about the impact of particular z-variables on the output y. The obtained bandwidths are then used to estimate the kernel function in equation 1 to compute the kernel probabilities. The firms closely located to company i in terms of z thereby receive higher kernel probabilities, whereas small (or even zero) kernel probabilities are assigned to firms facing very different operating environments than company i.[5]

The conditional DEA efficiency measure $\hat{\theta}_{DEA}(x, y|z)$ for a single observation is defined as (Daraio and Simar, 2007b)

[4]For a detailed presentation, see Hall et al. (2004)

[5]We note that the estimated efficiency scores depend on the kernel smoothing procedure. Other symmetric kernels with compact support and bandwidths selection procedures can be applied, see e.g., Cazals et al. (2002); Daraio and Simar (2005, 2007b).

$$\hat{\theta}_{DEA}(x,y|z) = max \ \{\theta > 0 \mid \theta y \leq \textstyle\sum_{i|z-h\leq z_i\leq z+h}^{n} \lambda_i y_i \qquad (2)$$

$$x \geq \textstyle\sum_{i|z-h\leq z_i\leq z+h}^{n} \lambda_i x_i$$

$$\textstyle\sum_{i|z-h\leq z_i\leq z+h}^{n} \lambda_i = 1, \quad i = 1, ..., n\},$$

where h represents the bandwidths. For each observation, the bandwidths determine the range of z in which other observations are considered as being similar. Hence, only observations within this range are considered as potential peers for the unit of interest and are selected into the respective reference group. For implementation, the reference set of firm i is restricted to those firms with positive kernel probabilities. The frontier reference point of firm i is obtained by $\hat{y}_i = \hat{\theta}_{i,DEA}(x_i, y_i|z_i) * y_i$.

2.2 Latent class SFA

The latent class stochastic frontier (LC-SFA) estimator, proposed by Greene (2002), Caudill (2003) and Orea and Kumbhakar (2004), belongs to class of stochastic and parametric approaches, and tries to account for technological heterogeneity, often with a focus on panel data. It is, to the authors knowledge, so far not used in regulation, but gathered interest in scientific publications also in regulated sectors such as electricity distribution (e.g., Cullmann, 2012; Llorca et al., 2014).

The LC-SFA accounts for heterogeneity among firms by endogenously sorting them into a prespecified number of groups. For each group, a separate frontier is estimated, and each firm gets a probability to belong to each group. The estimated frontiers of the different groups are allowed to differ in their parameters, and, thus, in their shape. Therefore, contrary to standard SFA with environmental factors, LC-SFA accounts not only for a shift in the technology induced by the environmental factor, but also for technological heterogeneity.

LC-SFA tries to estimate the group specific parameters for n observations with $J(j = 1, ..., J)$ groups of the form $\ln y_i = f(x_i, \beta_j) - u_{i|j} + v_{i|j} = f(x_i, \beta_j) + \varepsilon_{i|j}$. Similar to standard SFA, LC-SFA can be estimated via Maximum Likelihood, and needs several further assumptions: a functional form for $f(x)$ needs to be specified in advance (e.g., Cobb-Douglas or Translog). Distributional assumptions are necessary for the noise and inefficiency components, which typically enter the likelihood function as normal noise, $v \sim N(0, \sigma_v)$, and half-normal inefficiency, $u \sim N^+(0, \sigma_u)$. Given these assumptions, a log-density for firm i for each group j can be imposed with the standard parameterization $\sigma^2 = \sigma_{uj}^2 + \sigma_{vj}^2$, $\lambda = \sigma_{uj}^2/\sigma_{vj}^2$ and $\varepsilon = \varepsilon(\beta_j) = \log y - logf(x_i, \beta_j)$ such that

$$\log LF_{ij} = -\frac{1}{2} * \log\frac{\pi}{2} - \frac{1}{2}\log\sigma^2 + \log\Phi\left(-\frac{\lambda\varepsilon}{\sqrt{\sigma^2}}\right) - \frac{\varepsilon^2}{\sigma^2} \tag{3}$$

The contribution of each firm to the final likelihood function is a function of the likelihoods of each observation in each class weighted with the firms probability to belong to class j, P_{ij}. The total contribution of firm i to the likelihood function is

$$LF_i = \sum_{j=1}^{J} LF_{ij} * P_{ij} \tag{4}$$

The probability to belong to a certain group is modeled with standard assumptions on probabilities ($0 \leq P_{ij} \leq 1; \sum_{j=1}^{J} P_{ij} = 1$) using a multinomial logit model. At this point, the environmental variables enter the likelihood function and determine the probabilities of each firm to belong to each of the J groups, such that

$$P_{ij}(\zeta_j) = \frac{\exp(\zeta_j' z_i)}{\sum_{j=1}^{J}\exp(\zeta_j' z_i)} \tag{5}$$

with ζ as logit parameters to estimate with $\zeta_J = 0$. Given this parameterization, the final log-likelihood function to be maximized is obtained as

$$LF = \sum_{i=1}^{n}\left\{\sum_{j=1}^{J} LF_{ij} * P_{ij}(\zeta_j)\right\} \tag{6}$$

Note that each observation enters the likelihood function J times. Thus, each observation is influences the shape of each group to a certain degree depending on the weight $P_{ij}(\zeta_j)$. As a result, each firm has a reference point on each frontier. There are two possible ways to obtain one final reference point (Orea and Kumbhakar, 2004): One can either use the group frontier with the highest probability, or calculate a weighted reference point using the conditional posterior class probabilities $P(j|i)$. Following Greene (2002) $P(j|i)$ can be calculated as

$$P(j|i) = \frac{LF_{ij} * P_{ij}(\delta_j)}{\sum_{j=1}^{J} LF_{ij} * P_{ij}(\delta_j)} \tag{7}$$

This paper uses the weighting approach because it incorporates more information about the underlying data structure. Thus, a frontier reference point is calculated as weighted reference point from the J frontiers such that

$$\ln \hat{y}_i = \sum_{j=1}^{J} P(j|i) * \ln \hat{f}(x, \beta_j) \tag{8}$$

Finally, the maximization of the likelihood needs the a priori specification of the number of groups. As the number of parameters to estimate increases considerably with the number of classes, information criteria, such as the Bayesian Information Criterion (BIC), are used to account for the higher likelihood in a model with more parameters as well as to penalize the larger number of parameters.

2.3 StoNEZD

The stochastic semi-nonparametric envelopment of z variables data (StoNEZD) proposed by Johnson and Kuosmanen (2011) is a semi-nonparametric approach that extends the standard StoNED estimator (Kuosmanen and Kortelainen, 2012) to account for environmental factors. So far, limited empirical experience has been gathered with this estimator, but it is currently applied in the regulation of the Finish electricity distribution sector (Kuosmanen, 2012).

StoNEZD tries to estimate a production frontier in two stages: In the first stage, an average production function $g(x)$ is estimated using concave non-parametric least squares (CNLS, Hildreth, 1954) taking into account the presence of z. In the second stage, $g(x)$ is shifted upwards to obtain a frontier estimate. This shift is based on the expected value of inefficiency derived from the residuals from the first stage based on parametric assumptions.[6]

Johnson and Kuosmanen (2011) show that a multiplicative production model $y_i = f(x_i) * e^{\varepsilon}$ can be estimated with the two stage approach. For the first stage, a quadratic programming problem (QP) estimates the shape of the average production function $g(x)$ without any assumptions on a functional form but establishes concavity and monotonicity of the production function. Further, the QP directly takes into account that the firms deviation from this average is influenced by the existence of z. No distributional assumptions for u and v are necessary in this stage, but u, v and z are assumed to be uncorrelated. To estimate $g(x)$, Johnson and Kuosmanen (2011) propose a minimization of the squared residual accounting for z using the following

[6] The approach is similar to modified ordinary least squares (MOLS, cp. Greene, 2007), but uses a very flexible non-parametric first stage, rather than a standard least squares estimate.

constrained QP:

$$\min_{\alpha,\beta,\delta,\phi} \sum_{i=1}^{n} (\ln y_i - \ln \hat{\phi}_i - z_i'\delta) \tag{9}$$
$$\begin{aligned} s.t. \quad & \hat{\phi}_i = \alpha_i + x_i\beta_i && \forall i = 1,..,n \\ & \alpha_i + x_i\beta_i \leq \alpha_h + x_i\beta_h && \forall h, i = 1,..,n \\ & \beta_i \geq 0 && \forall i = 1,..,n \end{aligned}$$

This QP estimates firm-specific α and β coefficients that create linear hyperplanes $\hat{\phi}$, which are tangent to the average production function and deliver the fitted values on $g(x)$ for each observation. α and β can be interpreted as the marginal products of the inputs. Microeconomic requirements on production functions are imposed as constraints: The first constraint establishes a linear form of the hyperplanes. The second constraint imposes concavity using Afriats inequalities (Afriat, 1967). These concavity constraints relate the piece-wise linear hyperplanes for all observations against each other leading to n^2 separate constraints. The third constraint ensures monotonicity of the estimated average production function. Further, the QP estimates the impact of the environmental factor z, $\hat{\delta}$, which is identical for all firms. Finally, a residual for each observation containing only noise and inefficiency is given by $\hat{\eta}_i = \ln y_i - \ln \hat{\phi}_i - \hat{\delta} z_i = \hat{\varepsilon}_i - \hat{\delta} z_i$.
In the second stage, to shift the average production function $\hat{g}(x)$ to the frontier, the residuals $\hat{\eta}_i$ of the QP are used to estimate the expected value of inefficiency μ. Further distributional assumptions for noise and inefficiency are necessary. Following Kuosmanen and Kortelainen (2012), we assume a half-normal distribution for the inefficiency term, $u \sim N^+(0, \sigma_u)$, and a normal distribution for the noise term, $v \sim N(0, \sigma_v)$. Kuosmanen and Kortelainen (2012) suggest using a method of moments (MM) estimator following Aigner et al. (1977) to derive the expected value of inefficiency.[7] This approach uses the property of the third central moment of a normal-half-normally distributed residual to be a function of only one parameter, σ_u. Using the empirical third moment of the residuals, $\hat{M}_3$, an estimate $\hat{\sigma}_u$ can be recovered by calculating $\hat{\sigma}_u = \sqrt[3]{\frac{\hat{M}_3}{\sqrt{\frac{2}{\pi}} * [1 - \frac{4}{\pi}]}}$. Subsequently, the expected value of inefficiency is calculated as $\hat{\mu} = \hat{\sigma}_u * \sqrt{2/\pi}$. Now, the frontier is derived as $\hat{f}(x) = \hat{g}(x) * exp(\hat{\mu})$. Firm-specific frontier reference points can be estimated from the shifted average production function accounting for the impact of z, such that $\hat{y}_i = \hat{g}(x_i) * exp(\hat{\delta} z_i) * exp(\hat{\mu})$ for each observation.

[7]Kuosmanen and Kortelainen (2012) also propose a pseudo-likelihood approach (PSL) following Fan et al. (1996) to estimate the expected value of inefficiency. Throughout this paper, we use the MM estimator due to its computational ease.

2.4 Comparison of the estimators

The three estimators differ in their characteristics considerably, leading also to different strength and weaknesses. First, their a priori assumptions on the production process differ since cDEA is a non-parametric methodology, StoNEZD a semi-parametric and LC-SFA a parametric estimator. While cDEA and StoNEZD need only few assumptions on the technology set, as, e.g., monotonicity, convexity of the production set and certain scaling assumptions, LC-SFA demands a functional form to be specified in advance (e.g., Cobb-Douglas or Translog). As a result, cDEA and StoNEZD are more flexible in the frontier estimation; however, LC-SFA also allows for non-convex production sets. This difference in the nature of the estimators is also reflected in their asymptotic properties, and in particular in their rates of convergence: LC-SFA converges with the standard parametric rate of convergence, $n^{-1/2}$. On the contrary, DEA converges with $n^{-2/(m+q+1)}$ with m and q being the number of inputs and outputs, respectively. Thus convergence slows down considerably with an increasing number of dimensions. Further, the conditional version is slower by a factor $n^{-4/4+r}$ with r being the number of z-variables (Kneip et al., 1998; Jeong et al., 2010). For StoNEZD, Johnson and Kuosmanen (2011) show that δ converges with the standard parametric rate, $n^{-1/2}$, while results for the non-parametric CNLS estimation remains open (Kuosmanen et al., 2015). However, following Stone (1980, 1982), Johnson and Kuosmanen (2011) suggest an upper limit $n^{-2d/(2d+m)}$ with d being the degree of differentiability and m as the number of inputs. Thus, both, cDEA and StoNEZD suffer the "curse of dimensionality" with increasing data demand for additional dimensions on the input and output side.

Second, the estimators differ in the treatment of inefficiency and noise. cDEA is purely deterministic and does not consider the existence of noise, i.e., $\sigma_v^2 = 0$. This makes the efficiency estimates prone to outliers, which can lead to an overestimation of the frontier. On the contrary, LC-SFA and StoNEZD allow a differentiated treatment based on distributional assumptions with $\sigma_v^2 \geq 0$, which makes them less prone to noise but may lead to an underestimated frontier in settings with very low noise. Further, estimated frontiers and efficiency point estimates may vary with the distributional assumption for inefficiency and noise.

Third, the three estimators vary considerably in their treatment of the environmental factors. cDEA constructs observation-specific reference sets depending of the realization of z and the estimated corresponding bandwidth. Thus, the estimated effect of the z variable on the frontier is also observation-specific. On the contrary, StoNEZD uses the whole sample as a reference set resulting in an effect of z on the frontier common to all observations. LC-SFA uses all observation in the frontier estimation, and the reference sets are weighted with the probabilities of group membership. As a result, with LC-SFA the effect of the z variable on the frontier is also weighted and varies from one firm to the another.

Fourth, StoNEZD is a two-stage approaches, cDEA is a one-stage approach estimated in two stages, and LC-SFA is a one-stage approach. Under cDEA, inefficiency and potential noise may influence the bandwidth estimate, which ultimately influence the reference sets. Similarly, in the first stage of the StoNEZD the δ coefficient might catch up effects from the so-far neglected inefficiency term. These problems should not be present in the LC-SFA estimation, since LC-SFA considers environmental factors, noise and inefficiency in one estimation.

And fifth, the approaches differ with respect to the identification of peer units, which is a desirable property from a regulatory perspective because it provides guidance to imitate best practice. cDEA clearly identifies the reference set with similar units, and the units that span the frontier. StoNEZD allows to identify at least the observations that share a facet of the frontier. Such identification is not possible with LC-SFA. Further, the weighting approach in the LC-SFA can project all firms on different levels between multiple group frontiers.

3 Simulation design

3.1 The DGP

This paper aims on evaluating the methods presented in section 2 using Monte Carlo simulation techniques with a focus on environmental variables in regulatory benchmarking. This allows to assess the estimators' performance as the true position of the frontier and its characteristics are known. By varying the parameters of the frontier and components of the observations, such as environmental factors, noise, and inefficiency, different set-ups likely to be observed in reality can be simulated. Our data generating processes (DGPs) try to mimic real regulatory data to simulate cases that could be encountered by regulatory authorities. Therefore, regulatory data of electricity distribution firms from Finland (Kuosmanen, 2012), Norway (Bjørndal et al., 2010), and Germany (Agrell et al., 2013b) was analyzed.

To construct our datasets, we calculate a one-dimensional output variable y_i for each of the $n(i = 1, ..., n)$ observations. For each firm, y_i is a function of the input vector x_i collecting M inputs $(x_{i1}, ..., x_{iM})$ that are transformed into outputs using a production function $f(x)$. Each firm operates under certain environmental conditions $z_{i1}...z_{iL}$ that influence the maximum output $f(x_i)$ with an impact vector $\delta = (\delta_1, ..., \delta_L)$ that is common to all firms. Each firms observed output may further deviate from this maximum due to inefficiency u_i and noise v_i. The output y_i is calculated as

$$y_i = f(x_i) * exp(\delta z_i) * exp(v_i - u_i) \tag{10}$$

Functional form

For the production function $f(x)$ we assume a Translog specification such that

$$f(x) = p_o \prod_{m=1}^{M} x_m^{(p_m)} \prod_{m=1}^{M} x_m^{(1/2)*\sum_{l=1}^{M} p_{ml} ln x_l} \tag{11}$$

We specify decreasing returns to scale with the following parameters: $p_0 = 1$, $p_m = 0.15$, $p_{ml} = 0.05$ for $m = l$ and $p_{ml} = 0.01$ for $m \neq l$.

Sample sizes

The sample sizes takes into account that regulatory authorities typically face a wide spread in terms of firm size with concentration among the smaller firms, and a small number of considerably larger firms. We set the number of small firms in the sample to $n_{small} = 25, 50, 100, 150, 250$. Additionally, we assume that 4% of the firms are at least twice as large as the largest "small" firm in terms of the upper bound of the inputs, such that $n_{large} = \lceil 0.04 * n_{small} \rceil$. Thus, the total number of observations in the sample is 26, 52, 104, 156, 260.[8]

Noise and inefficiency

We assume noise to be normally distributed and inefficiency to follow a half-normal, i.e., $v \sim N(0, \sigma_v^2)$ and $u \sim |N(0, \sigma_u^2)|$. For σ_v^2 and σ_u^2 we assume $\sigma_u = \{0.05, 0.1, 0.15\}$ and $\sigma_v = \{0, 0.05\}$ for a total of nine potential noise/inefficiency set-ups with noise-to-signal ratios varying between 0 and 2. For the different levels of σ_u, the expected value of inefficiency, μ, varies between 4 and 12%. We refrain from higher levels of noise due to the typically strict standards for data collection and measurement for regulatory data. Following Badunenko et al. (2012), we discard draws with the wrong (positive) skewness of the composite error term $(v - u)$.

[8] Larger sample sizes are not considered because simulation with a large number of replications is computationally burdensome if not impossible.

σ_v \ σ_u	0.05	0.1	0.15
0	0	0	0
0.05	1	0.5	$0.33\bar{3}$
0.1	2	1	$0.66\bar{6}$
μ	4.0%	8.0%	12.0%

Table 1: Noise-to-signal ratios and expected inefficiency for different σ_u and σ_v

Inputs

The number of inputs considered varies by regulator, and, e.g., Finland bases their regulation of electricity distribution system operators (DSOs) on three inputs, Norway uses five, and Germany uses even nine. We assume that each firm employs four inputs ($M = 4$), which are correlated uniform variables. We define the range of the inputs to vary between $[0, 10]$ for small firms and $[20, 30]$ for large firms. Correlations among the inputs are given by the following correlation matrix indicating moderate to high correlation between 0.55 and 0.85.

$$\rho_X = \begin{bmatrix} 1 & 0.55 & 0.65 & 0.75 \\ & 1 & 0.7 & 0.8 \\ & & 1 & 0.85 \\ & & & 1 \end{bmatrix} \tag{12}$$

The inputs, however, are still random numbers with variation in the observed empirical correlations in the generated data. To ensure comparability of the data sets, we discard sets in which at least one correlation deviates from this matrix by ± 0.05.

Environmental variables

We consider four environmental factors, $z_1, ..., z_4$, of which three are randomly drawn variables, and one is correlated with the inputs. z_1 is drawn from $N(1, 0.15^2)$. Thus, z_1 is symmetrically distributed with mean one, and values below zero are basically ruled out. The second environmental factor, z_2, is drawn from an exponential distribution with a rate of 5.5 truncated at 1, i.e., $z_2 \sim TExp(5.5, 1)$. Thus, only realizations in $(0, 1]$ are possible and very small values of z_2 are very likely compared to values close to one. This variable resembles an operational condition measured in percentage, e.g., the underground cabling in an electricity distribution network. The third environmental

Variable	Distribution	Support	Mean	ρ_X
z_1	$N(1, 0.15^2)$	$(-\infty, \infty)$	1	0
z_2	$TExp(5.5, 1)$	$(0, 1]$	0.178	0
z_3	$\Gamma(2, 1)$	$(0, \infty)$	2	0
z_4	$\Gamma(2, 1)$	$(0, \infty)$	2	0.5

Table 2: Properties of considered environmental variables

factor, z_3, is drawn from a Gamma distribution $\Gamma(2, 1)$ with a mean of 2. Again, values below zero cannot be observed, but the distribution is not truncated at the upper end. The fourth environmental factor, z_4, also stems from a $\Gamma(2, 1)$ distribution, but is correlated with the inputs with $\rho_X = 0.5$. z_3 and z_4 resemble, for example, weather conditions, which are extreme in a few cases.

3.2 Scenarios considered

Table 3 lists the twelve scenarios we consider. First, we construct four baseline scenarios $BL1$ to $BL4$ which include one environmental factor each, z_1 to z_4. For these baseline scenarios, the impact of the environmental variables on the frontier is small to moderate, with an average frontier shift of about 5% (cp. Table 3).
Second, we construct four high impact scenarios ($HI1, ..., HI4$), which triple the impact of the environmental variables, δ_l, on the frontier. For these high impact scenarios, the average frontier shift varies between 14.7 and 15%. Thus, the firms' average deviation from the frontier is driven more by the environmental variables than by inefficiency.
Third, we construct four scenarios with multiple z variables ($HI1, ..., HI4$), for which two environmental factors impact the frontier. Four scenarios with multiple z's are constructed, $MZ1$ to $MZ4$. For these scenarios, the average frontier shift varies between 9.8 and 19.6%. We use z_1 and z_2 to influence the firms' production potentials. In the scenarios $MZ1$ and $MZ2$, the environmental variables are uncorrelated, whereas in $MZ3$ and $MZ4$ we set $\rho_{z_1,z_2} = 0.5$.

3.3 Estimation procedure

To cover a large spectrum of potential datasets, we estimate each of the 12 scenarios for each sample size and for each of the noise-to-signal scenarios in Table 1, which gives a total of 12*5*9=540 cases to simulate. We set the number of replications at $R = 100$ with fixed seeds. This gives a total of 54.000 estimations for each of the three estimators. This allows obtaining replicable results with considerable generality with a solvable number of cases to analyze.

	$BL1$	$BL2$	$BL3$	$BL4$
z_l	z_1	z_2	z_3	z_4
$\delta_l * \bar{z}_l$	-0.05	-0.3	-0.025	-0.025
1-exp($\delta_l * \bar{z}_l$)	4.9%	5%	5%	5%
	$HI1$	$HI2$	$HI3$	$HI4$
z_l	z_1	z_2	z_3	z_4
$\delta_l * \bar{z}_l$	-0.15	-0.9	-0.075	-0.075
1-exp($\delta_l * \bar{z}_l$)	14,7%	15%	15%	15%
	$MZ1$	$MZ2$	$MZ3$	$MZ4$
z_l	z_1, z_2	z_1, z_2	z_1, z_2	z_1, z_2
ρ_{z_1,z_2}	0	0	0.5	0.5
$\delta_l * \bar{z}_l$	-0.05, -0.3	-0.1, -0.6	-0.05, -0.3	-0.1, -0.6
1-exp($\sum_l \delta_l * \bar{z}_l$)	9.8%	19.6%	9.8%	19.6%

Table 3: Scenarios

For each of the 540 cases, we generate one set of inputs and environmental factors. For each of these 540 cases, we generate R random draws of noise and inefficiency and then calculate the observed output. Therefore, variation among the R draws of one case stems only from variation in noise and inefficiency, and not from variation in the inputs or the environmental conditions.

For the StoNEZD estimator, estimation of the expected value of inefficiency is based on a Method of Moments estimator. Following Kuosmanen and Kortelainen (2012), if wrong skewness occurs in the estimation of the expected inefficiency, i.e., $\hat{M}_3 > 0$, we set $\hat{M}_3 = -0.0001$. For the LC-SFA estimator, we use a Cobb-Douglas specification for all sample sizes. To choose the optimal number of groups, LC-SFA is estimated for $J = \{2, 3, 4\}$ and the estimation with the optimal BIC is chosen and reported. Further, to reduce the risk of local optima in the ML procedure, for each J optimization is carried out five times with random starting values and the solution with the best BIC is reported. For the cDEA approach, we use an Epanechnikov kernel following Daraio and Simar (2005) and Daraio and Simar (2007b). Bandwidths are computed using least squares cross validation proposed by Hall et al. (2004), Li and Racine (2007) and Li and Racine (2008).

For all estimators, if the estimations fail for some reason, we exclude the reported results for these cases. We implement LC-SFA and cDEA using R (R Core Team, 2015) with the packages Benchmarking, np, minqa, and lpSolveAPI. We implement StoNEZD using GAMS version 24.2.1. We run all simulations with a 32 CPU 2.8 MHz

AMD with 512 GB memory.

3.4 Performance measures

We evaluate the performance of the different approaches based on the estimated frontier reference points.[9] We consider the bias and the mean squared error (MSE) as the criteria following the literature on simulation design in efficiency analysis (e.g., Andor and Hesse, 2014; Kuosmanen et al., 2013). The bias delivers the average total deviation from the true frontier in percentage. A positive sign indicates overestimation, i.e., the firms targets are set to high. The MSE, which is the average squared deviation from the frontier, penalizes larger deviations more strongly, and a higher value indicates a stronger deviation from the true frontier.
We calculate a firm's optimal output given its environmental conditions, $\hat{f}^z(x_i)$, using the true value on the frontier $f(x_i)$ corrected by the effect of the environmental variable $f^z(x_i) = f(x_i) * exp(\sum_l \delta_l z_{li})$. Based on the estimated frontier reference point $\hat{f}(x|z)$ for n observation and R simulation replications, the performance measures are defined as follows:

$$BIAS = \frac{1}{nR}\sum_{r=1}^{R}\sum_{i=1}^{n}\frac{\hat{f}(x|z) - f^z(x_i)}{f^z(x_i)} \tag{13}$$

$$MSE = \frac{1}{nR}\sum_{r=1}^{R}\sum_{i=1}^{n}(\hat{f}(x|z) - f^z(x_i))^2 \tag{14}$$

4 Results

4.1 Baseline scenarios

Results of the BL scenarios in terms of bias and MSE are shown in Figures 1 to 4. Detailed numerical results are provided in the appendix, Tables 4 to 7.
BL1, the baseline scenario with a normally distributed operational condition, shows very different patterns for the three estimators. Common to all of them, however, is that although the average bias does not necessarily increase with increasing noise, a decline in accuracy occurs in terms of MSE. This indicates declining accuracy, but not over- or underestimation.

[9]Alternatively, efficiency scores of the DMUs could be evaluated, but this requires additionally selecting an efficiency estimator. Using frontier reference points avoids, e.g., the widely used but inconsistent JLMS point efficiency estimate (Jondrow et al., 1982) for StoNEZD and LC-SFA. As Kuosmanen (2012) points out, a consistently estimated frontier could be more suitable then an inconsistent point estimate of efficiency.

cDEA shows that the bias increases with the number of observations, increases strongly with the level of noise, but is stable with the level of inefficiency in the model. Under nearly all simulated conditions with noise, cDEA tends to overestimate the reference points leading to unreasonably high output requirements for the firms. This indicates that cDEA undercompensates for the environmental conditions if noise is present.

LC shows the lowest bias and MSE among the three estimators, irrespective of sample size and noise-to-signal ratio. The estimated absolute bias is low already for small samples, is nearly identical for different values of σ_u, and decreases with σ_v, whereas the MSE decreases with sample size, but increases with noise. LC overestimates the true frontier if low inefficiency is present, but slightly underestimates the true frontier if inefficiency increases, thus leading to an overcompensation of environmental conditions.

StoNEZD shows a good performance with small samples, and no reduction of the absolute bias but shrinking MSE with increasing sample size in most cases. Results indicate a better performance with lower values of inefficiency, whereas the level of noise seems to be of minor importance. The MSE indicates that StoNEZD leads to the comparatively worst results if no noise but moderate to high inefficiency is present, whereas under high noise levels StoNEZD can compete with LC in terms of MSE, but tends to underestimate the frontier and overcompensates for environmental conditions.

$BL2$ to $BL4$, the other baseline scenarios, show strikingly similar patters. cDEA shows an underestimation of the frontier only for no-noise scenarios with small samples, whereas it overestimates the frontier in all other cases. StoNEZD generally underestimates the frontier and overcompensates for the environmental conditions. LC, which shows the lowest absolute bias, performs well in all settings with some noise, whereas there is a downward bias for data without noise. StoNEZD competes with LC in terms of MSE in high noise scenarios. Thus, the distribution of the z-variables seems to be of minor importance as the results are similar for $BL1$ to $BL3$. Comparing $BL3$ and $BL4$ also indicates that a high correlation of inputs and operational conditions does not influence the estimators' accuracy.

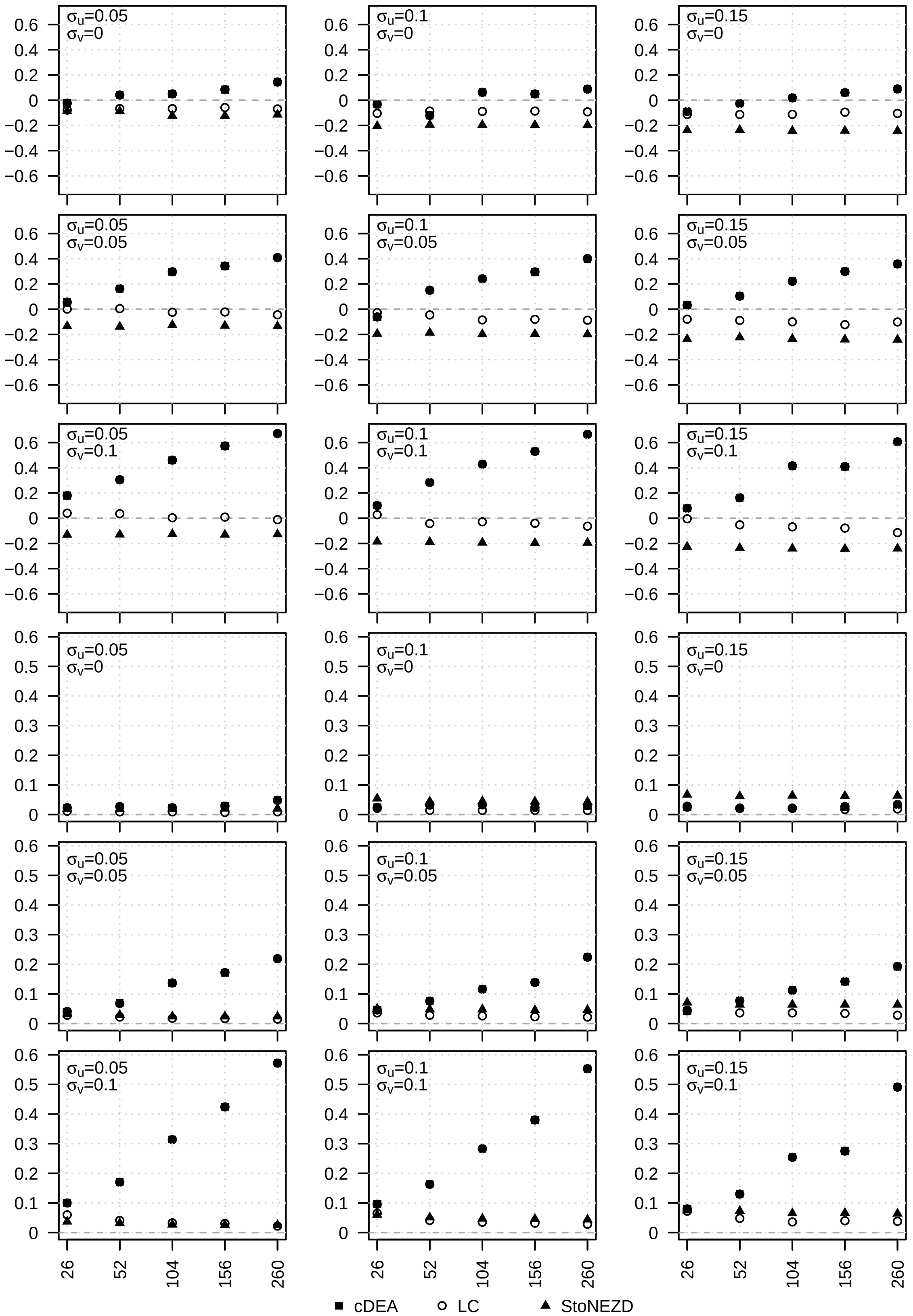

Figure 1: BL1 - Bias (upper 3*3 panels) and MSE (lower 3*3 panels)

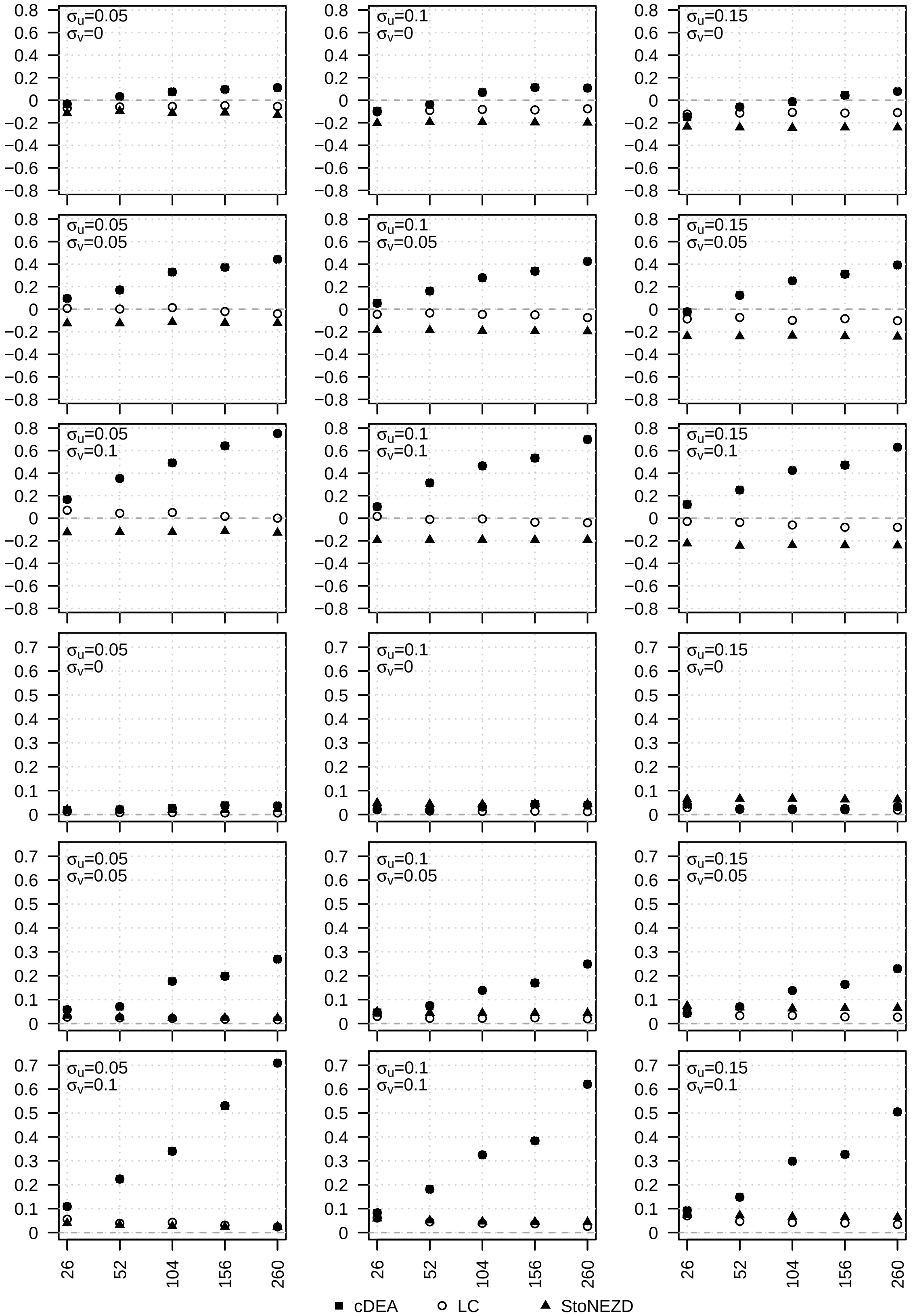

Figure 2: BL2 - Bias (upper 3*3 panels) and MSE (lower 3*3 panels)

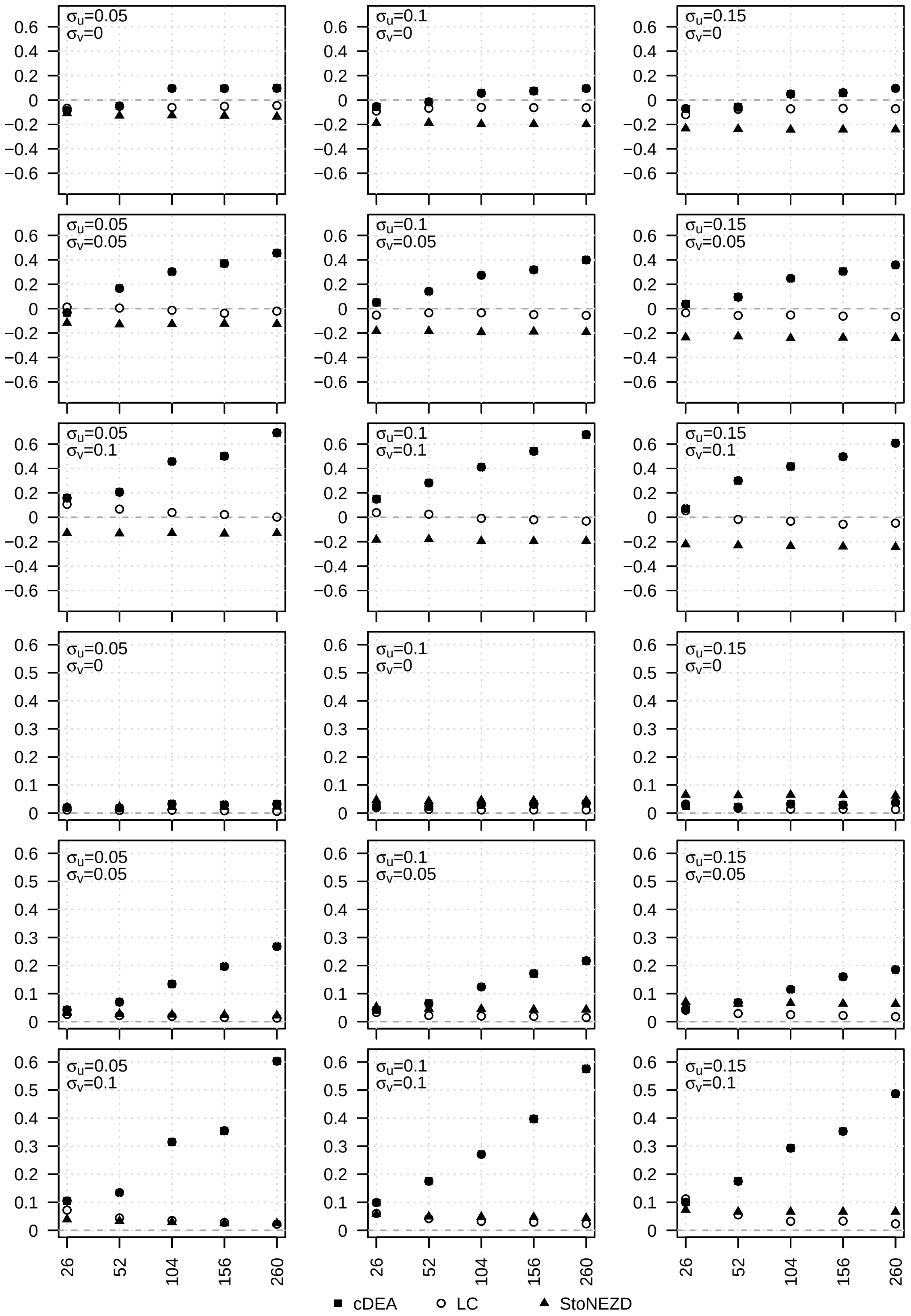

Figure 3: BL3 - Bias (upper 3*3 panels) and MSE (lower 3*3 panels)

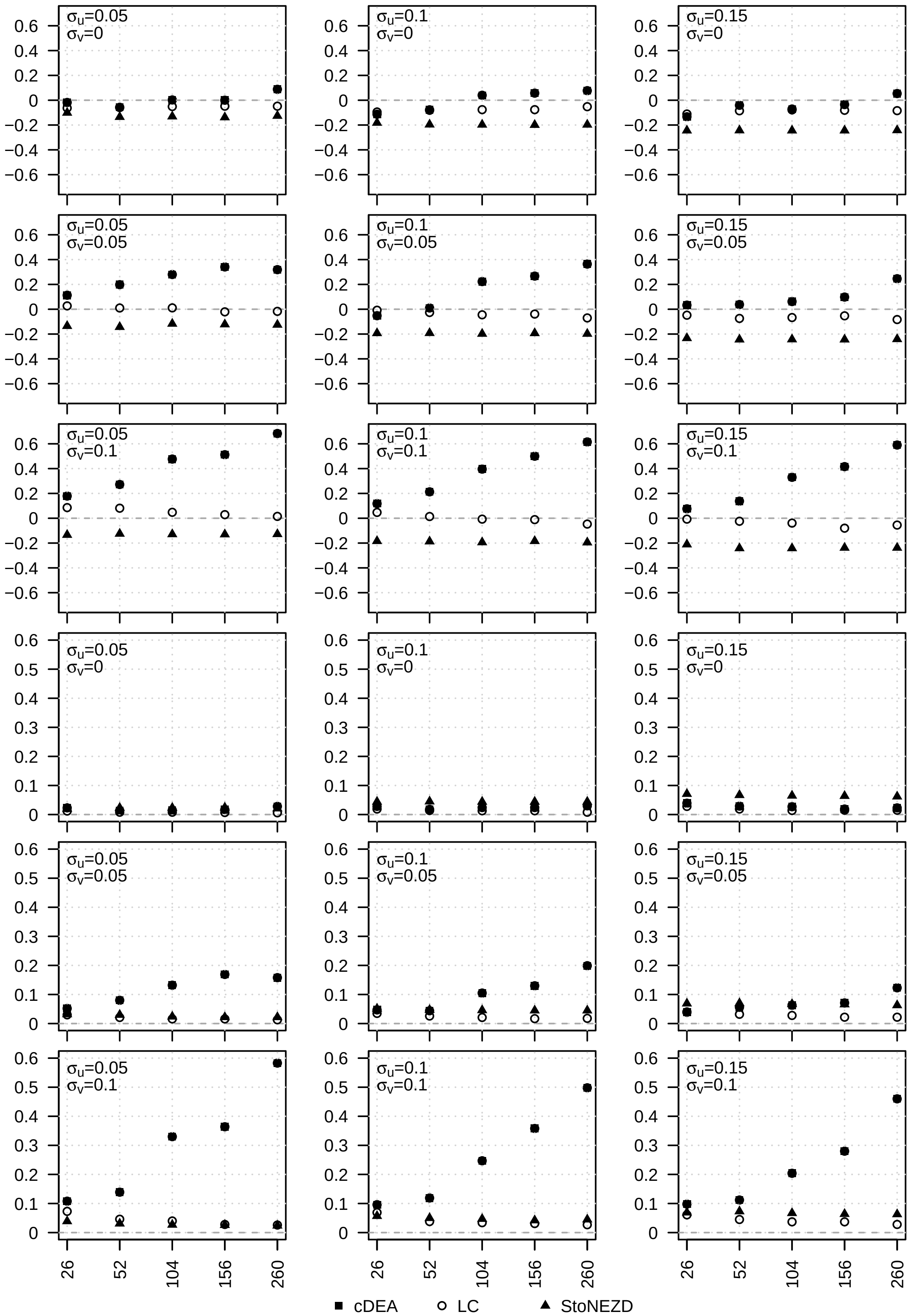

Figure 4: BL4 - Bias (upper 3*3 panels) and MSE (lower 3*3 panels)

4.2 High Impact scenarios

Next, we double the impact of the operational conditions on the output potential of the firms by doubling the δ-coefficient. Results for these four high impact scenarios $HI1$ to $HI4$ in terms of bias and MSE are shown in Figures 5 to 8. Detailed numerical results are provided in the appendix, Tables 8 to 11.

Again, the three estimators react differently to the changing conditions in terms of noise, sample size, and inefficiency. Generally, HI1 shows deteriorating accuracy of the estimators in terms of MSE with increasing noise.

cDEA shows an increase of bias and MSE with increasing sample size and with increasing noise levels, whereas the level of inefficiency seems to be of minor importance. Again in nearly all cases, cDEA overestimates the output potential of the firms and undercompensates for the environmental conditions. The effect, which is more pronounced in noisy settings, indicates that cDEA controls for the environment, but suffers from its deterministic nature.

LC shows a reduction of bias with increasing noise, but no considerable effect of inefficiency and sample sizes. LC performs best in nearly all considered noise-to-signal ratios in terms of bias and MSE.

StoNEZD shows a general tendency to underestimate the frontier, a negative influence of increasing inefficiency on model accuracy, but no clear impact of noise levels and sample sizes on estimation accuracy. StoNEZD performs well with moderate and high noise and accounts well for the stronger impact of environmental factors, but performs poorly when inefficiency increases. Only under high noise and high inefficiency can StoNEZD compete with LC in terms of MSE, but with on average higher absolute bias.

A comparison of the four different high impacts scenarios indicates that the patterns for the different estimators are similar irrespective of the distribution of the operational conditions, and irrespective of the correlation among inputs and z-variable. cDEA, however, shows a higher estimated bias and MSE higher for the non-normal cases, $HI2$, $HI3$ and $HI4$.

Compared to the baseline scenarios, the three estimators show the same patterns. However, while the results for the normally distributed environmental variables are similar (compare $BL1$ and $HI1$), a stronger impact of the truncated exponentially distributed environmental factor has a negative effect on cDEA's accuracy (compare $BL2$ and $HI2$). The same applies to the Gamma-distributed environmental factor (compare $BL3$ and $HI3$).

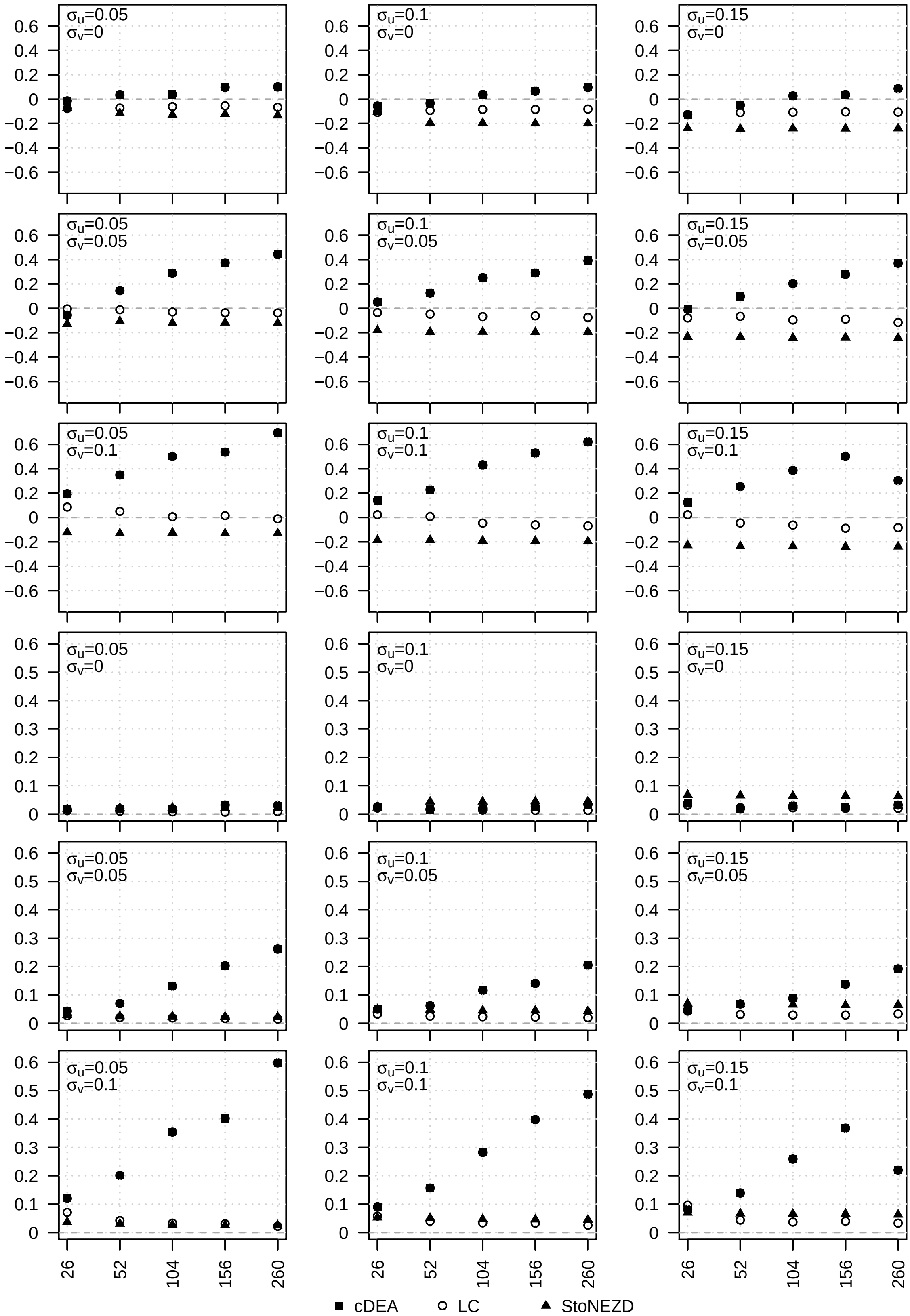

Figure 5: HI1 - Bias (upper 3*3 panels) and MSE (lower 3*3 panels)

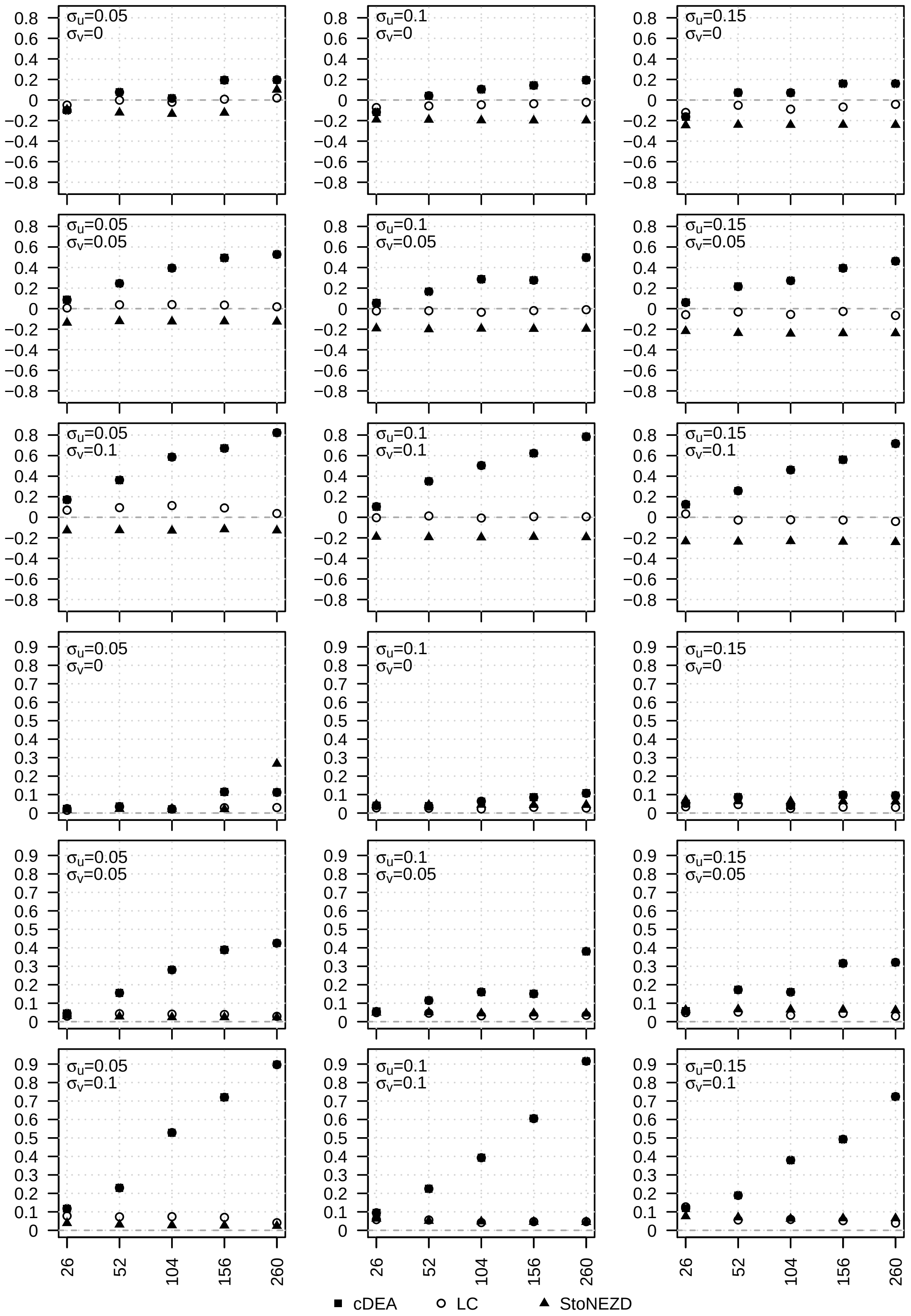

Figure 6: HI2 - Bias (upper 3*3 panels) and MSE (lower 3*3 panels)
Note: StoNEZD with n=260, $\sigma_u = \sigma_v = 0.05$ failed to converge

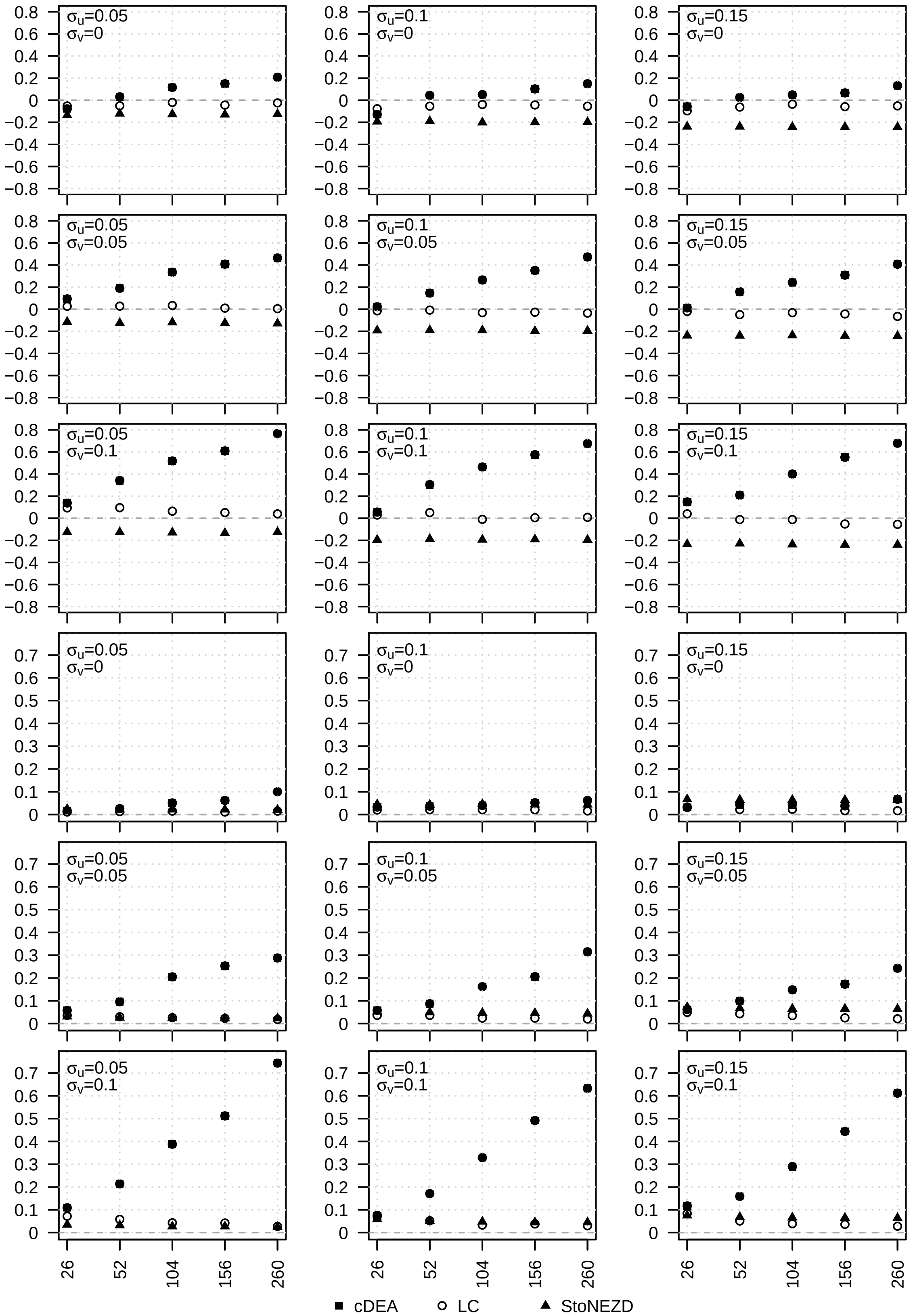

Figure 7: HI3 - Bias (upper 3*3 panels) and MSE (lower 3*3 panels)

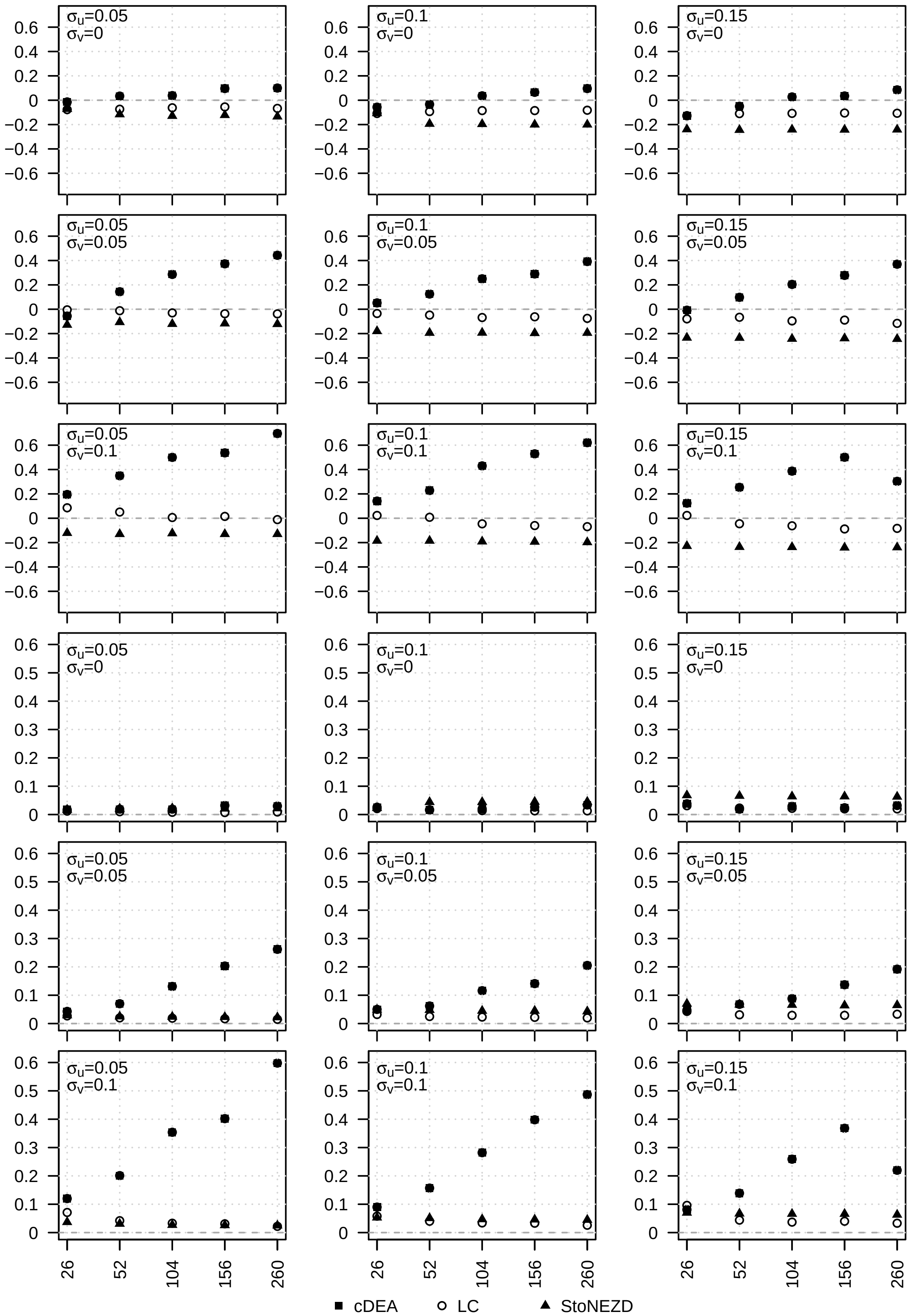

Figure 8: HI4 - Bias (upper 3*3 panels) and MSE (lower 3*3 panels)

4.3 Multiple-z scenarios

Next, we increase the number of z-variables that influence firms' output potential from one to two variables. One of the variables follows a normal, the other a truncated exponential distribution; different levels of impact and correlation are considered. Results for the four multiple-z scenarios $MZ1$ to $MZ4$ in terms of bias and MSE are shown in Figures 9 to 12. Detailed numerical results are provided in the appendix, Tables 12 to 15.

The results of the multiple-z scenarios show generally similar patterns as the baseline and high impact scenarios. For $MZ1$, cDEA performs well in no-noise scenarios, but shows tendencies to overestimate output potentials if noise is included in the data generation. Further, the bias and MSE of cDEA increases with sample size.

LC shows the best results in noisy scenarios, whereas the performance deteriorates compared to the other estimators if no noise is considered. However, in all cases with a certain level of noise and inefficiency, LC outperforms the other two estimators in terms of both performance measures.

StoNEZD shows a general tendency to underestimate the frontier. This downward bias, however, is stable irrespective of the sample sizes, but tends to increase with considered inefficiency. Nevertheless, despite this downward bias, overall accuracy measured with MSE indicates a good performance of StoNEZD in cases with moderate and high noise and all levels of inefficiency.

A comparison among the different MZ scenarios shows that these patterns are stable across different levels of impact of the operational conditions (compare $MZ1$ and $MZ2$), but also across different correlations among z-variables and inputs (compare $MZ1$ and $MZ3$, and $MZ2$ and $MZ4$, respectively). Therefore, the patterns revealed by the simulation results are generalizable for a large number of different settings.

Compared to BL and HI, cDEA shows a deteriorating performance in the scenarios with multiple z's. However, this negative effect can be due to including the exponentially distributed z-variable, having a second environmental factor, or having an overall higher impact compared to BL. LC and StoNEZD show no considerable differences in terms of bias and MSE.

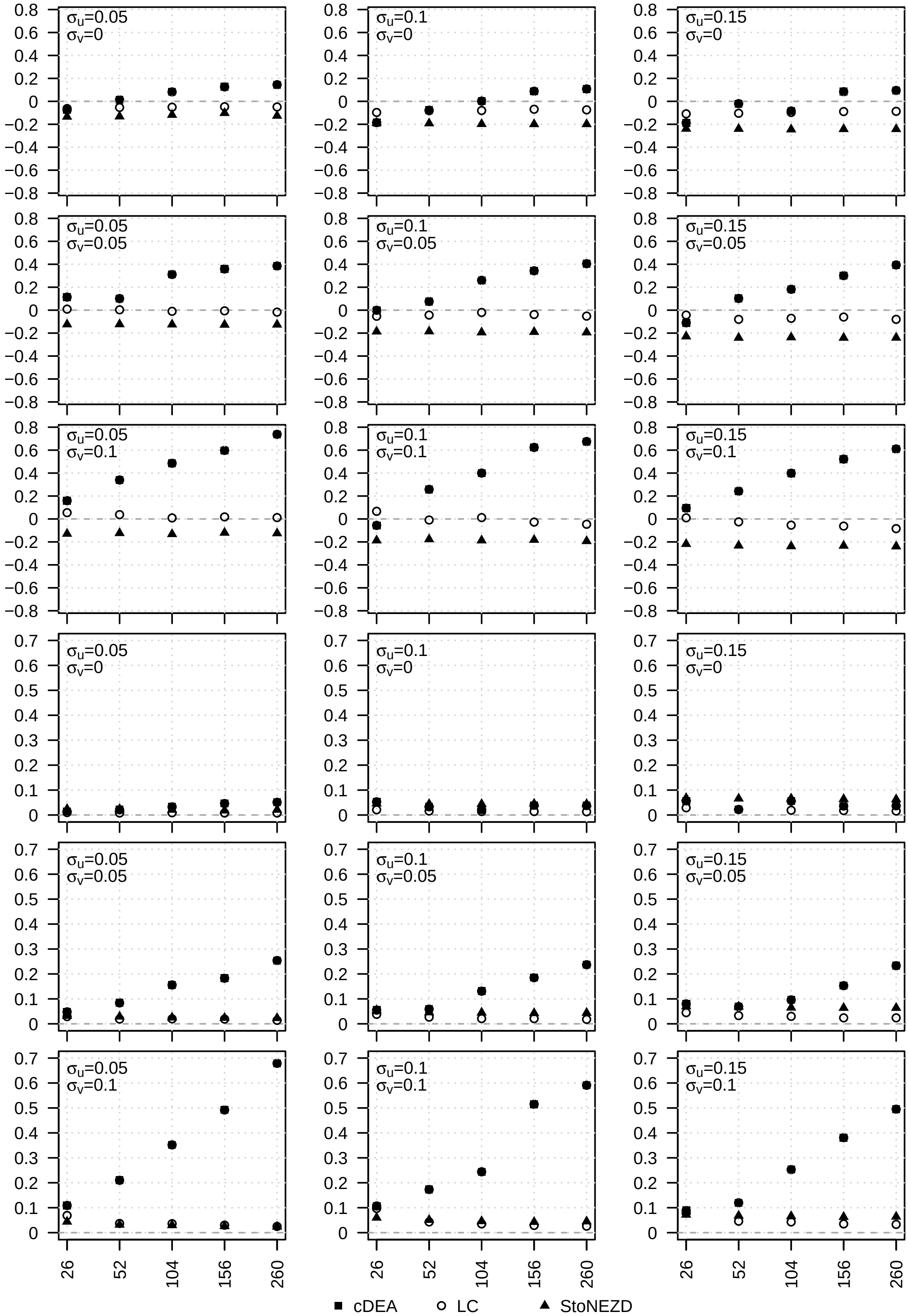

Figure 9: MZ1 - Bias (upper 3*3 panels) and MSE (lower 3*3 panels)

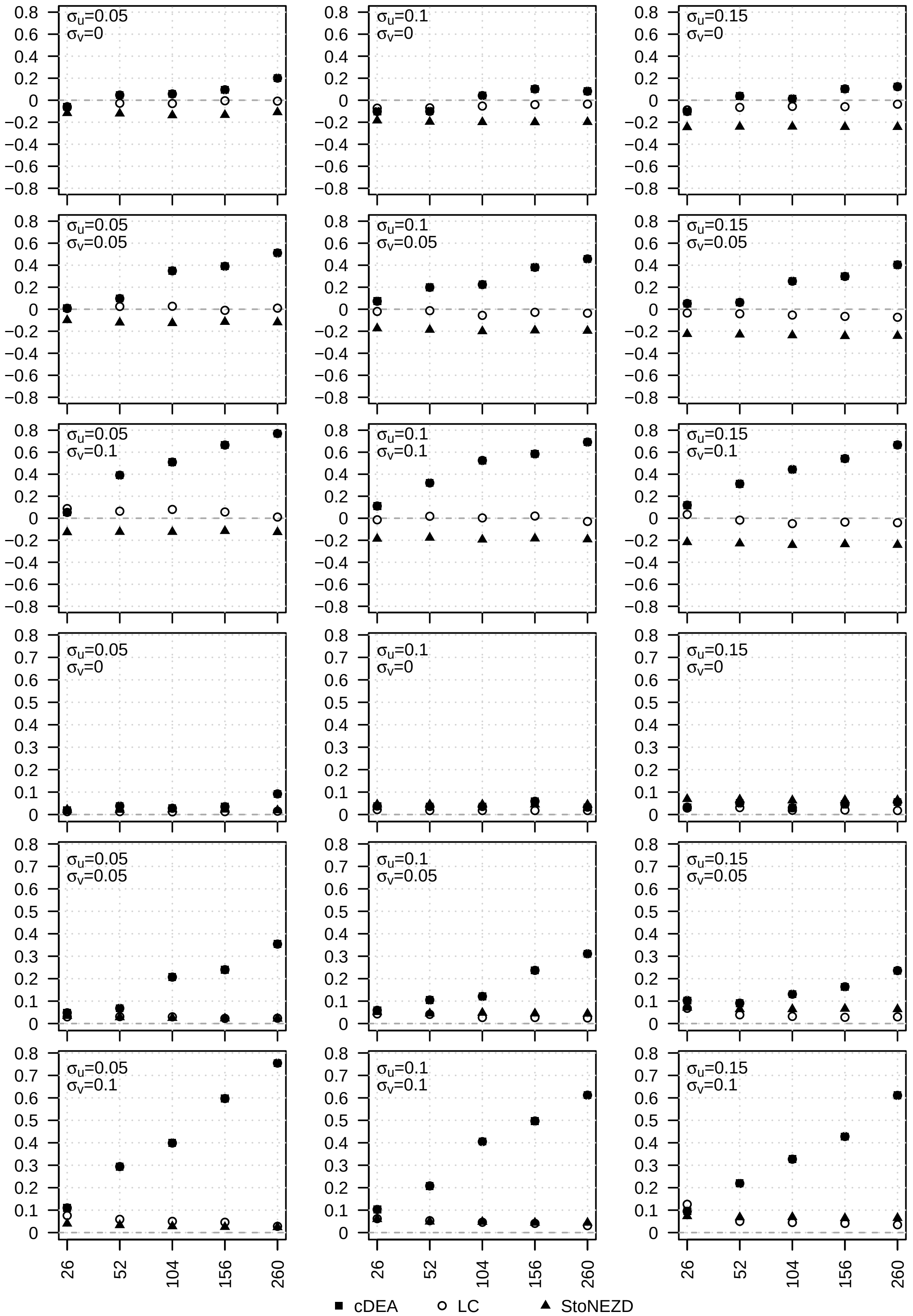

Figure 10: MZ2 - Bias (upper 3*3 panels) and MSE (lower 3*3 panels)

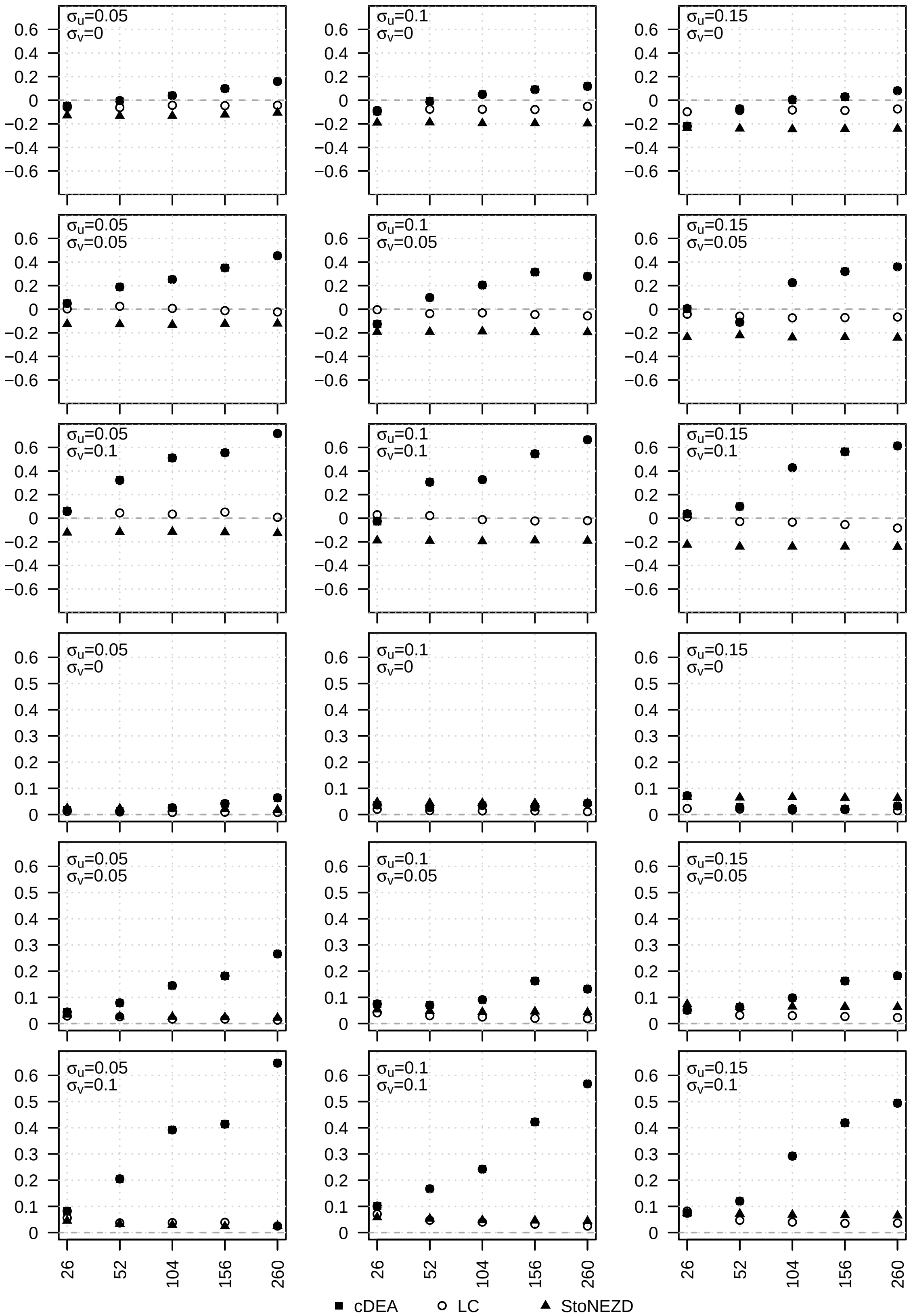

Figure 11: MZ3 - Bias (upper 3*3 panels) and MSE (lower 3*3 panels)

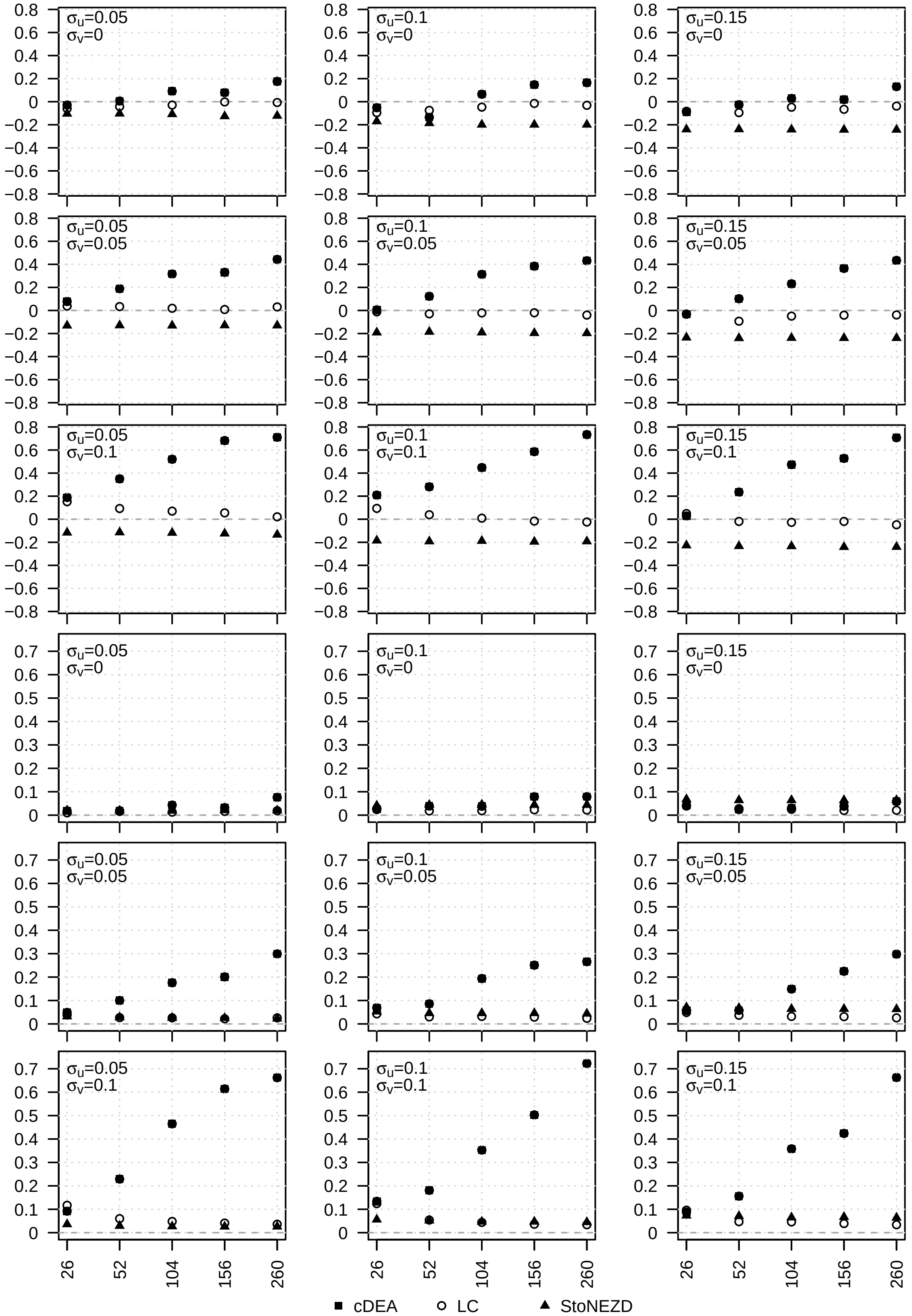

Figure 12: MZ4 - Bias (upper 3*3 panels) and MSE (lower 3*3 panels)

5 Discussion & conclusion

This paper compares conditional DEA (Daraio and Simar, 2005, 2007b), latent class SFA (Orea and Kumbhakar, 2004; Greene, 2005), and StoNEZD (Johnson and Kuosmanen, 2011), in terms of their ability to identify production frontiers in the presence of environmental factors. In a Monte Carlo simulation, data generation processes replicate regulatory data from the energy sector in terms of sample size, sample dispersion and distribution, and correlations of variables. Twelve scenarios modeling different impacts of environmental factors on production are considered. In terms of estimation accuracy, our results show several patterns that hold across the different scenarios.

cDEA performs well in cases without noise, but the results do not indicate consistency of the estimator. Further, results indicate a general tendency of cDEA to overestimate production potentials and, therefore, undercompensation for the impact of the operational conditions, in noisy and non-noisy settings. This is surprising, given the typical finding of DEA to show a downward bias (see, e.g., Simar and Wilson, 1998, 2000). While inferring that the reference set restriction based on kernel estimation may not sufficiently separate observations according to their environmental factors, kernel estimates do not necessarily lead to an excessive reference set restriction. The results suggest that regulators can use cDEA in cases that demand clearly identified reference units, comprise very low noise, and preferably with small samples up to 100 observations.

LC outperforms the other two estimators in all cases considered except for no-noise scenarios. It should be noted that the results are obtained using a Cobb-Douglas specification of the estimator, whereas the DGP follows a Translog specification. Thus, although the initial purpose of LC is to account for (unobserved) technological heterogeneity, i.e., changes in the parameters of the production function, LC-SFA is also suitable to account for differences in operational conditions. However, two caveats should be kept in mind. First, LC models have strong predictive power, but the coefficients may have counterintuitive signs. Therefore, the chosen and reported results often include estimations with negative marginal products of some inputs. Such estimation results could be discarded by researchers since they violate microeconomic theory. Second, contrary to cDEA, LC-SFA does not generate reference units, i.e., the used frontier reference points in the models do not indicate which firms are used to set the benchmarks. In regulatory benchmarking models, such peer units may be beneficial to provide guidance for firms apart from the frontier. Nonetheless, the results of the simulation indicate that LC is the regulators' low-risk choice, since bias and MSE indicate most often the best results, whereas overestimation of the frontier, i.e., setting overly optimistic output targets, occurs in only a few cases.

StoNEZD performs well in cases with considerable noise, and especially in cases with a strong noise-to-signal ratio, i.e., low inefficiency, which is in line with the finding by Andor and Hesse (2014) for the standard StoNED without z-variables. StoNEZD performs worse than its competitors in cases with high inefficiency and very low noise. These results are independent of the sample size, and although MSEs indicate consistency of the estimator, the estimator performs quite well already with small samples. Generally, StoNEZD tends to considerably underestimate the frontier. The deviations, however, are most often in a small range as indicated by low MSE. The results suggest that StoNEZD is the choice of the regulators expecting overall low potential efficiency improvements but rather strong noise, e.g., by impact from unobserved environmental factors. Further, the general downward bias of the frontier makes the estimator appealing to ensure feasible output targets.

All our result show that increasing noise negatively affects estimators' accuracy in terms of MSE. Hence, regulators need to scrutinize data quality, set strict standards for data collection, and account for potential heterogeneity on the firm level before applying frontier estimation. Reliable estimation results can help to set feasible improvement targets, and possibly support the acceptance of regulatory schemes by regulated firms. However, the results need to be interpreted with some care because the simulation is limited to the parameter space covered by the DGP. The DGP considered in this simulation models a large number of potential realizations, but cannot account for the entirety of datasets regulators may face. Future studies including technological heterogeneity induced by environmental factors and less well-behaved functional forms in the DGP may provide additional insights.

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6 Appendix

6.1 BL - Bias and MSE

BL1		cDEA			LC			StoNEZD		
n	σ_v \ σ_u	0.05	0.1	0.15	0.05	0.1	0.15	0.05	0.1	0.15
26	0	-0.024	-0.033	-0.089	-0.078	-0.104	-0.113	-0.086	-0.205	-0.238
		(0.023)	(0.024)	(0.025)	(0.011)	(0.021)	(0.028)	(0.019)	(0.054)	(0.067)
	0.05	0.056	-0.061	0.032	0.001	-0.026	-0.079	-0.134	-0.195	-0.237
		(0.041)	(0.046)	(0.043)	(0.028)	(0.036)	(0.045)	(0.031)	(0.050)	(0.071)
	0.1	0.180	0.101	0.079	0.040	0.028	-0.004	-0.131	-0.185	-0.226
		(0.100)	(0.096)	(0.080)	(0.060)	(0.065)	(0.072)	(0.037)	(0.060)	(0.075)
52	0	0.041	-0.121	-0.026	-0.066	-0.086	-0.113	-0.086	-0.195	-0.236
		(0.027)	(0.032)	(0.021)	(0.009)	(0.014)	(0.022)	(0.018)	(0.044)	(0.062)
	0.05	0.163	0.151	0.104	0.005	-0.045	-0.089	-0.138	-0.186	-0.223
		(0.068)	(0.076)	(0.077)	(0.022)	(0.028)	(0.036)	(0.029)	(0.048)	(0.064)
	0.1	0.305	0.284	0.163	0.036	-0.042	-0.052	-0.129	-0.188	-0.236
		(0.170)	(0.163)	(0.130)	(0.041)	(0.041)	(0.048)	(0.032)	(0.051)	(0.073)
104	0	0.050	0.063	0.019	-0.067	-0.089	-0.112	-0.123	-0.196	-0.243
		(0.023)	(0.033)	(0.021)	(0.009)	(0.014)	(0.022)	(0.021)	(0.045)	(0.064)
	0.05	0.297	0.241	0.222	-0.024	-0.085	-0.100	-0.125	-0.198	-0.235
		(0.137)	(0.116)	(0.112)	(0.018)	(0.026)	(0.036)	(0.024)	(0.048)	(0.064)
	0.1	0.461	0.429	0.416	0.004	-0.028	-0.068	-0.125	-0.193	-0.240
		(0.314)	(0.283)	(0.254)	(0.033)	(0.035)	(0.036)	(0.027)	(0.047)	(0.065)
156	0	0.085	0.050	0.060	-0.058	-0.086	-0.095	-0.123	-0.197	-0.242
		(0.029)	(0.024)	(0.027)	(0.007)	(0.014)	(0.017)	(0.020)	(0.044)	(0.063)
	0.05	0.342	0.296	0.300	-0.022	-0.080	-0.122	-0.131	-0.196	-0.240
		(0.172)	(0.139)	(0.142)	(0.017)	(0.023)	(0.034)	(0.024)	(0.045)	(0.064)
	0.1	0.573	0.530	0.410	0.009	-0.040	-0.078	-0.130	-0.196	-0.243
		(0.424)	(0.380)	(0.275)	(0.031)	(0.032)	(0.040)	(0.026)	(0.046)	(0.066)
260	0	0.144	0.088	0.089	-0.069	-0.092	-0.105	-0.114	-0.197	-0.243
		(0.048)	(0.030)	(0.034)	(0.009)	(0.014)	(0.019)	(0.020)	(0.043)	(0.064)
	0.05	0.410	0.402	0.359	-0.044	-0.087	-0.101	-0.135	-0.199	-0.241
		(0.219)	(0.224)	(0.193)	(0.015)	(0.022)	(0.028)	(0.024)	(0.046)	(0.064)
	0.1	0.672	0.665	0.606	-0.011	-0.063	-0.114	-0.127	-0.194	-0.240
		(0.572)	(0.553)	(0.491)	(0.022)	(0.028)	(0.038)	(0.025)	(0.044)	(0.064)

Table 4: Bias (MSE) for Scenario BL1

BL2		cDEA			LC			StoNEZD		
n	σ_v \ σ_u	0.05	0.1	0.15	0.05	0.1	0.15	0.05	0.1	0.15
26	0	-0.033 (0.017)	-0.097 (0.022)	-0.148 (0.042)	-0.071 (0.012)	-0.103 (0.020)	-0.123 (0.029)	-0.116 (0.021)	-0.203 (0.049)	-0.234 (0.064)
	0.05	0.096 (0.058)	0.053 (0.047)	-0.023 (0.043)	0.008 (0.027)	-0.045 (0.030)	-0.086 (0.043)	-0.125 (0.033)	-0.185 (0.050)	-0.239 (0.074)
	0.1	0.167 (0.109)	0.103 (0.083)	0.122 (0.093)	0.071 (0.056)	0.017 (0.061)	-0.028 (0.070)	-0.125 (0.040)	-0.193 (0.059)	-0.224 (0.073)
52	0	0.032 (0.021)	-0.038 (0.020)	-0.060 (0.024)	-0.059 (0.008)	-0.091 (0.015)	-0.114 (0.022)	-0.096 (0.018)	-0.194 (0.044)	-0.242 (0.066)
	0.05	0.171 (0.071)	0.162 (0.075)	0.123 (0.070)	0.002 (0.024)	-0.033 (0.022)	-0.073 (0.033)	-0.125 (0.026)	-0.186 (0.045)	-0.241 (0.068)
	0.1	0.353 (0.224)	0.315 (0.181)	0.251 (0.148)	0.044 (0.039)	-0.010 (0.045)	-0.037 (0.047)	-0.122 (0.032)	-0.191 (0.051)	-0.244 (0.072)
104	0	0.076 (0.026)	0.069 (0.032)	-0.012 (0.023)	-0.055 (0.008)	-0.082 (0.013)	-0.108 (0.020)	-0.114 (0.020)	-0.194 (0.043)	-0.245 (0.066)
	0.05	0.329 (0.177)	0.280 (0.139)	0.253 (0.138)	0.014 (0.022)	-0.046 (0.022)	-0.099 (0.034)	-0.114 (0.023)	-0.192 (0.044)	-0.234 (0.063)
	0.1	0.492 (0.340)	0.465 (0.325)	0.425 (0.298)	0.051 (0.043)	-0.006 (0.039)	-0.060 (0.042)	-0.123 (0.027)	-0.191 (0.046)	-0.238 (0.066)
156	0	0.096 (0.039)	0.113 (0.043)	0.043 (0.025)	-0.048 (0.007)	-0.086 (0.014)	-0.114 (0.020)	-0.110 (0.019)	-0.197 (0.045)	-0.242 (0.063)
	0.05	0.373 (0.198)	0.338 (0.170)	0.311 (0.164)	-0.020 (0.018)	-0.050 (0.024)	-0.084 (0.028)	-0.122 (0.024)	-0.195 (0.044)	-0.240 (0.064)
	0.1	0.642 (0.531)	0.534 (0.384)	0.471 (0.327)	0.017 (0.031)	-0.035 (0.037)	-0.080 (0.040)	-0.115 (0.024)	-0.192 (0.045)	-0.240 (0.065)
260	0	0.112 (0.037)	0.108 (0.040)	0.079 (0.033)	-0.055 (0.007)	-0.076 (0.012)	-0.110 (0.019)	-0.131 (0.022)	-0.199 (0.044)	-0.242 (0.063)
	0.05	0.442 (0.269)	0.425 (0.249)	0.392 (0.230)	-0.040 (0.016)	-0.074 (0.020)	-0.102 (0.027)	-0.123 (0.023)	-0.196 (0.044)	-0.243 (0.065)
	0.1	0.752 (0.709)	0.699 (0.620)	0.630 (0.505)	0.001 (0.024)	-0.040 (0.026)	-0.080 (0.034)	-0.129 (0.024)	-0.191 (0.044)	-0.242 (0.064)

Table 5: Bias (MSE) for Scenario BL2

BL3		cDEA			LC			StoNEZD		
n	σ_v \ σ_u	0.05	0.1	0.15	0.05	0.1	0.15	0.05	0.1	0.15
26	0	-0.087	-0.054	-0.070	-0.068	-0.090	-0.120	-0.108	-0.188	-0.233
		(0.020)	(0.025)	(0.027)	(0.011)	(0.019)	(0.032)	(0.020)	(0.046)	(0.065)
	0.05	-0.032	0.052	0.037	0.013	-0.053	-0.034	-0.116	-0.183	-0.236
		(0.042)	(0.043)	(0.045)	(0.026)	(0.033)	(0.041)	(0.030)	(0.052)	(0.070)
	0.1	0.158	0.149	0.073	0.106	0.038	0.054	-0.128	-0.184	-0.222
		(0.105)	(0.099)	(0.100)	(0.072)	(0.060)	(0.112)	(0.039)	(0.057)	(0.073)
52	0	-0.049	-0.015	-0.058	-0.051	-0.067	-0.076	-0.128	-0.186	-0.237
		(0.018)	(0.023)	(0.021)	(0.009)	(0.013)	(0.017)	(0.022)	(0.042)	(0.063)
	0.05	0.166	0.142	0.095	0.005	-0.034	-0.056	-0.129	-0.184	-0.227
		(0.070)	(0.065)	(0.068)	(0.022)	(0.022)	(0.029)	(0.028)	(0.046)	(0.064)
	0.1	0.206	0.282	0.300	0.067	0.025	-0.017	-0.132	-0.180	-0.230
		(0.134)	(0.175)	(0.175)	(0.044)	(0.042)	(0.055)	(0.033)	(0.049)	(0.066)
104	0	0.096	0.056	0.049	-0.061	-0.060	-0.072	-0.126	-0.198	-0.243
		(0.033)	(0.030)	(0.032)	(0.010)	(0.011)	(0.014)	(0.022)	(0.045)	(0.065)
	0.05	0.302	0.274	0.248	-0.013	-0.034	-0.052	-0.127	-0.193	-0.242
		(0.134)	(0.124)	(0.115)	(0.019)	(0.020)	(0.025)	(0.025)	(0.045)	(0.066)
	0.1	0.457	0.410	0.415	0.039	-0.009	-0.032	-0.129	-0.195	-0.236
		(0.315)	(0.271)	(0.293)	(0.035)	(0.032)	(0.032)	(0.029)	(0.048)	(0.066)
156	0	0.095	0.074	0.060	-0.052	-0.062	-0.068	-0.129	-0.197	-0.242
		(0.030)	(0.031)	(0.029)	(0.008)	(0.011)	(0.014)	(0.022)	(0.044)	(0.064)
	0.05	0.369	0.318	0.305	-0.038	-0.049	-0.061	-0.123	-0.188	-0.238
		(0.197)	(0.172)	(0.160)	(0.015)	(0.019)	(0.022)	(0.024)	(0.043)	(0.064)
	0.1	0.501	0.541	0.496	0.022	-0.020	-0.057	-0.134	-0.196	-0.240
		(0.355)	(0.397)	(0.353)	(0.028)	(0.029)	(0.033)	(0.025)	(0.047)	(0.066)
260	0	0.097	0.095	0.096	-0.045	-0.063	-0.071	-0.136	-0.199	-0.241
		(0.032)	(0.035)	(0.037)	(0.006)	(0.011)	(0.013)	(0.023)	(0.044)	(0.062)
	0.05	0.456	0.400	0.358	-0.020	-0.055	-0.064	-0.126	-0.192	-0.240
		(0.268)	(0.217)	(0.186)	(0.013)	(0.015)	(0.018)	(0.022)	(0.043)	(0.063)
	0.1	0.693	0.678	0.607	0.002	-0.031	-0.048	-0.130	-0.194	-0.244
		(0.603)	(0.576)	(0.487)	(0.022)	(0.023)	(0.023)	(0.025)	(0.044)	(0.066)

Table 6: Bias (MSE) for Scenario BL3

BL4		cDEA			LC			StoNEZD		
n	σ_v \ σ_u	0.05	0.1	0.15	0.05	0.1	0.15	0.05	0.1	0.15
26	0	-0.018	-0.113	-0.133	-0.063	-0.095	-0.111	-0.101	-0.182	-0.244
		(0.023)	(0.028)	(0.039)	(0.012)	(0.019)	(0.028)	(0.020)	(0.044)	(0.071)
	0.05	0.113	-0.052	0.033	0.027	-0.008	-0.048	-0.135	-0.193	-0.233
		(0.052)	(0.047)	(0.039)	(0.030)	(0.035)	(0.040)	(0.032)	(0.051)	(0.069)
	0.1	0.179	0.118	0.076	0.086	0.047	-0.007	-0.134	-0.184	-0.211
		(0.108)	(0.096)	(0.098)	(0.073)	(0.069)	(0.061)	(0.039)	(0.057)	(0.069)
52	0	-0.057	-0.077	-0.041	-0.059	-0.081	-0.085	-0.135	-0.196	-0.243
		(0.014)	(0.018)	(0.029)	(0.008)	(0.014)	(0.019)	(0.023)	(0.045)	(0.067)
	0.05	0.198	0.010	0.038	0.010	-0.026	-0.074	-0.143	-0.192	-0.244
		(0.080)	(0.044)	(0.056)	(0.021)	(0.026)	(0.032)	(0.030)	(0.047)	(0.070)
	0.1	0.272	0.213	0.138	0.081	0.014	-0.024	-0.125	-0.187	-0.242
		(0.139)	(0.119)	(0.112)	(0.046)	(0.038)	(0.045)	(0.031)	(0.050)	(0.073)
104	0	0.002	0.040	-0.071	-0.051	-0.076	-0.078	-0.131	-0.197	-0.244
		(0.015)	(0.024)	(0.027)	(0.008)	(0.013)	(0.014)	(0.023)	(0.044)	(0.065)
	0.05	0.280	0.223	0.062	0.011	-0.045	-0.067	-0.117	-0.198	-0.243
		(0.132)	(0.105)	(0.063)	(0.016)	(0.021)	(0.028)	(0.024)	(0.046)	(0.067)
	0.1	0.477	0.396	0.330	0.048	-0.007	-0.039	-0.129	-0.194	-0.242
		(0.330)	(0.247)	(0.204)	(0.040)	(0.034)	(0.037)	(0.027)	(0.047)	(0.067)
156	0	0.000	0.057	-0.035	-0.048	-0.076	-0.081	-0.138	-0.199	-0.244
		(0.017)	(0.024)	(0.019)	(0.007)	(0.013)	(0.015)	(0.024)	(0.044)	(0.064)
	0.05	0.341	0.267	0.098	-0.021	-0.039	-0.053	-0.122	-0.193	-0.244
		(0.169)	(0.130)	(0.071)	(0.016)	(0.017)	(0.022)	(0.022)	(0.045)	(0.066)
	0.1	0.512	0.500	0.416	0.029	-0.011	-0.080	-0.130	-0.184	-0.238
		(0.364)	(0.358)	(0.280)	(0.028)	(0.031)	(0.037)	(0.026)	(0.042)	(0.064)
260	0	0.088	0.077	0.054	-0.048	-0.052	-0.084	-0.126	-0.197	-0.242
		(0.028)	(0.030)	(0.023)	(0.006)	(0.008)	(0.015)	(0.021)	(0.044)	(0.062)
	0.05	0.319	0.364	0.246	-0.017	-0.070	-0.083	-0.125	-0.198	-0.241
		(0.158)	(0.199)	(0.123)	(0.013)	(0.018)	(0.022)	(0.022)	(0.045)	(0.063)
	0.1	0.683	0.615	0.590	0.015	-0.047	-0.055	-0.128	-0.195	-0.238
		(0.583)	(0.498)	(0.460)	(0.026)	(0.027)	(0.028)	(0.024)	(0.045)	(0.063)

Table 7: Bias (MSE) for Scenario BL4

6.2 HI - Bias and MSE

HI1		cDEA			LC			StoNEZD		
n	σ_v \ σ_u	0.05	0.1	0.15	0.05	0.1	0.15	0.05	0.1	0.15
26	0	-0.014	-0.057	-0.128	-0.077	-0.109	-0.127	-0.072	-0.107	-0.239
		(0.017)	(0.026)	(0.038)	(0.012)	(0.021)	(0.031)	(0.017)	(0.023)	(0.068)
	0.05	-0.056	0.052	-0.008	-0.005	-0.035	-0.079	-0.128	-0.180	-0.234
		(0.043)	(0.050)	(0.047)	(0.027)	(0.032)	(0.043)	(0.029)	(0.049)	(0.070)
	0.1	0.195	0.141	0.124	0.086	0.023	0.023	-0.121	-0.185	-0.228
		(0.120)	(0.090)	(0.081)	(0.071)	(0.058)	(0.096)	(0.037)	(0.053)	(0.070)
52	0	0.034	-0.034	-0.050	-0.074	-0.093	-0.110	-0.116	-0.194	-0.244
		(0.019)	(0.016)	(0.019)	(0.010)	(0.017)	(0.023)	(0.021)	(0.044)	(0.066)
	0.05	0.144	0.125	0.098	-0.012	-0.048	-0.066	-0.106	-0.194	-0.235
		(0.070)	(0.062)	(0.068)	(0.020)	(0.025)	(0.031)	(0.025)	(0.048)	(0.067)
	0.1	0.349	0.228	0.254	0.051	0.008	-0.045	-0.130	-0.185	-0.236
		(0.201)	(0.157)	(0.139)	(0.042)	(0.040)	(0.044)	(0.031)	(0.051)	(0.067)
104	0	0.038	0.036	0.027	-0.062	-0.085	-0.108	-0.129	-0.196	-0.241
		(0.019)	(0.020)	(0.029)	(0.008)	(0.014)	(0.022)	(0.022)	(0.044)	(0.064)
	0.05	0.286	0.250	0.204	-0.030	-0.068	-0.096	-0.121	-0.193	-0.243
		(0.131)	(0.116)	(0.088)	(0.019)	(0.023)	(0.029)	(0.024)	(0.045)	(0.066)
	0.1	0.500	0.430	0.387	0.006	-0.046	-0.062	-0.124	-0.191	-0.237
		(0.354)	(0.282)	(0.259)	(0.033)	(0.034)	(0.037)	(0.027)	(0.047)	(0.066)
156	0	0.096	0.065	0.035	-0.055	-0.085	-0.105	-0.122	-0.200	-0.242
		(0.032)	(0.026)	(0.024)	(0.007)	(0.013)	(0.020)	(0.021)	(0.045)	(0.064)
	0.05	0.373	0.290	0.278	-0.037	-0.062	-0.089	-0.117	-0.196	-0.239
		(0.203)	(0.141)	(0.137)	(0.017)	(0.022)	(0.029)	(0.023)	(0.045)	(0.064)
	0.1	0.537	0.529	0.501	0.015	-0.060	-0.088	-0.130	-0.193	-0.241
		(0.402)	(0.398)	(0.368)	(0.031)	(0.033)	(0.040)	(0.026)	(0.046)	(0.066)
260	0	0.100	0.096	0.085	-0.067	-0.082	-0.107	-0.134	-0.200	-0.241
		(0.030)	(0.034)	(0.032)	(0.009)	(0.013)	(0.020)	(0.022)	(0.045)	(0.063)
	0.05	0.443	0.392	0.370	-0.038	-0.075	-0.116	-0.122	-0.194	-0.244
		(0.262)	(0.205)	(0.192)	(0.015)	(0.020)	(0.033)	(0.022)	(0.043)	(0.065)
	0.1	0.696	0.621	0.303	-0.011	-0.069	-0.083	-0.130	-0.197	-0.239
		(0.598)	(0.487)	(0.220)	(0.022)	(0.026)	(0.033)	(0.025)	(0.045)	(0.063)

Table 8: Bias (MSE) for Scenario HI1

n	HI2 σ_v \ σ_u	cDEA 0.05	cDEA 0.1	cDEA 0.15	LC 0.05	LC 0.1	LC 0.15	StoNEZD 0.05	StoNEZD 0.1	StoNEZD 0.15
26	0	-0.100 (0.024)	-0.118 (0.041)	-0.163 (0.052)	-0.049 (0.015)	-0.073 (0.028)	-0.121 (0.034)	-0.093 (0.018)	-0.190 (0.045)	-0.246 (0.070)
	0.05	0.086 (0.045)	0.054 (0.055)	0.060 (0.057)	0.007 (0.030)	-0.022 (0.049)	-0.059 (0.049)	-0.137 (0.031)	-0.192 (0.049)	-0.218 (0.064)
	0.1	0.171 (0.118)	0.104 (0.095)	0.126 (0.120)	0.069 (0.078)	-0.004 (0.058)	0.033 (0.127)	-0.128 (0.039)	-0.190 (0.063)	-0.234 (0.076)
52	0	0.076 (0.035)	0.042 (0.038)	0.073 (0.086)	-0.001 (0.035)	-0.057 (0.026)	-0.051 (0.046)	-0.122 (0.022)	-0.192 (0.044)	-0.241 (0.064)
	0.05	0.245 (0.156)	0.166 (0.115)	0.215 (0.173)	0.038 (0.043)	-0.020 (0.046)	-0.032 (0.052)	-0.122 (0.028)	-0.201 (0.051)	-0.237 (0.067)
	0.1	0.363 (0.230)	0.350 (0.225)	0.258 (0.189)	0.094 (0.073)	0.013 (0.056)	-0.027 (0.056)	-0.126 (0.031)	-0.194 (0.051)	-0.237 (0.070)
104	0	0.017 (0.021)	0.105 (0.064)	0.071 (0.041)	-0.021 (0.022)	-0.046 (0.022)	-0.089 (0.025)	-0.135 (0.023)	-0.198 (0.045)	-0.242 (0.064)
	0.05	0.395 (0.281)	0.286 (0.161)	0.273 (0.160)	0.040 (0.041)	-0.035 (0.032)	-0.056 (0.035)	-0.125 (0.024)	-0.195 (0.045)	-0.243 (0.066)
	0.1	0.586 (0.529)	0.504 (0.393)	0.461 (0.380)	0.114 (0.074)	-0.007 (0.042)	-0.024 (0.059)	-0.130 (0.027)	-0.196 (0.048)	-0.232 (0.063)
156	0	0.194 (0.115)	0.143 (0.085)	0.160 (0.098)	0.008 (0.029)	-0.036 (0.031)	-0.068 (0.032)	-0.124 (0.021)	-0.199 (0.044)	-0.241 (0.064)
	0.05	0.493 (0.389)	0.277 (0.151)	0.394 (0.316)	0.035 (0.039)	-0.019 (0.033)	-0.027 (0.045)	-0.124 (0.024)	-0.197 (0.045)	-0.239 (0.065)
	0.1	0.671 (0.720)	0.623 (0.605)	0.561 (0.493)	0.091 (0.070)	0.006 (0.047)	-0.027 (0.052)	-0.117 (0.026)	-0.191 (0.046)	-0.238 (0.065)
260	0	0.197 (0.112)	0.194 (0.107)	0.160 (0.095)	0.020 (0.030)	-0.022 (0.027)	-0.042 (0.031)	0.100 (0.267)	-0.199 (0.044)	-0.242 (0.063)
	0.05	0.528 (0.425)	0.497 (0.381)	0.463 (0.321)	0.018 (0.029)	-0.010 (0.035)	-0.066 (0.030)	-0.126 (0.023)	-0.196 (0.044)	-0.239 (0.062)
	0.1	0.823 (0.897)	0.784 (0.916)	0.717 (0.724)	0.037 (0.041)	0.005 (0.047)	-0.040 (0.039)	-0.128 (0.024)	-0.194 (0.045)	-0.241 (0.065)

Table 9: Bias (MSE) for Scenario HI2

HI3		cDEA			LC			StoNEZD		
n	σ_v \ σ_u	0.05	0.1	0.15	0.05	0.1	0.15	0.05	0.1	0.15
26	0	-0.078 (0.017)	-0.129 (0.033)	-0.058 (0.031)	-0.052 (0.011)	-0.077 (0.020)	-0.096 (0.033)	-0.135 (0.024)	-0.193 (0.045)	-0.239 (0.067)
	0.05	0.094 (0.058)	0.023 (0.058)	0.012 (0.062)	0.027 (0.036)	-0.014 (0.036)	-0.020 (0.049)	-0.114 (0.031)	-0.192 (0.053)	-0.238 (0.072)
	0.1	0.138 (0.109)	0.056 (0.072)	0.148 (0.117)	0.094 (0.072)	0.029 (0.076)	0.040 (0.084)	-0.125 (0.035)	-0.195 (0.059)	-0.235 (0.075)
52	0	0.031 (0.026)	0.044 (0.037)	0.025 (0.044)	-0.050 (0.013)	-0.054 (0.021)	-0.062 (0.022)	-0.121 (0.022)	-0.189 (0.043)	-0.239 (0.065)
	0.05	0.190 (0.096)	0.146 (0.087)	0.159 (0.099)	0.028 (0.030)	-0.008 (0.036)	-0.049 (0.043)	-0.125 (0.026)	-0.190 (0.049)	-0.239 (0.067)
	0.1	0.342 (0.214)	0.305 (0.171)	0.209 (0.159)	0.096 (0.058)	0.051 (0.052)	-0.012 (0.051)	-0.126 (0.032)	-0.188 (0.051)	-0.229 (0.068)
104	0	0.116 (0.051)	0.051 (0.040)	0.048 (0.044)	-0.020 (0.015)	-0.039 (0.021)	-0.036 (0.023)	-0.127 (0.022)	-0.201 (0.046)	-0.242 (0.064)
	0.05	0.335 (0.205)	0.265 (0.162)	0.242 (0.148)	0.034 (0.026)	-0.031 (0.024)	-0.031 (0.035)	-0.119 (0.024)	-0.191 (0.047)	-0.236 (0.065)
	0.1	0.518 (0.388)	0.464 (0.329)	0.400 (0.290)	0.064 (0.043)	-0.010 (0.033)	-0.012 (0.039)	-0.129 (0.027)	-0.194 (0.048)	-0.237 (0.066)
156	0	0.149 (0.062)	0.102 (0.052)	0.066 (0.038)	-0.044 (0.011)	-0.043 (0.021)	-0.058 (0.017)	-0.130 (0.022)	-0.199 (0.044)	-0.242 (0.064)
	0.05	0.408 (0.253)	0.351 (0.206)	0.309 (0.173)	0.010 (0.023)	-0.027 (0.024)	-0.043 (0.025)	-0.125 (0.023)	-0.198 (0.046)	-0.241 (0.065)
	0.1	0.609 (0.512)	0.575 (0.492)	0.552 (0.444)	0.051 (0.042)	0.005 (0.038)	-0.052 (0.036)	-0.134 (0.027)	-0.191 (0.045)	-0.240 (0.065)
260	0	0.208 (0.100)	0.149 (0.062)	0.132 (0.067)	-0.024 (0.015)	-0.054 (0.016)	-0.051 (0.017)	-0.125 (0.021)	-0.198 (0.044)	-0.243 (0.064)
	0.05	0.464 (0.288)	0.473 (0.315)	0.407 (0.243)	0.005 (0.018)	-0.035 (0.020)	-0.065 (0.021)	-0.129 (0.023)	-0.195 (0.044)	-0.242 (0.064)
	0.1	0.767 (0.744)	0.676 (0.633)	0.679 (0.612)	0.040 (0.027)	0.008 (0.030)	-0.055 (0.028)	-0.125 (0.024)	-0.195 (0.045)	-0.240 (0.064)

Table 10: Bias (MSE) for Scenario HI3

HI4		cDEA			LC			StoNEZD		
n	σ_v \ σ_u	0.05	0.1	0.15	0.05	0.1	0.15	0.05	0.1	0.15
26	0	-0.040	-0.100	-0.148	-0.067	-0.083	-0.097	-0.123	-0.205	-0.239
		(0.022)	(0.027)	(0.046)	(0.013)	(0.024)	(0.027)	(0.023)	(0.050)	(0.068)
	0.05	0.049	-0.001	0.042	0.012	-0.001	-0.012	-0.138	-0.197	-0.224
		(0.041)	(0.044)	(0.068)	(0.033)	(0.038)	(0.057)	(0.034)	(0.053)	(0.067)
	0.1	0.188	0.214	0.003	0.100	0.064	0.063	-0.129	-0.183	-0.234
		(0.139)	(0.129)	(0.093)	(0.083)	(0.086)	(0.088)	(0.041)	(0.056)	(0.074)
52	0	-0.022	-0.112	0.001	-0.019	-0.066	-0.087	-0.116	-0.198	-0.243
		(0.016)	(0.030)	(0.035)	(0.017)	(0.018)	(0.027)	(0.021)	(0.046)	(0.066)
	0.05	0.196	0.019	0.031	0.029	0.001	-0.048	-0.109	-0.192	-0.239
		(0.090)	(0.057)	(0.065)	(0.027)	(0.036)	(0.038)	(0.025)	(0.047)	(0.068)
	0.1	0.284	0.243	0.225	0.088	0.011	0.029	-0.124	-0.197	-0.237
		(0.178)	(0.143)	(0.143)	(0.050)	(0.046)	(0.059)	(0.032)	(0.054)	(0.069)
104	0	0.054	0.085	0.054	-0.026	-0.057	-0.070	-0.135	-0.197	-0.242
		(0.025)	(0.039)	(0.037)	(0.015)	(0.016)	(0.022)	(0.023)	(0.044)	(0.065)
	0.05	0.262	0.183	0.030	0.032	-0.034	-0.061	-0.120	-0.192	-0.239
		(0.125)	(0.092)	(0.066)	(0.028)	(0.025)	(0.030)	(0.025)	(0.046)	(0.065)
	0.1	0.330	0.464	0.452	0.064	0.006	-0.001	-0.124	-0.197	-0.242
		(0.231)	(0.351)	(0.311)	(0.048)	(0.043)	(0.040)	(0.028)	(0.048)	(0.068)
156	0	0.133	0.030	0.051	-0.034	-0.060	-0.082	-0.134	-0.197	-0.244
		(0.064)	(0.024)	(0.028)	(0.013)	(0.018)	(0.021)	(0.023)	(0.043)	(0.065)
	0.05	0.366	0.235	0.281	0.002	-0.038	-0.062	-0.127	-0.198	-0.238
		(0.199)	(0.114)	(0.152)	(0.023)	(0.020)	(0.027)	(0.024)	(0.047)	(0.064)
	0.1	0.626	0.582	0.513	0.050	-0.001	-0.021	-0.129	-0.191	-0.235
		(0.503)	(0.479)	(0.377)	(0.036)	(0.039)	(0.036)	(0.026)	(0.045)	(0.064)
260	0	0.149	0.100	0.070	-0.034	-0.058	-0.072	-0.138	-0.199	-0.244
		(0.054)	(0.034)	(0.029)	(0.012)	(0.014)	(0.019)	(0.023)	(0.044)	(0.063)
	0.05	0.414	0.430	0.148	-0.009	-0.046	-0.062	NA	-0.196	-0.243
		(0.238)	(0.260)	(0.081)	(0.018)	(0.021)	(0.021)	NA	(0.044)	(0.065)
	0.1	0.735	0.728	0.651	0.030	-0.015	-0.053	-0.131	-0.193	-0.242
		(0.668)	(0.698)	(0.556)	(0.027)	(0.028)	(0.034)	(0.025)	(0.044)	(0.064)

Table 11: Bias (MSE) for Scenario HI4

6.3 MZ - Bias and MSE

MZ1		**cDEA**			**LC**			**StoNEZD**		
n	σ_v \ σ_u	0.05	0.1	0.15	0.05	0.1	0.15	0.05	0.1	0.15
26	0	-0.076	-0.185	-0.188	-0.063	-0.097	-0.109	-0.134	-0.187	-0.239
		(0.014)	(0.053)	(0.057)	(0.010)	(0.021)	(0.029)	(0.024)	(0.046)	(0.068)
	0.05	0.114	-0.001	-0.110	0.009	-0.053	-0.043	-0.124	-0.186	-0.229
		(0.048)	(0.055)	(0.080)	(0.029)	(0.038)	(0.045)	(0.032)	(0.054)	(0.070)
	0.1	0.160	-0.056	0.094	0.055	0.067	0.010	-0.129	-0.187	-0.218
		(0.109)	(0.107)	(0.089)	(0.069)	(0.097)	(0.078)	(0.044)	(0.060)	(0.072)
52	0	0.013	-0.077	-0.020	-0.053	-0.081	-0.104	-0.132	-0.192	-0.240
		(0.021)	(0.033)	(0.022)	(0.008)	(0.016)	(0.022)	(0.024)	(0.044)	(0.066)
	0.05	0.101	0.076	0.102	0.003	-0.043	-0.080	-0.123	-0.185	-0.241
		(0.084)	(0.059)	(0.069)	(0.019)	(0.027)	(0.033)	(0.029)	(0.046)	(0.069)
	0.1	0.340	0.258	0.243	0.038	-0.009	-0.025	-0.123	-0.177	-0.232
		(0.210)	(0.173)	(0.120)	(0.037)	(0.043)	(0.046)	(0.032)	(0.051)	(0.068)
104	0	0.082	0.002	-0.084	-0.051	-0.080	-0.097	-0.118	-0.198	-0.245
		(0.033)	(0.022)	(0.056)	(0.009)	(0.014)	(0.019)	(0.021)	(0.044)	(0.066)
	0.05	0.311	0.260	0.182	-0.010	-0.020	-0.071	-0.125	-0.194	-0.237
		(0.156)	(0.131)	(0.096)	(0.020)	(0.021)	(0.030)	(0.025)	(0.045)	(0.065)
	0.1	0.486	0.400	0.399	0.009	0.012	-0.054	-0.132	-0.187	-0.238
		(0.352)	(0.244)	(0.253)	(0.036)	(0.035)	(0.043)	(0.030)	(0.046)	(0.066)
156	0	0.126	0.088	0.084	-0.047	-0.069	-0.089	-0.102	-0.199	-0.242
		(0.046)	(0.039)	(0.036)	(0.008)	(0.014)	(0.018)	(0.018)	(0.045)	(0.064)
	0.05	0.359	0.343	0.301	-0.006	-0.038	-0.060	-0.127	-0.190	-0.241
		(0.183)	(0.185)	(0.153)	(0.019)	(0.021)	(0.024)	(0.024)	(0.043)	(0.064)
	0.1	0.598	0.624	0.522	0.018	-0.027	-0.062	-0.120	-0.182	-0.233
		(0.492)	(0.515)	(0.381)	(0.030)	(0.029)	(0.035)	(0.026)	(0.043)	(0.063)
260	0	0.145	0.108	0.095	-0.049	-0.074	-0.087	-0.126	-0.199	-0.242
		(0.051)	(0.038)	(0.037)	(0.008)	(0.013)	(0.016)	(0.021)	(0.044)	(0.063)
	0.05	0.385	0.405	0.394	-0.017	-0.052	-0.080	-0.126	-0.194	-0.240
		(0.254)	(0.237)	(0.233)	(0.014)	(0.018)	(0.024)	(0.023)	(0.044)	(0.064)
	0.1	0.738	0.675	0.610	0.012	-0.046	-0.084	-0.125	-0.193	-0.239
		(0.679)	(0.591)	(0.495)	(0.025)	(0.026)	(0.033)	(0.024)	(0.045)	(0.064)

Table 12: Bias (MSE) for Scenario MZ1

MZ2		cDEA			LC			StoNEZD		
n	σ_v \ σ_u	0.05	0.1	0.15	0.05	0.1	0.15	0.05	0.1	0.15
26	0	-0.059	-0.102	-0.102	-0.065	-0.072	-0.088	-0.117	-0.184	-0.245
		(0.018)	(0.038)	(0.033)	(0.013)	(0.022)	(0.029)	(0.022)	(0.045)	(0.069)
	0.05	0.009	0.073	0.052	0.007	-0.020	-0.034	-0.099	-0.174	-0.225
		(0.048)	(0.059)	(0.103)	(0.030)	(0.043)	(0.068)	(0.034)	(0.051)	(0.071)
	0.1	0.053	0.111	0.119	0.088	-0.014	0.034	-0.127	-0.186	-0.217
		(0.110)	(0.103)	(0.095)	(0.076)	(0.062)	(0.126)	(0.040)	(0.060)	(0.073)
52	0	0.047	-0.101	0.037	-0.027	-0.069	-0.065	-0.121	-0.196	-0.241
		(0.038)	(0.036)	(0.051)	(0.013)	(0.018)	(0.031)	(0.021)	(0.045)	(0.067)
	0.05	0.097	0.199	0.062	0.025	-0.013	-0.041	-0.121	-0.187	-0.230
		(0.068)	(0.105)	(0.091)	(0.031)	(0.041)	(0.039)	(0.028)	(0.045)	(0.064)
	0.1	0.391	0.321	0.313	0.064	0.018	-0.017	-0.124	-0.177	-0.229
		(0.294)	(0.208)	(0.219)	(0.059)	(0.053)	(0.050)	(0.033)	(0.049)	(0.068)
104	0	0.057	0.042	0.013	-0.030	-0.053	-0.056	-0.137	-0.199	-0.240
		(0.028)	(0.036)	(0.030)	(0.012)	(0.018)	(0.019)	(0.023)	(0.045)	(0.063)
	0.05	0.349	0.224	0.255	0.027	-0.056	-0.053	-0.126	-0.200	-0.237
		(0.207)	(0.121)	(0.131)	(0.030)	(0.027)	(0.032)	(0.025)	(0.048)	(0.064)
	0.1	0.510	0.525	0.444	0.080	0.003	-0.049	-0.124	-0.194	-0.243
		(0.399)	(0.405)	(0.327)	(0.050)	(0.046)	(0.045)	(0.028)	(0.048)	(0.069)
156	0	0.095	0.102	0.103	-0.005	-0.040	-0.059	-0.134	-0.200	-0.243
		(0.035)	(0.059)	(0.046)	(0.013)	(0.018)	(0.020)	(0.022)	(0.045)	(0.064)
	0.05	0.390	0.381	0.298	-0.010	-0.028	-0.065	-0.115	-0.193	-0.244
		(0.240)	(0.237)	(0.164)	(0.023)	(0.027)	(0.028)	(0.023)	(0.045)	(0.066)
	0.1	0.665	0.584	0.542	0.057	0.020	-0.035	-0.116	-0.184	-0.236
		(0.597)	(0.497)	(0.428)	(0.046)	(0.041)	(0.041)	(0.025)	(0.044)	(0.064)
260	0	0.201	0.082	0.123	-0.008	-0.035	-0.036	-0.109	-0.198	-0.243
		(0.092)	(0.033)	(0.056)	(0.015)	(0.018)	(0.018)	(0.019)	(0.044)	(0.064)
	0.05	0.511	0.457	0.403	0.010	-0.036	-0.073	-0.119	-0.196	-0.242
		(0.354)	(0.311)	(0.236)	(0.024)	(0.025)	(0.030)	(0.022)	(0.044)	(0.064)
	0.1	0.770	0.692	0.666	0.011	-0.029	-0.041	-0.126	-0.192	-0.242
		(0.755)	(0.612)	(0.611)	(0.028)	(0.031)	(0.035)	(0.024)	(0.044)	(0.065)

Table 13: Bias (MSE) for Scenario MZ2

MZ3		cDEA			LC			StoNEZD		
n	σ_v \ σ_u	0.05	0.1	0.15	0.05	0.1	0.15	0.05	0.1	0.15
26	0	-0.048	-0.095	-0.220	-0.060	-0.086	-0.098	-0.129	-0.190	-0.235
		(0.017)	(0.037)	(0.072)	(0.012)	(0.020)	(0.023)	(0.024)	(0.047)	(0.067)
	0.05	0.049	-0.126	0.005	0.003	-0.004	-0.042	-0.125	-0.192	-0.237
		(0.044)	(0.075)	(0.051)	(0.029)	(0.041)	(0.052)	(0.034)	(0.054)	(0.074)
	0.1	0.060	-0.026	0.037	0.057	0.030	0.010	-0.122	-0.188	-0.224
		(0.082)	(0.101)	(0.074)	(0.056)	(0.071)	(0.082)	(0.045)	(0.058)	(0.075)
52	0	-0.003	-0.009	-0.073	-0.062	-0.076	-0.087	-0.133	-0.188	-0.240
		(0.013)	(0.027)	(0.028)	(0.010)	(0.015)	(0.021)	(0.023)	(0.044)	(0.065)
	0.05	0.189	0.099	-0.110	0.025	-0.037	-0.059	-0.128	-0.192	-0.221
		(0.079)	(0.070)	(0.063)	(0.026)	(0.030)	(0.032)	(0.027)	(0.048)	(0.063)
	0.1	0.322	0.306	0.100	0.045	0.022	-0.028	-0.117	-0.192	-0.240
		(0.205)	(0.167)	(0.120)	(0.037)	(0.047)	(0.047)	(0.033)	(0.053)	(0.072)
104	0	0.039	0.049	0.004	-0.043	-0.077	-0.083	-0.133	-0.196	-0.246
		(0.026)	(0.035)	(0.022)	(0.008)	(0.014)	(0.017)	(0.023)	(0.044)	(0.066)
	0.05	0.252	0.204	0.225	0.007	-0.031	-0.073	-0.132	-0.188	-0.239
		(0.145)	(0.091)	(0.098)	(0.017)	(0.025)	(0.030)	(0.026)	(0.044)	(0.065)
	0.1	0.511	0.327	0.428	0.035	-0.012	-0.033	-0.114	-0.195	-0.240
		(0.392)	(0.242)	(0.292)	(0.038)	(0.040)	(0.040)	(0.029)	(0.047)	(0.068)
156	0	0.099	0.090	0.029	-0.046	-0.079	-0.087	-0.122	-0.196	-0.244
		(0.042)	(0.029)	(0.021)	(0.009)	(0.014)	(0.019)	(0.021)	(0.043)	(0.064)
	0.05	0.351	0.314	0.320	-0.012	-0.045	-0.071	-0.124	-0.195	-0.237
		(0.182)	(0.163)	(0.163)	(0.017)	(0.020)	(0.027)	(0.024)	(0.046)	(0.064)
	0.1	0.555	0.546	0.564	0.052	-0.023	-0.054	-0.119	-0.188	-0.240
		(0.414)	(0.422)	(0.419)	(0.039)	(0.032)	(0.035)	(0.024)	(0.046)	(0.066)
260	0	0.159	0.118	0.080	-0.043	-0.052	-0.074	-0.106	-0.197	-0.242
		(0.064)	(0.043)	(0.033)	(0.008)	(0.011)	(0.015)	(0.018)	(0.043)	(0.063)
	0.05	0.453	0.278	0.360	-0.023	-0.056	-0.066	-0.122	-0.195	-0.241
		(0.266)	(0.132)	(0.183)	(0.013)	(0.019)	(0.023)	(0.022)	(0.043)	(0.063)
	0.1	0.718	0.665	0.613	0.008	-0.020	-0.083	-0.127	-0.191	-0.242
		(0.647)	(0.568)	(0.494)	(0.025)	(0.025)	(0.036)	(0.025)	(0.044)	(0.065)

Table 14: Bias (MSE) for Scenario MZ3

MZ4		**cDEA**			**LC**			**StoNEZD**		
n	σ_v \ σ_u	0.05	0.1	0.15	0.05	0.1	0.15	0.05	0.1	0.15
26	0	-0.029	-0.052	-0.088	-0.060	-0.095	-0.083	-0.104	-0.170	-0.240
		(0.018)	(0.026)	(0.042)	(0.010)	(0.024)	(0.039)	(0.020)	(0.041)	(0.068)
	0.05	0.078	0.005	-0.033	0.038	-0.012	-0.031	-0.131	-0.191	-0.235
		(0.049)	(0.069)	(0.057)	(0.038)	(0.043)	(0.049)	(0.032)	(0.053)	(0.071)
	0.1	0.189	0.209	0.028	0.152	0.094	0.049	-0.116	-0.185	-0.227
		(0.092)	(0.134)	(0.088)	(0.117)	(0.124)	(0.096)	(0.036)	(0.056)	(0.073)
52	0	0.006	-0.136	-0.026	-0.042	-0.075	-0.095	-0.103	-0.186	-0.239
		(0.019)	(0.039)	(0.024)	(0.016)	(0.018)	(0.028)	(0.019)	(0.044)	(0.064)
	0.05	0.189	0.123	0.102	0.034	-0.029	-0.093	-0.129	-0.185	-0.240
		(0.100)	(0.086)	(0.058)	(0.027)	(0.030)	(0.037)	(0.028)	(0.046)	(0.068)
	0.1	0.350	0.281	0.235	0.093	0.039	-0.019	-0.114	-0.192	-0.233
		(0.229)	(0.181)	(0.156)	(0.060)	(0.054)	(0.047)	(0.029)	(0.052)	(0.071)
104	0	0.091	0.065	0.028	-0.029	-0.047	-0.048	-0.108	-0.199	-0.241
		(0.043)	(0.038)	(0.030)	(0.013)	(0.019)	(0.025)	(0.019)	(0.045)	(0.064)
	0.05	0.316	0.313	0.231	0.019	-0.021	-0.049	-0.131	-0.191	-0.238
		(0.176)	(0.194)	(0.149)	(0.026)	(0.032)	(0.032)	(0.026)	(0.046)	(0.064)
	0.1	0.520	0.448	0.474	0.070	0.009	-0.027	-0.118	-0.188	-0.234
		(0.465)	(0.353)	(0.358)	(0.048)	(0.044)	(0.046)	(0.027)	(0.047)	(0.065)
156	0	0.079	0.147	0.019	-0.002	-0.015	-0.066	-0.126	-0.199	-0.243
		(0.032)	(0.079)	(0.038)	(0.016)	(0.023)	(0.020)	(0.021)	(0.044)	(0.064)
	0.05	0.331	0.384	0.365	0.008	-0.020	-0.041	-0.129	-0.196	-0.239
		(0.201)	(0.251)	(0.225)	(0.022)	(0.029)	(0.031)	(0.025)	(0.046)	(0.064)
	0.1	0.682	0.586	0.528	0.055	-0.016	-0.019	-0.124	-0.195	-0.241
		(0.614)	(0.503)	(0.424)	(0.041)	(0.036)	(0.039)	(0.026)	(0.047)	(0.067)
260	0	0.176	0.165	0.131	-0.007	-0.031	-0.038	-0.122	-0.199	-0.243
		(0.076)	(0.079)	(0.059)	(0.019)	(0.022)	(0.021)	(0.020)	(0.044)	(0.064)
	0.05	0.443	0.431	0.434	0.031	-0.040	-0.039	-0.130	-0.197	-0.239
		(0.299)	(0.266)	(0.298)	(0.026)	(0.024)	(0.026)	(0.023)	(0.044)	(0.063)
	0.1	0.711	0.734	0.707	0.021	-0.024	-0.048	-0.134	-0.192	-0.240
		(0.662)	(0.723)	(0.663)	(0.036)	(0.034)	(0.034)	(0.026)	(0.045)	(0.064)

Table 15: Bias (MSE) for Scenario MZ4