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Restrictions Search for Panel VARs

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Restrictions Search for Panel VARs

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Abstract

As panel vector autoregressive (PVAR) models can include several countries and variables in one system, they are well suited for global spillover analyses. However, PVARs require restrictions to ensure the feasibility of the estimation. The present paper uses a selection prior for a data-based restriction search. It introduces the stochastic search variable selection for PVAR models (SSVSP) as an alternative estimation procedure for PVARs. This extends Koop and Korobilis's stochastic search specification selection (S^4) to a restriction search on single elements. The SSVSP allows for incorporating dynamic and static interdependencies as well as cross-country heterogeneities. It uses a hierarchical prior to search for data-supported restrictions. The prior differentiates between domestic and foreign variables, thereby allowing a less restrictive panel structure. Absent a matrix structure for restrictions, a Monte Carlo simulation shows that SSVSP outperforms S^4 in terms of deviation from the true values. Furthermore, the results of a forecast exercise for G7 countries demonstrate that forecast performance improves for the SSVSP specifications which focus on sparsity in form of no dynamic interdependencies.

Keywords: model selection, stochastic search variable selection, PVAR
JEL Classification: C11, C33, C52

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Contents

1 Introduction

Intensifying international goods and knowledge flows, as well as trade agreements, show the importance of international interdependencies among economies. With these inter-linkages, considering spillovers in real and financial variables across countries is essential. Shocks are likely to propagate internationally with asymmetric effects across various economies. Global spillover analyses require taking both the interdependencies and heterogeneities across countries into account. Analyses disregarding country specific information and global dependencies could end up with biased results regarding spillover effects and transmission channels.[1]

One tool that is able to consider dynamic and static global interdependencies as well as cross-section heterogeneities is the unrestricted panel vector autoregressive (PVAR) model. A PVAR includes several countries and several variables in one model. Thus, lagged foreign variables can impact domestic variables, meaning that dynamic interdependencies exist. Static interdependencies between two variables of two countries occur if the covariance between the two is unequal to zero. Finally, the PVAR accounts for heterogeneity across countries since the coefficient matrices can vary across economies. This strength of PVARs to be able to take into account interdependencies and heterogeneities across countries in one model comes at the cost of a large number of parameters to estimate. The unrestricted PVAR model which includes N countries, G variables, and P lags has $(NG)^2P$ parameters of the coefficient matrix and $\frac{NG(NG+1)}{2}$ parameters of the covariance to estimate. This is set against a usually relatively low number of time series observations for macroeconomic variables. To overcome this problem, the researcher has to set restrictions on the PVAR.

Despite the high potential of PVAR models for international spillover analyses, estimation strategies for PVARs are still limited. Papers implementing PVAR models often use assumptions on homogeneity and no dependencies to ensure the feasibility of the estimation.[2] Others follow the cross sectional shrinkage approach proposed by Canova and Ciccarelli (2009), which factorize the coefficients. A third and straightforward way of setting restrictions is to use the inherent panel structure in the data to assume that there are only interdependencies and heterogeneities

[1] Compare to Canova and Ciccarelli (2009), Canova and Ciccarelli (2013), Luetkepohl (2014) and Georgiadis (2015).

[2] Estimation procedures for these models are described in Canova and Ciccarelli (2013) and Breitung (2015).

across countries for specific country and variable combinations. The aim of this paper is to conduct a data-based restriction search by using a selection prior.

The paper specifies a selection prior for PVAR models that differentiates between domestic and foreign variables for each country. The algorithm, based on the selection prior, will search for dynamic interdependencies by checking whether the impact of lagged foreign variables is zero. Further, it will assess static interdependencies and use the restrictions as additional zero restrictions on a recursively identified structural PVAR model. This will be achieved by searching for zero restrictions on the upper triangular decomposition matrix of the covariance matrix. Finally, the algorithm searches for homogeneity between coefficients of domestic variables of different countries. It follows closely the selection prior for PVAR models of Koop and Korobilis (2015), which is called stochastic search specification selection (S^4), but extends the approach from a matrix wide search to single elements, as George et al. (2008) do in their stochastic search variable selection (SSVS) for VAR models. In order to distinguish the algorithm from S^4 and SSVS, the algorithm is called stochastic search variable selection for PVAR models (SSVSP).

The SSVSP extends the estimation procedure for PVAR models, contributing to the existing literature on PVARs. The paper adds to the selection prior literature, in particular by extending the S^4 algorithm. By implementing their prior on country matrices, Koop and Korobilis (2015) assume a specific panel structure; namely, all variables of one country are treated in a similar way: either restricted or not. The SSVSP allows for a less restrictive panel structure. It does not restrict variables on a country basis but searches for dynamic and static interdependencies for each foreign variable as well as for homogeneity for each domestic variable. Thus, the underlying panel structure separates domestic and foreign variables, although foreign variables are not separated on a country basis.

This less restrictive panel structure has the advantages that, firstly, the SSVSP prior has a wider range for empirical application than does the more rigid S^4. Applications, including financial and real variables, can especially benefit from a less restrictive form since the SSVSP can incorporate variable specific restrictions. For example, the prior allows for the possibility that only foreign financial variables have a dynamic impact on a domestic variable while real variables have no impact. Secondly, the SSVSP is able to provide a clear ranking of posterior probabilities of which variables to include in the model and which coefficients are homogeneous for each equation. Using the S^4 prior of Koop and Korobilis (2015) has the problem that the decision for excluding a single variable depends on the results for a matrix-wide search. Thirdly, compared to the commonly used Litterman prior for Large Bayesian VAR models, which assumes a specific shrinkage depending on the lag number, the SSVSP differentiates between domestic and foreign variables, thus taking the panel structure into account.[3]

These advantages are reflected in the results of both a Monte Carlo simulation and a forecasting exercise. Firstly, the results of the Monte Carlo studies show that especially when a more flexible panel structure is present, the posterior estimates of the SSVSP deviate less from the true values than the ones of S^4. Furthermore, the SSVSP is accurate in the selection of the restrictions displayed in the posterior probabilities for no interdependencies and homogeneity. Secondly, the results of the empirical application demonstrate that forecast performance improves for the SSVSP specifications which focus on sparsity in form of no dynamic interdependencies. The very large number of restrictions searched for in the SSVSP - dynamic and static interdependencies as well as homogeneity restrictions - leads to relatively weak forecast performance. In addition, the impulse responses to a shock in the US interest rate show plausible results which are in line with the literature. Overall, the results regarding the use of the SSVSP for PVAR models are encouraging.

2 Literature

So far, the literature basically has two ways to overcome the curse-of-dimensionality problem in PVAR models. One strand of the literature using PVAR models makes the assumption of either homogeneity or a lack of dynamic or static interdependencies.[4] These assumptions should be based on a solid theory. One common re-

[3] In addition to PVARs, large Bayesian VAR and Global VAR models are also potential tools to analyze international spillovers. Detailed descriptions of the two models are in Banbura et al. (2010), Pesaran et al. (2004), and Dees et al. (2007). However, these two types of models come with some limitations. Large Bayesian VAR models are limited in terms of neglecting the existence of a panel dimension in the data. BVAR models usually assume identical priors for each country. Thus, large BVAR models are especially applicable for analyzing intra-country spillovers, including a large number of variables. Global VARs, however, are restrictive in the way that they impose a particular structure on interdependencies by the chosen weights for aggregating the foreign component. GVAR models are especially useful for studies focusing on aggregated impacts or on spillovers from one large economy. In contrast to macroeconomic panel regressions, PVAR models allow to focus on effects on each single country and variable in the structural analysis while standard panel regressions deliver averaged or pooled results.

[4] Examples include Love and Zicchino (2006), assuming homogeneity and no dynamic interdependencies, and Ciccarelli et al. (2013), restricting for no dynamic interdependencies.

striction is to block exogeneity based on the small-open-economy assumption. The second strand of literature follows the cross sectional shrinkage approach proposed by Canova and Ciccarelli (2009).[5] The authors reduce the number of coefficients to be estimated, generating a common, country-specific, and variable-specific factor.

For global spillover analyses, it is hard to justify exogeneity, homogeneity, and no dependence assumptions. However, using the estimation strategy of Canova and Ciccarelli (2009) complicates structural shock identification because their model has two potential types of impulses. The first type is an impulse to the factors, the other to the variables. These two types come from the estimated evolution of the factors and from the regression in which the coefficients depend on a number of factors. To be able to only focus on impulses to the variables, the impulse response analysis must be done conditionally on shocks to the factors and vice versa. This paper follows a different approach to estimate the PVAR model by using a selection prior. An advantage of this prior is that it can easily account for the panel structure of the data and can handle both an over-parameterized unrestricted model as well as a large number of restricted models.

The selection prior literature starts with the paper of George and McCulloch (1993), who developed the prior for multiple regression models. The procedure, which the authors call stochastic search variable selection (SSVS), selects the variables that should be included in the regression model. This is achieved by using a hierarchical prior for the coefficients of the right hand side variables. The variables that should be included in the model occur more frequently when sampling from the conditional posterior distributions in the Gibbs sampler. George et al. (2008) further develop the SSVS, extending it for use with VAR models. They set a hierarchical prior on the autoregressive coefficients and find the elements that equal zero. Additionally, the authors use the prior for structural identification. They decompose the covariance matrix into two upper triangular matrices and let the SSVS algorithm find additional zero restrictions by searching for the elements of the decomposition matrix that are zero. Korobilis (2008) and Jochmann et al. (2010) show that forecast performance is improved for VAR models when using SSVS. The first paper uses SSVS in a factor model that includes a large number of macroeconomic variables for the United States. The second paper allows for structural breaks. Using data for the United States, the authors show that forecasts improve mainly due to the usage of SSVS and not due to the consideration of structural breaks. Subsequently,

[5]Examples include Canova et al. (2012) and Ciccarelli et al. (2012).

Korobilis (2013) extend the selection priors further to nonlinear set-ups.

Koop and Korobilis (2015) are the first to develop a selection prior for PVAR models. Their stochastic search specification selection (S^4) builds closely on George et al. (2008) but adds a restriction search for homogeneity of domestic autoregressive coefficients across countries. Further, in contrast to SSVS, they run the restriction search on whole matrices including all variables of one country and, thusly, assume a specific matrix panel structure. Therefore, the authors called their procedure specification search. Koop and Korobilis (2015) show with their Monte Carlo simulation that S^4 performs better than the OLS estimates. On average, the S^4 estimates are closer to the true values than the OLS estimates. Using data for sovereign bond yields, industrial production, and bid-ask spread for euro area countries from January 1999 through December 2012, they show that model fit improves when taking the characteristics of a panel model into account compared to a BVAR model without restriction search. Thus, the results of Koop and Korobilis (2015) show clearly that a prior for the PVAR model has to account for the inherent panel dimension within the data. Korobilis (2016) comes to the same conclusion. He compares different prior specifications for PVAR models. For larger PVAR models, priors taking the panel dimension into account deviate less from the true values than other VAR priors. In addition, priors with a panel dimension improve the forecasting performance that he demonstrates for the same empirical application as in Koop and Korobilis (2015). For small samples, however, the Bayesian shrinkage priors cannot outperform the OLS estimates.

One main drawback of S^4 is that the results lose detail since the S^4 algorithm is applied to matrices. Koop and Korobilis (2015) assume a specific country grouping for the restrictions. The authors can only make statements about interdependencies and heterogeneities between the countries but not which variables are driving the linkages and country specific coefficients. But this detail is essential for further interpretation of the results. Doing the restriction search for matrices can also lead to the exclusion of potentially important variables since decisions can only be made for whole matrices. Instead, the SSVSP makes a restriction search for each variable and, thus, can provide evidence supporting the exclusion of a single lag of a variable. In addition, in a set-up where country grouping for restrictions does not hold, Korobilis (2016) shows that the absolute deviation from the true value is lower for SSVS than it is for S^4. This result contributes to the argument for a restriction search on single elements.

One problematic issue is that SSVSP requires the SUR form of a VAR model, leading to the inversion of large matrices. This leads to a computationally demanding algorithm for medium and large size VARs.[6] To overcome this problem, Koop (2013) develops a natural conjugate selection prior for VARs. Here, no MCMC methods must be used. However, the natural conjugate selection prior has two disadvantages.[7] Firstly, each variable can only be either included or excluded in the whole VAR system. Secondly, the natural conjugate specification requires a specific covariance prior. Thus, a restriction search for the covariance elements of the VAR is not possible. Hence, for the purpose, being able to include static interdependencies and to allow for dynamic interdependencies that are not homogeneous across countries, the natural conjugate SSVS prior is not an alternative. Instead, the computational burden is accepted for having a differentiated prior that is able to account for the characteristics of a PVAR model, which should be less of a problem with increasing computational capacities.

3 PVAR Restrictions

A PVAR model for country i at time t with $i = 1, ..., N$ and $t = 1, ..., T$ is given by

$$y_{it} = A_{i1}Y_{t-1} + A_{i2}Y_{t-2} + ... + A_{iP}Y_{t-P} + u_{it}, \tag{1}$$

where $Y_{t-1} = (y'_{1t-1}, ..., y'_{Nt-1})'$ and y_{it} denotes a vector of dimension $[G \times 1]$.[8] The number of variables is defined as G. All A_{ip} have dimension $[G \times NG]$ for lag $p = 1, ..., P$. The index i denotes that the matrices are country specific for country i. The u_{it} are uncorrelated over time and normally distributed with mean zero and covariance matrix Σ_{ii}. The covariance matrix between errors of different countries is defined as $E(u_{it}u'_{jt}) = \Sigma_{ij} \ \forall \ i \neq j$ with dimension $[G \times G]$.

The PVAR model for all N countries can then be written as

$$Y_t = A_1Y_{t-1} + A_2Y_{t-2} + ... + A_PY_{t-P} + U_t. \tag{2}$$

The Y_t and U_t are $[NG \times 1]$-vectors. The U_t is normally distributed with mean zero and covariance matrix Σ that is of dimension $[NG \times NG]$. The $[NG \times NG]$-matrix

[6] Both Koop (2013) and Korobilis (2013) elaborate further on this issue.

[7] Koop (2013) explains the disadvantages of the natural conjugate prior in detail.

[8] Although this specification does not include a constant, it can be extended to include one.

A_p for one lag p, $p = 1, ..., P$, is defined as

$$A_p = \begin{pmatrix} \alpha_{p,11}^{11} & \cdots & \alpha_{p,1j}^{1k} & \cdots & \alpha_{p,1N}^{1G} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \alpha_{p,i1}^{l1} & \cdots & \alpha_{p,ij}^{lk} & \cdots & \alpha_{p,iN}^{lG} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \alpha_{p,N1}^{G1} & \cdots & \alpha_{p,Nj}^{Gk} & \cdots & \alpha_{p,NN}^{GG} \end{pmatrix}.$$

The element $\alpha_{p,ij}^{lk}$ refers to the coefficient of lag p of variable k of country j in the equation of variable l of country i. Thus, it measures the impact of lag p of variable k of country j on variable l of country i.

The following simple example will make the notation of α clear. The exemplary PVAR is a model with one lag and includes 3 countries and 2 variables ($N = 3$, $G = 2$).[9] The A matrix for the model with one lag will have the following form:

$$A = \begin{pmatrix} \alpha_{11}^{11} & \alpha_{11}^{12} & \alpha_{12}^{11} & \alpha_{12}^{12} & \alpha_{13}^{11} & \alpha_{13}^{12} \\ \alpha_{11}^{21} & \alpha_{11}^{22} & \alpha_{12}^{21} & \alpha_{12}^{22} & \alpha_{13}^{21} & \alpha_{13}^{22} \\ \alpha_{21}^{11} & \alpha_{21}^{12} & \alpha_{22}^{11} & \alpha_{22}^{12} & \alpha_{23}^{11} & \alpha_{23}^{12} \\ \alpha_{21}^{21} & \alpha_{21}^{22} & \alpha_{22}^{21} & \alpha_{22}^{22} & \alpha_{23}^{21} & \alpha_{23}^{22} \\ \alpha_{31}^{11} & \alpha_{31}^{12} & \alpha_{32}^{11} & \alpha_{32}^{12} & \alpha_{33}^{11} & \alpha_{33}^{12} \\ \alpha_{31}^{21} & \alpha_{31}^{22} & \alpha_{32}^{21} & \alpha_{32}^{22} & \alpha_{33}^{21} & \alpha_{33}^{22} \end{pmatrix},$$

where the first two rows are the equations for country 1, then rows 3 and 4 are the equations for country 2, and the last two rows belong to country 3. Thus, α_{13}^{21}, for example, measures the impact of variable 1 of country 3 on variable 2 of country 1.

A structural form of the PVAR model is derived by decomposing the covariance matrix Σ into $\Sigma = \Psi^{-1'}\Psi^{-1}$ where Ψ is a upper triangular matrix. Therefore, the structural identification is based on a recursive order. An element ψ_{ij}^{lk} of the upper triangular matrix Ψ defines the static relation between variable l of country i and variable k of country j.

This structural PVAR model can account for dynamic interdependencies (DI), static interdependencies (SI), and cross-section heterogeneities (CSH).[10] Firstly, the model allows lagged variables of foreign countries to impact domestic variables.

[9]The notation is changed for simplicity from now on. If a model with only one lag is considered, the index of the coefficient matrix and of single coefficients referring to the lag order is omitted.

[10]Canova and Ciccarelli (2013) provide a survey of the PVAR restrictions.

Secondly, there are static interdependencies between two variables of two countries if the element of the upper triangular decomposition matrix of the covariance matrix is equal to zero. Thus, the search for static interdependencies allows for a data-based structural identification of a PVAR model using additional zero restrictions on top of a recursive order. Thirdly, the PVAR accounts for heterogeneity across countries since the A_{ip} matrices can vary across countries.

The strength of PVARs to account for interdependencies and heterogeneities comes at the costs of many parameters to estimate. To overcome this problem the researcher must set restrictions on the PVAR. A straightforward way of setting restrictions is to use the panel structure inherent in the data. Thus, one can expect that there are only interdependencies and heterogeneities across countries for specific country and variable combinations. For example, expectations could be that the short term interest rate of the United States impacts the Eurozone interest rate or that strong GDP growth in France also impacts German exports. On the other hand, one would assume that the Canadian GDP growth does not influence the Eurozone's short term interest rate or that Japanese GDP growth is dynamically independent of changes in Italian GDP growth. We would also expect that the sign and magnitude of the impact of Portugal's and Spain's GDP growth on their domestic GDP growth is fairly similar, while it would differ from the impact of United States' GDP growth on itself.

Therefore, for some coefficients the following restrictions can be found in the data:

1. **No dynamic interdependencies (DI)**: no lagged impact from variable l of country i to variable k of country j if $\alpha^{lk}_{1,ij} = ... = \alpha^{lk}_{p,ij} = 0$ for $j \neq i$ and $\forall p = 1, ..., P$
2. **No static interdependencies (SI)**: no correlation between the error term of equation l of country i, u^l_{it}, with the error term of equation k of country j, u^k_{jt}, if $\psi^{lk}_{ij} = 0$ for $j \neq i$
3. **No cross-section heterogeneities (CSH)**: homogeneous coefficient across the economies if $\alpha^{lk}_{p,jj} = \alpha^{lk}_{p,ii}$ for $j \neq i$ and $\forall p = 1, ..., P$

We can define $[(NG - G)NG]$ DI, $[(N(N-1)/2)G^2]$ SI, and $[(N(N-1)/2)G^2]$ CSH restrictions.[11] The essential part is to determine for which country and variable

[11]Note that while SI restrictions are symmetric, DI restrictions cannot be symmetric.

combinations these restrictions hold. The SSVSP algorithm is able to search for PVAR restrictions that are supported by the data. The SSVSP of this paper follows closely Koop and Korobilis (2015).

4 Selection Prior for PVAR

The stochastic search variable selection algorithm for PVARs works with the unrestricted PVAR model. In the following, the PVAR model is simplified to a model including only one lag. If the researcher includes several lags, the restriction search for dynamic interdependencies would provide guidance as to which lags should be included in the model. Thus, the DI restriction search can be used as a lag length selection criterion. The full unrestricted model with one lag can be rewritten as

$$Y_t = Z_{t-1}\alpha + U_t, \tag{3}$$

where α is the vectorized matrix A and $Z_{t-1} = (I_{NG} \otimes Y_{t-1})$.

The basic idea of a selection prior is that the selection of a variable is made by a hierarchical prior. Each element of α is drawn from a mixture of two normal distributions centering around the restriction, either with a small or large variance. Depending on a hyperparameter, γ, which is Bernoulli distributed, the coefficient shrinks to the restriction (small variance case) or is estimated with a looser prior (larger variance case). Thus, the algorithm imposes soft restrictions by allowing for a small variance. In contrast to Koop and Korobilis (2015), the restriction search is completed for each single element and not on the whole matrices that includes all variables for a given country. A Gibbs sampler is used to obtain the posterior distributions.

The SSVSP algorithm has now specific priors for the parameters of A_1 and for the covariance matrix building with the DI, SI, and CSH restrictions. The DI restrictions impose limits on the coefficients of the lagged foreign endogenous variables. The DI prior is given by

$$\begin{aligned} \alpha_{ij}^{lk} \mid \gamma_{DI,ij}^{lk} &\sim (1-\gamma_{DI,ij}^{lk})\mathcal{N}(0,\tau_1^2) + \gamma_{DI,ij}^{lk}\mathcal{N}(0,\tau_2^2) \\ \gamma_{DI,ij}^{lk} &\sim Bernoulli(\pi_{DI,ij}^{lk}). \end{aligned}$$

The prior distribution of α_{ij}^{lk} is conditional on the hyperparameter $\gamma_{DI,ij}^{lk}$. This hyperparameter also has a distribution. That is why the prior is called a hierarchical prior. $\gamma_{DI,ij}^{lk}$ is Bernoulli distributed.[12] Thus, it takes either the value one or zero. If $\gamma_{DI,ij}^{lk}$ is equal to zero, α_{ij}^{lk} is drawn from the first part of the normal distribution with mean zero and variance τ_1^2. If $\gamma_{DI,ij}^{lk}$ is equal to one, α_{ij}^{lk} is drawn from the second part of the normal distribution with mean zero and variance τ_2^2. The values of τ_1^2 and τ_2^2 must be chosen such that τ_1^2 is smaller than τ_2^2. Thus, if $\gamma_{DI,ij}^{lk} = 0$, the prior is tight in the sense that the parameter is shrunk to zero. Whereas the prior is loose for $\gamma_{DI,ij}^{lk} = 1$ since the prior variance is larger. Hence, if $\gamma_{DI,ij}^{lk} = 0$, no dynamic interdependency is supported by the data and the coefficient will be estimated with a small variance around zero. Going back to the simple 3-country-2-variable example, the coefficients of A, which are checked for dynamic interdependencies, are now marked with DI:

$$A = \begin{pmatrix} & & DI & DI & DI & DI \\ & & DI & DI & DI & DI \\ DI & DI & & & DI & DI \\ DI & DI & & & DI & DI \\ DI & DI & DI & DI & & \\ DI & DI & DI & DI & & \end{pmatrix}.$$

The covariance matrix of the PVAR model is decomposed into two upper triangular matrices Ψ, $\Sigma = \Psi^{-1'}\Psi^{-1}$. This ensures a recursive order to identify the structural shocks of the PVAR model. The SI prior is set on the elements of the upper triangular matrix. If SI restrictions are found, the structural PVAR is overidentified since additional zero restrictions can be set on top of the recursive ordering. The SSVSP prior is a data-based method to structurally identify the system. It has the drawback that the identification is limited since it only allows for a recursive structure. A clear advantage of this decomposition is that it assures that by construction every simulated Σ is positive definite.[13]

[12] How $\pi_{DI,ij}^{lk}$ is set is described in detail in the Appendix. This holds also for the CSH and SI priors.

[13] Compare also to Koop and Korobilis (2015).

The prior for SI restrictions follows the same logic as the DI prior:

$$\begin{aligned}\psi_{ij}^{lk} \mid \gamma_{SI,ij}^{lk} &\sim (1-\gamma_{SI,ij}^{lk})\mathcal{N}(0,\kappa_1^2) + \gamma_{SI,ij}^{lk}\mathcal{N}(0,\kappa_2^2)\\ \gamma_{SI,ij}^{lk} &\sim Bernoulli(\pi_{SI,ij}^{lk}).\end{aligned}$$

The prior is for all $j \neq i$. To assure positive variance elements, the $(\psi_{ii}^{kk})^2$ are gamma distributed, $(\psi_{ii}^{kk})^2 \sim \mathcal{G}(a,b)$. The elements for the same country, ψ_{ii}^{lk} for $l \neq k$, are normally distributed with mean zero and variance κ_2^2. All ψ_{ij}^{lk} elements $(j \neq i)$ are drawn from the specified hierarchical prior. Thus, the parameters are drawn from a weighted normal distribution with weights $\gamma_{SI,ij}^{lk} \in \{0,1\}$. The parameter κ_1^2 is smaller than κ_2^2. If $\gamma_{SI,ij}^{lk}$ is equal to zero, the parameter shrinks to zero showing that the data do not support static interdependency. The SI restrictions are symmetric. Back to the example, the following elements of the covariance matrix are checked for static interdependencies:

$$\Psi = \begin{pmatrix} & & SI & SI & SI & SI \\ & & SI & SI & SI & SI \\ & & & & SI & SI \\ & & & & SI & SI \\ & & & & & \\ & & & & & \end{pmatrix}.$$

The selection prior can also easily be used to only estimate the reduced form of a PVAR model by not defining the Ψ-matrix and not searching for static interdependency restrictions but by assuming a standard distribution for the PVAR variance, e.g. an inverse Wishart distribution.

Searching for homogeneity across countries is not as straightforward as searching for the zero restrictions for dynamic and static interdependencies. The main contribution of Koop and Korobilis (2015) is the development of a procedure how to search for CSH restrictions. Possible homogeneity across countries is assessed for the coefficients measuring the impact of domestic variables on domestic variables. Economies have homogeneous coefficient if α_{jj}^{lk}, the coefficient of variable k of country j in the equation of variable l of the same country j, is equal to α_{ii}^{lk}, the coefficient of the same variable k but of country i in the equation of variable l of country i. The restricted part of the distribution assumes that in mean the coefficients are equal,

$\alpha_{jj}^{lk} = \alpha_{ii}^{lk}$ in mean, while the unrestricted part has mean zero. The CSH prior is given by

$$\begin{aligned} \alpha_{jj}^{lk} \mid \gamma_{CSH}^{w} &\sim (1-\gamma_{CSH}^{w})\mathcal{N}(\alpha_{ii}^{lk}, \xi_1^2) + \gamma_{CSH}^{w}\mathcal{N}(0, \xi_2^2) \\ \gamma_{CSH}^{w} &\sim Bernoulli(\pi_{CSH}^{w}). \end{aligned}$$

The prior is for all $j \neq i$. There are $(N(N-1)/2)G^2 = K$ combinations of coefficients that are checked for homogeneity. The index $w = 1, ..., K$ refers to a specific combination. Again, ξ_1^2 is smaller than ξ_2^2. The main difference to the DI and SI prior is that instead of shrinking the parameter to zero in the first part of the normal distribution, the mean is equal to the coefficient for which homogeneity is being checked. Going back to the 3-countries-2-variables example, the coefficients that are marked with CSH are checked for homogeneity:

$$A = \begin{pmatrix} CSH & CSH & & & & \\ CSH & CSH & & & & \\ & & CSH & CSH & & \\ & & CSH & CSH & & \\ & & & & CSH & CSH \\ & & & & CSH & CSH \end{pmatrix}.$$

The algorithm checks, for example, whether the coefficient of the first variable of country 1 (in the equation of the first variable for country 1) is equal to the coefficient of the first variable of country 2 (in the equation of the first variable for country 2), the same between country 1 and 3 as well as between 2 and 3. Thus, for the first variable in the equation of the same variable three combinations are checked for homogeneity. The restrictions are $\alpha_{11}^{11} = \alpha_{22}^{11}$, $\alpha_{11}^{11} = \alpha_{33}^{11}$, and $\alpha_{22}^{11} = \alpha_{33}^{11}$.

To be able to check all possible combinations, the procedure of Koop and Korobilis (2015) is followed, who define a selection matrix

$$\Gamma = \prod_{w=1}^{K} \Gamma_w.$$

The number of Γ_w matrices equals the number of possible combinations to check for homogeneity, $w = 1, ..., K$. Each Γ_w has the dimension $[NG \times NG]$. The matrix Γ_w is an identity matrix with two exceptions. The diagonal element at the position α_{jj}^{lk}

is set equal to γ^w_{CSH} and the off-diagonal element referring to the element α^{lk}_{ii} is set equal to $(1-\gamma^w_{CSH})$. Back to the example, if $\alpha^{11}_{11} = \alpha^{11}_{22}$ is checked, the restriction matrix is an identity matrix of dimension $[NG \times NG] = [36 \times 36]$. The element in the first row and first column is replaced by γ^1_{CSH} and the element in the 15th row and first column, referring to the position of α^{11}_{22} in the vectorized A matrix, by $(1-\gamma^1_{CSH})$. If γ^1_{CSH} equals zero, α^{11}_{11} and α^{11}_{22} are homogeneous. If all coefficients are heterogeneous, all Γ_w are identity matrices. To impose the CSH restrictions, the posterior mean of α is multiplied by the selection matrix Γ.

The posterior distributions are simulated using a Gibbs sampler. The prior specification, normal mixture distributions, has the advantage that it allows the usage of the Gibbs sampler which is easily solvable.[14] The means of the posterior distributions are used as point estimates for the coefficients.

The outcome of the algorithm can be interpreted in two ways.[15] Based on the results of the algorithm, the researcher can select one specific restricted PVAR model. Hence, the algorithm is used as a model selection criterion. The posterior probabilities $\gamma_{DI}, \gamma_{SI}, \gamma_{CSH}$ give the information whether a variable is included in the model or not and whether it is homogeneous or not. These probabilities are calculated as the proportion of γ_{DI}, γ_{SI}, or γ_{CSH} draws that equal one over all draws. Based on the estimated $\gamma_{DI}, \gamma_{SI}, \gamma_{CSH}$ values, it is possible to provide a ranking for DI, SI and CSH restrictions. The posterior probabilities $\gamma_{DI}, \gamma_{SI}, \gamma_{CSH}$ can be sorted in descending order. The researcher can set the restrictions successively starting with the variable for which the posterior probability of γ_{DI}, γ_{SI}, or γ_{CSH} being zero is highest or for which the probability being one is lowest. The researcher can set the restrictions successively until the model with the best fit is found.

Another way to make the selection is via a threshold value. The selection prior literature often uses 0.5 as a threshold value to determine whether a restriction is set. Using the results as a model selection criterion shows particularly well the strong advantages of the SSVSP prior for PVAR compared to the S^4. While Koop and Korobilis (2015) can only make statements about including or excluding a whole country, based on the SSVSP it is possible to make clear decisions on exclusion for every single variable. Using the SSVS of George et al. (2008) would also allow the researcher to make clear statements about single variables, but it neglects the pos-

[14]The Gibbs sampler algorithm is described in detail in the Appendix.

[15]Compare to the general survey in Koop and Korobilis (2010) or the specific explanation for the S^4 in Koop and Korobilis (2015).

sibility of cross-sectional homogeneities as an important characteristic of PVARs. Alternatively, the outcome of the algorithm can be used as a Bayesian model averaging (BMA) result. Thus, the posterior means averaged over all draws are taken as coefficient estimates. Since each draw leads to a specific restricted model, the BMA results average over all possible restricted models.

5 Monte Carlo Simulation

5.1 Simulation Set-up

In order to evaluate the prior, two Monte Carlo simulations are conducted.[16] For both Monte Carlo simulations data are generated from a panel VAR model which includes three countries, two variables for each country, one lag, and 100 observations. Assume for an international spillover analysis that both dynamic and static interdependencies as well as cross-sectional heterogeneities exist for specific variable and country combinations. Firstly, assume that country 2 has a dynamic impact on country 1 and country 1 on country 3. Country 3 does not impact the other two countries dynamically. Coefficients are homogeneous between countries 2 and 3. Static interdependencies exist between country 1 and 2. This example has a clear country grouping structure. Hence, all variables of one country have either an impact on all variables of a second country or not. The same holds for homogeneity across countries. A scenario like this is given by the first Monte Carlo simulation where the following parameter values are set:

$$A^{true} = \begin{pmatrix} 0.8 & 0 & 0.2 & 0.2 & 0 & 0 \\ 0 & 0.7 & 0.3 & 0.3 & 0 & 0 \\ 0 & 0 & 0.6 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0.3 & -0.4 & 0 & 0 & 0.6 & 0.5 \\ 0.2 & 0.4 & 0 & 0 & 0 & 0.5 \end{pmatrix}, \Psi^{true} = \begin{pmatrix} 1 & 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 1 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

[16] 100 samples, each with a length of 100 are simulated. The Gibbs sampler is done with 55000 draws, of which 5000 draws are disregarded as draws of the burn-in-phase. The calculation is based on a further development of the MATLAB code provided by Koop and Korobilis (2015) (https://sites.google.com/site/dimitriskorobilis/matlab/panel_var_restrictions).

Separating A_1 and Ψ into $[2 \times 2]$ matrices that only include variables of one country shows the clear country grouping structure.

Secondly, assume that the interdependency and homogeneity structure is not automatically similar for all variables of one country. Hence, a less restrictive panel structure exists. Thus, the first variable of country 2 and 3 has a dynamic impact on country 1's variables, but not the second variable of the foreign countries. Assume that variable 1 of country 3 is dynamically influenced by both variables of country 1, while there are no such interdependency structures for variable 2. Static interdependencies and homogeneity across coefficients also only exist for special country and variable pairs. The second Monte Carlo simulation incorporates these properties and has the following true parameters:

$$A^{true} = \begin{pmatrix} 0.8 & 0 & 0.2 & 0 & 0.2 & 0 \\ 0 & 0.7 & 0.2 & 0 & 0.2 & 0 \\ 0 & 0 & 0.6 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0.3 & 0 & 0 \\ 0.3 & -0.4 & 0 & 0 & 0.6 & 0.5 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \end{pmatrix}, \Psi^{true} = \begin{pmatrix} 1 & 0 & 0.5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Here the interdependency structure between the countries and homogeneity across parameters varies across variables. There is no clear country grouping.

The performance of SSVSP will be compared to the performance of different prior specifications. The benchmark model is a model with no restriction search, referred to as unrestricted VAR. This model is estimated using the SSVSP prior but with fixed γ values such that each parameter is drawn from the distribution with the higher variance. Furthermore, the SSVSP will be compared to the S^4 and to the SSVS of George et al. (2008).[17] The SSVS prior sets the DI prior on all lagged values and the SI prior on all covariance elements. Thus, no distinction between domestic and foreign variables is made. Additionally, two specific specifications of the SSVSP will be compared. Firstly, the SSVSP will only search for DI restrictions (abbreviated with SSVSP_DI), meaning that the γ values for the SI and CSH priors are set to one (coefficients are drawn from the looser parts of the distributions). Secondly, the restriction search will only be done for CSH restrictions (SSVSP_CSH).

The performance of each estimator is checked via the Absolute Percentage De-

[17] The prior hyperparameters used in the Monte Carlo simulations for all different priors are given in the Appendix.

Table 1: Absolute percentage deviation for estimated coefficient matrix A from the true value

	Simulation 1	**Simulation 2**
SSVSP	0.043	0.036
SSVSP_DI	0.026	0.023
SSVSP_CSH	0.041	0.037
S^4	0.048	0.056
SSVS	0.067	0.066
unrest VAR	0.027	0.027

APD: deviation of the estimated coefficients from the true value, average over 100 MC draws and all coefficients. Coefficient estimates are the posterior means averaged over all MC draws. SSVSP_DI: SSVSP with only DI restrictions. SSVSP_CSH: SSVSP with only CSH restrictions. S^4: prior of Koop and Korobilis (2015). SSVS: prior of George et al. (2008). Unrest VAR: parameters drawn from unrestricted part of distributions. Simulation 1: DGP has matrix panel structure. Simulation 2: DGP has flexible panel structure.

viation (APD) statistic:[18]

$$APD = \frac{1}{(NG)^2} \sum_{i=1}^{(NG)^2} \mid \alpha_i - \alpha_i^{true} \mid .$$

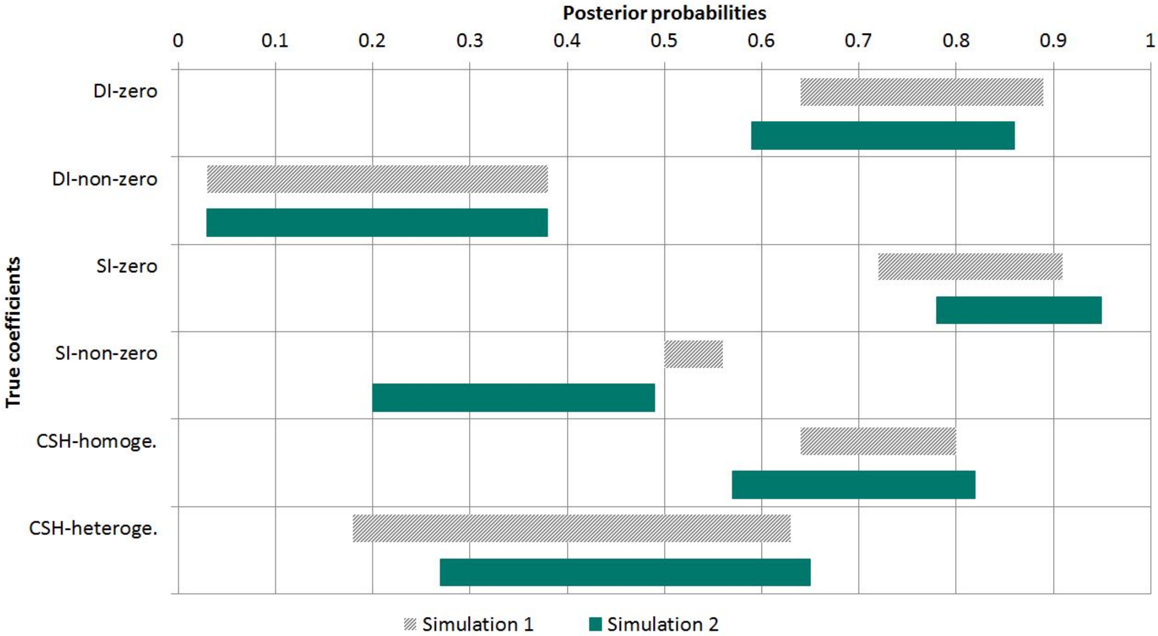
The statistic measures the absolute deviation of the estimated coefficient α_i, given by the posterior mean averaged over all simulation draws, from the true value, α_i^{true}. Furthermore, the accuracy of the SSVSP to find the restrictions is checked. This is achieved by comparing the restrictions' probabilities to the true values. Thus, the probabilities that $\alpha_{ij}^{lk} = 0$, $\psi_{ij}^{lk} = 0$, and $\alpha_{jj}^{lk} = \alpha_{ii}^{lk}$ are compared among themselves and in relation to the true values. These posterior probabilities are calculated as the proportion of $\gamma_{DI,ij}^{lk}, \gamma_{SI,ij}^{lk}$, and γ_{CSH}^{w} draws that equal zero averaged over all Gibbs sampler draws and all simulated samples. The higher the proportion of γ draws that equal zero is, the higher the probability is that no dynamic and no static interdependencies exist and coefficients are homogeneous.

5.2 Results

The results of the Monte Carlo study demonstrate that, firstly, the SSVSP outperforms the S^4 in terms of closeness to the true coefficients. This especially holds when

[18] Both Koop and Korobilis (2015) and Korobilis (2016) use mean deviation statistics to evaluate the performance of estimators in Monte Carlo simulations.

Figure 1: Range of posterior probabilities $\alpha_{ij}^{lk} = 0$, $\psi_{ij}^{lk} = 0$, and $\alpha_{jj}^{lk} = \alpha_{ii}^{lk}$



Posterior probabilities, $p(\alpha_{ij}^{lk} = 0)$, $p(\psi_{ij}^{lk} = 0)$, and $p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$, are calculated as the proportion of $\gamma_{DI,ij}^{lk}, \gamma_{SI,ij}^{lk}$, and γ_{CSH}^{w} draws that equal zero averaged over all Gibbs sampler draws and all simulated samples. Simulation 1: DGP has matrix panel structure. Simulation 2: DGP has flexible panel structure.

a less restrictive panel structure is present. Secondly, the SSVSP accurately selects the restrictions. This is validated by the higher posterior probabilities for no interdependencies and homogeneity for parameters which are truly zero or homogeneous compared to the probabilities for non-zero and heterogeneous parameters.

As table 1 shows, the estimated coefficients which are the posterior means averaged over all simulation draws from the SSVSP are on average slightly closer to the true values compared to S^4 for both simulations.[19] The APD for the SSVSP takes a lower value, $APD_{SSVSP} = 0.043$ for simulation one and $APD_{SSVSP} = 0.036$ for simulation two, compared to the value for the S^4, $APD_{S^4} = 0.048$ for simulation one and $APD_{S^4} = 0.056$ for simulation two. This holds also for the SSVSP_DI and SSVSP_CSH. In particular, the S^4 performs weaker in simulation two, where a less restrictive panel structure is present, since the grouping structure on which the restriction search is done is not present in the data. However, the unrestricted VAR outperforms the SSVSP and S^4. But, the estimated coefficients from SSVSP_DI deviate less from the true values, $APD_{SSVSP\_DI} = 0.026$ for simulation one and

[19]SSVSP estimates for A and Σ are given in the Appendix for both simulations.

Table 2: Accuracy of selecting DI restrictions: posterior probabilities for restrictions $p(\alpha_{ij}^{lk} = 0)$

Simulation 1						**Simulation 2**					
-	-	**0.2**	**0.2**	**0**	**0**	-	-	**0.2**	**0**	**0.2**	**0**
		0.26	*0.38*	*0.88*	*0.78*			*0.30*	*0.62*	*0.38*	*0.59*
-	-	**0.3**	**0.3**	**0**	**0**	-	-	**0.2**	**0**	**0.2**	**0**
		0.06	*0.18*	*0.83*	*0.76*			*0.31*	*0.70*	*0.30*	*0.70*
0	**0**	-	-	**0**	**0**	**0**	**0**	-	-	**0**	**0**
0.71	*0.75*			*0.85*	*0.73*	*0.82*	*0.73*			*0.84*	*0.65*
0	**0**	-	-	**0**	**0**	**0**	**0**	-	-	**0**	**0**
0.74	*0.77*			*0.89*	*0.79*	*0.84*	*0.75*			*0.86*	*0.71*
0.3	**-0.4**	**0**	**0**	-	-	**0.3**	**-0.4**	**0**	**0**	-	-
0.15	*0.03*	*0.79*	*0.64*			*0.19*	*0.03*	*0.76*	*0.67*		
0.2	**0.4**	**0**	**0**	-	-	**0**	**0**	**0**	**0**	-	-
0.25	*0.05*	*0.78*	*0.66*			*0.86*	*0.79*	*0.83*	*0.71*		

True values are in bolt, probabilities for restrictions, $p(\alpha_{ij}^{lk} = 0)$, are in italic. Posterior probabilities, $p(\alpha_{ij}^{lk} = 0)$, are calculated as the proportion of $\gamma_{DI,ij}^{lk}$ draws that equal zero averaged over all Gibbs sampler draws and all simulated samples. Simulation 1: DGP has matrix panel structure. Simulation 2: DGP has flexible panel structure.

$APD_{SSVSP\_DI} = 0.023$ for simulation two, than the estimates of the unrestricted VAR. Doing the restriction search only for CSH, however, reduces the average deviation from the true values only in simulation one compared to the SSVSP. This indicates that the gain cannot be explained by the reduced number of restrictions on which the search is done for but rather by searching for no dynamic interdependencies. Thus, the use of a prior which incorporates no dynamic interdependencies is beneficial for the DGPs of both simulations.

Furthermore, the SSVSP algorithm is accurate in selecting the restrictions. This is true because posterior probabilities that no interdependencies or heterogeneities exist are higher for true zero or homogeneous values compared to the probabilities for true non-zero or true heterogeneous values shown in figure 1. The graph presents the range of posterior probabilities for simulation one and two for true zero or homogeneous coefficients and true non-zero or true heterogeneous coefficients. The posterior probabilities, $p(\alpha_{ij}^{lk} = 0)$, $p(\psi_{ij}^{lk} = 0)$, and $p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$, are calculated as one minus the posterior means for $\gamma_{DI,ij}^{lk}, \gamma_{SI,ij}^{lk}$, and γ_{CSH}^{w} averaged over all MC draws. Since the γ parameters are Bernoulli distributed, the posterior probabilities measure the proportion of $\gamma_{DI,ij}^{lk}, \gamma_{SI,ij}^{lk}$, and γ_{CSH}^{w} draws that equal zero averaged

Table 3: Accuracy of selecting SI restrictions: posterior probabilities for restrictions $p(\psi_{ij}^{lk} = 0)$

Simulation 1						**Simulation 2**					
-	-	**0.5**	**0.5**	**0**	**0**	-	-	**-0.5**	**0**	**0**	**0**
		0.56	*0.54*	*0.72*	*0.80*			*0.49*	*0.95*	*0.78*	*0.95*
-	-	**-0.5**	**-0.5**	**0**	**0**	-	-	**0**	**-0.5**	**0**	**0**
		0.56	*0.50*	*0.74*	*0.76*			*0.91*	*0.20*	*0.89*	*0.91*
-	-	-	-	**0**	**0**	-	-	-	-	**0**	**0**
				0.84	*0.86*					*0.90*	*0.93*
-	-	-	-	**0**	**0**	-	-	-	-	**0**	**0**
				0.91	*0.91*					*0.92*	*0.93*

True values are in bolt, probabilities for restrictions, $p(\psi_{ij}^{lk} = 0)$, are in italic. Posterior probabilities, $p(\psi_{ij}^{lk} = 0)$, are calculated as the proportion of $\gamma_{SI,ij}^{lk}$ draws that equal zero averaged over all Gibbs sampler draws and all simulated samples. Simulation 1: DGP has matrix panel structure. Simulation 2: DGP has flexible panel structure.

over all Gibbs sampler draws and all simulated samples. Looking at simulation one and DI restrictions, the probabilities that $\alpha_{ij}^{lk} = 0$ are considerably higher for true zero parameters than for true non-zero values. The first are in a range between 0.64 and 0.89 while the latter one are between 0.03 and 0.38. Turning to simulation two, if no dynamic interdependencies occur in truth, shown by zero values for the parameters, the probabilities that $\alpha_{ij}^{lk} = 0$ are between 0.59 and 0.86. These values are all higher than the probabilities for the parameters which dynamically affect the dependent variables. These probability values are between 0.03 and 0.38. Table 2 shows the posterior probabilities for $\alpha_{ij}^{lk} = 0$ and 3 for $\psi_{ij}^{lk} = 0$ in detail. The true values of the simulations are presented in bolt, probabilities for the restrictions in italic. Results for simulation one are shown in the left column, results for simulation two in the right column. Focusing on simulation one, the probability for no dynamic impact of country 2 on country 1 for variable 2, for example, shown in the second row of the table, is 0.06 for variable 1 and 0.18 for variable 2. The true values, each 0.3, show that dynamic interdependencies exist. The variables of country 3, however, have no dynamic impact on variable 2 of country 1, shown by the zero values. The algorithm finds here a substantially higher probability for no dynamic interdependencies with values of 0.83 for variable 1 and 0.76 for variable 2.

The SSVPS also selects accurately the SI restrictions in both simulations. This is true since for both simulations the probabilities that $\psi_{ij}^{lk} = 0$ are higher for true

Table 4: Accuracy of selecting CSH restrictions: posterior probabilities for restrictions $p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$

	Simulation 1			Simulation 2		
coefficients	**true** α_{jj}^{lk}	**true** α_{ii}^{lk}	$p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$	**true** α_{jj}^{lk}	**true** α_{ii}^{lk}	$p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$
$\alpha_{11}^{11} = \alpha_{22}^{11}$	0.8	0.6	*0.61*	0.8	0.6	*0.65*
$\alpha_{11}^{21} = \alpha_{22}^{21}$	0	0	*0.73*	0	0	*0.79*
$\alpha_{11}^{12} = \alpha_{22}^{12}$	0	0.5	*0.19*	0	0.5	*0.28*
$\alpha_{11}^{22} = \alpha_{22}^{22}$	0.7	0.5	*0.54*	0.7	0.3	*0.30*
$\alpha_{11}^{11} = \alpha_{33}^{11}$	0.8	0.6	*0.63*	0.8	0.6	*0.64*
$\alpha_{11}^{21} = \alpha_{33}^{21}$	0	0	*0.75*	0	0	*0.82*
$\alpha_{11}^{12} = \alpha_{33}^{12}$	0	0.5	*0.18*	0	0.5	*0.27*
$\alpha_{11}^{22} = \alpha_{33}^{22}$	0.7	0.5	*0.57*	0.7	0.5	*0.40*
$\alpha_{22}^{11} = \alpha_{33}^{11}$	0.6	0.6	*0.79*	0.6	0.6	*0.75*
$\alpha_{22}^{21} = \alpha_{33}^{21}$	0	0	*0.80*	0	0	*0.80*
$\alpha_{22}^{12} = \alpha_{33}^{12}$	0.5	0.5	*0.64*	0.5	0.5	*0.57*
$\alpha_{22}^{22} = \alpha_{33}^{22}$	0.5	0.5	*0.65*	0.3	0.5	*0.48*

Probabilities for CSH restrictions, $p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$, are in italic. Posterior probabilities, $p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$, are calculated as the proportion of γ_{CSH}^{w} draws that equal zero averaged over all Gibbs sampler draws and all simulated samples. Simulation 1: DGP has matrix panel structure. Simulation 2: DGP has flexible panel structure.

zero compared to non-zero parameters. The results for simulation one show that probabilities are in a range of 0.72 and 0.91 for zero values while for the existing static interdependencies between country 1 and 2 for both variables probabilities are between 0.50 and 0.56. For simulation two the probabilities for no static interdependencies, between 0.78 and 0.95, are clearly higher for the true zero values compared to the probabilities for non-zero values, 0.20 and 0.49.

Moreover, the SSVSP is mostly accurate in the selection of the cross-section heterogeneity restrictions. The detailed results for $p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$ are presented in table 4. For both simulations probabilities that the coefficients are homogeneous are higher for true homogeneous coefficients (with few exceptions for simulation two). However, especially for true values which are close to each other but not equal the probabilities for homogeneity are relatively high with values above 0.5. For example, $\alpha_{22}^{11} = \alpha_{33}^{11}$, true values 0.6 and 0.6, has a higher posterior probability for homogeneity, 0.79 for simulation one and 0.75 for simulation two, than the clearly heterogeneous coefficients α_{11}^{12} and α_{33}^{12}, 0 and 0.5, with probabilities of 0.18 for simulation one and 0.27 for simulation two. However, the coefficients α_{11}^{11} and α_{33}^{11}, with true values 0.8 and

0.6 have a relatively high posterior probability for homogeneity, 0.63 for simulation one and 0.64 for simulation two. This slightly weaker performance of the SSVSP to pick the correct CSH restrictions compared to DI and SI was already visible in the APD results for the SSVSP_CSH.

6 Empirical Application

6.1 Data and Procedure

The SSVSP is now applied to a simple empirical application. The analysis consists of three key macroeconomic variables: a growth rate of industrial production (IP), a CPI growth rate (CPI), and a short term interest rate (IR). The model includes the G7 countries.[20] The application can be used to study cross-country spillovers in macroeconomic variables. The variables can show synchronized business cycles or spillovers from monetary policy. The data are from the OECD and have monthly frequency from 1990:1 through 2015:2. The PVAR model includes one lag.[21] The model which is considered here serves as an illustration for the performance of SSVSP. In many ways it will not be the best model for the DGP as it, for example, takes into account only a fraction of variables which could be of interest for assessing the question of global spillovers. Furthermore, the lag order one is set by assumption and not further validated.

The variables are ordered in a recursive way. Thus, the upper triangular matrix Ψ has the following simplified form focusing on the country order:

$$\begin{matrix} CA \\ I \\ UK \\ F \\ J \\ D \\ US \end{matrix} \begin{pmatrix} \times & \times & \times & \times & \times & \times & \times \\ & \times & \times & \times & \times & \times & \times \\ & & \times & \times & \times & \times & \times \\ & 0 & & \times & \times & \times & \times \\ & & & & \times & \times & \times \\ & & & & & \times & \times \\ & & & & & & \times \end{pmatrix}.$$

[20]The countries are Canada (CA), Italy (I), United Kingdom (UK), France (F), Japan (J), Germany (D), and United States (US).

[21]The hyperparameters of the prior distributions are set as in the Monte Carlo simulations. Detailed information is provided in the Appendix. 110,000 draws are computed for the Gibbs sampler, the first 10,000 are disregarded as burn-in-phase.

For each country the three macroeconomic variables are included. The industrial production growth rate is ordered first, CPI growth rate second, and the short term interest rate third. The monetary policy shock for one country is thus identified by the assumption that the interest rate does not react contemporaneously to unexpected changes in real variables while a monetary policy shock instantaneously impacts the two real variables. The recursive country ordering is based on the openness of a country. Openness is measured based on yearly import and export data for the economies. The higher the trade of a country is, the more open it is. The countries are arranged in ascending order meaning that the most open country, the United States, is ordered last. Thus, US variables can influence all other countries contemporaneously but are not affected by the variables of the remaining G7 countries.

Using the empirical application as an example, the SSVSP is validated based on its forecasting performance, on the restriction posterior probabilities, and on an impulse response analysis. At first, forecasts are provided for 12 horizons for the period beginning from January 2005 through the end of the sample.[22] To obtain the forecasts a predictive distribution is simulated based on the reduced form of the PVAR model with normal distributed error terms. The reduced form model is the model where no SI restriction search is done and the covariance matrix is drawn from an inverse Wishart distribution.[23] The forecasts are evaluated using the mean squared forecast error (MSFE) and the average predictive likelihoods (PL). The MSFE is calculated as the difference between the estimated forecast and the true value given by the data. The MSFEs for a specific variable and horizon are averaged over all forecasts. The PL is the posterior predictive density evaluated at the true observation y_{t+h}.

The forecast performance is compared to the unrestricted VAR, SSVSP_DI, SSVSP_CSH, S^4, and SSVS. Furthermore, two specifications are added which access the selection property of the SSVSP: SSVSP_setDI_v1 and SSVSP_setDI_v2. SSVSP_setDI_v1 uses the outcome of the SSVSP and sets zeros whenever the posterior probability for a DI restriction is larger than 0.99. Based on this model the forecasts are produced. SSVSP_setDI_v2 also sets these coefficients to zero but uses 0.5 as a threshold value. Furthermore, the SSVSP is validated based on the

[22]The forecasts for the included 21 variables are generated iteratively. Forecasts start conditional on the data from January 1990 to December 2004.

[23]The covariance matrix is drawn from an inverse Wishart distribution with T degrees of freedom and identity matrix plus sum of squared residuals as scaling matrix.

Table 5: MSFEs relative to unrestricted VAR

	number of MSFEs ≤ 1(in %)				
horizon	**1**	**2**	**4**	**6**	**12**
SSVSP	38.10	42.86	33.33	28.57	33.33
SSVSP_DI	100.00	85.71	85.71	95.24	90.48
SSVSP_CSH	33.33	38.10	19.05	23.81	52.38
SSVSP_setDI_v1	0.00	0.00	52.38	57.14	66.67
SSVSP_setDI_v2	4.76	4.76	52.38	71.43	80.95
S^4	42.86	33.33	38.10	33.33	47.62
SSVS	19.05	9.52	66.67	95.24	76.19

MSFE: difference between the estimated forecast and the true value given by the data, relative to unrestricted VAR. Forecasts are provided for 12 horizons for the period beginning from January 2005 through the end of the sample. Unrestricted VAR: parameters drawn from unrestricted part of distributions. S^4: prior of Koop and Korobilis (2015). SSVS: prior of George et al. (2008). SSVSP_DI: SSVSP with only DI restrictions. SSVSP_CSH: SSVSP with only CSH restrictions. SSVSP_setDI_v1: threshold 0.99 to set zero DI restrictions. SSVSP_setDI_v2: threshold 0.5 to set zero DI restrictions. Σ drawn from inverse Wishart distribution with T degrees of freedom and identity matrix plus sum of squared residuals as scaling matrix. 110,000 Gibbs draws, 10,000 disregarded as burn-in-phase.

posterior probabilities of the SSVSP for DI, SI and CSH restrictions. Finally, an impulse response analysis is conducted based on the recursive identification system.

6.2 Results

The results of the empirical application demonstrate three key findings. Firstly, the MSFEs and PLs results favor the SSVSP_DI and the two selection models, SSVSP_setDI_v1 and SSVSP_setDI_v2, indicating that restrictions search is beneficial since sparsity in form of no dynamic interdependencies exist. However, the very large number of restrictions searched for in the SSVSP leads to relatively weak forecast performance. Secondly, the posterior probabilities for the restrictions indicate that domestic interest rates and inflation evolve unaffected by lagged foreign industrial production growth rates, validated by high posterior probabilities for no dynamic interdependencies. The interest rate of a country depends likely statically and dynamically on foreign interest rates. No heterogeneities are in particular found for the effect of domestic industrial production growth on the domestic interest rate and inflation. Thirdly, the impulse response analysis supports the reliability of the

Table 6: PLs compared to unrestricted VAR

	number of PLs ≥ 0 (in %)				
horizon	**1**	**2**	**4**	**6**	**12**
SSVSP	33.33	28.57	47.62	23.81	38.10
SSVSP_DI	57.14	42.86	42.86	42.86	61.90
SSVSP_CSH	28.57	19.05	38.10	23.81	57.14
SSVSP_setDI_v1	28.57	28.57	47.62	47.62	42.86
SSVSP_setDI_v2	28.57	52.38	52.38	61.90	66.67
S^4	47.62	47.62	33.33	33.33	38.10
SSVS	47.62	38.10	57.14	71.43	61.90

PL: posterior predictive density evaluated at the true observation y_{t+h}, compared to unrestricted VAR. Forecasts are provided for 12 horizons for the period beginning from January 2005 through the end of the sample. Unrestricted VAR: parameters drawn from unrestricted part of distributions. S^4: prior of Koop and Korobilis (2015). SSVS: prior of George et al. (2008). SSVSP_DI: SSVSP with only DI restrictions. SSVSP_CSH: SSVSP with only CSH restrictions. SSVSP_setDI_v1: threshold 0.99 to set zero DI restrictions. SSVSP_setDI_v2: threshold 0.5 to set zero DI restrictions. Σ drawn from inverse Wishart distribution with T degrees of freedom and identity matrix plus sum of squared residuals as scaling matrix. 110,000 Gibbs draws, 10,000 disregarded as burn-in-phase.

results. In the following the key findings are explained in more detail.

Table 5 shows in percent the number of MSFEs which are smaller or equal to one averaged over all variables for forecast horizon 1, 2, 4, 6, and 12. The MSFEs are given relative to the unrestricted VAR. Thus, a MSFE smaller than one indicates improved forecast performance to the unrestricted VAR. Between 28.57% and 42.86% of the MSFEs of the SSVSP are below or equal to the MSFEs of the unrestricted VAR.[24] Thus, the SSVSP cannot improve the forecasts compared to the unrestricted VAR. This is quite similar to the performance of the S^4. However, the SSVSP_DI performs particularly well. It outperforms the unrestricted model at the best in 100.00% of the cases (horizon one) and at worst in 85.71% of the cases (horizons two and four). Since the number of restrictions which are examined in the SSVSP are high, the information in the data might not be enough for the estimation. Thus, the improved performance of SSVSP_DI could be a result of the reduced number of restrictions. However, only searching for CSH restrictions does not lead to improvements compared to the SSVSP. The SSVSP_DI captures the high probabilities for no dynamic interdependencies which are present in the data. The probability

[24]Detailed results are given in the Appendix.

Table 7: MSFEs for SSVSP relative to S^4

horizon	**1**	**2**	**4**	**6**	**12**
IP_CA	0.98	0.95	0.95	0.95	0.99
CPI_CA	0.98	1.01	0.98	0.90	1.00
IR_CA	0.99	0.86	1.00	0.99	0.91
IP_I	1.06	0.95	1.04	1.11	1.11
CPI_I	0.67	1.05	1.05	1.03	1.03
IR_I	1.44	1.38	1.40	1.39	1.39
IP_UK	0.90	0.99	0.94	0.91	0.97
CPI_UK	1.06	1.07	0.98	1.06	1.07
IR_UK	0.99	0.91	0.91	0.92	0.95
IP_F	1.02	0.98	1.00	1.06	1.01
CPI_F	1.00	1.04	1.01	1.04	1.15
IR_F	1.70	1.35	1.34	1.51	1.51
IP_J	0.80	0.97	1.02	1.07	1.01
CPI_J	0.94	0.94	1.01	1.00	1.00
IR_J	3.35	2.25	2.16	2.00	1.95
IP_D	0.99	0.98	1.01	1.01	0.99
CPI_D	1.00	0.98	1.00	1.01	1.00
IR_D	1.45	1.17	0.97	0.98	0.98
IP_US	1.00	0.93	0.93	0.98	1.03
CPI_US	1.00	1.02	1.01	1.00	1.01
IR_US	1.00	0.99	0.97	0.95	0.98
MSFE $\leq$ 1(in %)	66.67	57.14	52.38	42.86	33.33

MSFE: difference between the estimated forecast and the true value given by the data, relative to S^4. Forecasts are provided for 12 horizons for the period beginning from January 2005 through the end of the sample. Σ drawn from inverse Wishart distribution. 110,000 Gibbs draws, 10,000 disregarded as burn-in-phase.

for homogeneity seems to be lower. Excluding dynamic interdependencies based on a specific threshold improves the forecast performance for the higher horizons. The two specifications, SSVSP_setDI_v1 and SSVSP_setDI_v2, can particularly well pick up the sparsity in the data. The model with a lower threshold value, SSVSP_setDI_v2, leads to higher improvements. The performance of the SSVS is volatile, ranging from 9.52% to 95.24% of MSFEs below or equal one. It performs well for the last three reported horizons. The SSVS also searches for dynamic interdependencies, thus, it is similar to the SSVSP_DI specification. With the exception that the SSVSP_DI distinguishes between domestic foreign variables. The results of the SSVS support the finding that the MSFEs favor priors which capture the

Table 8: PLs for SSVSP compared to S^4

horizon	**1**	**2**	**4**	**6**	**12**
IP_CA	0.00	-0.30	0.21	0.21	0.01
CPI_CA	-0.25	-0.23	-0.35	0.41	0.26
IR_CA	0.07	0.72	0.98	0.60	-0.55
IP_I	0.09	-0.12	0.07	0.27	0.32
CPI_I	0.56	0.07	-0.21	0.27	0.20
IR_I	0.05	-2.00	-0.83	-0.32	-1.60
IP_UK	0.71	-0.11	0.03	-0.12	-0.48
CPI_UK	-0.32	-0.79	-0.04	0.24	0.16
IR_UK	-0.01	0.62	0.13	0.64	0.48
IP_F	-0.71	0.12	-0.01	0.11	0.10
CPI_F	-1.26	-0.47	0.66	0.10	0.14
IR_F	-4.84	-2.22	-0.83	-1.22	-0.48
IP_J	0.00	0.11	-0.08	0.05	0.17
CPI_J	0.03	-0.54	-0.01	-0.41	-0.67
IR_J	-3.28	-2.02	-1.75	-1.84	-1.99
IP_D	0.00	0.08	0.07	-0.19	0.12
CPI_D	-0.04	0.04	0.06	0.09	-0.33
IR_D	-1.23	-0.74	-0.33	-0.81	0.79
IP_US	-0.01	-0.27	-0.62	0.09	-0.44
CPI_US	0.01	0.37	0.16	-0.08	0.17
IR_US	0.00	-0.38	0.17	0.55	0.17
PL $\geq$ 0 (in %)	52.38	38.10	47.62	61.90	61.90

PL: posterior predictive density evaluated at the true observation y_{t+h}, compared to S^4. Forecasts are provided for 12 horizons for the period beginning from January 2005 through the end of the sample. Σ drawn from inverse Wishart distribution. 110,000 Gibbs draws, 10,000 disregarded as burn-in-phase.

possibility of no dynamic interdependencies. The S^4 also includes DI but assumes a specific matrix structure which does not seem to be supported by the data.

Table 6 presents in percent the number of PL, in difference to the unrestricted model, which are higher or equal zero. In general, a higher PL indicates a better performance since the posterior predictive density covers the true observation with a higher probability. The results are generally in line with the findings based on the MSFEs but differ in magnitude and also in horizon. In particular, the PL results favor the SSVSP_DI, SSVSP_setDI_v1, and SSVSP_setDI_v2 as well as the SSVS. In contrast to the extremely volatile MSFE results, the SSVS outperforms the SSVSP at all horizons. However, the SSVSP_DI exceeds the SSVS at two and

Table 9: Posterior probabilities for DI restrictions, $p(\alpha_{ij}^{lk} = 0)$

		CA			I			UK			F			J			D			US		
		IP	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**
CA	**IP**				-	-	-	-	-	-	-	-	-	1	-	-	1	-	-	0	-	0
	CPI				1	-	-	1	-	-	1	-	-	1	-	0	1	-	-	1	0	-
	IR				1	-	0	1	-	-	1	-	-	1	-	0	1	-	-	-	-	-
I	**IP**	-	-	0				-	-	-	-	-	-	-	-	-	-	-	-	0	-	-
	CPI	1	-	-				1	1	-	1	-	-	1	-	-	1	-	-	1	-	-
	IR	1	-	-				1	-	-	1	-	-	1	-	-	1	-	0	-	-	-
UK	**IP**	-	-	-	-	-	-				-	-	-	-	-	-	-	-	-	-	-	-
	CPI	1	-	-	1	-	-				1	-	-	1	-	-	1	-	-	1	-	-
	IR	1	-	-	1	-	1				1	-	-	1	-	-	1	-	-	1	-	-
F	**IP**	-	-	-	-	-	-	-	-	0				-	-	0	-	-	-	0	-	0
	CPI	1	-	-	1	-	-	1	-	-				1	-	-	1	-	-	1	0	-
	IR	1	-	-	1	-	-	1	-	-				1	-	-	1	-	0	1	-	-
J	**IP**	-	-	-	-	-	-	-	-	0	-	-	-				-	-	0	-	-	-
	CPI	1	-	-	1	-	-	1	-	-	1	-	-				1	-	-	-	0	-
	IR	1	1	1	1	1	-	1	1	1	1	-	-			-	1	1	-	1	1	1
D	**IP**	-	-	-	-	-	-	-	-	0	-	-	-	0	-	-				-	-	-
	CPI	1	-	-	1	-	-	1	-	-	1	-	-	1	-	-				1	-	-
	IR	1	1	1	1	-	1	1	1	-	1	-	-	1	1	-				1	-	-
US	**IP**	-	-	-	-	-	-	1	-	-	1	-	-	-	-	-	1	-	-			
	CPI	1	-	-	1	-	-	1	-	-	1	-	-	1	-	-	1	-	-			
	IR	1	1	-	1	-	1	1	-	-	1	-	-	1	-	-	1	-	-			

Posterior probabilities for DI restrictions, $p(\alpha_{ij}^{lk} = 0)$. 1: $1 \leq p(\alpha_{ij}^{lk} = 0) \leq 0.99$. 0: $0.1 \leq p(\alpha_{ij}^{lk} = 0) \leq 0$. -: $0.99 < p(\alpha_{ij}^{lk} = 0) < 0.1$. The posterior probabilities, $p(\alpha_{ij}^{lk} = 0)$, are calculated as one minus the posterior means for $\gamma_{DI,ij}^{lk}$.

Table 10: Posterior probabilities for SI restrictions, $p(\psi_{ij}^{lk} = 0)$

		CA			I			UK			F			J			D			US		
		IP	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**
CA	**IP**				-	1	-	-	1	-	1	-	-	1	-	-	1	1	-	-	-	-
	CPI				-	0	-	-	-	-	-	0	-	-	-	-	-	-	-	-	0	-
	IR				-	-	-	-	-	-	-	-	0	-	-	-	-	-	-	-	-	0
I	**IP**							-	-	1	-	-	1	-	1	-	-	1	-	-	-	1
	CPI							-	0	-	-	0	-	-	-	-	-	0	-	-	0	-
	IR							-	-	0	-	-	0	-	-	-	-	-	-	-	-	-
UK	**IP**										-	-	-	-	-	-	-	-	-	-	-	-
	CPI										-	0	-	-	0	-	-	-	-	-	-	-
	IR										-	-	0	-	-	-	-	-	0	-	-	0
F	**IP**													-	1	1	-	1	1	1	-	-
	CPI													-	-	-	-	0	-	-	0	-
	IR													-	-	-	-	-	-	-	-	-
J	**IP**																1	1	1	1	1	1
	CPI																-	-	-	-	-	-
	IR																-	0	-	-	-	-
D	**IP**																			1	1	1
	CPI																			-	-	-
	IR																			-	-	0
US	**IP**																					
	CPI																					
	IR																					

Posterior probabilities for SI restrictions, $p(\psi_{ij}^{lk} = 0)$. 1: $1 \leq p(\psi_{ij}^{lk} = 0) \leq 0.99$. 0: $0.1 \leq p(\psi_{ij}^{lk} = 0) \leq 0$. -: $0.99 < p(\psi_{ij}^{lk} = 0) < 0.1$. The probabilities, $p(\psi_{ij}^{lk} = 0)$, are calculated as one minus the posterior means for $\gamma_{SI,ij}^{lk}$.

Table 11: Posterior probabilities for CSH restrictions, $p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$

		CA			I			UK			F			J			D			US		
		IP	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**
CA	**IP**	-	-	0 (1)																		
	CPI	1 (6)	-	-																		
	IR	1 (5)	-	-																		
I	**IP**				-	-	0 (1)															
	CPI				1 (6)	-	-															
	IR				1 (6)	-	-															
UK	**IP**							-	-	0 (2)												
	CPI							1 (6)	-	-												
	IR							1 (6)	-	-												
F	**IP**										-	-	0 (1)									
	CPI										1 (6)	0 (1)	-									
	IR										1 (6)	-	-									
J	**IP**													-	-	0 (6)						
	CPI													1 (6)	-	0 (1)						
	IR													1 (6)	1 (1)	-						
D	**IP**																0 (1)	-	0 (1)			
	CPI																1 (6)	0 (2)	-			
	IR																1 (6)	1 (1)	-			
US	**IP**																			0 (1)	-	0 (1)
	CPI																			1 (6)	0 (2)	-
	IR																			1 (6)	-	-

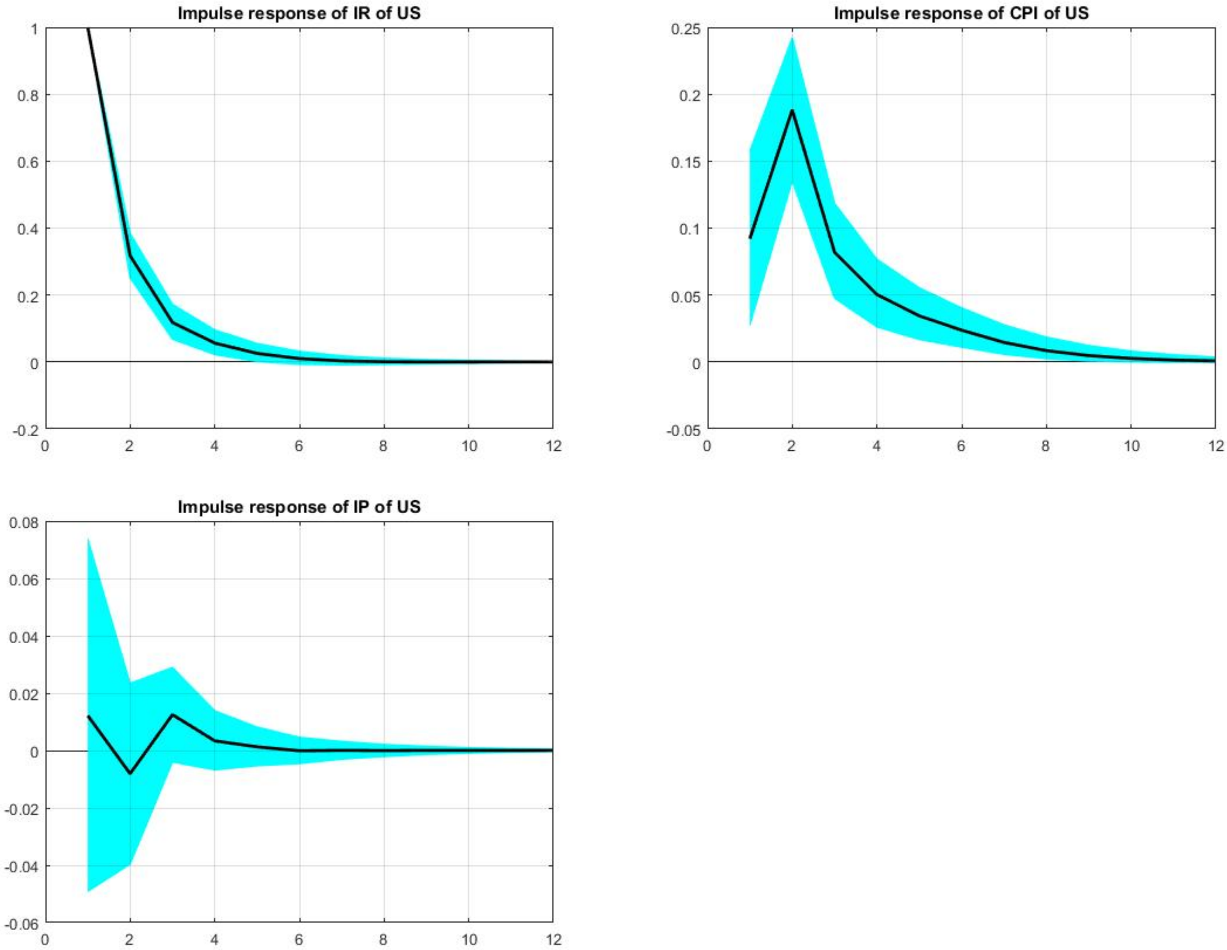
Posterior probabilities for CSH restrictions. 1: $1 \leq p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk}) \leq 0.99$. 0: $0.1 \leq p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk}) \leq 0$. -: $0.99 < (\alpha_{jj}^{lk} = \alpha_{ii}^{lk}) < 0.1$. The probabilities, $p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$, are calculated as one minus the posterior means for γ_{CSH}^{w}. Values in parentheses count how often the probability is equal to one or zero for a coefficient in all possible combinations, $\alpha_{jj}^{lk} = \alpha_{ii}^{lk}$.

is equally good at one out of five horizons. Again, the results point to the direction that no dynamic interdependencies are present in the data and a prior which can pick up these characteristics performs well. Compared to the findings based on the MSFEs the prior specifications are less often able to outperform the unrestricted VAR. This could be explained by a higher parameter uncertainty of the selection priors since they are a mixture of two distributions. The higher uncertainty is reflected in the posterior predictive density. The results are in general in line with Korobilis (2016) who also shows a high volatility in the performance as well as improved forecasting results for the SSVS compared to the S^4. However, combining the advantages of both priors, the panel dimension of the S^4 and the single restriction search of the SSVS, in the SSVSP does not seem to pay of due to the large number of restrictions to search for. As Korobilis (2016) shows the approach of Canova and Ciccarelli (2009) has no clear advantage, measured by forecasting performance, over the selection priors.

Table 7 and 8 report for horizons 1, 2, 4, 6, and 12 MSFEs and PLs relative to S^4. On average, over these five horizons the forecast of SSVSP are equally good compared to the forecasts of S^4. In 50.48% the MSFEs relative to S^4 are below one and in 52.38% the PLs compared to S^4 are above zero. There is no clear pattern whether SSVSP can outperform the forecasts of S^4 for a specific forecast horizon. 66.67% of the first horizon forecasts have MSFE below one, indicating that the forecast performance improves by using SSVSP. The forecast performance, measured by MSFEs, seem to decrease over the forecast horizon. At horizon twelve 33.33% of the MSFE are below one. However, evaluated on the average predictive likelihood the SSVSP offers the highest gain in forecast performance for higher horizons, 61.90% for horizon six and twelve.

Tables 9, 10, and 11 provide posterior probabilities for the restrictions. The probabilities, $p(\alpha_{ij}^{lk} = 0)$, $p(\psi_{ij}^{lk} = 0)$, and $p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$, are calculated as one minus the posterior means for $\gamma_{DI,ij}^{lk}, \gamma_{SI,ij}^{lk}$, and γ_{CSH}^{w}. The probabilities measure the proportion of $\gamma_{DI,ij}^{lk}, \gamma_{SI,ij}^{lk}$, and γ_{CSH}^{w} draws that equal zero averaged over all Gibbs sampler draws meaning that the coefficients are drawn from the restricted part of the distribution. Tables 9, 10, and 11 show that the SSVSP can provide a detailed ranking on how likely a restriction should be set based on the data on a variable basis. The algorithm is able to detect a nuanced structure of the restrictions present in the data. Since the presented PVAR model serves as an illustration, the economic findings should not be over-interpreted.

Figure 2: Responses of US variables to a shock to US interest rate

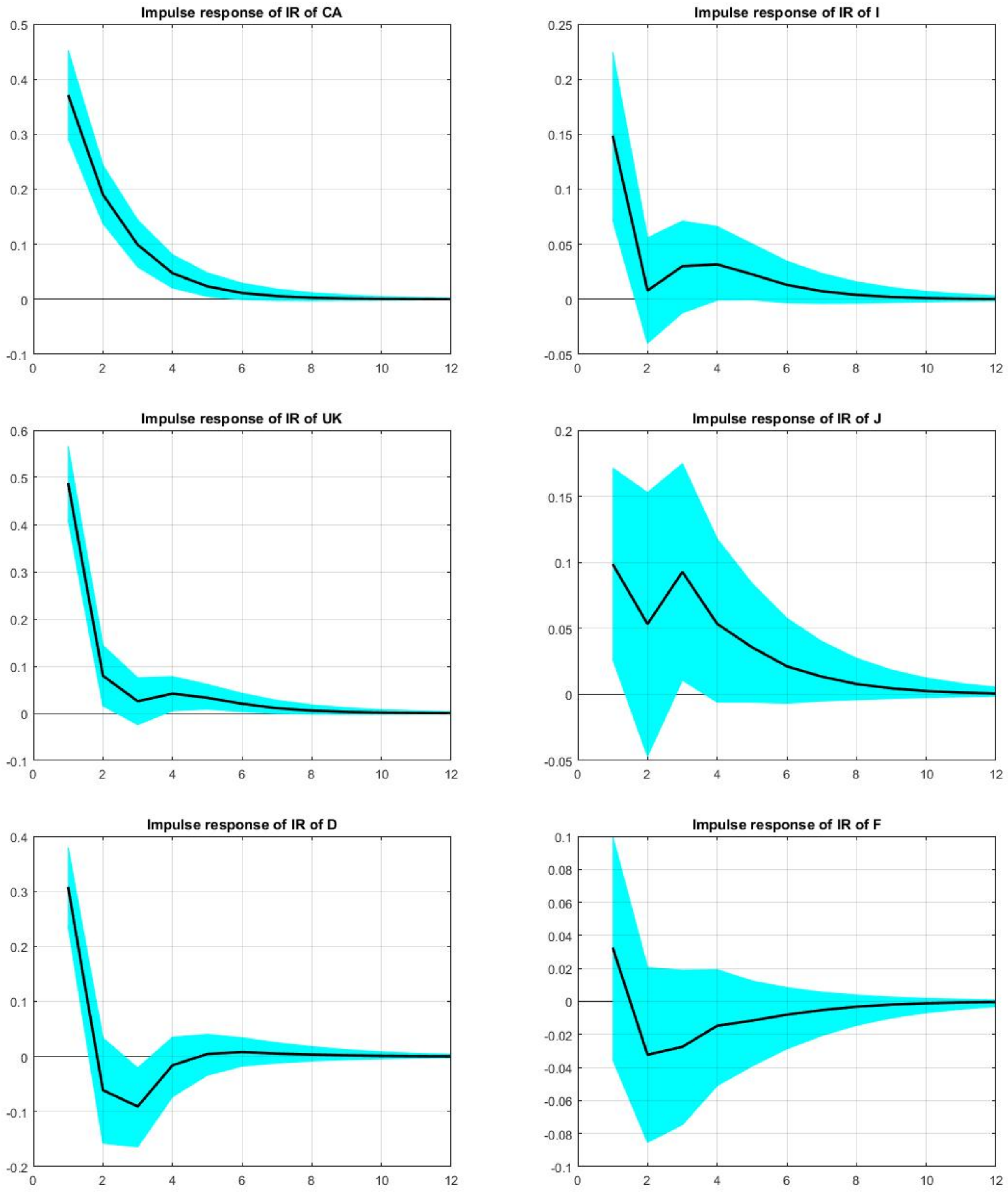


Solid line shows response, shaded area 68% Bayesian credible interval.

The posterior probabilities show that restrictions are especially supported for the industrial production variable while it is vice versa for interest rates.[25] Table 9 provides the posterior probabilities for no dynamic interdependencies. A one indicates that the probability for no DI is between one and 0.99, $1 \leq p(\alpha_{ij}^{lk} = 0) \leq 0.99$, a zero that $0.1 \leq p(\alpha_{ij}^{lk} = 0) \leq 0$ and the diagonal stroke that $0.99 < p(\alpha_{ij}^{lk} = 0) < 0.1$. The results show that the probabilities are high, indicated by ones, that no dynamic impacts of foreign lagged IP on interest rates and CPI growth exist. Lagged foreign interest rates seem to affect the domestic variables. Furthermore, US variables have a dynamic impact on other countries' variables. Both findings are supported by a low probabilities for no DI, meaning a zero in table 9.

[25]Detailed results for all parameters are given in the Appendix.

Figure 3: Responses of foreign interest rates to a shock to US interest rate



Solid line shows response, shaded area presents 68% Bayesian credible interval.

The lowest probabilities for the SI restrictions are found for combinations of the same variable. Results are shown in table 10. Additionally, industrial production seems to be fairly independent from other variables, shown by the high probabilities for no static interdependencies between IP and other variables. Finally, the probabilities for homogeneity of coefficients are especially high for the industrial production variables in other equations, shown in table 11. The values in parentheses count how often the posterior probability took the value one or zero for a specific coefficient in the comparison $\alpha_{jj}^{lk} = \alpha_{ii}^{lk}$. Heterogeneity is favored - low probability for no CSH - for the effect of the interest rate on industrial production growth.

The impulse response analysis sheds light on the reliability of the findings. Exemplary, the responses to a shock to the US interest rate, presented in figure 2 for US variables and in figure 3 for foreign interest rates, will be discussed. A contractionary US monetary policy, shown by an increase in the US interest rate, leads to a rise in US CPI in this system. The response of industrial production growth is insignificant. The increase of inflation in response to a tightening in the monetary policy is in line with the price puzzle. The price puzzle - first mentioned by Sims (1992) - refers to this result contradicting theoretical models and empirical findings which would claim that a rise in the interest rate leads to a decline in inflation. The puzzle is expected for VAR models which just include industrial production growth, inflation, and a short term interest rate and have a structural identification based on a recursive system. The finding of the price puzzle underlines that the here estimated PVAR model can only serve as an illustration and has its clear limitations. The foreign interest rates immediately raise in response to a tightening in the US monetary policy. The increases in the interest rates are lower, below 0.5, than the initial raise in the US interest rate, which is normalized to one. The UK interest rate is initially affected most, followed by the Canadian and German interest rate responses. After around two horizons the effect of the US shock is insignificant for the interest rate of the United Kingdom, Germany, and Italy. The responses of the interest rates of Japan and France are lowest. For Japan the response is insignificant after the first horizon while for France the response is insignificant for all horizons. The raise in the Canadian interest rate lasts longest and comes to zero after six horizons. To sum up, the impulse repose functions show that the results based on SSVSP are in line with theoretically expected responses from a recursively identified system with the three included variables. The illustrative model shows that the results obtained using SSVSP are plausible.

7 Concluding Remarks

This paper introduces the SSVSP as an extension of the Bayesian S^4 proposed by Koop and Korobilis (2015). The SSVSP is an alternative Bayesian estimation procedure for PVARs that is able to fully incorporate dynamic and static interdependencies as well as cross-country heterogeneities. It allows for a flexible panel structure since it only distinguishes between domestic and foreign variables. Using a hierarchical prior, the SSVSP searches for restrictions that are supported by the data.

The results of the Monte Carlo simulations show that the SSVSP outperforms the S^4 in terms of deviation from the true values in particular when a less restrictive panel structure is present. The average deviation of the estimated parameters from the true values for the simulation with a flexible panel structure is less for the SSVSP, $APD_{SSVSP} = 0.036$, than for S^4, $APD_{S^4} = 0.056$. The SSVSP_DI, where a mixture prior is only set on the parameters measuring dynamic interdependencies, has the smallest deviation from the true values of all models. Furthermore, the accuracy of the SSVSP in selecting the restrictions is proven by the posterior probabilities for no interdependencies and homogeneity.

The results of the empirical application are summarized in three main findings. Firstly, the forecast performance is especially good for the SSVSP_DI and the two selection models, SSVSP_setDI_v1 and SSVSP_setDI_v2. Thus, restrictions search for no dynamic interdependencies is beneficial. However, the performance of the SSVSP is limited by the very large number of restrictions searched for. Secondly, posterior probabilities for DI and SI restrictions show that interest rates likely depend on foreign interest rates while foreign industrial production growth does not impact other domestic variables. Thirdly, responses to a shock in the US interest rate are in line with expected response functions.

The SSVSP prior can be further developed. The SI restriction search, based on data, is an initial way to achieve structural identification, but it is limited by the fact it is built on a recursive system. For just identified systems, the BMA result of the reduced form can be used combined with the clear mapping between reduced form covariance matrix and a short run restriction matrix to obtain the structural form. For overidentified systems, however, the draws of the coefficient matrices have to be from the structural form. This means a selection prior for A, conditional on a restriction matrix A_0, must be stated and a valid Gibbs sampler has to be derived.

One critical issue is the selection of hyperparameters. In this specification, the hyperparameters are fixed for all parameters that are estimated. George et al. (2008) propose a default semi-automatic approach to select the hyperparameters. The values are not fixed but vary for each coefficient. For example $\tau_{1,i} = c_1\sqrt{var(\alpha_i)}$ and $\tau_{2,i} = c_2\sqrt{var(\alpha_i)}$, whereby $c_1 = 0.1$ and $c_2 = 10$. The $var(\alpha_i)$ is the estimated variance of the OLS estimate for α_i in a model without restriction search. The κ and ξ are set in an equal manner. Trying this approach leads to hyperparameters that tend to be so small that the majority of values are drawn from the loose part of the prior. Koop and Korobilis (2015) specify distributions for the hyperparameters as also suggested in Giannone et al. (2015). This allows them to have varying hyperparameters and a less subjective choice of hyperparameters. The issues regarding structural identification and choices of hyperparameters can be addressed in further research.

To sum up, the findings of the Monte Carlo simulations conducted and the exemplary empirical application encourage the use of the SSVSP to estimate PVAR models. However, further research regarding both the recursive structural identification and the specified hyperparameters should be undertaken.

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Gibbs Sampler Algorithm

The full unrestricted PVAR model with one lag including N countries and for each country G variables can be written as

$$Y_t = Z_{t-1}\alpha + U_t,$$

where α is the vectorized $[NG \times NG]$-coefficient matrix A for lag one. The $Z_{t-1} = (I_{NG} \otimes Y_{t-1})$ where $Y_{t-1} = (y'_{1t-1}, ..., y'_{Nt-1})'$ and y_{it} denotes a vector of dimension $[G \times 1]$. The Y_t and U_t are $[NG \times 1]$-vectors. The U_t is normally distributed with mean zero and covariance matrix Σ that is of dimension $[NG \times NG]$. The element α_{ij}^{lk} refers to the coefficient of variable k of country j in the equation of variable l of country i.

The Gibbs sampler algorithm has the following three steps:

Step 1:

Sample α from a normal posterior conditional on Σ, γ_{DI}, γ_{CSH}.

$$\alpha \mid \Sigma, \gamma_{DI}, \gamma_{CSH} \sim \mathcal{N}(\Gamma\mu_\alpha, V_\alpha),$$

where $V_\alpha = ((D'D)^{-1} + \Sigma^{-1} \otimes X'X)^{-1}$ with $X = Y_{t-1}$ and $\mu_\alpha = V_\alpha((\Sigma^{-1} \otimes X'X)\alpha_{OLS})$. D is a diagonal matrix with $D = diag(h_{11}^{11}, ..., h_{NN}^{GG})$. The value of h depends on γ_{DI} and γ_{CSH}: $h_{ij}^{lk} = \begin{cases} \tau_1, & if\ \gamma_{DI,ij}^{lk} = 0 \\ \tau_2, & if\ \gamma_{DI,ij}^{lk} = 1 \end{cases}$ for the parameters, where DI restriction search is done $(i \neq j)$ and $h_{jj}^{lk} = \begin{cases} \xi_1, & if\ \gamma_{CSH}^{w} = 0 \\ \xi_2, & if\ \gamma_{CSH}^{w} = 1 \end{cases}$ for the block diagonal parameters where CSH restriction search is done. α_{OLS} is the OLS estimate of α. The posterior mean is restricted with the selection matrix Γ.

Step 2:

Update γ_{DI} and γ_{CSH} from Bernoulli distribution:

$$\gamma^{lk}_{DI,ij} \sim Bernoulli(\pi^{lk}_{DI,ij})$$
$$\pi^{lk}_{DI,ij} = \frac{u2^{lk}_{DI,ij}}{u1^{lk}_{DI,ij} + u2^{lk}_{DI,ij}}$$
$$\gamma^{w}_{CSH} \sim Bernoulli(\pi^{w}_{CSH})$$
$$\pi^{w}_{CSH} = \frac{v2^{w}_{CSH}}{v1^{w}_{CSH} + v2^{w}_{CSH}}.$$

Hereby, $u1^{lk}_{DI,ij} = f(\alpha^{lk}_{ij} \mid 0, \tau_1^2) prob_{DI}$ and $u2^{lk}_{DI,ij} = f(\alpha^{lk}_{ij} \mid 0, \tau_2^2)(1 - prob_{DI})$. $f()$ denotes the p.d.f. of the normal distribution with mean zero and variance τ_1^2 or τ_2^2 evaluated at α^{lk}_{ij}. The parameter $prob_{DI}$ is set equal to 0.5. This shows that *a priori* the researcher assumes that it is equally likely that a dynamic interdependency between two variables of country i and j are zero or nonzero. $v1^{w}_{CSH} = f(\alpha^{lk}_{jj} \mid \alpha^{lk}_{ii}, \xi_1^2) prob_{CSH}$ and $v2^{w}_{CSH} = f(\alpha^{lk}_{jj} \mid 0, \xi_2^2)(1 - prob_{CSH})$. Again, $prob_{CSH}$ is set equal 0.5. Depending on γ^{w}_{CSH} the elements in Γ_w are updated.

Step 3:

Update $\Sigma = \Psi^{-1'}\Psi^{-1}$ and γ_{SI}. The variance elements, ψ^{kk}_{ii}, are drawn from a Gamma distribution:

$$(\psi^{kk}_{ii})^2 \sim \mathcal{G}(a + \frac{T}{2}, B_n),$$

where $n = 1, ..., NG$ and

$$B_n = \begin{cases} b + 0.5SSE_{nn} & n = 1 \\ b + 0.5(SSE_{nn} - s'_n(S_{n-1} + (R'R)^{-1})^{-1} s_n) & n = 2, ..., NG \end{cases}.$$

Note that ψ^{11}_{11} is assigned to B_2, ψ^{22}_{11} to B_2, ..., and ψ^{GG}_{NN} to B_{NG}. T is defined as the length of the time series and SSE as the sum of squared residuals. S_n is the upper-left $n \times n$ submatrix of SSE, and $s_n = (s_{1n}, ..., s_{n-1,n})'$ contains the upper diagonal elements of SSE. R is a diagonal matrix with $R = diag(r^{11}_{11}, ..., r^{GG}_{NN})$. The value of r depends on γ_{SI}: $r^{lk}_{ij} = \begin{cases} \kappa_1, & if\ \gamma^{lk}_{SI,ij} = 0 \\ \kappa_2, & if\ \gamma^{lk}_{SI,ij} = 1 \end{cases}.$

Define the vector $\psi = (\psi^{11}_{12}, ..., \psi^{GG}_{N-1,N})'$. Thus, ψ contains the covariance elements, ψ^{lk}_{ij} for all $i \neq j$ and has the dimension $n_{SI} \times 1$, where $n_{SI} = 1, .., N_{SI}$ and N_{SI} is the length equal to the number of SI restrictions. The elements of ψ are updated from a normal distribution:

$$\psi_{n_{SI}} \mid \alpha, \psi, \gamma_{SI} \sim \mathcal{N}(\mu_{n_{SI}}, V_{n_{SI}}).$$

Hereby, $\mu_{n_{SI}} = -\psi_{ii}^{kk}(S_{n_{SI}-1} + (R'R)^{-1})^{-1} s_{n_{SI}}$ and $V_{n_{SI}} = (S_{n_{SI}-1} + (R'R)^{-1})^{-1}$. The element ψ_{ii}^{kk} is the variance element in the same row of Psi as $\psi_{ij}^{lk} = \psi_{n_{SI}}$ for all $i \neq j$. The off-diagonal elements of the covariance matrix that belong to one country are drawn from a normal distribution with mean zero and variance κ_2.

Hyperparameter

Table 12: Hyperparameters

τ_1	τ_2	ξ_1	ξ_2	κ_1	κ_2	a	b
0.2	4	0.2	4	0.3	4	0.01	0.01

A value of $\tau_1 = 0.2$ and $\tau_2 = 4$ means that the variance of the tight prior equals 0.04 and 16 for the loose prior. The criterion that the variance of the first part of the normal distribution is smaller than the second part is clearly fulfilled. Several other specifications are also checked. The accuracy of the algorithm in selecting the restrictions varies with the specification of the hyperparameters. If the τ_1, κ_1, and ξ_1 are chosen too small, the majority of values is drawn from the second part of the normal distribution (γ equals one with a very high probability). Still, γ equals more often one in the cases no restriction is set in the true specification of the Monte Carlo simulation. Values for hyperparameters smaller than or equal to 0.1 prove to be too small, resulting in the difficulties mentioned. George et al. (2008) propose a default semi-automatic approach to selecting the hyperparameters. The values are not fixed, but varying for each coefficient. For example $\tau_{1,i} = c_1\sqrt{var(\alpha_i)}$ and $\tau_{2,i} = c_2\sqrt{var(\alpha_i)}$ whereby $c_1 = 0.1$ and $c_2 = 10$. $var(\alpha_i)$ is a OLS estimated of the variance of the coefficient in an unrestricted model. κ and ξ are set in an equal manner. Trying this approach also leads to hyperparameters smaller than 0.1. The hyperparameters of the other priors are set to the proposed default values of the authors. For S^4 the small variance values are set to 0.1 and the high variance values to square root of 10, for the SSVS small variance values are set to 0.1 and high variance values to 5 as used by Koop and Korobilis (2015) and George et al. (2008).

Estimates - Monte Carlo Simulation

The simulation is done with 100 samples, each with a length of 100. The Gibbs sampler has 55,000 draws of which 5,000 draws are disregarded as draws in the burn-in-phase. The posterior means for the first Monte Carlo Simulation based on the SSVSP prior are the following:

$$A^{SSVSP} = \begin{pmatrix} 0.59 & 0.00 & 0.03 & 0.02 & 0.28 & 0.23 \\ 0.11 & 0.49 & 0.01 & -0.02 & -0.41 & 0.38 \\ 0.21 & 0.31 & 0.52 & -0.02 & -0.03 & -0.04 \\ 0.17 & 0.28 & 0.47 & 0.40 & -0.03 & -0.03 \\ -0.02 & -0.04 & -0.02 & -0.01 & 0.51 & -0.03 \\ 0.02 & -0.01 & -0.05 & -0.02 & 0.48 & 0.41 \end{pmatrix}$$

$$\Sigma^{SSVSP} = \begin{pmatrix} 1.42 & 0.15 & -0.36 & -0.45 & 0.25 & 0.22 \\ 0.15 & 1.48 & -0.35 & -0.47 & 0.17 & 0.26 \\ -0.36 & -0.35 & 1.65 & 0.49 & 0.12 & 0.16 \\ -0.45 & -0.47 & 0.49 & 1.58 & -0.01 & 0.03 \\ 0.25 & 0.17 & 0.12 & -0.01 & 1.45 & 0.23 \\ 0.22 & 0.26 & 0.16 & 0.03 & 0.23 & 1.37 \end{pmatrix}$$

The estimated values for the second Monte Carlo Simulation are given by:

$$A^{SSVSP} = \begin{pmatrix} 0.56 & 0.00 & 0.01 & 0.00 & 0.25 & 0.00 \\ 0.17 & 0.53 & -0.02 & 0.00 & -0.41 & -0.02 \\ 0.21 & 0.19 & 0.51 & -0.01 & 0.00 & -0.01 \\ 0.02 & -0.01 & 0.44 & 0.27 & 0.00 & 0.01 \\ 0.17 & 0.18 & 0.01 & 0.00 & 0.51 & 0.00 \\ -0.01 & -0.01 & -0.02 & 0.00 & 0.45 & 0.40 \end{pmatrix}$$

$$\Sigma^{SSVSP} = \begin{pmatrix} 1.70 & 0.15 & -0.45 & 0.02 & 0.30 & 0.04 \\ 0.15 & 1.25 & 0.03 & -0.50 & 0.11 & 0.00 \\ -0.45 & 0.03 & 1.34 & -0.03 & 0.01 & 0.00 \\ 0.02 & -0.50 & -0.03 & 1.33 & -0.02 & 0.01 \\ 0.30 & 0.11 & 0.01 & -0.02 & 1.30 & 0.01 \\ 0.04 & 0.00 & 0.00 & 0.01 & 0.01 & 1.08 \end{pmatrix}$$

Empirical Application

The Gibbs sampler has 110,000 draws of which 10,000 draws are disregarded as draws in the burn-in-phase. The probabilities, $p(\alpha_{ij}^{lk} = 0)$, $p(\psi_{ij}^{lk} = 0)$, and $p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$, are calculated as one minus the posterior means for $\gamma_{DI,ij}^{lk}, \gamma_{SI,ij}^{lk}$, and γ_{CSH}^{w}. The probabilities measure the proportion of $\gamma_{DI,ij}^{lk}, \gamma_{SI,ij}^{lk}$, and γ_{CSH}^{w} draws that equal zero averaged over all Gibbs sampler draws meaning that the coefficients are drawn from the restricted part of the distribution. The MSFE is calculated as the difference between the estimated forecast and the true value given by the data. The MSFEs for a specific variable and horizon are averaged over all forecasts. The PL is the posterior predictive density evaluated at the true observation y_{t+h}.

Table 13: MSFE of SSVSP relative to unrestricted VAR

horizon	1	2	4	6	12
IP_CA	1.02	0.98	0.94	1.06	1.13
CPI_CA	1.07	1.00	1.04	1.00	0.96
IR_CA	1.22	1.18	1.26	1.40	1.37
IP_I	1.12	0.99	1.05	1.10	1.11
CPI_I	0.99	1.00	1.02	0.99	0.98
IR_I	1.46	1.44	1.57	1.56	1.52
IP_UK	1.13	0.99	1.04	1.03	1.02
CPI_UK	1.10	1.03	1.02	1.02	1.01
IR_UK	1.17	1.07	1.03	1.10	1.00
IP_F	1.02	0.98	0.99	1.08	1.00
CPI_F	1.00	1.01	1.01	1.01	1.02
IR_F	1.70	1.33	1.34	1.53	1.51
IP_J	0.99	1.00	1.01	1.03	1.02
CPI_J	1.04	1.01	0.99	1.01	1.02
IR_J	3.80	3.60	3.73	4.00	3.36
IP_D	0.99	1.01	1.01	1.04	0.99
CPI_D	1.00	0.99	0.99	1.01	0.97
IR_D	1.45	1.14	0.98	1.00	0.98
IP_US	1.00	0.97	0.97	1.00	1.01
CPI_US	1.00	1.00	1.01	1.00	0.99
IR_US	1.00	1.00	0.99	0.99	1.01
MSFE $\leq$ 1(in %)	38.10	42.86	33.33	28.57	33.33

MSFE: difference between the estimated forecast and the true value given by the data, relative to unrestricted VAR. Unrestricted VAR: parameters drawn from unrestricted part of distributions. Forecasts are provided for 12 horizons for the period beginning from January 2005 through the end of the sample. Σ drawn from inverse Wishart distribution. 110,000 Gibbs draws, 10,000 disregarded as burn-in-phase.

Table 14: PL of SSVSP relative to unrestricted VAR

horizon	1	2	4	6	12
IP_CA	0.00	-0.69	0.14	0.06	-0.25
CPI_CA	-0.26	-0.49	0.25	0.20	0.30
IR_CA	-2.52	-1.05	-1.12	-1.16	-1.71
IP_I	-0.25	0.22	0.02	-0.02	0.28
CPI_I	-0.98	-0.13	-0.38	-0.20	-0.03
IR_I	0.98	-1.45	-1.16	-0.99	-1.70
IP_UK	-0.45	-0.76	-0.45	-0.01	-0.12
CPI_UK	0.59	0.02	0.20	0.36	0.45
IR_UK	-0.95	-1.65	-0.38	-1.31	-0.64
IP_F	-0.72	-0.03	-0.01	-0.05	0.09
CPI_F	-1.24	-0.13	0.03	0.29	0.17
IR_F	-4.82	-2.18	-0.89	-1.53	-0.86
IP_J	0.00	0.18	-0.13	-0.10	0.06
CPI_J	0.21	-0.32	0.03	-0.41	-0.38
IR_J	-3.58	-4.45	-3.95	-4.15	-4.44
IP_D	0.00	0.12	0.16	-0.18	-0.02
CPI_D	-0.04	-0.59	0.17	0.69	-0.08
IR_D	-1.22	-0.67	-0.30	-0.71	0.77
IP_US	0.00	0.16	0.32	-0.12	-0.37
CPI_US	-0.01	0.35	0.10	-0.01	0.32
IR_US	0.02	-0.87	-0.76	-0.47	-0.54
PL $\geq$ 0(in %)	33.33	28.57	47.62	23.81	38.10

PL: posterior predictive density evaluated at the true observation y_{t+h}, compared to unrestricted VAR. Unrestricted VAR: parameters drawn from unrestricted part of distributions. Forecasts are provided for 12 horizons for the period beginning from January 2005 through the end of the sample. Σ drawn from inverse Wishart distribution. 110,000 Gibbs draws, 10,000 disregarded as burn-in-phase.

Table 15: Posterior probabilities for DI restrictions, $p(\alpha_{ij}^{lk} = 0)$

		CA			**I**			**UK**			**F**			**J**			**D**			**US**		
		IP	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**
	IP				0.81	0.34	0.68	0.95	0.62	0.45	0.98	0.49	0.29	0.99	0.68	0.34	0.99	0.17	0.35	0.09	0.45	0.04
CA	**CPI**				0.99	0.79	0.96	1.00	0.96	0.77	1.00	0.69	0.79	1.00	0.95	0.08	1.00	0.91	0.21	0.99	0.06	0.57
	IR				1.00	0.90	0.00	1.00	0.99	0.87	1.00	0.90	0.31	1.00	0.99	0.05	1.00	0.98	0.15	0.98	0.95	0.11
	IP	0.91	0.19	0.10				0.82	0.20	0.37	0.96	0.20	0.47	0.69	0.56	0.28	0.43	0.55	0.14	0.09	0.40	0.39
I	**CPI**	1.00	0.99	0.79				1.00	0.99	0.95	1.00	0.83	0.98	1.00	0.99	0.51	1.00	0.82	0.83	1.00	0.66	0.64
	IR	1.00	0.82	0.75				1.00	0.91	0.45	1.00	0.75	0.87	1.00	0.82	0.11	1.00	0.87	0.01	0.91	0.69	0.57
	IP	0.83	0.64	0.62	0.89	0.53	0.39				0.88	0.50	0.63	0.98	0.60	0.35	0.97	0.65	0.31	0.11	0.60	0.47
UK	**CPI**	1.00	0.49	0.56	1.00	0.77	0.83				1.00	0.72	0.82	1.00	0.58	0.35	1.00	0.30	0.42	0.99	0.12	0.73
	IR	1.00	0.98	0.99	1.00	0.96	0.99				1.00	0.78	0.66	1.00	0.87	0.72	1.00	0.99	0.33	1.00	0.33	0.94
	IP	0.55	0.32	0.36	0.95	0.44	0.60	0.73	0.58	0.03				0.74	0.53	0.07	0.97	0.52	0.15	0.05	0.46	0.04
F	**CPI**	1.00	0.54	0.97	1.00	0.63	0.95	1.00	0.97	0.85				1.00	0.95	0.72	1.00	0.98	0.75	0.99	0.04	0.48
	IR	1.00	0.93	0.92	1.00	0.84	0.95	1.00	0.95	0.83				1.00	0.96	0.68	1.00	0.90	0.00	0.99	0.90	0.83
	IP	0.65	0.24	0.36	0.27	0.25	0.17	0.48	0.39	0.08	0.47	0.27	0.31				0.41	0.32	0.08	0.40	0.23	0.19
J	**CPI**	0.99	0.89	0.87	1.00	0.20	0.96	1.00	0.71	0.78	1.00	0.82	0.93				1.00	0.86	0.47	0.94	0.06	0.80
	IR	1.00	0.99	0.99	1.00	0.99	0.99	1.00	1.00	0.99	1.00	0.99	0.74				1.00	0.99	0.17	1.00	1.00	0.99
	IP	0.83	0.44	0.47	0.92	0.39	0.52	0.63	0.46	0.01	0.60	0.19	0.46	0.07	0.53	0.12				0.23	0.27	0.20
D	**CPI**	1.00	0.46	0.94	1.00	0.39	0.41	1.00	0.98	0.85	1.00	0.39	0.43	1.00	0.51	0.57				1.00	0.86	0.32
	IR	1.00	0.99	1.00	1.00	0.99	1.00	1.00	1.00	0.99	1.00	0.99	0.96	1.00	1.00	0.72				1.00	0.61	0.98
	IP	0.52	0.17	0.77	0.98	0.62	0.38	0.99	0.83	0.22	0.99	0.62	0.79	0.98	0.77	0.40	1.00	0.75	0.49			
US	**CPI**	1.00	0.95	0.89	1.00	0.29	0.97	1.00	0.97	0.87	1.00	0.79	0.94	1.00	0.98	0.54	1.00	0.29	0.69			
	IR	1.00	0.99	0.90	1.00	0.84	0.99	1.00	0.98	0.95	1.00	0.95	0.99	1.00	0.98	0.84	1.00	0.93	0.48			

Table 16: Posterior probabilities for SI restrictions $p(\psi_{ij}^{lk} = 0)$

		CA			I			UK			F			J			D			US		
		IP	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**	**IP**	**CPI**	**IR**
CA	**IP**				0.80	0.99	0.99	0.91	0.99	0.99	0.99	0.99	0.99	0.99	0.98	0.99	0.99	0.99	0.99	0.38	0.99	0.90
	CPI				0.87	0.04	0.84	0.85	0.78	0.80	0.73	0.00	0.82	0.84	0.84	0.81	0.21	0.75	0.82	0.62	0.00	0.28
	IR				0.68	0.34	0.46	0.76	0.26	0.72	0.55	0.47	0.00	0.73	0.70	0.31	0.72	0.73	0.69	0.71	0.69	0.00
I	**IP**							0.97	0.99	0.99	0.62	0.99	0.99	0.97	0.99	0.99	0.87	0.99	0.99	0.97	0.99	0.99
	CPI							0.59	0.06	0.27	0.48	0.01	0.34	0.46	0.54	0.38	0.56	0.05	0.27	0.38	0.05	0.46
	IR							0.90	0.79	0.00	0.87	0.79	0.00	0.30	0.68	0.66	0.86	0.73	0.48	0.72	0.84	0.77
UK	**IP**										0.49	0.98	0.99	0.84	0.98	0.98	0.98	0.97	0.93	0.99	0.99	0.89
	CPI										0.72	0.00	0.41	0.85	0.00	0.57	0.63	0.84	0.75	0.69	0.47	0.77
	IR										0.37	0.25	0.08	0.60	0.21	0.43	0.21	0.42	0.00	0.53	0.26	0.00
F	**IP**													0.98	0.99	0.99	0.93	0.99	0.99	0.99	0.99	0.98
	CPI													0.13	0.24	0.44	0.35	0.00	0.54	0.59	0.04	0.55
	IR													0.75	0.76	0.73	0.75	0.74	0.10	0.58	0.72	0.73
J	**IP**																0.99	0.99	0.99	0.99	0.99	0.99
	CPI																0.85	0.60	0.75	0.65	0.19	0.78
	IR																0.21	0.09	0.26	0.41	0.24	0.13
D	**IP**																			0.99	0.99	0.99
	CPI																			0.69	0.78	0.77
	IR																			0.43	0.13	0.00
US	**IP**																					
	CPI																					
	IR																					

Table 17: Posterior probabilities for CSH restrictions, $p(\alpha_{jj}^{lk} = \alpha_{ii}^{lk})$

	CA, I	**CA, UK**	**CA, F**	**CA, J**	**CA, D**	**CA, US**	**I, UK**	**I, F**	**I, J**	**I, D**	**I, US**
IP in IP	0.73	0.70	0.66	0.41	0.53	0.35	0.74	0.68	0.35	0.58	0.29
IP in CPI	1.00	0.99	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00
IP in IR	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
CPI in IP	0.28	0.34	0.27	0.26	0.29	0.32	0.27	0.26	0.25	0.29	0.33
CPI in CPI	0.70	0.67	0.61	0.29	0.58	0.13	0.67	0.58	0.35	0.54	0.17
CPI in IR	0.73	0.93	0.73	0.93	0.92	0.84	0.75	0.63	0.76	0.75	0.71
IR in IP	0.29	0.25	0.25	0.07	0.16	0.24	0.25	0.25	0.07	0.16	0.24
IR in CPI	0.86	0.58	0.76	0.45	0.48	0.39	0.63	0.85	0.47	0.51	0.40
IR in IR	0.83	0.84	0.82	0.81	0.75	0.71	0.78	0.77	0.76	0.70	0.68
	UK, F	**UK, J**	**UK, D**	**UK, US**	**F, J**	**F, D**	**F, US**	**J, D**	**J, US**	**D, US**	
IP in IP	0.70	0.35	0.59	0.29	0.31	0.60	0.25	0.13	0.81	0.06	
IP in CPI	1.00	1.00	1.00	0.99	1.00	1.00	1.00	1.00	1.00	1.00	
IP in IR	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	
CPI in IP	0.25	0.26	0.27	0.30	0.25	0.30	0.35	0.26	0.28	0.41	
CPI in CPI	0.62	0.27	0.62	0.11	0.21	0.66	0.07	0.10	0.45	0.00	
CPI in IR	0.76	0.96	0.96	0.90	0.77	0.75	0.71	0.99	0.94	0.93	
IR in IP	0.21	0.09	0.17	0.29	0.06	0.13	0.18	0.09	0.10	0.21	
IR in CPI	0.58	0.40	0.43	0.41	0.46	0.50	0.38	0.38	0.32	0.32	
IR in IR	0.81	0.79	0.72	0.70	0.80	0.73	0.71	0.74	0.71	0.69	